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Measuring Counterparty Credit Exposure to a Margined Counterparty

Abstract: Firms active in OTC derivative markets increasingly use margin agreements to reduce counterparty credit risk. Making several simplifying assumptions, I use both a quasi-analytic approach and a simulation approach to quantify how margining reduces counterparty credit exposure. Margining reduces counterparty credit exposure by over 80 percent, using baseline parameter assumptions. I show how expected positive exposure (EPE) depends on key terms of the margin agreement and the current mark-to-market value of the portfolio of contracts with the counterparty. I also discuss a possible shortcut that could be used by firms that can model EPE without margin but cannot achieve the higher level of sophistication needed to model EPE with margin.

Keywords: counterparty risk, collateral, margin, derivatives

1 Introduction

Firms active in OTC derivative markets increasingly use margin agreements to reduce counterparty credit risk. Making several simplifying assumptions, I use both a quasi-analytic approach and a simulation approach to quantify how margining reduces counterparty credit exposure. I show how expected positive exposure (EPE) depends on key terms of the margin agreement and the current mark-to-market value of the portfolio of contracts with the counterparty. I also discuss a possible shortcut that could be used by firms that can model EPE without margin but cannot achieve the higher level of sophistication needed to model EPE with margin.

1.1 Counterparty credit risk

A firm that uses OTC derivatives is exposed to counterparty credit risk because a counterparty may default when the portfolio of OTC derivative contracts with the counterparty has positive value. The value of an OTC derivatives portfolio, which depends on market variables such as interest rates or exchange rates, will change when those variables change. As a result, counterparty credit exposure will change in the future even if no new positions are added to the portfolio.

Because future counterparty credit exposure is uncertain, measuring it requires a statistical forecast of future moves in market variables. With such a forecast in hand, a dealer can estimate the probability distribution of counterparty credit exposure on one or more future dates. Summary measures of counterparty credit exposure can be computed from these probability distributions.

Three commonly used summary measures, as defined by BCBS (2005), are

- Potential future exposure (PFE): the maximum exposure estimated to occur on a future date at a high level of statistical confidence.
- Expected exposure (EE): the probability-weighted average exposure estimated to exist on a future date.
- Expected positive exposure (EPE): the time-weighted average of individual expected exposures estimated for given forecasting horizons (e.g. one year).

Both industry experts and regulators agree that EPE is the conceptually correct measure of exposure to be used in a calculation of economic or regulatory capital for counterparty credit risk.[1] For that reason, I focus on measuring the effect of margining on EPE. However, the techniques presented here could be adapted to measure PFE.

1.2 Margin agreements

More and more participants in OTC derivatives markets use collateral and margin agreements to reduce counterparty credit risk. According to a recent survey, 55 percent of derivatives

[1] Canabarro and Duffie (2003), Picoult and Lamb (2004), BCBS (2005).

transactions are covered by margin agreements as of December 31, 2004, up from 30 percent as of December 31, 2002.[2]

A margin agreement contains rules for computing the amount of collateral to be passed between parties on any given day. There are several key terms that are individually negotiated by counterparties. The models I present below consider four key terms:

Threshold: Exposure amount below which no margin is held.

Grace period: Number of days after default until the counterparty's position is liquidated or replaced.

Remargin period: Interval (in days) at which margin is monitored and called for.

Minimum transfer amount: Amount below which no margin transfer is made.

The grace period used in a model should be based on market practice and experience in closing out defaulted counterparties, not solely on the grace period written into the margin agreement.

While margin agreements can reduce counterparty credit risk, they pose a challenge to modelers. Models to measure counterparty credit risk are already notoriously complicated and difficult, because they must forecast future moves in market variables into the distant future (as long as 30 years, for long-dated swap contracts) and they must be able to revalue the portfolio of derivative contracts given arbitrary changes in market variables. Margin agreements add another layer of complexity, because future collateral amounts and margin calls must also be modeled.[3]

While a great deal of work has been done in recent years to improve understanding of EPE for unmargined counterparties, little has been done for margined counterparties.[4] By building a simplified, stylized model of EPE for a margined counterparty, I aim to fill this gap in the literature by establishing how EPE varies with a few key terms of a margin agreement as well as with the current mark-to-market value of the contracts with the counterparty. I also discuss what shortcuts may be appropriate for firms that can model EPE without margin but cannot achieve the higher level of sophistication needed to model EPE with margin.

2 Measuring EPE for a margined counterparty

The most common models for measuring EPE are simulation models. These models have four steps. First, simulate a sample path for the future values of the market variables underlying the portfolio of derivative contracts with a counterparty.[5] Second, compute the mark-to-market value of the portfolio along the path. Third, compute exposure as the mark-to-market

[2] ISDA (2005)

[3] Margin agreements also give rise to operational and legal risks that must be managed.

[4] Work on EPE for unmargined counterparties includes ISDA (2001, pp. 63–69), Canabarro, Picoult and Wilde (2003), Canabarro and Duffie (2004).

[5] The correct portfolio to use when computing counterparty exposure is a "netting set." As defined in BCBS (2005), "a 'netting set' is a group of transactions with a single counterparty that are subject to a legally enforceable bilateral netting arrangement . . . If a transaction with a counterparty is not subject to a bilateral netting agreement, it comprises its own netting set."

value, if positive, or zero otherwise. Fourth, average exposure across sample paths and over time to compute EPE.[6]

A margin agreement requires additional modeling.[7] At each time step along the sample path, the model must test whether a margin call is required or whether excess collateral should be returned to the counterparty. If a margin call is made, the model must track the delivery of the collateral. When considering counterparty default, the model must consider whether margin calls made on the previous day have been received yet. All these add complexity to the EPE model.

I build a simplified, stylized model of EPE for a margined counterparty. I consider both a quasi-analytic model and a slightly richer simulation model. The model assumes that the mark-to-market value of the portfolio of contracts with the counterparty follows a random walk with Gaussian increments. The model considers only four key terms of a margin agreement. The mechanics of margin calls are simplified for tractability. The model does not allow for initial margin or non-cash collateral.[8] Sections 3 and 4 describe the analytic and simulation models in more detail.

The "base case" values that I assume for the four key terms of a margin agreement, as well as for the current mark-to-market (MTM) value of the contracts with the counterparty, are shown in the table below. In addition, I set the annualized standard deviation of the future MTM value of the contracts with the counterparty equal to one. Because EPE scales with the standard deviation, this is simply a normalization.

Parameter	Base case value
Current MTM	0
Threshold	0
Grace period	10 days
Remargin period	1 day
Minimum transfer amount*	0

* = used in simulations, not used in analytic approximation

2.1 Results

One summary measure of the effect of margining is the ratio of EPE taking margining into account to EPE without margining. For the base case parameters given in the table above, this ratio equals 0.17 in both the analytic approximation and the simulations. Put another way, a margin agreement with standard terms can reduce counterparty credit exposure by over 80 percent.

Figures 1 and 2 show how EPE depends on the key terms of the margin agreement and the current MTM. Each panel varies one parameter while holding the other four at their

[6]The basic structure of an EPE model is described in Canabarro and Duffie (2004).

[7]I consider one-sided margin agreements, where only one party to the margin agreement (the counterparty) is ever required to provide collateral.

[8]Allowing non-cash collateral would make the model more realistic. However, according to ISDA (2005), cash makes up 73 percent of collateral held against OTC derivative exposures.

base case values. Figure 1 plots the ratio of EPE with margin to EPE without margin while Figure 2 plots the levels of EPE with and without margin (blue solid and red dashed lines, respectively).[9] The figures show both the analytic approximation (left column) and simulations (right column).

The top panels of Figures 1 and 2 show how EPE varies with the current MTM of the portfolio. Comparing EPE without and with margining (the red and blue lines in the top panel) in Figure 2, margining removes nearly all of the strong dependence of EPE on current MTM that exists without margining. Figure 1 shows that portfolios with large current MTM show the greatest reduction in EPE from margining.

The second row of panels in Figures 1 and 2 shows how EPE varies with the collateral threshold. EPE with margining increases strongly with the threshold, with the effect only tapering off when the threshold is so high that EPE with margining nears EPE without margining.

The two top rows of figures 1 and 2 show how EPE varies individually with the current MTM and the collateral threshold. There are interesting interactions when these are varied simultaneously, as shown in Figure 3. In this figure, all other parameters are at their base case values. At high levels of current MTM, the threshold has a linear effect on EPE. At low levels of current MTM, the threshold has almost no effect on EPE. At low thresholds, current MTM has almost no effect on EPE. At high thresholds, EPE increases linearly with current MTM up to the threshold, then flattens out.

The third and fourth rows of Figures 1 and 2 show how the grace period and the remargin period affect EPE. In general, EPE increases as both the grace period and remargin period get longer. However, the effect is not identical, and the difference can be understood as due to the timing of a default within the period. A default always occurs at the beginning of the grace period, and exposure can rise during the entire grace period until, at the end of the grace period, the position is closed. In contrast, a default will occur at a random time during the remargin period. On average, exposure will have risen without remargining for half of the remargin period before a default.

For both the grace period and the remargin period, Figure 1 shows the square root of time function, plotted as a red dashed line. The square root of time is added to the plot to reflect the idea that EPE with margin could be approximated by multiplying 1-year EPE without margin by the square root of the grace period. This approximation appears to work well. For the reasons discussed above, EPE rises more slowly than the square root of time for the remargin period.

Figure 4 varies the grace period and remargin period simultaneously. The lack of much curvature in the EPE surface in Figure 4 suggests that there is little interaction between the two in their effect on EPE.

The results in Figures 1, 2, and 4 set both the current MTM and threshold to zero. More generally, when the current MTM or threshold is not zero, EPE will reflect both current and

[9] EPE without margin (the denominator of the ratios shown in Figure 1) is EPE computed at the base case values without margining, except that the EPE without margin is recomputed for each value of current MTM.

future exposure. Only the future exposure piece, but not the total EPE, would be expected to increase with the grace period or remargin period.

The bottom panel in Figures 1 and 2 shows how EPE varies with the minimum transfer amount. Only the simulations allow for a non-zero minimum transfer amount. As expected, EPE rises with the minimum transfer amount, but quite slowly.

2.2 Summary, and a possible shortcut EPE

Some firms may be able to model EPE without margining but may not have the higher level of sophistication to model EPE with margining. In looking for a possible shortcut EPE for such firms, the figures lead to the following conclusions about the effects of each of the variables:

- The collateral threshold and the current mark-to-market have important effects on EPE for margined counterparties. These effects should be taken into account in any shortcut EPE calculation.
- The grace period has an effect on EPE that is roughly proportional to the square root of time (when the current mark-to-market and threshold are zero, as they are in the Base Case).
- The remargin period has an effect similar to the grace period, but slightly weaker.
- If the minimum transfer amount is small, it has little effect on EPE and, if a small amount of conservatism were added on to the shortcut EPE to cover it, it may not be material.

Analyzing Figure 3 suggests using the threshold plus the 10-day expected exposure (EE) computed with no margining as a shortcut EPE with margining.[10] Note that the shortcut uses expected exposure after 10 days (EE), not 10-day expected positive exposure (EPE), which is the average exposure over the first 10 days. For a margined counterparty, exposure is only relevant at the end of the 10 day grace period, so expected exposure—which measures exposure on a single future date—is the relevant concept, not EPE—which averages exposure over many future dates. As part of the shortcut calculation, if the EPE computed without any threshold (implying no margining would ever take place) were smaller than the proposed shortcut EPE, it could be used instead. It would not make sense to have a higher loan-equivalent amount for a margined counterparty than for an otherwise-identical unmargined counterparty.

The suggested shortcut EPE formula is shown in Table 1. Panel A shows the shortcut formula, while Panel B shows the analytic EPE numbers from Figure 3. As hoped for, the shortcut EPE shown in Panel A is conservative – it is always greater than the actual EPE in Panel B – but it is fairly sensitive to the two key risk drivers (current mark-to-market and threshold).

[10] If the grace period were more than 10 days, "10-day EE" would be replaced with the EE for the longer grace period. If remargining were less frequent than daily, the 10-day period should be lengthened accordingly.

3 The analytic approximation

By making some simplifying assumptions, it is possible to obtain a quasi-analytic approximation for EPE to a margined counterparty as a function of the parameters given in the table above.[11]

3.1 Definitions and notation

- time t runs from today, $t = 0$, to the EPE horizon, $t = T$
- $V(t)$ = mark-to-market at time t
- $C(t)$ = collateral held at t
- $E(t)$ = exposure at $t = \max(0, V(t) - C(t))$
- D = collateral threshold
- m = grace period
- rm = remargin period
- $EE(t)$ = expected exposure at t conditional on default at t = average of $E(t|\text{default})$ over possible values of $V(t)$
- EPE = average of $EE(t)$ over $(0, T)$

For a non-defaulting counterparty, collateral held is defined by

$$C(t) = \max(0, V(s) - D) \tag{1}$$

where s is the remargin date at or before t. Assuming that today ($t = 0$) is a remargin date, $s = t - t \mod rm$. For the base case of daily remargining, $rm = 1$ and $s = t$. I also assume for the analytic approximation that collateral is monitored, called for, and delivered on each remargin date.[12]

Exposure at default is defined by

$$E(t|\text{default}) = \max(0, V(t + m) - C(t)) \tag{2}$$

I assume that the stochastic behavior of $V(t)$ is unaffected by the counterparty's default, in effect assuming no wrong-way risk.

3.2 Deriving expected exposure

Substituting (1) into (2) gives

$$E(t|\text{default}) = \max(0, V(t + m) - \max(0, V(s) - D)) \tag{3}$$

Using (3), the expected exposure at t can be written as

$$EE(t) = \int \max(0, V(t + m) - \max(0, V(s) - D))\, dF \tag{4}$$

[11] I use the term "quasi-analytic" because solving the model requires numerical integration.
[12] The simulation approach below assumes that collateral called on t is delivered on $t + 1$.

where $F = F(V(t+m), V(s))$ is the joint distribution of $V(t+m)$ and $V(s)$.

Looking at (3), the various max operators lead to four possible values of the exposure at default, depending on $V(t+m)$ and $V(s)$, summarized in the following table:

		$E(t\|\text{default})$
$V(s) < D$	$V(t+m) < 0$	0
$V(s) < D$	$V(t+m) > 0$	$V(t+m)$
$V(s) > D$	$V(t+m) < V(s) - D$	0
$V(s) > D$	$V(t+m) > V(s) - D$	$V(t+m) - V(s) + D$

Using this table, (4) can be rewritten as

$$\begin{aligned} EE(t) = &\int_{V(s)<D, V(t+m)>0} V(t+m)\, dF + \\ &\int_{V(s)>D, V(t+m)>V(s)-D} (V(t+m) - V(s) + D)\, dF \end{aligned} \tag{5}$$

To establish the joint distribution $F(V(t+m), V(s))$, I assume that $V(t)$ follows a random walk with Gaussian increments:

$$V(s) = V(0) + \sigma\sqrt{s}X \tag{6}$$

$$V(t+m) = V(0) + \sigma\sqrt{s}X + \sigma\sqrt{t+m-s}Y \tag{7}$$

where X and Y are independent standard normal random variables.

Using this assumption and setting $V(0) = V$ for ease of notation, (5) can be rewritten as

$$\begin{aligned} EE(t) = &\int_{-\infty}^{\frac{D-V}{\sigma\sqrt{s}}} \int_{\frac{-V-\sigma\sqrt{s}x}{\sigma\sqrt{t+m-s}}}^{\infty} (V + \sigma\sqrt{s}x + \sigma\sqrt{t+m-s}y)\, \phi(x)\phi(y)\, dy\, dx \\ &+ \int_{\frac{D-V}{\sigma\sqrt{s}}}^{\infty} \int_{\frac{-D}{\sigma\sqrt{t+m-s}}}^{\infty} (\sigma\sqrt{t+m-s}y + D)\, \phi(x)\phi(y)\, dy\, dx \end{aligned} \tag{8}$$

where $\phi()$ is the standard normal density function.

Simplifying (8) a little bit to elimnate the double integrals yields

$$\begin{aligned} EE(t) = &\int_{-\infty}^{\frac{D-V}{\sigma\sqrt{s}}} \left[(V + \sigma\sqrt{s}x) N\left(\frac{V + \sigma\sqrt{s}x}{\sigma\sqrt{t+m-s}} \right) + \frac{\sigma\sqrt{t+m-s}}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{V+\sigma\sqrt{s}x}{\sigma\sqrt{t+m-s}}\right)^2} \right] \phi(x)\, dx \\ &+ N\left(\frac{V-D}{\sigma\sqrt{s}} \right) \left[DN\left(\frac{D}{\sigma\sqrt{t+m-s}} \right) + \frac{\sigma\sqrt{t+m-s}}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{D}{\sigma\sqrt{t+m-s}}\right)^2} \right] \end{aligned} \tag{9}$$

where $N()$ is the standard normal cumulative distribution function.

Equation (9) is the expression used to compute $EE(t)$.[13] Using (9), EPE is computed as

$$EPE = \frac{1}{T}\int_0^T EE(t)\,dt \tag{11}$$

I used numerical integration to produce the analytic results shown in Figures 1–4, setting the volatility parameter $\sigma = 1$.

4 The simulation model

The simulation model also assumes that the mark-to-market value of contracts in the netting set follows a random walk with Gaussian increments.[14] As changes in the mark-to-market value cause changes in exposure, margin is called for when the mark-to-market exceeds the threshold (or returned if the mark-to-market falls below the threshold).

The simulation algorithm uses the following notation: V_0 is the current mark-to-market. E is the exposure gross of collateral, D is the threshold. C_0, the initial collateral held, equals $\max(0, V_0 - D)$. The grace period (days to liquidate a defaulted counterparty's positions) is m. Collateral called on day t is assumed to be delivered on day $t+1$. $C_{it}|$default is collateral available conditional on the counterparty's defaulting at t. $E_{it}|$default is mean exposure, net of collateral, conditional on default at t and taking into account the movement in the mark-to-market over the grace period. ϵ is a standard normal random variable. The endpoint of the simulation, T, equals 250 days (one year). The number of simulations, N, and the number of simulations of exposure within the grace period, M, are both set to 400.

The simulation algorithm is:

for $i = 1$ to N **do**
  $V_{i0} = V_0$, $C_{i0} = C_0$
  for $t = 1$ to T **do**
    $V_{it} = V_{it-1} + \epsilon\sqrt{\frac{1}{250}}$
    $E_{it} = \max(0, V_{it})$
    $C_{it} = C_{it-1} + Call_{it-1}$
    if t is a remargining day **then**
      $Call_{it} = \max(E_{it} - D, 0) - C_{it}$
      if $|Call_{it}| <$ minimum transfer amount **then**
        $Call_{it} = 0$

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[13] If $s = 0$, equation (9) simplifies to

$$EE(t) = \begin{cases} VN\left(\frac{V}{\sigma\sqrt{t+m}}\right) + \frac{\sigma\sqrt{t+m}}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{V}{\sigma\sqrt{t+m}}\right)^2} & \text{if } V < D \\ DN\left(\frac{D}{\sigma\sqrt{t+m}}\right) + \frac{\sigma\sqrt{t+m}}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{D}{\sigma\sqrt{t+m}}\right)^2} & \text{if } V \geq D \end{cases} \tag{10}$$

[14] Since real-world contracts need not follow a random walk, a useful extension would be to repeat the simulation exercise with a more realistic model for the mark-to-market value.

11: **end if**
12: **end if**
13: $C_{it}|\text{default} = C_{it}$
14: $E_{it}|\text{default} = \frac{1}{M}\sum_{j=1}^{M} \max\left(0, V_{it} + \epsilon\sqrt{\frac{m}{250}} - C_{it}|\text{default}\right)$
15: **end for**
16: $EPE_i = \frac{1}{T}\sum_{t=1}^{T} \max(0, E_{it}|\text{default})$
17: **end for**
18: $EPE = \frac{1}{N}\sum_{i=1}^{N} EPE_i$

The definition of $C_{it}|\text{default}$ in line 13 assumes that collateral that was posted on $t-1$ is delivered on t despite the counterparty's default on t. An alternative would be to assume that this collateral would be clawed back by the bankruptcy court before it is delivered on t. The alternative would change this line to $C_{it}|\text{default} = \min(C_{it}, C_{it-1})$. The alternative has a small effect on the results shown in Figures 1 and 2. In the base case, it increases the EPE with margin by about 0.007, or 3 percent of EPE without margin.

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Figure 1. (EPE with margin)/(EPE without margin) as a function of current mark-to-market (MTM), threshold, grace period, remargin period and minimum transfer amount (the red dashed line plots the square root of time function)

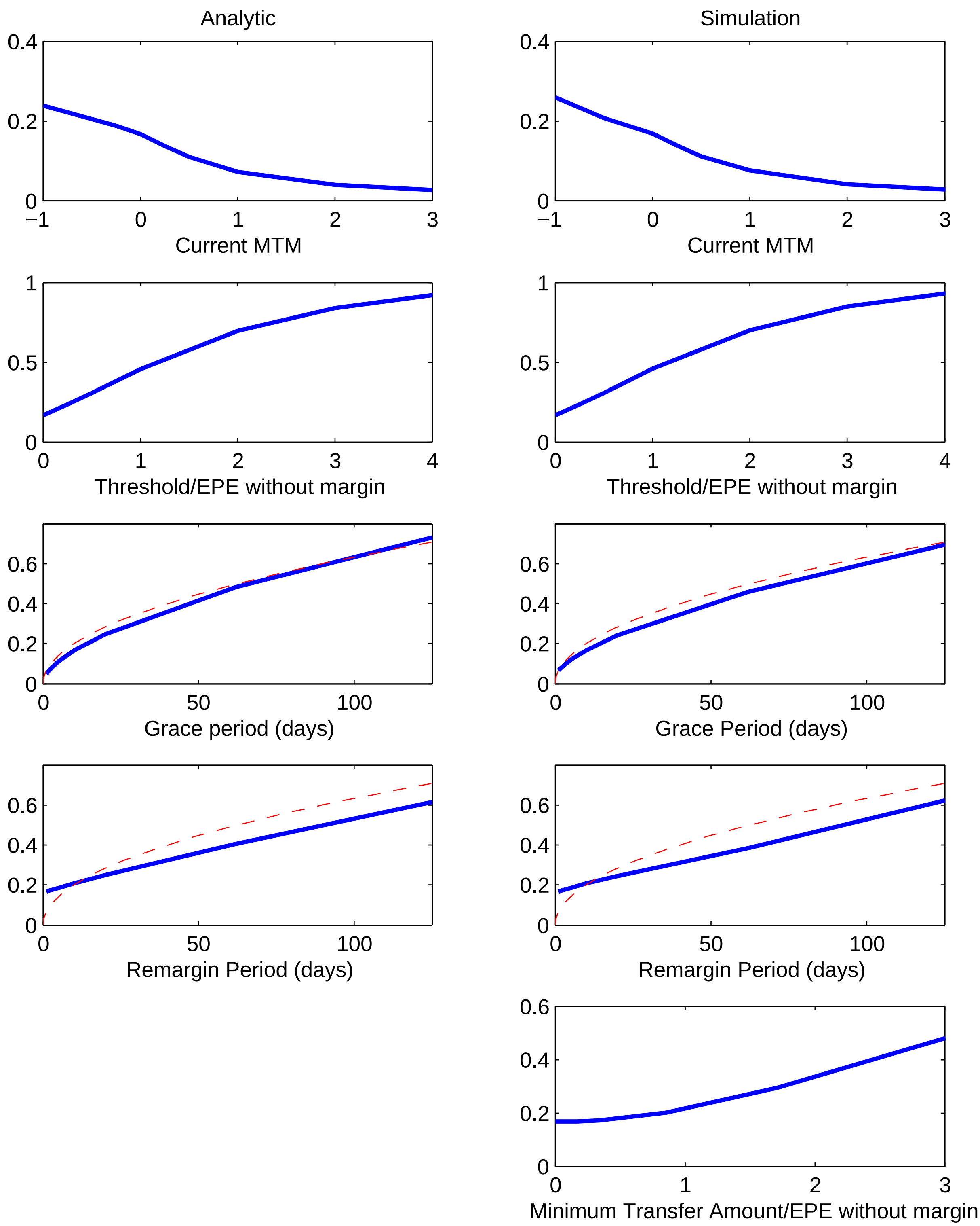

Figure 2. EPE for a margined counterparty as a function of current mark-to-market (MTM), threshold, grace period, remargin period and minimum transfer amount (red dashed line shows EPE without margin)

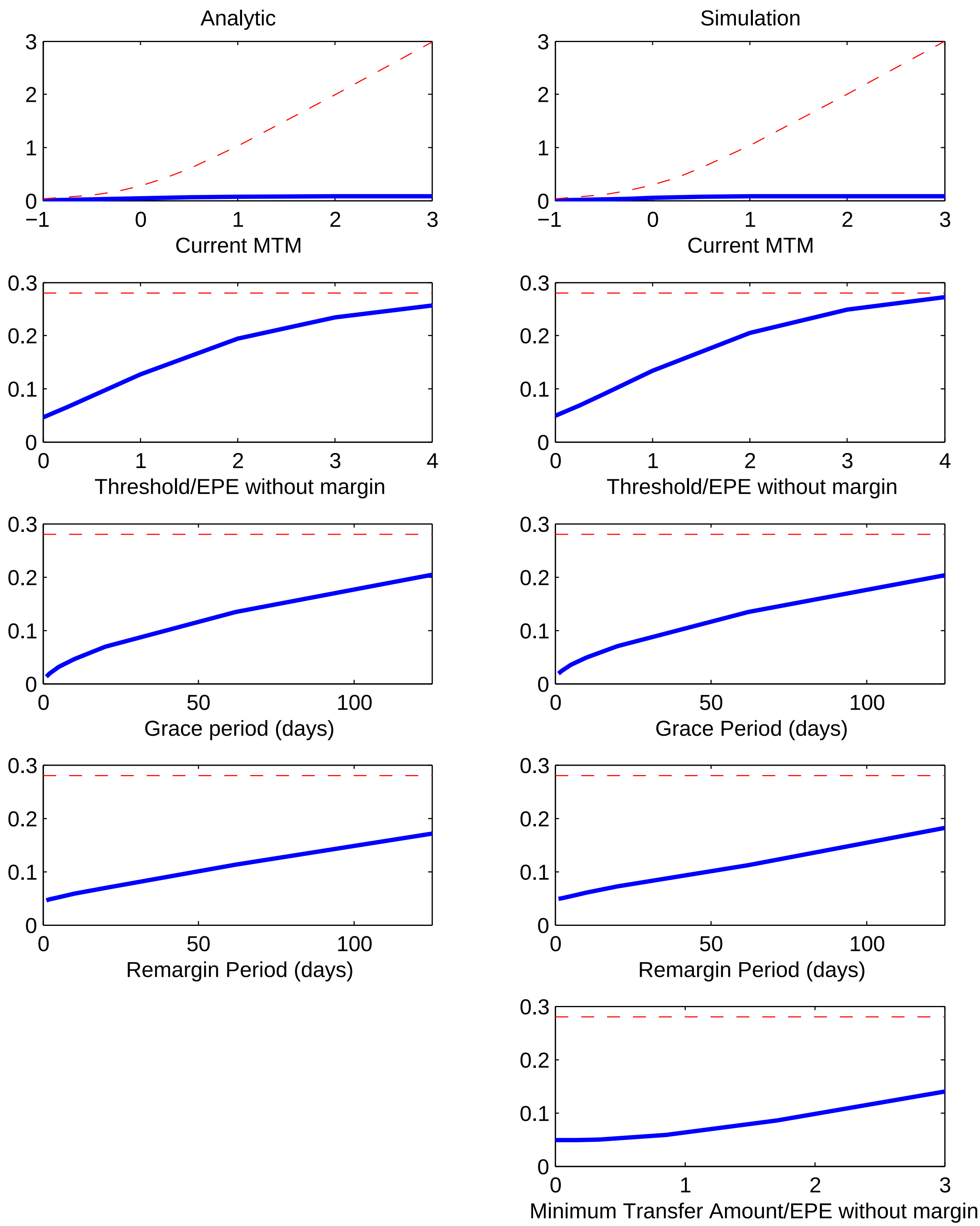

Figure 3. Analytic approximation to EPE with margin as a function of both threshold and current mark-to-market

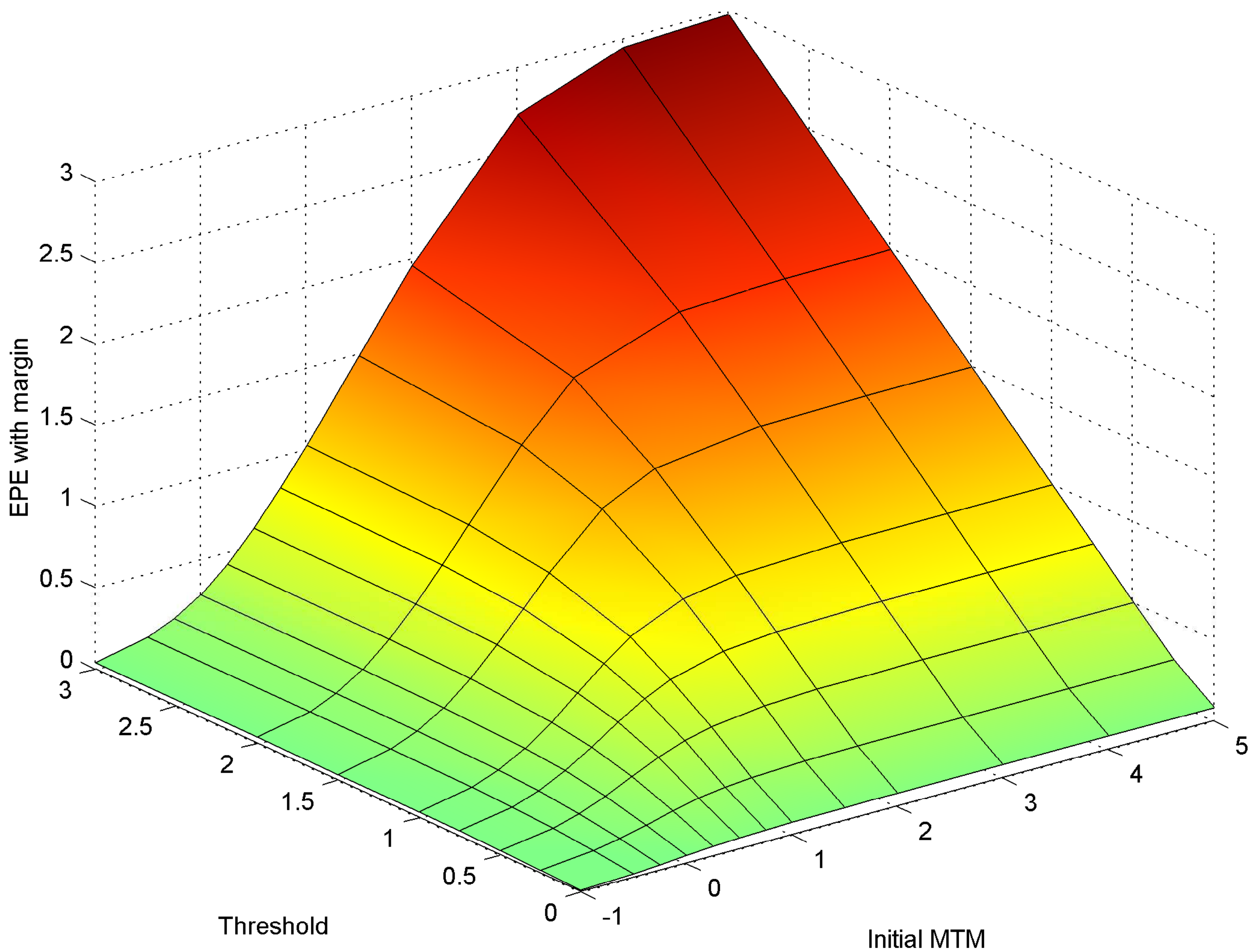

Figure 4. EPE with margin as a function of grace period and remargin period

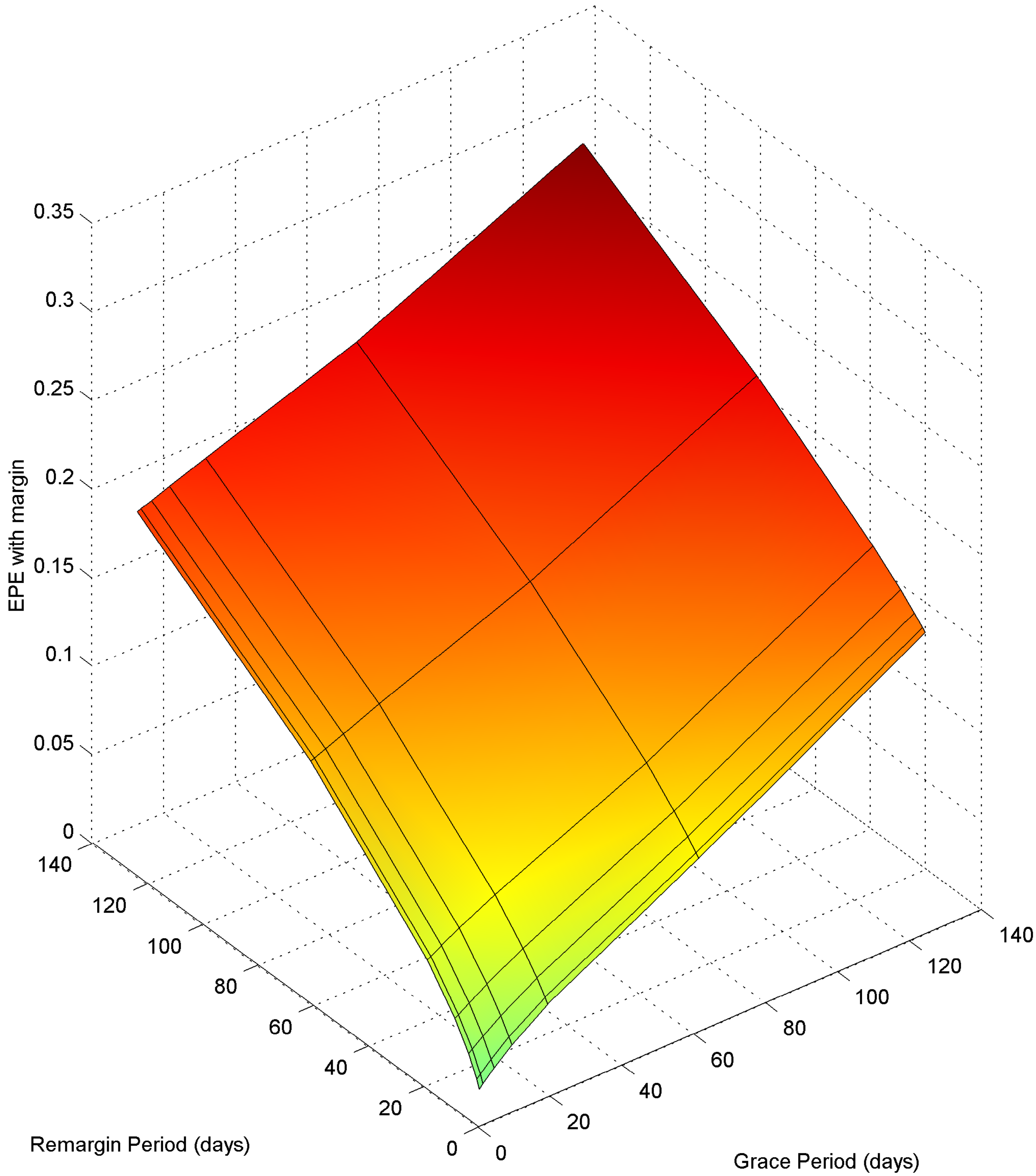

Table 1. Shortcut EPE as a function of current mark-to-market and threshold

	Current mark-to-market						
Threshold	-1	0	1	2	3	4	5
	Panel A: Shortcut EPE						
0	0.034	0.080	0.080	0.080	0.080	0.080	0.080
1	0.034	0.279	1.024	1.080	1.080	1.080	1.080
2	0.034	0.279	1.024	1.982	2.080	2.080	2.080
3	0.034	0.279	1.024	1.982	2.970	3.080	3.080
	Panel B: EPE measured with analytic approximation						
0	0.008	0.046	0.074	0.079	0.079	0.079	0.079
1	0.032	0.249	0.758	0.962	0.988	0.990	0.990
2	0.034	0.277	0.993	1.716	1.950	1.978	1.980
3	0.034	0.279	1.022	1.952	2.704	2.940	2.968
Memo:							
No threshold	0.034	0.279	1.024	1.982	2.970	3.960	4.950

Note: Shortcut EPE equals threshold plus 10-day expected exposure (EE) or EPE for an unmargined counterparty (no threshold), whichever is smaller. Panel B uses daily remargining and a 10-day grace period.