Finance and Economics Discussion Series
Divisions of Research & Statistics and Monetary Affairs
Federal Reserve Board, Washington, D.C.

The Elusive Capital-User Cost Elasticity Revisited

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2007-25

The 'Elusive' Capital-User Cost Elasticity Revisited

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Board of Governors of the Federal Reserve System

May 15, 2009

Abstract

This paper sheds new light on the estimation of the long-run elasticity of the demand for business capital— for a measure that includes both equipment and structures—to changes in its user cost using a quarterly panel of two-digit manufacturing industries from South Africa from 1970 to 2000. For a variety of regression specifications, we find highly significant estimates of the user cost elasticity in the vicinity of -1.0 as implied by a Cobb-Douglas production function. These estimates contrast sharply with many previous studies that obtained small and/or statistically insignificant estimates of the user cost elasticity. This difference in findings may owe to the fact that the capital demand curve is better identified in a small open economy because shocks to capital supply are more likely to be exogenous. The economic embargo imposed on South Africa from 1985 to early 1994 temporarily forced its economy to become more closed and therefore provides a unique opportunity to assess the importance of identification in the estimation of the user cost elasticity. We find that the estimated magnitude of the user cost elasticity is considerably smaller over the embargo period. These findings suggest that the true elasticity is in the vicinity of the Cobb-Douglas benchmark, and that identification is important to uncovering this estimate.

Keywords: user cost elasticity; fixed investment; capital accumulation; price of capital; interest rate

JEL classifications: C23, E22, E44, E62

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An earlier version of this paper was entitled, "*Estimating the Long-Run User Cost Elasticity of Capital for a Small Open Economy: Evidence Using Data from South Africa.*" We thank the South African Trade and Industrial Policy Strategies, and Coen Pretorius, Nthabiseng Molemoeng, Stefaans Walters of the Reserve Bank of South Africa for providing the data set used in the study. We would also like to thank Masao Ogaki and other seminar participants at Ohio State University, the Midwest Economics Association Meetings, and the Board of Governors of Federal Reserve System for useful comments and suggestions. The views expressed in the paper are those of the authors and do not necessarily reflect those of the Board of Governors or the Federal Reserve System.

1 Introduction

Economists have long had a keen interest in knowing the degree that businesses wish to adjust capital holdings in response to shifts in the supply of capital—the user cost elasticity of capital. The attention paid to this parameter is well justified, as its magnitude is of central importance for calibrating macroeconomic models that, in turn, are used for forecasting, for evaluating economic hypotheses, and examining fiscal and monetary policy alternatives. For example, this elasticity is relevant for assessing the effectiveness of investment tax incentives, such as the bonus depreciation allowances enacted by the U.S. federal government in 2002, 2003 and 2008. Despite its significance, econometricians have generally found it difficult to identify this elasticity in empirical work, leading to estimates that vary substantially in magnitude from study to study. This reflects the familiar econometric challenge of identifying the slope of a demand curve in an environment where both demand and supply can be shifted by sources that cannot always be isolated.[1] This challenge is complicated further in this context by adjustment costs—both internal and external. These costs prolong the period needed for capital to fully adjust to a given disturbance and tend to make the demand response depend on the anticipated persistence of the shock (Tevlin and Whelan [2003]).[2]

Recently, new methods have been applied to this topic that point to a fairly large aggregate response of capital demand to changes in capital supply. Much of this work has followed from the insights of Caballero [1994], who argued that the user cost elasticity can be identified more effectively using a cointegration approach that emphasizes the low-frequency variation in the data, thereby de-emphasizing transitory distortions associated with adjustment costs and possibly sidestepping endogeneity problems. Using this strategy for aggregate data for equipment capital, Caballero [1994] and Schaller [2006] obtain statistically significant estimates of the long-run user cost elasticity that are close to the Cobb-Douglas benchmark of -1.0.[3] However, the estimates in these studies become insignificant when structures are included in the measure of capital. This is not a negligible omission, as structures constitute a huge fraction of the nominal stock of business capital, and

[1] In principle, this problem could be addressed using instrumental variables. As noted by Hassett and Hubbard [2002], conventional instrumental variables, such as lagged endogenous variables or sales-to-capital ratios, have not proven successful.

[2] The presense of fixed costs and the distribution of capital across firms can also affect also the response of capital changes in the user cost (Caballero, Engel and Haltiwanger [1995]).

[3] A number of other papers have also reported significant negative estimates of the user cost elasticity for firm level data. These include Cummins, Hassett and Hubbard [1994], Chirinko, Fazzari and Meyer [1999], Chrinko, Fazzari and Meyer [2004], Guiso et al. [2002] and, more recently, Gilchrist and Zakrajšek [2007].

therefore should be an important component of the overall response of capital to changes in its user cost.[4] More recent work by Smith [2007] uses cointegration methods for a panel of United Kingdom industries, but with a measure of capital that includes both equipment and structures. He finds a user cost elasticity that is substantially smaller— around -0.40. Using a stationary specification and aggregate data for the United States, Tevlin and Whelan [2003] find estimates of the user cost elasticity around -0.20.

This study revisits the subject using a unique quarterly dataset of manufacturing industries from South Africa for the period between 1970 and 2000. We think that the South African experience over this period is particularly pertinent to the user cost elasticity debate. The country is sufficiently small and open that its economy likely takes interest rates and capital goods prices as given (Schaller [2006]), and is sufficiently isolated that we can conjecture that these prices are also exogenous. This allows us to bypass some of the challenges posed by endogeneity. Also, our industry level dataset has a large number of observations and a cross-sectional dimension, which allows us to control for sources of endogeneity stemming from latent aggregate and industry effects and may make our results less susceptible to small sample bias than some previous studies. Finally, the embargo imposed on South Africa provides a unique opportunity to assess the importance of endogeneity for estimates of the user cost elasticity since the embargo forced the country's economy to transition from open toward autarky, and then back to open. Under our working assumption that the user cost is exogenous in a small and isolated open economy, we would expect the user cost elasticity to be smaller during the embargo period— when the endogeneity problem was likely more severe—than in the non-embargo periods.

Using both cointegration and distributed lag specifications that are derived directly from optimal investment behavior in the presence of adjustment costs, we find highly significant estimates of the user cost elasticity in the range of -0.80 to -1.0. In most cases, these estimates are statistically indistinguishable from the Cobb-Douglas benchmark of -1.0. Unlike previous studies (Caballero [1994] or Schaller [2006]) that also find user cost elasticity estimates in this range, our estimates are for measures of business fixed capital that includes both equipment and structures. To our knowledge, this study is one of the first to document such a large user cost elasticity for a broad

[4] According to estimates from the Bureau of Economic Analysis for the postward period, structures on average accounted for almost two-thirds of the nominal stock of private nonresidential fixed capital and about one-third of nominal private fixed investment in the United States.

measure of business capital that includes both equipment and structures, and the first to show that similar estimates can be obtained using both stationary and cointegrating regression specifications. Interestingly, we find that controlling for the effect of the embargo period— when the economy was less open and more likely to have interest rates and prices of capital goods that are determined endogenously— leads to a statistically significant increase in the absolute value of the user cost elasticity estimate. The estimated elasticity during the embargo period is much smaller, in the range of estimates found in the majority of previous studies. These latter results are particularly intriguing as they suggest the importance of endogeneity as a possible explanation for why previous studies—which largely employ data from large economies—have often failed to obtain estimates of the user cost elasticity of a substantial magnitude.

2 The Embargo: Some Background

As mentioned above, a key feature of the South African economic history that we exploit in this study is the country's unique revision toward autarky that began around 1985 and ended early in 1994. During this period, the world imposed economic sanctions on South Africa to encourage an end to its apartheid regime—a political system that granted different rights to citizens based on race. As a result of the embargo, several foreign public and private entities operating in South Africa decided to disinvest and/or stop making new investments or reinvestments of earnings in the country.[5] In addition to these restrictions on capital flows, several countries also restricted or banned trade with South Africa. These restrictions limited the country's ability to trade goods and financial claims with the rest of the world.

South Africa's trade-to-GDP ratio dropped from an average of 23 percent during the pre-embargo period to an average of 19 percent during the embargo, then snapped back to an average of 25 percent after the embargo was lifted. The country's current account balance, shown in Figure

[5] The International Monetary Fund estimated that the embargo cost South Africa $8 billion in foregone foreign investment between 1985 and 1991, about 3 percent of the country's cumulative GDP from 1985 to 1991. The U.S. General Accounting Office (GAO) estimated that $10.8 billion flowed out of South Africa from January 1985 through June 1989, including $3.7 billion in loan repayments to banks, $7.1 billion in other debt repayments and capital flight (GAO 1990, 12, 17). Similarly, Trust Bank (a South African commercial bank) calculated that the country had forgone nearly $14 billion in loans and direct investments between 1985 and 1990 in comparison to what loans and direct investments would have been had money flowed in at the rates that had prevailed before 1985 (The Economist, 10 February 1990, 69).

1, also follows a pattern consistent with these restrictions.[6] Before 1985, the country registered current account deficits that averaged 2 percent of GDP. However, when economic sanctions intensified between 1985 and 1993, the current account balance swung to a surplus that averaged about 2.4 percent of GDP. When economic sanctions were lifted in early 1994, the current account balance reversed again to a deficit as the country re-integrated into the world economy.[7]

For estimation purposes, we interpret the beginning of the embargo as September 1985, when official sanctions were enacted against South Africa by the European Community and the United States, and the end of the embargo as April 1994, when the country held its first all-race democratic elections.[8] These dates are also consistent with the swings in South Africa's current account balance discussed above.

3 Theoretical Motivation

We assume that each industry can be represented by a forward-looking representative firm that operates in perfectly competitive markets and that faces internal costs for adjusting its capital stock. Each of these firms maximizes its market value by choosing its labor input for the current period and its capital stock for the following period. The investment decision balances costs of adjustment against the costs associated with deviating from capital holdings that would be optimal in the absence of internal adjustment costs. As a preliminary step, we define *frictionless capital* as:

$$k_{i,t}^{*} \equiv y_{i,t} - \sigma u_{i,t} + (\sigma - 1)\, a_{i,t}^{k} = \begin{bmatrix} 1 & -\sigma & (\sigma - 1) \end{bmatrix} \begin{bmatrix} y_{i,t} \\ u_{i,t} \\ a_{i,t}^{K} \end{bmatrix} \tag{1}$$

for industries $i = 1, ..., N$, where $y_{i,t}$ and $u_{i,t}$ are *log* of output and *log* of the user cost for frictionless capital in industry i, and $-\sigma$ is the user cost elasticity of capital.[9] The final term

[6] Detailed historical accounts of the economic embargo and the disinvestment can be obtained from the Institute for International Economics website at http://www.petersoninstitute.org/research/topics/sanctions/southafrica.cfm.

[7] See Coulibaly [2009].

[8] Shortly after these elections, the United Nations adopted a resolution for all of its members to end economic sanctions against the country.

[9] This equation is inspired by the standard first order condition for capital for a case where there are no adjustment frictions and where the production function takes the following constant elasticity of substitution (CES) form:

$$F(A_{i,t}^{K}, A_{i,t}^{L}, K_{i,t,}, L_{i,t}) = \left[\omega_i \left(A_{i,t}^{K} K_{i,t}\right)^{\frac{\sigma-1}{\sigma}} + (1-\omega_i)\left(A_{i,t}^{L} L_{i,t}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

involves $a^k_{i,t}$, which summarizes the extent that technology *directly* augments capital in production. If such a term exists, we assume it is known in the current period by firms, but is not observed by econometricians. The frictionless user cost in each industry is given by:

$$U_{i,t} = p^K_{i,t-1}\left(\frac{1-\tau z_t}{1-\tau}\right)\left[r^f_{t-1} + \zeta + \delta_i - \frac{E_{t-1}\left[\Delta p^K_{i,t}\right]}{p^K_{i,t-1}}\right] \tag{2}$$

where τ_t is the corporate tax rate, $p^K_{i,t}$ is the real price of capital goods, z is the present value of the depreciation allowances associated with one unit of capital, r^f_t is the risk free real interest rate, ζ is an appropriate risk premium, and δ_i is the depreciation rate.[10]

We assume that the determinants of $k^*_{i,t}$ in equation (1) in each industry evolve according to a joint stochastic process that competitive firms treat as given.[11] A key issue for identification is whether there is capital-augmenting technological progress so that $a^K_{i,t}$ shows up as a determinant of $k^*_{i,t}$. Our specification allows for technology to have independent effects on all factors of production, thereby nesting as special cases a number of important alternatives. One special case is Hicks-Neutral technological progress, where a common technological factor $a_{it} = a^K_{i,t} = a^L_{i,t}$ augments all inputs to the same extent. A second important special case is for technology to solely augment labor so that $a^K_{i,t} = 0$. As King, Plosser, and Rebelo [1988] show, this case is theoretically appealing because it allows for the existence of a balanced growth steady state. A third possibility is that production takes a Cobb-Douglas form, which imposes the restriction that $\sigma = 1$ so that the final term involving the unobserved technology factor drops out of equation (1). A final, more general, possibility is a case where both capital- and labor-augmenting technology are allowed to have distinct trajectories.

We assume that the representative firm in each industry i seeks to maximize the present value of its cash flows by choosing optimal trajectories of capital and labor in the face of adjustment

where σ is the elasticity of input substitution, $K_{i,t}$ is the level of capital, and $L_{i,t}$ is the level of the variable input, $A^K_{i,t}$ and $A^L_{i,t}$represent the degree that technology augments capital and labor, respectively. For simplicity, we suppress all constants and industry fixed effects.

[10] We include a risk premium in the cost of capital to allow for the possibility that the firm's stakeholders are risk averse, so that they require additional compensation for variation in returns around their expected value.

[11] In the context of our competitive model, an individual firm's choice of output is essentially determined by its predetermined level of capital and by the given market-determined wages that determines its usage of variable inputs.

costs for capital:

$$\sum_{j=0}^{\infty} R_t^{t+j} E_t \left\{ (1-\tau_{t+j}) \left\{ F\left(K_{i,t+j}, L_{i,t+j}\right) - w_{i,t+j} L_{i,t+j} \right\} - \overline{p}_{i,t+j}^k \left[I_{i,t+j} + J\left(K_{t+1+i}, ..., K_{t+1+i-M}\right) \right] \right\} \tag{3}$$

where the optimization is subject to the capital accumulation constraint $K_{i,t+1} = K_{i,t+1}(1-\delta_{i,t}) + I_{i,t}$. In this formulation, $w_{i,t}$ is the given real market wage, $0 < R_t^{t+j} \leq 1$ is a risk-adjusted factor that discounts between periods t and $t+j$, and $\overline{p}_{i,t+j}^k = \frac{p_{t+j}^K}{p_{t+j}^Y}(1-\tau_{t+j} z_{t+j})$ is the real price of capital after deducting the present value of capital depreciation allowances. $J(\cdot)$ is a convex and linearly homogeneous function that captures internal costs associated with adjusting the capital stock. Since adjustment costs should mainly be an issue when the capital stock is not static, we restrict this function so that, for any fixed $K > 0$, $J(K, ..., K) = 0$ and $\frac{\partial}{\partial K_j} J(K, ..., K) = 0$ for all j. This function is a generalization of the usual adjustment cost function in which $M = 1$, which implies that firms choose their path of investment in order to smooth capital growth over time. As argued by Tinsley [2002], costs could be a function of many lags of capital growth and, *a priori*, it is difficult to rule out cases where firms smooth capital adjustment according to criteria that put weight on higher-order changes. The functional form shown above is sufficiently flexible that it allows for these more-general forms of smoothing in capital accumulation while still allowing for the more familiar case where $M = 1$.[12]

In Appendix A, we generalize the approach described in Tevlin and Whelan [2003] to show that, up to a linear approximation, the optimal capital stock will follow a distributed lag of the form:

$$k_{i,t+1} = \begin{bmatrix} 1 & -\sigma & (\sigma-1) \end{bmatrix} G(B) \begin{bmatrix} y_{i,t} \\ u_{i,t} \\ a_{i,t}^K \end{bmatrix} \tag{5}$$

where $G(B) = G_0 + G_1 B + G_2 B^2 + \ldots$, is a matrix polynomial in the backshift operator B, and G_j is a 3×3 matrix for any whole number j.[13] This equation serves as the structural motivation for the regression specifications in the remainder of the paper. The matrix lag polynomials

[12] For concreteness, one example of an adjustment cost function that satisfies these properties is:

$$J\left(K_{t+1}, ..., K_{t+1-M}\right) = K_t \sum_{m=1}^{M} \frac{\theta_m}{2} \left[(1-B)^m \left(\frac{\Delta K_{t+1}}{K_t} \right) \right]^2, \tag{4}$$

where $B$is the backshift operator. This form clearly nests the familiar case of capital adjustment costs ($M = 1$), but also allows for costs that entail higher-order smoothing—such as adjustment costs that impose direct costs for changing the investment rate ($M = 2$).

[13] Note that these matrices are not generally diagonal. This is because current and lagged values of a given

that multiply each of the right-hand-side variables trace out, in reduced form, how the capital stock evolves in response to innovations to each of the determinants of frictionless capital. The attributes of these responses depend not only on the long-run frictionless elasticities, but on the matrix lag polynomial $G(B)$ which, in turn, reflects both the magnitude of adjustment costs and the anticipated persistence of shocks to fundamentals. These two characteristics have important implications for estimation. For example, Chirinko, Fazzari and Meyer [1999] and others have recognized that proper identification of the user cost elasticity should allow for the possibility that capital may not fully reflect the effects of a given shock to the user cost elasticity for quite some time. Also, findings in Tevlin and Whelan [2003] suggest that long-run changes to the capital stock are driven primarily by shocks that are more persistent.[14] This latter issue is particularly important for identification. To illustrate using equation (5) above, let $g_{lh}(B)$ denote element *(l,h)* of the matrix polynomial $G(B)$. In the absence of restrictions on $G(1)$, the long-run coefficient on the user cost is $g_{12}(1) - \sigma g_{22}(1) + (\sigma - 1)g_{32}(1)$, so that the long run response of the user cost does not identify the parameter σ unless $g_{22}(1) = 1$ and $g_{12}(1) = g_{32}(1) = 0$. In Appendix B, we extend results in Tevlin and Whelan [2003] to show that when the process for a given fundamental has a unit root, the frictionless elasticity corresponding to that variable is at least partially identified. When all the frictionless fundamentals have unit roots, then $G(1) = I_3$ and all three of the frictionless demand elasticities—if these variables were observable— could be identified using their long-run responses.

In this model, the potential presence of the unobserved factor $a_{i,t}^K$ means that cointegration between capital, output and the user cost may not hold in general, but may hold for the special cases (discussed above) in which this term disappears.[15] If cointegration does hold, then a levels specification could be a particularly efficient identification strategy because the estimated parameters from this regression are super-consistent even in the presence of endogeneity. However, if cointegration fails to hold, then estimates using levels specifications could yield spurious results.[16]

fundamental may help predict the future values other fundamentals that become relevant determinants of investment in the face of sluggish adjustment.

[14] Simulation evidence in Caballero [1994] shows that adjustment costs can be a huge source of small sample bias in such regressions for the timespans similar to what we normally observe in the data.

[15] It will also hold if $a_{i,t}^K$ is stationary.

[16] The empirical evidence for a cointegration specification of this nature is mixed. Using aggregate U.S. data, Tevlin and Whelan [2003] cannot reject no cointegration for specifications using equipment capital. By contrast, Schaller [2006] finds evidence for cointegration for equipment capital after adjusting their estimates to account for small sample bias. Though Caballero [1994] and Smith [2007] use levels specifications, they do not formally test for

We regard the potential presence of such a relationship as an empirical issue, and test the null of no cointegration using formal panel cointegration tests. In addition, we estimate the user cost elasticity using difference specifications that do not rely on cointegration.

As one would expect, identification in these specifications hinges on our ability to isolate exogenous movements in the user cost while controlling for changes in output—the relevant shift factor for demand. We think that our South African data may be particularly useful in this regard, because the user cost is likely to be exogenous during the non-embargo portion of our dataset. Taken together, estimates using these alternative approaches should provide a fairly robust sense of the range of elasticity estimates that can be supported by the data.

4 Data

Our dataset consists of a quarterly panel of 24 two-digit manufacturing industries over the period from 1970:Q1 to 2000:Q4.[17] Quarterly industry-level estimates of the real capital stock (K_{it}), fixed investment, gross value added (Y_{it}), consumption of fixed capital, and industry-specific price deflators for investment (p_{it}^K) and output (p_{it}^Y) were obtained from South Africa Trade and Industrial Policy Strategies (TIPS). Quarterly data for prime borrowing rates (r_t) and the average corporate tax rate (τ_t) were obtained from the South African Reserve Bank. The user cost of capital for each industry in each quarter ($U_{i,t}$) was calculated using equation (2). The cost of capital component of the user cost (the bracketed term in the user cost equation) is calculated as the sum of the nominal borrowing cost in the preceding quarter (r_{t-1}), a fixed risk premium of 10 percentage points, the estimated depreciation rate ($\delta_{i,t}$), less a proxy for anticipated capital gains for investment goods ($E_{t-1}\left[\Delta p_{i,t}^k\right]/p_{i,t-1}^k$) in that quarter.[18] For each industry, we proxy for expected capital gains using the conditional forecast from an OLS regression that projects the four-quarter rate of increase in the investment price deflator onto variables in the time t information set: Namely, current and

cointegration.

[17] We excluded four industries from our sample (tobacco, leather products, glass products, and communications equipment) because their investment data were questionable or did not exist. Taken together, these four industries account for an average of about $3\frac{1}{2}$percent of quarterly nominal output and about $1\frac{3}{4}$percent of the nominal capital stock for the manufacturing sector during our sample.

[18] Depreciation rates for each industry in each quarter were calculated by dividing the consumption of fixed capital by the capital stock at the end of the previous quarter, then converting this figure to an annual rate.

lagged values of the nominal interest rate and lags of the dependent variable.[19] The capital price component of the user cost ($p^k_{i,t}$) is formed as the ratio of the industry's investment deflator and its output deflator, and is multiplied by our estimates of the relevant tax terms.[20]

Figures 2 and 3 show time-series plots of the capital-output ratio and user costs for each industry in our panel. Note that all three of our primary variables of interest (capital, output and the user cost) vary both in the time and cross-section dimensions. In turn, since corporate tax rates and risk-free rates do not vary across industries, cross-sectional variation in the user cost owes almost entirely to differences across industries in the relative price of capital, anticipated capital gains, and capital depreciation rates.[21] In reality, our user cost measure may miss some variations in the user cost stemming from changes in the risk premium, which could vary idiosyncratically over time and across industries and may not be adequately reflected in our measure of borrowing costs. But we think it is quite likely that risk premiums are not integrated processes, so this variation is not an issue—at least asymptotically—in our levels specifications. In the difference specifications, we attempt to limit the influence of potential variations in the risk premium by differencing, and by controlling for fixed and aggregate effects.

[19] We use the four-quarter rate of change, rather than the one-quarter change that matches the frequency of our sample, in order to state capital gains at an annual frequency and to minimize variations owing to price seasonalities. Our proxy for expected capital gains is large enough to make the cost of capital negative in some quarters for a few industries. Negative user costs are ruled out by optimization and, from a more practical standpoint, make it impossible to take logs. To deal with this problem, we chose a fairly large risk premium. This ruled out negative user costs for all observations but the furniture industry between 1980Q3 and 1981Q1–when the investment price deflator grew at an annual rate of more than 30 percent. In order to maintain a balanced panel, we set the cost of capital for these three observations to the average of the values in 1980Q2 and 1981Q2. Dropping these observations has no meaningful effect on our estimates.

[20] We calculate the the present value of future depreciation allowances in each year z_{it}using the formula: $z_t = \frac{\delta_{i,t}}{r_t+\delta_{i,t}} = \sum_{j=1}^{\infty} \delta_{i,t}\left(1+r_t\right)^{-j}\left(1-\delta_{i,t}\right)^{j-1}$ Note that this formula implicitly assumes that firms expect interest rates, the corporate tax rate, and the rate of depreciation to remain constant in the coming periods–i.e. all tax changes are surprises.

[21] It is possible that taxes could still drive cross-sectional variations in the user cost due to cross-industry differences in the depreciation rate (which factors into z_t). But industry depreciation rates vary little across time, so that a first order approximation, fixed and aggregate effects should temper the effect of this variation on our estimators.

5 Estimation and Results

5.1 Unit Root Tests

Before proceeding to estimation, we formally test for unit roots in our measures of capital, output, and the user cost. As mentioned earlier, unit roots are necessary to identify frictionless elasticities from long-run responses. We also test for a unit root in the ratio of capital to output, which is a precondition for a cointegrating relation between capital, output, and the user cost. Starting with the single time series tests, Figure 4 shows the results of Dickey-Fuller GLS tests by industry.[22] To correct for small-sample size distortions, we augmented these equations with lag difference terms using the lag selection criterion described in Ng and Perron [2001] to choose an appropriate lag order.[23] These tests fail to reject unit roots at 5 percent significance in all but six of the twenty-four industries in our panel. At 10 percent significance, we fail to reject unit roots in all but nine industries.

A potential issue for our analysis is the presence of structural breaks. In our dataset, there is a strong rationale to believe that there may be structural breaks at the time that the embargo was imposed and removed. These structural breaks, if present, could make it difficult to draw inferences about the existence of unit roots, an important characteristic for our estimation. To assess the effect of potential structural breaks, we conduct unit roots tests following the procedure in Clemente, Montañes and Reyes [1998], which is robust for two structural breaks. Figure 5 shows the results of this test for each industry in our sample. The break-robust tests fail to reject a unit root for capital, output, and the capital-to-output ratio. For the user cost, the tests fails to reject a unit root at 5 percent significance for all but two of the industries in our panel—close to what we would expect from type I error.[24]

We also tested for the presence of unit roots using panel tests, which have the additional benefit

[22] The estimation equations for each test included a drift term and are augmented with lagged differences to correct for small-sample size distortions.

[23] We fit ARMA(1,1) processes to the user cost of each industry and found that about one-half of the industries had negative estimated MA coefficients. As is well understood, this property tends to cause unit root tests to overreject the null when the estimation equation includes an insufficient number of lagged difference terms. Ng and Perron [2001] show that their modified information criterion is much more effective than other lag-selection criteria–such as the AIC and SIC– for mitigating this problem.

[24] We obtained very similar findings when we used the the Zivot-Andrews test (Andrews and Zivot [1992]), which is robust for one structural break. When estimating elasticities in the remainder of the paper, we control for aggregate effects, which should limit the effect of potential breakpoints on our estimates.

of exploiting information contained in the cross-sectional dimension of the dataset. Table 1 shows the results for a number of alternative specifications. The first line shows p-values for the panel unit root tests developed by Pesaran [2007], which maintains a null hypothesis that a given variable has a unit root for all industries against the alternative of no unit root in *at least one* industry.[25] The second line of the table shows p-values obtained by applying the Hadri [1999] test to our full dataset, which maintains the *null of stationarity* for all 24 industries against the *alternative* of a unit root. For robustness, we conducted all of these tests both on our full panel of 24 industries and for an "$I(1)$ subsample" of industries in which single-series tests failed to reject the presence of a unit root in the user cost at the 10 percent significance level. Taken together, these tests provide support for maintaining that the relevant variables in our full panel have unit roots at the 5 percent significance level. However, at the 10 percent significance level, the evidence for unit roots in the user cost for all 24 industries in the full panel is mixed. For the $I(1)$ subsample, the panel tests do support the existence of unit roots in the user cost. Since unit roots in the user cost are preconditions for identification, we err on the side of caution by also showing elasticity estimates obtained by restricting the dataset to just the $I(1)$ subsample.

5.2 Estimates Using Panel Cointegration Techniques

We begin by estimating a cointegrating specification between capital, output and the user cost. Equation (5) can be rearranged to obtain:

$$k_{i,t+1} - y_{i,t} = -\sigma_i u_{i,t} + e_{i,t}, \tag{6}$$

where

$$e_{it} = \begin{bmatrix} 1 & -\sigma_i & (\sigma_i - 1) \end{bmatrix} [G(B) - I_3] \begin{bmatrix} y_{i,t} \\ u_{i,t} \\ a^K_{i,t} \end{bmatrix}$$

is an unobserved cointegrating residual. This specification follows previous studies (Caballero [1994], Schaller [2006], Smith [2007], and others) in that it restricts the capital-output elasticity to unity. We estimate the long-run user cost elasticity using this specification for the full sample and

[25] There are a number of alternative panel unit root tests available, including the tests developed by Levin, Lin and Chu [2002], Levin, Lin and Chu [2002], and Im, Pesaran and Shin [2003]. We chose the Pesaran [2007] test because it is more robust than these other tests for generic forms of cross-sectional correlation between the residuals across groups.

for our $I(1)$ subsample.[26] With output and the user cost containing unit roots, the stationarity condition for the error term is an empirical question that can be verified using cointegration tests. In our model, this condition boils down to a claim that the factor $a_{i,t}^K$ follows a stationary process or is non-existent.[27]

Small sample bias remains an important econometric issue when testing for cointegration because our sample may not be sufficiently large to overcome finite-sample correlation between the regressors and the structural error term in equation (6).[28] We estimate our cointegrating relationship using pooled DOLS (Kao and Chiang [2000]) which assumes homogeneity, and mean-group DOLS (Pedroni [2001]) which allows for heterogeneity in the true elasticity across industries. Both of these specifications include dynamic OLS terms that correct for biases that arise in finite samples when there is correlation between the error term and our regressors.[29] The structural form of the error in equation (6) provides some useful guidance about what variables to include as dynamic correction terms. Specifically, when the conditions for cointegration hold, the error term will generally include lagged differences in both the user cost and output. For this reason, we include first-differenced lags and leads of both of these variables in all of our specifications.[30] In addition, we include time dummies and fixed effects in order to correct for biases that arise in the presence of contemporaneous correlation between residuals across industries (Pedroni [2001]).

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[26] In our analysis, we found allowing the output elasticity to be freely estimated did not affect our estimates of the user cost elasticity. The user cost elasticity was also little affected when the capital-output elasticity to take a wide range of alternative values.

[27] These conditions ensure that $e_{i,t}$ is stationary as follows. As shown in the Appendix , the first two columns of $G(1)$ will be the same as the first two columns of the identity matrix I_3if the processes for output and the user cost contain unit roots and the right-hand side variables are not cointegrated. This implies that the first two columns of $G(1) - I_3$ are zeros, so that the lag polynomials in the error term that multiply the I(1) processes for output and the user cost must contain unit roots. Therefore, all of these terms are $I(0)$. By contrast, the lack of a unit root for $a_{i,t}^K$ ensures that the third column of $G(1) - I_3$ will not be nonzero, ensuring that the lag polynomials that multiply the $I(0)$ process $a_{i,t}^K$ contain no unit roots. Therefore, these terms are also $I(0)$.

[28] Simulations in Caballero [1994] show that the degree of small-sample bias can be considerable when estimating single-equation cointegrating regressions. We repeated these simulation experiments in a panel context (not shown) and came to a similar conclusion.

[29] Kao and Chiang [2000] show that estimates of the coefficients in a cointegrating regression from an uncorrected panel OLS estimator have a biased asymptotic distribution. Their simulations show that a pooled dynamic panel OLS (DOLS) estimator has only a small bias for samples with cross-section and time dimensions similar to our panel, and that this estimator outperforms alternative estimators such as pooled FMOLS.

[30] These terms should have no effect asymptotically. The inclusion of output had very little effect if we included many leads and lags of our error correction terms, but the estimates tended to be more stable and converged more rapidly with output included.

We begin by testing the validity of the cointegrating relation in equation (6) using both our full dataset and the $I(1)$ subsample. We conduct two sets of tests. The first set is a homogeneous cointegration specification that uses cointegrating residuals from pooled DOLS specifications, while the second set is a heterogenous specification that uses cointegrating residuals fitted using our mean-group DOLS specifications. All of our tests assess the null hypothesis of no cointegration using test statistics described by Pedroni [1999].[31] We conduct these tests for pooled *within-dimension* ("*panel*") statistics that maintain the null that the residuals in all industries have unit roots against the alternative that these residuals have a common stable autoregressive parameter, and for *between-dimension* ("*group*") statistics that maintain the null that the cointegrating residuals in all industries have a unit root against the alternative that the residuals in all industries have stable—but not necessarily common—autoregressive roots.[32] The results, shown in Table 2, show that our tests are able reject the null of no cointegration, and provide fairly strong empirical support for both our homogeneous and heterogeneous cointegrating specifications.

Table 3 reports our user elasticity estimates from these two cointegration specifications. The first and second columns of the table report estimates using our full panel of 24 industries and for the $I(1)$ subset of industries, respectively. Results for our pooled DOLS specifications—shown in the top portion of the table—all point to highly significant estimates of the user cost elasticity that are in the neighborhood of the Cobb-Douglas benchmark of -1.0. All of these pooled specifications include 25 leads and lags of first-differences in output and the user cost, the order of which was determined using a sequential t-test procedure similar to that described by Ng and Perron [1995].[33] This number of dynamic correction terms is in line with specifications used by Caballero [1994] and Schaller [2006] for quarterly equipment capital. Figure 6 shows how varying the number of included leads and lags affects our estimates. Estimates of the user cost elasticity—shown in the top panels of the figure—tended to increase in absolute magnitude as we added more dynamic correction terms, but generally remained in the range of -1.0 for a wide range of possible specifications. The bottom panels of this figure show that our lag/lead length roughly corresponds with what would

[31] To calculate the statistics, we used a modified version of Pedroni's RATS code, which is available at: http://econpapers.repec.org/software/bocbocode/.

[32] The first three statistics are analogous to the panel unit root tests developed by Levin, Lin and Chu [2002], while the fourth and fifth tests are akin to the panel tests in Im, Pesaran and Shin [2003].

[33] Specifically, we started by estimating a specification that included 32 leads and lags of first-differenced output and user costs and then tested the joint significance of the coefficients on the last included lead and lag of the user cost. If they were not significant at 10 percent, we reestimated after dropping one lead and lag.

be suggested by the Akaike Information Criterion.[34]

The bottom portion of Table 3 summarizes our estimation results using group-mean DOLS. This method estimates separate DOLS specifications for each industry and then forms an estimate of the aggregate elasticity using a weighted average of the industry estimates. To correct for finite-sample bias, we include in each DOLS regression 8 leads and 16 lags of the first-differences in output and the user cost.[35] We find a user-cost elasticity estimate of -0.54 for the full panel, but the estimate rises in magnitude to -0.85 for the $I(1)$ subsample.

In the following section, we present and estimate a distributed-lag specification. The motivation for this alternative specification is two-fold. First, it provides some assurance that our results are robust to alternative econometric specifications. Second, it allows us to test the importance, for estimation, of the exogeneity of shocks to the user cost. We estimate the user cost elasticity during the embargo period and compare it to the elasticity in the non-embargo period. Such an analysis could not be appropriately carried out with the cointegration specification which, under the maintained assumptions, should be consistent even in the presence of such endogeneity.

5.3 Distributed Lag Specification

When shocks to the user cost are exogenous, and both output and the user cost contain unit roots, the first two columns of the matrix $G(1)$ in equation (5) are identical to the first two columns of a three-dimensional identity matrix I_3. Given these assumptions, we can estimate distributed lag specifications of the form:

$$\Delta k_{i,t} = \eta_i + T_t + N_i^y(B)\Delta y_{i,t} + N_i^u(B)\Delta u_{i,t} + e_{i,t} \tag{7}$$

for industries $i = 1, ..., N$, where η_i is an industry fixed effect, T_t is a dummy for year t that controls for aggregate effects, and $e_{i,t}$ is the portion of the structural residual that remains after controlling for these effects. This specification is obtained by taking the first difference of equation (5), where we let $\eta_i + T_t + e_{it}$ collectively denote the components of the structural residual:

[34] We also explored the Schwarz criterion.

[35] Lag selection results for the homogeneous specification suggest that we might consider including more dynamic correction terms. We are reluctant to do so because the asymptotics of the industry-level estimators are solely in the time dimension, which is limited in our dataset. Specifically, our chosen lead/lag length eliminates 24 of the 117 available time series observations in each industry. The industry-level regressions then include about 50 parameters, leaving only about 40 degrees of freedom.

$$[g_{13}(B) - \sigma_i g_{23}(B) + (1 - \sigma_i) g_{33}(B)] \, \Delta a_{i,t}^{K}.$$

Under our maintained unit root assumptions, the long run lag polynomials in the above equation take the structural form:

$$N_i^y(B) = g_{11}(B) - \sigma_i g_{21}(B) + (1 - \sigma_i) g_{31}(B), \qquad \text{and}$$
$$N_i^u(B) = g_{12}(B) - \sigma_i g_{22}(B) + (1 - \sigma_i) g_{32}(B),$$

where the long-run responses are restricted such that $N_i^y(1) = 1$ and $N_i^u(1) = -\sigma_i$. To estimate this equation, we assume that the terms in these lag polynomials become small beyond some finite lag order, so that the long run responses of capital to each fundamental can be obtained by cumulating the estimated parameters of the relevant distributed lag function.[36] Though we maintain that innovations to the user cost are exogenous in our data, we take some additional precautions to guard against the possibility that the user cost may not be entirely exogenous. Our panel estimators allow us to control for latent aggregate effects, which (among other things) protects against variations in the user cost stemming from technological progress that augments capital demand to the same extent in all industries, and latent fixed effects that (among other things) defend against fixed differences in technological growth across industries that might be reflected in their user cost.[37]

We start by including a large number of lags in our regressions and then apply the sequential t-test procedure described in Ng and Perron [2001] to successively eliminate lags of the user cost that—at the margin—do not contain statistically relevant information about the overall magnitude of the response.[38] Figure 7 shows the results of this lag order selection exercise for two alternative specifications. The circles that denote the estimate at each lag show the outcome of our sequential t-test at that lag: Dark circles denote that the coefficient on the last included lag can be statistically

[36] Our benchmark specification shown does not include any lagged values of the dependent variable as regressors as in some specifications. Including lags of the depedent variables had little effect on our results. In principle, the two approaches should provide equivalent answers in a large sample if the true distributed lag representation is "well behaved" in the sense that the lagged responses decay geometrically. Our specification has the additional benefits of allowing for a more general form of the response function.

[37] As mentioned earlier, these controls may also eliminate measurement error in the user cost stemming from pure time-series and pure cross-sectional variation in the risk premium.

[38] Specifically, starting from a large number of included lags, we successively eliminate the last included lag until the estimated coefficient on this last lag is statistically significant at 10 percent.

distinguished from zero at 10 percent significance level. The outcome of these sequential t-tests suggest that we include 27 quarterly lags in our baseline specification.[39]

Table 4 shows estimates from our distributed lag specifications for both the heterogeneity specification that assumes an identical user cost elasticity across industries ("*HOM*"). We also show results of a seemingly unrelated specification that estimates separate elasticities for each industry, and construct the aggregate elasticity by averaging the industry-specific elasticities ("HET") using as weights each industry's average share of the total nominal capital stock over the sample period.[40]

The estimated long-run user cost elasticities for these specifications range between -0.48 (heterogeneous specification) and -0.62 (homogeneous specification) for the full sample, both of which are highly significant. For the subsample of industries for which unit root in the user cost is more robust, the estimates of user cost elasticity are larger, ranging from -0.77 to -0.83. In the following section, we assess the importance of identification for estimation of the user cost elasticity by examining the effect of embargo on our estimates.

5.4 Economic Embargo, Endogeneity, and User Cost Elasticity

Presumably when the economy became less open during the embargo period (1985-1994), one would expect shocks to the user cost to have become less exogenous because the embargo introduced differences between South Africa's domestic interest rates and the world interest rates, and between domestic and world prices of capital goods. This presumption is borne out in the data. Figure 8 shows the aggregate user cost for fixed business capital in the United States (a proxy for the world) and for South Africa.[41] Though the composition of the business capital stock likely differs in these

[39] When calculating these criterion, we adjust the sample so that it remains fixed as we add more lags.

We also conduct lag order specifications using the Akaike and Schwarz information criteria for each number of included lags. The results did not provide additional useful guidance.

[40] In order to allow for estimation error in these shares, we include in our system a second set of regressions that estimates the time average of the industry shares for each industry in the panel. The formula for each weighted-average elasticity was then calculated using estimates from these entire set of regressions. The standard errors of the estimated aggregate elasticitywere determined using the delta method where we account for cross-correlation of residuals across the two regressions.

[41] For South Africa, we constructed the user cost using a variant of equation (2), where we substituted the industry-specific price deflators for output and investment with deflators formed by chain-aggregating across all 24 industries in our panel. The U.S. user cost was calculated using BEA aggregates and a quarterly average of the prime lending rate. To ensure that the levels of these two user costs were broadly comparable, we assumed that U.S. capital was also subject to a fixed risk premium of 10 percentage points. We use the real exchange rate to converted the U.S. user cost so that it is denominated in terms of South African goods and services.

two countries, broad patterns in these user costs suggest that the cost of capital in South Africa became detached from the rest of the world during the embargo period. The contemporaneous correlation between the two user costs series is highly positive both before and after the embargo, but fell significantly during the embargo. This pattern is consistent with the conjecture that shocks to the user cost during the embargo period were less influenced by exogenous factors.[42] In light of this, the identification problem would be heightened during the embargo period, making it more difficult to estimate the user cost elasticity. We formally test this hypothesis by estimating the user elasticity during the embargo and non-embargo periods.

To capture the potential effect on our estimates of this heightened endogeneity, we augment the lagged-difference specification in the preceding section to include terms that interact the observable explanatory variables with a dummy variable for the embargo period. Our formulation for this regression is:

$$\begin{aligned}\Delta k_{i,t} = \eta_i + T_t + N^y(B)\Delta y_{i,t} + M^y(B)\left(d_t \Delta y_{i,t}\right) \\ +N^u(B)\Delta u_{i,t} + M^u(B)(d_t \Delta u_{i,t}) + \epsilon_{i,t},\end{aligned} \tag{8}$$

so that the embargo affects the entire long run relationship between capital and its fundamentals, but only for observations of capital growth that occur within the embargo period. As in our previous distributed lag specification, we estimate these regressions using contemporaneous observations and 27 lags of each the fundamentals (including the interactions with the embargo dummy). Given these estimates, we determine the long-run elasticity of capital with respect to the user cost by calculating $N^u(1)$. The marginal effect of the embargo-period data on our long-run user cost elasticity is calculated as $M^u(1)$, while the long-run elasticity to during the embargo-period is $N^u(1) + M^u(1)$.

Results using this specification are shown in Table 5. The first and second columns show estimates using our full panel of 24 industries and the $I(1)$ subsample. We find that the magnitude of the user cost elasticity outside of the embargo period increases from the -0.45 to -0.62 range (in previous section) to -0.75. For the $I(1)$ sample, the estimate increases from the -0.77 to -0.83 range to -0.86. Interestingly, the user cost elasticity for the embargo period is significantly lower and in the -0.25 to -0.27 range, consistent with estimates in many previous studies that document a small and, often, insignificant user cost elasticity. These results are consistent with the hypothesis that the user cost became more endogenous during the embargo period, leading to

[42] These patterns are even stronger when we exclude tax terms from the calculations of the user cost.

a greater role of simultaneity bias. These findings may help explain why studies of the capital-user cost elasticity using stationary specifications and data from large economies often find very little role for the user cost in determining the size of the capital stock.[43] The similarity results from this specification after correcting for the embargo period to those shown earlier from the cointegration specification is particularly intriguing and suggests that our estimates are fairly robust across econometric specifications.

To better describe how capital adjusts to innovations in the user cost, we show the marginal and cumulative responses of capital to a one percent increase in the user cost in figures 9 and 10, respectively. These responses are for our embargo-corrected estimates using the $I(1)$ subsample—those for the full sample have similar contours. These marginal responses for our embargo corrected estimates show a distinct hump-shape that reaches maximum response at about twelve quarters and then gradually attenuates to about zero by the twenty eighths quarter. These responses suggest a rather slow and non-monotonic adjustment process that extends over about seven years. This helps document the importance of focusing on the long-run in order to capture the full response of the capital stock to effect of a shock to the user cost, and the importance of using a regression specification that is flexible enough to allow the shape of the response to be non-monotonic over time.

Our estimation assumed that the embargo started in 1985 and ended in early 1994. The exact dating of the embargo period could be uncertain as it is with most event studies. The choice of these dates is supported by anecdotal evidence and narratives on the imposition and removal of the embargo as well as in the aggregate economic data shown earlier. Nonetheless, we conduct sensitivity analyses that vary the embargo's starting and ending dates within a two-year window of our chosen dates. Figure 11 shows in the bottom panels, the log-likelihood of various starting and ending dates, and the top panels show the corresponding user cost elasticities. The bottom panels show the log-likelihood of each alternative, holding all else equal, while the top and middle panels show corresponding elasticity estimates for the non-embargo and embargo periods, respectively. Our user cost elasticity estimate is not sensitive to reasonable changes in the starting and ending dates of the embargo as shown in the top panels and our starting date (1985) maximizes the log-likelihood.

[43] For instance, see the review by Chirinko [1993].

6 Concluding Remarks

In this study we estimate a statistically significant user cost elasticity between -0.80 and -1.0 which, in most cases, are statistically indistinguishable from the Cobb-Douglas benchmark of -1.0. This study is one of the first to document such a large user cost elasticity for a broad measure of business capital that includes both equipment and structures, and the first to show that similar estimates can be obtained using both stationary and cointegrating regression specifications.

One explanation for our ability to identify a large user cost elasticity is that exogenous shocks to the user cost are better identified in a small and isolated economy like South Africa. In a closed economy or in a large open economy, the capital stock and the user cost of capital are jointly determined by a domestic demand and supply equilibrium that equates the marginal product and marginal opportunity cost of capital services. This simultaneity introduces inconsistency into estimates of the user cost elasticity. In a small open economy like South Africa, however, shocks to the user cost are largely influenced by world interest rates and prices of capital goods that the country takes as given.

The economic embargo that the world imposed on South Africa between about 1985 and early 1994 forced their economy to revert toward autarky. This provides a unique opportunity to determine the extent that endogeneity attenuates estimates of the capital-user cost elasticity. We find a robust correlation between the user cost for South Africa and that of the United States (a proxy for world user costs) before the embargo. This correlation falls significantly during the embargo period, and goes back up after the sanctions are lifted and South Africa is re-integrated into the world economy. We find that during the embargo period, when the user cost became less influenced by exogenous factors, the estimated user cost elasticity fell considerably—to magnitudes that are more in line with those obtained in many previous studies. These results underscore the importance of identification to uncovering the 'elusive' user cost elasticity of capital.

A Appendix: Derivation of Estimation Equation

We start by finding a linearized solution for the firm's capital stock for some arbitrary $M > 0$ for the generalized, linearly homogeneous adjustment cost function $J(\cdot)$ that satisfies the restrictions described in the main text. After substituting out investment using the capital accumulation constraint in equation (3), the first order condition for the capital stock in this case—after some rearranging—is:

$$E_t\left[\frac{\partial F(A^L_{t+1}L_{t+1}, A^K_{t+1}K_{t+1})}{\partial K_{t+i}}\right] = \overline{p}^K_t\left[r+\delta-\frac{E_t\left[\Delta p^K_{t+1}\right]}{p^K_t}\right]+ \tag{9}$$

$$\sum_{m=0}^{M} E_t\left[\overline{p}^K_{t+m}\right](1+r)^{1-m}J_{1+m}(K_{t+1+m},...,K_{t+1+m-M}) \tag{10}$$

where $J_l(\cdot)$ denotes the partial derivative of J with respect to its lth argument, for $l = 1,...,M+1$.[44] Noting that the first term on the right-hand side of this equation is the frictionless user cost U_{t+1} from equation (2) and assuming that $F(\cdot,\cdot)$ takes the standard CES form with elasticity of substitution σ, the equation above can be manipulated to form:

$$E_t\left[\left(\frac{Y_{t+1}}{K_{t+1}}\right)^{\frac{1}{\sigma}}\frac{\left(A^K_{t+1}\right)^{\frac{\sigma-1}{\sigma}}}{U_{t+1}}\right] = E_t\left[\left(\frac{K^*_{t+1}}{K_{t+1}}\right)^{\frac{1}{\sigma}}\right]$$

$$= 1+\frac{\sum_{m=0}^{M}\frac{E_t\left[p^k_{t+m}\right]}{p^K_t}(1+r)^{1-m}J_{1+m}(K_{t+1+m},...,K_{t+1+m-M})}{r+\delta-\frac{E_t\left[\Delta p^k_{t+1}\right]}{p^K_t}},$$

where we have used the definition of frictionless capital from (1). Given an assumption $J_l(K,...K) = 0$ for any fixed $K > 0$, the second term on the right-hand-side simplifies to zero in a no-growth state where K remains fixed at K^*. Since $J(\cdot)$ is homogeneous of degree one in its arguments, Euler's homogeneous function theorem ensures that the partial derivatives $J_l(\cdot)$ are homogeneous of degree zero. Therefore, $J_{1+l}(K_{t+1+m}, K_{t+m}, ..., K_{t+1}, ..., K_{t+1+m-M}) = J_{1+l}\left(\frac{K_{t+1+m}}{K_{t+1}}, \frac{K_{t+m}}{K_{t+1}}..., 1, ..., \frac{K_{t+1+m-M}}{K_{t+1}}\right)$ for $l = 0,...,M$ and $m = 0,...,M$. Using this property, we can linearize around a no-growth state where $1 = \frac{K_{t+1}}{K_t} = \frac{K_{t-1}}{K_t} = ... = \frac{K_{t+1-M}}{K_t}$ and where $E_t\left[p^k_{t+m}/p^k_t\right] = \left(1+\pi^k\right)^m$[45] to obtain:

[44] For simplicity, we suppress all industry subscripts, and drop time subscripts for τ, z, δ and r.

[45] The steady state that we require can be described in more detail. Noting that $k^* = y - \sigma u + (\sigma-1)a^K$, it can be shown that the growth rate of the frictionless stock is governed by the following approximation: $\Delta k^*_t \approx \Delta k_t + (\sigma - s^*_L)\Delta a^K_t + s^*_L\left(\Delta l_t + \Delta a^L_t - \Delta k_t\right) - \sigma\Delta u_t$, where s^*_L is the previous period's labor share and l is the log

$$E_t\left[k_{t+1}^*\right] \approx k_{t+1} + \sum_{m=1}^{M} \overline{d}_m \left(k_{t+1+m} - k_{t+1}\right) + \sum_{m=1}^{M} d_m \left(k_{t+1-m} - k_{t+1}\right), \tag{11}$$

where

$$d_m \equiv \sigma \left(\frac{1+r}{r+\delta-\pi^k}\right) \sum_{j=0}^{M-m} \left(\frac{1+\pi^K}{1+r}\right)^j J_{1+j,m+1+j}^*, \text{ and}$$
$$\overline{d}_m \equiv \sigma \left(\frac{1+r}{r+\delta-\pi^k}\right) \sum_{j=0}^{M-m} \left(\frac{1+\pi^K}{1+r}\right)^{j-m} J_{m+1+j,1+j}^* \text{ for } m = 1, \ldots, M,$$

and where $J_{j,k}^*$ is the partial derivative of $J(\cdot)$ with respect to its *jth* and *kth* arguments, evaluated at the zero-growth state.[46] Define $\rho \equiv \frac{1+r}{1+\pi^k}$, which amounts to a discount factor that adjusts for the risk premium embedded in the discount rate r and for the relative rate of capital goods inflation π^k. We assume that $\rho > 1$. Applying this definition and noting that $J_{j,k}^* = J_{k,j}^*$ by Young's Theorem, it can be shown that $\overline{d}_m = \rho^{-m} d_m$, for $m = 1, ..., M$. Using this fact and collecting terms, equation (11) simplifies to the following *2Mth* order difference equation:

$$E_t\left[k_{t+1}^*\right] \approx \sum_{m=1}^{M} d_m \rho^{-m} k_{t+1+m} + d_0 k_{t+1} + \sum_{m=1}^{M} d_m k_{t+1-m} \tag{12}$$
$$= D(B) k_{t+1} \tag{13}$$

where we have defined the following polynomial in the backshift operator:

$$D(B) = d_M (\rho B)^{-M} + ... + d_1 (\rho B)^{-1} + d_0 + d_1 B + ... + d_M B^M$$

and let $d_0 \equiv 1 - \sum_{m=1}^{M} d_m (1 + \rho^m)$. This function satisfies the restriction that $D(1) = 1$, so that the optimal capital stock tracks its frictionless target in the long run. In addition, since this backshift polynomial $D(B)$ is symmetric in B and $(\rho B)^{-1}$, each backward-stable root λ_m will have a corresponding forward-stable root $\lambda_m \rho^{-1}$. Taken together, these properties imply that $D(B)$ can be restated as the following product of lead and lag polynomials:

$$D(B) = \frac{a(B)}{a(1)} \frac{a\left(\rho^{-1} B^{-1}\right)}{a\left(\rho^{-1}\right)},$$

of L. Given this condition, the steady state we describe for capital requires $\sigma \Delta u = (\sigma - s_L^*) \Delta a^K + s_L^* \left(\Delta l + \Delta a^L\right)$, which ensures that the supply and demand for capital shift in tandem to keep the equilibrium quantity of capital constant.

[46] Note that all terms involving r and $\frac{E_t\left[\Delta p_{t+1}^k\right]}{p_t^k}$ drop out in the vicinity of the steady state since $J_j^* = 0$.

where we have defined $a(x) \equiv \prod_{m=1}^{M}(1-\lambda_m x) = 1 + a_1 x + ... + a_M x^M$. Using this representation, the difference equation for the capital stock in equation (12) simplifies to:

$$E_t\left[a(B)k_{t+1} - a(1)\,a\left(\rho^{-1}\right)a\left(\rho^{-1}B^{-1}\right)^{-1}k^*_{t+1}\right] = 0. \tag{14}$$

Note that this the same general form as the solutions derived by Tinsley [2002] in which firms minimize a quadratic loss objective in which adjustment costs are a function of M lags of capital, and much of the remainder of this derivation closely resembles his setup and results.[47]

At this stage, we let f_t denote $a\left(\rho^{-1}\right)E_t\left[a\left(\rho^{-1}B^{-1}\right)^{-1}k^*_{t+1}\right]$, so that $a(B)k_{t+1} = a(1)\,f_t$. This factor, which constitutes a moving target toward which capital error-corrects over time, amounts to a weighted average of anticipated future frictionless stocks where the weights are determined implicitly by the discount factor and the eigenvalues that are embedded in in the lead polynomial $a\left(\rho^{-1}B^{-1}\right)$.[48] Letting g_t denote the $(M-1)\times 1$ lead vector $[f_{t+M-1},\ldots,f_t]'$, the forward motion of this target can be described by the companion system:

$$\begin{aligned} g_t &= E_t\left[\mathbf{A}g_{t+1} + a\left(\rho^{-1}\right)k^*_{t+1}\right], \\ &= a\left(\rho^{-1}\right)E_t\left[\sum_{i=0}^{\infty}\mathbf{A}^i\iota_M k^*_{t+1+i}\right] \end{aligned} \tag{15}$$

where $\mathbf{A}$ as the $M\times M$ bottom row companion matrix of the lead polynomial $a\left(\rho^{-1}B^{-1}\right)$:

[47] Note that for the $M=1$ case, this solution is fully consistent with what Tevlin and Whelan [2003] obtain using a much simpler reduced-form approach in which firms chose capital holdings to minimize the quadratic loss function of the form:

$$\sum_{l=1}^{\infty}\theta^l E_t\left[\gamma\left(\Delta k_{t+l}\right)^2 + \left(k_{t+l} - k^*_{t+l}\right)^2\right].$$

Indeed, the solution using this formulation is identical to our model if we allow $d_1 = \gamma$ and $\rho^{-1} = \theta$.

[48] This can be seen more clearly by noting from equation (15) that

$$f_t = \iota_M' g_t = a\left(\rho^{-1}\right)E_t\left[\sum_{i=0}^{\infty}\iota_M'\mathbf{A}^i\iota_M k^*_{t+1+i}\right],$$

so that the weight on the anticipated capital stock at on the capital stock at horizon $1+i$ is $\iota_M'\mathbf{A}^i\iota_M$. The matrix $\mathbf{A}^i$ can be decomposed as $T\Lambda^i U$, where Λ is a diagonal matrix composed of the M forward eigenvalues $\rho^{-1}\lambda_m$, T is the $M\times M$ matrix of the M corresponding eigenvectors, and $U = T^{-1}$. Applying the rules of matrix algebra, it can be shown that $\iota_M'\mathbf{A}^i\iota_M = \sum_{m=1}^{M}(\rho^{-1}\lambda_m)^i T_{M,m}U_{m,M}$. Note that the sum $\sum_{m=1}^{M}T_{M,m}U_{m,M}$ is the M,M element of the identity matrix TU, so that the weight given to frictionless capital at horizon i amounts to a weighted average of the M "discount factors" $(\rho^{-1}\lambda_m)^i$.

$$\mathbf{A} = \begin{bmatrix} 0 & & I_{M-1} & \\ -a_M \rho^{-M} & -a_{M-1}\rho^{-(M-1)} & \dots & -a_1\rho^{-1} \end{bmatrix}, \tag{16}$$

and ι_M is an $M \times 1$ selection vector that has one as its Mth element and zeros elsewhere.

Tevlin and Whelan [2003] show that the forward-looking nature of the capital target is crucial for empirical estimation because the long-run response of capital to an unanticipated change in fundamentals is determined by the degree that this new information affects the effective capital target f_t.[49] In turn, this response is closely related to the anticipated persistence of the disturbances. To allow for these expectation effects, we assume that firms forecast the determinants of the frictionless capital stock in equation (1) using the following *VAR(p)* process:

$$v_{t+1} = C(B)v_t + e_{t+1}, \qquad \text{where} \qquad C(B) = \sum_{j=0}^{p-1} C_{j+1}B^j, \tag{17}$$

where v_t is the vector of q determinants of k^*_{t+1}, C_j is a $q \times q$ matrix of VAR coefficients for lag j, and e_t is a serially uncorrelated vector of covariance-stationary forecast errors such that $E_t[e_{t+i}] = \mathbf{0}_{q\times 1}$ for any whole number i.[50] Defining the $pq \times 1$ vector $z_t = [x'_t, ..., x'_{t-p+1}]'$, this VAR can be restated in companion form as $z_{t+1} = \mathbf{H}z_t + [e'_{t+1}\ \mathbf{0}_{1\times(p-1)q}]'$, where

$$\mathbf{C} = \begin{bmatrix} C_1 & & C_p \\ I_{(p-1)q} & \dots & \mathbf{0}_{(p-1)q\times p} \end{bmatrix}, \tag{18}$$

is the $pq \times pq$ companion matrix for the VAR, so that $E_t[z_{t+i}] = \mathbf{H}^i z_t$ for any whole number i.[51] To rule out explosive dynamics, we assume that q_0 of the eigenvalues of matrix $\mathbf{C}$ are exactly equal to one, and that the remaining $pq - q_0$ eigenvalues have modulus less than one.

After restating the frictionless capital stock in vector form as $k^*_{t+1} = b'v_{t+1}$, equation (14) can be solved to yield that:

$$f_t = b'\mathbf{D}z_t \tag{19}$$

[49] The VAR form used here allows for greater generality than Tevlin and Whelan [2003], who assumed that each frictionless fundamental follows its own *AR(p)* process.

[50] For brevity, we abstract from constant terms. Allowing for them does not substantively affect our results.

[51] We allow our notation to accomodate an arbitrary number of frictionless fundamentals to account for the possibility that the capital-augmenting technology term does not exist. In addition, this allows for the possibility that the VAR could be augmented to include non-fundamental factors that improve the forecasts of these relevant fundamentals.

where we have defined the $q \times pq$ matrix:

$$\mathbf{D} \equiv a\left(\rho^{-1}\right)\left[I_q \ \mathbf{0}_{q\times(p-1)q}\right]\sum_{i=0}^{\infty}\left(\iota_M^{\prime}\mathbf{A}^i\iota_M\right)\mathbf{C}^{i+1}, \tag{20}$$

where $\mathbf{D}$ is composed of p adjacent $q\times q$ matrices such that $\mathbf{D} = [D_1 \ \cdots \ D_p]$. Using this partition, and the fact that $z_t = [x_t^{\prime}, ..., x_{t-p+1}^{\prime}]^{\prime}$, equation (19) can be restated in the following lag polynomial representation:

$$f_t = b^{\prime}D(B)x_t, \qquad \text{where} \qquad D(B) \equiv D_1 + D_2B + \ldots + D_pB^{p-1}. \tag{21}$$

Inserting this representation into equation (14), it can be shown that the optimal capital stock is determined by the equation:

$$a(B)k_{t+1} = a(1)b^{\prime}D(B)x_t, \tag{22}$$

or, after inverting the backshift polynomial $a(B)$, by the MA representation:

$$k_{t+1} = b^{\prime}G(B)x_t, \qquad \text{where} \qquad G(B) \equiv a(1)a(B)^{-1}D(B). \tag{23}$$

Since $G(1) = D(1)$ by construction, both the forward-looking target f_t and the optimal capital stock k_t share the same long-run sensitivity to shocks. This shows that the long-run sensitivity of capital to an unanticipated change in frictionless fundamentals is governed by the anticipated persistence of this effect, which is embodied in the polynomial $D(B)$.

B Appendix: First Proof

Recall from the previous appendix the definition of the VAR matrix polynomial $C(L)$ shown in equation (17). We begin by showing that if $C(1) = I_q$—so that the *VAR(p)* in equation (17) can be restated as a *VAR(p-1)* system in Δv_t—then $G(1) = I_q$. A necessary condition to establish that $C(1) = I_q$ is that all q of the variables in the *VAR(p)* contain unit roots.[52] Once we establish this result, we turn our attention to cases with q_0 unit roots, where $1 \leq q_0 < q$.

[52]Sufficiency requires that we rule out the existence of cointegrating relations between the q variables in the *VAR*. It is well known that a cointegrated system can never be represented by a finite-order *VAR* in first differences (for instance, see Hamilton [1994]). Since we can think of no good theoretical argument that would impose a long-run relation between the determinants of frictionless capital, this assumption seems reasonable.

Begin by defining the matrix $\mathbf{S}$ as a $pq \times q$ matrix of stacked identity operators $[I_q \ \cdots \ I_q]'$. Using this definition, equation (20), and the fact that $G(1) = D(1)$ from equation (23), this means that:

$$G(1) = \mathbf{DS} = a\left(\rho^{-1}\right)[I_q \ \mathbf{0}_{q\times q(p-1)}]\sum_{i=0}^{\infty}\left(\iota_M' \mathbf{A}^i \iota_M\right)\mathbf{C}^{i+1}\mathbf{S} \tag{24}$$

Straightforward matrix algebra using equation (18) establishes that:

$$\mathbf{CS} = [C(1) \ I_q \ \cdots \ I_q]' = \mathbf{S}, \tag{25}$$

when $C(1) = C_1 + \ldots + C_p = I_q$, a fact that can be applied iteratively to show that $\mathbf{C}^{i+1}\mathbf{S} = \mathbf{S}$ for all non-negative integers i. Noting that $\left(\iota_M' \mathbf{A}^i \iota_M\right)$ is a scalar, we can then simplify equation (24) to become:

$$G(1) = a\left(\rho^{-1}\right)[I_q \ \mathbf{0}_{q\times(p-1)q}]\mathbf{S}\sum_{i=0}^{\infty}\left(\iota_M' \mathbf{A}^i \iota_M\right) = I_q a\left(\rho^{-1}\right)\sum_{i=0}^{\infty}\left(\iota_M' \mathbf{A}^i \iota_M\right), \tag{26}$$

so our desired result boils down to proving that $\sum_{i=0}^{\infty}\left(\iota_M' \mathbf{A}^i \iota_M\right) = a\left(\rho^{-1}\right)^{-1}$. Straightforward calculations show that the summation in this expression simplifies to:

$$\sum_{i=0}^{\infty}\iota_M' \mathbf{A}^i \iota_M = \iota_M'\left(\sum_{i=0}^{\infty}\mathbf{A}^i\right)\iota_M = \iota_M'\left(I - \mathbf{A}\right)^{-1}\iota_M, \tag{27}$$

since all the roots of the bottom row companion matrix are stable. Note that the expression on the right hand side of this equation is pre- and postmultiplied by the selection vector ι_M, so this summation is simply the bottom right-hand element of the inverted matrix $(I_M - \mathbf{A})^{-1}$, Using equation (16), this matrix takes the form:

$$(I_M - \mathbf{A})^{-1} = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \\ a_M\rho^{-M} & a_{M-1}\rho^{-(M-1)} & a_{M-2}\rho^{-(M-2)} & \cdots & a_2\rho^{-2} & 1 + a_1\rho^{-1} \end{bmatrix}^{-1}.$$

The bottom diagonal element of this matrix can be calculated by block inversion. Dividing $(I_M - \mathbf{A})^{-1}$ into the blocks:

$$N_1 \equiv \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}_{M-1 \times M-1}, \quad N_2 \equiv \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -1 \end{bmatrix}_{M-1 \times 1},$$

$$N_3 \equiv \begin{bmatrix} a_M \rho^{-M} & a_{M-1}\rho^{-(M-1)} & a_{M-2}\rho^{-(M-2)} & \cdots & a_2\rho^{-2} \end{bmatrix}_{1 \times M-1}$$

and $N_4 = 1 + a_1\rho^{-1}$, and noting that:

$$(N_1)^{-1} \equiv \begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \\ 0 & 1 & \cdots & 1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 1 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix},$$

the standard formula for block inversion suggests that the bottom diagonal element is $\left(N_4 - N_3 (N_1)^{-1} N_2\right)^{-1}$. Straightforward calculations using this formula show that $\iota_M' (I - \mathbf{A})^{-1} \iota_M = a\left(\rho^{-1}\right)^{-1}$. This proves the first result.

We now turn our attention to cases where the *VAR(p)* in equation 17 has fewer than q unit roots. Without loss of generality, assume that the first $q_0 < q$ of the variables in vector v_t have unit roots. Given this setup, we need to show that the columns of the matrix $G(1)$ that multiply the first q_0 elements of v_t are the same as the first q_0 columns of the identity matrix I_q. As a first step, Appendix C shows that, in the absence of cointegrating relations, the first q_0 columns of the summed VAR polynomial $C(1)$ must be the same as the first q_0 columns of an identity matrix I_q. Now we proceed to calculate $G(1) = D(1)$ using equation (24). Equation (25) still holds, and the matrix product $\mathbf{CS}$ can be decomposed as follows:

$$\mathbf{CS} = \mathbf{S} + \begin{bmatrix} C(1) - I_q \\ \mathbf{0}_{(p-1)q} \end{bmatrix},$$

where the first q_0 columns of the matrix in brackets must contain only zeros. By successively pre-multiplying this matrix by $\mathbf{C}$, one can show that

$$\mathbf{C}^{i+1}\mathbf{S} = \mathbf{S} + \begin{bmatrix} H_{i+1}\left(C(1) - I_q\right) \\ \mathbf{0}_{(p-1)q} \end{bmatrix},$$

for any non-negative integer i, where we have defined $H_i = I_q + C_1 H_{i-1}$ for any whole number i, and let $H_0 \equiv \mathbf{0}_{q\times q}$. Inserting this expression into equation (24) and simplifying, we obtain that:

$$G(1) = I_q a\left(\rho^{-1}\right)\sum_{i=0}^{\infty}\left(\iota_M^{\prime}\mathbf{A}^i\iota_M\right) + a\left(\rho^{-1}\right)\left[\sum_{i=0}^{\infty}\left(\iota_M^{\prime}\mathbf{A}^i\iota_M\right)H_{i+1}\right](C(1)-I_q). \tag{28}$$

Finally, using our results for the q unit root case (above) for the first term of this equation, we can simplify this expression to become:

$$G(1) = I_q + a\left(\rho^{-1}\right)\left[\sum_{i=0}^{\infty}\left(\iota_M^{\prime}\mathbf{A}^i\iota_M\right)H_{i+1}\right](C(1)-I_q). \tag{29}$$

Since the first q_0 columns of the second set of terms of the matrix $C(1) - I_q$ contain only zeros, one can confirm that—regardless of the form of the $q \times q$ matrix in brackets—the first q_0 columns of the second expression on the right-hand-side of this equation must also contain only zeros. By implication, the first q_0 columns of $D(1)$ must be the first q_0 columns of the identity matrix I_q.

C Appendix: Second Proof

Partitioning the vector v_t so that the first q_0 variables are $I(1)$ and the remaining $q - q_0$ variables are $I(0)$. We wish to prove that, when a nonstationary *VAR(p)* representation of the form shown in equation (17) exists, the first q_0 columns of the matrix $C(1)$ must be the same as the corresponding columns of the identity matrix I_q.

We begin by forming a basis of the space of $q \times 1$ vectors a such that $a'v_t$ is $I(0)$. Since the first q_0 elements of v_t are assumed to be $I(1)$ and there are no cointegrating relations linking these variables, one such basis is the $(q - q_0) \times q$ matrix:

$$A' = \left[\mathbf{0}_{(q-q_0)xq_0}\; I_{q-q_0}\right]. \tag{30}$$

Therefore, the space of $I(0)$ linear combinations $a'v_t$ is spanned by the space of vectors $h'A'$ for any nonzero $q \times 1$ vector h. Since the elements of Δv_t must be stationary, it can always be written using a Wold representation of the form:

$$(1-B)v_t = W(B)e_t, \qquad \text{where} \qquad W(B) = W_0 + W_1 B + W_2 B^2 + ..., \tag{31}$$

and W_j is a $q\times q$ matrix for $j = 0, 1, 2, ...$ Using this representation, one can perform a multivariate Beveridge-Nelson decomposition to determine that:

$$v_t = v_0 + W(1)\sum_{s=1}^{t} e_s + \eta_t - \eta_0, \qquad \text{where} \qquad \eta_t \equiv -\sum_{s=0}^{\infty}(W_{s+1} + W_{s+2} + ...)e_{t-s}$$

is a stationary variable.[53] Multiplying this relation through by A', it is evident that, in order to ensure that the linear combinations of v_t in the space spanned by A' are stationary, it must be true that the basis A' satisfies:

$$A'W(1) = \mathbf{0}_{q\times q}, \tag{32}$$

so that any rotation of $A\prime$ must also satisfy this condition.

Now note that the *VAR(p)* can be written as $[I_q - C(B)B]\, v_t = e_t$. Premultipling both sides of equation (31) by the expression $[I_q - C(B)B]$ and simplifying, one obtains the restriction that:

$$(1 - x)I_q = [I_q - C(x)x]\, W(x)$$

for all values x. Evaluating this expression at $x = 1$, we find that:

$$[I_q - C(1)]\, W(1) = \mathbf{0}_{q\times q},$$

which shows that all the rows of $[I_q - C(1)]$ are in the space spanned by the basis A'. From equation (30), it is clear that all the $1 \times q$ vectors spanned by this basis must have zeros in their first q_0 columns, which implies that the first q_0 columns of the matrix $[I_q - C(1)]$ must be composed of zeros. In turn, this requires that the first q_0 columns of $C(1)$ are identical to the corresponding columns of the identity matrix I_q.

[53] We assume the normality condition that the sequence $\{sW_s\}_{s=0}^{\infty}$ is absolutely summable.

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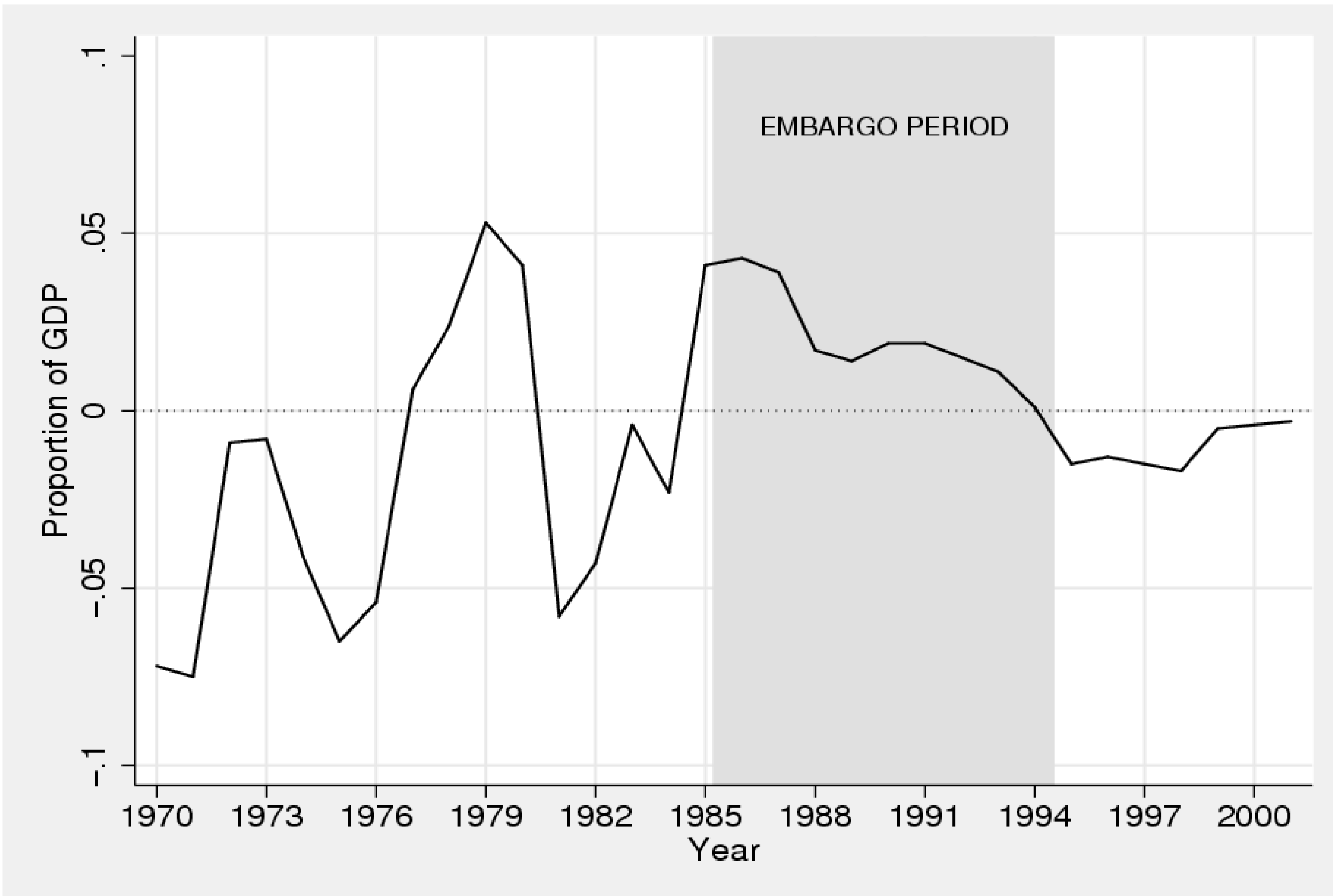

Figure 1: Ratio of Current Account to Gross Domestic Product for South Africa, 1970 to 2001.

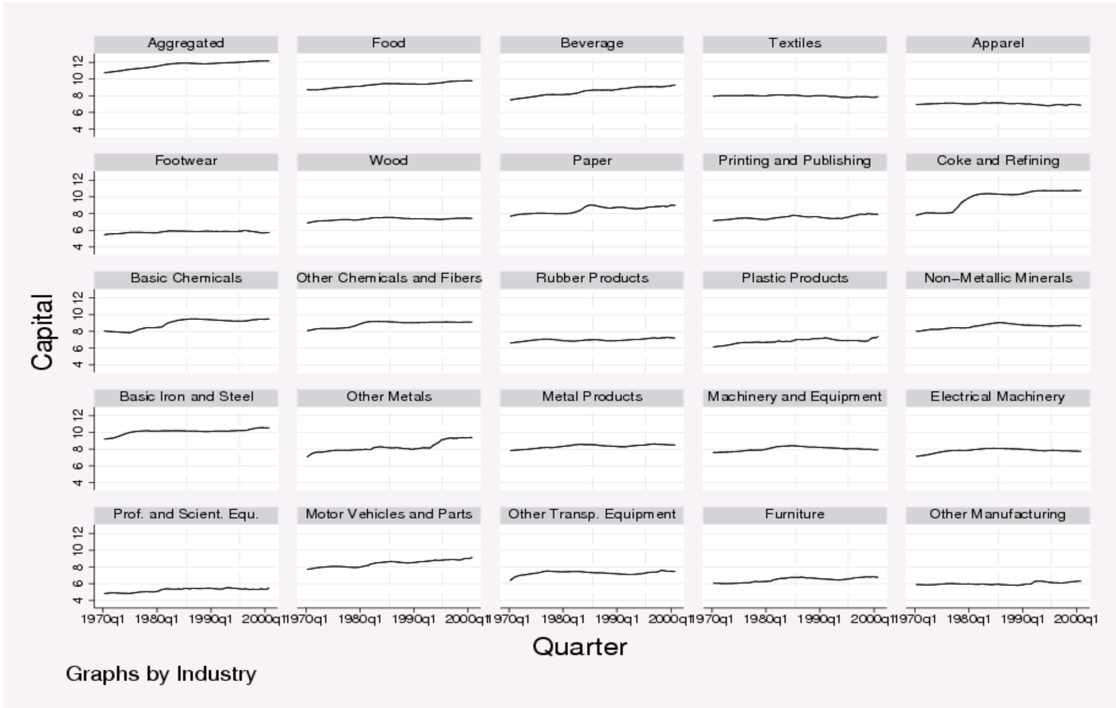

Figure 2: Ratio of real capital to output for twenty four South African manufacturing industries, 1970Q1 to 2000Q4.

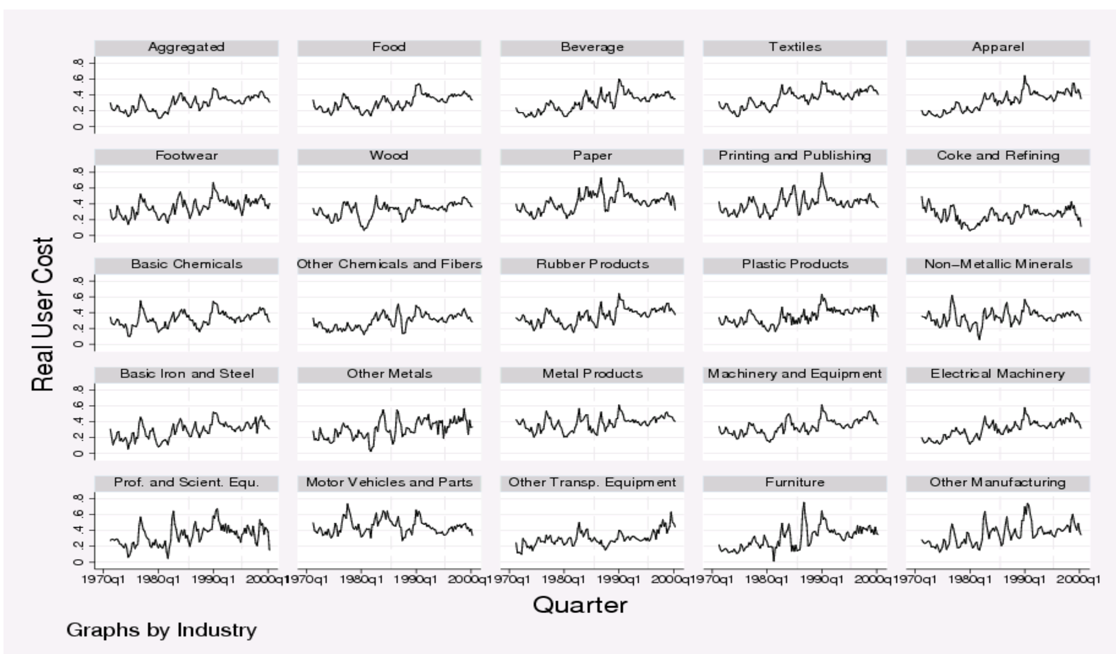

Figure 3: Real user cost for twenty four South African manufacturing industries, 1970q1 to 2000q4.

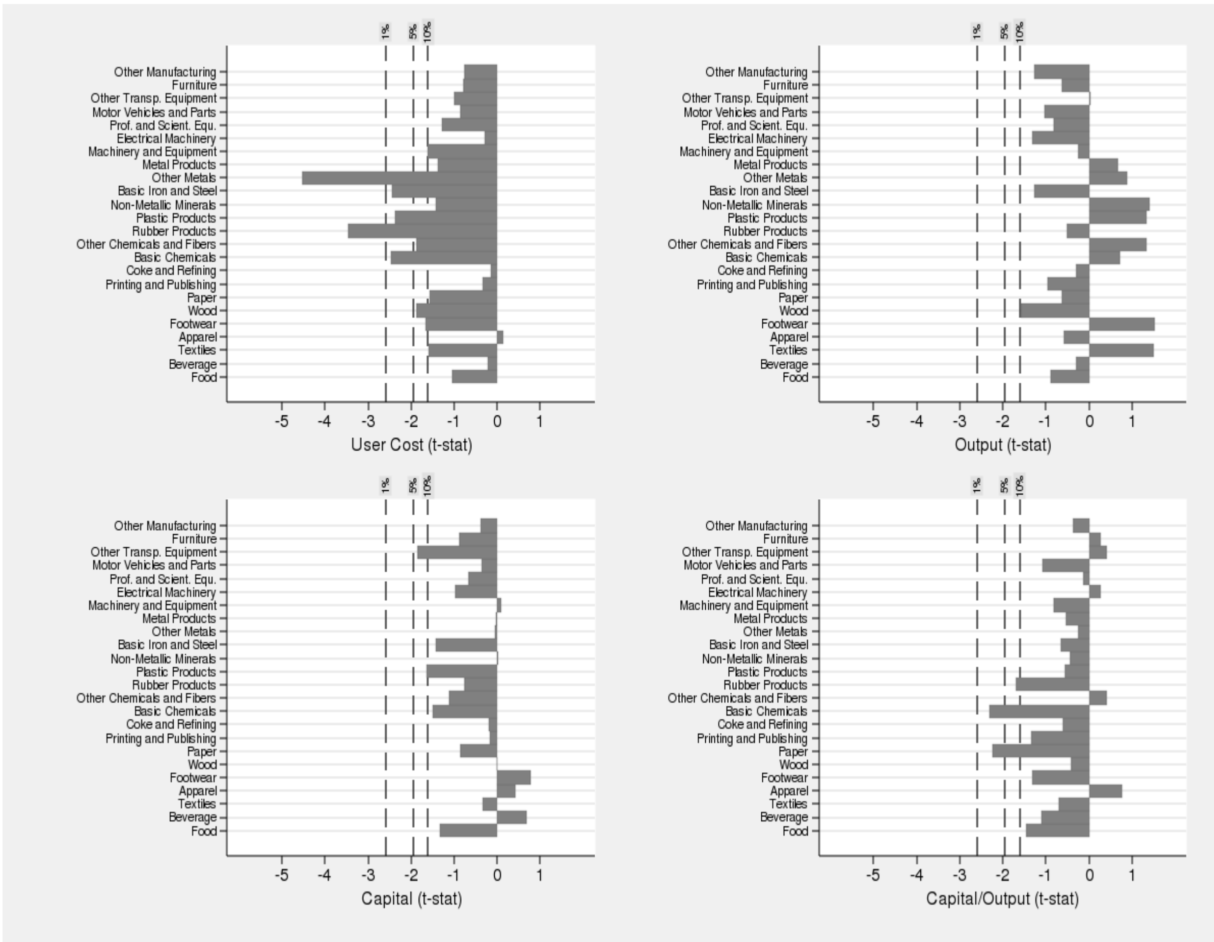

Figure 4: Bars for each industry denote test statistics for a Dickey-Fuller GLS test, where we control for aggregate effects using a preliminary regression. The number of included lag correction terms was chosen by the lag selection criterion in Ng and Perron [2001].

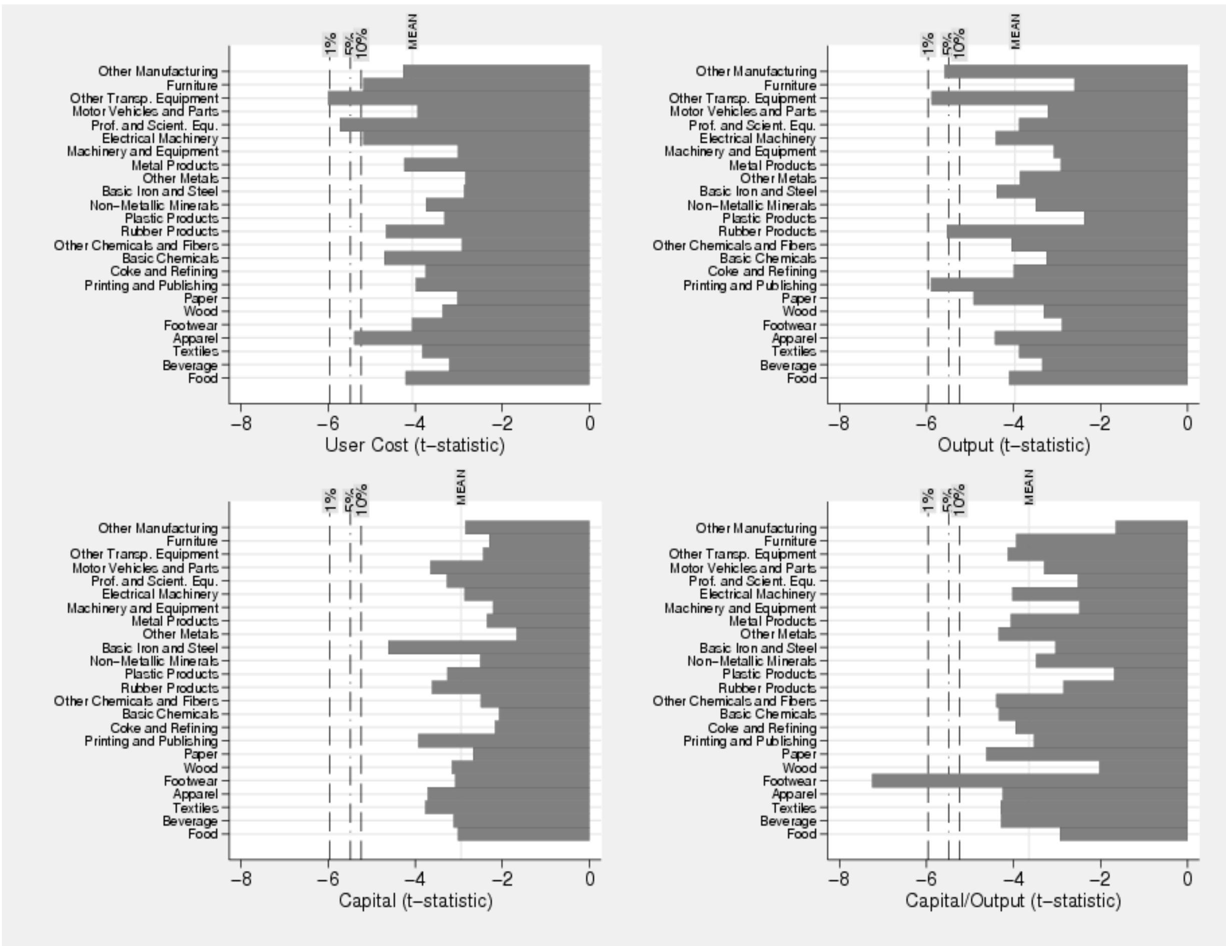

Figure 5: Bars for each industry denote test statistics for a Clemente-Montanes-Reyes additive outlier test, which maintains the null of a unit root and allows for two structural breaks.

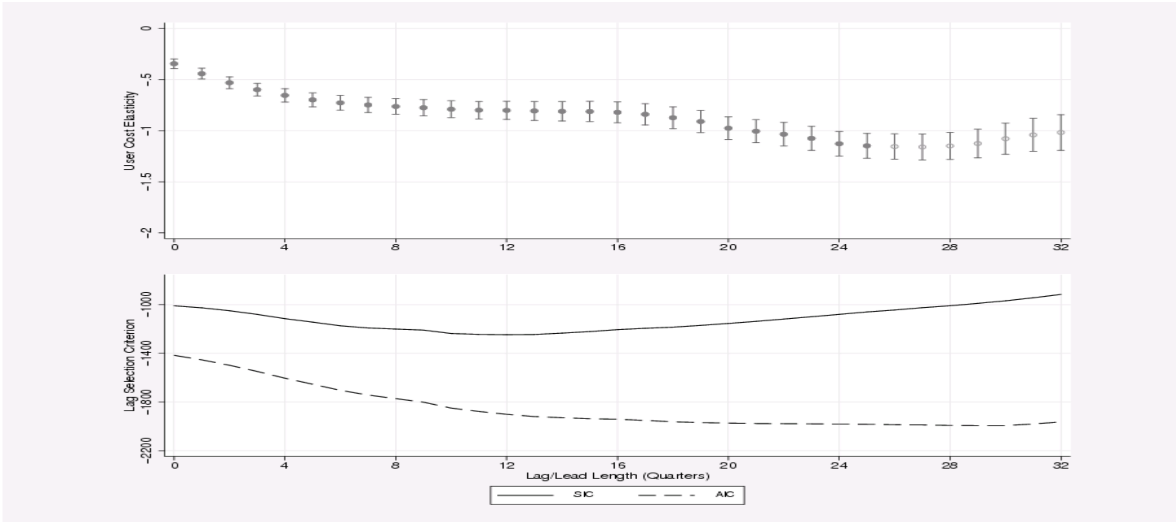

Figure 6: Sensitivity of user cost elasticity estimates from homogeneous panel DOLS to number of included lags and leads, for the I(1) subsample. The top panel show the user cost elasticity estimate for given number of DOLS leads/lags. The bottom panel shows the Information criteria for each lag/lead specification.

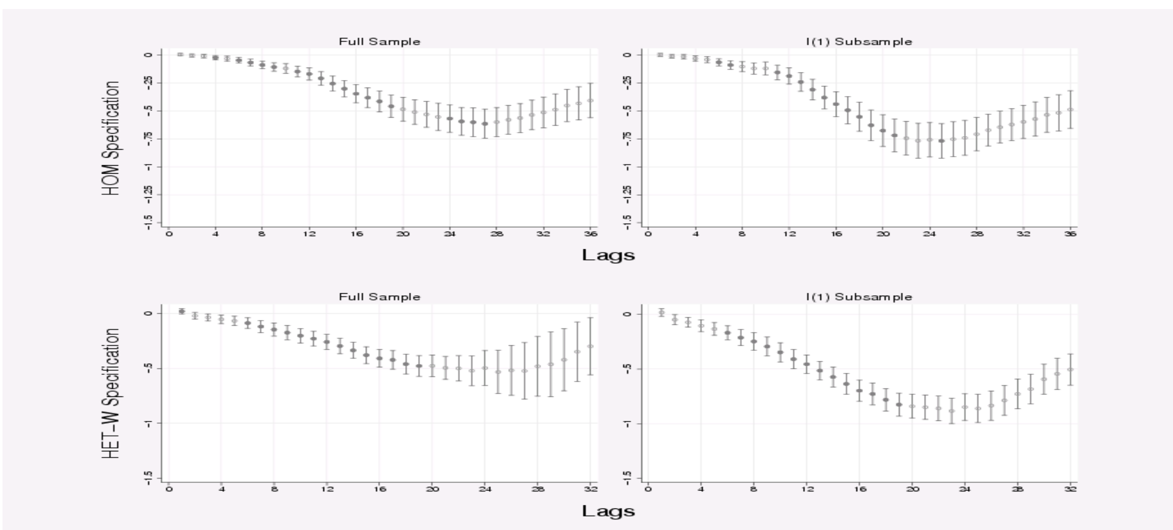

Figure 7: Long-run user cost elasticity estimate from the HOM and HET-W distributed lag specifications as a function of the number of included lags. Dots denote the point estimate at each lag, and bars denote a 95 percent confidence interval. Shaded dots denote that a t-test for the joint significance of the last included lag of the user cost is significant at 10 percent.

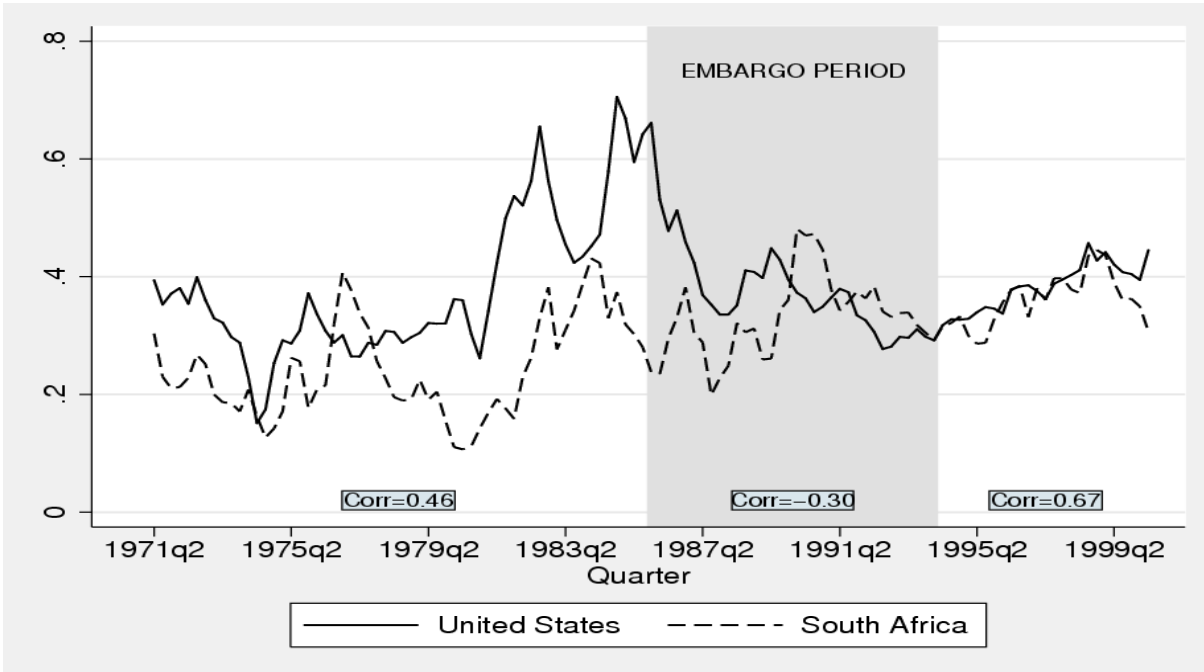

Figure 8: Comparative analysis of the user cost elasticity for the United States and South Africa.

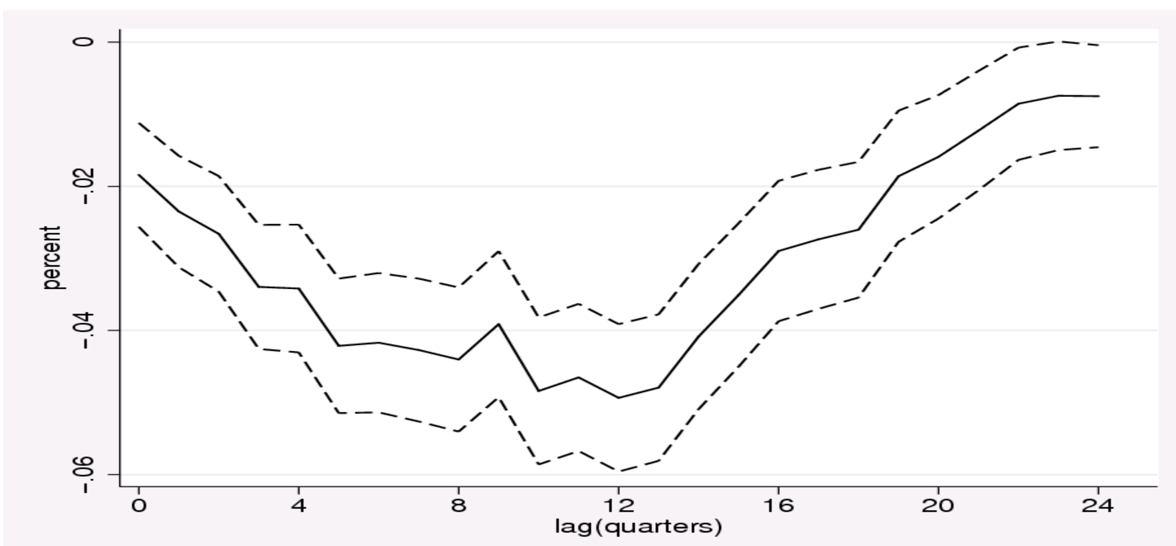

Figure 9: Estimated marginal response of capital to a 1 percent increase in the user cost with 95 percent confidence band.

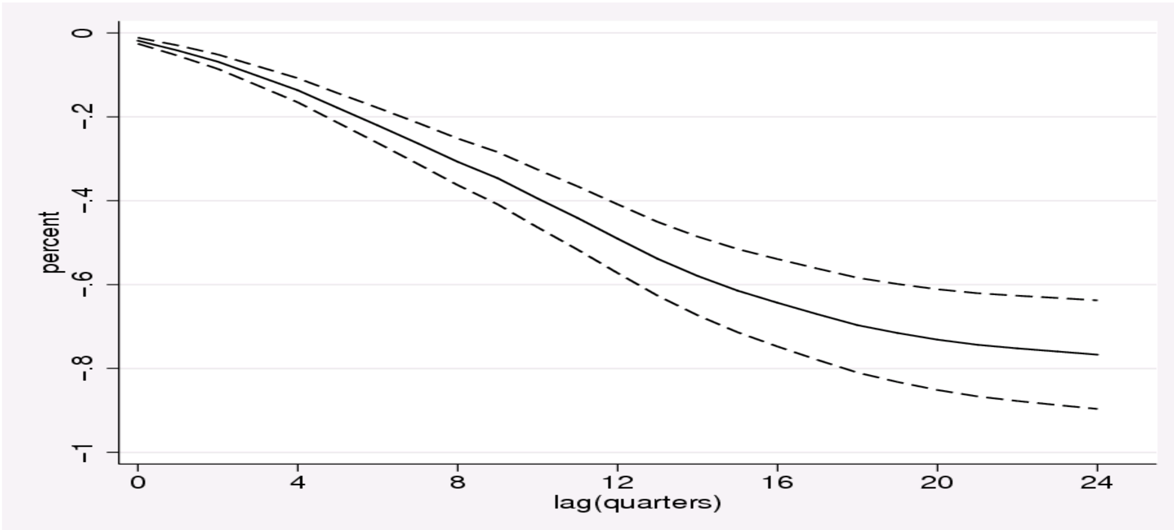

Figure 10: Estimated cumulative response of capital to a 1 percent increase in the user cost with 95 percent confidence band.

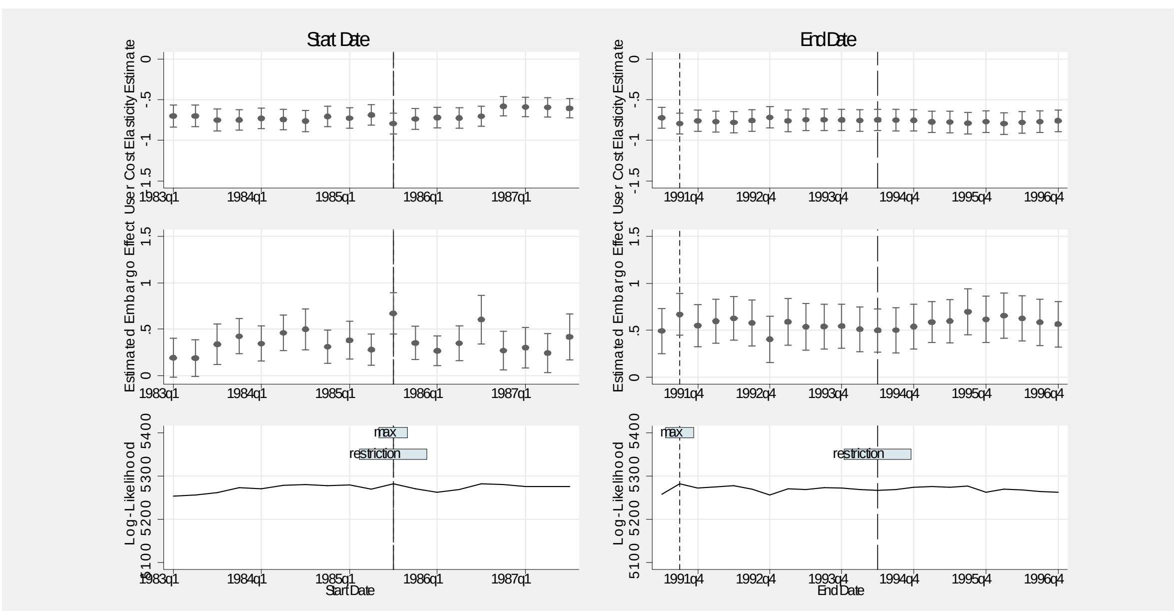

Figure 11: Sensitivity of results from embargo specification to beginning and end dates of the embargo, for specifications that are restricted so that the long-run capital-output elasticity is one. Left panels show sensitivity to the begin date, while right panels show sensitivity to the ending date. Top panels: Estimates of corrected user user cost elasticity with 95 percent confidence interval. Middle panels: Estimated effect of embargo on elasticity with 95 percent confidence interval. Bottom panels: Log-likelihood for given start or end date.

Table 1: Panel Unit Root and Stationarity Tests

Dataset	H_0	H_A	User Cost	Output	Capital	$\frac{\text{Capital}}{\text{Output}}$
Full Sample	All I(1)	At least one I(0)	0.093	0.485	1.000	0.924
	All I(0)	All I(1)	0.000	0.000	0.000	0.000
I(1) Subsample	All I(1)	At least one I(0)	0.363	0.101	1.000	0.903
	All I(0)	All I(1)	0.000	0.000	0.000	0.000

For each sample, the top line reports the p-value for the null hypothesis of a unit root using a version of the CADF panel unit root test devised in Pesaran (2007). In our test, we constructed a "Ztbar" statistic by taking the average of normalized t-statistics across industries, using simulated first and second moments. Asymptotically, this statistic is normally distributed under the null (see Im, Pesaran and Shin [2007]) , a fact that we use to compute the p-value under the null.. Results are not meaningfuly affected when we instead use the critical values in Pesaran[2007]. In an alternative specification, we include a "Trend" to allow for trend-stationary under the alternative hypothesis, doing so does not affect our tests results. These tests are robust for cross-sectional dependence and serial correlation. The number of included lagged differences for each industry was chosen by a sequential t-test criterion in which lags were dropped until that lag was significant at 10 percent for more than 10 percent of the industries. The bottom line for each dataset reports the p-value for the Hadri (1999) panel test, which maintains the null of stationarity. All tests control for fixed and aggregate effects. The Pesaran test is robust for serial correlation and generic forms of cross-sectional error correlation. The results of the Hadri test are robust to serially correlated errors and heteroskedasticity.

Table 2: Panel Cointegration Tests (p-value under H_0 of No Cointegration)

Test	*Full Panel*	*I(1) Panel*
HOM: Pooled DOLS		
panel t-stat (p)	0.000	0.000
group t-stat (p)	0.000	0.000
HET: Group-Mean DOLS		
panel t-stat (p)	0.000	0.000
group t-stat (p)	0.000	0.000

Listed p-values are for the null of no cointegration for the statistics described in Pedroni(2001). For the *homogeneous* specification, residuals are from a cointegrating vector that was fitted using a pooled DOLS specification that included 25 leads and lags of the first-difference of the independent variables. The *heterogeneous* specifications were estimated using a group-mean DOLS estimator that included 8 leads and 16 lags of the first-differenced independent variables. All estimators control for group fixed effects and and time effects. The results of these tests were unchanged when we restricted the output elasticity to unity.

Table 3: Estimates from Panel Cointegration Specifications

Long Run Elasticity of Capital to	*Full Panel*	*I(1) Panel*
HOM: Pooled DOLS		
User Cost	-0.965	-1.000
	(0.064)	(0.072)
Sample Size	$1,560$	975
HET: Group Mean DOLS		
User Cost	-0.536	-0.845
	(0.039)	(0.056)
Sample Size	$2,208$	$1,380$

Pooled DOLS: Specifications include 25 leads and lags of changes in output and the user cost, along with current values. Standard errors are robust for cross-sectional correlation in the error term and autocorrelation. *Mean-Group DOLS:* Specifications include 8 leads and 16 lags of the first-differences in the user cost and output. Standard errors are robust to cross-sectional correlation and autocorrelation. All specifications control for fixed and aggregate effects.

Table 4: Estimates from OLS Panel Regression using Difference Specification (robust standard error)

Long Run Elasticity of Capital to	*Full Panel*		*I(1) Panel*	
	HOM	*HET-W*	*HOM*	*HET-W*
User Cost ($N^u(1)$)	-0.615	-0.477	-0.767	-0.826
	(0.067)	(0.048)	(0.079)	(0.054)
Lags Included	27	19	25	19
Sample Size	2,160	2,160	1,350	1,350

All regressions include the contemporaneous observation of the dependent variable and lags of the independent variables in addition to industry fixed effects,and aggregate effects. In all specifications, the long-run capital output elasticity is constrained to be one. The estimates denoted "HOM" restrict so that all industries have the same elasticity. The "HET-W" estimates separate user cost elasticities for each industry, then forms an estimate of the aggregate elasticity that weights each industry's estimate by its average share of the total nominal capital stock over the sample period. The number of included lags in each specification was determined using a sequential T test procedure in which the lag length was successively shortened until the coefficient(s) on the last included lag(s) was (were jointly) significant. Standard errors are robust to both cross-sectional correlation and heteroskedasticity in the residuals.

Table 5: Estimates from OLS Panel Regression using Difference Specification (robust standard error)

Long Run Elasticity of Capital to	Period	*Full Panel*	*I(1) Panel*
User Cost	Non-Embargo ($N^u(1)$)	-0.749	-0.858
		(0.074)	(0.085)
	Embargo ($N^u(1) + M^u(1)$)	-0.250	-0.265
		(0.108)	(0.134)
	Difference ($M^u(1)$)	0.499	0.593
		(0.115)	(0.142)
	Lags Included	27	27
	Sample Size	2,160	1,350

All regressions include the contemporaneous observation of the dependent variable and 27 lags, along with fixed industry fixed effects, and aggregate effects. The number of included lags was determined by performing a sequential T-test procedure for the entire dataset. The robust standard errors account for both cross-sectional correlation and heteroskedasticity in the residuals. All specifications are restricted so that the long-run capital output elasticity is one. Standard errors were calculated using the entire covariance matrix for all estimated parameters.