Monetary Policy in a Forward-Looking Input-Output Economy

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Monetary Policy in a Forward-Looking Input-Output Economy*

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Abstract

This paper examines the implications for monetary policy of sticky prices in both final and intermediate goods in a New Keynesian model. Both optimal policy under commitment and discretionary policy, which is the minimization of a simple loss function, are studied. Consumer utility losses under alternative simple loss functions are compared, including their robustness to model and shock misperceptions, and parameter uncertainty. Targeting inflation in both consumer and intermediate goods performs better than targeting a single price index; price-level targeting of both consumer and intermediate goods prices performs significantly better. Moreover, targeting prices in both sectors yields superior robustness properties.

Keywords: Inflation Targeting, Price-level Targeting, Intermediate Goods

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1 Introduction

Recent years have witnessed significant innovations in monetary policy as many central banks have adopted inflation targeting as their objective. Bernanke et al. (1999) and King (2005) have noted that, in general, those countries that have adopted inflation targeting were able to draw down inflation while experiencing economic performance that was favorable. These favorable results have motivated researchers to ask a number of questions regarding the design of monetary policy regimes, including: What goals should the government assign the central bank? What are the potential benefits if central banks were to commit to future actions based on current or future shocks to the economy? If such commitments are not realistic, how can policy be designed in order to closely approximate a commitment regime? How should policy be judged?

Researchers have made extensive use of New Keynesian macroeconomic models to study issues in monetary policy design. Most models assume that one production sector is subject to imperfect competition and sticky prices. These models can be seen as essentially equivalent to one-sector models in which a large number of differentiated final goods are produced by firms engaging in monopolistic competition.\footnote{Often these models will label the differentiated goods as intermediate inputs that are then combined into one composite good. The composite good is sold by a perfectly competitive firm that charges the "price index" of the intermediate goods prices. This can be seen as equivalent to final goods that enter into the household’s utility function via the linearly homogeneous aggregator, and so is essentially the same as a one-sector model.} Within these models, the general conclusion has emerged that the central bank should seek to stabilize inflation in order to reduce household utility losses that occur over the business cycle.

When these policy studies are applied to the real world, inflation is typically identified with a change in some form of a consumer price index (CPI). However, one might ask: Is something like the CPI the best price index in guiding monetary policy? To pursue this matter further, Mankiw and Reis (2003) examine how characteristics of different sectors of the economy such as size, sensitivity to cyclicality, and sluggishness of price adjustment should enter into the price index of an inflation targeting central bank that wants to stabilize economic activity. They find that to reach this objective, a central bank may need to place a greater emphasis on nominal wages. This complements the studies by Ercog, Henderson, and Levin (2000) and Givens (2006), who also find
that nominal rigidities in the labor market should lead central banks to target wages.

Huang and Liu (2005) develop a New Keynesian input-output model in which intermediate goods are used solely to produce consumption goods. Prices in both sectors are sticky. They show that including the inflation rate of intermediate goods prices improves the performance of simple instrument rules. However, many questions remain regarding monetary policy in a New Keynesian input-output model, such as: What are the characteristics of optimal policy in such an environment? Do policy recommendations from simpler one-sector models perform well in the more complex model? How should policy be designed if commitment to a loss function or instrument rules is not possible? Can discretionary policy be designed so that it approximates optimal policy? If there is uncertainty about parameter values characterizing the different sectors, how much of a difference do different central bank loss functions make?

I use the model from Huang and Liu (2005) with some minor extensions to examine in greater detail the characteristics of optimal commitment policies and the design of discretionary monetary policy when the input-output structure is characterized by sticky prices at both stages of production. I employ a second-order approximation to the household’s utility function as a natural criterion by which to judge alternative policy prescriptions within the model. Under a parameterization consistent with the modern U.S. economy, I first study the equilibrium paths of key macroeconomic variables when a central bank can commit to a loss function based on the household’s utility function. Compared to an otherwise equivalent sticky-price one-sector model, commitment in this two-sector model leads to a richer understanding of how policy makers should respond to shocks.

Next, under the more realistic assumption that central banks cannot commit to future policy actions or rules, I consider policy regimes that are pursued under discretion. I characterize monetary policy regimes by the loss function that they are assigned to minimize. I consider loss functions that include inflation targets and price-level targets in addition to other variables. I judge the performance of a regime by its effects on a model-consistent household loss function. I find that regimes that focus solely on CPI-based targets perform poorly. On the other hand, regimes that target price indices in both sectors perform the best among alternative inflation-targeting and price-level targeting regimes.
In related work on price-level targeting, Vestin (2006) shows that assigning a simple loss function that stabilizes a single price level like the CPI nearly replicates the commitment solution. In benchmark version of the richer input-output model, CPI price-level targeting no longer replicates the commitment solution in a standard one-sector New Keynesian model. Not only does a CPI price-level targeting regime perform worse than other price-level targeting regimes, its performance is nearly equaled by targeting inflation in both sectors instead of price levels. Furthermore, I show that a CPI price-level targeting regime that is calibrated in a one-sector world and then run in the true input-output world performs worse than the best inflation targeting regime.

I also show that targeting price indices in both sectors endows the regime with superior robustness properties. If there are potential misperceptions about the sources of shocks, a price-level targeting regime that includes prices of both sectors is very robust across a wide range of specifications of the sources of true shocks. I also show that regimes that only target one of the price indices perform much worse if actual parameter values differ from the assumed values when the loss function is set. Regimes that target both price indices perform very well over a wide range of parameter values that differ from the ones used in setting the initial policy regime.

The remainder of the paper proceeds in the following manner. Section 2 sets up the model used for the analysis. Section 3 examines the characteristics of the optimal policy under commitment within the model. Section 4 carries out a detailed analysis and comparison of various discretionary regimes. Section 5 examines the robustness properties of the different discretionary policy regimes. I examine the implications for policy of assuming a simple one-sector model, of misperceiving the sources of shocks, and parameter uncertainty. Section 6 considers the performance of various policy regimes under economies that differ from the benchmark economy in the importance of the intermediate goods sector. Section 7 concludes.

2 Model of an Input-Output Economy

The model I employ is in the tradition of New Keynesian models. I follow Huang and Liu (2005) in characterizing the production side of the economy as a production chain with sticky prices
throughout. Specifically, differentiated final goods are produced using labor and a composite of differentiated intermediate goods. Both sectors exhibit sticky prices. However, I model the policy side of the economy very differently than in Huang and Liu (2005). I assume that the government assigns goals to an independent central bank in the form of an intertemporal loss function. The central bank acts to minimize this loss function.

2.1 Households

The economy is populated by an infinite number of identical households. These households maximize expected lifetime utility, which is given by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [u(C_t) - v(N_t)] \right\}.$$  \hfill (1)

Households take wages as given and are assumed to provide labor to all firms in the economy. In this paper I assume that $u(C_t) = \log(C_t)$. Log utility in consumption and additively separable preferences allow the model to be consistent with a balanced growth path. I also assume linear disutility in labor, which simplifies some of the derivations (it can also be seen as a proxy for indivisibilities in the labor supplied by households). The consumption good, $C_t$, is a Dixit-Stiglitz aggregate of a continuum of different non-durable final goods, $y_{ft}(i)$, given by

$$C_t \equiv \int_0^1 y_{ft}(i)^{\theta_{ft}/(\theta_{ft} - 1)} \, di^{\theta_{ft}/(\theta_{ft} - 1)}$$  \hfill (2)

where $\theta_{ft}$ is the time-varying elasticity of substitution (assumed always to be greater than one). I assume complete financial markets and the necessary transversality conditions so that households face identical lifetime budget constraints and well defined optimization problems.

\footnote{Due to this model’s similarity with Huang and Liu (2005), I present the model setup here and present the complete description and derivation in a technical appendix that is available upon request. One of the main differences in the production side of the model in this paper lies in the addition of shocks that lead to cost-push shocks, which were absent in Huang and Liu (2005). Other studies have taken different approaches to modeling the input-output nature of production, including Long and Plosser, 1983, and Basu, 1995.}
2.2 Firms

On the production side there are two sectors, each consisting of a continuum of firms producing
differentiated goods. The first sector produces goods that households consume, while the second
sector produces intermediate goods that are needed to produce the final goods. All firms are price
takers in their input markets. I assume a competitive labor market in which one wage obtains in
the entire economy. Firms engage in monopolistic competition in each sector. I assume that firms
have the opportunity to adjust their prices via the standard Calvo framework.

2.2.1 Final Goods

The final goods firms have access to a constant-returns-to-scale Cobb-Douglas production function
with labor-augmenting technology,

$$y_{ft}(i) = Y_{mt}(i)\varphi \left(A_{ft}N_{ft}(i)^{1-\varphi}\right),$$

where $Y_{mt}(i)$ represents the amount of the composite intermediate good that firm $i$ purchases and
uses in period $t$. The labor-augmenting technology shock is given by $A_{ft}$, while the labor used
by firm $i$ is given by $N_{ft}(i)$. This composite intermediate good is related to the differentiated
intermediate goods via a Dixit-Stiglitz aggregator,

$$Y_{mt}(i) \equiv \left[\int_{0}^{1} y_{mt}(i, j)^{(\theta_{mt}-1)/\theta_{mt}} dj\right]^{\theta_{mt}/(\theta_{mt}-1)},$$

where $\theta_{mt} > 1$ is the time-varying elasticity of substitution of differentiated intermediate goods,
and $y_{mt}(i, j)$ is the amount of differentiated intermediate good $j$ demanded by firm $i$.

Final goods firms can adjust their prices with probability $1 - \alpha_f$ each period. When they are
able, they set a price that maximizes expected discounted profits. Each firm meets the demand for
its goods given its stated price. The markup that firms charge varies as the elasticity of substitution
varies. This can be seen as a proxy for any change in the economic environment that affects the
markup that monopolistic firms could charge above their marginal cost in a hypothetical flexible
price environment. For example, if households were to substitute differentiated goods for one another more readily due to preference changes, increased varieties of goods, or any other reason, firms would experience a shock on the markup that they could charge over their marginal costs.\(^3\) The profit-maximization problem can be represented as

\[
\max_{P_{ft(i)}} E_d \left\{ \sum_{s=i}^{\infty} \alpha_f^{s-t} Q_{ts} [P_{ft(i)} (1 + \tau_f) - V_{fs}(i)] y_{fs}^d(i) \right\}, \tag{5}
\]

where \(\tau_f\) is a subsidy to final-goods producers, \(V_{fs}(i)\) is the marginal cost of production in period \(s\), and \(y_{fs}^d(i)\) is the total demand for firm \(i\)'s output in period \(s\). When the firm has the option to reset its price, it maximizes its objective function by choosing an appropriate \(P_{ft(i)}\).

### 2.2.2 Intermediate Goods

The intermediate goods firms face an optimization problem similar to the one faced by final goods producers. Intermediate goods firms have access to a constant-returns-to-scale production function of the form

\[
y_{mt}(j) = A_{mt} N_{mt}(j), \tag{6}
\]

where \(A_{mt}\) represents the labor-augmenting technology shock and \(N_{mt}(j)\) represents the labor used by firm \(j\). They adjust their prices each period with a probability of \(1 - \alpha_m\). The interpretation of the sources of the elasticity of substitution (and hence markup) shocks is slightly different for intermediate goods, given that they do not directly respond to household preferences. The ease with which differentiated intermediate goods can be substituted for one another arise from a number of sources. For example, if final goods producers are able to more easily substitute the differentiated intermediate goods for one another in the production process, this can be seen as a different form of a technology shock. Alternatively, changes in the business structure of the industry could affect monopoly power enjoyed by the firms. Once again, in addition to allowing for these sources of fluctuations, a stochastic elasticity of substitution introduces cost-push shocks into the reduced-form

\(^3\) Additionally, this allows for a distinct effect on inflation in the form of a cost-push in a reduced-form Phillips curve, which is often assumed in models that consider only one source of nominal rigidities.
Phillips curve of the intermediate goods sector. The profit maximization problem for intermediate goods firms is similar to that of final goods firms given in (5).

2.3 Government

The government serves two purposes in this model. First, it assigns a loss function to an independent central bank. The central bank sets the nominal interest rate to achieve its assigned goals. I assume that the central bank can react to and affect state variables in the current period. I solve for Markov Perfect equilibria under the assumption that the central bank does not engage in complicated strategies to support other possible equilibria in a dynamic game with households.

The second task of the government is to tax households and provide subsidies to firms so that the steady-state equilibrium is not distorted due to the inefficiencies arising from monopolistic competition. I assume that the government does this via lump-sum taxes and always maintains a balanced budget. A natural extension of this setup is to study how more realistic fiscal and monetary policies can affect one another in this environment. This interaction may affect the loss function that the government would assign to the central bank. I ignore these possible complications for the remainder of the paper and leave their study for future research.

2.4 Reduced-Form Model

I log linearize the model using log deviations from a hypothetical efficient equilibrium (the equilibrium that would obtain if prices were flexible and there were no elasticity-of-substitution shocks). I define the natural rates or values as those values that would arise in the efficient equilibrium. The log-linear approximation to the model will be valid as long as shocks to the system are sufficiently small relative to the size of the state variables. Table 1 lists the key variables and symbols.

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4This is not an innocuous assumption. Svensson and Woodford (2005) show that if current state variables are completely predetermined, the central bank actually minimizes a loss function that features projections of the target variables.

5Dixit and Lambertini (2003) and Lambertini (2006) show that the coordination issues between monetary and fiscal authorities are not trivial. Many other studies have shown that effective monetary policy depends on concurrent actions by the fiscal authority (e.g. see Woodford, 2001, Benigno and Woodford, 2003, and Sims, 2005).

6The details of the log linearization of the model in this paper are given in the technical appendix, which is available upon request. Huang and Liu (2005) provide details of the log linearization of their model, which is similar to mine.
### Symbol | Meaning
--- | ---
$\pi_{kt}$ | inflation in sector $k$, $k \in \{f, m\}$ (final, intermediate)
$\tilde{c}_t$ | output gap (sticky relative to natural rate)
$\tilde{q}_t$ | relative price gap of intermediate to final goods
$q_t^*$ | relative price level in the efficient equilibrium
$i_t$ | nominal interest-rate deviation; $\log[(1 + i_t) / (1 + \bar{i})]$ 
$\hat{r}_t^*$ | log deviation of natural real interest rate
$u_{kt}$ | cost-push shock in sector $k$
$\hat{a}_{kt}$ | technology shock in sector $k$
$\rho_k$ | autocorrelation of technology shock in sector $k$
$\varphi$ | measure of intmd. goods importance in production
$\beta$ | subjective time discount factor
$\sigma^{-1}$ | intertemporal elasticity of substitution in consumption
$\alpha_k$ | probability that a firm in $k$ keeps its previous price
$\theta_{kt}$ | elasticity of substitution between differentiated goods in $k$
$\theta_k$ | steady-state value of $\theta_{kt}$
$E_t x_{t+1}$ | the expected value of $x_{t+1}$ at time $t$

#### Table 1: List of Symbols

The intertemporal IS equation is obtained from the household’s first-order condition. Its log-linearized version is given by

$$
\tilde{c}_t = E_t \tilde{c}_{t+1} - \frac{1}{\sigma} (\hat{u}_t - E_t \pi_{f,t+1} - \hat{r}_t^*),
$$

(7)

where $\hat{r}_t^* = -[\varphi (1 - \rho_m) \hat{a}_{mt} + (1 - \varphi) (1 - \rho_f) \hat{a}_f]$. I assume that the technology shocks are stationary ($|\rho_k| < 1$ for $k \in \{f, m\}$) and evolve according to

$$
\hat{a}_{k,t+1} = \rho_k \hat{a}_{kt} + \sigma_{ak} \epsilon_{k,t+1},
$$

(8)

where $\epsilon_{k,t+1}$ represents a white noise process that is uncorrelated with all other stochastic variables (its variance is normalized to unity). I interpret $\tilde{c}_t$ as analogous to the output gap in standard one-sector models. This is valid since only households purchase final goods and services, which implies that final output equals consumption. The Phillips curves for the two sectors are given by

$$
\pi_{ft} = \beta E_t \pi_{f,t+1} + \kappa_f (\varphi \tilde{q}_t + (1 - \varphi) \sigma \tilde{c}_t) + u_{ft}
$$

(9)

9
and

$$
\pi_{mt} = \beta \mathbb{E}_t \pi_{m,t+1} + \kappa_m (\sigma \tilde{c}_t - \tilde{q}_t) + u_{mt}.
$$

(10)

The cost-push shocks, $u_{kt}$ for $k \in \{f, m\}$, are reduced-form expressions of fluctuations in the time-varying elasticities of substitution of the differentiated goods. I assume that each cost-push shock is given by

$$
u_{kt} = \sigma_{uk} \eta_{kt}, \ k \in \{f, m\},
$$

(11)

where $\eta_{kt}$ represents a white noise process that is uncorrelated with all other stochastic variables (its variance is normalized to unity). The term $\varphi \tilde{q}_t + (1 - \varphi) \sigma \tilde{c}_t$ is equivalent to the real marginal cost gap for final goods producers, while $\sigma \tilde{c}_t - \tilde{q}_t$ is equivalent to the real marginal cost gap for intermediate goods producers. The coefficients $\kappa_f$ and $\kappa_m$ are given by

$$
k_f = \frac{(1 - \alpha_f \beta) (1 - \alpha_f)}{\alpha_f},
$$

and

$$
k_m = \frac{(1 - \alpha_m \beta) (1 - \alpha_m)}{\alpha_m}.
$$

They emerge endogenously from the model and link current inflation to the current real marginal cost of the firms in their respective sectors. These slope coefficients imply that a higher subjective discount factor, $\beta$, or a higher level of price stickiness, $\alpha_k$, lowers the value of the slope coefficient as firms put less weight on their current marginal cost of production relative to future expected marginal costs. The relative price gap of the intermediate goods price index over the price index of final goods evolves, by definition, according to

$$
\tilde{q}_t = \tilde{q}_{t-1} + \pi_{mt} - \pi_{ft} + (1 - \varphi) (\Delta \hat{a}_{mt} - \Delta \hat{a}_{ft}).
$$

(12)

The final goods Phillips curve in this setting gives an indication of why the coefficient of the output gap in a fully specified, reduced-form empirical Phillips curves might be smaller than otherwise expected if the intermediate goods sector is important. The coefficient for this term in
the final goods Phillips curve, given by $\kappa_f (1 - \varphi) \sigma$, decreases with $\varphi$ (while the coefficient of the relative price term increases at the same time). Thus, in the input-output model, higher levels of importance of the intermediate goods sector have similar effects on the coefficient of the output gap as parameter values in a one-sector model that imply higher degrees of strategic complementarities (e.g., see Kimball, 1995, and Woodford, 2003).\footnote{By strategic complementarity I mean that if other producers increase the price they charge for their goods, any producer finds it optimal to charge a higher price for its own good. This has also been labeled as a real rigidity by Ball and Romer (1990).}

In order to judge households’ preferences over different policy options, I find it useful to use a second-order Taylor approximation to the household utility function. For the present model, this is given by:

$$L = \left( -\frac{1}{2} u_x (\mathbf{C}) \mathbf{C} \right) E_0 \sum_{t=0}^{\infty} \beta^t L_t^* + \text{t.i.p.} + O (\|\xi\|^3),$$

where

$$L_t^* = \sigma \tilde{r}_t^2 + \varphi (1 - \varphi) (\sigma \tilde{r}_t - \tilde{q}_t)^2 + \tilde{\theta}_f \kappa_f \pi^2_{ft} + \varphi \tilde{\theta}_m \kappa_m^{-1} \tilde{\pi}_{mt}^2,$$

"t.i.p" represents terms independent of policy, and $O (\|\xi\|^3)$ represents terms of third order or higher. The period loss function resulting from a second-order approximation to the representative household’s utility function in the input-output model contains additional terms besides the standard inflation and output gap terms.\footnote{For details of the derivation see the technical appendix for this paper, which is available from the author. Huang and Liu (2005) provide a derivation under the assumption of constant elasticities of substitution for differentiated goods in the two production sectors.} The interpretation of the loss function is straightforward. The first term confirms that the standard output gap affects household utility. The second term shows that, in addition to the output gap, deviations in the real marginal cost for intermediate goods firms affect the utility function of agents in the economy. Deviations in the real marginal cost of intermediate goods represent an inefficient allocation of labor across sectors of the economy. Inflation rates in the two sectors enter the loss function as they are linked to inefficient labor usage across firms within the respective sectors.

When solving the model I treat $\tilde{c}_t$ as the instrument of the central bank. This is an innocuous simplification since the interest rate does not appear in the objective function of the central bank and the IS equation is not a constraint in the central bank’s optimization problem. Since the
nominal interest rate can be uniquely determined through the IS curve, I obtain the solution in which the nominal interest rate is the policy instrument. I compute price levels using the identity, 
\[ p_{k,t+1} = p_{kt} + \pi_{k,t+1} \] for \( k \in \{ f, m \} \), where \( p_{kt} \) is the log-deviation in the price level from its initial value as determined in a steady state. I represent the structural equations of the economy as

\[
\begin{bmatrix}
X_{t+1} \\
HE_{t}y_{t+1}
\end{bmatrix} = A \begin{bmatrix}
X_{t} \\
y_{t}
\end{bmatrix} + B \hat{\eta}_{t} + \begin{bmatrix}
C \\
0
\end{bmatrix} \varepsilon_{t+1}
\] (15)

where

\[
X_{t} = \begin{bmatrix}
u_{ft} \\
u_{mt} \\
\hat{\alpha}_{ft} \\
\hat{\alpha}_{mt} \\
\hat{\alpha}_{f,t-1} \\
\hat{\alpha}_{m,t-1} \\
\hat{q}_{t-1}
\end{bmatrix}, \quad y_{t} = \begin{bmatrix}
\hat{q}_{t} \\
\pi_{ft} \\
\pi_{mt}
\end{bmatrix}, \quad \varepsilon_{t+1} = \begin{bmatrix}
\eta_{f,t+1} \\
\eta_{m,t+1}
\end{bmatrix}.
\]

The matrices \( A, B, C, \) and \( H \) are given in the appendix.

3 Optimal Policy with Commitment

In this section I study policy under the assumption that the central bank can be assigned a loss function that is derived from the household utility function, and that it can commit to future actions. This policy regime provides a benchmark with which to compare the results of discretionary monetary policy. Additionally, the characteristics of impulse responses to shocks and long-run outcomes under the commitment solution provide further means of judging the desirability of particular policy prescriptions.

In state-space form the policy period loss function is given by

\[
L_{t}^{*} = \frac{1}{2} Y_{t}' \Lambda Y_{t}
\] (16)
where

\[
Y_t = D \begin{bmatrix} X_t \\ y_t \\ \tilde{c}_t \end{bmatrix} = \begin{bmatrix} \pi_{ft} \\ \pi_{mt} \\ \tilde{c}_t \\ \sigma \tilde{c}_t - \tilde{q}_t \end{bmatrix},
\]

The matrix of loss coefficients, \( \Lambda^* = \text{diag}(\lambda_f^*, \lambda_m^*, \lambda_c^*, \lambda_e^*) \), is derived from (14): \( \lambda_f^* = \bar{f} \kappa_f^{-1} \), \( \lambda_m^* = \varphi \bar{m} \kappa_m^{-1} \), \( \lambda_c^* = \sigma \), and \( \lambda_e^* = \varphi (1 - \varphi) \). The matrix \( D \) is given in the appendix. With the period loss function defined as above, I write the intertemporal loss function as

\[
E_0 \sum_{t=0}^{\infty} (1 - \beta) \beta^t L_t^e.
\]

Without loss of generality, I have multiplied the loss function by \((1 - \beta)\) to convert the loss to a permanent period loss (meaning the loss that would be constant in every period forever).

Under commitment, the central bank solves the standard Lagrangian

\[
\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} (1 - \beta) \beta^t \left\{ L_t^e + \begin{bmatrix} \xi'_{t+1} \\ \nu_t' \end{bmatrix} \left( \begin{bmatrix} X_{t+1} \\ y_{t+1} \\ \tilde{c}_{t+1} \end{bmatrix} - \tilde{A} \begin{bmatrix} X_t \\ y_t \\ \tilde{c}_t \end{bmatrix} \right) \right. \\
- \begin{bmatrix} C \\ 0 \\ 0 \end{bmatrix} \xi_{t+1} + \left. \frac{1 - \beta}{\beta} \xi_0' (X_0 - \bar{X}_0), \right\}
\]

where I have used the law of iterated expectations to consolidate the Lagrangian. I have written the vector of Lagrangian multipliers relating to the forward-looking variables as \( \nu_t' \) in order to emphasize the point that these variables depend on information available at time \( t \). The Lagrangian multipliers for the predetermined variables, contained in \( \xi_{t+1}' \), are forward-looking variables and therefore depend on information revealed at time \( t + 1 \).
3.1 Benchmark Calibration of the Model

To complete the model setup, in this section I describe the parameter values that characterize the benchmark economy. The assumption of log utility in consumption implies that $\sigma = 1$. I set $\beta = 0.99$, which implies that the annual real interest rate in the steady state is approximately 4 percent, given that I interpret a time period to be a quarter. Consistent with Blinder et al. (1998) and Carlton (1986), I set the average price contract equal to a year, which means setting $\alpha_f = 0.75$ and $\alpha_m = 0.75$.\footnote{Though Bils and Klenow (2004) find a shorter length of contract for final goods prices, I have chosen to make the price stickiness equal in the two sectors, so as not to give either sector an apparent advantage of needing attention (see Aoki, 2001).} The steady-state elasticities of substitution for the differentiated goods, $\theta_f$ and $\theta_m$, are set to 10, which implies a steady-state markup of 11 percent.

The intermediate goods sector is the crucial feature of the paper’s analysis, and therefore it is important to be careful in setting $\varphi$, the parameter that indexes its importance. Given that I consider a constant-returns-to-scale Cobb-Douglas production function, I can interpret $\varphi$ as the share of total input costs of intermediate goods. Jorgenson et al. (1987) and Hornstein and Praschnik (1997) examine the role of intermediate inputs in the production in final goods. Their results suggest that $\varphi$ could be reasonably interpreted as falling in the interval $[0.4, 0.75]$. For the benchmark calibration, I assume that $\varphi = 0.6$, which puts the benchmark economy in the middle of the range of reasonable values.\footnote{Huang and Liu (2005) also set $\varphi = 0.6$ in their analysis. Basu (1995) argues for higher values, but choosing 0.6 allows me to avoid the criticism that I have weighted the calibration to favor my results.} Following the analysis of the benchmark economy, I examine the consequences of setting $\varphi$ to be greater or lower than this value.

I set the AR(1) coefficients for technology shocks to $\rho_f = \rho_m = 0.95$. The standard deviations of the innovations to productivity shocks are set to 0.02. These values are consistent with empirical studies that show productivity shocks to be small, but highly persistent (for example, see Cooley and Prescott, 1995, Kim, 2000, and Huang and Liu, 2005). There is less agreement on the characterization of cost-push shocks. For simplicity, I assume that the cost-push shocks are purely transitory by setting $\rho_{uf} = \rho_{um} = 0$, and set the standard deviations of their white noise shock components to 0.02. Considering purely transitory shocks simplifies the analysis and allows for
an easy comparison with other purely forward-looking studies. My calibration of the standard deviation of the cost-push shocks is similar to Walsh (2003b) and Jensen (2002).

3.2 Optimal Policy under Commitment in the Benchmark Economy

The equilibrium dynamics following a one-standard-deviation shock to productivity in the final goods sector are given in Figure 1, while the dynamics after a similar shock in the intermediate goods sector are given in Figure 2.

![Diagram of equilibrium dynamics](image)

**Figure 1: Impulse Response to a Final Goods Productivity Shock**

The bottom panel of Figure 1 shows the behavior of the real interest rate gap and output gap, both of which indicate the stance of monetary policy. Contractionary monetary policy is undertaken in response to the productivity shock as indicated by the positive real interest rate gap. The contractionary policy move is accompanied by a decrease in the absolute level of the

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11 This also allows me to avoid the issue of the source of persistent inflation. One popular method to account for persistent inflation is to make cost-push shocks correlate over time (for example, Givens, 2006). Others, such as Walsh (2003b) and Jensen (2002) make the shocks transitory, but build in persistence through firm behavior.
equilibrium nominal interest rate, which slowly rises thereafter.

The second panel of Figure 1 shows the behavior of inflation in the two sectors in response to a productivity shock in the final goods sector. The productivity shock induces inflation rates in each sector to move in opposite directions. The real marginal cost gaps in the two sectoral Phillips curves open a window to the transmission mechanism for shocks affecting inflation in the two sectors. As shown in (12), the productivity shock directly affects the relative price gap. Equations (9) and (10) show that the relative price gap pushes the real marginal cost gaps, and hence inflation, in opposite directions in the two sectors. Furthermore, the intermediate goods sector is more sensitive to productivity shocks than is the final goods sector, and therefore intermediate goods inflation will be greater in magnitude unless the central bank acts to alleviate this effect.

![Graph showing impulse response to an Intmd. Goods Productivity Shock](image-url)

Figure 2: Impulse Response to an Intmd. Goods Productivity Shock

The central bank must account for the different sensitivities of the two sectors to the shock and to the nominal interest rate in order to bring about the optimal path of the nominal interest rate. After a productivity shock in the final goods sector, the central bank relieves the excess
pressure on the intermediate goods sector through contractionary policy, inducing slightly greater deflation in the final goods sector, and less inflation in the intermediate goods sector. The balancing of inflation across the two sectors allows for a more efficient distribution of resources within and across the sectors. The third panel shows that in the long run the price levels are brought back to their original levels. The price level’s long-run neutrality provides a reason to believe that some form of price-level targeting in this environment may prove to be optimal under discretion.

Figure 3: Impulse Response to a Final Goods Cost-Push Shock

Figure 2 shows that a one-standard-deviation shock to the productivity of the intermediate goods sector leads to similar conclusions; one must simply reverse all of the directions in which the variables move with one notable exception. The model predicts that under optimal policy the central bank initially decreases the nominal interest rate regardless of the sector from which the productivity shock arises. The nominal interest rate decreases after productivity shocks in both sectors despite the fact that policy is contractionary following a final goods productivity shock and expansionary following an intermediate goods productivity shock. The seemingly disjoint behavior
between the nominal interest rate and the policy stance occurs because the equilibrium real interest rate always falls below its steady state value following productivity shocks, though by different amounts depending on the sector in which the shock arises.

Figure 3 displays the equilibrium dynamics following a cost-push shock in the final goods sector. If the central bank were to engage in neutral policy (zero real-interest-rate gap), both sectors would experience inflation; however, inflation in the final goods sector would be extremely high. The central bank engages in contractionary policy to reduce inflation in both sectors, balancing the higher sensitivity of inflation in the intermediate goods sector to the nominal interest rate with the large initial effect on final goods inflation from the cost-push shock.

Figure 4 shows that the impulse responses to a cost-push shock in the intermediate goods sector retain the same qualitative characteristics as those in Figure 3, but carry the opposite sign in inflation rates and price levels. Once again both sectors experience inflationary pressure, with the intermediate goods sector experiencing a stronger initial push in this instance. Just as in the
previous case, the central bank pursues contractionary policy in response to the cost-push shock. Nevertheless, since inflation in the intermediate goods sector is more sensitive to the nominal interest rate and is affected more by the cost-push shock, the central bank will set the magnitude of its response differently than when responding to a cost-push shock in the final goods sector.

4 Policy under Discretion

Under the more realistic assumption that central banks cannot commit to future actions, I study policy regimes that minimize assigned loss functions under discretion.\textsuperscript{12} Moreover, I focus on regimes that admit simple loss functions since they are easier to communicate to the public in a transparent and understandable way. Inflation and price-level targeting have been leading candidates in the study of the performance of simple loss functions under discretion. A next logical step would be to combine inflation and price-level targeting into a hybrid regime that includes targets of both kinds in the loss function. However, in results not reported, I find that there is no discernible difference in performance between simple price-level targeting and a hybrid scheme that includes both inflation and price-level targets. Some studies such as Walsh (2003b) and Néssen and Vestin (2005) have found that alternative classes of regimes, such as "speed limit" policies (regimes that include changes in the output gap) and average inflation targeting perform better than pure price-level or inflation targeting regimes, but \textit{only} when there is persistence in inflation.\textsuperscript{13} I therefore concentrate on simple price-level and inflation targeting regimes.

I use the household loss function as the model-consistent metric for assessing the performance of different regimes. Furthermore, I use the household losses under the optimal commitment policy as a reference for the discretionary policy regimes. The idea that the assigned loss function may differ from the household loss function has been introduced into many studies following the initial suggestion in Barro and Gordon (1983) and further development in Rogoff (1985). I rank candidate

\textsuperscript{12} Since this economy is linearized around a zero-inflation steady state, policies consistent with this linearization must imply equilibrium inflation rates sufficiently close to zero. I consider policies that imply zero inflation in the steady-state.

\textsuperscript{13} In Strum (2008) I extend the model in the present paper to allow both intermediate and final goods inflation to exhibit persistence. As part of the analysis, I show that speed-limit targeting does not outperform price-level targeting when the model is purely forward-looking (as it is in this paper).
inflation and price-level targeting regimes by their implied effects on expected household utility.

4.1 Characterization of Policy Regimes

Policy regimes are defined by the type of loss function that is assigned to the central bank. Loss functions take the same general form as the household loss function, but the target variables and weights are determined by the government. I define inflation targeting regimes as having the common feature that inflation rates enter the assigned loss function as target variables, regardless of additional variables that may enter the loss function. The period loss function for an inflation targeter takes the same form as (16) and (17), where $\Lambda^*$ is replaced by a set of weights that is assigned by the government and represented by

$$\Lambda = \text{diag}(\lambda_f, \lambda_m, \lambda_c, \lambda_v).$$

Within this class of inflation targeting regimes, pure discretion (PD) is the regime in which the central bank is assigned the household loss-function weights (given in $\Lambda^*$). I consider three additional inflation targeting regimes whose weights do not arise directly from the household utility function: CPI inflation targeting (CIT), intermediate goods inflation targeting (PIT), and dual inflation targeting (DIT). Under CIT, the weight on intermediate goods inflation, $\lambda_m$, is set to zero, as is the weight for final goods inflation, $\lambda_f$, under PIT. Under DIT both weights are positive.

I follow Svensson (1999a) and consider a price-level targeting regime to be characterized by loss functions of a similar form as above in which squared deviations of the price-level from some target value replace inflation terms. I consider three analogous price-level targeting regimes: CPI price-level targeting (CPT), intermediate goods price-level targeting (PPT), and dual price-level targeting (DPT). Under CPT the weight on the price level of intermediate goods, $\lambda_m$, is set to zero, as is the weight on the price level of final goods, $\lambda_f$, under PPT. Under DPT both coefficients are positive.
4.2 Policy Regime Comparisons

To compare the relative performances of the various discretionary policy regimes, I compute the expected utility losses from their respectively induced equilibrium paths. In each case I report the implied household loss as a percentage of quarterly steady-state output and as a percent deviation of the household loss compared to the household loss under the hypothetical optimal commitment policy (COM). It is important to note that this methodology differs from that utilized in other studies that compare discretionary policy regimes using ad hoc loss functions as the metric (for example, see Walsh, 2003b).

Since assigned loss functions do not come from the model, some care must be taken in setting the weights on the target variables in the loss functions. Often in the literature weights are arbitrarily chosen. These weights are then used across regimes in order to rank the regimes’ effectiveness at minimizing the overall loss function. However, proceeding in such a manner may bias the test in favor of one of the regimes due to accidentally favorable weights. Moreover, one would not know which regimes would receive a favorable bias. Some authors do include a range of weights, which alleviates this problem to some degree. Nevertheless, with a large parameter space, doubt can still remain as to hidden biases. In order to circumvent this problem, I use numerical methods to find the joint set of weights for each policy regime which then leads to the minimum household loss for that regime type, subject to the constraint that each weight be nonnegative.\footnote{The numerical method utilized to find the optimal weights starts with an initial guess of weights close to those in the household loss function and then examines how the household losses change as a function of the joint set of weights. The candidate optimal set of weights was found by following the decreasing household losses for a variety of curves. In all cases, every path led to the same minimal household loss. To check the result I employed two techniques. First, I evaluated household losses over a much larger range of weights (moving further away from the candidate weights). Second, I employed a simulated annealing technique given in Yang et al. (2005). In all cases the candidate minimized points were reconfirmed.}

Each period the central bank minimizes its assigned loss function subject to (15). To solve for a Markov Perfect equilibrium, I apply standard dynamic programming techniques. This linear-quadratic setup leads to a quadratic value function, given by

\[
\frac{1}{2} [(1 - \beta) X_t' V_t X_t + \beta z_t].
\]

(20)
The scalar $z_t$ depends on $V_t$ and the covariance matrix of the stochastic variables. This value function will satisfy the Bellman equation

$$ \frac{1}{2} \left[ (1-\beta) X_t'V_tX_t + \beta z_t \right] = (1-\beta) \min_{\xi_t} \left\{ L_t + \beta \mathbb{E}_t \left\{ \frac{1}{2} \left[ X_{t+1}'V_{t+1}X_{t+1} + \frac{\beta}{1-\beta} z_{t+1} \right] \right\} \right\}. \tag{21} $$

Table 2 reports the optimal target weights for the different regimes as well as the expected household utility losses. I report the unconditional household loss as a percentage of permanent quarterly steady-state consumption, calculated as

$$ \% \text{ of } \bar{C} = 100 \frac{\mathbb{E}_t \left[ \mathcal{L}^{regime} \right]}{\mathbb{E}_t \left[ (C)^C \right]}, $$

where $\bar{C}$ is the steady state value of consumption (final goods output) and $\mathbb{E}_t \left[ \mathcal{L}^{regime} \right]$ is the expected permanent period household loss less the loss that would occur in the efficient equilibrium. I also report the loss as a percentage of the loss that would be incurred under the optimal commitment policy.

The optimal weights for a dual inflation targeting regime are close to the weights in the household loss function itself (assigned in PD and COM). The dual inflation targeting regime places more relative weight on inflation in the intermediate goods sector and on the real marginal cost in the intermediate sector than does the household loss function. Regimes that target both price indices put more weight on final goods prices than on intermediate goods prices.

Within both inflation targeting and price-level targeting regimes, targeting both indices produces the best results.$^{15}$ Exclusively targeting the final goods index produces the worst results in each regime class. The class of price-level targeting regimes outperforms the class of inflation targeting regimes.$^{16}$ However, the difference between the best inflation targeting regime, DIT, and the worst

---

$^{15}$Similar to Aoki (2001), if one of the two sectors is characterized by much greater price stickiness than the other, then the loss function will put almost all of the weight on inflation in the sticky sector.

$^{16}$Strum (2008) extends the model in this paper to incorporate inflation persistence through backward-looking price setters. Compared to models with sticky prices in one sector, price-level targeting is more robust in the presence of inflation persistence. In the extended model, price-level targeting continues to outperform inflation targeting even if inflation persistence is high in the final goods sector as long as it is not too high in the intermediate goods sector.
price-level targeting regime, CPT, is small.

<table>
<thead>
<tr>
<th></th>
<th>$\pi_{f,t}$</th>
<th>$\pi_{m,t}$</th>
<th>$p_{f,t}$</th>
<th>$p_{m,t}$</th>
<th>$e_t$</th>
<th>$c_t - q_t$</th>
<th>$q_t$</th>
<th>% of $C$</th>
<th>% above COM</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD</td>
<td>1</td>
<td>0.60</td>
<td>0</td>
<td>0</td>
<td>0.009</td>
<td>0.002</td>
<td>0</td>
<td>2.44</td>
<td>14.33</td>
</tr>
<tr>
<td>CIT</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.007</td>
<td>0.005</td>
<td>0</td>
<td>2.73</td>
<td>27.88</td>
</tr>
<tr>
<td>PIT</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.01</td>
<td>0</td>
<td>2.59</td>
<td>21.32</td>
</tr>
<tr>
<td>DIT</td>
<td>1</td>
<td>0.79</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.01</td>
<td>0</td>
<td>2.43</td>
<td>13.88</td>
</tr>
<tr>
<td>CPT</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.02</td>
<td>0</td>
<td>0</td>
<td>2.34</td>
<td>9.62</td>
</tr>
<tr>
<td>PPT</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0.02</td>
<td>0</td>
<td>2.28</td>
<td>6.70</td>
</tr>
<tr>
<td>DPT</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.62</td>
<td>0.01</td>
<td>0</td>
<td>0</td>
<td>2.14</td>
<td>14.15</td>
</tr>
</tbody>
</table>

Table 2: Optimal Loss-Function Weights and Losses in the Benchmark Economy

These results obtain in a model in which all sectors are buffeted by both productivity and cost-push shocks. What if the economy has the input-output structure, but is only buffeted by a subset of the shocks posited in the benchmark economy? I study four alternative shock scenarios in which there are only productivity shocks, cost-push shocks, final goods shocks, and intermediate goods shocks. I assume that policy makers understand the true shock processes and choose the optimal loss function weights given the true shock processes. Table 3 reports the losses for the discretionary regimes that use loss functions crafted under the four alternative true-shock-source scenarios (each column contains the minimal losses that the various regimes can attain under the conditional shock mechanism). The losses are reported as a percent of permanent quarterly steady-state consumption given the particular assumption of the source of shocks.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>PD</td>
<td>1.63</td>
<td>0.82</td>
<td>0.0384</td>
<td>2.41</td>
</tr>
<tr>
<td>CIT</td>
<td>1.56</td>
<td>0.79</td>
<td>0.0381</td>
<td>2.69</td>
</tr>
<tr>
<td>PIT</td>
<td>1.56</td>
<td>0.89</td>
<td>0.0380</td>
<td>2.54</td>
</tr>
<tr>
<td>DIT</td>
<td>1.49</td>
<td>0.73</td>
<td>0.0380</td>
<td>2.39</td>
</tr>
<tr>
<td>CPT</td>
<td>1.46</td>
<td>0.79</td>
<td>0.0380</td>
<td>2.28</td>
</tr>
<tr>
<td>PPT</td>
<td>1.56</td>
<td>0.71</td>
<td>0.0381</td>
<td>2.20</td>
</tr>
<tr>
<td>DPT</td>
<td>1.44</td>
<td><strong>0.69</strong></td>
<td><strong>0.0378</strong></td>
<td><strong>2.10</strong></td>
</tr>
<tr>
<td>COM</td>
<td>1.44</td>
<td>0.69</td>
<td>0.0378</td>
<td>2.10</td>
</tr>
</tbody>
</table>

Table 3: Losses in Four Economies that differ by Sources of Shocks

Table 3 shows that the same general patterns of policy effectiveness obtain under various assump-
tions of the sources of shocks.\textsuperscript{17} The main difference is that sometimes only targeting intermediate
good price inflation or price levels proves less effective. Similar to the benchmark model, price-level
targeting generally outperforms inflation targeting, targeting prices in both sectors outperforms tar-
geting prices in only one sector, and dual price-level targeting is always the best-performing regime.
In fact, dual price-level targeting performs nearly as well as the optimal commitment policy.

5 Robustness of Discretionary Policies

In the previous section, I assumed that the government was aware of the correct model and correctly
perceived the different shocks that hit the economy when formulating the loss function for the central
bank. However, given the uncertainty that governments and central banks face in determining the
correct model and the sources of shocks, it is worthwhile to explore the following questions: What
happens if the government erroneously bases its policy based on the assumption of a one-sector
economy? What happens if policy makers do not correctly identify the sectors from which shocks
arise? How robust are the results to parameter values that differ from those on which the original
policy is based (either due to mismeasurement or changes in the structure of the economy following
the adoption of the policy regime)?

In this section I examine these three general areas of concern about policy robustness. First I
examine the consequences of the government incorrectly basing policy on a one-sector model. I
then consider the implications for household losses when the government misperceives the sources
of shocks to the economy. Finally I consider the consequences when the parameters characterizing
the importance of intermediate goods to final goods production and price stickiness in the two
sectors differ from those used in the policy calibration.

5.1 Implementation of Policies from a Simple One-Sector Model

Suppose the government assigns loss functions based on a one-sector model, despite the underlying
reality of the input-output model. Within the one-sector framework I consider two competing

\textsuperscript{17}I assume that the variances, persistence, and covariances of the shocks remain as they are in the benchmark
calibration.
regimes: (C)IT (inflation targeting), and (C)PT (price-level targeting), in which I identify inflation and price-level targets with consumer price indices. I calculate the optimal weights for inflation and price-level targeting in the one-sector world. The loss functions are assigned to the central bank, which minimizes the loss functions subject to (15) in the true economy. I report the losses that households would experience in the benchmark economy under these one-sector-based policies in Table 4 (these losses are directly comparable to those in Table 2).

<table>
<thead>
<tr>
<th></th>
<th>% of C</th>
<th>% above COM</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C)IT</td>
<td>2.77</td>
<td>29.63</td>
</tr>
<tr>
<td>(C)PT</td>
<td>2.49</td>
<td>16.38</td>
</tr>
</tbody>
</table>

Table 4: One-Sector Policy Results in the Input-Output Model

Table 4 confirms that the order of performance of price-level and inflation targeting regimes does not change when they are incorrectly based on a one-sector model. However, a comparison of these results with those in Table 2 shows that carrying policy prescriptions straight over from the one-sector model can lead to unintended outcomes. The one-sector (CPI) inflation targeting prescription performs worse than the other inflation targeting regimes when implemented within the full input-output model. Furthermore, not only does (C)PT perform worse than the other price-level targeting regimes, it performs worse than the dual inflation targeting regime, DIT. Thus, it is possible to design an inflation targeting regime within the input-output model that outperforms the optimal price-level targeting regime that has been carried over from the one-sector model.

5.2 Policy Robustness under Misperceptions of Shocks

Suppose now that the central bank realizes that the input-output model represents a more accurate view of the world, but constructs its policy reaction function based on misperceptions regarding shocks. I examine the implications for household losses if the central bank cannot distinguish the sector or type of shock that hits the economy. Specifically, I allow the government to choose loss-function coefficient values based on its belief regarding the source of shocks. I then determine the unconditional expected household losses when the policy based on a misperception of shock sources is used within the benchmark economy. Table 5 reports the losses for each regime as a
percentage of the permanent quarterly steady-state consumption, which means that these losses can be compared directly to those reported in Table 2. In the column labeled "Benchmark," I have reproduced the losses from Table 2 in which the correctly formulated benchmark loss functions are run in the benchmark world.

<table>
<thead>
<tr>
<th>Shock Perceptions</th>
<th>Benchmark</th>
<th>Final Goods</th>
<th>Intmd. Goods</th>
<th>Productivity</th>
<th>Cost-pushing</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIT</td>
<td>2.73</td>
<td>3.03</td>
<td>22.61</td>
<td>3.01</td>
<td>2.73</td>
</tr>
<tr>
<td>PIT</td>
<td>2.59</td>
<td>2.87</td>
<td>2.63</td>
<td>2.98</td>
<td>2.60</td>
</tr>
<tr>
<td>DIT</td>
<td>2.43</td>
<td>4.66</td>
<td>14.65</td>
<td>2.87</td>
<td>2.43</td>
</tr>
<tr>
<td>CPT</td>
<td>2.34</td>
<td>2.43</td>
<td>22.61</td>
<td>2.91</td>
<td>2.34</td>
</tr>
<tr>
<td>PPT</td>
<td>2.28</td>
<td>2.28</td>
<td>2.34</td>
<td>3.01</td>
<td>2.28</td>
</tr>
<tr>
<td>DPT</td>
<td>2.14</td>
<td>2.18</td>
<td>2.19</td>
<td>2.41</td>
<td>2.14</td>
</tr>
</tbody>
</table>

Table 5: Losses in the Benchmark Economy under Regimes based on Shock Misperceptions

The central bank’s performance deteriorates when its misperception of the sources of shocks leads it to put too much weight on stabilizing prices in either of the two sectors (not only relative to prices in the other sector, but also relative to the output gap and intermediate goods real marginal cost gap). Under the assumption of only final goods shocks the weights for DIT are especially affected. Under the assumption of only intermediate goods shocks, the weights in CIT, DIT, and CPT lead to poor performances in the benchmark model.

The most robust result that emerges from this exercise is that even if the policy makers misperceive the sources of shocks to the economy, they minimize their losses by choosing a DPT regime. Even when the DPT loss function is formulated under misperceptions, it outperforms not only all of the other regimes that were formed under the same misperception of shocks, but also almost all of the other regimes that were formulated under a correct perception of the sources of shocks.

Suppose, instead, that the government erroneously assumes that shocks occur as posited in the benchmark model and uses the loss functions based on the benchmark model in economies with different sources of shocks. I examine the household losses under the benchmark-formulated discretionary regimes in each of the four alternative economies: only final goods shocks, only intermediate goods shocks, only productivity shocks, and only cost-push shocks. Table 6 reports the losses as the percent of permanent quarterly steady-state consumption (where the steady state
consumption is formulated for an economy with the conditional shock structure). The losses in each column can be compared directly to the corresponding columns in Table 3.

<table>
<thead>
<tr>
<th>True Shock Sources:</th>
<th>Final Goods</th>
<th>Intmd. Goods</th>
<th>Productivity</th>
<th>Cost-push</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Benchmark Regimes</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CIT</td>
<td>1.73</td>
<td>1.00</td>
<td>0.0419</td>
<td>2.69</td>
</tr>
<tr>
<td>PIT</td>
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<td>0.95</td>
<td>0.0510</td>
<td>2.54</td>
</tr>
<tr>
<td>DIT</td>
<td>1.54</td>
<td>0.89</td>
<td>0.0466</td>
<td>2.39</td>
</tr>
<tr>
<td>CPT</td>
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<td>0.88</td>
<td>0.0646</td>
<td>2.28</td>
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<tr>
<td>PPT</td>
<td>1.56</td>
<td>0.72</td>
<td>0.0811</td>
<td>2.20</td>
</tr>
<tr>
<td>DPT</td>
<td><strong>1.45</strong></td>
<td><strong>0.69</strong></td>
<td><strong>0.0381</strong></td>
<td><strong>2.10</strong></td>
</tr>
</tbody>
</table>

Table 6: Losses in Alternative Economies under an Incorrect Benchmark Shock Assumption

Once again, the dual price-level targeting regime clearly outperforms all of the other specifications, and, with a few exceptions, price-level targeting generally outperforms inflation targeting. Dual inflation targeting always outperforms single-sector inflation targeting of either type. These two exercises jointly show that the policy makers can best avoid unpleasant ex post policy results when there are potential misperceptions about shocks by targeting prices in both sectors, especially when they target price levels instead of inflation rates.

Shock misperceptions can make important differences regarding the actual performance of policies that are thought ex ante to be good. Assuming that shocks hit both sectors and are both of the labor-augmenting productivity type and cost-push shock type prevents the central bank from being exposed to the extreme risk that can occur with policies based on the assumption of only intermediate goods shocks are implemented in a benchmark world. Benchmark-based regimes experience their highest decreases in performance when they are implemented in worlds in which there are only intermediate goods shocks or only productivity shocks (as judged by the percentage increase in loss compared to when they are calibrated and run according to the correct perceptions of shocks). If the DIT regime is set in the benchmark world, it is fairly robust (meaning that it never occupies a lower relative rank to other policies in non-benchmark worlds). However, it is much less robust when formulated on the assumption of restricted sets of shocks. One clear result emerges: DPT always performs well, no matter what set of shocks were assumed to be true when formulating its loss function and no matter what the true world is in which it is implemented.
5.3 Policy Robustness to Parameter Changes

Besides model and shock uncertainty, policy makers must contend with parameter uncertainty. In this model, the intensity of the use of intermediate goods and the price stickiness in the two sectors constitute key features relevant for policy analysis. In order to understand the effect of uncertainty regarding the intensity of use of intermediate goods in the production process, I examine the losses that occur as φ ranges over values that differ from the value used to formulate policy. In this and subsequent robustness exercises, I assume that the stochastic nature of the shocks does not change across these different environments. I also assume that the central bank can correctly identify the source of the shocks. To accomplish this, I first find the optimized loss-function coefficients for the various policy regimes based on the benchmark calibration. I then run this policy in economies in which the value of φ differs from the benchmark value.

![Figure 5: Relative Losses as φ varies: Benchmark Economy](image)

Figure 5 reports the losses of benchmark discretionary regimes relative to the conditional optimal commitment policy as the true or post-policy value of φ varies from 0 to 1. The top panel displays the results for the inflation targeting regimes, while the bottom panel shows the results for the price-level targeting regimes. PD differs from the other policies in that its weights implicitly change as
the parameter values change.\textsuperscript{18} PD is plotted in both panels to facilitate an easier comparison of the inflation and price-level targeting regimes. DPT remains the best regime for a very large portion of the parameter space. It becomes the second-best regime if $\varphi$ turns out to be less than 0.25. CPT proves to be much more robust than PPT, especially at lower values of $\varphi$. Among the inflation-targeting regimes, DIT exhibits the best robustness. CIT performs especially well if $\varphi$ turns out to be lower than expected, while PIT performs well if $\varphi$ is higher than expected.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure6.png}
\caption{Relative Losses as $\alpha_f$ and $\alpha_m$ vary with $\alpha_f = \alpha_m$: Benchmark Economy}
\end{figure}

Figure 6 displays the results if price stickiness turns out to be different than expected (but still equal in the two sectors).\textsuperscript{19} If price stickiness turns out to be lower than expected, all four regimes that target either prices or inflation in only one sector perform worse than the dual targeting regimes. DIT performs very well only if price stickiness turns out to be less than the values used to calibrate the loss function. DPT performs well over the entire range and proves to be the most robust regime with respect to uncertainty in price stickiness.

These results support the idea that dual price-level targeting under discretion allows the central bank to achieve results similar to an optimal commitment regime in forward-looking New Keynesian

\textsuperscript{18}The optimal commitment policy loss function coefficients also vary as the true model parameters vary.
\textsuperscript{19}Since cost-push shocks are derived from elasticity-of-substitution shocks, their reduced-form variances change as price stickiness changes. Also, as noted earlier, if price stickiness in one sector turns out to be much greater than the other sector, the central bank will want to target the sticky sector.
models with little risk due to uncertainty in these parameter values. However, while many modern central banks can be characterized as operating within a framework similar to CIT, no modern central bank targets price levels. Given the lack of experience with price-level targeting, a move to something like DIT first, and then to DPT may provide a smoother path towards the optimally performing regime. This would allow central banks to gain experience in monitoring and reacting to intermediate goods price indices while they prepare to switch to price-level targeting.

6 The Importance of Intermediate Goods

Though the benchmark economy serves as a good basis by which to judge an economy similar to that of the United States today, it is natural to ask how these results would be different if the intermediate goods sector were more or less important. This question is important for at least two reasons. First, economies at different stages of development may utilize intermediate goods with different levels of intensity. Hanes (1996) has documented the fact that modern consumption goods in the U.S. pass through more stages of production today than they did in earlier times. Second, economies at similar stages of development may nevertheless differ in their use of intermediate inputs, and therefore would be characterized by different values of $\varphi$.

6.1 High Importance of Intermediate Goods

For an economy that exhibits an intense use of intermediate goods, I assume the same parameterization as the benchmark model, except that I now set $\varphi = 0.8$. This corresponds to the upper end of the range for $\varphi$ in the United States, and a correspondingly high importance of the intermediate goods sector in the production process of final goods. Table 7 reports the weights for the optimized loss coefficients, the loss as a percentage of steady-state output, and the losses relative

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20Sweden in the 1930’s seems to be the lone example; see Berg and Jonung (1999).
21Of course, to make a rigorous statement about the optimal transition path, one would have to account for the effects of the transition itself—a task I leave for future study.
22This idea can be found in the early writings of Eugen von Böhm-Bawerk (1884). In his magnum opus, Capital and Interest, von Böhm-Bawerk discusses the idea that as economies become more advanced, they utilize an ever expanding process of “roundabout” production. Though he has capital in mind in his exposition, the kernel of his idea easily generalizes to a process of using goods to produce goods—an intermediate goods sector.
to the optimal commitment policy in the right-most column.

Once again DPT is the top performing regime. However, the weight placed on the price level of the intermediate good has increased substantially. Furthermore, PPT is now very close to DPT and much better than the remaining regimes. CPT is actually marginally worse than DIT. These results come as no surprise since a larger value for $\varphi$ means that the intermediate goods are more important in the production process. DIT’s ability to outperform other inflation targeting regimes when $\varphi$ is high reinforces the idea that it serves as a good intermediate regime.

<table>
<thead>
<tr>
<th></th>
<th>$\pi_{f,t}$</th>
<th>$\pi_{m,t}$</th>
<th>$p_{f,t}$</th>
<th>$p_{m,t}$</th>
<th>$\hat{c}_t$</th>
<th>$\hat{c}_t - \hat{q}_t$</th>
<th>$%$ of $C$</th>
<th>$%$ above COM</th>
</tr>
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<td>PD</td>
<td>1</td>
<td>0.80</td>
<td>0</td>
<td>0</td>
<td>0.009</td>
<td>0.001</td>
<td>2.64</td>
<td>14.40</td>
</tr>
<tr>
<td>CIT</td>
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<td>0</td>
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<td>0.009</td>
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<td>3.07</td>
<td>33.30</td>
</tr>
<tr>
<td>PIT</td>
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<td>0.002</td>
<td>2.70</td>
<td>17.02</td>
</tr>
<tr>
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<td>0.005</td>
<td>0.007</td>
<td>2.63</td>
<td>13.99</td>
</tr>
<tr>
<td>CPT</td>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>0.02</td>
<td>0</td>
<td>2.63</td>
<td>14.07</td>
</tr>
<tr>
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<td>0.24</td>
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<td>COM</td>
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<td>0</td>
<td>0</td>
<td>0.009</td>
<td>0.001</td>
<td>2.31</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 7: Optimal Weights and Losses: High Importance of Intmd. Goods

Figure 7: Relative Losses as $\varphi$ varies: High Importance of Intmd. Goods

I repeat the earlier robustness analysis and plot the relative losses under these regimes for
different true or post-policy values of \( \varphi \) in Figure 7. These results complement the results obtained for policies crafted in the benchmark economy. Except for very low values of \( \varphi \), dual price-level and inflation targeting continue to be robust, outperforming other regimes in each class. Compared to the benchmark case, CIT and CPT demonstrate more robustness than they did under the benchmark calibration. On the other hand, PIT and PPT perform very poorly if \( \varphi \) is significantly lower than expected.

### 6.2 Low Importance of Intermediate Goods

In order to consider an economy that exhibits a low intensity of intermediate goods use, I assume that \( \varphi = 0.2 \). Table 8 reports the results the loss function weights and implied household losses under this assumption. Again, DPT is not only the top performer, but very nearly replicates the performance of the optimal commitment solution. CPT performs much better than all other regimes and nearly as well as DPT. Targeting only intermediate prices proves to be less successful, with PPT performing worse than both PD and CIT. Figure 8 shows that the dual-targeting regimes continue to exhibit good robustness properties when the true or post-policy values of \( \varphi \) turn out to be much higher than an assumed low value. DPT is the best of all regimes, while DIT performs better than the other inflation-targeting regimes.

<table>
<thead>
<tr>
<th></th>
<th>( \pi_{f,t} )</th>
<th>( \pi_{m,t} )</th>
<th>( p_{f,t} )</th>
<th>( p_{m,t} )</th>
<th>( \tilde{c}_t )</th>
<th>( \tilde{c}_t - \hat{q}_t )</th>
<th>% of C</th>
<th>% above COM</th>
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<tr>
<td>PD</td>
<td>1</td>
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<td>0.009</td>
<td>0.001</td>
<td>1.79</td>
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</tr>
<tr>
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<td>0.009</td>
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<td>0.02</td>
<td>0.01</td>
<td>2.26</td>
<td>50.56</td>
</tr>
<tr>
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<td>0</td>
<td>0.003</td>
<td>0.007</td>
<td>1.75</td>
<td>16.90</td>
</tr>
<tr>
<td>CPT</td>
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<td>0.01</td>
<td>0</td>
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<td>2.38</td>
</tr>
<tr>
<td>PPT</td>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>0.07</td>
<td>2.13</td>
<td>41.72</td>
</tr>
<tr>
<td>DPT</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.19</td>
<td>0.008</td>
<td>0.001</td>
<td>1.50</td>
<td>0.01</td>
</tr>
<tr>
<td>COM</td>
<td>1</td>
<td>0.20</td>
<td>0</td>
<td>0</td>
<td>0.009</td>
<td>0.001</td>
<td>1.50</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 8: Optimal Weights and Losses: Low Importance of Intmd. Goods
Figure 8: Relative Losses as $\varphi$ varies: Low Importance of Intmd. Goods

7 Conclusion

In this paper I have examined how accounting for nominal rigidities in the final and intermediate goods sectors in New Keynesian models can affect the design of monetary policy. In distinction from simple one-sector models, policy makers face policy trade-offs when faced with technology shocks when there are multiple levels of price stickiness in an input-output production structure. Furthermore, under an optimal commitment policy in the New Keynesian input-output model, the central bank responds very differently to technology shocks, depending on the sector from which a particular shock originates. On the other hand, cost-push shocks from either sector lead to the same qualitative responses by the central bank.

An examination of the performance of alternative discretionary regimes reveals a large advantage to targeting prices in both final and intermediate goods sectors. Targeting prices in both sectors allows the central bank to balance inflation more effectively across the two sectors, and thus lower household losses compared to the results of just targeting prices in one sector. Targeting prices in both sectors leads to a more efficient allocation of resources both within and across the two sectors. These advantages lead not only to dual-targeting regimes’ outperforming other single-sector-targeting regimes under perfect information and model awareness, but also to superior
robustness characteristics across model and parameter uncertainties. Specifically, dual-targeting regimes perform better when policy makers misperceive the sources of shocks or when they calibrate loss functions based on parameters values that differ from those that characterize the true economic environment.

Consistent with Vestin (2006), price-level targeting outperforms inflation targeting in this forward-looking New Keynesian model. However, in order to realize the full benefits of moving from inflation targeting to price-level targeting, the central bank must target both price indices. Furthermore, the dual price-level targeting regime displays even better robustness properties than the dual inflation-targeting regime.

Given the lack of experience with price-level targeting and the interest in targeting both price indices in the future, this analysis suggests that a dual inflation-targeting regime may be an excellent intermediate step in the procession of successive policy regimes that a country could adopt. In the model employed in this paper, policy performance would improve in the short run following the adoption of a dual inflation-targeting regime. At the same time the central bank could study price-level targeting while it gains experience at targeting intermediate goods inflation.

The general results from the benchmark parameterization obtain for economies that differ greatly in their use of intermediate goods in the production of final goods. However, the weights on the various target variables depend directly on the importance of the intermediate goods sector in final goods production. This suggests that an important goal of future empirical work should be an accurate measure of parameters characterizing the usage of intermediate inputs within the production process. This may have the added effect of helping to craft policies that are more appropriate for varying levels of development.

The conclusions in this paper must be tempered by the fact that the empirical relevance of the forward-looking New Keynesian model is controversial. Fuhrer and Moore (1995) and Fuhrer (1997) criticize forward-looking New Keynesian models for their poor empirical fit. Rudebusch (2002) and others find empirical evidence that lagged inflation enters into the Phillips curve. Nevertheless, Gali and Gertler (1999) and Sbordone (2002) argue that the forward-looking New Keynesian Phillips curve does provide a good model for inflation dynamics. Their results indicate that empirical
failures may have more to do with the fact that the output gap enters the model as a proxy for the theoretically pure real marginal cost term. Indeed, the model in this paper suggests using marginal costs that incorporate intermediate goods.

Furthermore, many authors have also found that the degree of inflation persistence is critical in evaluating the potential performance of alternative monetary policy regimes. Néssen and Vestin (2005) show that average inflation targeting can outperform price-level targeting when there is sufficient inflation persistence. Walsh (2003b) finds that speed-limit targeting is optimal within a model that permits significant inflation persistence. Thus, another important avenue of research is to understand how the potential of inflation persistence in the expanded model affects the conclusions reached in this paper.

Other extensions to the model used in this paper could improve its ability to shed light on monetary policy questions. Expanding the demand side of the model and allowing for capital on the production side would add important elements. As suggested earlier, a more fundamental mechanism for price stickiness would allow for an accounting of endogenous changes in price stickiness. Extending the model to allow for sticky wages would allow for a comparison of the relative importance of these two rigidities. The complications that arise when fiscal and monetary policy are conducted simultaneously, noise in the data, and issues of model uncertainty could also provide additional useful insights.

Despite these caveats, the central messages from this paper remain clear. Incorporating nominal rigidities in final and intermediate goods sectors into models used to study policy design is important. Including inflation in both final and intermediate goods into a central bank’s loss function can lead to welfare gains on the order of moving from an inflation target to a price-level target in standard one-sector New Keynesian models. Moreover, targeting prices in both sectors endows the policy regime with very beneficial robustness properties. Since price-level targeting is less familiar to policy makers and targeting prices in both sectors was shown to be the most desirable type within each class of regimes in this model, these results suggest that first moving to a generalized inflation targeting regime, and then to a generalized price-level targeting regime, may be a desirable path for monetary policy evolution.
A Structural Matrices for the Economy

Recall that I can represent the structural equations of the economy as

\[
\begin{bmatrix}
X_{t+1} \\
HE_{t} y_{t+1}
\end{bmatrix} = A \begin{bmatrix}
X_{t} \\
y_{t}
\end{bmatrix} + B \hat{c}_{t} + \begin{bmatrix}
C \\
0
\end{bmatrix} \varepsilon_{t+1}
\]

where

\[
X_{t} = \begin{bmatrix}
\hat{u}_{ft} \\
\hat{u}_{mt} \\
\hat{\pi}_{ft} \\
\hat{\pi}_{mt} \\
\hat{\alpha}_{f,t-1} \\
\hat{\alpha}_{m,t-1} \\
\hat{\eta}_{t-1}
\end{bmatrix},
\quad
y_{t} = \begin{bmatrix}
\tilde{q}_{t} \\
\pi_{ft} \\
\pi_{mt}
\end{bmatrix},
\quad
\varepsilon_{t+1} = \begin{bmatrix}
\eta_{f,t+1} \\
\eta_{m,t+1} \\
\epsilon_{f,t+1} \\
\epsilon_{m,t+1}
\end{bmatrix}.
\]

The matrices that describe the exact state-space structural relations for the economy are given by

\[
H = \begin{bmatrix}
0 & 0 & 0 \\
0 & \beta & 0 \\
0 & 0 & \beta
\end{bmatrix}
\]
\[ A = \begin{bmatrix} 
\rho_{uf} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \rho_{um} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \rho_f & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \rho_m & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & - (1 - \phi) & (1 - \phi) & (1 - \phi) & - (1 - \phi) & 1 & -1 & -1 & 1 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & -\kappa_f \phi & 1 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & \kappa_m & 0 & 1 
\end{bmatrix}, \]

\[ B = \begin{bmatrix} 
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
-\kappa_f (1 - \phi) \sigma \\
-\kappa_m \sigma 
\end{bmatrix}, \]

37
and

\[
C = \begin{bmatrix}
\sigma_{uf} & 0 & 0 & 0 \\
0 & \sigma_{um} & 0 & 0 \\
0 & 0 & \sigma_f & 0 \\
0 & 0 & 0 & \sigma_m \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix},
\]

The target variables are collected via

\[
Y_t = D \begin{bmatrix}
X_t \\
y_t \\
\tilde{c}_t \\
\end{bmatrix} = \begin{bmatrix}
\pi_{ft} \\
\pi_{mt} \\
\tilde{c}_t \\
\sigma\tilde{c}_t - \tilde{q}_t
\end{bmatrix},
\]

where

\[
D = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & \sigma
\end{bmatrix}.
\]
References


Woodford, Michael. (1986). "Stationary Sunspot Equilibria, the Case of Small Fluctuations around a Deterministic Steady State." Manuscript, University of Chicago.

