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Abstract

TIPS breakeven inflation rate, defined as the difference between nominal and TIPS yields of comparable maturities, is potentially useful as a real-time measure of market inflation expectations. In this paper, we provide evidence that a fairly large TIPS liquidity premium existed until recently, using a multifactor no-arbitrage term structure model estimated with nominal and TIPS yields, inflation and survey forecasts of interest rates. Ignoring the TIPS liquidity premiums leads to counterintuitive implications for inflation expectations and inflation risk premium, and produces large pricing errors for TIPS. In contrast, models incorporating a TIPS liquidity factor generate much better fit for these variables and reveal a TIPS liquidity premium that was until recently quite large ($\sim 1\%$) but has come down in recent years, consistent with the common perception that TIPS market grew and liquidity conditions improved. Our results indicate that after taking proper account of the liquidity conditions in the TIPS market, the movement in TIPS breakeven inflation rate can provide useful information for identifying real yields, expected inflation and inflation risk premium.

1 Introduction

This paper presents a joint study of yields on nominal Treasury securities and those on Treasury Inflation-Protected Securities (TIPS) in a no-arbitrage asset pricing framework. Since its inception in 1997, the market for TIPS has grown substantially and now comprises about 8% of the outstanding Treasury debt market. More than a decade's TIPS data thus accumulated is a rich source of information to academic researchers and market participants alike. Because TIPS are securities whose coupon and principal payments are indexed to the price level, information about yields on these "real bonds" has direct implications for asset pricing models, many of which are written in terms of real consumption. Meanwhile, real-time TIPS data has attracted much attention from policy makers and market participants as a source of information about the state of the economy. In particular, the differential between yields on nominal Treasury securities and on TIPS of comparable maturities, often called the "breakeven inflation rate" (BEI) or "inflation compensation", has been used by policy makers and market participants as a proxy for market's inflation expectation. For example, the minutes of FOMC meetings often take note of changes in TIPS yields since the previous meeting,[1] and explicit references to TIPS breakeven rates in speeches by Fed officials are common.[2] Similarly, financial press frequently cite TIPS breakeven rates when discussing inflation expectations.

However, two difficulties arise in such interpretation of the TIPS breakeven inflation. First, TIPS breakeven inflation contains the inflation risk premium, which is the extra compensation investors in nominal bonds demand for bearing inflation risks. Second, TIPS has only been introduced recently and during its existence has been a less liquid instrument compared to nominal Treasury securities. The additional "liquidity premium" TIPS investors require for holding such instruments will drive up TIPS yields and depress the TIPS breakeven inflation. While the inflation risk premium has been studied by many researchers,[3] the liquidity pre-

[1] For example, the minutes of the June 2006 FOMC meeting includes the following sentence: "Yields on inflation-indexed Treasury securities increased by more than those on nominal securities, and the resulting decline in inflation compensation retraced a substantial share of the rise that had occurred over the preceding intermeeting period."

[2] Fed Vice Chairman Kohn (2006)'s speech on October 4, 2006, for example, includes the following remark: "In financial markets, the spread of nominal over indexed yields has also retreated substantially at the near end of the yield curve."

[3] See Pennacchi (1991), Buraschi and Jiltsov (2005), Ang, Bekaert, and Wei (2008) and Chernov and Mueller (2007), among others.

mium embedded in TIPS breakeven inflation has to our knowledge never been examined in the asset pricing literature.

The goal of this paper is to document the existence of liquidity premium in TIPS yields and to quantitatively characterize its behavior. To this end, we estimate and contrast several models for real and nominal yields, where the liquidity premium is either ignored or modeled as following different processes. All models we use are from the affine-Gaussian no-arbitrage term structure family and allow a rich dynamics in both the inflation risk premium and in nominal and real term premia.

Our main findings can be summarized as follows. First, a general 3-factor affine term structure model that ignores the liquidity premium generates large pricing errors for TIPS as well as counterfactual implications for inflation expectations and inflation risk premiums. In comparison, after incorporating an additional liquidity factor, the three 4-factor models that we estimate lead to notably smaller TIPS pricing errors, generate reasonable inflation risk premiums, and produce model-implied inflation expectations that agree well with survey inflation forecasts. Second, the liquidity premium estimates from all 4-factor models share the feature that it was large (1-2%) in the early years but declined in recent years, consistent with the notion that TIPS market liquidity conditions have improved over time and that TIPS pricing has possibly become more "efficient". In particular, around 80% of the variations in our estimates of TIPS liquidity premiums can be explained by variables related to the liquidity conditions in the TIPS market. Third, our best model for TIPS liquidity premiums has three parts, including a deterministic trend that captures the gradual but steady decline in TIPS liquidity premiums from the early years, a TIPS-specific factor that is independent of the nominal bond factors, as well as a component that is correlated with the rest of the economy. Finally, a variance decomposition shows that TIPS liquidity premiums explain more than 40% of variations in TIPS breakeven inflation, while that percentage declines to about one quarter in the long run.

The results in this paper shows that one needs to be careful in using the TIPS breakeven inflation rate as a proxy for inflation expectation, since an economically significant TIPS liquidity premium, on top of the inflation risk premium, could drive a large wedge between the TIPS breakeven inflation and inflation expectation. This problem seems to be especially severe in the early years of the TIPS market.

The remainder of this paper is organized as follows. In Section 2, we provide evidence that TIPS yields and TIPS breakeven inflation contain an additional factor, likely reflecting the illiquidity of TIPS, beyond those driving the nominal interest rates. Section 3 spells out the details of the no-arbitrage models we use, including the specification of the additional liquidity factor, and Section 4 explains the empirical methodology. Section 5 presents the main empirical results based on one model assuming zero TIPS liquidity premium and three models in which the TIPS liquidity premium is assumed to follow different specifications. Section 6 provides further discussions on the model estimates of the TIPS liquidity premiums and shows that they are indeed linked to the liquidity conditions in the TIPS markets. Section 7 provides some variance decomposition results for TIPS yields, TIPS breakeven inflation and nominal yields. Finally, Section 8 concludes.

2 A TIPS Liquidity Factor: Simple Analysis

In this section we present some simple analysis suggesting there exists a factor that is important for explaining the variations in TIPS yields but not as crucial for modeling nominal interest rates. We further argue that this factor is related to the illiquidity of the TIPS market. This serves as the motivation for introducing a TIPS-specific factor when we model nominal and TIPS yields jointly in later sections.

Simple Regression Analysis

In our first analysis, we regress the 10-year TIPS breakeven inflation rate, defined as the spread between the 10-year nominal yield and the 10-year TIPS yield, on 3-month, 2-year and 10-year nominal yields plus a constant:[4]

$$BEI^{\mathcal{T}}_{t,10} = \alpha + \beta_1 y^N_{t,0.25} + \beta_2 y^N_{t,2} + \beta_3 y^N_{t,10} + e_t. \tag{1}$$

Standard finance theory suggests that nominal yield of any maturity can be decomposed into the underlying real yield, inflation expectation and the inflation risk premium:

$$y^N_{t,\tau} = y^R_{t,\tau} + I_{t,\tau} + \wp_{t,\tau},$$

where $y^N_{t,\tau}$ and $y^R_{t,\tau}$ are the τ-period nominal and real yields, respectively, $I_{t,\tau}$ is the expected inflation over the next τ periods and $\wp_{t,\tau}$ is the inflation risk premium. If TIPS yields are a

[4] We thank Greg Duffee for this suggestion. Results using three different nominal yields or using the first principal components of nominal yields are similar. Nominal and TIPS yields are from fitted Svensson yield curves maintained by the staff at the Federal Reserve Board of Governors.

good measure of the underlying real yields, the TIPS breakeven inflation rate is simply the sum of expected inflation and inflation risk premiums, which are also parts of the nominal yields. Therefore, a regression of TIPS breakeven inflation onto nominal yield curve factors in this case can be expected to result in a high R^2. On the other hand, variations in TIPS yields that are unrelated to those in the nominal yields could lead to a low R^2 in such a regression. The results from running Regression (1) over the full sample of Jan. 6, 1999 to Mar. 14, 2007 are reported in Panel A of Table 1. The R^2 from this regression is a mere 32%, suggesting that a large portion of variations in the 10-year breakeven inflation cannot be explained by factors underlying the nominal interest rate variations. We have also examined this regression in first-differences, and obtained an R^2 of about 60%. This is much lower than the R^2's from a comparable regression of the first-difference of a nominal yield onto the first-differences of other nominal yields, which typically give an R^2 in excess of 95%.

Principal Components Analysis

Next, we conduct a principal component analysis (PCA) of the cross section of the nominal and TIPS yields. It is well known that, in the case of nominal yields, three factors explain most of the nominal yield curve movements. This is confirmed in Panel B of Table 1, which shows that more than 97% of the variations in weekly changes of 3- and 6-month and 1-, 2-, 4-, 7- and 10-year nominal yields can be explained by the first three principal components. However, once we add the 5-, 7- and 10-year TIPS yields, at least four factors are needed to explain the same amount of variance. Panel C of Table 1 reports the correlations between the first four PCA factors extracted from nominal yields alone and the first four PCA factors extracted from nominal and TIPS yields combined. It is interesting to note that, once we add TIPS yields to the analysis, the first, the second and the fourth factors largely retain their interpretations as the level, slope and curvature of the nominal yields curve, as can be seen from their high correlation with the first, the second and the third nominal factors, respectively. However, the third PCA factor extracted from nominal and TIPS yields combined is not highly correlated with any of the nominal PCA factors. The results shown here and the simple regression analysis shown above have an interesting parallel with the literature on the unspanned stochastic volatility (USV) effect: using a simple regression analysis, Collin-Dufresne and Goldstein (2002) argued that bond derivatives contain a factor that is not spanned by the yield curve factors, and Heidari and Wu (2003) reported evidence for unspanned stochastic volatility using a

PCA analysis.[5]

A Case for TIPS Liquidity Premium

A promising interpretation of the TIPS-specific factor we found above is that it reflects a "liquidity premium": investors would demand a compensation for holding a relative new and illiquid instrument like TIPS, especially in the early years.

Indeed, several measures related to TIPS market liquidity conditions, as well as anecdotal reports, indicate that the liquidity in TIPS market was much poorer than that of nominal securities, and that TIPS market liquidity improved over time, although this improvement was not a smooth, steady process. The top panel of Figure 1 shows the gross TIPS issuance over the period 1997-2007. The TIPS issuance dipped slightly in 2000-2001 before rising substantially in 2004. According to Sack and Elsasser (2004), there were talks around 2001 that the Treasury might discontinue the TIPS due to the relatively weak demand for TIPS. TIPS outstanding, shown in the bottom panel of Figure 1, began to grow at a faster pace from 2004 onward and now exceeds 400 billion dollars. Figure 2 tells a similar story from the demand side: TIPS transaction volumes grew by sixfold during our sample period and TIPS mutual funds also experienced significant growth.[6] In view of this institutional history, it is not unreasonable to suppose that TIPS contained a significant liquidity premium at least in its early years and that the liquidity premium have edged lower over time.

[Insert Figure 1 about here.]

[Insert Figure 2 about here.]

A liquidity premium in TIPS can help resolve a seeming inconsistency between survey inflation forecasts (10-year SPF survey and Michigan long-term survey) and the 10-year TIPS breakeven inflation, all of which are plotted in Figure 3. Recall that the true breakeven inflation, defined as the yield differential between nominal and real bonds of comparable maturities and liquidity features, is the sum of expected inflation and the inflation risk premium, and can be considered as a good measure of the former if the second term is relatively small and does not vary too much over time. However, Figure 3 shows that this is not the case: the

[5] Note, however, that the presence of USV is still debated. See, e.g., Joslin (2008) and Jacobs and Karoui (2009).

[6] Data on TIPS mutual fund is from the Investment Company Institute.

TIPS breakeven inflation lied below both measures of survey inflation forecasts almost all the time before 2004.[7 8 9] Such disparity cannot be attributed solely to the existence of inflation risk premium, as such an explanation would require the inflation risk premium to be mostly negative in the 1999-2007 period and highly volatile, which stands in contrast with most studies in the literature that find inflation risk premiums to be positive on average and relatively smooth.[10]

On the other hand, a positive TIPS liquidity premium would push the TIPS-based breakeven inflation below the true breakeven inflation and, if the TIPS liquidity premium exceeds the inflation risk premium in absolute terms, even lower than survey inflation forecasts. Furthermore, part of the volatility of the TIPS breakeven inflation rate may be due to the volatility of the TIPS liquidity premium.

In order to study these issues quantitatively, we need a framework for identifying and measuring the relevant components, including the TIPS liquidity premium, inflation expectations, and inflation risk premium. For this purpose, we use the no-arbitrage term structure modeling framework, to which we now turn.

[Insert Table 1 about here.]

[Insert Figure 3 about here.]

[7] This result is not specific to the use of survey inflation as a proxy for inflation expectations. Other measures of inflation expectation based on time-series models also tend to be above the TIPS breakeven inflation in early years.

[8] Similar points are made by Shen and Corning (2001) and Shen (2006).

[9] This is in contrast to the U.K. experience. The U.K. inflation-linked gilts were first issued in early 1980s, when inflation risk premiums were presumably still quite high after the recent experience of double-digit inflation. For example, Risa (2001) estimates the inflation risk premiums to be above 2% until late 1980s. The high inflation risk premiums evidently more than offset any potential liquidity premiums, resulting in a 10-year U.K. breakeven inflation that lay consistently above survey inflation forecasts throughout the 1980s (see Shen and Corning (2001)).

[10] See Campbell and Shiller (1996), Foresi, Penati, and Pennacchi (1997), Veronesi and Yared (1999), Buraschi and Jiltsov (2005), Ang, Bekaert, and Wei (2008), among others. Hördahl and Tristani (2009) provides a nice overview of some recent development.

3 A Joint Model of Nominal and TIPS yields

This section details the no-arbitrage framework that we use to model nominal and TIPS yields jointly. The no-arbitrage approach has the benefit of avoiding the tight assumptions that go into structural, utility-based models while still allowing term structure variations to be modeled in a dynamically consistent manner by requiring the cross section of yields to satisfy the no-arbitrage restrictions.

3.1 State Variable Dynamics and the Nominal Pricing Kernel

We assume that real yields, expected inflation and nominal yields are driven by a vector of three latent variables, $x_t = [x_{1t}, x_{2t}, x_{3t}]'$, which follows a multivariate Gaussian process,

$$dx_t = \mathcal{K}(\mu - x_t)dt + \Sigma dB_t, \tag{2}$$

where B_t is an n-dimensional vector of standard Brownian motion, μ is a 3×1 constant vector, and $\mathcal{K}, \Sigma$ are 3×3 constant matrices.

The nominal short rate is specified as

$$r^N(x_t) = \rho_0^N + \rho_1^{N\prime} x_t, \tag{3}$$

where ρ_0^N is a constant and ρ_1^N is a 3×1 vector.

The nominal pricing kernel takes the form

$$dM_t^N / M_t^N = -r^N(x_t)dt - \lambda^N(x_t)' dB_t, \tag{4}$$

where the vector of nominal prices of risk is given by

$$\lambda^N(x_t) = \lambda_0^N + \Lambda^N x_t, \tag{5}$$

in which λ_0^N is a 3×1 vector and Λ^N is a 3×3 matrix. Note that the nominal term structure in this paper falls into the "essentially affine" $A_0(3)$ category as described in Duffee (2002).

3.2 Inflation and the Real Pricing Kernel

The price level processes takes the form:

$$d\log Q_t = \pi(x_t)dt + \sigma_q' dB_t + \sigma_q^{\perp} dB_t^{\perp}. \tag{6}$$

where the instantaneous expected inflation, $\pi(x_t)$, is also an affine function of the state variables in the form of

$$\pi(x_t) = \rho_0^\pi + \rho_1^{\pi\prime} x_t, \tag{7}$$

and the unexpected inflation, $\sigma_q' dB_t + \sigma_q^\perp dB_t^\perp$, is allowed to load both on shocks that move the nominal interest rates and expected inflation, dB_t, and on an orthogonal shock $dB_t^\perp$ with $dB_t dB_t^\perp = 0$. The orthogonal shock is included to capture short-run inflation variations that may not be spanned by yield curve movements.

A real bond can be thought of as a nominal asset paying realized inflation upon maturity. Therefore, the real and the nominal pricing kernels are linked by the no-arbitrage relation

$$M_t^R = M_t^N Q_t. \tag{8}$$

Applying Ito's lemma to Equation (8) and using Equations (3) to (7), the real pricing kernel can be derived as following the process

$$\begin{aligned} dM_t^R/M_t^R &= dM_t^N/M_t^N + dQ_t/Q_t + (dM_t^N/M_t^N)\cdot(dQ_t/Q_t) && (9)\\ &= -r^R(x_t)dt - \lambda^R(x_t)' dB_t - (\cdot) dB_t^\perp && (10) \end{aligned}$$

where the real short rate is given by

$$r^R(x_t) = \rho_0^R + \rho_1^{R\prime} x_t, \tag{11}$$

and the vector of real prices of risk is given by

$$\lambda^R(x_t) = \lambda_0^R + \Lambda^R x_t, \tag{12}$$

in which the coefficients are linked to their nominal counterparts by

$$\begin{aligned} \rho_0^R &= \rho_0^N - \rho_0^\pi - \frac{1}{2}(\sigma_q'\sigma_q + \sigma_q^{\perp 2}) + \lambda_0^{N\prime}\sigma_q && (13)\\ \rho_1^R &= \rho_1^N - \rho_1^\pi + \Lambda^{N\prime}\sigma_q && (14)\\ \lambda_0^R &= \lambda_0^N - \sigma_q && (15)\\ \Lambda^R &= \Lambda^N. && (16) \end{aligned}$$

3.3 Nominal and Real Bond Prices

By the definition of nominal and real pricing kernels, the time-t prices of τ-period nominal and real bonds, $P_{t,\tau}^N$ and $P_{t,\tau}^R$, are given by

$$P_{t,\tau}^i = E_t(M_{t+\tau}^i)/M_t^i, \quad i = N, R. \tag{17}$$

The bond prices can be also expressed in terms of a risk-neutral expectation as

$$P_{t,\tau}^{i}=E_{t}^{Q}\left(\exp\left(-\int_{t}^{t+\tau}r_{s}^{i}\,ds\right)\right),\quad i=N,R. \tag{18}$$

where the superscript Q denotes the risk-neutral measure.

Following the standard literature,[11] it is straightforward to derive a closed-form solution for the bond prices:

$$P_{t,\tau}^{i}=\exp\left(A_{\tau}^{i}+B_{\tau}^{i\prime}x_{t}\right),\quad i=N,R, \tag{19}$$

where

$$\frac{dA_{\tau}^{i}}{d\tau}=-\rho_{0}^{i}+B_{\tau}^{i\prime}\left(\mathcal{K}\mu-\Sigma\lambda_{0}^{i}\right)+\frac{1}{2}B_{\tau}^{i\prime}\Sigma\Sigma^{\prime}B_{\tau}^{i} \tag{20}$$

$$\frac{dB_{\tau}^{i}}{d\tau}=-\rho_{1}^{i}-\left(\mathcal{K}+\Sigma\Lambda^{i}\right)^{\prime}B_{\tau}^{i} \tag{21}$$

with initial conditions $A_{0}^{i}=0$ and $B_{0}^{i}=0_{3\times1}$.

Nominal and real yields therefore both take the affine form,

$$y_{t,\tau}^{i}=a_{\tau}^{i}+b_{\tau}^{i\prime}x_{t},\quad i=N,R, \tag{22}$$

where the factor loadings a^{i} and b^{i} are given by

$$a_{\tau}^{i}=-A_{\tau}^{i}/\tau,\;\; b_{\tau}^{i}=-B_{\tau}^{i}/\tau, \tag{23}$$

3.4 Inflation Expectations and Inflation Risk Premiums

In this model, inflation expectations also take an affine form,

$$I_{t,\tau}\triangleq E_{t}(\log(Q_{t+\tau}/Q_{t}))/\tau=a_{\tau}^{I}+b_{\tau}^{I\prime}x_{t}, \tag{24}$$

where the factor loadings a^{I} and b^{I} are given by

$$\begin{aligned} a_{\tau}^{I} &= \rho_{0}^{\pi}+(1/\tau)\rho_{1}^{\pi\prime}\int_{0}^{\tau}ds(I-e^{-\mathcal{K}s})\mu \\ b_{\tau}^{I} &= (1/\tau)\int_{0}^{\tau}ds\,e^{-\mathcal{K}^{\prime}s}\rho_{1}^{\pi}, \end{aligned}$$

From equations (22)-(24), it can be seen that both the breakeven inflation rate, defined as the difference between zero coupon nominal and real yields of identical maturities, and the

[11] See Duffie and Kan (1996) and Dai and Singleton (2000), among others.

inflation risk premium, defined as the difference between the breakeven inflation rate and the expected log inflation over the same horizon, are affine in the state variables:

$$BEI_{t,\tau} \triangleq y^N_{t,\tau} - y^R_{t,\tau} = a^N_\tau - a^R_\tau + (b^N_\tau - b^R_\tau)' x_t. \tag{25}$$

and

$$\wp_{t,\tau} \triangleq y^N_{t,\tau} - y^R_{t,\tau} - I_{t,\tau} = a^N_\tau - a^R_\tau - a^I_\tau + (b^N_\tau - b^R_\tau - b^I_\tau)' x_t. \tag{26}$$

(27)

Using Equation (8) we can write the price of a τ-period nominal bond as

$$P^N_{t,\tau} = E_t(M^R_{t+\tau} Q^{-1}_{t+\tau}) / (M^R_t Q^{-1}_t). \tag{28}$$

It is then straightforward to show that the inflation risk premium $\wp_{t,\tau}$ consists of a covariance term, $c_{t,\tau}$, and a Jensen's inequality term, $J_{t,\tau}$:

$$\wp_{t,\tau} = c_{t,\tau} + J_{t,\tau}, \tag{29}$$

where

$$\begin{aligned} c_{t,\tau} &\equiv -(1/\tau)\log[1 + cov_t(M^R_{t+\tau}/M^R_t, Q_t/Q_{t+\tau})/(E_t(M^R_{t+\tau}/M^R_t)E_t(Q_t/Q_{t+\tau}))] \\ J_{t,\tau} &\equiv -(1/\tau)[\log(E_t(Q_t/Q_{t+\tau})) - E_t(\log(Q_t/Q_{t+\tau}))]. \end{aligned}$$

In practice, the Jensen's inequality term is fairly small, and the inflation risk premium is mainly determined by the covariance between the real pricing kernel and inflation,[12] and can assume either a positive or a negative sign depending on how the two terms covaries over time.

3.5 A Four-Factor Model of TIPS Yields

Given the evidence presented in Section 2 on the existence of a TIPS-specific factor, we allow the TIPS yield to deviate from the true underlying real yield. The spread between the TIPS yields and the true real yield,

$$L_{t,\tau} = y^{\mathcal{T}}_{t,\tau} - y^R_{t,\tau}, \tag{30}$$

[12] An alternative definition of inflation risk premium used in the literature is $\hat{\wp}_{t,\tau} = y^N_{t,\tau} - \left(y^R_{t,\tau} - \frac{1}{\tau}\ln E_t\left(Q_t/Q_{t+\tau}\right)\right)$ (See Buraschi and Jiltsov (2005)). The two definitions differ by the Jensen's inequality term $J_{t,\tau}$.

mainly captures the liquidity premium TIPS investors demand for holding a less liquid instrument, but could also reflect other factors that can potentially drive a wedge between the TIPS yield and the true real yield.[13] Since the relative illiquidity of TIPS would lower TIPS prices and raise TIPS yields, we would in general expect $L_{t,\tau}$ to be positive.

We model $L_{t,\tau}$ as containing a stochastic component and a deterministic component. To model the stochastic part, we assume that the investors discount TIPS cash flows by adjusting the true instantaneous real short rate with a positive liquidity spread, resulting in a TIPS yield that exceeds the true real yield by

$$L^s_{t,\tau} = -(1/\tau)\log E^Q_t\left(\exp\left(-\int_t^{t+\tau}(r^R_s + l_s)ds\right)\right) - y^R_t, \tag{31}$$

where $L^s_{t,\tau}$ denotes the stochastic part of the TIPS liquidity premium, l_t is the instantaneous liquidity spread, and the superscript Q represents expectation taken under the risk-neutral measure.[14] This is analogous to the corporate bond pricing literature,[15] where defaultable bonds are priced by discounting future cash flows using a default- and liquidity-adjusted short rate. Note that, without the instantaneous liquidity spread l in Equation (31), the TIPS yield becomes identical to the true real yield y^R and the stochastic part of the TIPS liquidity premium becomes zeros (see Equation (18)).

The instantaneous liquidity spread l_t is given by

$$l_t \;=\; \gamma' x_t + \tilde{\gamma}\tilde{x}_t, \tag{32}$$

where γ is a 3×1 constant matrix, $\tilde{\gamma}$ is a constant and $\tilde{x}_t$ follows the Vasicek (1977) process and is independent of all other state variables contained in x_t:

$$d\tilde{x}_t = \tilde{\kappa}(\tilde{\mu} - \tilde{x}_t)dt + \tilde{\sigma}dW_t, \tag{33}$$

in which $dW_t dB_t = 0_{3\times1}$. A non-zero γ allows the liquidity premium to be correlated with the state of economy. We assume that the independent liquidity factor $\tilde{x}_t$ carries a market price of risk of

$$\tilde{\lambda}_t = \tilde{\lambda}_0 + \tilde{\lambda}_1\tilde{x}_t. \tag{34}$$

[13] Such factors include indexation lags and seasonal and short-run variations in headline CPI prices.

[14] Although our treatment is motivated by the liquidity consideration, our model of TIPS yields could be also viewed more generally as a model in which TIPS yields of all maturities are affected by a common TIPS-specific factor.

[15] See Duffie and Singleton (1999), Longstaff, Mithal, and Neis (2005), Driessen (2005).

Appendix B shows that the stochastic part of the TIPS liquidity premium takes the affine form

$$L^s_{t,\tau} = \left[\tilde{a}_\tau + (a^{\mathcal{T}}_\tau - a^R_\tau)\right] + \left[\begin{array}{cc} (b^{\mathcal{T}}_\tau - b^R_\tau)' & \tilde{b}_\tau \end{array}\right] \left[\begin{array}{c} x_t \\ \tilde{x}_t \end{array}\right] \quad (35)$$

Note that although we focus on a one-factor specification for the liquidity factor $\tilde{x}_t$, it is straightforward to extend the model to incorporate multiple liquidity factors.

To accommodate potential nonstationarities associated with the inception of the TIPS market, we also allow for a maturity-independent downward-trending deterministic component in the liquidity premium, which takes the form

$$L^d_t = (c_1/2)\left[1 - \tanh\left(c_2\left(t - c_3\right)\right)\right], \quad (36)$$

where c_1 controls the initial level of TIPS liquidity premium, c_2 controls the speed that the liquidity premium comes down over time, and c_3 controls the time when the decline is the steepest. The backwards S-shaped hyperbolic tangent function is designed to yield a liquidity premium that was high when the TIPS was first introduced but decreases over time and asymptotes towards zero. This is meant to capture the depressed demand in a fledgling market as well as its gradual adjustment towards the equilibrium.

The total liquidity premium on a τ-year TIPS in our "full model" is then the sum of the two components:

$$L_{t,\tau} = L^d_t + L^s_{t,\tau}. \quad (37)$$

with the TIPS yields given by

$$y^{\mathcal{T}}_{t,\tau} = y^R_{t,\tau} + L_{t,\tau}. \quad (38)$$

We shall refer to this model as Model L-IId. In the empirical part, we also estimate three restricted versions of the full model. The model with the least restrictions, which we shall call Model L-II, sets c_1 and hence the deterministic part of the liquidity premium to zero. The next restricted model, which we shall term Model L-I, further sets $\gamma = 0_{3\times 1}$ in Equation (32), so that the liquidity premiums on TIPS are not correlated with the nominal term structure factors. This results in a model similar to those in Driessen (2005) and Longstaff, Mithal, and Neis (2005), which model the liquidity premium in corporate bonds as a one-factor process that is independent of the credit and interest rate factors. Finally, the most restricted model, which we shall call Model NL, simply equate TIPS yields with the true real yields, i.e.,

$$y^{\mathcal{T}}_{t,\tau} = y^R_{t,\tau}. \quad (39)$$

This is the model studied by Chen, Liu, and Cheng (2005), although their specification differ from ours along other dimensions.

3.6 Discussions and Related Literature

Besides its tractability, the affine-Gaussian bond pricing framework used here allows for a flexible correlation structure between the factors. As inflation risk premiums arise from the correlation between the real pricing kernel and inflation, it is important to allow for a general correlation structure. On the other hand, the affine-Gaussian setup does not capture the *time-varying* inflation uncertainty and therefore cannot further decompose inflation risk premiums into the part due to time-varying inflation risks and time-varying prices of inflation risk. Nonetheless, the affine-Gaussian model could still provide a reasonable estimate of inflation risk premium, similar to the way it generates reasonable measures of term premia despite its inability to capture time-varying interest rate volatilities. We view the general affine-Gaussian model as an important benchmark to be investigated before more sophisticated models can be explored.

Some of the models studied in the earlier literature, such as Pennacchi (1991) and Campbell and Viceira (2001), can be viewed as a special case of this model. For example, Pennacchi (1991)'s model corresponds to a two-factor version of our model with constant market price of risk. Campbell and Viceira (2001) is also a special case of this model, but their real term structure has a lower dimension than the nominal term structure (nominal yields are described by 2 factors and real yields are described by 1 factor). In this paper, we let the real term structure have as many factors as the nominal term structure; if the real term structure is truly lower-dimensional than nominal term structure, we let the data decide on that. A related point is that in a reduced-form setup like ours, one cannot make a distinction between *real* and *nominal* factors, as correlation effects in the general model make such a distinction meaningless.

To our knowledge, this is the first paper to model liquidity premium in TIPS in a no-arbitrage framework. There is a large literature studying the pricing implications of indexed bonds using data from other countries with longer histories of issuing inflation linked securities,[16] There have also been studies of US real yields and inflation risk premia that use

[16] See Woodward (1990), Barr and Campbell (1997), Evans (1998), Remolona, Wickens, and Gong (1998) and Risa (2001) for the UK, Kandel, Ofer, and Sarig (1996) for Isreal and Hördahl and Tristani (2007) for the Euro

realized inflation[17] or survey inflation forecasts[18] without incorporating information from indexed bonds. Due to the short data history, studies using U.S. inflation-linked assets, including TIPS or inflation swaps, are fairly recent and relatively few.[19] In addition, most of these studies take TIPS at their face value, and typically find a real yield that seems too high as well as inflation risk premium estimates that are insignificant or negative in the early sample when TIPS was first introduced. In contrast, this paper shows that there is a persistent liquidity premium component in TIPS prices, which, if not properly taken account of, will bias the results.

4 Empirical Methodology

4.1 Identification and Summary of Models

We only impose restrictions that are necessary for achieving identification so as to allow a maximally flexible correlation structure between the factors, which has shown to be critical in fitting the rich behavior in risk premiums that we observe in the data.[20] In particular, we employ the following normalization:

$$\mu = 0_{3\times1}, \quad \Sigma = \begin{bmatrix} 0.01 & 0 & 0 \\ \Sigma_{21} & 0.01 & 0 \\ \Sigma_{31} & \Sigma_{32} & 0.01 \end{bmatrix}, \quad \mathcal{K} = \begin{bmatrix} \mathcal{K}_{11} & 0 & 0 \\ 0 & \mathcal{K}_{22} & 0 \\ 0 & 0 & \mathcal{K}_{33} \end{bmatrix}, \quad \tilde{\sigma} = 0.01. \tag{40}$$

and leave all other parameters unrestricted. It can be shown that any specification of the affine Gaussian model that has a $\mathcal{K}$ matrix with all-real eigenvalues can be transformed to this form.[21]

area.

[17] See, e.g., Ang, Bekaert, and Wei (2008).

[18] See, e.g., Pennacchi (1991), Foresi, Penati, and Pennacchi (1997) and Brennan, Wang, and Xia (2004).

[19] To our knowledge, this paper and a contemporaneous study by Chen, Liu, and Cheng (2005) are the first two to study TIPS in a no-arbitrage framework. Other papers analyzing TIPS or inflation swaps include Chernov and Mueller (2007), Adrian and Wu (2008), Haubrich, Pennacchi, and Ritchken (2008), Christensen, Lopez, and Rudebusch (2008), Grishchenko and Huang (2008).

[20] See Duffie and Kan (1996) and Dai and Singleton (2000).

[21] With normalization (40), the specification we estimate in this paper can be shown to be equivalent to that of Sangvinatsos and Wachter (2005). The main difference between their paper and ours is empirical: they use a much longer sample, which would be desirable if the relationship between inflation and interest rates can be assumed to be stable.

To summarize, we estimate four models that differ in how TIPS liquidity premium is modeled, including one model that equates TIPS yields with true real yields and assumes zero liquidity premium on TIPS (Model NL), a model with an independent liquidity factor (Model L-I), a model allowing the TIPS liquidity premium to be correlated with other state variables (Model L-II), and the most general model (Model L-IId) in which TIPS liquidity premiums also contain a deterministic trend. Table 2 summarizes the models and the parameter restrictions. Two things are worth noting here: First, as shown in Section 3.5, Models NL, L-I and L-II can all be considered as special cases of Model L-IId. Second, Model NL has a 3-factor representation of TIPS yields, while in all other models TIPS yields have a 4-factor specification.

[Insert Table 2 about here.]

4.2 Data

We use 3- and 6-month, 1-, 2-, 4-, 7-, and 10-year nominal yields and CPI-U data from January 1990 to March 2007. In contrast, our TIPS yields are restricted by data availability and cover a shorter period from January 1999 to March 2007, with the earlier period without TIPS data (1990-1998) treated as missing observations. Both nominal and TIPS yields, shown in Figure 4, are based on zero-coupon yield curves fitted at the Federal Reserve Board[22] and are sampled at the weekly frequency, while CPI-U inflation is available monthly and assumed to be observed on the last Wednesday of the current month.[23] As discussed in more details in Appendix A, shorter-maturity TIPS yields are affected to a larger degree by the problem of indexation lags. In addition, no more than one TIPS with a maturity of below 5 years existed before 2002, hence near-maturity zero-coupon TIPS yields cannot be reliably estimated during that period. We therefore only use 5-, 7-, and 10-year TIPS yields in our estimation. Because the models we estimate do not accommodate seasonalities, we use seasonally-adjusted CPI inflation in the estimation. TIPS are indexed to non-seasonally-adjusted CPI; however, our use of seasonally-adjusted CPI is not expected to matter much for the relatively long maturities

[22] Nominal yields are based on the Svensson (1994) curve specification for the entire sample; TIPS yields are based on the Nelson and Siegel (1987) curve specification prior to January 2004 and the Svensson (1995) curve specification thereafter. See Gürkaynak, Sack, and Wright (2007, 2009) for more details.

[23] Here we abstract from the real-time data issue by assuming that investors correctly infer the current inflation rate in a timely fashion.

that we examine.

The sample period 1990-2007 was chosen as a compromise between having more data in order to improve the efficiency of estimation, and having a more homogeneous sample so as to avoid possible structural breaks[24] in the relation between term structure variables and inflation. This sample period roughly spans Greenspan's tenure and a little bit of Bernanke's as well.

Results from Kim and Orphanides (2006) suggest that the standard technique of estimating our models using only nominal and TIPS yields and inflation data for a relative short sample period of 1990-2007 will almost surely run into small sample problems: variables like $\mathcal{K}$ and Λ^N may not be reliably estimated, and the estimated model typically generates a path of expected future short rate over the next 5 to 10 years that is too smooth compared to survey-based measures of market expectations.[25] Therefore, we supplement the aforementioned data with survey forecasts of 3-month T-bill yields to help stabilize the estimation and to better pin down some of the model parameters. Note that survey *inflation* forecast data are *not* used in the estimation, as we would like to get a measure of inflation expectations from yields data, independently of other sources of information about inflation expectation.

To be specific, we use the 6-month- and 12-month-ahead forecasts of the 3-month T-bill yield from Blue Chip Financial Forecasts, which are available monthly, and allow the size of the measurement errors to be determined within the estimation. We also use long-range forecast of 3-month T-bill yield over the next 5 to 10 years from the same survey, which are available twice a year, with the standard deviation of its measurement error fixed at a fairly large value of 0.75% at an annual rate. This is done to prevent the long-horizon survey forecasts from having unduly strong influence on the estimation, and can be viewed as similar to a quasi-informative prior in a Bayesian estimation.

We denote the short-horizon survey forecasts by $f_{t,6m}$ and $f_{t,12m}$ and the long-range forecast by $f_{t,long}$. The standard deviation of their measurement errors are denoted denoted $\delta_{f,6m}, \delta_{f,12m}$ and $\delta_{f,long}$, respectively. These survey-based forecasts are taken as noisy measures of corresponding true market expectations. More specifically, we have that for the short-

[24] The 1979-83 episode of Fed's experiment with reserve targeting is a well known example.

[25] Results for our models based on the conventional estimation method are available upon request.

term survey forecasts,

$$f_{t,\tau} = E_t(y^N_{t+\tau,3m}) + \epsilon^f_{t,\tau}, \qquad \epsilon^f_{t,\tau} \sim N(0, \delta^2_{f,\tau}), \tag{41}$$

for $\tau = 6m$ or $12m$, and for long-range forecasts,

$$f_{t,long} = E_t\left(\frac{1}{5}\int_{5y}^{10y} y^N_{t+\tau,3m} d\tau\right) + \epsilon^f_{t,long}, \qquad \epsilon^f_{t,long} \sim N(0, 0.0075^2), \tag{42}$$

where the corresponding model forecasts, $E_t(y^N_{t+\tau,3m})$, can be solved from Equations (2) and (22) and can be shown to be

$$E_t(y^N_{t+\tau,3m}) = a^f_\tau + b^{f\prime}_\tau x_t, \tag{43}$$

where the factor loadings a^f_τ, b^f_τ are given by

$$a^f_\tau = a^N_{3m} + b^{N\prime}_{3m}\left(I_{3\times 3} - e^{-\kappa\tau}\right) \tag{44}$$

$$b^f_\tau = e^{-\kappa'\tau} b^N_{3m} \tag{45}$$

[Insert Figure 4 about here.]

4.3 Estimation Methodology

We rewrite the model in a state-space form and estimate it by the Kalman filter. Denote by $\mathrm{x}_t = [q_t, x_t', \tilde{x}_t]'$ the augmented state vector including the log price level, $q_t \equiv \log(Q_t)$, and the TIPS liquidity factor, $\tilde{x}_t$. The state equation is derived as Euler discretization of equations (2), (6) and (33) and can be written in a matrix form as

$$\mathrm{x_t = G_h + \Gamma_h x_{t-h} + \eta^x_t}. \tag{46}$$

where

$$G_h = \begin{bmatrix} \rho_0^\pi h \\ \mathcal{K}\mu h \\ \tilde{\kappa}\tilde{\mu} h \end{bmatrix}, \ \Gamma_h = \begin{bmatrix} 1 & \rho_1^{\pi\prime} h & 0 \\ 0 & I_{3\times 3} - \mathcal{K}h & 0 \\ 0 & 0 & 1 - \tilde{\kappa}h \end{bmatrix} \text{ and } \eta^{\mathrm{x}}_t = \begin{bmatrix} \sigma_q'\eta_t + \sigma_q^\perp \eta_t^\perp \\ \Sigma\eta_t \\ \tilde{\sigma}\tilde{\eta}_t \end{bmatrix}$$

in which η_t, $\eta_t^\perp$, and $\tilde{\eta}_t$ are independent of each other, and have the distribution

$$\eta_t \sim N(0, hI_{n\times n}), \ \ \eta_t^\perp \sim N(0, h), \ \ \tilde{\eta}_t \sim N(0, h). \tag{47}$$

We specify the set of nominal yields as $Y_t^N = \{y_{t,\tau_i}^N\}_{i=1}^7$, and the set of TIPS yields as $Y_t^{\mathcal{T}} = \{y_{t,\tau_i}^{\mathcal{T}}\}_{i=1}^3$, and collect in y_t all data used in the estimation, including the log price level q_t, all nominal yields Y_t^N, all TIPS yields $Y_t^{\mathcal{T}}$, and 6 month-ahead, 12 month-ahead, and long-horizon forecasts of future 3-month nominal yield:

$$\mathrm{y}_t = [q_t, Y_t^N, Y_t^{\mathcal{T}}, f_{t,6m}, f_{t,12m}, f_{t,long}]'. \tag{48}$$

We assume that all nominal and TIPS yields and survey forecasts of nominal short rate are observed with error. The observation equation therefore takes the form

$$\mathrm{y}_t = A + B\mathrm{x}_t + \mathrm{e}_t \tag{49}$$

where

$$A = \begin{bmatrix} 0 \\ A^N \\ \tilde{a} + A^{\mathcal{T}} \\ a_{6m}^f \\ a_{12m}^f \\ a_{long}^f \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & B^{N\prime} & 0 \\ 0 & B^{\mathcal{T}\prime} & \tilde{b}' \\ 0 & b_{6m}^{f\prime} & 0 \\ 0 & b_{12m}^{f\prime} & 0 \\ 0 & b_{long}^{f\prime} & 0 \end{bmatrix}, \quad \mathrm{e}_t = \begin{bmatrix} 0 \\ e_t^N \\ e_t^{\mathcal{T}} \\ e_{t,6m}^f \\ e_{t,12m}^f \\ e_{t,long}^f \end{bmatrix}, \tag{50}$$

in which A^i and B^i, $i = N, \mathcal{T}$ collect the nominal and TIPS yield loadings on x_t, respectively, in obvious ways, and $\tilde{a}$ and $\tilde{b}$ collects the TIPS yield loadings on the independent liquidity factor $\tilde{x}_t$. We assume a simple i.i.d. structure for the measurement errors so that

$$e_{t,\tau_i}^N \sim N(0, \delta_{N,\tau_i}^2), \quad e_{t,\tau_i}^{\mathcal{T}} \sim N(0, \delta_{\mathcal{T},\tau_i'}^2), \quad e_{t,\tau_i}^f \sim N(0, \delta_{f,\tau_i}^2) \tag{51}$$

Based on the state equation (46) and the observation equation (49), it is straightforward to implement the Kalman-filter and perform the maximum likelihood estimation. The details are given in Appendix C. Two aspects are worth noting here: first, the log price level q_t is nonstationary, so we use a diffuse prior for q_t when initializing the Kalman filter. Second, inflation, survey forecast and TIPS yields are not available for all dates, which introduces missing data in the observation equation and are handled in the standard way by allowing the dimensions of the matrices A and B in Equation (49) to be time-dependent (see, for example, Harvey (1989, sec. 3.4.7)).

Note that all four versions of our models can be accommodated in the above setup. To implement Model NL without the liquidity premium, one simply set $\tilde{\gamma} = 0$ and $\gamma = 0_{3\times1}$, and fix $\tilde{\mu}$, $\tilde{\kappa}$, $\tilde{\lambda}_0$ and $\tilde{\lambda}_1$ at arbitrary values as those variables do not enter the likelihood function of Model NL. To implement Model L-I with the independent liquidity factor only, one would fix $\gamma = 0_{3\times 1}$.

To facilitate the estimation and also to make the results easily replicable, we follow the following steps in estimating all our models:

1. We first perform a "pre"-estimation where a set of preliminary estimates of the parameters governing the nominal term structure is obtained using nominal yields and survey forecasts of 3-month T-bill yield alone.
2. Based on these estimates and data on nominal yields, we can obtain a preliminary estimate of the state variables, x_t.
3. A regression of the monthly inflation onto estimates of x_t obtained in the second step gives a preliminary set of estimates of the parameters governing the inflation dynamics.
4. Finally, these preliminary estimates are used as starting values in the full, one-step estimation of all model parameters.

5 Empirical Results

In this Section, we discuss and compare the empirical performance of the various Models. As we shall see, there are notable differences between the model equating TIPS yields with the true real yields (Model NL) and the models that allow the two sets of yields to differ by a liquidity premium component (Models L-I, L-II and L-IId).

5.1 Parameter Estimates and Overall Fit

Parameter Estimates

Table 3 reports the parameter estimates for all four models. Four things are worth noting here: First, estimates of parameters governing the nominal term structure are almost identical across the three models; under our set-up, these parameters seem to be fairly robustly estimated. In particular, all estimations uncover a very persistent factor with a half life of about

13 years. Note also that all four models exhibit a similar fit to nominal yields and survey forecasts of nominal short-term interest rates. For example, the nominal yield fitting errors ($\delta_{N,\tau}$) are fairly small in all four models: except for the 3-month yield which has δ_N of about 10 basis points, other maturity yields have δ_N of 5 basis points or less.

Second, there are notable differences in the estimates of parameters governing the expected inflation process. In particular, the loading of the instantaneous inflation on the second and the most persistent factor is negligible in the model without a TIPS liquidity factor (Model NL) but becomes positive and significant in the three models with a TIPS liquidity factor (Models L-I, L-II and L-IId). As a result, the monthly autocorrelation of one-year-ahead inflation expectation is about 0.9 in Model NL but above 0.99 in all other models. As we will see later, the lack of persistence in the inflation expectation process prevents Model NL from generating meaningful variations in longer-term inflation expectations as we observe in the data.

Third, parameter estimates for the TIPS liquidity factor process are significant in both Models L-I and L-II and assume similar values. The estimated market price of liquidity risk carries the expected negative sign, as one would generally expect any deterioration of liquidity conditions to occur during bad economic times. In Models L-II and L-IId, the loading of the instantaneous TIPS liquidity premium on each of the three state variables, γ, is estimated to be indistinguishable from zero; however, a Wald test shows that they are jointly significant.

Finally, the fit to TIPS yields are significantly better in models with a TIPS liquidity factor, as can be seen from the smaller estimates of the standard deviations of TIPS measurement errors. For example, for the 10-year TIPS, the fitting errors from the L-I and L-II models are 6 basis points, while the fitting error from Model NL is 35 basis points. The fitting errors are found to have a substantial serial correlation. For example, in the case of the 5-year TIPS, we obtain a weekly AR(1) coefficient of 0.96, 0.91, and 0.91 for Model NL, L-I model, and L-II model, respectively. The finding of serial correlation in term structure fitting errors are however a fairly common phenomenon, and have been noted by Chen and Scott (1993) and others.

[Insert Table 3 about here.]

Overall Fit

Panel A of Table 4 reports some test statistics that compare the overall fit of the three models. We first report two information criteria commonly used for model selection, the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). Both information criteria are minimized by the most general model, Model L-IId.

We also report results from likelihood ratio (LR) tests of the three restricted models, Models NL, L-I and L-II, against their more general counterparts, Model L-I, L-II and L-IId, respectively. Compared to Model L-I, Model NL imposes the restriction $\tilde{\gamma} = 0$. The standard likelihood ratio test does not apply here because the nuisance parameters, $\tilde{\kappa}$, $\tilde{\mu}$, $\tilde{\lambda}_0$ and $\tilde{\lambda}_1$, are not identified under the null (Model NL) and appear only under the alternative (Model L-I).[26] Here we follow Garcia and Perron (1996) and calculate a conservative upper bound for the significance of the likelihood ratio test statistic as suggested by Davies (1987). In particular, denote by θ the vector of nuisance parameters of size s, and define the likelihood ratio statistic as a function of θ:

$$LR(\theta) = 2\left[\log L_1(\theta) - \log L_0\right],$$

where $L_1(\theta)$ is the likelihood value of the alternative model for any admissible values of the nuisance parameters $\theta \in \Omega$, and L_0 is the maximized likelihood value of the null model. For an estimated LR value of M, Davies (1987) derives an upper bound for its significance as

$$\Pr\left[\sup_{\theta \in \Omega} LR(\theta) > M\right] < \Pr\left[LR(\theta) > M\right] + VM^{\frac{1}{2}(s-1)} \exp^{-(M/2)} \frac{2^{-s/2}}{\Gamma(s/2)}$$

where $\Gamma(.)$ represents the Gamma function and V is defined as

$$V = \int_{\Omega} \left| \frac{\partial LR(\theta)}{\partial \theta} \right| d\theta.$$

Garcia and Perron (1996) further assumes that the likelihood ratio statistic has a single peak at $\widehat{\theta}$, which reduces V to $2M^{\frac{1}{2}}$. Apply this procedure to testing the null of Model NL against the alternative of Model L-I gives an estimate of the maximal p value of essentially zero, hence Model NL is overwhelmingly rejected in favor of Model L-I. With the LR statistic estimated as $-2\log\left[L(\mathrm{NL})/L(\mathrm{L{-}I})\right] = 4617.67$ with 1 degree of freedom, we feel confident that using alternative econometric procedures to deal with the nuisance parameter problem is unlikely to overturn the rejection.

The LR test of Model L-I against Model L-II, on the other hand, is fairly standard. Based on the likelihood estimates of the two models, we obtain a LR statistic of

$$-2\log\left[L(\mathrm{L{-}I})/L(\mathrm{L{-}II})\right] = 8.6,$$

[26] For discussions on testing with nuisance parameters, see, for example, Davies (1977, 1987, 2002) and Andrews and Ploberger (1994, 1995).

and are able to reject Model L-I in favor of Model L-II at the 5% level based on a χ_3^2 distribution.

Finally, Model L-II is rejected in favor of the full model, Model L-IId, based on the Davies (1987) procedure, with a large LR test value of 433.41.

[Insert Table 4 about here.]

5.2 Fitting TIPS Yields and TIPS Breakeven Inflation

The estimated standard deviations of TIPS measurement errors reported in the previous section suggest that Model NL has trouble fitting the TIPS yields. This section looks at the fit of TIPS yields and TIPS breakeven inflation across models in more details.

The three left (right) panels of Figures 5 plot the actual and the model-implied TIPS yields (TIPS breakeven inflation) based on Models NL, L-I and L-IId, respectively, together with the real yields (the true breakeven inflation) as implied by the three models.[27] By construction, the model-implied TIPS yields and the model-implied real yields coincide under Model NL.

The top left panel of Figures 5 shows that, according to Model NL, the downward path of 10-year TIPS yields from 1999 to 2004 is part of a broader decline in real yields since the early 1990's, with real yields estimated to have come down from a level as high as 7% in the early 90's to about 2% around 2003. During the same period, the 10-year nominal yield declined from around 9% to a little over 4%. Therefore, Model NL attributes the decline of 10-year nominal yield entirely to a lower real yield, leaving little room for lower inflation expectation or lower inflation risk premiums. While there is some empirical evidence suggesting that long-term inflation expectations may have edged down during this period,[28] it is hard to imagine economic mechanisms that would generate such a large decline in long-term real yields. Furthermore, although Model NL matches the general trend of TIPS yields during this period, it has problem fitting the time variations, frequently resulting in large fitting errors. In contrast, the decline in real yields as implied by Models L-I and L-IId, plotted in the middle

[27] Model-implied true breakevens are calculated as the difference between model-implied nominal yields and model-implied real yields of comparable maturities. Model-implied values are calculated using smoothed estimates of the state variables. Results for Model L-II are similar to those for Model L-IId and are not reported.

[28] See Kozicki and Tinsley (2006), for example.

and the bottom left panels, is less pronounced and more gradual. With the flexibility brought by the additional liquidity premium factor, these two models are also able to fit TIPS yields almost perfectly.

The problem with Model NL can be further seen by looking at the model-implied 10-year breakeven inflation, as shown in the upper right panel of Figures 5. The 10-year breakeven rate implied by Model NL, which by construction should equal the 10-year TIPS breakeven inflation, appears too smooth compared to its data counterpart and misses most of the short-run variations in the actual series. The poor fitting of the TIPS breakeven inflation rates highlights the difficulty that the 3-factor model has in fitting nominal and TIPS yields simultaneously.[29] In contrast, the 10-year breakeven inflation rates implied by Models L-I and L-IId, shown in the middle and bottom left panels, show substantial variations that roughly match those of the actual TIPS breakeven inflation rate. In particular, the model-implied and the TIPS-based breakeven inflation rates peak locally at the beginning of 2000, in the middle of 2001 and 2002, and so on, and the magnitude of their variation are also similar. In these two models, the gap between the model-implied and the TIPS-based breakeven inflation rates is the sum of TIPS liquidity premium and TIPS measurement errors.

To quantify the improvement in terms of the model fit, Panels B and C of Table 4 provide three goodness-of-fit statistics for TIPS yields at the 5-, 7- and 10-year maturities and TIPS breakeven inflation at the 7- and 10-year maturities, respectively. The first statistic, CORR, is simply the sample correlation between the fitted series and its data counterpart. Consistent with the visual impression from Figure 5, allowing a TIPS liquidity premium component improves the model fit for raw TIPS yields and even more so for TIPS breakeven inflation, with the correlation between model-implied 10-year TIPS breakeven and the data counterpart rising from 32% to over 90% once we move from Model NL to the other models. The next two statistics are based on one-step-ahead model prediction errors from the Kalman Filter, v_t, defined in Equation (C-15) in Appendix C, and are designed to capture how well each model can explain the data without resorting to large exogenous shocks or measurement errors. More specifically, the second statistic is the root mean squared prediction errors (RMSE), and the third statistic is the coefficient of determination (R^2), defined as the percentage of in-sample

[29] Given the flexible nature of latent-factor model used in this paper, it is possible that there may exist another local maximum of the likelihood function under which the TIPS yields are fitted better, producing a closer match between the model-implied and the TIPS-based breakeven inflation rates. However, such a fit would certainly come at the expense of other undesirable features of the model.

variations of each data series explained by the model:

$$R^2 = 1 - \frac{\sum_{t=2}^{T} v_t^2}{\sum_{t=2}^{T} (y_t - \overline{y})^2}, \tag{52}$$

where y_t is the observed series and $\overline{y}$ denotes its sample mean.[30] As we can see from Panels B and C of Table 4, based on these two metrics, the improvement moving from Model NL to models with a liquidity factor is notable even for TIPS yields. In other words, the seemingly reasonable fit of Model NL for raw TIPS yields is only achieved by assuming large exogenous shocks to the state variables. The fit of Model NL for TIPS breakevens is even worse, with a R^2 of -18.12% at the 10-year maturity. In comparison, all other models with a TIPS liquidity factor explain more than 88% of the time variations of TIPS breakevens at both maturities.

[Insert Figure 5 about here.]

5.3 Matching Survey Inflation Forecasts

It is conceivable that a model with more parameters like Model L-IId could generate smaller in-sample fitting errors for variables whose fit is explicitly optimized, but produce undesirable implications for variables not used in the estimation. To check this possibility, we examine the model fit for a variable that is not used in our estimation but is of enormous economic interest, the expected inflation. In particular, we examine how closely the model-implied inflation expectations mimic survey-based inflation forecasts. Ang, Bekaert, and Wei (2007) recently provide evidence that survey inflation forecasts outperforms various other measures of inflation expectations in predicting future inflation. In addition, survey inflation forecast has the benefit of being a real-time, model-free measure, and hence not subject to model estimation errors or look-ahead biases that could affect measures based on in-sample fitting of realized inflation.[31]

[30] Unlike in a regression setting, a negative value of R^2 could arise because the model expectation and the prediction errors are not guaranteed to be orthogonal in a small sample.

[31] Alternatively, we could compare the out-of-sample forecasting performance of various models. However, we doubt the usefulness of such an exercise for two reasons. First, the sample available for carrying out such an exercise is extremely limited due to the relatively short sample of TIPS. In addition, the large idiosyncratic fluctuations associated with commonly used price indices would lead to substantial sampling variability in any metric of forecast performance we use and further complicate the inference problem.

Panel D of Table 4 reports the three goodness-of-fit statistics, CORR, RMSE and R^2, for 1- and 10-year ahead inflation forecasts from the SPF. Among the models, Model NL generates inflation expectations that agree least well with survey inflation forecasts, producing large RMSEs and small R^2 statistics at both horizons. This poor fit is especially prominent at the 1-year horizon: the RMSE is large, the correlation between the model and survey forecast is essentially 0, and the R^2 is highly negative at -52%. In contrast, all other models, which have a liquidity factor, generate a reasonable fit with the survey forecasts at both horizons. The best fit is achieved by Models L-II and L-IId, both of which generate correlations above 80% and small RMSEs at both horizons and explain a large amount of sample variations in survey inflation forecasts. Models L-II and L-IId also improve notably upon Model L-I, suggesting that some cyclical variations in TIPS yields might not be due to movements in the real yields. Overall, Model L-IId does not seem to suffer from an overfitting problem. As we will see from later sections, this model also generate sensible implications for TIPS liquidity premiums and inflation risk premiums, further supporting our conclusion.

A visual comparison of the model-implied inflation expectations and survey forecasts can help us understand the results in Table 4. The left panels of Figure 6 plot the 1-year inflation expectation based on Models NL, L-I and L-IId, together with the survey forecast. It can be seen that the Model-NL-implied 1-year inflation expectation contains a large amount of short-run fluctuations that are not shared by its survey counterpart. It also fails to capture the downward trend in survey inflation forecasts during much of the sample period. In comparison, implied 1-year inflation expectation based on the other models show a visible downward trend, consistent with the survey evidence. It is interesting that although Models L-I and L-IId exhibit similar fit to TIPS yields and TIPS breakevens, as can be seen from Figure 5, they are more differentiable based on their implications for inflation expectations. In particular, while the 1-year inflation expectation implied by Model L-IId bears a high resemblance to the 1-year survey forecast, the same series implied by Model L-I appears to be much more variable than the survey counterpart.

It is also not surprising that Model NL produces a larger RMSEs for 10-year inflation expectations than the L-I and L-II models: the upper middle panel of Figure 6 shows that Model NL completely misses the downward trend in the 10-year survey inflation forecast since the early 1990s and implies a 10-year inflation expectations that moved little over the sample period. This is the flip side of the discussions in Section 5.2, where we see a Model-

NL-implied 10-year real rate that is too variable and is used by the model to explain the entire decline in nominal yields in the 1990s. Overall, the near-constancy of the long-term inflation expectation is the most problematic feature of Model NL. Models L-I and L-IId, on the other hand, produce 10-year inflation expectations that are clearly downward trending, though the model-implied values are a bit lower than the survey forecast in the early 1990s, as shown in the two lower panels in the middle column of Figure 6. As can be recalled from Figure 5, the long-term real yields in these models also display a downward trend, but a much weaker one compared to that in Model NL.

Finally, the three right panels of Figure 6 plot the model-implied inflation risk premiums at the 1- and 10-year horizons for the three models under consideration. One immediately notable feature is that Model-NL implies an inflation risk premium, shown in the upper right panel, that is negative and increasing over time in the 1990-2007 period. In contrast, as mentioned in Section 2, most of the existing studies not using TIPS find that average inflation risk premium has been positive historically. Furthermore, studies such as Clarida and Friedman (1984) indicate that the inflation risk premium likely was positive and substantial in the early 1980s and probably has come down since then. As can be seen from Figure 7, which plots the 10-year inflation risk premium estimates together with the 95% confidence bands for the three models, even after we take into account sampling uncertainties, long-term inflation risk premiums implied by Model NL remain negative over most of the sample period. In comparison, the two models that allow for a liquidity premium, Models L-I and L-IId, both generate 10-year inflation risk premiums that are positive and fluctuate in the 0 to 1% range over the same sample period. The short-term inflation risk premiums implied by these two models, on the other hand, are fairly small, consistent with our intuition.

[Insert Figure 6 about here.]

[Insert Figure 7 about here.]

5.4 Summary

In summary, we find that Model NL, which equates TIPS yields with true underlying real yields, fares poorly along a number of dimensions, including generating a poor fit with the TIPS data as well as unreasonable implications for inflation expectations and inflation risk premiums. This underscores the need to take into account a liquidity premium in modeling TIPS yields. In contrast, models that allows for a TIPS liquidity premium, Models L-I and L-II, improves upon Model NL in all three aspects and in particular produce long-term inflation expectations that agree quite well with survey forecasts.

Among models allowing a liquidity premium in TIPS yields, Models L-II and Model L-IId generate short-term inflation expectations that matches survey counterparts better than Model L-I, suggesting it is important to allow for a systematic component in TIPS liquidity premiums.

Finally, a likelihood ratio test rejects Models L-II in favor of our preferred model, Model L-IId, which features a deterministic trend in TIPS liquidity premium that is designed to capture the "newness effect" during the early years of TIPS. In the remainder of our analysis we'll be mainly focusing on this model.

6 TIPS Liquidity Premium

6.1 Model Estimates of TIPS Liquidity Premiums

Once we estimate the model parameters and the state variables, we can calculate the TIPS liquidity premiums at various maturities based on Equation (35). The top and the middle panels of Figure 8 plot the 5-, 7- and 10-year liquidity premiums implied by Models L-I and L-II, respectively, while the bottom panel shows the the deterministic and the stochastic components of the liquidity premiums based on Model L-IId.

Three things are worth noting from this graph: First, all three panels show that liquidity premiums exhibit substantial time variations at all maturities. The substantial variabilities at maturities as long as 10 years are in part due to the fact that the independent liquidity factor is estimated to be very persistent under the risk-neutral measure. As can be seen from Table 3, the risk-neutral persistence of the liquidity factor, $\tilde{\kappa}^* = \tilde{\kappa} + \tilde{\sigma}\tilde{\lambda}_1$, is estimated to be very small at around 0.1 in all models and is tightly estimated, with a standard error of about 0.006. In contrast, the persistence parameter under the physical measure, $\tilde{\kappa}$, is not as precisely estimated, with typical values of around 0.20 and typical standard errors of around 0.27.

Second, the term structure of TIPS liquidity premiums is relatively flat at all times under Model L-I, while under Model L-II, the term structure has a mild downward-sloping behavior during the 2001-2004 period. Technically, a market price of risk on the independent liquidity factor that is on average positive, as is the case in all three models here, would contribute to a downward-sloping term structure of TIPS liquidity premium.[32] This is in contrast to the standard one-factor interest rate models, where the market price of interest rate risk is typically found to be negative. Although the TIPS liquidity premiums in Models L-II and L-IId are also driven by the nominal bond factors, x_t, in addition to $\tilde{x}_t$, a variance decomposition indicates that the TIPS-specific factor $\tilde{x}_t$ drives most of the variations in TIPS liquidity premium. Nonetheless, as we've seen in Section 5, allowing the TIPS liquidity premiums to depend on nominal bond factors seems important in explaining the dynamics of TIPS yields.

Finally, all three models imply that the TIPS liquidity premiums were fairly high (1-2 %) when TIPS were first introduced, had been on a downward trajectory until around 2004, and have stayed at a relatively low level from 2005 onwards. The deterministic trend implied by Model L-IId came down from about 120 basis points in 1999 to below 10 basis points by the end of 2004. After removing the downward trend, the stochastic liquidity premiums appear more stationary and largely vary between -50 and 50 basis points.

[Insert Figure 8 about here.]

6.2 What Drives the TIPS Liquidity Premiums?

The behavior of the liquidity premiums we have seen in Figure 8 seems broadly consistent with the perception that TIPS market liquidity conditions have improved over time. In this section, we examine this issue more closely. One difficulty in this regard is the lack of precise, real-time measures of the TIPS market liquidity. One measure that has been used in the literature[33] is the 13-week moving average of weekly TIPS turnover, defined as the weekly average of

[32] For example, it is straightforward to show that under Model L-I, the slope of the TIPS liquidity premium curve is given by

$$\frac{\partial L_{t,\tau}}{\partial \tau}|_{\tau=0} = 0.5\tilde{\kappa}^*(\tilde{\mu}^* - \tilde{x}_t),$$

the unconditional mean of which is given by

$$0.5\tilde{\kappa}^*(\tilde{\mu}^* - \tilde{\mu}) = -0.5\tilde{\sigma}(\tilde{\lambda}_0 + \tilde{\lambda}_1\tilde{\mu}).$$

where the equality comes from Equation (B-6). Therefore, if the average market price of liquidity risk, $\tilde{\lambda}_0 + \tilde{\lambda}_1\tilde{\mu}$, is positive, the term structure of the liquidity premium will be on average downward-sloping.

[33] See Sack and Elsasser (2004).

daily TIPS transaction volumes divided by the TIPS outstanding at the end of the current month.[34] As can be seen in the top panel of Figure 3, the turnover in TIPS remained low up to mid 2002 and then rose substantially, suggesting an improvement in the liquidity conditions of the TIPS market in recent years.[35] The rise coincides roughly with the decline in our TIPS liquidity premium estimates. In particular, all our model-based TIPS liquidity premiums (Model L-I, L-II, L-IId) have correlations with this measure more negative than -73%.

However, the turnover may be affected by factors other than the liquidity conditions in the TIPS market. For example, there is a large empirical literature documenting a positive contemporaneous correlation between price volatility and trading volumes, independent of current market liquidity conditions.[36] In particular, as shown in the middle panel of Figure 9, interest rate volatilities declined markedly during the latter half of the sample period, which might have contributed to the drop in TIPS turnover after 2005. Indeed, evidence from TIPS bid-ask spreads, arguably a better measure of liquidity conditions in the TIPS market, indicates that the early improvement in TIPS liquidity may not have been largely reversed in the most recent sample period, as one might assume based on the rapid decline in TIPS turnover since 2005. For example, two informal surveys of the primary dealers conducted by the Federal Reserve Bank of New York in 2003 and 2007, shown in Table 5, find that the average bid-ask spreads on TIPS continue to narrow across all maturities during this period.[37] Unfortunately, a long history of this measure is unavailable.

[Insert Table 5 about here]

To quantify the effects of various factors on our estimates of TIPS liquidity premiums, we therefore regress the 5-, 7- and 10-year TIPS liquidity premiums from Model L-IId onto three explanatory variables, the first of which being the TIPS turnover ratio.

$$L_{t,\tau}^{L-IId} = \alpha + \beta_1 \mathit{TURNOVER}_t + \beta_2 \mathit{IMPVOL}_t + \beta_3 \mathit{ASW}_t^{nom} + \varepsilon_t^L, \;\; \tau = 5, 7, 10. \quad (53)$$

[34] TIPS transaction volumes are reported by primary dealers in Government Securities Dealers Reports (FR-2004) collected by the Federal Reserve Bank of New York, and the amount of TIPS outstanding are based on Monthly Statement of the Public Debt issued by the Treasury.

[35] The decline in the liquidity premium during the 2003-2004 period may also be driven by the increased market attention to inflation risk amid a booming economy and rising oil prices.

[36] See Karpoff (1987) for a review of the empirical evidence.

[37] Results from the 2003 survey are quoted in Sack and Elsasser (2004).

The second explanatory variable, *IMPVOL*, is the implied volatility from options on 10-year Treasury note futures and is included to control for the positive contemporaneous correlation between price volatility and trading volumes. The last variable, *ASW*, represents the difference between the on-the-run and the off-the-run 10-year Treasuries par asset swap spreads and can be considered as a market-based measure of the liquidity premiums on the nominal Treasury market. In a fixed-income asset swap, one party exchanges the fixed-rate cash flows from the underlying security for a floating-rate cash flow where the floating rate is typically quoted as 6-month LIBOR plus a spread, the asset swap spread. Asset swap spreads varies over time and across securities according to the perceived default and liquidity risk of the underlying security. Because both nominal Treasury and TIPS are usually considered free of default risks, their asset swap spreads can be regarded as a good measure of the liquidity premiums in those assets. Consistent with their relative liquidity, we usually observe increasingly more negative spreads as we move across asset classes in the order of TIPS, off-the-run nominal Treasuries and on-the-run nominal Treasuries. For our purposes, an ideal measure of the relative illiquidity of TIPS compared to off-the-run nominal Treasuries would be the difference between the TIPS and the off-the-run nominal asset swap spreads.[38] Unfortunately TIPS asset swaps only started trading in 2006; we therefore use the difference between the off-the-run and the on-the-run 10-year nominal Treasuries as an approximation. The daily correlation between the two spreads is 0.90 over the period of March 16, 2006 to November 13, 2009.

Figure 9 plots all three explanatory variables; their correlations with TIPS liquidity premium estimates from Model LII-d and with each other are reported in Panel B of Table 6. The three variables are not highly correlated with each other, but all have large correlations with the liquidity premium estimates. The results from this regression are reported in Panel A of Table 6. The coefficients on all three variables are statistically significant and carry the expected sign. In particular, a lower TIPS turnover raises the TIPS liquidity premiums, but the effect will be smaller if the lower turnover is accompanied by a lower volatility.[39] Together these three variables explain about 80% of the variations in TIPS liquidity premium estimates at all maturities.[40] It remains an interesting topic for future research to see whether

[38] Such a measure is used in a recent study by Campbell, Shiller, and Viceira (2009) as they focus on a more recent sample of July 2007 to April 2009.

[39] Results using measures of realized volatilities are similar.

[40] We note, however, that this type of regression should be viewed only as a rough gauge of the relationship; quantities like turnover are not expected to have a simple linear relationship to the liquidity premium. For example, although the turnover for nominal Treasury securities have also risen substantially in our sample period,

our model-based measures of TIPS liquidity premiums correlate with other measures of TIPS market liquidity.[41]

[Insert Table 6 about here]

[Insert Figure 9 about here]

The results in Table 6 suggest one simple way to adjust the TIPS breakeven inflation for the liquidity effects. Consider the difference between the SPF forecast of average inflation over the next ten years, $SPF_{t,10}$, and the 10-year TIPS breakeven inflation, $BEI_{t,10}$, and regress it on the same three right-hand-side variables as in Equation (53):[42]

$$SPF_{t,10} - BEI^{\mathcal{T}}_{t,10} = \alpha + \beta_1 \textbf{\textit{TURNOVER}}_t + \beta_2 \textbf{\textit{IMPVOL}}_t + \beta_3 \textbf{\textit{ASW}}^{nom}_t + \varepsilon_t \tag{54}$$

Assuming that the 10-year inflation risk premiums do not vary too much over time, the fitted values from this regression can be thought of as a rough gauge of the liquidity premium component in the difference between the TIPS breakeven inflation and the survey counterpart, the other components being inflation risk premiums and measurement errors. Using the coefficient estimates, one can generate a real-time estimate of TIPS liquidity premiums, which can be added to the observed TIPS breakeven inflation to produce a liquidity-adjusted series. The results from such an exercise are plotted in Figure 10, which shows the adjusted 10-year TIPS breakeven inflation together with the raw series as well as the model-implied true breakeven inflation. The adjusted BEI using this simple method is much closer to the true BEI based on our full model. It is much more variable than the true BEI, especially in the early years, although the difference has narrowed towards the end of our sample period.

that probably had a very small effect on the liquidity premium, as the turnover was already high and liquidity premiums were likely negligible for these securities.

[41] For example, Fleming and Krishnan (2009) develops several measures of liquidity conditions in the TIPS market using high-frequency trading data. However, their measures are only available for a very short period of time due to data availability.

[42] The Cleveland Fed used a similar regression to adjust TIPS BEI, in which they regress the BEI-survey forecast differential on the level and the squared level of nominal Treasury off-the-run premiums plus a constant. Our analysis suggests that such a regression might be biased as it misses the persistent downward trend in the TIPS liquidity premiums in the early years. The Cleveland Fed stopped updating this adjustment in October 2008 citing “the extreme rush to liquidity is affecting the accuracy of the estimates” (http://www.clevelandfed.org/research/data/tips/index.cfm).

While the qualitative behavior of TIPS liquidity premium thus seems reasonable, our estimates of L_t does seem large in comparison with that of the corporate bonds. Corporate bonds, including those with the highest credit rating (AAA/AA), tend to trade infrequently at once a day or less; TIPS, in comparison, trade more often than AAA/AA corporate bonds even during the early years when liquidity was the poorest. The bid-ask spread in TIPS has also been substantially smaller than those of corporate bonds. In this regard, our estimates of TIPS liquidity premium around 1.5% in the early years seems puzzlingly high, in comparison with investment-grade corporate bonds which typically trade at a liquidity premiums of about 50 basis points.[43] The high value of the TIPS liquidity premium in the early years may partly reflect a depressed demand for TIPS due to the newness of the security and the relative complexity of TIPS payoff structure. Furthermore, a popular belief that TIPS are tax-disadvantageous for taxable investors[44] may have further depressed the demand for TIPS.

7 Variance Decomposition

7.1 Decomposing TIPS Yields and TIPS Breakeven Inflation

In this section we examine how much variations in TIPS yields and TIPS breakeven inflation rate can be attributed to variations in TIPS liquidity premiums. Using Equations (2) and (30), we can decompose TIPS yields, $y^{\mathcal{T}}_{t,\tau}$, and TIPS breakven inflation, $BEI^{\mathcal{T}}_{t,\tau}$, into different components:

$$y^{\mathcal{T}}_{t,\tau} = y^{R}_{t,\tau} + L_{t,\tau}, \qquad BEI^{\mathcal{T}}_{t,\tau} = I_{t,\tau} + \wp_{t,\tau} - L_{t,\tau}, \tag{55}$$

where $y^{R}_{t,\tau}$ is the true underlying real yields, $L_{t,\tau}$ is the TIPS liquidity premiums, $I_{t,\tau}$ is the expected inflation over the next τ periods, and $\wp_{t,\tau}$ is the inflation risk premiums.

Table 7 reports the variance decomposition results based on Equation (55) and Model L-IId estimates, using either the unconditional (Panel A) or the instantaneous (Panel B) variance-covariance matrix of the state variables. A time series plot of the decomposition is shown in Figure 11.

[Insert Table 7 about here.]

[43] See Longstaff, Mithal, and Neis (2005), de Jong and Driessen (2006) for example.

[44] See, for example, the discussion in Hein and Mercer (2003).

For TIPS yields, real yields dominate TIPS liquidity premiums in accounting for the time variations both unconditionally and instantaneously. In comparison, TIPS liquidity premiums are more important in driving TIPS breakeven inflation, explaining about 23% of its unconditional variations at all three maturities, although expected inflation still accounts for the majority of time variations in TIPS breakeven inflation. The contribution of TIPS liquidity premiums is even larger instantaneously and explains about 43% of the short-run variations in TIPS breakeven inflation. The last observation suggests that one should be especially cautious in interpreting short-run variations in TIPS breakeven inflation solely in terms of changes in inflation expectation or inflation risk premiums.

7.2 Decomposing Nominal Yields

Although it is not the focus of the current paper, our models can also be used to separate nominal yields into real yields, expected inflation and inflation risk premiums:

$$y_{t,\tau}^{N} = y_{t,\tau}^{R} + I_{t,\tau} + \wp_{t,\tau}. \tag{56}$$

Figure 12 plots the 1- and 10-year nominal yields and their constituents, whereas Table 8 reports the variance decomposition results.

These results indicate that, at least during our sample period, real yield changes explain more than half of the variations in nominal yields at all maturities. Inflation expectation explains about 40% (20%) of the variations in the 1-quarter (10-year) nominal yield. Inflation risk premiums account for the remaining 10% of the nominal yield changes. This stands in contrast to previous studies using a longer sample period but not using TIPS yields, which typically find relatively smooth real yields but volatile inflation expectation or inflation risk premiums.[45] The limited evidence we have so far from TIPS seems to suggest that real yields may also vary considerably over time.

8 Conclusion

In this paper, we document that there exists a TIPS-specific factor that is important for explaining TIPS yield variations but not as crucial for explaining nominal interest rate movement, and

[45] See Ang, Bekaert, and Wei (2008, Figure 2) and Chernov and Mueller (2007, Figure 7) for example.

provide evidence that this factor might be reflecting a liquidity premium in TIPS yields.

We then develop a joint no-arbitrage term structure model of nominal and TIPS yields incorporating a rich specification of the TIPS liquidity premiums. The main findings can be summarized as the following. First, we find that our estimated liquidity premium was quite large ($\sim 1\%$) until about 2003 but has come down in recent years, consistent with the common perception that TIPS market liquidity has improved in recent years. Second, our TIPS liquidity premium estimates contain a persistent downward trend in its early years, that is best modeled as a deterministic trend reflecting gradual market acceptance of TIPS as a new debt instrument. Finally, we show that ignoring the liquidity premium components leads to a poor model fit of TIPS yields, TIPS breakeven inflation and survey inflation forecasts.

TIPS breakeven inflation has been increasingly gaining attention as a measure of investors' inflation expectations that is available in real-time and at high frequencies. However, our results raise caution in interpreting movement in TIPS breakeven inflation solely in terms of changing inflation expectations, as substantial liquidity premiums and inflation risk premiums could at times drive a large wedge between between the two. In an encouraging note, the reduced liquidity premium that we find for the most recent period (2005-2007) raises the possibility that, going forward, the TIPS yields may be a better gauge of the true real yields. However, given that TIPS is less liquid than nominal Treasury securities, we caution that TIPS liquidity premiums might rise again in times of financial market stress. A better understanding of the determinants of TIPS liquidity premiums and the sources of its variation remains an interesting topic for future research.

Appendix

A More on the TIPS Data

This appendix is devoted to a more detailed discussion of the TIPS data. Figure A1 shows the smoothed TIPS par yield curves on June 9, 2005 in the top panel and on June 9, 1999 in the bottom panel, together with the actual traded TIPS yields that were used to create the smoothed TIPS par-yield and zero-coupon curves. The smoothing is done by assuming that the zero-coupon TIPS yield curve follows the 4-parameter Nelson-Siegel (1987) functional form up to the end of 2003 and the 6-parameter Svensson (1994) functional form thereafter,[46] and minimizing the fitting error for the actual traded TIPS securities. The substantial increase in the number of points in the top panel reflects the growth of the TIPS market. Note that in 1999 there is essentially one data point on the curve between the 0 and 5 years maturity (corresponding to the 5-year TIPS issued in 1997), thus the TIPS term structure in the short-maturity region of 0-5 years cannot be determined reliably. With more points across the maturity spectrum in 2005, shorter maturity TIPS yields can be determined more reliably than in 1999.

[insert Figure A1 about here]

Still, the analysis of the short-maturity TIPS are complicated by the indexation lag and seasonality issues. Note that the TIPS payments are indexed to the CPI 2.5 months earlier, thus TIPS yields contain some amount of realized inflation, often referred to as the "carry effect". The yield that is more relevant to policy makers is the one that takes out this realized inflation – the so-called carry-adjusted yields, which can be heuristically represented as

$$y_{t,\tau}^{\mathcal{T},CA} = y_{t,\tau}^{\mathcal{T}} + (\delta/\tau)\pi_{t-\delta,t}, \tag{A-1}$$

where $\pi_{t-\delta,t} = \log(Q_t/Q_{t-\delta})/\delta$ denotes the inflation realized between time $t-\delta$ and t, with $\delta = 2.5$ months.[47] Because the realized inflation $\pi_{t-\delta,t}$ can be quite volatile, the carry-unadjusted yield $y^{\mathcal{T}}$ and the carry-adjusted yield $y^{\mathcal{T},CA}$ can differ significantly, though the difference becomes smaller with increasing maturity, due to the δ/τ factor in Equation (A-1). Figure A2 shows the carry-adjusted and unadjusted TIPS yields for 5-year and 10-year maturities. It can be seen that indeed the 5-year carry-adjusted and unadjusted TIPS yields show greater discrepancies than the 10-year ones. This has been

[46] In comparison, the zero-coupon nominal yield curve is assumed to follow the 6-parameter Svensson (1994) functional form during the entire sample period. In the case of TIPS, however, there were not enough securities in the early years to pin down as many parameters. See Gurkaynak, Sack, and Wright (2007a, 2007b) for details.

[47] Note that equation (A-1) takes out realized inflation in the previous 2.5 months but makes no adjustment for the lack of inflation protection during the last 2.5 months prior to the maturity date.

particularly the case in 2005, during which large fluctuations in oil prices caused sharp short-term fluctuations in inflation. The expression (A-1) for the carry adjustment is only a schematic one. The actual carry-adjustment is further complicated by the fact that TIPS is indexed to the seasonally-unadjusted CPI, rather than the seasonally-adjusted CPI. While one could in principle explicitly model seasonality (and carry effects) within the dynamic model of inflation and term structure, such a procedure may be more prone to specification errors than the case in which these effects are corrected at the input stage.[48]

[insert Figure A2 about here]

As noted in the main text, since data reliability and indexation lags pose larger problems to shorter-maturity TIPS, in this paper we focus on long-maturity TIPS yield for which the effects of indexation lag and seasonality are less important. While the analysis of shorter-maturity TIPS yields is an important problem in itself, it deserves a fuller treatment elsewhere.[49] The 5-, 7- and 10-year carry-unadjusted TIPS yields used in this analysis can be viewed as the carry-corrected TIPS yield plus a measurement error, as suggested by Equation (A-1).

B Stochastic TIPS Liquidity Premium

Since $\tilde{x}_t$ is independent of the other state variables in x_t, the first term on the right-hand side of Equation (31) can be written as the sum of two components:

$$\begin{aligned} & -(1/\tau)\log E_t^Q\left(\exp\left(-\int_t^{t+\tau}(r_s^R+l_s)ds\right)\right) \\ = & -(1/\tau)\log E_t^Q(e^{-\int_t^{t+\tau}\tilde{\gamma}\tilde{x}_s ds}) - (1/\tau)\log E_t^Q(e^{-\int_t^{t+\tau}(\rho_0^R+(\rho_1^R+\gamma)'x_s)ds}) \end{aligned} \tag{B-2}$$

The first component can be solved to be

$$-(1/\tau)\log E_t^Q(e^{-\int_t^{t+\tau}\tilde{\gamma}\tilde{x}_s ds}) = \tilde{a}_\tau + \tilde{b}_\tau \tilde{x}_t \tag{B-3}$$

where $\tilde{a}$ and $\tilde{b}$ has the familiar form of factor loadings in a one-factor Vasicek model:

$$\tilde{a}_\tau = \tilde{\gamma}\left[(\tilde{\mu}^* - \frac{\tilde{\sigma}^2}{2\tilde{\kappa}^*})(1-\tilde{b}_\tau) + \frac{\tilde{\sigma}^2}{4\tilde{\kappa}^*}\tau\tilde{b}_\tau^2\right] \tag{B-4}$$

$$\tilde{b}_\tau = \tilde{\gamma}\frac{1-\exp(-\tilde{\kappa}^*\tau)}{\tilde{\kappa}^*\tau}, \tag{B-5}$$

[48] See Ghysels (1993) for a recent discussion of the Sims (1974)-Sargent (1978) debate that bears on this issue.

[49] Taking a proper account of the seasonality and carry effects is important to TIPS traders, but here in this paper we are concerned with more basic questions.

in which the risk-neutral $\tilde{\kappa}^*, \tilde{\mu}^*$ are given by

$$\tilde{\kappa}^* = \tilde{\kappa} + \tilde{\sigma}\tilde{\lambda}_1, \qquad \tilde{\mu}^* = (\tilde{\kappa}\tilde{\mu} - \tilde{\sigma}\tilde{\lambda}_0)/\tilde{\kappa}^*, \tag{B-6}$$

The second component can be shown to take the form

$$-(1/\tau)\log E_t^Q(e^{-\int_t^{t+\tau}(\rho_0^R+(\rho_1^R+\gamma)'x_s)ds}) = a_\tau^{\mathcal{T}} + b_\tau^{\mathcal{T}\prime} x_t \tag{B-7}$$

where $a^{\mathcal{T}}$, $b^{\mathcal{T}}$ are given by

$$a_\tau^{\mathcal{T}} = -A_\tau^{\mathcal{T}}/\tau, \tag{B-8}$$

$$b_\tau^{\mathcal{T}} = -B_\tau^{\mathcal{T}}/\tau, \tag{B-9}$$

where

$$\frac{dA_\tau^{\mathcal{T}}}{d\tau} = -\rho_0^R + B_\tau^{\mathcal{T}\prime}\left(\mathcal{K}\mu - \Sigma\lambda_0^R\right) + \frac{1}{2}B_\tau^{\mathcal{T}\prime}\Sigma\Sigma' B_\tau^{\mathcal{T}} \tag{B-10}$$

$$\frac{dB_\tau^{\mathcal{T}}}{d\tau} = -\left(\rho_1^R + \gamma\right) - \left(\mathcal{K} + \Sigma\Lambda^R\right)' B_\tau^{\mathcal{T}} \tag{B-11}$$

with initial conditions $A_0^{\mathcal{T}} = 0$ and $B_0^{\mathcal{T}} = 0_{n\times1}$.

Taken together, we have that

$$\begin{aligned} L_{t,\tau}^s &= \left(\tilde{a} + a_\tau^{\mathcal{T}}\right) + \begin{bmatrix} b_\tau^{\mathcal{T}\prime} & \tilde{b} \end{bmatrix} \begin{bmatrix} x_t \\ \tilde{x}_t \end{bmatrix} - y_t^R \\ &= \left[\tilde{a}_\tau + (a_\tau^{\mathcal{T}} - a_\tau^R)\right] + \begin{bmatrix} (b_\tau^{\mathcal{T}} - b_\tau^R)' & \tilde{b}_\tau \end{bmatrix} \begin{bmatrix} x_t \\ \tilde{x}_t \end{bmatrix} \end{aligned} \tag{B-12}$$

where the second equality comes from Equation (23).

C Kalman Filter and Likelihood Function

We use the Kalman filter to compute optimal estimates of the unobservable state factors based on all available information. For example, given the initial guess for the state factors $\widehat{\mathrm{x}}_0$, it follows from the state equation (46) that the optimal estimate of the state factor x_t at time $t = h$ is given by

$$\widehat{\mathrm{x}}_{h,0} \triangleq E(\mathrm{x_h} \mid \Im_0) = \mathrm{G_h} + \Gamma_\mathrm{h}\widehat{\mathrm{x}}_0,$$

which implies that we carry the error of the initial guess to all subsequent estimations. More generally, we have

$$\widehat{\mathrm{x}}_{t,t-h} \triangleq E(\mathrm{x_t} \mid \Im_{\mathrm{t-h}}) = \mathrm{G_h} + \Gamma_\mathrm{h}\widehat{\mathrm{x}}_{\mathrm{t-h,t-h}}. \tag{C-13}$$

for any time period t.

The variance-covariance matrix of the estimation error can be derived as

$$\begin{aligned} Q_{t,t-h} &\triangleq E\left[(\mathrm{x_t} - \widehat{\mathrm{x}}_{\mathrm{t,t-h}})(\mathrm{x_t} - \widehat{\mathrm{x}}_{\mathrm{t,t-h}})'\right] \\ &= E\left\{\left[\Gamma_h(\mathrm{x_{t-h}} - \widehat{\mathrm{x}}_{\mathrm{t-h}}) + \eta_t^{\mathrm{x}}\right]\left[\Gamma_h(\mathrm{x_{t-h}} - \widehat{\mathrm{x}}_{\mathrm{t-h}}) + \eta_t^{\mathrm{x}}\right]'\right\} \\ &= \Gamma_h Q_{t-h,t-h}\Gamma_h' + \Omega_{t-h}^{\mathrm{x}}, \end{aligned} \tag{C-14}$$

where $\Omega_{t-h}^{\mathrm{x}} = E\left[\eta_t^{\mathrm{x}}\eta_t^{\mathrm{x}\prime}\right]$.

Given any forecast for $\mathrm{x_t}$, we can compute a forecast for the observable variables based on all time $t-h$ information:

$$\widehat{\mathrm{y}}_{t,t-h} \triangleq E\left[\mathrm{y_t} \mid \Im_{\mathrm{t-h}}\right] = A + B\widehat{\mathrm{x}}_{t,t-h},$$

the forecast error of which is given by

$$\mathrm{v_t} \triangleq \mathrm{y_t} - \widehat{\mathrm{y}}_{t,t-h} = B(\mathrm{x_t} - \widehat{\mathrm{x}}_{t,t-h}) + \mathrm{e_t}. \tag{C-15}$$

The conditional variance-covariance matrix of this estimation error can then be computed as

$$\begin{aligned} V_{t,t-h} &\triangleq E\left\{(\mathrm{y_t} - \widehat{\mathrm{y}}_{\mathrm{t,t-h}})(\mathrm{y_t} - \widehat{\mathrm{y}}_{\mathrm{t,t-h}})'\right\} \\ &= BQ_{t,t-h}B' + \Omega_t^e \end{aligned} \tag{C-16}$$

where

$$\Omega_t^e = E\left[\mathrm{e_t}\mathrm{e_t}'\right].$$

The next step is to update the equation for the state variables. Before doing this, we need to recover the distribution of $\mathrm{x_t} \mid \mathrm{y_t}$. The conditional covariance between the forecasting errors for state variables and that for observation variables takes the form of

$$\begin{aligned} V_{t,t-h}^{xy} &\triangleq E\left\{(\mathrm{y_t} - \widehat{\mathrm{y}}_{\mathrm{t,t-h}})(\mathrm{x_t} - \widehat{\mathrm{x}}_{\mathrm{t,t-h}})'\right\} \\ &= BE\left[(\mathrm{x_t} - \widehat{\mathrm{x}}_{\mathrm{t,t-h}})(\mathrm{x_t} - \widehat{\mathrm{x}}_{\mathrm{t,t-h}})'\right] \\ &= BQ_{t,t-h} \end{aligned} \tag{C-17}$$

The conditional joint distribution for $\mathrm{y_t}$ and $\mathrm{x_t}$ is therefore

$$\begin{bmatrix} \mathrm{y_t} \\ \mathrm{x_t} \end{bmatrix} \mid \Im_{t-h} \sim N\left(\begin{bmatrix} a + F\widehat{\mathrm{x}}_{t,t-h} \\ \widehat{\mathrm{x}}_{t,t-h} \end{bmatrix}, \begin{bmatrix} BQ_{t,t-h}B' + \Omega_t^e & BQ_{t,t-h} \\ Q_{t,t-h}B' & Q_{t,t-h} \end{bmatrix}\right),$$

which implies that conditional on $\mathrm{y_t}$, $\mathrm{x_t}$ is also distributed normal:

$$\mathrm{x_t} \mid \mathrm{y_t} \sim N\left(\widehat{\mathrm{x}}_{t,t}, Q_{t,t}\right),$$

where

$$\widehat{\mathrm{x}}_{t,t} = \widehat{\mathrm{x}}_{t,t-h} + Q_{t,t-h} B' V_{t,t-h}^{-1} \left(\mathrm{y_t} - \widehat{\mathrm{y}}_{\mathrm{t,t-h}} \right) \tag{C-18}$$

$$Q_{t,t} = Q_{t,t-h} - Q_{t,t-h} B' V_{t,t-h}^{-1} B Q_{t,t-h}; \tag{C-19}$$

The variance-covariance matrix of the updated state vector, $Q_{t,t}$, will be smaller than the previous estimate, $Q_{t,t-h}$, due to the new information coming in through the observation of $\mathrm{y_t}$.

We estimate the parameters by maximizing the log-likelihood function

$$\begin{aligned}\sum_{t=h}^{Th} \log f\left(\mathrm{y_t} | \Im_{\mathrm{t-h}}\right) = & -\frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{i=h}^{t}\log|V_i| \\ & -\frac{1}{2}\sum_{i=h}^{t}\left[\left(\mathrm{y_t} - \mathrm{A} - \mathrm{B}\widehat{\mathrm{x}}_{\mathrm{t,t-h}}\right)' V_t^{-1}\left(\mathrm{y_t} - \mathrm{A} - \mathrm{B}\widehat{\mathrm{x}}_{\mathrm{t,t-h}}\right)\right].\end{aligned} \tag{C-20}$$

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Table 1: TIPS Liquidity Factor

Panel A: Regression Analysis

Coefficients				Adj.
Constant	3-month	2-year	10-year	R^2
		In Level		
3.4637	-0.4519	0.6516	-0.4384	32.3%
(0.1138)	(0.0345)	(0.0479)	(0.0346)	
		In Weekly Changes		
0.0032	0.0110	-0.0850	0.5687	59.0%
(0.0025)	(0.0334)	(0.0409)	(0.0366)	

Panel B: Variance Explained by Principal Components

PC	nominal yields only	nominal and TIPS yields
1st	75.17	71.11
2nd	93.25	87.26
3rd	97.44	94.72
4th	99.36	97.58

Panel C: Correlation of Principal Components

		nominal and TIPS yields			
		PC1	PC2	PC3	PC4
nominal	PC1	0.97	-0.21	-0.15	0.01
yields	PC2	0.12	0.86	-0.49	0.02
alone	PC3	0.04	0.08	0.20	0.97
	PC4	-0.01	-0.03	-0.06	0.01

Panel A regresses 10-year TIPS breakevens on 3-month, 2-year and 10-year nominal yields using weekly data from Jan. 6, 1999 to Mar. 14, 2007. Panel B report the cumulative percentage of total variance of weekly yield changes explained by the first four principal components, where the second and the third column report results for nominal yields alone and nominal and TIPS yields combined, respectively. The in-sample correlation between the two sets of principal components are reported in Panel C.

Table 2: Summary of Models

Model	Restrictions and Idenfications
Model NL	$\gamma = 0_{3\times1}, \quad \tilde{\gamma} = 0, \quad c_1 = 0, \quad \tilde{\kappa}, \tilde{\mu}, \tilde{\lambda}_0, \tilde{\lambda}_1$, c_2 and c_3 unidentified
Model L-I	$\gamma = 0_{3\times1}, \quad \tilde{\gamma}, \tilde{\kappa}, \tilde{\mu}, \tilde{\lambda}_0, \tilde{\lambda}_1$ unrestricted, $\quad c_1 = 0, \quad c_2$ and c_3 unidentified
Model L-II	$\gamma, \tilde{\gamma}, \tilde{\kappa}, \tilde{\mu}, \tilde{\lambda}_0, \tilde{\lambda}_1$ unrestricted, $\quad c_1 = 0, \quad c_2$ and c_3 unidentified
Model L-IId	$\gamma, \tilde{\gamma}, \tilde{\kappa}, \tilde{\mu}, \tilde{\lambda}_0, \tilde{\lambda}_1$, c_1, c_2 c_3 unrestricted

Table 3: Parameter Estimates

	Model NL		Model L-I		Model L-II		Model L-IId	
State Variables Dynamics								
$dx_t = \mathcal{K}(\mu - x_t)dt + \Sigma dB_t$								
$\mathcal{K}_{11}$	0.8587	(0.5206)	0.8676	(0.5227)	0.8573	(0.4948)	0.8374	(0.4606)
$\mathcal{K}_{22}$	0.0529	(0.0787)	0.0573	(0.0793)	0.0555	(0.0794)	0.0529	(0.0781)
$\mathcal{K}_{33}$	1.5219	(0.8048)	1.5321	(0.8040)	1.5394	(0.7729)	1.5672	(0.7574)
$100 \times \Sigma_{21}$	-0.3346	(0.3200)	-0.3098	(0.3105)	-0.3232	(0.3101)	-0.3175	(0.3101)
$100 \times \Sigma_{31}$	-4.5682	(9.1343)	-4.5353	(9.0421)	-4.4144	(8.1903)	-4.0846	(6.7681)
$100 \times \Sigma_{32}$	-0.5524	(0.2581)	-0.5449	(0.2522)	-0.5494	(0.2538)	-0.5362	(0.2469)
Nominal Pricing Kernel								
$dM_t^N/M_t^N = -r^N(x_t)dt - \lambda(x_t)'dB_t$,								
$r^N(x_t) = \rho_0^N + \rho_1^{N'} x_t, \quad \lambda(x_t) = \lambda_0^N + \Lambda^N x_t$								
ρ_0^N	0.0419	(0.0135)	0.0434	(0.0116)	0.0428	(0.0125)	0.0422	(0.0132)
$\rho_{1,1}^N$	2.8343	(5.2462)	2.8318	(5.2564)	2.7671	(4.7530)	2.5726	(3.9118)
$\rho_{1,2}^N$	0.4825	(0.1249)	0.4797	(0.1239)	0.4809	(0.1248)	0.4675	(0.1207)
$\rho_{1,3}^N$	0.6089	(0.0378)	0.6177	(0.0403)	0.6180	(0.0387)	0.6195	(0.0377)
$\lambda_{0,1}^N$	0.4362	(0.2228)	0.4107	(0.1807)	0.4147	(0.1946)	0.4236	(0.2449)
$\lambda_{0,2}^N$	-0.1818	(0.8732)	-0.2933	(0.7752)	-0.2447	(0.8142)	-0.2049	(0.8544)
$\lambda_{0,3}^N$	-0.0471	(3.3761)	-0.4308	(2.8805)	-0.2597	(3.1019)	-0.1136	(3.3049)
$[\Sigma\Lambda^N]_{11}$	-0.5330	(1.7491)	-0.5489	(1.7874)	-0.5288	(1.6390)	-0.4592	(1.3462)
$[\Sigma\Lambda^N]_{21}$	1.7508	(4.9179)	1.7894	(4.9932)	1.7104	(4.5060)	1.5341	(3.7315)
$[\Sigma\Lambda^N]_{31}$	3.7651	(15.9132)	3.8379	(16.1348)	3.6471	(14.3685)	3.1187	(11.0801)
$[\Sigma\Lambda^N]_{12}$	-0.0274	(0.2452)	-0.0277	(0.2406)	-0.0272	(0.2370)	-0.0419	(0.2252)
$[\Sigma\Lambda^N]_{22}$	-0.2865	(0.1339)	-0.2854	(0.1318)	-0.2849	(0.1330)	-0.2752	(0.1326)
$[\Sigma\Lambda^N]_{32}$	-0.6948	(0.8960)	-0.6677	(0.8587)	-0.6854	(0.8420)	-0.6296	(0.6714)
$[\Sigma\Lambda^N]_{13}$	-0.0717	(0.3145)	-0.0749	(0.3224)	-0.0725	(0.3155)	-0.0551	(0.3238)
$[\Sigma\Lambda^N]_{23}$	0.6171	(0.2355)	0.6329	(0.2429)	0.6303	(0.2397)	0.6314	(0.2404)
$[\Sigma\Lambda^N]_{33}$	0.6551	(2.1043)	0.6812	(2.1309)	0.6634	(1.9803)	0.5887	(1.7096)

Table 3 Continued

	Model NL		Model L-I		Model L-II		Model L-IId	
Log Price Level								
$d\log Q_t = \pi(x_t)dt + \sigma_q' dB_t + \sigma_q^{\perp} dB_t^{\perp},\ \pi(x_t) = \rho_0^{\pi} + \rho_1^{\pi'} x_t$								
ρ_0^{π}	0.0271	(0.0035)	0.0280	(0.0069)	0.0239	(0.0082)	0.0236	(0.0084)
$\rho_{1,1}^{\pi}$	-0.7843	(2.8227)	0.8836	(1.4332)	0.0746	(0.6436)	0.0729	(0.5890)
$\rho_{1,2}^{\pi}$	0.0766	(0.1062)	0.2563	(0.0707)	0.2575	(0.1262)	0.2443	(0.1248)
$\rho_{1,3}^{\pi}$	-0.5094	(0.3937)	0.1216	(0.1276)	-0.0165	(0.2079)	-0.0221	(0.2107)
$100 \times \sigma_{q,1}$	-0.1724	(0.0651)	-0.1176	(0.0720)	-0.1050	(0.0763)	-0.1076	(0.0757)
$100 \times \sigma_{q,2}$	-0.0231	(0.0803)	0.0597	(0.0750)	0.0651	(0.0757)	0.0506	(0.0750)
$100 \times \sigma_{q,3}$	-0.0016	(0.0656)	0.0354	(0.0594)	0.0306	(0.0611)	0.0358	(0.0609)
$100 \times \sigma_q^{\perp}$	0.7795	(0.0288)	0.7279	(0.0241)	0.7144	(0.0245)	0.7144	(0.0245)
TIPS Liquidity Premium								
$l_t = \tilde{\gamma}\tilde{x}_t + \gamma' x_t,\ d\tilde{x}_t = \tilde{\kappa}(\tilde{\mu} - \tilde{x}_t)dt + \tilde{\sigma} dW_t,\ \tilde{\lambda}_t = \tilde{\lambda}_0 + \tilde{\lambda}_1 \tilde{x}_t.$								
$\tilde{\gamma}$			0.6114	(0.0411)	0.6152	(0.0415)	1.2545	(0.0914)
$\tilde{\kappa}$			0.2083	(0.2655)	0.2206	(0.2630)	0.6037	(0.3973)
$\tilde{\mu}$			0.0218	(0.0113)	0.0157	(0.0115)	0.0003	(0.0105)
$\tilde{\lambda}_0$			0.3213	(0.6657)	0.2851	(0.5090)	-0.0263	(0.3356)
$\tilde{\sigma}\tilde{\lambda}_1$			-0.1091	(0.2652)	-0.1209	(0.2627)	-0.1472	(0.4020)
γ_1					-0.6765	(1.2459)	-2.9521	(5.7532)
γ_2					-0.0179	(0.1547)	0.2739	(0.1717)
γ_3					-0.0833	(0.2509)	-1.0137	(0.3285)
c_1							1.1871	(0.0310)
c_2							0.0014	(0.0001)
c_3							731467.911	(25.2593)

Table 3 Continued

	Model NL		Model L-I		Model L-II		Model L-IId	
Measurement Errors: Nominal Yields								
$100 \times \delta_{N,3m}$	0.1005	(0.0026)	0.1012	(0.0027)	0.1012	(0.0027)	0.1013	(0.0027)
$100 \times \delta_{N,6m}$	0.0231	(0.0016)	0.0221	(0.0016)	-0.0222	(0.0016)	-0.0224	(0.0017)
$100 \times \delta_{N,1y}$	0.0532	(0.0017)	0.0530	(0.0018)	0.0530	(0.0018)	0.0531	(0.0017)
$100 \times \delta_{N,2y}$	0.0000	(140.6008)	-0.0000	(59.2489)	-0.0000	(50.6589)	-0.0000	(27.7162)
$100 \times \delta_{N,4y}$	0.0293	(0.0012)	0.0294	(0.0012)	0.0294	(0.0013)	0.0294	(0.0012)
$100 \times \delta_{N,7y}$	0.0000	(120.1913)	0.0000	(0.9395)	0.0000	(44.6608)	-0.0000	(0.9058)
$100 \times \delta_{N,10y}$	0.0489	(0.0018)	0.0490	(0.0019)	0.0490	(0.0019)	0.0489	(0.0018)
Measurement Errors: TIPS Yields								
$100 \times \delta_{\mathcal{T},5y}$	0.4307	(0.0953)	0.0654	(0.0059)	0.0657	(0.0060)	-0.0000	(4.8466)
$100 \times \delta_{\mathcal{T},7y}$	0.3511	(0.0832)	-0.0021	(0.0414)	-0.0004	(0.2385)	-0.0428	(0.0035)
$100 \times \delta_{\mathcal{T},10y}$	0.3578	(0.0802)	0.0647	(0.0060)	-0.0643	(0.0059)	-0.0520	(0.0038)
Measurement Errors: Survey Forecasts of Nominal Short Rate								
$100 \times \delta_{f,6m}$	0.1760	(0.0135)	0.1758	(0.0134)	0.1758	(0.0134)	0.1758	(0.0134)
$100 \times \delta_{f,12m}$	0.2261	(0.0197)	0.2260	(0.0196)	0.2260	(0.0197)	0.2261	(0.0196)

This table reports parameter estimates and standard errors for all four models we estimate. Standard errors are calculated using the BHHH formula and are reported in parentheses.

Table 4: Specification Tests

		Model NL	Model L-I	Model L-II	Model L-IId
		Panel A: Overall model fit			
No. of parameters		42	47	50	53
Log likelihood		53,663.65	55,972.49	55,976.70	56,193.41
AIC		-107,243.30	-111,850.97	-111,853.40	-112,280.81
BIC		-107,041.70	-111,625.37	-111,613.39	-112,026.41
LR p-value		0.00*	0.04	0.00*	
		Panel B: Fitting TIPS yields			
5-year	CORR (in %)	93.14	99.41	99.42	99.53
	RMSE	0.43	0.13	0.13	0.11
	R^2 (in %)	83.93	98.61	98.62	99.06
7-year	CORR (in %)	92.99	99.45	99.46	99.50
	RMSE	0.36	0.10	0.10	0.10
	R^2 (in %)	85.96	98.91	98.92	98.93
10-year	CORR (in %)	92.52	99.19	99.20	99.42
	RMSE	0.37	0.12	0.11	0.10
	R^2 (in %)	80.90	98.18	98.21	98.76
		Panel C: Fitting TIPS Breakeven Inflation			
7-year	CORR (in %)	51.61	97.21	97.24	97.35
	RMSE	0.35	0.09	0.09	0.10
	R^2 (in %)	23.44	94.47	94.54	94.21
10-year	CORR (in %)	32.08	94.76	94.80	95.11
	RMSE	0.35	0.11	0.11	0.10
	R^2 (in %)	-18.12	88.71	88.85	89.74
		Panel D: Matching survey inflation forecasts			
1-year	CORR (in %)	2.07	62.94	88.65	88.08
	RMSE	0.78	0.58	0.33	0.34
	R^2 (in %)	-52.21	17.45	72.93	70.71
10-year	CORR (in %)	73.34	72.72	83.86	83.92
	RMSE	0.51	0.40	0.42	0.42
	R^2 (in %)	17.36	49.46	44.35	42.22

This table reports various diagnostic statistics for the four models estimated. Panel A reports the number of parameters, the log likelihood, the Akaike information criterion (AIC), the Bayesian information criterion (BIC) values, and the p-value from a Likelihood Ratio test of the current model against the more general Model to its right, where the p-values reported for Models NL and L-II are the Davie (1987) upper bounds. Panels B to D report three goodness-of-fit statistics for the 5-, 7- and 10-year TIPS yields, 7- and 10-year TIPS breakeven inflation and 1- and 10-year survey inflation forecasts, respetively, including the root mean squared fitted errors (RMSE), the correlation between the fitted series and the data counterpart (CORR), and the coefficient of determination (R^2) as defined in Equation (52).

Table 5: TIPS Bid-Ask Spreads Across Maturities (in ticks)

	Less than five years	Five to Ten Years	Above ten years
2003	1 to 2	2	4 to 16

	Two-year	Five-year	Ten-Year	Twenty-Year	Thirty-year
2007	1/2 to 1	1	1-2	4-6	6-10

This table reports the TIPS bid-ask spread at various maturities based on two informal survey conducted by the Federal Reserve Bank of New York in 2003 and 2007, respectively. One tick is 1/32s of a point where a point roughly equals one percent of the security's par value.

Table 6: What Drives the TIPS Liquidity Premiums

Panel A: Regression Analysis

	Coefficients				Adjusted
Maturity	Constant	Turnover	Imp Vol	ASW Spread	R^2
5-year	0.3074	-0.4309	0.1196	0.0258	79.52%
	(0.0771)	(0.0275)	(0.0095)	(0.0025)	
7-year	0.4139	-0.3752	0.0824	0.0253	80.95%
	(0.0641)	(0.0229)	(0.0079)	(0.0020)	
10-year	0.5076	-0.3337	0.0465	0.0251	82.30%
	(0.0541)	(0.0193)	(0.0067)	(0.0017)	

Panel B: In-Sample Correlations

	Liquidity Premiums			TIPS	10-Year
	5-year	7-year	10-year	Turnover	Imp Vol
TIPS Turnover	-0.7286	-0.7547	-0.7850		
10-year implied volatility	0.5515	0.5098	0.4449	-0.1314	
On/off ASW spread	0.7996	0.8189	0.8340	-0.6374	0.4742

Panel A regresses 5, 7- and 10-year TIPS liquidity premium estimates based on Model L-IId on TIPS turnover, implied volatility of 10-year nominal Treasury future options and the difference between the on-the-run and the off-the-run 10-year Treasuries par asset swap spreads using weekly data from either Jan. 6, 1999 to Mar. 14, 2007. Their in-sample pairwise correlations are reported in Panel B.

Table 7: Variance decomposition of TIPS Yields and TIPS BEI

Panel A: Unconditional Variance Decomposition

	TIPS yield		TIPS BEI		
Maturity	real yield	liq prem	inf exp	inf risk prem	liq prem
5-year	1.1717	-0.1717	0.5870	0.1890	0.2240
	(0.2836)	(0.2836)	(0.3050)	(0.2386)	(0.3170)
7-year	1.1819	-0.1819	0.5659	0.1994	0.2347
	(0.2690)	(0.2690)	(0.3065)	(0.2616)	(0.3131)
10-year	1.1910	-0.1910	0.5453	0.2090	0.2458
	(0.2581)	(0.2581)	(0.3192)	(0.2944)	(0.3095)

Panel B: Instantaneous Variance Decomposition

	TIPS yield		TIPS BEI		
Maturity	real yield	liq prem	inf exp	inf risk prem	liq prem
5-year	1.1596	-0.1596	0.5447	0.0167	0.4386
	(0.2963)	(0.2963)	(0.2473)	(0.2064)	(0.2639)
7-year	1.2285	-0.2285	0.5500	0.0239	0.4261
	(0.3040)	(0.3040)	(0.2710)	(0.2397)	(0.2597)
10-year	1.3024	-0.3024	0.5431	0.0348	0.4221
	(0.3073)	(0.3073)	(0.2984)	(0.2740)	(0.2556)

Note: This table reports the unconditional and the instantaneous variance decompositions of TIPS yields into real yields and TIPS liquidity premiums, and of nominal yields into expected inflation, the inflation risk premiums and the negative of TIPS liquidity premiums, all based on Model L-IId estimates. The variance decompositions of TIPS yields are calculated according to

$$1 = \frac{cov\left(y^{\mathcal{T}}_{t,\tau}, y^{R}_{t,\tau}\right)}{var\left(y^{\mathcal{T}}_{t,\tau}\right)} + \frac{cov\left(y^{\mathcal{T}}_{t,\tau}, L_{t,\tau}\right)}{var\left(y^{\mathcal{T}}_{t,\tau}\right)},$$

while the variance decompositions of the TIPS breakeven inflation are calculated according to

$$1 = \frac{cov\left(BEI^{\mathcal{T}}_{t,\tau}, I_{t,\tau}\right)}{var\left(BEI^{\mathcal{T}}_{t,\tau}\right)} + \frac{cov\left(BEI^{\mathcal{T}}_{t,\tau}, \wp^{I}_{t,\tau}\right)}{var\left(BEI^{\mathcal{T}}_{t,\tau}\right)} + \frac{cov\left(BEI^{\mathcal{T}}_{t,\tau}, -L_{t,\tau}\right)}{var\left(BEI^{\mathcal{T}}_{t,\tau}\right)},$$

where the results are based on either the unconditional variance-covariance matrix of the state variables (Panel A) or the instantaneous variance-covariance matrix of the state variables, $\Sigma\Sigma'$ (Panel B). Standard errors calculated using the delta method are reported in parentheses.

Table 8: Variance decomposition of Nominal Yields

Panel A: Unconditional Variance Decomposition

Maturity	real yield	inf exp	inf risk prem
1-quarter	0.5108	0.4156	0.0736
	(0.2541)	(0.2281)	(0.0927)
1-year	0.5715	0.3497	0.0787
	(0.1930)	(0.1843)	(0.0924)
5-year	0.6503	0.2609	0.0888
	(0.1486)	(0.1417)	(0.1146)
10-year	0.6715	0.2347	0.0938
	(0.1401)	(0.1429)	(0.1362)

Panel B: Instantaneous Variance Decomposition

Maturity	real yield	inf exp	inf risk prem
1-quarter	0.7719	0.2252	0.0029
	(0.1090)	(0.1009)	(0.0312)
1-year	0.7692	0.2172	0.0137
	(0.1082)	(0.0915)	(0.0365)
5-year	0.7132	0.2496	0.0372
	(0.1231)	(0.1154)	(0.0970)
10-year	0.6892	0.2494	0.0614
	(0.1331)	(0.1438)	(0.1345)

Note: This table reports the unconditional and the instantaneous variance decompositions of nominal yields into real yields, expected inflation, the inflation risk premiums, all based on Model L-IId estimates. The variance decomposition is calculated according to

$$1 = \frac{cov\left(y_{t,\tau}^{N}, y_{t,\tau}^{R}\right)}{var\left(y_{t,\tau}^{N}\right)} + \frac{cov\left(y_{t,\tau}^{N}, I_{t,\tau}\right)}{var\left(y_{t,\tau}^{N}\right)} + \frac{cov\left(y_{t,\tau}^{N}, \wp_{t,\tau}^{I}\right)}{var\left(y_{t,\tau}^{N}\right)},$$

where the results are based on either the unconditional variance-covariance matrix of the state variables (Panel A) or the instantaneous variance-covariance matrix of the state variables, $\Sigma\Sigma'$ (Panel B). Standard errors calculated using the delta method are reported in parentheses.

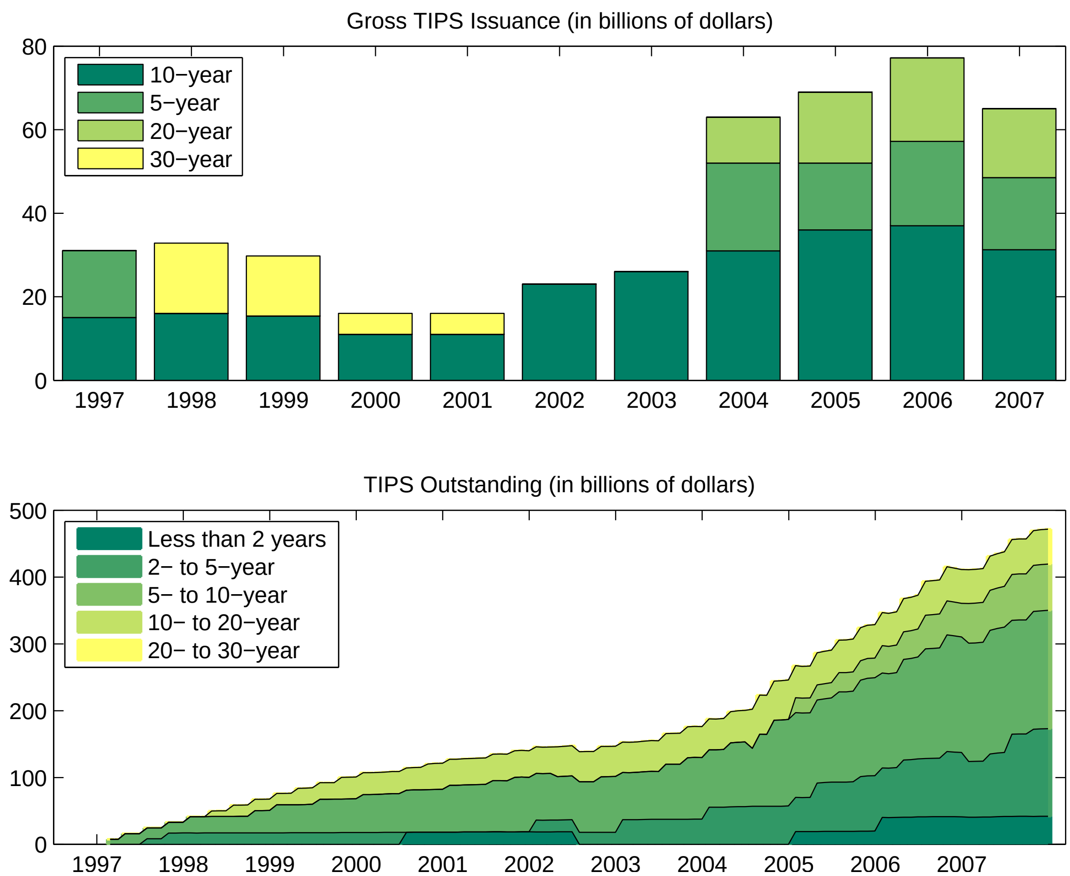


Figure 1: TIPS Issuance and Outstanding

The top panel plots gross TIPS issuance broken down by initial maturities of 10, 5, 20 and 30 years. The bottom panel plots TIPS outstanding broken down by remaining maturities, based on data reported in the Treasury's Monthly Statement of the Public Debt (MSPD).

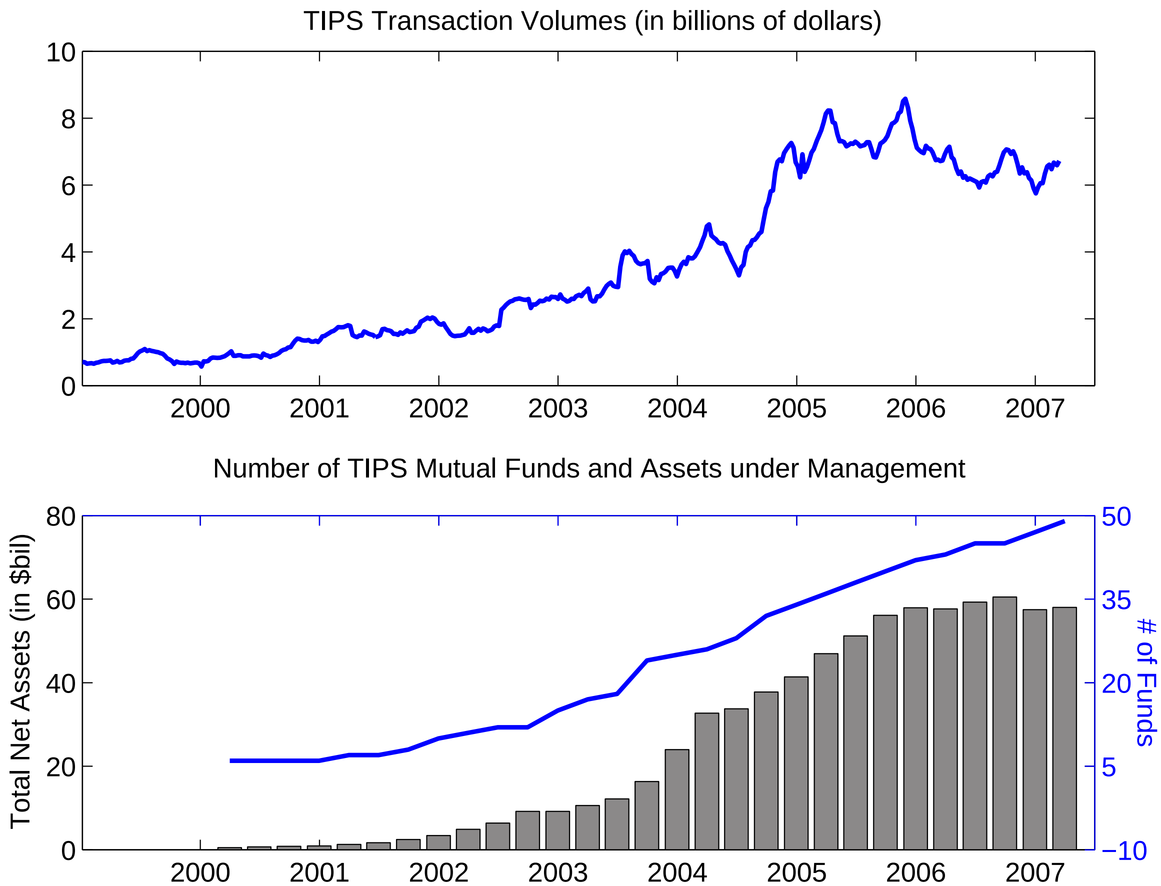


Figure 2: TIPS Transaction Volumes and TIPS Mutual Funds

Top top panel plots the weekly TIPS transaction volumes, defined as 13-week moving average of weekly averages of daily TIPS transaction volumes reported by primary dealers in Government Securities Dealers Reports (FR-2004). The bottom panels plots number of TIPS mutual funds (left axis) and the total net assets under management (left axis).)

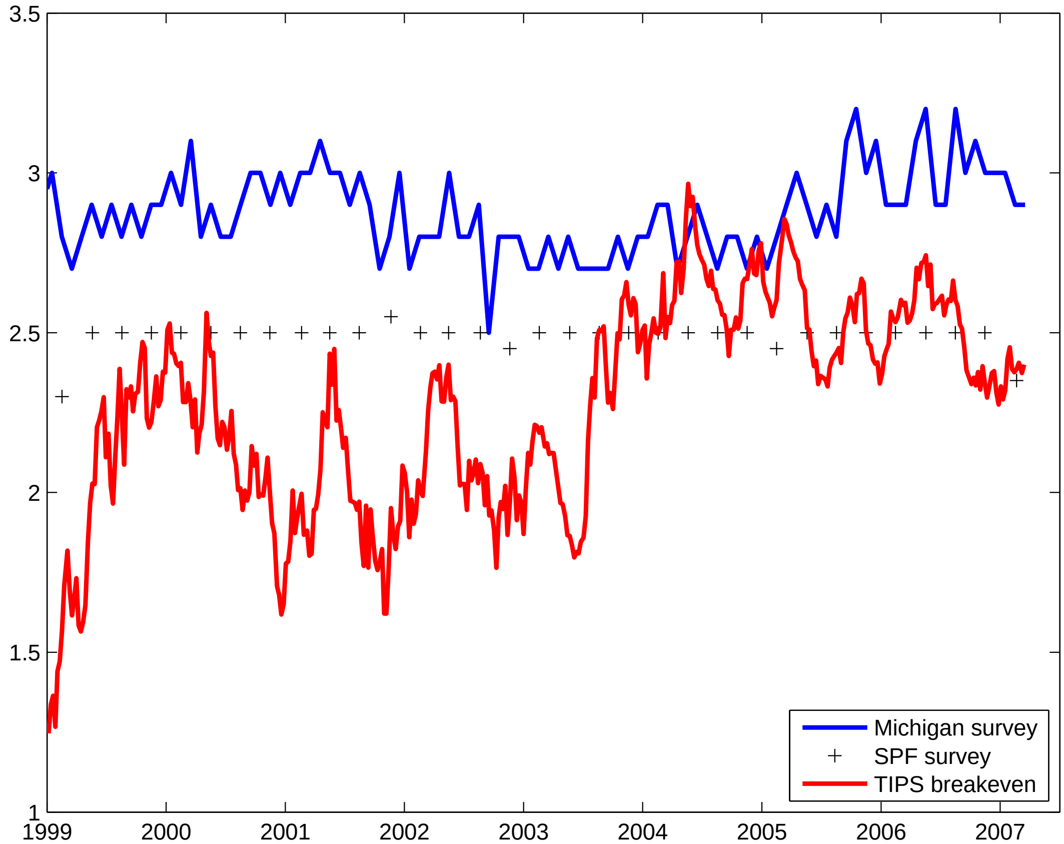


Figure 3: Survey Inflation Forecasts and TIPS Breakeven Inflation

This chart shows the 10-year TIPS breakeven inflation (red line), long-horizon Michigan inflation forecast (blue line), and 10-year SPF inflation forecast (black pluses).

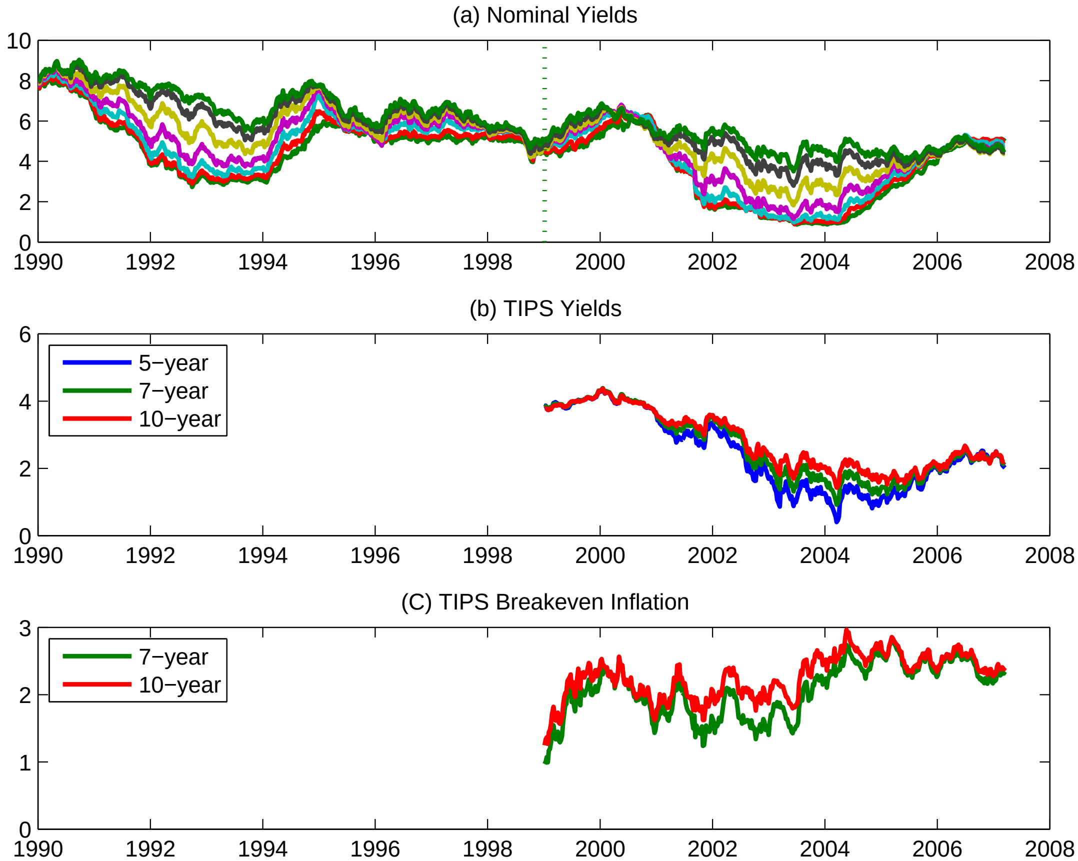


Figure 4: Nominal and TIPS Yields and TIPS Breakeven Inflation

Top top panel plots the 3- and 6-month, 1-, 2-, 4-, 7- and 10-year nominal yields. The middle panel plots the 5-, 7- and 10-year TIPS yields. The bottom panels plots the 5- and 7-year TIPS breakeven inflation.)

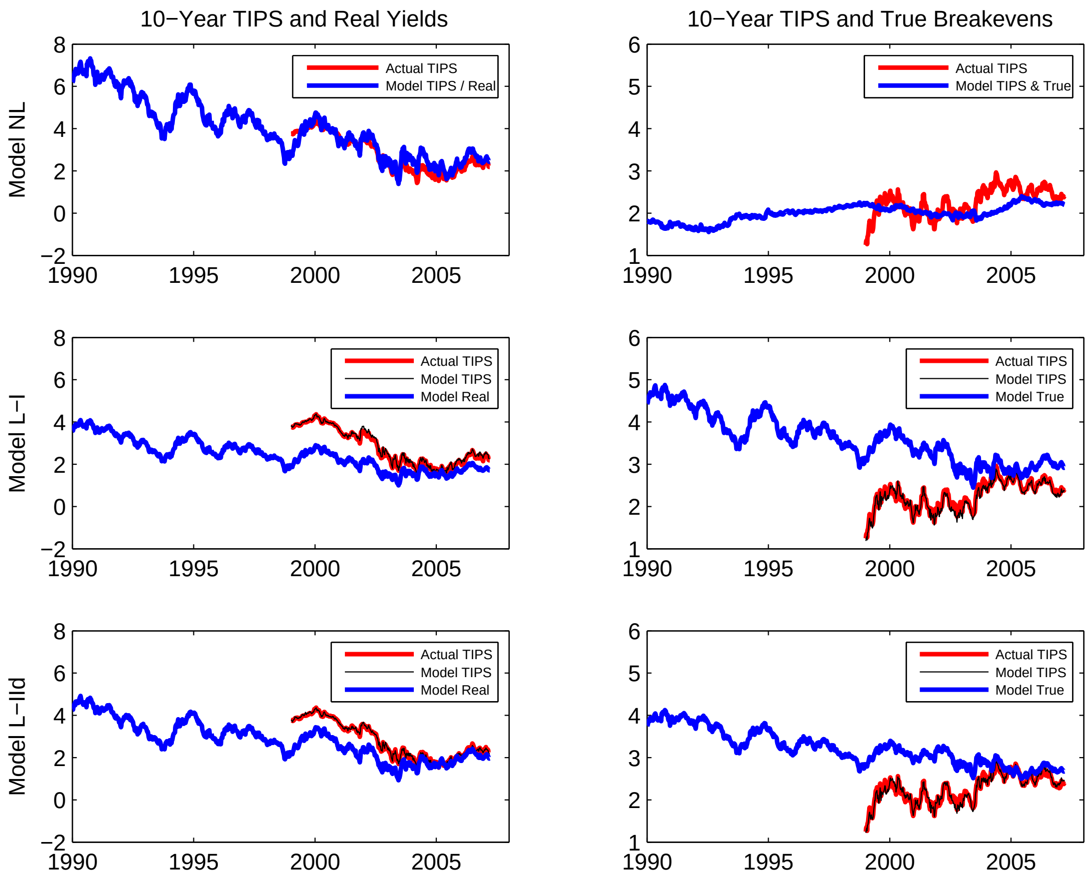
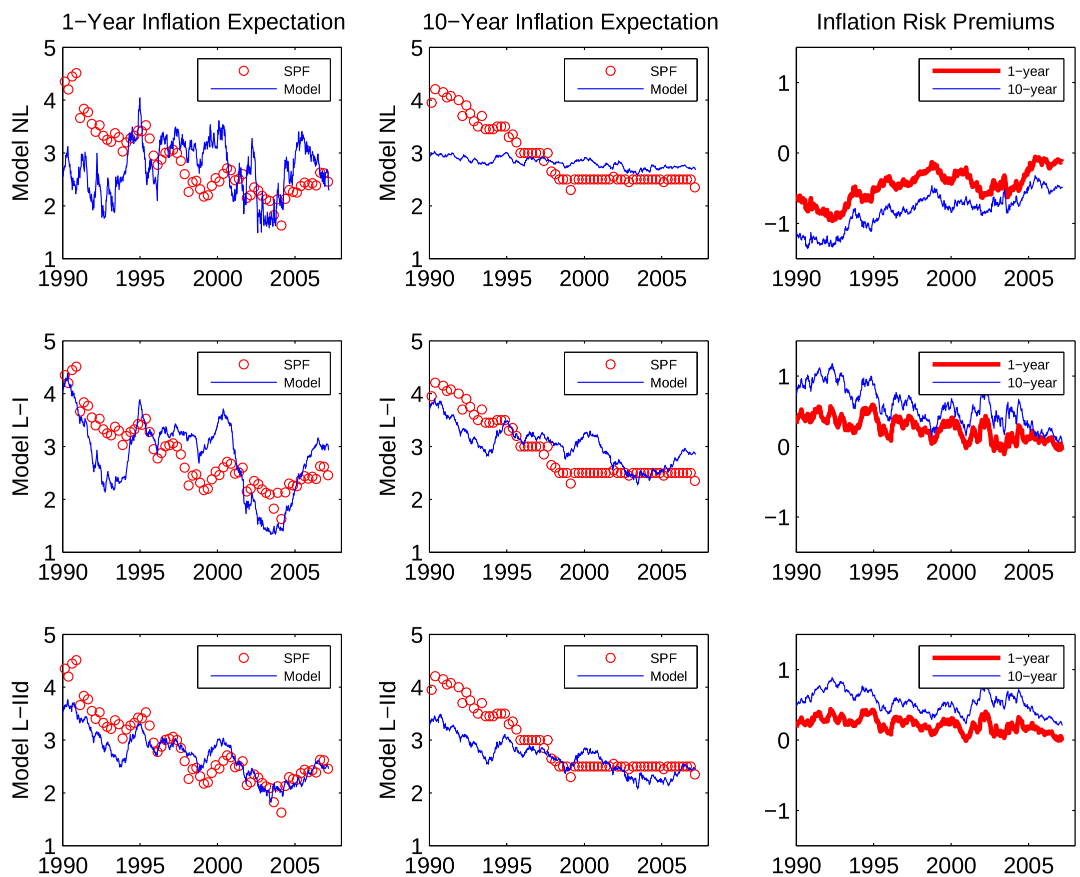


Figure 5: Actual and Model-Implied TIPS Yields and Breakevens

The panels on the left plot the 10-year actual TIPS yields (red), the 10-year model-implied TIPS yields (black) and the 10-year model-implied real yields (blue). The panels on the right plot the 10-year actual TIPS breakevens (red), the 10-year model-implied TIPS breakevens (black) and the 10-year model-implied true breakevens (blue). The model estimates are based on Model NL (upper panels), Model L-I (middle panels), and Model L-IId (lower panels), respectively.



The left and middle panels plot the 1- and 10-year model-implied inflation expectation, respectively, together with the survey counterparts from Survey of Professional Forecasters (SPF), while the right panels plot the 1- and 10-year model-implied inflation risk premiums. The model estimates are based on Model NL (upper panels), Model L-I (middle panels), and Model L-IId (lower panels), respectively.

Figure 6: Inflation Expectations and Inflation Risk Premiums

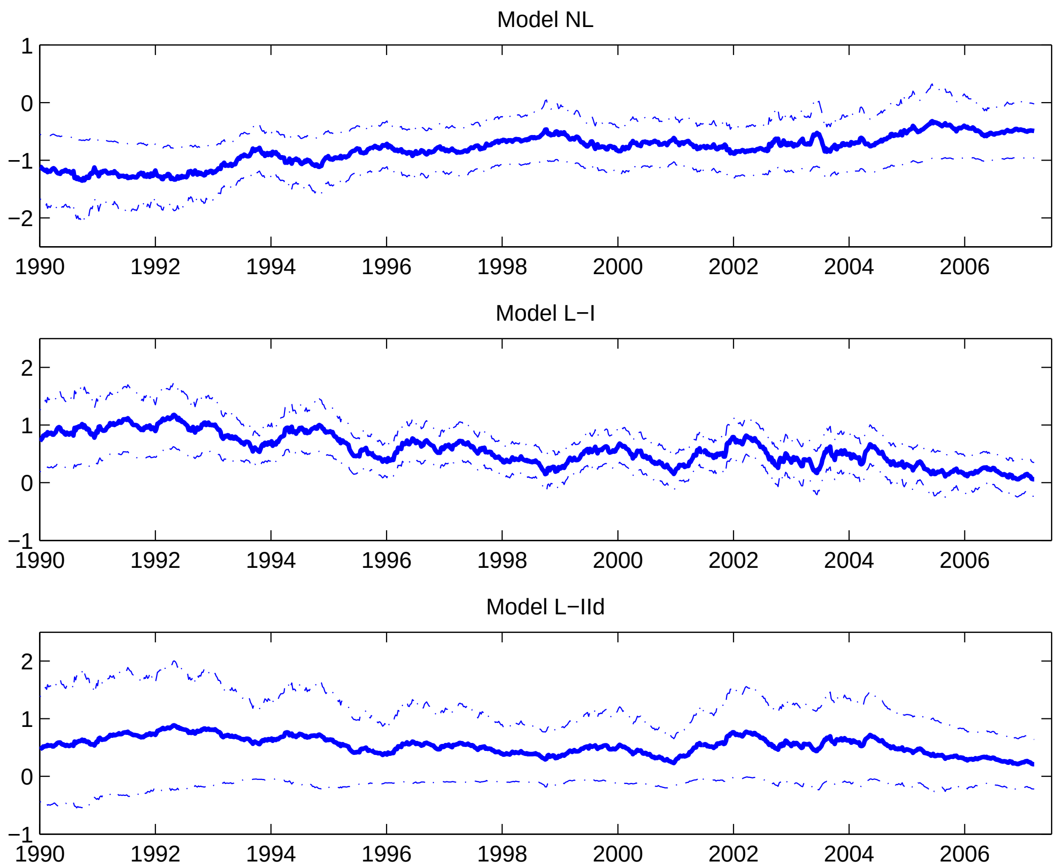


Figure 7: 10-year Inflation Risk Premium with Confidence Bands

The three panels plot the model-implied 10-year inflation risk premiums with 2 BHHH standard error bands based on Model NL (top panel), Model L-I (middle panel) and Model L-IId (bottom panel), respectively.

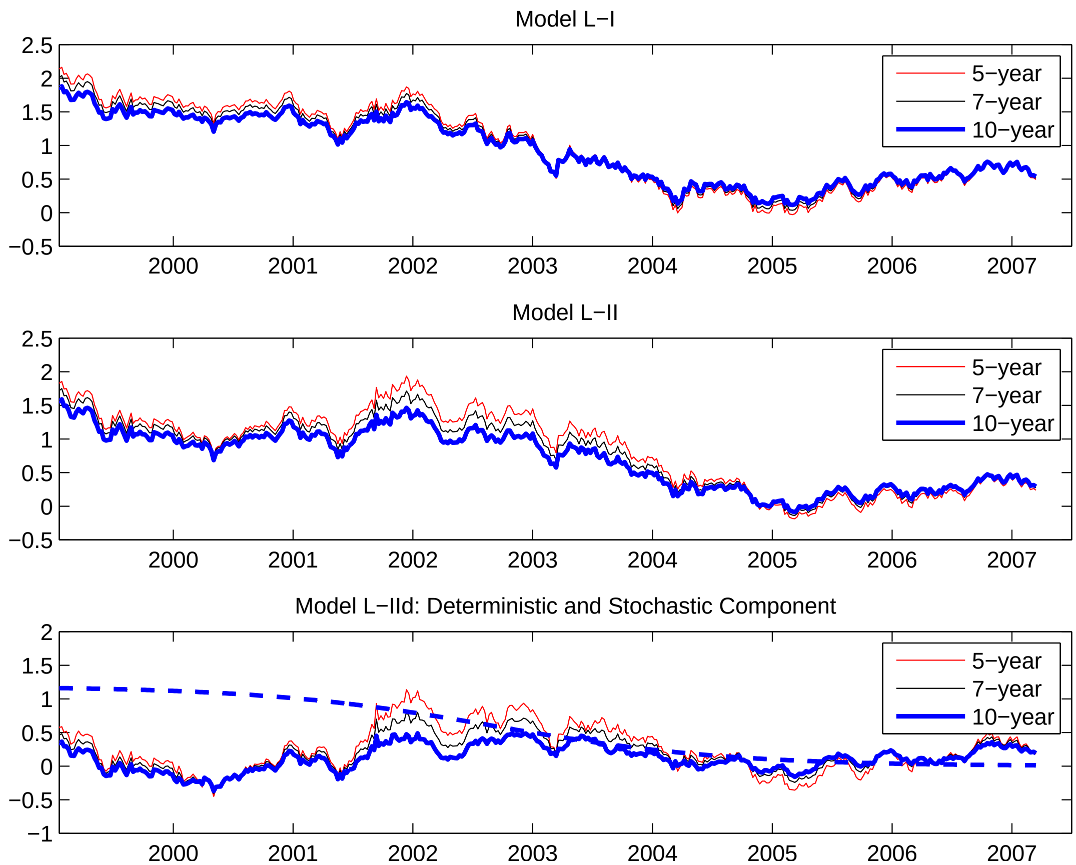


Figure 8: TIPS Liquidity Premiums

The top panel plot the 5-, 7- and 10-year TIPS liquidity premiums based on Model NL estimates. The bottom two panels plots the same series based on Model L-IId estimates, as well as a decomposition of these series into a deterministic component (dashed line in the middle panel) and a stochastic component (bottom panel).

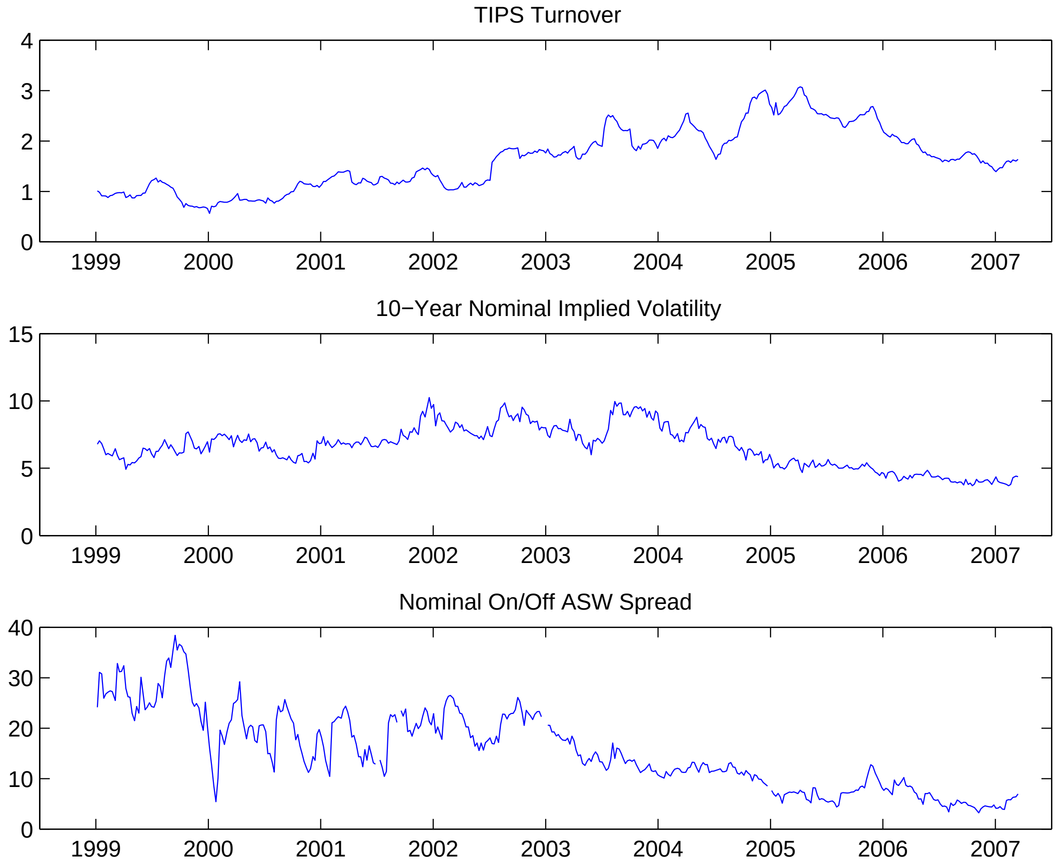


Figure 9: Measures Related to TIPS Liquidity

This chart plots various measures that potentially reflect liquidity conditions in the TIPS market, including the TIPS turnover ratio as defined in Section 6.2 (top panel), implied volatilities from options on the 10-year nominal Treasury note futures (middle panel) and the difference between the on-the-run and the off-the-run 10-year Treasuries par asset swap spreads (bottom panel).

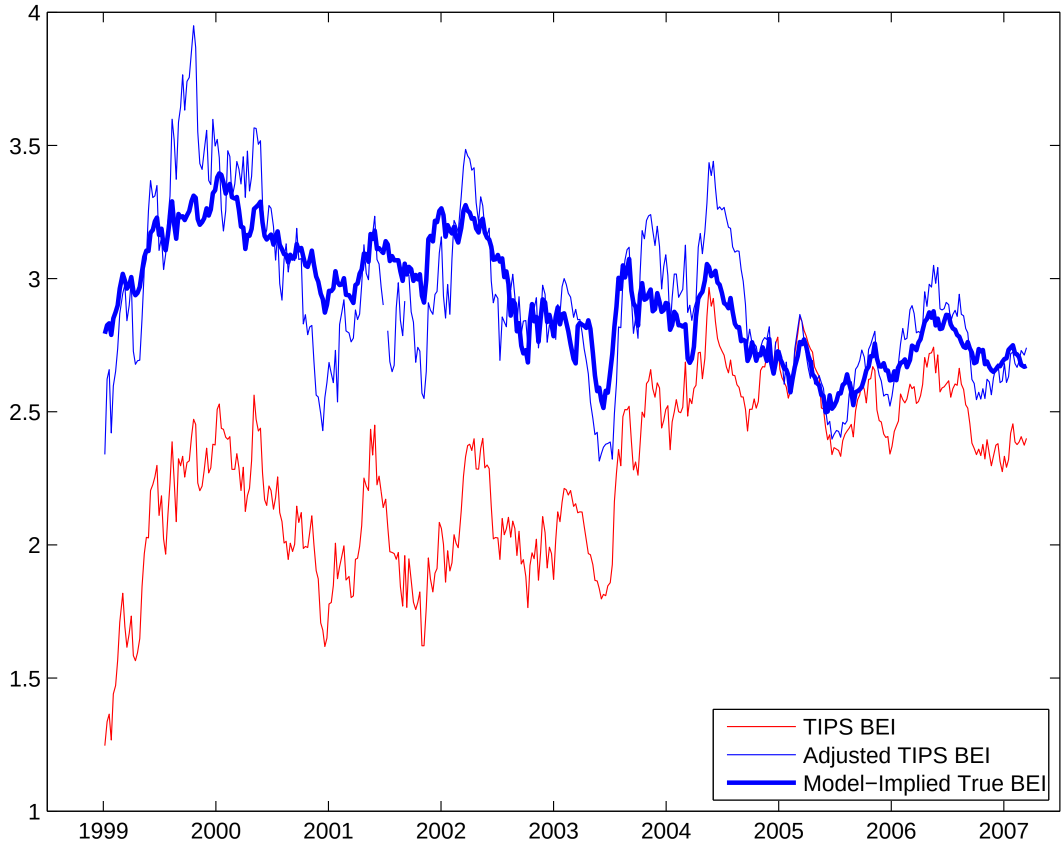


Figure 10: A Simple Liquidity Adjustment for TIPS BEI

This chart plots the liquidity-adjusted 10-year TIPS BEI base on Equation (54) (thin blue line) together with the unadjusted series (red line) and the model-implied true TIPS BEI from Model L-IId (thick blue line).

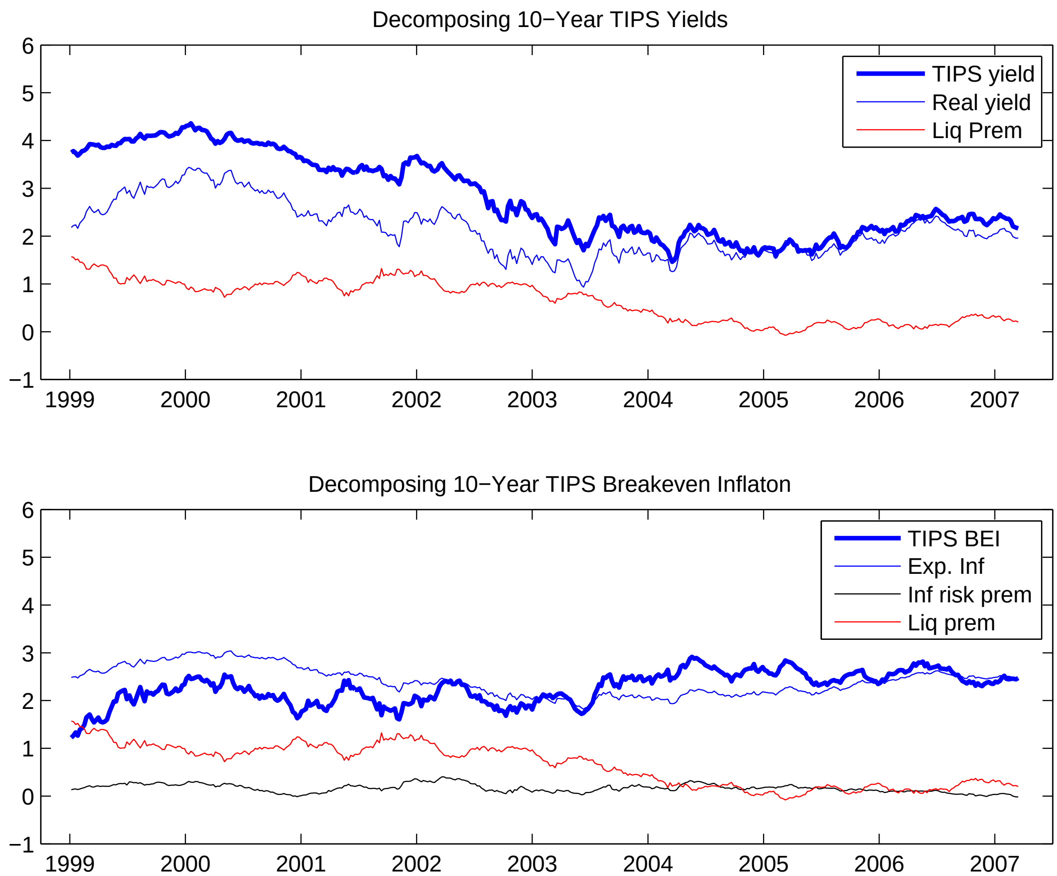


Figure 11: Decomposing TIPS Yields and TIPS Breakeven Inflation

The top panel decomposes the 10-year TIPS yields into the real yield and the TIPS liquidity premiums, while the bottom panel decomposes the 10-year TIPS breakeven inflation into the expected inflation, the inflation risk premium and the TIPS liquidity premium, all according to Equation (55).

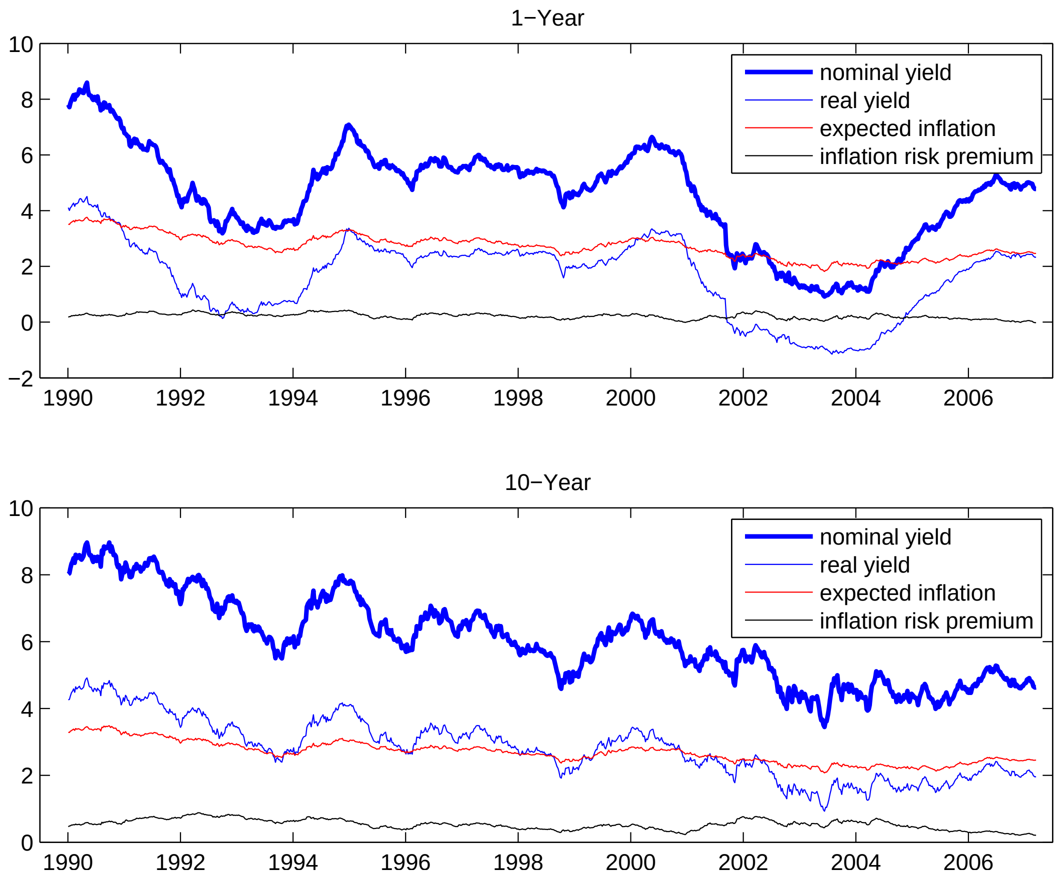
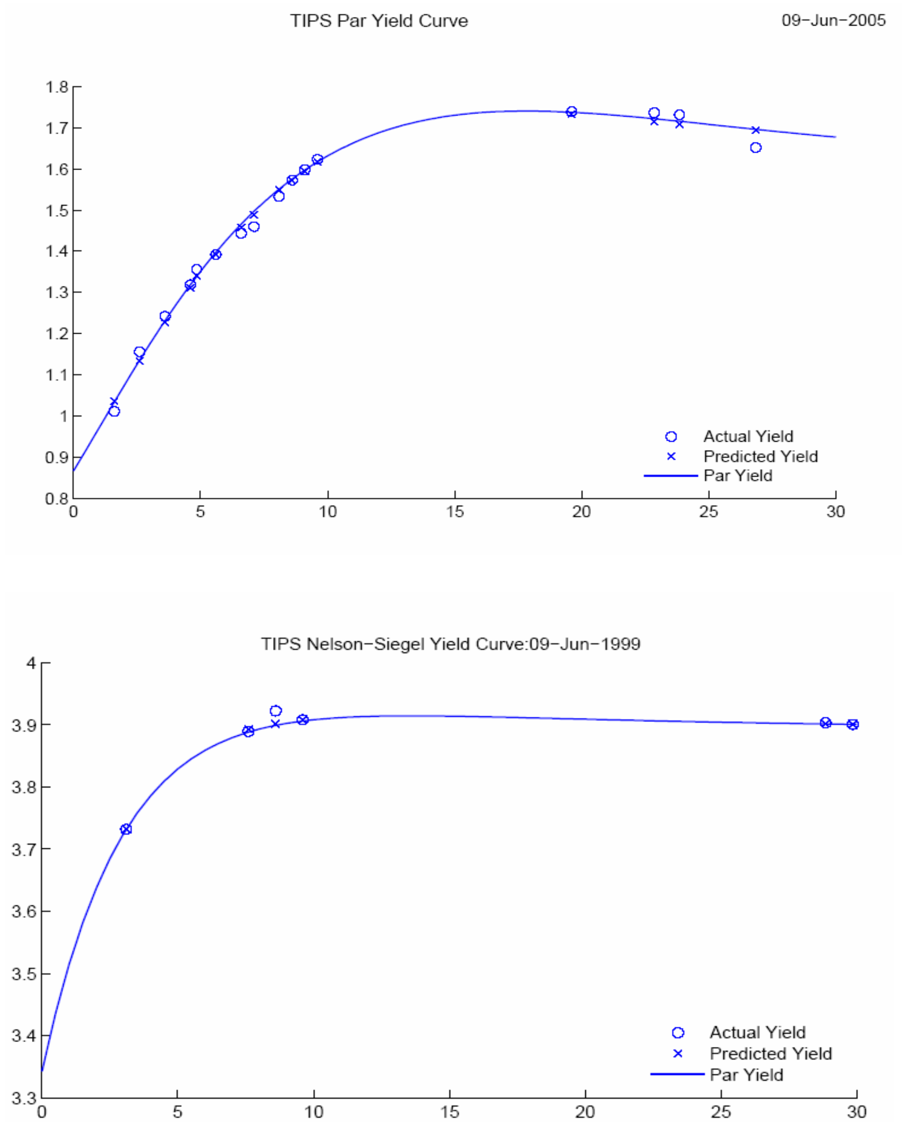


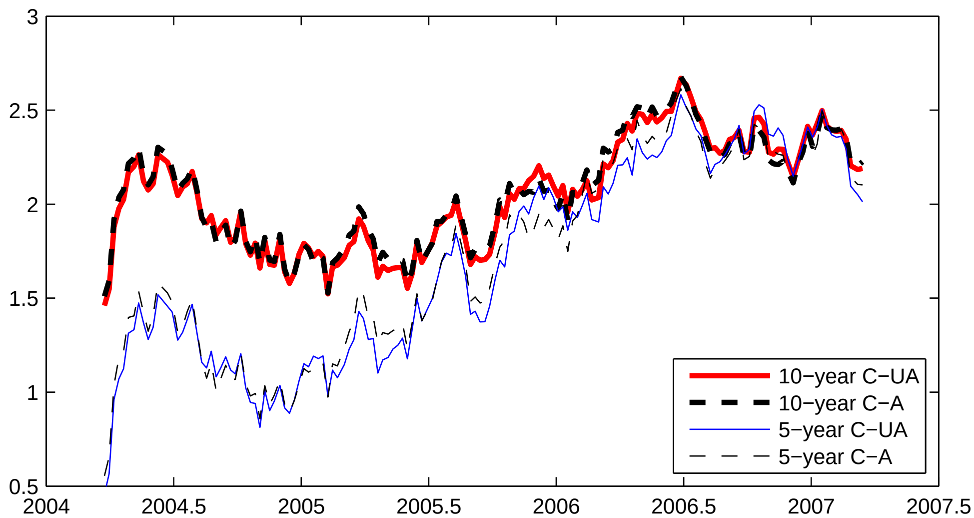
Figure 12: Decomposing Nominal Yields

This chart decomposes the 1- and 10-year nominal yields into real yields, expected inflation and inflation risk premiums according to Equation (56).



Note: The top (bottom) panel plots the fitted TIPS par yield curve together with individual TIPS yields on June 9, 2005 (June 9, 1999).

Figure A1: TIPS Yield Curves



Note: This figure plots 10-year carry-unadjusted (carry-adjusted) TIPS yields in red solid (black dashed) line and 5-year carry-unadjusted (carry-adjusted) TIPS yields in blue solid (gray dashed) line.

Figure A2: TIPS Yields with and without Carry Adjustment