Output Gaps

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Abstract

What is the output gap? I discuss three alternative definitions: the deviation of output from its long-run stochastic trend (i.e., the "Beveridge-Nelson cycle"); the deviation of output from the level consistent with current technologies and normal utilization of capital and labor input (i.e., the "production-function approach"); and the deviation of output from "flexible-price" output (i.e., its "natural rate"). Estimates of each concept are presented from a dynamic-stochastic-general-equilibrium (DSGE) model of the U.S. economy used at the Federal Reserve Board. Four points are emphasized: The DSGE model's estimate of the gap (for each definition) is very similar to gaps from policy institutions, but the model's estimate of potential growth has a higher variance and substantially different covariance with GDP growth; the change in the Beveridge-Nelson trend covaries negatively with the change in the gap in the DSGE model, providing a structural model estimate of a controversial parameter; in this model, estimates of the natural-rate concept are similar to those based on the Beveridge-Nelson and production function approaches; and the estimate of the output gap, irrespective of definition, is closely related to unemployment fluctuations.

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1 Introduction

"The world is complicated enough without introducing further confusion and ambiguities because two different names are unknowingly being used for the same thing, or because the same word is being applied to quite different phenomena" Paul Samuelson (1948), page 6.

What is the output gap? The question is simple. But the answer is not, in part because economist's employ quite different definitions; indeed, Paul Samuelson noted more than half a century ago the troublesome tendency of economists to talk at, rather than to, each other by using the same term for different phenomena.

I consider three definitions of the output gap:

- The deviation of output from its long-run stochastic trend (i.e., the "Beveridge-Nelson cycle", as defined in Beveridge and Nelson (1981));
- The deviation of output from the level consistent with current technologies and normal utilization of capital and labor input (i.e., the "production-function approach", as employed, for example, by the Congressional Budget Office (CBO) (CBO (2001)));
- The deviation of output from a "flexible-price" or "natural rate" level.[1]

This research presents estimates of each concept from a dynamic-stochastic-general-equilibrium (DSGE) model developed at the Federal Reserve Board and discusses the

[1] Notions related to the last three concepts have a long intellectual pedigree. For example, Plosser and Schwert (1979) highlight the difficulty of using the production-function approach in policy applications, as its focus on the "supply-side" of the economy does not allow consideration of the multitude of factors that influence equilibrium production and hence affect the desirability of alternative paths for production, employment, etc. Gordon (1979) (in the same volume as Plosser and Schwert (1979)) suggested "natural" output as corresponding to his particular view of a desirable economic outcome – namely, stable inflation (thereby anticipating future work, e.g., Woodford (2003) and Edge, Kiley, and (2008)).

correspondence between these estimates and those produced elsewhere; the uses of each concept are also highlighted. The EDO model can produce estimates of each concept. However, the concepts, in and of themselves, do not play a central role in EDO (where decisions are based on utility and profit maximization problems, and the gap measures are determined from definitions "outside the model" which do not enter private agents' decisions). As a result, it is important to consider the question at hand before deciding on a given output gap concept: The Beveridge-Nelson gap provides answers to some questions, the production-function approach provides answers to other questions, and the flexible-price/natural-rate gaps provide yet again different information.

Key results from the analysis are the the following.

- The DSGE model's estimates of the output gap (irrespective of approach) is similar to gaps from the Congressional Budget Office or the Federal Reserve's large-scale macro-econometric model (FRB/US) model, but the DSGE model's estimate of potential growth is considerably more variable. The latter result stems from the significant degree of fluctuation in aggregate technology estimated by the DSGE model, a result consistent with the significant role such fluctuations play in model's descended from those of the real-business-cycle tradition (from Kydland and Prescott (1982)). Moreover, potential growth from the DSGE model co-varies with GDP growth to a much more significant degree than does potential from either the CBO or the FRB/US model; this is again consistent with the model's roots in the quantitative tradition initiated by Kydland and Prescott (1982), and closely related to a large time-series literature on the covariance between potential and actual GDP growth.

- The substantial variance of the change in the Beveridge-Nelson trend in EDO is consistent with the overall variance of the change in real GDP because the change in trend covaries negatively with the change in the gap in the DSGE model. Such a negative covariance has been a subject of controversy (Morley, Nelson, and Zivot (2003)). The DSGE model provides a structural interpretation – improvements in productivity lead to a widening of the output gap, as New-Keynesian frictions slow the economy's adjustment to technology shocks (e.g., Boivin, Kiley, and Mishkin (2011)).

- The flexible-price/natural-rate gaps are highly dependent on modeling assumptions, but the natural rate gap from the EDO model developed at the Federal Reserve Board is similar to traditional gap estimates such as those from the CBO, in large part because the business cycle variation in the unemployment rate is attributed, by the model, primarily to shocks to "aggregate" demand or "the natural rate of interest".

- The DSGE model's estimate of the output gaps, again largely irrespective of definition, is closely related to unemployment fluctuations, similar to gaps from policy institutions based on Okun's law.

Before proceeding to the main discussion, a brief introduction to the primary model considered in the analysis helps frame the discussion. The DSGE model employed is the EDO (Estimated, Dynamic, Optimization-based) model developed in (see Edge, Kiley, and (2007, 2008, 2010) and Chung, Kiley, and Laforte (2010, 2012)). This model is ideally suited to address the questions of interest, as it includes

- A two-sector structure emphasizing the importance of fluctuations in aggregate and investment-specific technology for explaining long-run growth and cyclical fluctuations – implying a stochastic trend in production and expenditure;

- A rich array of other shocks to pricing decisions, preferences, and the desirability of smoothing consumption over time through capital accumulation that allow for temporary fluctuations in activity;

- A New-Keynesian structure with nominal rigidities in both prices and wages and variable utilization of capital input; and

- A New-Keynesian description of unemployment, emphasizing the discrepancy between households' willingness to work and firms' willingness to hire created by traditional wage and price rigidities.

- An empirical approach that ensures the model can provide a reasonable characterization of the sources of fluctuations over history.

Each of these features allows the model to capture commonly emphasized features of each output gap concept: Stochastic trends imply a rich Beveridge-Nelson decomposition; variable utilization and stochastic technology imply a sophisticated modeling of the production-function approach; and nominal rigidities yield a New-Keynesian model of unemployment fluctuations around its natural rate. Moreover, this combination of features, and the day-to-day use of the model in a policy environment, is unique in policy and academic circles.

Section 2 summarizes some related recent research. The third discusses the structure of the EDO model. The fourth section defines each output gap concept. The fifth section presents estimation results that highlight features of the model. The sixth section presents estimates of output gaps. The seventh section compares gap estimates from EDO to those of the CBO and from the Federal Reserve's FRB/US model – a traditional, large-scale econometric model. A concluding section provides some thoughts for future research.

2 Related Research

Despite the ambiguity associated with the concept of the output gap suggested by the quite different concepts highlighted above, the related literature is deep and influential in policy circles.

Perhaps the cleanest definition of an output gap is that associated with the cycle in output introduced by Beveridge and Nelson (1981). According to this definition, the cycle in output, or the output gap, is the deviation of output from the level expected to prevail in the long run (i.e., as time, t, approaches ∞), with the effect of any deterministic aspects of the growth in output on its long-run level removed. This definition of the output gap has several advantages. First, it is not model-specific: Any approach capable of capturing the reduced-form time-series characteristics of output can provide an estimate of this definition of the output gap; as a result, univariate or multivariate time-series methods as well as structural economic models can, in principle, estimate the output gap.[2] Second, the Beveridge-Nelson approach provides information related to a very specific forecasting problem – in particular, the current estimate of the Beveridge-Nelson gap provides an estimate of how much economic growth will deviate from its deterministic (or, in some models, "steady-state") level going forward: An answer to this forecasting problem forms the basis of a long-standing academic approach to discussions of the gap (e.g., the discussions in Nelson (1964) and Thurow and Taylor (1966) of the growth outlook for the 1960s in the United States, or the discussion in CBO (2010) of the growth outlook from 2010-2020). Finally, this method often provides estimates of the output gap that correspond to widely held views on what is "reasonable", although this result depends on assumptions (e.g., the discussion in Morley, Nelson, and Zivot (2003)).

The EDO model provides an excellent framework for computation of the Beveridge-Nelson gap. As highlighted in the introduction, the inclusion of stochastic trends in both aggregate and investment-specific technology implies the model has a non-trivial Beveridge-Nelson permanent component to output, and connects the model to the burgeoning literature on empirical two-sector growth models (see Edge, Kiley, and (2008, 2010)). The neoclassical core of the model ensures that the model captures the long-run relationships between output, investment, and consumption: King, Plosser, Stock, and Watson (1991) demonstrated the usefulness of such information for estimation of the Beveridge-Nelson gap in a multivariate context; similarly, Cochrane

[2]The related literature using time-series methods is large; notable contributions include Clark (1987), Stock and Watson (1988), and Morley, Nelson, and Zivot (2003).

(1994) emphasized the potential role for consumption in identifying trends in output (due to the dependence of consumption on permanent income). The estimation of EDO includes information on output and several components of consumption and investment, yielding a rich, multivariate information set for output gap estimation. Indeed, the information set is quite broad, including expenditure variables, labor input, real wages, inflation, and the federal funds rate. Previous work has highlighted how time-series decompositions of output into permanent and transitory components can exploit information on hours (e.g., Rotemberg and Woodford (1996)) and inflation (e.g., Kuttner (1994)).

While the Beveridge-Nelson concept is perhaps the cleanest definition, the most prevalent definition at policy institutions appears to be the production-function approach (e.g., the approach used at the CBO (2001) and in the Federal Reserve's FRB/US model). In this approach, the output gap is defined as the deviation of output from the level that would occur if capital and labor input were utilized at "normal" rates, given current technology. In general, this approach, as employed at policy institutions, involves some smoothing of measured total factor productivity in order to estimate "true" technology. In contrast, the level of true technology in EDO is estimated to be consistent with the observed behavior of a broad range of economic variables and the underlying structure of the model. In principle, EDO could deliver very volatile estimated levels of "true" technology; however, this result is not predetermined, as the EDO model includes features that could cause measured total factor productivity to deviate from underlying technology (such as variable capital utilization, which could cause measured capital input to deviate from true capital input, and imperfect competition which could distort total factor productivity computations). The primary advantage of considering the EDO estimate of this concept is to compare it to estimates of the same concept produced by policy institutions. While the preferences of policy institutions for this approach is quite clear from their modeling efforts, it is not obvious what question motivates this particular focus.

The last approach considered (the flexible-price/natural-rate approach) is the focus of some discussions in the research literature using DSGE models. Most prominently, the "natural-rate" approach in Woodford (2003) and Edge, Kiley, and (2008) has been widely discussed. The natural-rate approach defines the gap as one that would arise in the absence of nominal rigidities and shocks to "markups"; this approach is motivated in simple New-Keynesian models by the structure of such simple models, in which nominal rigidities are the only (significant) distortion, implying that the removal of such distortions provides an estimate of the level of output that is economically efficient. However, a focus on flexible-price or natural output will not, in general, be directly related to economic efficiency. Specifically, the behavior of a flexible-price/natural-rate gap and its relationship to an efficiency gap measure will be highly dependent on the model's structure – specifically, on the types of imperfections and shocks included in the model. For example, EDO has two sectors, and nominal rigidities in the prices and wages prevailing in both sectors; as a result, its estimate of the natural rate of output may differ substantially from those of a model with one sector or with only nominal rigidities in prices or wages. Justiniano and Primiceri (2008) also discuss the factors that influence efficiency gaps relative to those that simply remove wage and price rigidities in a simpler model.

As an important aside, this discussion highlights how consideration of the implications of the efficiency gap are complex. In a simple one-sector model without markup or other distortionary shocks, stabilization of the natural-rate gap would imply stabilization of inflation (e.g., Woodford (2003)). In EDO, the world is more complicated. There are two sectors, both with nominal rigidities and which are not symmetric (as one produces only a capital good and the other produces a good that can be used for consumption and as a capital good). As a result, stabilization of aggregate output at the natural level level is not, by itself, sufficient for efficiency (as the distribution of output across sectors matters as well, e.g. Aoki (2001) and Erceg and Levin (2006)). In addition, EDO has nominal wage rigidities in each sector, again implying that stabilization of the efficiency gap need not imply efficiency (e.g., Erceg, Henderson, and Levin (2000)). Finally, EDO has markup shocks, which also induce a tradeoff between output gap and inflation stabilization (e.g., Clarida, Gali, and Gertler (1999)). Each of these features seems at least potentially important, implying that great care should be taken when considering the implications of a given estimate of the natural rate gap for economic efficiency.

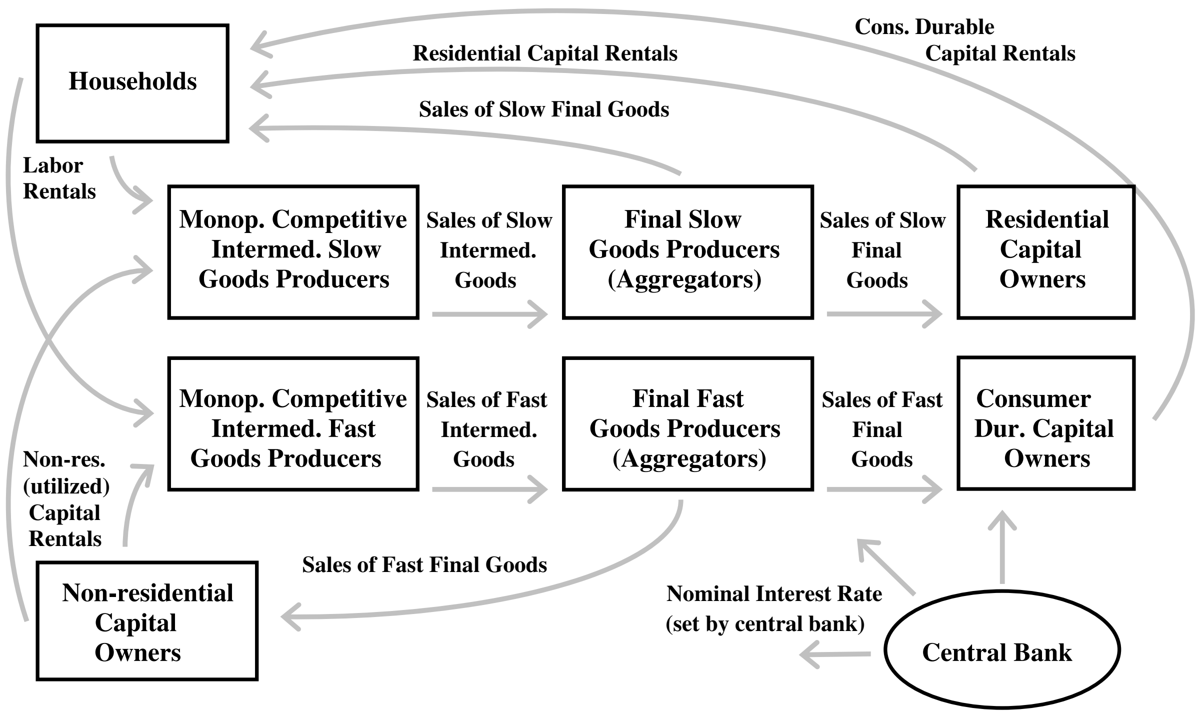
Finally, as discussed in the next section and in Chung, Kiley, and Laforte (2012), the EDO model includes a New-Keynesian model of unemployment: Workers desire to work more hours than firms demand because of imperfect competition in the labor market, and variations in the wage markup – that is, changes in the degree to which imperfect competition drives a wedge between the willingness to work and to hire – lead to fluctuations in unemployment, as in Gali (2011) and Gali, Smets, and Wouters (2011). As shown below, the information in the unemployment rate helps bring into alignment gap estimates from the natural rate approach with estimates of the output gap based on a notion of long-run trends, like the Beveridge-Nelson definition.

3 The EDO Model

The EDO model contains a detailed description of domestic production and expenditures decisions. The heart of the model is a two-sector production structure and a New-Keynesian model of unemployment. In particular, the economy consists of a consumption goods and an investment goods sector. The motivation for this basic structure is discussed in detail in Edge, Kiley, and Laforte (2007, 2008, 2010) and Chung, Kiley, and Laforte (2012); the latter provides a more detailed description of the model – which is only sketched below, as the model includes in the neighborhood of 100 equations.

Figure 1 provides a graphical overview of the economy described by the model. The model possesses two final good sectors in order to capture key long-run growth

Figure 1: Model Overview



facts and to differentiate between the cyclical properties of different categories of durable expenditure (e.g., housing, consumer durables, and nonresidential investment). For example, technological progress has been faster in the production of business capital and consumer durables (such as computers and electronics). The first sector is the slow-growing sector—called “CBI” because *most* of these goods are used for consumption (C) and because they are produced by the business and institutions (BI) sector—and the second is the fast-growing sector—called “KB” because these goods are used for capital (K) accumulation and are produced by the business

(B) sector. The goods are produced in two stages by intermediate- and then final-goods producing firms (shown in the center of the figure). As in most new-Keynesian models, the introduction of intermediate and final goods producers facilitates the specification of nominal rigidities.

The disaggregation of production (aggregate supply) leads naturally to some disaggregation of expenditures (aggregate demand). EDO moves beyond the typical model with just two categories of (private domestic) demand (consumption and investment) and distinguishes between four categories of private demand: consumer non-durable goods and non-housing services, consumer durable goods, residential investment, and non-residential investment. The boxes surrounding the producers in the figure illustrate the sources of each demand for each category. Consumer non-durable goods and services are sold directly to households; consumer durable goods, residential capital goods, and non-residential capital goods are intermediated through capital-goods intermediaries (owned by the households), who then rent these capital stocks to households. Consumer non-durable goods and services and residential capital goods are purchased (by households and residential capital goods owners, respectively) from the first of economy's two final goods producing sectors, while consumer durable goods and non-residential capital goods are purchased (by consumer durable and residential capital goods owners, respectively) from the second sector. In addition to consuming the non-durable goods and services that they purchase, households supply labor to the intermediate goods-producing firms in both sectors of the economy.

Finally, the EDO model assumes that labor input consists of both employment and hours per worker. Workers differ in the disutility they associate with employment. Moreover, the labor market is characterized by monopolistic competition. As a result, unemployment arises in equilibrium – some workers are willing to be employed at the prevailing wage rate, but cannot find employment because firms are unwilling to hire additional workers at the prevailing wage. As emphasized by Gali (2011), this framework for unemployment is simple and implies that the unemployment rate reflects wage pressures: When the unemployment rate is unusually high, the prevailing wage rate exceeds the marginal rate of substitution between leisure and consumption, implying that workers would prefer to work more.

The remainder of this section provides an overview of the decisions made by each of the agents in the economy. Given some of the broad similarities between the model and others, the presentation is selective.

3.1 The Final Goods Producers' Problem

The economy produces two final goods and services: slow-growing "consumption" goods and services, X_t^{cbi}, and fast-growing "capital" goods, X_t^{kb}. These final goods are produced by aggregating (according to a Dixit-Stiglitz technology) an infinite number of sector-specific differentiated intermediate inputs, $X_t^s(j)$ for $s = cbi, kb$, distributed over the unit interval. The representative firm in each of the consumption and capital goods producing sectors chooses the optimal level of each intermediate input, taking as given the prices for each of the differentiated intermediate inputs, $P_t^s(j)$, to solve the cost-minimization problem:

$$\min_{\{X_t^s(j)\}_{j=0}^1} \int_0^1 P_t^s(j) X_t^s(j) dj \text{ subject to } \left(\int_0^1 \left(X_t^s(j)\right)^{\frac{\Theta_t^s - 1}{\Theta_t^s}} dj \right)^{\frac{\Theta_t^s}{\Theta_t^s - 1}} \geq X_t^s, \text{ for } s = cbi, kb. \tag{1}$$

The term Θ_t^s is the stochastic elasticity of substitution between the differentiated intermediate goods inputs used in the production of the consumption or capital goods sectors. Letting $\theta_t^s \equiv \ln \Theta_t^s - \ln \Theta_*^s$ denote the log-deviation of Θ_t^s from its steady-state value of Θ_*^s, we assume that

$$\theta_t^s = \epsilon_t^{\theta,s}, \text{ for } s = cbi, kb, \tag{2}$$

where $\epsilon_t^{\theta,s}$ is an i.i.d. shock process. A stochastic elasticity of substitution introduces transitory markup shocks into the pricing decisions of intermediate-goods producers.

3.2 The Intermediate Goods Producers' Problem

The intermediate goods entering each final goods technology are produced by aggregating (according to a Dixit-Stiglitz technology) an infinite number of differentiated labor inputs, $L_t^s(j)$ for $s = cbi, kb$, distributed over the unit interval and combining this aggregate labor input (via a Cobb-Douglas production function) with utilized non-residential capital, $K_t^{u,nr,s}$. Each intermediate-good producing firm effectively solves three problems: two factor-input cost-minimization problems (over differentiated labor inputs and the aggregate labor and capital) and one price-setting profit-maximization problem.

In its first cost-minimization problem, an intermediate goods producing firm chooses the optimal level of each type of differential labor input, taking as given the wages for each of the differentiated types of labor, $W_t^s(i)$, to solve:

$$\min_{\{L_t^s(i,j)\}_{i=0}^1} \int_0^1 W_t^s(i) L_t^s(i,j) di \text{ subject to } \left(\int_0^1 \left(L_t^s(i,j)\right)^{\frac{\Theta_t^l - 1}{\Theta_t^l}} di \right)^{\frac{\Theta_t^l}{\Theta_t^l - 1}} \geq L_t^s(j), \text{ for } s = cbi, kb. \tag{3}$$

The term Θ_t^l is the stochastic elasticity of substitution between the differentiated labor inputs. Letting $\theta_t^l \equiv \ln \Theta_t^l - \ln \Theta_*^l$ denote the log-deviation of Θ_t^l from its steady-state value of Θ_*^l, we assume that

$$\theta_t^l = \epsilon_t^{\theta,l}. \tag{4}$$

where $\epsilon_t^{\theta,l}$ is an i.i.d. shock process. A stochastic elasticity of substitution introduces transitory wage markup shocks into the wage decisions of households.

In its second cost-minimization problem, an intermediate-goods producing firm chooses the optimal levels of aggregated labor input and utilized capital, taking as given the wage, W_t^s, for aggregated labor, L_t^s (which is generated by the cost function derived the previous problem), and the rental rate, $R_t^{nr,s}$, on utilized capital, $K_t^{u,nr,s}$, to solve:

$$\min_{\left\{L_t^s(j), K_t^{u,nr,s}(j)\right\}} W_t^s L_t^s(j) + R_t^{nr,s} K_t^{u,nr,s}(j)$$
$$\text{subject to } \left(Z_t^m Z_t^s L_t^s(j)\right)^{1-\alpha} \left(K_t^{u,nr,s}(j)\right)^{\alpha} \geq X_t^s(j), \text{ for } s = cbi, kb, \text{ with } Z_t^{cbi} \equiv 1. \quad (5)$$

The parameter α is the elasticity of output with respect to capital, while the Z_t variables denote the level of productivity. The level of productivity has two components. The first, Z_t^m, is common to both sectors and thus represents the level of economy-wide technology. The second, Z_t^s, is sector specific; Z_t^{cbi} is normalized to one, while Z_t^{kb} is not restricted.

The exogenous productivity terms contain a unit root, that is, they exhibit permanent movements in their levels. The stochastic processes Z_t^m and Z_t^{kb} evolve according to

$$\ln Z_t^n - \ln Z_{t-1}^n = \ln \Gamma_t^{z,n} = \ln\left(\Gamma_*^{z,n} \cdot \exp[\epsilon_t^{z,n}]\right) = \ln \Gamma_*^{z,n} + \epsilon_t^{z,n}, \ n = kb, m \quad (6)$$

where $\Gamma_*^{z,n}$ and $\epsilon_t^{z,n}$ are the steady-state and stochastic components of $\Gamma_t^{z,n}$. The stochastic component $\epsilon_t^{z,n}$ is an i.i.d shock process.

The unit-root in technology in both sectors yields a non-trivial Beveridge-Nelson permanent/transitory decomposition. The presence of capital-specific technological progress allows the model to generate differential trend growth rates in the economy's two production sectors. In line with historical experience, a more rapid rate of technological progress in capital goods production is accommodated by calibrating $\Gamma_*^{z,kb} > 1$, where (as is the case for all model variables) an asterisk on a variable denotes its steady-state value.

In its price-setting (or profit-maximization) problem, an intermediate goods producing firm chooses its optimal nominal price and the quantity it will supply consistent with that price. In doing so it takes as given the marginal cost, $MC_t^s(j)$, of producing a unit of output, $X_t^s(j)$, the aggregate price level for its sector, P_t^s, and households' valuation of a unit of nominal profits income in each period, which is given by $\Lambda_t^{cnn}/P_t^{cbi}$ where Λ_t^{cnn} denotes the marginal utility of non-durables and non-housing services consumption. Specifically, firms solve:

$$\max_{\{P_t^s(j),X_t^s(j)\}_{t=0}^{\infty}} \mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t^{cnn}}{P_t^{cbi}} \{P_t^s(j)X_t^s(j) - MC_t^s(j)X_t^s(j) - \frac{100 \cdot \chi^p}{2} \left(\frac{P_t^s(j)}{P_{t-1}^s(j)} - \eta^p \Pi_{t-1}^{p,s} - (1-\eta^p)\Pi_*^{p,s} \right)^2 P_t^s X_t^s \Big\}$$

subject to $X_\tau^s(j) = (P_\tau^s(j)/P_\tau^s)^{-\Theta_\tau^s} X_\tau^s$ for $\tau = 0, 1, ..., \infty$ and $s = cbi, kb$. (7)

The profit function reflects price-setting adjustment costs (the size which depend on the parameter χ^p and the lagged and steady-state inflation rate). The constraint against which the firm maximizes its profits is the demand curve it faces for its differentiated good, which derives from the final goods producing firm's cost-minimization problem. This type of price-setting decision delivers a new-Keynesian Phillips curve. Because adjustment costs potentially depend upon lagged inflation, the Phillips curve can take the "hybrid" form in which inflation is linked to its own lead and lag as well as marginal cost.

3.3 The Capital Owners' Problem

I now shift from producers' decisions to spending decisions. There exists a unit mass of non-residential capital owners (individually denoted by k, with k distributed over the unit interval) who choose investment in non-residential capital, E_t^{nr}, the stock of non-residential capital, K_t^{nr} (which is linked to the investment decision via the capital accumulation identity), and the amount and utilization of non-residential capital in each production sector, $K_t^{nr,cbi}$, U_t^{cbi}, $K_t^{nr,kb}$, and U_t^{kb}. (Recall, that the firm's choice variables in equation 5 is utilized capital $K_t^{u,nr,s} = U_t^s K_t^{nr,s}$.) The mathematical representation of this decision is described by the following maximization problem (in which capital owners take as given the rental rate on non-residential capital, R_t^{nr}, the price of non-residential capital goods, P_t^{kb}, and households' valuation of nominal capital income in each period, $\Lambda_t^{cnn}/P_t^{cbi}$, and the exogenous risk premium specific to non-residential investment, A^{nr}_τ):

$$\max_{\{E^{nr}_t(k),K^{nr}_{t+1}(k),K^{nr,cbi}_t(k),K^{nr,kb}_t(k)U^{cbi}_t(k),U^{kb}_t(k)\}_{t=0}^{\infty}}$$

$$\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda^{cnn}_t}{A^{nr}_\tau P^{cbi}_t} \Big\{ R^{nr}_t U^{cbi}_t(k) K^{nr,cbi}_t(k) + R^{nr}_t U^{kb}_t(k) K^{nr,kb}_t(k) - P^{kb}_t E^{nr}_t(k)$$
$$-\kappa \left(\frac{U^{cbi}_t(k)^{1+\psi} - 1}{1+\psi} \right) Q^{nr}_t K^{nr,cbi}_t - \kappa \left(\frac{U^{kb}_t(k)^{1+\psi} - 1}{1+\psi} \right) Q^{nr}_t K^{nr,kb}_t \Big\}$$

subject to

$$K^{nr}_{\tau+1}(k) = (1-\delta^{nr}) K^{nr}_\tau(k) + E^{nr}_\tau(k) - \frac{100 \cdot \chi^{nr}}{2} \left(\frac{E^{nr}_\tau(k) - E^{nr}_{\tau-1}(k)\Gamma^{x,kb}_t}{K^{nr}_\tau} \right)^2 K^{nr}_\tau \text{ and}$$
$$K^{nr,cbi}_\tau(k) + K^{nr,kb}_\tau(k) = K^{nr}_\tau(k) \text{ for } \tau = 0, 1, ..., \infty. \quad (8)$$

The parameter δ^{nr} in the capital-accumulation constraint denotes the depreciation rate for non-residential capital, while the parameter χ^{nr} governs how quickly investment adjustment costs increase when $(E^{nr}_\tau(k) - E^{nr}_{\tau-1}(k)\Gamma^{x,kb}_t)$ rises above zero; note that these adjustment costs include a term for the stochastic growth rate of the trend in the level of the output in sector KB, $\Gamma^{x,kb}_t$ equal to $\Gamma^{z,m}_t \Gamma^{z,kb}_t$. The variable A^{nr}_t is a stochastic element reflecting a risk premium on non-residential investment. Letting $a^{nr}_t \equiv \ln A^{nr}_t$ denote the log-deviation of A^{nr}_t from its steady-state value of unity, we assume that:

$$a^{nr}_t = \rho^{nr} a^{nr}_{t-1} + \epsilon^{a,nr}_t. \quad (9)$$

Higher rates of utilization incur a cost (reflected in the last two terms in the capital owner's profit function). Utilization is unity in the steady-state, implying $\kappa = R^{nr}_* / Q^{nr}_*$. The time-variation in utilization, along with the imperfect competition in product and labor markets, implies that direct measurement of total factor productivity may not provide an accurate estimate of technology; as a result, the EDO model can deliver smoother estimates of technology that might be implied by a real-business-cycle model.

The problems solved by the consumer durables and residential capital owners are slightly simpler than the non-residential capital owner's problems. Since utilization rates are not variable for these types of capital, their owners make only investment and capital accumulation decisions. Taking as given the rental rate on consumer durables capital, R_t^{cd}, the price of consumer-durable goods, P_t^{kb}, and households' valuation of nominal capital income, $\Lambda_t^{cnn}/P_t^{cbi}$, and the exogenous risk premium specific to consumer durables investment, A_τ^{cd}, the capital owner chooses investment in consumer durables, I_t^{cd}, and its implied capital stock, K_t^{cd}, to solve:

$$\max_{\{E_t^{cd}(k),K_{t+1}^{cd}(k)\}_{t=0}^{\infty}} \mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t^{cnn}}{A_\tau^{cd} P_t^{cbi}} \left\{ R_t^{cd} K_t^{cd}(k) - P_t^{kb} E_t^{cd}(k) \right\}$$

subject to

$$K_{\tau+1}^{cd}(k) = (1-\delta^{cd}) K_\tau^{cd}(k) + E_\tau^{cd}(k) - \frac{100 \cdot \chi^{cd}}{2} \left(\frac{E_\tau^{cd}(k) - E_{\tau-1}^{cd}(k)\Gamma_\tau^{x,kb}}{K_\tau^{cd}} \right)^2 K_\tau^{cd}$$

for $\tau = 0, 1, ..., \infty.$ (10)

The residential capital owner's decision is analogous:

$$\max_{\{E_t^{r}(k),K_{t+1}^{r}(k)\}_{t=0}^{\infty}} \mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t^{cnn}}{A_\tau^{r} P_t^{cbi}} \left\{ R_t^{r} K_t^{r}(k) - P_t^{cbi} E_t^{r}(k) \right\}$$

subject to

$$K_{\tau+1}^{r}(k) = (1-\delta^{r}) K_\tau^{r}(k) + E_\tau^{r}(k) - \frac{100 \cdot \chi^{r}}{2} \left(\frac{E_\tau^{r}(k) - E_{\tau-1}^{r}(k)\Gamma_\tau^{x,cbi}}{K_\tau^{cd}} \right)^2 K_\tau^{cd}$$

for $\tau = 0, 1, ..., \infty.$ (11)

The notation for the consumer durables and residential capital stock problems parallels that of non-residential capital. In particular, the asset-specific risk premia shocks, A_t^{cd} and A_t^{r}, follow an autoregressive process similar to that given in equation (9).

3.4 The Households' Problem

The final group of private agents in the model is the household who makes both expenditure and labor-supply decisions. There is a representative household with a continuum of members represented by the unit square and indexed by a pair (i, k). The first dimension indexed by i represents the type of labor service in which a given member of the household is specialized. The second dimension, indexed by k, determines her disutility of work. For an individual (i,k), we posit that the disutility of working is given by $\varsigma^N k^{\sigma_N} + \varsigma^h \frac{(h_t(i))^{1+\sigma_h}}{1+\sigma_h}$ if the worker is employed, and zero otherwise. Note that aggregate labor input is the product of hours per worker and employment.

Full risk sharing of consumption among household members is assumed. The household derives utility from four sources: purchases of the consumer non-durable goods and non-housing services, the flow of services from their rental of consumer-durable capital, the flow of services from their rental of residential capital, and leisure time, which is equal to what remains of their time endowment after labor is supplied to the market. Preferences are separable over all arguments of the utility function. The utility that the household derives from nondurable goods and services consumption is influenced by the habit stock for of this consumption component, which equals a factor h multiplied by its consumption last period E_{t-1}^{cnn}.

Household preferences over leisure are also subject to an exogenous shifter (Λ_t^{Lpref}), where

$$ln\Lambda_t^{Lpref} = \rho^{Lpref} ln\Lambda_{t-1}^{Lpref} + \epsilon_t^{Lpref}.$$

The labor supply shock equals 1 in the steady state.

Formally, household utility is given by

$$\begin{aligned} U_t = & \varsigma^{cnn} \ln(E_t^{cnn} - hE_{t-1}^{cnn}) + \varsigma^{cd} \ln(K_t^{cd}) \\ & + \varsigma^r \ln(K_t^r) - \Lambda_t^{Lpref} \sum_{s=cbi,kb} \int_0^1 \int_0^{N_t(i)} \left(\varsigma^{N,s} k^{\sigma_N} + \varsigma^{h,s} \frac{(h_t(i))^{1+\sigma_h}}{1+\sigma_h} \right) dkdi. \end{aligned}$$

or

$$\begin{aligned} U_t = & \varsigma^{cnn} \ln(E_t^{cnn} - hE_{t-1}^{cnn}) + \varsigma^{cd} \ln(K_t^{cd}) \\ & + \varsigma^r \ln(K_t^r) - \Lambda_t^{Lpref} \sum_{s=cbi,kb} \int_0^1 \left(\varsigma^{N,s} \frac{N_t^s(i)^{1+\sigma_N}}{1+\sigma_N} + \varsigma^{h,s} N_t^s(i) \frac{(h_t^s(i))^{1+\sigma_h}}{1+\sigma_h} \right) di. \end{aligned}$$

with $N_t^s(i)$ the fraction of household members specialized in type i labor in sector s.

In the utility function the parameter β is the household's discount factor, σ_h denotes its inverse hours supply elasticity, σ_N denotes its inverse employment supply elasticity, while ς^{cnn}, ς^{cd}, ς^{r}, and ς^{l} are scale parameter that tie down the ratios between the household's consumption components.

The stationary, unit-mean, stochastic variable Ω_t represents an aggregate risk-premium shock that drives a wedge between the policy short-term interest rate and the return to bonds received by a household. Letting $\omega_t \equiv \ln \Omega_t - \ln \Omega_*$ denote the log-deviation of Ω_t from its steady-state value of Ω_*, the process is

$$\omega_t = \rho^{\omega} \omega_{t-1} + \epsilon_t^{\omega}. \tag{12}$$

The variable ϵ_t^{ω} is an i.i.d. shock process, and ρ^{ω} represents the persistence of Ω_t.

The household's budget constraint reflects wage setting adjustment costs, which depend on the parameter χ^{w} and the lagged and steady-state wage inflation rate, and the costs in changing the mix of labor supplied to each sector, which depend on the parameter χ^{l}. The costs incurred by households when the mix of labor input across sectors changes may be important for sectoral co-movements.

3.5 Unemployment

As presented formally in Chung, Kiley, and Laforte (2012), households' marginal disutility of employment, in consumption units, is below the prevailing real wage rate – implying households would prefer a higher level of employment, as in Gali (2011). Consequently, there exists unemployment in equilibrium. Moreover, the unemployment rate fluctuates in equilibrium, as fluctuations in the wage markup (from wage and price rigidities and from markup shocks) imply fluctuations in the gap between workers' willingness to work and firms' willingness to hire.

Specifically, employment is determined (ignoring nominal rigidities) by the optimality condition:

$$\frac{MU_t^l}{MU_t^c} = \frac{\Lambda_t^{Lpref} \varsigma^N N_t(i)^{\sigma_N}}{MU_t^c} = \frac{\Theta_t^l - 1}{\Theta_t^l} \frac{W_t(i)}{P_t}.$$

For labor type k (noted above), labor force participation is the level of labor that the household would be willing to provide absent monopolistic competition

$$\frac{MU_t^l}{MU_t^c} = \frac{\Lambda_t^{Lpref}\varsigma^N L_i(i)^{\sigma_N}}{MU_t^c} = \frac{W_t(i)}{P_t}$$

Unemployment is defined as the difference between workers' willingness to work, as captured in the labor force participation condition, and firms willingness to hire, as captured in the optimal employment condition. Most importantly, the presence of a wage markup implies that unemployment exists and fluctuates with the markup.[3]

The role of markup fluctuations in determining unemployment carries over to the sticky-wage framework in EDO. While this particular notion of unemployment is obviously stylized and incomplete (in emphasizing market power and markup fluctuations in the labor market, rather than search frictions and other factors), it emphasizes one key factor – Keynesian-style wage rigidities – and allows the model to confront unemployment data. As emphasized in Gali (2011) and Gali, Smets, and Wouters (2011), these benefits are sizable, although future research will surely wish to explore other factors.

3.6 Gross Domestic Product

The demand and production aspects of the model are closed through the exogenous process for demand other than private domestic demand. $\widetilde{X}_t^{HG}$ represents exogenous demand (i.e., GDP other than private domestic demand, the aggregate of E_t^{cnn}, E_t^{cd}, E_t^r, and E_t^{nr}). In this formulation, $\widetilde{X}_t^{HG}$ represents the level of expenditure relative to the stochastic long-run trend, i.e., the model assumes balanced growth, so exogenous demand for each sector fluctuates around its long-run trend; for example, the long-run trend for sector KB is given by $Z_t^m Z_t^{kb}$. Exogenous demand is assumed to follow the

[3]Manipulating these expressions to form an unemployment rate, it is easy to see that the natural rate of unemployment fluctuates in response to markup shocks, but not labor supply shocks – thereby addressing the identification issue in Chari, Kehoe, and Mcgrattan (2009), a point emphasized by Gali, Smets, and Wouters (2011). This issue is not a focus on this analysis.

process:

$$\ln \widetilde{X}_t^{HG} - \ln \widetilde{X}_*^{HG} = \rho^{HG} \left(\ln \widetilde{X}_t^{HG} - \ln \widetilde{X}_*^{HG} \right) + \epsilon_t^{HG}.$$

We assume that the exogenous demand impinges on each sector symmetrically, and specifically that the percent deviation of exogenous demand proportionally affects demand for each sector's ($s = cbi, kb$) output via the share of exogenous demand in total demand, ω_{HG}.

The rate of change of Gross Domestic Product (real GDP) equals the Divisia (share-weighted) aggregate of production in the two sectors (and of final spending across each expenditures category), as given by the identity:

$$H_t^{gdp} = \left(\left(\frac{X_t^{cbi}}{X_{t-1}^{cbi}} \right)^{P_*^{cbi} X_*^{cbi}} \left(\frac{X_t^{kb}}{X_{t-1}^{kb}} \right)^{P_*^{kb} X_*^{kb}} \right)^{\frac{1}{P_*^{cbi} X_*^{cbi} + P_*^{kb} X_*^{kb}}} . \tag{13}$$

3.7 Monetary Authority and A Long-term Interest Rate

The last important agent in our model is the monetary authority. It sets monetary policy in accordance with an Taylor-type interest-rate feedback rule. Policymakers smoothly adjust the actual interest rate R_t to its target level $\bar{R}_t$

$$R_t = (R_{t-1})^{\rho^r} \left(\bar{R}_t \right)^{1-\rho^r} \exp\left[\epsilon_t^r \right], \tag{14}$$

where the parameter ρ^r reflects the degree of interest rate smoothing, while ϵ_t^r represents a monetary policy shock. The central bank's target nominal interest rate, $\bar{R}_t$ depends the deviation of output from the level consistent with current technologies and "normal" (steady-state) utilization of capital and labor ($\tilde{X}^{pf}$, the "production function" output gap–defined more formally below). Consumer price inflation also enters the target. The target equation is:

$$\bar{R}_t = \left(\tilde{X}_t^{pf} \right)^{r^y} \left(\frac{\Pi_t^c}{\Pi_*^c} \right)^{r^\pi} R_*. \tag{15}$$

In equation (15), R_* denotes the economy's steady-state nominal interest rate, and ϕ^y and ϕ^π denote the weights in the feedback rule. Consumer price inflation, Π_t^c, is the weighted average of inflation in the nominal prices of the goods produced in each sector, $\Pi_t^{p,cbi}$ and $\Pi_t^{p,kb}$:

$$\Pi_t^c = (\Pi_t^{p,cbi})^{1-w_{cd}}(\Pi_t^{p,kb})^{w_{cd}}. \tag{16}$$

The parameter w^{cd} is the share of the durable goods in nominal consumption expenditures.

The model also includes a long-term interest rate (RL_t), which is governed by the expectations hypothesis subject to an exogenous term premia shock:

$$RL_t = \mathcal{E}_t \left[\Pi_{\tau=0}^{N} R_\tau\right] \cdot \Upsilon_t. \tag{17}$$

where Υ is the exogenous term premium, governed by

$$Ln\left(\Upsilon_t\right) = \left(1-\rho^{\Upsilon}\right) Ln\left(\Upsilon_*\right) + \rho^{\Upsilon} Ln\left(\Upsilon_{t-1}\right) + \epsilon_t^{\Upsilon}. \tag{18}$$

In this version of EDO, the long-term interest rate plays no allocative role; nonetheless, the term structure contains information on economic developments useful for forecasting (e.g., Edge, Kiley, and Laforte (2010)) and hence RL is included in the model and its estimation.

4 Definition of Output Gaps

Output gaps are variously defined as

- The deviation of output from its long-run level, as in Beveridge and Nelson (1981).
- The deviation of output from a level consistent with a production function, the current level of capital input, and labor input at long-run values (the production function approach of, for example, the CBO (2001)).
- The deviation of output from the level consistent with flexible prices and wages, as in the natural rate approach emphasized in New-Keynesian models (Woodford (2003) and Edge, Kiley, and Laforte (2008)).

The output gap as defined by Beveridge and Nelson (1981)) is given by (ignoring constant terms)

$$\tilde{X}_t^{bn} = \mathcal{E}_t \left[\sum_{\tau=-\infty}^{t} H_\tau^{gdp} - \sum_{\tau=-\infty}^{\infty} H_\tau^{gdp} \right]. \tag{19}$$

In words, the Beveridge-Nelson gap is the deviation of the level of output in period t from the level expected to prevail in the long-run (in period $infty$), with levels in equation 19 expressed as cumulative growth rates. Implicit in the definition of the Beveridge-Nelson trend above is its alternative representation

$$\tilde{X}_t^{bn} = \mathcal{E}_t \left[- \sum_{\tau=t+1}^{\infty} H_\tau^{gdp} \right], \tag{20}$$

i.e., the Beveridge-Nelson gap is the forecast of GDP growth in excess of its steady-state level going forward. This measure is computed for the EDO model given the implied reduced-form vector autoregressive/moving average representation of the model in terms of the observable variables used in its estimation. However, it is clear from this definition that a structural model is not needed to compute the Beveridge-Nelson gap; all that is required is an accurate description of the stochastic process followed by output, which makes the Beveridge-Nelson approach to estimation of output gaps one that can be tackled using a wide variety of empirical approaches.

Turning to the production function approach, production in each sector of the EDO model is governed by a Cobb-Douglas production function. In the production function approach to measuring the output gap, the gap is defined as the deviation of output from the level that would occur if labor input (per capita) and utilization rates equaled their steady-state values (where these steady-state values, denoted with a $*$, are constant, with the latter equal to one). As a result, the production-function gap is given by the Divisia-weighted (i.e., share-weighted) aggregate of the production-function gaps in each sector, which are defined by

$$\tilde{X}_t^{S,PF} = \ln((L_t^s/L_*)^{1-\alpha} (U_t^s)^\alpha); \;\; s = cbi, kb, \tag{21}$$

Several points are noteworthy. First, variable utilization of capital and capital adjustment costs, in addition to imperfect competition in product and labor markets, imply that simple "growth accounting" may not accurately measure the production-function gap. For EDO, the production function gap is inferred by imposing the model's structural restrictions and using the data on all observables to infer this gap. In addition, the production function gap, as written above, does not depend on any smoothing of technology: In EDO, the cyclical movements in total factor productivity (properly measured, after accounting for imperfect competition and the effects of variable utilization and capital adjustment costs) are solely a function of utilization, which enters equation 21; production-function based methods that do not rely on an entire model's structure to control for cyclical movements in total factor productivity, such as those of the CBO or the Federal Reserve's FRB/US model, may smooth their measures of total factor productivity according to some method, and such effects would enter equation 21 through the utilization term (although alternative presentations of the production-function method may include such adjustments as a separate term in their accounting). Because labor input and utilization move to their steady-state values in the long run and production always lies on the production function for each sector (by definition), the production function gap differs from the Beveridge-Nelson gap solely because of deviations of the (aggregate productive) capital stock from its long-run level. To the extent that the contribution of capital stock deviations from long-run levels contribute only moderately to overall deviations of production from its long-run level, it is reasonable to expect that the production function gap and the Beveridge-Nelson gaps will be similar.

The final definition is the natural-rate gap, which is the gap between output and the level that would prevail absent wage and price rigidities and markup shocks.[4]

[4] Edge, Kiley, and Laforte (2008) follow the literature in distinguishing between two natural-rate concepts: One is the level of output that would be achieved in the absence of nominal rigidities and variable markups, taking as given the current values for the economy's "state" variables (e.g., the capital stock, etc.); the second is the level of output that would prevail if nominal rigidities and markup fluctuations had been absent arbitrarily far in the past through current data – a situation that would imply a different "state" vector. Herein, the former definition is reported. Qualitative

This is the concept emphasized in Woodford (2003), largely because the analysis of Woodford considers only distortions associated with nominal rigidities and markup shocks (so that the natural rate is the efficient rate). The relationship between the natural rate of output and economic efficiency depends upon whether certain shocks are distortionary – a point on which theory is ambiguous, a standard feature of New-Keynesian DSGE models (discussed, for example, in Smets and Wouters (2007)). [5]. Edge, Kiley, and Laforte (2008) highlighted this issue with regard to "investment shocks": To the extent these shocks proxy for financial frictions (which may be the case for the risk premium shocks in EDO), it is not clear whether the resulting fluctuations are efficient.[6]

results do not depend on this distinction.

[5]Chari, Kehoe, and McGrattan (2009) are highly critical of this ambiguity, and suggest that DSGE models must find some method for identifying whether such shocks are distortionary or non-distortionary before such models can be used in policy applications; one possibility they mention is to explore microeconomic implications of these shocks/distortions. In this version of EDO, the New-Keynesian specification of unemployment distinguishes between labor supply and wage markup shocks, as in Gali, Smets, and Wouters (2011), addressing one concern of Chari, Kehoe, and McGrattan (2009).

[6]Chung, Kiley, and Laforte (2012) provide additional details on the links between the risk premium shocks in EDO and models of financial frictions. The risk premia wedges are related to the *investment wedge* of Chari, Kehoe, and McGrattan (2007), which represents a wedge between the marginal rates of substitution and technical substitution between current and future consumption.The central role of this efficiency condition is highlighted in standard microeconomic texts, e.g. Kreps (1990), page 167. Interpreting such wedges as inefficient is consistent with the idea that fluctuations in such premia reflect, for example, information imperfections that make external finance more costly than internal funds, as in Bernanke, Gertler, and Gilchrist (1999). Carlstrom and Fuerst (2009) present a model without capital, and hence their financial frictions enter in quite a different manner than the intertemporal wedges discussed above. Nonetheless, their presentation is a nice example of the difference between efficient allocations and the constrained optimal monetary policy.

5 Estimation Strategy and Results

Within EDO, fluctuations in all economic variables are driven by thirteen structural shocks. It is most convenient to summarize these shocks into five broad categories:

- Permanent technology shocks: This category consists of shocks to aggregate and investment-specific (or fast-growing sector) technology.

- A labor supply shock: This shock affects the willingness of to supply labor. As was apparent in our earlier description of the unemployment rate and in the presentation of the structural drivers below, this shock captures very persistent movements in unemployment that the model judges are not indicative of wage pressures. While EDO labels such movements labor supply shocks, an alternative interpretation would describe these as movements in unemployment that reflect persistent structural features not otherwise captured by the model.

- Financial, or intertemporal, shocks: This category consists of shocks to risk premia. In EDO, variation in risk premia – both the premium households' receive relative to the federal funds rate on nominal bond holdings and the additional variation in discount rates applied to the investment decisions of capital intermediaries – are purely exogenous. Nonetheless, the specification captures aspects of related models with more explicit financial sectors (e.g., Bernanke, Gertler, and Gilchrist (1999)), as we discuss in our presentation of the model's properties below. The term premium shock only affects the long-term interest rate.

- Markup shocks: This category includes the price and wage markup shocks.

- Other demand shocks: This category includes the shock to autonomous demand and a monetary policy shock.

Using this categorization, only technology shocks affect the Beveridge-Nelson *permanent component*. The Beveridge-Nelson *gap* reflects the influence of all shocks (and technology shocks imply movements in the gap, as the economy does not instantaneous adjust to the long-run implications of a shock to technology for standard neoclassical adjustment reasons and because of the short-run impediments to adjustment created by wage and price rigidities).

Several shocks do not influence the flexible-price or natural-rate of output (defined as the flexible price and constant markup outcome): markup shocks, by definition; the monetary policy shock, as such shocks are neutral under price and wage flexibility; and the aggregate risk premium shock driving a wedge between the household return to a nominal bond and the policy interest rate, which enters everywhere the nominal funds rate enters and hence affects the *natural rate of interest* but not the *natural rate of output* (as in related models, e.g., Smets and Wouters (2007)). The last point will be important in discussion prescriptions for policy from gaps and policy rules. It will also be quite important in examining the historical fluctuations in the *natural rate of interest* and the *natural rate of output*, as this is an important shock in EDO and was the shock that emerged as central in the 2008-2009 recession where the link between the funds rate and other bond yields broke due to a jump in risk spreads.

The empirical implementation of the model takes a log-linear approximation to the first-order conditions and constraints that describe the economy's equilibrium, casts this resulting system in its state-space representation for the set of (in our case 13) observable variables, uses the Kalman filter to evaluate the likelihood of the observed variables, and forms the posterior distribution of the parameters of interest by combining the likelihood function with a joint density characterizing some prior beliefs. Since we do not have a closed-form solution of the posterior, we rely on Markov-Chain Monte Carlo (MCMC) methods.

The model is estimated using 13 data series over the sample period from 1984:Q4 to 2011:Q4. The series are:

1. The civilian unemployment rate (U);
2. The growth rate of real gross domestic product (ΔGDP);

3. The growth rate of real consumption expenditure on non-durables and services (ΔC);
4. The growth rate of real consumption expenditure on durables (ΔCD);
5. The growth rate of real residential investment expenditure (ΔRes);
6. The growth rate of real business investment expenditure (ΔI);
7. Consumer price inflation, as measured by the growth rate of the Personal Consumption Expenditure (PCE) price index ($\Delta P_{C,total}$);
8. Consumer price inflation, as measured by the growth rate of the PCE price index excluding food and energy prices ($\Delta P_{C,core}$);
9. Inflation for consumer durable goods, as measured by the growth rate of the PCE price index for durable goods (ΔP_{cd});
10. Hours, which equals hours of all persons in the non-farm business sector from the Bureau of Labor Statistics (H);[7]
11. The growth rate of real wages, as given by compensation per hour in the non-farm business sector from the Bureau of Labor Statistics divided by the GDP price index (ΔRW);
12. The federal funds rate (R).
13. The yield on the 2-yr. U.S. Treasury security (RL).

Our implementation adds measurement error processes to the likelihood implied by the model for all of the observed series used in estimation except the short-term nominal interest rate series.

The model' calibrated parameters are presented in Table 1 and 2, while the estimated parameters are presented in Tables 3, 4, and 5. We based our decision on

[7]We remove a low-frequency trend from hours. We first pad the historical series by appending 40 quarterly observations which approach the most recent 40-quarter moving average of the data at a rate of 0.05 percent per quarter. We then extract a trend from this padded series via the Hodrick-Prescott filter with a smoothing parameter of 6400; our model is not designed to capture low frequency trends in population growth or labor force participation.

Table 1: Calibrated Parameters

β	α	ψ	δ^{nr}	δ^{cd}	δ^{r}
0.999	0.260	1	0.030	0.055	0.0035
Θ_*^{cbi}, Θ_*^{kb}, Θ_*^{l}	$\Gamma_*^{z,m}$	$\Gamma_*^{z,kb}$	ω_{HG}	Π_*^c	
7.000	1.000	1.011	0.20	1.005	

Table 2: Measurement Errors on Observable Variables

$ME_{\Delta gdp}$	$ME_{\Delta cns}$	$ME_{\Delta cd}$	$ME_{\Delta res}$	$ME_{\Delta bi}$	ME_{rw}
0.3	0.1	1.5	1.5	1.5	0.3
$ME_{\Delta p_{pce}}$	$ME_{\Delta p_{corepce}}$	$ME_{\Delta p_{cd}}$	ME_h	ME_U	ME_{rl}
0.5	0.05	0.2	0.3	0.1	0.1

whether to calibrate or estimate a parameter on several considerations. First, some determinants of steady-state behavior were calibrated to yields growth rates of GDP and associated price indexes that corresponded to "conventional" wisdom in policy circles, even though slight deviations from such values would have been preferred (in a "statistically significant" way) to our calibrated values. In other cases, parameters were calibrated based on how informative the data were likely to be on the parameter and/or identification and overparameterization issues. Finally, the standard deviations of the measurement error assumed in the observables was chosen to ensure a moderate contribution of such errors to the variability in the data (according to our model) while also preserving desirable forecast properties; we present the observables and the role of measurement error in the results below.

The first three columns of Table 3 and 4 outline our assumptions about the prior distributions of the estimated parameters, the remaining columns describe the parameters' posterior distributions. The habit-persistence parameter is moderate, near

Table 3: Estimated Behavioral Parameters

	Prior Distribution			Posterior Distribution				
Parameter	Type	Mean	S.D.	Mode	S.D.	10th perc.	50th perc.	90th perc.
h	U	0.000	0.577	0.715	0.014	0.719	0.742	0.756
σ_N	G	0.125	10.000	39.408	13.789	34.734	51.789	69.987
σ_h	G	1.000	10.000	21.886	8.834	18.471	28.787	40.795
r_π	N	1.500	0.062	1.463	0.060	1.377	1.440	1.538
r_y	N	0.125	0.125	0.263	0.023	0.186	0.224	0.253
ρ^r	N	0.500	0.250	0.833	0.015	0.819	0.847	0.862
χ^p	G	4.000	1.000	3.545	0.574	2.784	3.429	4.276
χ^w	G	4.000	1.000	5.494	1.597	2.460	4.087	6.414
χ^l	G	4.000	1.000	3.229	2.831	2.369	5.099	9.416
χ^{nr}	G	4.000	1.000	0.253	0.069	0.244	0.311	0.422
χ^{cd}	G	4.000	1.000	0.470	0.167	0.511	0.630	0.942
χ^r	G	4.000	1.000	9.140	2.581	8.209	11.458	14.893
η^p	N	0.000	0.250	0.314	0.116	0.202	0.357	0.503
η^w	N	0.000	0.250	-0.230	0.091	-0.320	-0.216	-0.109

Table 4: Estimated Autoregressive (and Mean) Parameters for Exogenous States

	Prior Distribution			Posterior Distribution				
Parameter	Type	Mean	S.D.	Mode	S.D.	10th perc.	50th perc.	90th perc.
ρ^ω	N	0.000	0.500	0.895	0.008	0.888	0.904	0.909
ρ^{nr}	N	0.000	0.500	0.938	0.006	0.919	0.931	0.937
ρ^{cd}	N	0.000	0.500	-0.240	0.117	-0.377	-0.206	-0.070
ρ^r	N	0.000	0.500	0.877	0.008	0.859	0.868	0.881
ρ^{Lpref}	N	0.500	0.250	0.979	0.005	0.966	0.970	0.978
ρ^{HG}	B	0.500	0.015	0.582	0.122	0.404	0.577	0.723
ρ^Υ	N	0.000	0.500	0.909	0.030	0.883	0.917	0.957
$Ln(\Upsilon_*)$	N	0.000	0.001	0.000	0.000	0.000	0.000	0.000

Table 5: Estimated Innovation Standard Errors for Exogenous States

	Prior Distribution			Posterior Distribution				
Parameter	Type	Mean	S.D.	Mode	S.D.	10th perc.	50th perc.	90th perc.
σ_{ω}	I	1.000	2.000	0.295	0.018	0.253	0.284	0.301
$\sigma_{a,nr}$	I	1.000	2.000	0.403	0.056	0.394	0.423	0.541
$\sigma_{a,cd}$	I	1.000	2.000	9.112	4.531	8.896	12.284	20.515
$\sigma_{a,r}$	I	1.000	2.000	0.515	0.136	0.450	0.725	0.851
σ_{lpref}	I	1.000	2.000	5.665	2.472	5.080	8.000	11.359
$\sigma_{\theta,l}$	I	1.000	2.000	2.493	0.674	1.110	1.879	2.862
σ_{HG}	I	1.000	2.000	0.579	0.105	0.456	0.581	0.733
$\sigma_{z,m}$	I	0.250	2.000	0.597	0.080	0.530	0.614	0.734
$\sigma_{z,kb}$	I	0.250	2.000	0.936	0.148	0.749	0.935	1.111
$\sigma_{\theta,cbi}$	I	0.200	2.000	0.452	0.075	0.365	0.430	0.548
$\sigma_{\theta,kb}$	I	0.200	2.000	0.685	0.179	0.444	0.658	0.914
σ_{Υ}	I	0.200	2.000	0.105	0.002	0.101	0.104	0.106
σ_{r}	I	0.200	2.000	0.124	0.004	0.121	0.127	0.130

0.7.[8] Investment adjustment costs are large for residential investment but small for business investment and consumer durables. The estimated value of the inverse of the labor supply elasticities for employment and hours given employment show that the hours margin exhibits somewhat more flexibility than the employment margin. Finally, the estimated quadratic costs in prices and wages are sizable, with somewhat higher costs for wage adjustment. We also find only a modest role for lagged inflation in our adjustment cost specification (around 1/3), equivalent to modest indexation to lagged inflation in other sticky-price specifications. Overall, while the EDO model is large, it has standard features, which are discussed more thoroughly in Chung, Kiley, and Laforte (2012).

With regard to exogenous disturbances in the model, the exogenous aggregate risk premium and those affecting the decisions to invest in productive capital and housing are all fairly persistent. Moreover, the labor supply shock is quite persistent, exceeding 0.95 at the posterior mode.

The role of these shocks within the model, on average, can be seen by looking at the variance decomposition of real GDP, the unemployment rate, core PCE inflation, and the federal funds rate implied by the model. Tables 6 present forecast error variance decompositions for selected shocks at various (quarterly) horizons at the posterior mode of the parameter estimates for these variables (results summarized and presented in more detail in Chung, Kiley, and Laforte (2012)). The variance of real GDP growth is accounted for primarily by the technology shocks in each sector ($\epsilon^{z,m}$ and $\epsilon^{z,k}$), although the economy-wide risk premium shock contributes non-negligibly at short horizons; as noted in the introduction, this result shows echoes of the models roots in the real-business cycle paradigm initiated by Kydland and Prescott (1982). However, quarterly moves in real GDP are not typically viewed as the best indicator of the overall state of the economy – indeed, information from the unemployment

[8]See Kiley (2010a) for a discussion of issues related to identification of the habit parameter using frequentist techniques.

Table 6: Forecast Error Variance Decomposition for Key Variables (at posterior mode of parameters)

Shocks	Horizon	Real GDP	Unempl. rate	Inflation (core)	Federal Funds Rate
ϵ^{ω}	1	(0.29,0.32,0.35)	(0.37,0.40,0.43)	(0.01,0.01,0.02)	(0.02,0.02,0.02)
	16	(0.03,0.04,0.06)	(0.32,0.35,0.39)	(0.00,0.01,0.02)	(0.49,0.51,0.54)
	∞	(0.00,0.00,0.00)	(0.15,0.21,0.25)	(0.01,0.01,0.03)	(0.61,0.66,0.69)
$\epsilon^{a,nr}$	1	(0.21,0.26,0.30)	(0.31,0.35,0.40)	(0.00,0.01,0.01)	(0.01,0.01,0.02)
	16	(0.26,0.31,0.43)	(0.36,0.42,0.47)	(0.00,0.01,0.02)	(0.21,0.25,0.28)
	∞	(0.00,0.00,0.00)	(0.24,0.29,0.39)	(0.03,0.04,0.07)	(0.05,0.06,0.08)
$\epsilon^{a,cd}$	1	(0.02,0.03,0.03)	(0.01,0.01,0.02)	(0.00,0.00,0.00)	(0.00,0.00,0.00)
	16	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)
	∞	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)
$\epsilon^{a,r}$	1	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)
	16	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.01,0.01,0.01)
	∞	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.00,0.00,0.00)	(0.02,0.02,0.03)
$\epsilon^{z,m}$	1	(0.20,0.24,0.29)	(0.00,0.01,0.01)	(0.10,0.12,0.14)	(0.01,0.01,0.01)
	16	(0.41,0.46,0.53)	(0.01,0.02,0.02)	(0.30,0.35,0.40)	(0.03,0.03,0.04)
	∞	(0.64,0.70,0.76)	(0.01,0.01,0.02)	(0.23,0.28,0.34)	(0.01,0.02,0.02)
$\epsilon^{z,kb}$	1	(0.07,0.09,0.11)	(0.00,0.00,0.00)	(0.01,0.01,0.01)	(0.00,0.00,0.00)
	16	(0.10,0.13,0.16)	(0.00,0.00,0.00)	(0.04,0.04,0.05)	(0.00,0.00,0.00)
	∞	(0.24,0.30,0.35)	(0.00,0.00,0.00)	(0.05,0.06,0.08)	(0.00,0.00,0.00)

rate is also viewed as very indicative of cyclical conditions. The variance of the unemployment rate is accounted for primarily by the economy-wide risk premium (ϵ^{ω}) and business investment risk premium ($\epsilon^{a,nr}$) shocks at horizons between one and sixteen quarters. Technology shocks in each sector contribute very little, while the labor supply shock contributes quite a bit a low frequencies. The large role for risk premia shocks in the forecast error decomposition at business cycle horizons illustrates the importance of this type of "demand" shock for volatility in the labor market. This result is notable, as the unemployment rate is the series most like a "gap" variable in the model – that is, the unemployment rate shows persistent cyclical fluctuations about its long-run value. It is also important to remember that the aggregate risk premium shock is a pure shock to the "natural rate of interest" (Woodford (2003)), and hence its role in unemployment fluctuations provides a clue that a natural-rate definition of the output gap may resemble gaps from other approaches. Finally, the variance of core inflation is accounted for primarily by the markup shocks, and the variance of the federal funds rate is accounted for primarily by the economy-wide risk premium (except in the very near term, when the monetary policy shock is important).

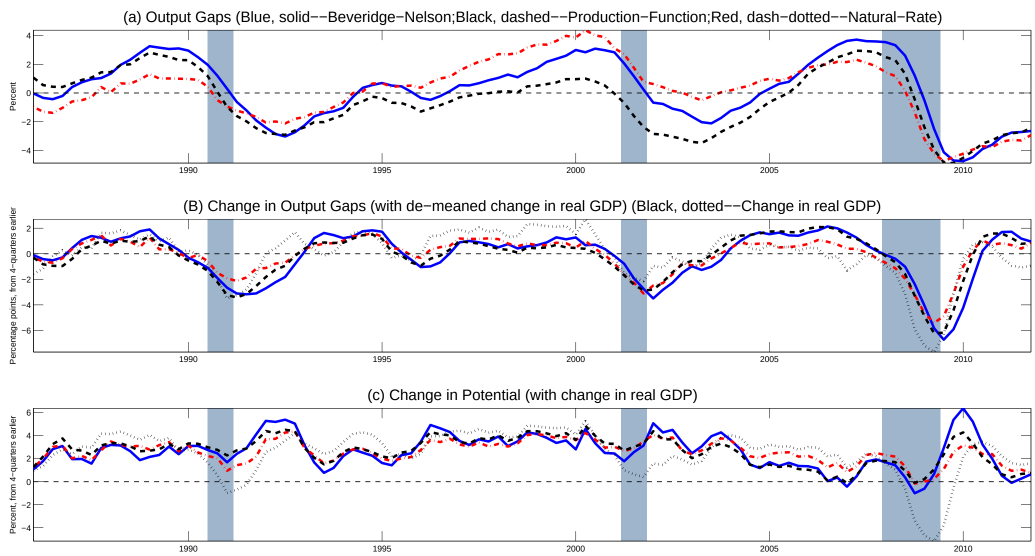
6 Output Gap Estimates

The analysis now presents estimates of alternative definitions of the output gap.

Figure 2 presents the estimate of the gap implied by each approach from the EDO model in the upper panel; the shading represents National Bureau of Economic Research (NBER) recession periods. It is clear that each measure of the gap captures the cyclical peaks in activity as identified by the NBER well.

It is also clear that the EDO Beveridge-Nelson output gap (the blue, solid line) has continue to widen following the NBER-identified end of recent recessions – consistent with the generally agreed upon view that these periods have been sluggish or

Figure 2: Output Gaps from the EDO Model



"jobless" recoveries. The picture of the Beveridge-Nelson gap implied by EDO also shows a fairly smooth evolution of the gap; this contrasts with much of the literature on univariate time-series estimates of the Beveridge-Nelson gap (e.g., the discussion in Morley, Nelson, and Zivot (2003)), but echoes the result from (at least some) multivariate time series approaches (e.g., Rotemberg and Woodford (1996)).

It is not surprising that the Beveridge-Nelson and production-function (black, dashed line) gaps move closely together: The difference between the two estimates reflects the use of the current, rather than long-run, capital stock in the production function approach, and business cycle fluctuations in the capital stock are modest relative to those in labor input of aggregate output.

It is probably more surprising to some readers that the natural-rate gap is closely related to the Beveridge-Nelson cycle (as some DSGE models, such as that of Edge, Kiley, and Laforte (2008)) showed more notable differences between a natural-rate gap and a traditional gap. These earlier DSGE models relied more heavily on investment-technology shocks, in contrast to shocks to the natural rate of interest; the importance of financial disturbances in recent fluctuations highlights how the shift toward models with financial disturbances may reconcile traditional views of the business cycle with the view from DSGE models, at least to a significant degree.

The middle panel of figure 2 presents the percent change from four-quarters earlier in the gap estimates, along with the (de-meaned) change in real GDP: As is clear, the cyclical movements in GDP associated with expansions and contractions are well captured by changes in the gap–confirming a traditional view of overall fluctuations in economic activity.

That said, there remains a sizable amount of variation in "potential" output according to each approach (the lower panel). As shown below, this variation is significantly different that that shown in estimates from policy institutions.

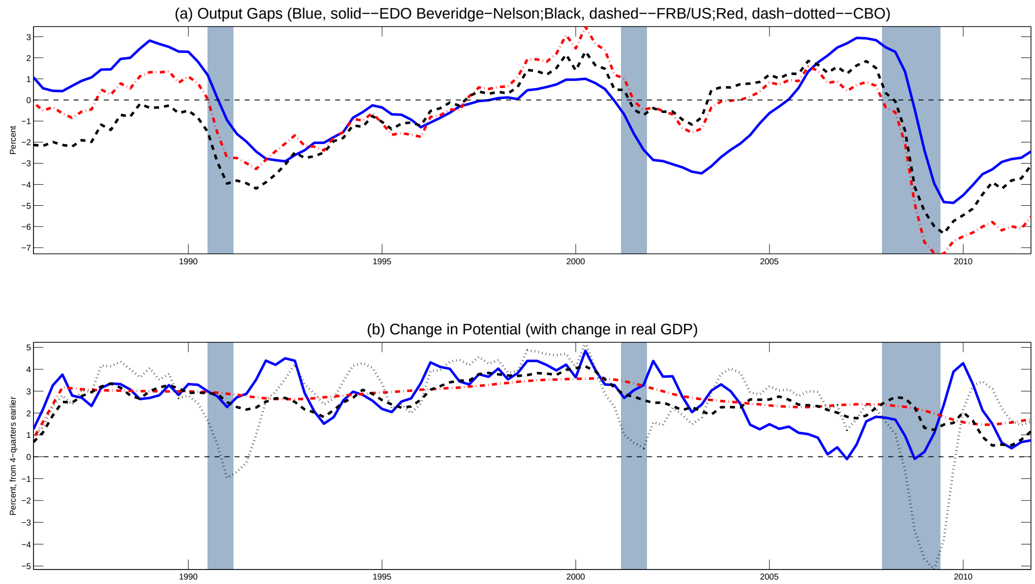
7 Comparing Gaps from EDO to Other Estimates

7.1 Estimates

Figure 3 present the production-function based output gaps from the CBO (CBO (2010) and from the Federal Reserve's FRB/US model, along with the Beveridge-Nelson gap from the EDO model, in the upper panels. The CBO gap is widely used by economists, and the FRB/US model has been used at the Federal Reserve Board for some time. These gaps are fairly strongly correlated with the Beveridge-Nelson cycle from the EDO model (with correlation coefficients exceeding 0.7 for both measures). In terms of broad cyclical swings, the CBO and FRB/US gaps share similarities with the gap from the DSGE model. But there are important differences as well–according to the DSGE model, output was more notably above potential in 2007, and the shortfall relative to potential following the financial crisis and recession, while sizable, is estimated to be somewhat more modest by the DSGE model.

The lower panel of the figure presents the percent change from the four-quarters

Figure 3: Output Gaps from the EDO Model



(at an annual rate), along with the change in the Beveridge-Nelson permanent component from EDO. While the gaps in the upper panels are highly correlated, the trend estimates from the FRB/US model and especially from the CBO are much smoother. In short, EDO has a much more variable "potential" growth rate.

Of course, the data on GDP is the same for the EDO Beveridge-Nelson gap, the CBO gap, and the FRB/US model gap. Moreover, the growth rate of GDP equals the sum of the change in the gap and potential GDP growth. As a result, similar movements in the gap for all three measures, and quite different movements in potential, must imply quite different *covariances* between actual growth, the change in the gap, and potential growth. For example, the variance of GDP growth equals the sum of the variance of the change in the gap, the variance of potential growth, and twice the covariance of the change in the gap and potential growth; similar mixes of variances and covariances could be done for other combinations of actual/gap/potential. In

EDO, the covariance between the change in the Beveridge-Nelson gap and the corresponding change in potential is negative, which explains the more variable estimate of trend – much like in Morley, Nelson, and Zivot (2003). In contrast, the covariance for the CBO and FRB/US measures is positive. The negative covariance in the EDO DSGE model is intuitive – it is well known that New-Keynesian features in DSGE models impede the adjustment of the economy toward its efficient level (e.g., the literature survey in Boivin, Kiley, and Mishkin (2011)).

The structural implications of DSGE models like EDO for the covariance between the Beveridge-Nelson trend and cycle could be used to inform the related time-series literature. For example, Beveridge and Nelson (1981) simply assumed an ARIMA representation of GDP growth, which imposes no restrictions on the covariances between innovations to the trend and cycle; Clark (1987) and other research assumed an unobserved components structure for GDP growth with an assumption of zero correlation between the (true) innovations to the trend and cycle; and Morley, Nelson, and Zivot (2003) discuss in detail the role of such assumptions and the strength of evidence for/against a correlation between innovations to trend and cycle.

7.2 Slack and Unemployment

A final subject that is important in policy discussions related to the output gap or economic slack concerns the relationship of slack, in a projection or reduced-form sense, to key economic variables - most significantly the unemployment rate. Indeed, such a correlations is among the most important reduced-form relationships in empirical macroeconomics (i.e., the unemployment/output gap relationship known as Okun's law (Okun (1962))).

Table 7 presents the correlations of the level of various gap measures from EDO and policy institutions with the unemployment rate, and the correlations of changes in the same gaps with changes in the unemployment rate.

All the measures of the output gap show the Okun's law relationship – that is,

Table 7: Correlation of Gaps with Unemployment Rate (1)

Gap measure	Levels	Changes
Beveridge-Nelson	-0.83	-0.82
Natural rate	-0.97	-0.86
CBO	-0.93	-0.62
FRB/US	-0.93	-0.60

1. Sample 1985q1-2011q4.

output gaps and their changes are strongly negatively correlated with the level or change in the unemployment rate. This is basic macroeconomics – deviations of output from long-run levels must be associated with movements in labor input, and the lead of hours over unemployment is a well-known regularity.

It may be somewhat surprising that the natural rate gap is even more strongly correlated with the unemployment rate than are traditional gaps from CBO and FRB/US. But it should not be: As highlighted earlier, fluctuations in the unemployment rate owe importantly, in EDO, to fluctuations in the natural rate of interest (that is, to the aggregate risk premium shock).

8 Conclusion

The review of alternative output gap definitions and estimates, both from the EDO DSGE model and from policy institutions, suggests that care must be taken in defining concepts in any discussion of economic slack and related policy implications. Four conclusions can be drawn from this analysis:

- The EDO model's estimate of the output gap (according to the Beveridge-Nelson, production-function, and natural-rate approach) is very similar to gaps from the Congressional Budget Office or the Federal Reserve's large-scale macro-econometric model (FRB/US) model, but the DSGE model's estimate of potential growth is considerably more variable.

- The substantial variance of the change in the Beveridge-Nelson trend in EDO is consistent with the overall variance of the change in real GDP because the change in trend covaries negatively with the change in the gap in the DSGE model. The DSGE model provides a structural interpretation – improvements in productivity lead to a widening of the output gap, as New-Keynesian frictions slow the economy's adjustment to technology shocks.

- The flexible-price/natural-rate gaps are highly dependent on modeling assumptions, but the natural rate gap from the EDO model developed at the Federal Reserve Board is similar to traditional gap estimates such as those from the CBO, in large part because the business cycle variation in the unemployment rate is attributed, by the model, primarily to shocks to "aggregate" demand or "the natural rate of interest".

- The DSGE model's estimate of gaps is as closely related to unemployment fluctuations as those from policy institutions (e.g., obeys Okun's law).

On balance, the results suggest that the ability of a DSGE model like EDO to capture the trend/cycle decomposition of output that drives much of the discussion of macroeconomic stabilization policy is quite good – as should be suspected given the forecasting performance of such models (e.g., Edge, Kiley, and Laforte (2010)). An interesting topic for future research involves using the structural model's implications for the covariances between trend and cycle innovations in order to bridge the distance between a structural analysis of output gaps such as herein and the time-series literature of, for example, Morley, Nelson, and Zivot (2003).

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