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Optimal Monetary Policy with State-Dependent Pricing

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Abstract

In an abstract economic model, we study optimal monetary policy from the timeless perspective under a general state-dependent pricing framework. We find that when firms are monopolistic competitors subject to idiosyncratic menu cost shocks, households have isoelastic preferences, and there is no government spending, strict price stability is optimal both in the long run and in response to aggregate shocks. Key to this finding is an "envelope" property: At zero inflation, a marginal increase in the rate of inflation has no effect on firms' profits and therefore it has no effect on the probability of price adjustment. Our results lend support to more informal statements about the suitability of the Calvo model for studying optimal monetary policy despite its apparent conflict with the Lucas critique. We offer an analytic solution that does not require local approximation or efficiency of the steady state.

Keywords: monetary policy, state-dependent pricing, monopolistic competition

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1 Introduction

A key normative question in monetary economics is the design of optimal monetary policy. An extensive amount of literature studies this question under the assumption that the timing of price changes is given exogenously, typically using the Calvo (1983) model with a constant adjustment rate.[1] Useful as it is as a first approximation, this literature nevertheless is subject to the Lucas

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[1]For example, Clarida, Galí, and Gertler (1999); Woodford (2002, 2003); Yun (2005); Benigno and Woodford (2005).

(1976) critique: In principle, the frequency of price changes should not be treated as a parameter which is independent of policy. Many economists, therefore, have argued against the use of the Calvo model, claiming that it provides a poor approximation to more elaborate models of price adjustment. For example, Golosov and Lucas (2007) show that the behavior of firms in the Calvo model is very different from that in a "menu cost" model, when firms are subject to idiosyncratic productivity shocks as well as aggregate money growth shocks.

This paper studies optimal monetary policy in a model of state-dependent pricing by monopolistically competitive firms. In models of this sort the frequency of adjustment is a statistic determined in equilibrium, not an exogenous parameter. In particular, we will work with a model in which individual prices are sticky because firms are subject to random idiosyncratic lump-sum costs of adjustment à la Dotsey, King, and Wolman (1999). Each firm would change its price only if the increase in the firm's value due to adjustment exceeds the "menu cost." As a result, the probability with which firms reoptimize prices depends on the gains from adjustment. This framework is very flexible because it nests a variety of pricing specifications, including the Calvo model and the fixed menu cost model as extreme limiting cases (Costain and Nakov, 2011).

Aside from pricing being state-dependent, our setup follows closely the standard New Keynesian model with Calvo pricing (for example, Benigno and Woodford, 2005). In particular, the monetary authority is assumed to set the nominal interest rate, with money's role being only that of a unit of account. An important distinction with Clarida, Galí, and Gertler (1999), Woodford (2002), and Yun (2005), is that we assume no production subsidy to offset the markup distortion due to monopolistic competition. This implies that the steady state level of output is inefficiently low. Hence, the central bank has a constant temptation to inflate the economy so as to bring output closer to its efficient level.

We derive the optimal plan from the timeless perspective, as in Woodford (2003).[2] We demonstrate analytically that, if preferences are isoelastic and there is no government spending, it is optimal to commit to zero inflation both in the long run and in reaction to shocks. Importantly, this result holds for a general specification of the menu cost distribution. In the optimal allocation, price markups are positive but constant, output is at its natural (flexible-price) level, and price dispersion is minimized. Perhaps surprisingly, this prescription coincides with the one obtained under Calvo pricing (Benigno and Woodford, 2005).

The reason why zero inflation is optimal in our model of state-dependent pricing is the following. Relative to Calvo pricing, our stochastic menu costs model implies two additional welfare effects of inflation. First, firms must spend real resources (menu costs) on adjusting nominal prices. This distortion is minimized at zero inflation because under such a policy all firms are at their optimal

[2] That is, the plan ignores policymakers' incentives to behave differently in the initial few periods, exploiting the private sector's expectations that had formed prior to the plan's starting date.

price. The second effect is somewhat more subtle. The main difference between exogenous-timing and state-dependent pricing models is that price adjustment frequencies are endogenous in the latter. A priori, the monetary authority could have an incentive to use inflation so as to affect the rate at which firms change their prices. If firms adjusted their prices faster in reaction to shocks, in principle one would expect that to have beneficial welfare effects. However, the fact that adjusting firms set *optimally* their prices implies that, in the timeless perspective regime with zero inflation, a marginal increase in the rate of inflation has no effect on firms' profits, and therefore it has no effect on the rate of adjustment. This *envelope* property implies that the monetary authority has no incentive to deviate from zero inflation in order to affect the speed of adjustment.

We also show that the same reasons for which zero inflation is optimal under Calvo pricing continue to hold under state-dependent pricing. First, inefficient price dispersion is minimized at zero inflation. Second, in the timeless perspective regime with zero inflation, the marginal welfare gain from raising output toward its socially efficient level (i.e. a movement along the Phillips curve) exactly cancels out with the marginal welfare loss from committing to and generating expectations of future inflation (i.e., an upward shift of the Phillips curve). This finding echoes Kydland and Prescott's "rules versus discretion". However, we find that it is independent of whether pricing is time- or state-dependent.

Our results thus lend support to more informal statements about the suitability of the Calvo model for studying optimal monetary policy despite its apparent conflict with the Lucas (1976) critique. In particular, we provide sufficient conditions under which, even though pricing is state-dependent and so the adjustment frequency is endogenously determined, it turns out that the probability of adjustment remains constant under the optimal policy. When these conditions are satisfied, which is what the literature usually assumes, the distinction between time-dependent and state-dependent pricing frameworks vanishes, provided that monetary policy is set optimally.[3]

The following section lays out the model and derives the conditions for equilibrium. Section 3 sets up the optimal monetary policy problem and obtains the main result regarding the optimality of zero inflation from the timeless perspective; it also formalizes the main intuition with a simplified version of the model (with the full proof in the Appendix). Section 4 analyzes numerically the case with positive government expenditure; for a plausible calibration of the model, we find that the optimal deviations from strict price stability in response both to productivity and to government spending shocks are indistinguishable from zero.[4] Section 5 concludes with a discussion of a possible extension.

[3] Independently, Lie (2009) studies numerically the optimal monetary policy in a New Keynesian model with stochastic menu costs and a monetary friction.

[4] Existing studies of optimal monetary policy with monopolistic distortions prove analytically the existence of a short-run tradeoff between inflation and output stabilization in the presence of positive government spending (Benigno and Woodford, 2005; Woodford, ch. 6, section 5). However, they do not quantify the importance of the tradeoff. We show that in a model such as ours the tradeoff is negligible.

2 Model

There are three types of agents: households, firms, and a monetary authority. We begin by describing the behavior of households and firms.

2.1 Households

A representative household maximizes the expected flow of period utility $u(C_t) - x(N_t; \chi_t)$, discounted by β, subject to

$$C_t = \left(\int_0^1 C_{it}^{(\epsilon-1)/\epsilon} di \right)^{\epsilon/(\epsilon-1)}$$

and

$$\int_0^1 P_{it} C_{it} di + R_t^{-1} B_t = W_t N_t + B_{t-1} + \Pi_t,$$

where C_t is a basket of differentiated goods $i \in [0, 1]$, of quantity C_{it} and price P_{it}; N_t denotes hours worked and W_t is the nominal wage rate; χ_t is an exogenous shock to the disutility of labor;[5] B_t are nominally riskless bonds with price R_t^{-1}, and Π_t are the profits of firms owned by the household, net of lump-sum taxes.

The first order conditions are

$$u'(C_t) w_t = x'(N_t; \chi_t), \tag{1}$$

$$R_t^{-1} = \beta E_t \frac{u'(C_{t+1})}{\pi_{t+1} u'(C_t)}, \tag{2}$$

where $w_t \equiv W_t / P_t$ is the real wage, $\pi_t \equiv P_t / P_{t-1}$ is the gross inflation rate, and the aggregate price index is given by

$$P_t \equiv \left(\int_0^1 P_{it}^{1-\epsilon} di \right)^{1/(1-\epsilon)}.$$

2.2 Firms

There is a continuum of firms on the unit interval. Firm i's production function is

$$y_{it} = z_t n_{it},$$

where z_t is an exogenous aggregate productivity process. The firm's labor demand thus equals $n_{it} = y_{it} / z_t$ and its real cost function is $w_t y_{it} / z_t$. The real marginal cost common to all firms is

[5] Our results hold also in the case when the utility of consumption is affected by a preference shock; here we ommit such a shock for simplicity.

therefore w_t/z_t. Optimal allocation of expenditure across product varieties by households implies that each individual firm faces a downward-sloping demand schedule for its good, given by $y_{it} = (P_{it}/P_t)^{-\epsilon} y_t$.

Following Dotsey et al. (1999), we assume that firms face random lump sum costs of adjusting prices ("menu costs"), distributed *i.i.d.* across firms and over time. Let $G(\kappa)$ and $g(\kappa)$ denote the cumulative distribution function and the probability density function, respectively, of the stochastic menu cost $\kappa \geq 0$. We assume that a positive random fraction of firms draw a zero menu cost, so that $G(0) > 0$.[6] Assuming that κ is measured in units of labor time, the total cost paid by a firm changing its price is $w_t\kappa$.[7]

Let v_{0t} denote the value of a firm that adjusts its price in period t before subtracting the menu cost. Let $v_{jt}(P)$ denote the value of a firm that has kept its nominal price unchanged at the level P in the last j periods. This firm will change its nominal price only if the value of adjustment, $v_{0t} - w_t\kappa$, exceeds the value of continuing with the current price, $v_{jt}(P)$. Therefore, from the set of firms that last reoptimized j periods ago (which we henceforth refer to as "vintage-j firms"), only those with a menu cost draw $\kappa \leq (v_{0t} - v_{jt}(P))/w_t$ will choose to change their price. The real value of an adjusting firm is given by

$$\begin{aligned} v_{0t} &= \max_P \left\{ \Pi_t(P) + \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} \left[G\left(\frac{v_{0,t+1} - v_{1,t+1}(P)}{w_{t+1}} \right) v_{0,t+1} - \Xi_{1,t+1}(P) \right] \right. \\ &\quad \left. + \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} \left[1 - G\left(\frac{v_{0,t+1} - v_{1,t+1}(P)}{w_{t+1}} \right) \right] v_{1,t+1}(P) \right\}, \end{aligned}$$

where $\beta u'(C_{t+s})/u'(C_t)$ is the stochastic discount factor between periods t and $t+s \geq t$,

$$\Pi_t(P) \equiv \left(\frac{P}{P_t} - \frac{w_t}{z_t} \right) \left(\frac{P}{P_t} \right)^{-\epsilon} Y_t$$

is the firm's real profit as a function of its nominal price P, and

$$\Xi_{j+1,t+1}(P) \equiv w_{t+1} \int_0^{(v_{0,t+1} - v_{j+1,t+1}(P))/w_{t+1}} \kappa g(\kappa)\, dk$$

is next period's expected adjustment cost for a firm currently in vintage j. The real value of a

[6] We make this technical assumption to ensure a unique stationary distribution of firms over price vintages in the case of zero inflation. See the Appendix for details.

[7] Alternatively, we can assume that κ is measured in terms of the basket of final goods, in which case the total cost paid by a firm changing its price is simply κ. The results are not dependent on this assumption.

firm in vintage j, as a function of its current nominal price P, is given by

$$\begin{aligned} v_{jt}(P) &= \Pi_t(P) + \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} \left[G\left(\frac{v_{0,t+1} - v_{j+1,t+1}(P)}{w_{t+1}} \right) v_{0,t+1} - \Xi_{j+1,t+1}(P) \right] \\ &\quad + \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} \left[1 - G\left(\frac{v_{0,t+1} - v_{j+1,t+1}(P)}{w_{t+1}} \right) \right] v_{j+1,t+1}(P). \end{aligned} \tag{3}$$

We assume that J periods after the last price adjustment, firms draw a zero menu cost.[8] This means that firms in vintage $J-1$ know that in the following period they will adjust their price with probability one at no cost. Therefore, expression (3) holds for vintages $j = 1, ..., J-2$, whereas for vintage-$(J-1)$ firms the corresponding value function is

$$v_{J-1,t}(P) = \Pi_t(P) + \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} v_{0,t+1}. \tag{4}$$

The optimal price setting decision is given by

$$0 = \Pi_t'(P_t^*) + \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} \left[1 - G\left(\frac{v_{0,t+1} - v_{1,t+1}(P_t^*)}{w_{t+1}} \right) \right] v_{1,t+1}'(P_t^*), \tag{5}$$

where

$$\Pi_t'(P) = \left[\epsilon \frac{w_t}{z_t} - (\epsilon - 1) \frac{P}{P_t} \right] (P)^{-\epsilon-1} P_t^{\epsilon} Y_t.$$

Iterating (5) forward, and using the implications of (3) and (4) for the terms $v_{j,t+j}'(P_t^*)$, $j = 1, ..., J-1$, we can express the pricing decision as

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{j=0}^{J-1} \beta^j E_t \prod_{k=1}^{j} (1 - \lambda_{k,t+k}) u'(C_{t+j}) P_{t+j}^{\epsilon} Y_{t+j} (w_{t+j}/z_{t+j})}{\sum_{j=0}^{J-1} \beta^j E_t \prod_{k=1}^{j} (1 - \lambda_{k,t+k}) u'(C_{t+j}) P_{t+j}^{\epsilon-1} Y_{t+j}},$$

where

$$\lambda_{jt} \equiv G\left(\frac{v_{0t} - v_{jt}}{w_t} \right) \tag{6}$$

denotes the period-t adjustment probability of firms in vintage $j = 1, ..., J-1$, and we define $v_{jt} \equiv v_{jt}(P_{t-j}^*)$ for short. As emphasized by Dotsey et al. (1999), this pricing decision is analogous to the one in the Calvo model. In particular, the term $\prod_{k=1}^{j} (1 - \lambda_{k,t+k})$ is the endogenous probability that the price chosen at t survives for the next j periods, thus replacing the exogenous probability $(1 - \lambda^C)^j$ where λ^C is the constant adjustment probability in the Calvo model. We can rewrite

[8] This is a tractability assumption which ensures a finite state space under zero inflation or when the support of the menu cost distribution is unbounded from above.

the price decision in terms of stationary variables as

$$p_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{j=0}^{J-1} \beta^j E_t \prod_{k=1}^{j} (1 - \lambda_{k,t+k}) \left(\prod_{k=1}^{j} \pi_{t+k}\right)^{\epsilon} u'(C_{t+j}) Y_{t+j} (w_{t+j}/z_{t+j})}{\sum_{j=0}^{J-1} \beta^j E_t \prod_{k=1}^{j} (1 - \lambda_{k,t+k}) \left(\prod_{k=1}^{j} \pi_{t+k}\right)^{\epsilon-1} u'(C_{t+j}) Y_{t+j}}, \tag{7}$$

where $p_t^* \equiv P_t^*/P_t$ is the optimal relative price and $\prod_{k=1}^{j} \pi_{t+k} = P_{t+j}/P_t$ is accumulated inflation between periods t and $t+j$.

2.3 Market clearing

Labor input is required both for the production of goods and for changing prices. Labor demand for production by firm i is $n_{it} = y_{it}/z_t = (P_{it}/P_t)^{-\epsilon} y_t/z_t$. Thus, total labor demand for production purposes equals $\Delta_t y_t/z_t$, where $\Delta_t \equiv \int_0^1 (P_{it}/P_t)^{-\epsilon} di$ denotes relative price dispersion. At the same time, the total amount of labor used by vintage-j firms for pricing purposes equals $\psi_{jt} \int_0^{(v_{0t}-v_{jt})/w_t} \kappa g(\kappa) dk$, where ψ_{jt} is the mass of firms in vintage j. Equilibrium in the labor market therefore implies

$$N_t = \frac{Y_t \Delta_t}{z_t} + \sum_{j=1}^{J-1} \psi_{jt} \int_0^{(v_{0t}-v_{jt})/w_t} \kappa g(\kappa) dk. \tag{8}$$

Also, equilibrium in the goods market requires that

$$Y_t = C_t + G_t, \tag{9}$$

where G_t denotes government expenditure, which follows an exogenous process.

2.4 Inflation, price dispersion, and price distribution dynamics

All firms adjusting at time t choose the same nominal price, P_t^*. Given that no nominal price survives for longer than J periods by assumption, the finite set of beginning-of-period prices at any time t is $\left\{P_{t-1}^*, P_{t-2}^*, ..., P_{t-J}^*\right\}$. Let ψ_{jt} denote the time-t fraction of firms with beginning-of-period nominal price P_{t-j}^*, for $j = 1, 2, ..., J$, with $\sum_{j=1}^{J} \psi_{jt} = 1$. The price level evolves according to

$$P_t^{1-\epsilon} = (P_t^*)^{1-\epsilon} \sum_{j=1}^{J} \lambda_{jt} \psi_{jt} + \sum_{j=1}^{J-1} \left(P_{t-j}^*\right)^{1-\epsilon} (1 - \lambda_{jt}) \psi_{jt},$$

where adjustment probabilities $\{\lambda_{jt}\}_{j=1}^{J-1}$ are given by (6), and where $\lambda_{J,t} = 1$. Rescaling both sides of the above equation by P_t, we obtain

$$1 = (p_t^*)^{1-\epsilon} \sum_{j=1}^{J} \lambda_{jt}\psi_{jt} + \sum_{j=1}^{J-1} \left(\frac{p_{t-j}^*}{\prod_{k=0}^{j-1} \pi_{t-k}} \right)^{1-\epsilon} (1-\lambda_{jt})\,\psi_{jt}. \tag{10}$$

This equation determines the inflation rate π_t, given $\left\{p_{t-j}^*\right\}_{j=0}^{J-1}$ and $\{\pi_{t-j}\}_{j=1}^{J-2}$. Similarly, price dispersion follows

$$\Delta_t = (p_t^*)^{-\epsilon} \sum_{j=1}^{J} \lambda_{jt}\psi_{jt} + \sum_{j=1}^{J-1} \left(\frac{p_{t-j}^*}{\prod_{k=0}^{j-1} \pi_{t-k}} \right)^{-\epsilon} (1-\lambda_{jt})\,\psi_{jt}, \tag{11}$$

where again $\lambda_{J,t} = 1$. The distribution of beginning-of-period prices evolves according to

$$\psi_{j,t} = (1-\lambda_{j-1,t-1})\,\psi_{j-1,t-1} \tag{12}$$

for $j = 2, ..., J$, and

$$\psi_{1t} = 1 - \textstyle\sum_{j=2}^{J} \psi_{j,t} = \lambda_{1,t-1}\psi_{1,t-1} + \lambda_{2,t-1}\psi_{2,t-1} + ... + \psi_{J,t-1}. \tag{13}$$

2.5 Equilibrium

There are $8 + 2J + (J-1) = 7 + 3J$ stationary endogenous variables: C_t, N_t, Y_t, R_t, π_t, p_t^*, w_t, Δ_t, $\left\{\psi_{jt}\right\}_{j=1}^{J}$, $\{v_{jt}\}_{j=0}^{J-1}$, and $\{\lambda_{jt}\}_{j=1}^{J-1}$. The equilibrium conditions are (1), (2), the $J-1$ equations (6), equations (7) to (11), the J laws of motion (12) and (13), the value functions

$$\begin{aligned} v_{jt} \;\; &= \;\; \left(\frac{p_{t-j}^*}{\prod_{k=0}^{j-1} \pi_{t-k}} - \frac{w_t}{z_t} \right) \left(\frac{p_{t-j}^*}{\prod_{k=0}^{j-1} \pi_{t-k}} \right)^{-\epsilon} Y_t \\ &\quad + \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} \left[\lambda_{j+1,t+1} v_{0,t+1} + (1-\lambda_{j+1,t+1})\, v_{j+1,t+1} - w_{t+1} \int_0^{(v_{0,t+1}-v_{j+1,t+1})/w_{t+1}} \kappa dG(\kappa) \right] \end{aligned}$$

for $j = 0, 1, ..., J-2$, and

$$v_{J-1,t} = \left(\frac{p_{t-(J-1)}^*}{\prod_{k=0}^{(J-1)-1} \pi_{t-k}} - \frac{w_t}{z_t} \right) \left(\frac{p_{t-(J-1)}^*}{\prod_{k=0}^{(J-1)-1} \pi_{t-k}} \right)^{-\epsilon} Y_t + \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} v_{0,t+1};$$

plus a specification of monetary policy. If we were to close the model with a Taylor rule, this would give us a total of $2 + (J-1) + 5 + J + J + 1 = 7 + 3J$ equations. Instead, we will study the optimal state-contingent monetary policy plan, which will essentially double the number of equations and variables.

2.5.1 Flexible-price equilibrium

It is instructive to derive the flexible-price equilibrium in this framework. In such an equilibrium, menu costs are zero and all firms choose the same nominal price $P_t^* = \frac{\epsilon}{\epsilon-1}\frac{w_t}{z_t}P_t$ in each period t. All relative prices are one: $p_t^* = P_t^*/P_t = 1$. The equilibrium conditions simplify to

$$\begin{aligned} u'(C_t^{fp})w_t^{fp} &= x'(N_t^{fp};\chi_t), \\ z_t N_t^{fp} &= Y_t^{fp}, \\ Y_t^{fp} &= C_t^{fp} + G_t, \\ z_t &= \frac{\epsilon}{\epsilon-1} w_t^{fp}, \end{aligned}$$

and so we obtain the classical decoupling of real and nominal variables. The flexible-price output Y_t^{fp} derived above is used in defining the output gap as the ratio between actual output and its flexible-price counterpart.

3 Optimal monetary policy

3.1 The general problem

For the purpose of deriving the optimality conditions of the Ramsey plan, it is useful to define

$$\pi_{jt}^{acc} \equiv \prod_{k=0}^{j-1} \pi_{t-k} = \frac{P_t}{P_{t-j}}, \quad j = 1, ..., J-1,$$

that is, the accumulated inflation between periods $t-j$ and t. This implies $\prod_{k=1}^{j} \pi_{t+k} = \pi_{j,t+j}^{acc}$. We also define

$$\theta_{jt} \equiv \prod_{k=0}^{j-1} \left(1 - \lambda_{j-k,t-k}\right), \quad j = 1, ..., J-1,$$

that is, the probability that a price chosen at $t-j$ survives until t, which in turn implies $\prod_{k=1}^{j} \left(1 - \lambda_{k,t+k}\right) = \theta_{j,t+j}$. These definitions allow us to express the optimal pricing decision in equation (7) in a more compact form,

$$p_t^* = \frac{\epsilon}{\epsilon-1} \frac{\sum_{j=0}^{J-1} \beta^j E_t \theta_{j,t+j} \left(\pi_{j,t+j}^{acc}\right)^{\epsilon} u'\left(C_{t+j}\right) Y_{t+j} \left(w_{t+j}/z_{t+j}\right)}{\sum_{j=0}^{J-1} \beta^j E_t \theta_{j,t+j} \left(\pi_{j,t+j}^{acc}\right)^{\epsilon-1} u'\left(C_{t+j}\right) Y_{t+j}}.$$

Similarly, we replace $\prod_{k=0}^{j-1} \pi_{t-k}$ by π_{jt}^{acc} in the laws of motion of inflation and price dispersion, and in the firms' value functions. It is useful to express the variables π_{jt}^{acc} and θ_{jt} recursively,

$$\pi_{jt}^{acc} = \pi_t \pi_{j-1,t-1}^{acc}, \quad j = 1, ..., J-1,$$

$$\theta_{jt}^{acc} = (1 - \lambda_{jt})\, \theta_{j-1,t-1}^{acc}, \quad j = 1, ..., J-1,$$

where the recursions start with $\pi_{0,t-1}^{acc} = 1$ and $\theta_{0,t-1}^{acc} = 1$, respectively. We use $w_t = x'(N_t; \chi_t)/u'(C_t)$ to substitute for the real wage in the equilibrium conditions. In addition, we use the constraint $Y_t = C_t + G_t$ to substitute for C_t. Finally, we define $\tilde{v}_{jt} \equiv v_{jt} u'(C_t)$, $j = 0, 1, ..., J-1$, such that $(v_{0t} - v_{jt})/w_t = (\tilde{v}_{0t} - \tilde{v}_{jt})/x'(N_t; \chi_t)$. At time 0, the central bank chooses the state-contingent path for all endogenous variables, which maximizes the following Lagrangian:

$$\begin{aligned}
\mathcal{L}_0 = {} & E_0 \textstyle\sum_{t=0}^{\infty} \beta^t \{ u(Y_t - G_t) - x(N_t; \chi_t) \\
& + \phi_t^{p^*} \left[p_t^* \sum_{j=0}^{J-1} \beta^j \theta_{j,t+j} (\pi_{j,t+j}^{acc})^{\epsilon - 1} Y_{t+j} u'(Y_{t+j} - G_{t+j}) - \frac{\epsilon}{\epsilon - 1} \sum_{j=0}^{J-1} \beta^j \theta_{j,t+j} (\pi_{j,t+j}^{acc})^{\epsilon} Y_{t+j} \frac{x'\left(N_{t+j}; \chi_{t+j}\right)}{z_{t+j}} \right] \\
& + \phi_t^N \left[N_t - Y_t \Delta_t / z_t - \textstyle\sum_{j=1}^{J-1} \psi_{jt} \int_0^{(\tilde{v}_{0t} - \tilde{v}_{jt})/x'(N_t;\chi_t)} \kappa g(\kappa)\, d\kappa \right] \\
& + \phi_t^{\pi} \left[(p_t^*)^{1-\epsilon} \textstyle\sum_{j=1}^{J} \lambda_{jt} \psi_{jt} + \sum_{j=1}^{J-1} \left(p_{t-j}^* / \pi_{jt}^{acc} \right)^{1-\epsilon} (1 - \lambda_{jt})\, \psi_{jt} \right] \\
& + \phi_t^{\Delta} \left[(p_t^*)^{-\epsilon} \textstyle\sum_{j=1}^{J} \lambda_{jt} \psi_{jt} + \sum_{j=1}^{J-1} \left(p_{t-j}^* / \pi_{jt}^{acc} \right)^{-\epsilon} (1 - \lambda_{jt})\, \psi_{jt} - \Delta_t \right] \\
& + \sum_{j=1}^{J-1} \phi_t^{\lambda_j} \left[\lambda_{jt} - G\left(\frac{\tilde{v}_{0t} - \tilde{v}_{jt}}{x'(N_t; \chi_t)} \right) \right] + \sum_{j=2}^{J} \phi_t^{\psi_j} \left[\psi_{j,t} - (1 - \lambda_{j-1,t-1})\, \psi_{j-1,t-1} \right] + \phi_t^{\psi_1} \left[\psi_{1t} + \sum_{j=2}^{J} \psi_{j,t} \right] \\
& + \sum_{j=0}^{J-2} \phi_t^{v_j} \left[\left(\frac{p_{t-j}^*}{\pi_{jt}^{acc}} u'(Y_t - G_t) - \frac{x'(N_t; \chi_t)}{z_t} \right) \left(\frac{p_{t-j}^*}{\pi_{jt}^{acc}} \right)^{-\epsilon} Y_t - \tilde{v}_{jt} \right] \\
& + \sum_{j=0}^{J-2} \phi_t^{v_j} \beta \left[\lambda_{j+1,t+1} \tilde{v}_{0,t+1} + (1 - \lambda_{j+1,t+1})\, \tilde{v}_{j+1,t+1} - x'\left(N_{t+1}; \chi_{t+1}\right) \int_0^{(\tilde{v}_{0t+1} - \tilde{v}_{j+1,t+1})/x'\left(N_{t+1};\chi_{t+1}\right)} \kappa g(\kappa)\, d\kappa \right] \\
& + \phi_t^{v_{J-1}} \left[\left(\frac{p_{t-(J-1)}^*}{\pi_{J-1,t}^{acc}} u'(Y_t - G_t) - \frac{x'(N_t; \chi_t)}{z_t} \right) \left(\frac{p_{t-(J-1)}^*}{\pi_{J-1,t}^{acc}} \right)^{-\epsilon} Y_t - \tilde{v}_{J-1,t} + \beta \tilde{v}_{0,t+1} \right] \\
& + \phi_t^{\pi_1^{acc}} \left[\pi_{1t}^{acc} - \pi_t \right] + \textstyle\sum_{j=2}^{J-1} \phi_t^{\pi_j^{acc}} \left[\pi_{jt}^{acc} - \pi_t \pi_{j-1,t-1}^{acc} \right] \\
& + \phi_t^{\theta_1} \left[\theta_{1t} - (1 - \lambda_{1t}) \right] + \textstyle\sum_{j=2}^{J-1} \phi_t^{\theta_j} \left[\theta_{jt} - (1 - \lambda_{jt})\, \theta_{j-1,t-1} \right] \}.
\end{aligned} \tag{14}$$

Since the nominal interest rate only appears in the consumption Euler equation, the latter is excluded from the set of constraints on the Ramsey problem. Instead, this equation is used residually to back out the nominal interest rate path consistent with the optimal allocation. The first-order conditions of the above problem are derived in the Appendix.

Our object of interest is optimal monetary policy from a "timeless perspective." As explained by Woodford (2003), this type of policy does not exploit the private sector's expectations that formed prior to the particular date on which the plan was implemented. Instead, the central bank commits itself to behave, from date 0, in a way consistent with the way it would have chosen to behave had it committed to the optimal policy in the infinite past. The interest is thus in optimality in the long run, once the economy has converged to its ergodic distribution.

The Appendix proves the following result:

Proposition 1 *Let functional forms for preferences be of the constant elasticity type and government expenditure be zero. Then the zero inflation policy ($\pi_t = 1$) is optimal from the timeless perspective.*

There are two important aspects of the above proposition. The first is that optimal *trend* inflation is zero. Therefore, the presence of monopolistic distortions does not justify a positive rate of trend inflation, and the optimal policy involves a commitment to eventually eliminating any inefficient price dispersion due to staggered price setting. This normative prescription is the same as the one implied by the standard New Keynesian model with Calvo price setting, as shown by Benigno and Woodford (2005).[9] The main insight of the Calvo framework, about the desirability of zero long-run inflation, thus continues to hold in a general model of state-dependent pricing. The key difference between exogenous-timing models of price adjustment such as Calvo's and state-dependent pricing models is the endogeneity of the timing of price adjustment in the latter. A priori, the central bank could have an incentive to use trend inflation to influence the speed at which firms change prices, if such a policy were to have beneficial effects on society. The above result implies that the endogeneity of price adjustment frequencies does not affect the optimality of zero trend inflation.

To understand the intuition for this result, let us consider the different channels through which trend inflation affects welfare. Two of these channels are common to exogenous-timing models such as Calvo or Taylor. One is that, in the presence of staggered prices, inflation increases the extent of price dispersion, distorting the economy's pricing system. This leads to inefficient allocation of resources across product lines, and increases the total amount of (labor) resources needed to produce a given consumption basket; hence, it lowers welfare. Notice that inefficient price dispersion attains a global minimum at zero inflation because, under such a policy, all relative prices end up being equal.

The other common channel, through which trend inflation affects welfare, works through its two opposing effects on the inflation-output tradeoff: On the one hand, holding constant inflation expectations, a rise in current inflation allows the central bank to raise output toward its socially

[9] The same result holds for another prominent exogenous-timing model of price adjustment, namely the Taylor model, where adjustment probabilities are zero for a number of periods after a price change and one afterwards. A proof of the latter result is available upon request from the authors.

efficient level, thus reducing the monopolistic distortion and improving welfare; intuitively, the economy moves *along* the New Keynesian Phillips curve (NKPC).[10] On the other hand, choosing higher inflation raises the inflation expectations of price-setters; the latter produces an upward *shift* of the NKPC, thus worsening the short-run tradeoff between inflation and output. As it turns out, at zero inflation, the marginal welfare cost of raising inflation expectations exactly offsets the marginal welfare benefit of exploiting the short-run inflation-output tradeoff.

While the former two welfare effects of trend inflation are common to exogenous-timing models, our framework with idiosyncratic menu cost shocks includes two additional channels through which trend inflation affects welfare. One is that inflation forces firms to spend real resources (menu costs) on adjusting their nominal prices; this distortion is minimized at zero inflation, because eventually all firms end up being at their optimal price. The other extra channel is more subtle; Namely, in the stochastic menu costs model, adjustment frequencies are endogenous. In particular, trend inflation affects the relative prices of different cohorts of firms $(p_{t-j}/\prod_{k=0}^{j-1}\pi_{t-k},\ j=0,...,J-1)$, which has an effect on their profits, on their value functions, and ultimately on the gains from adjustment. A priori, the central bank may be tempted to use trend inflation to influence the speed of price adjustment, so as to shift the NKPC in a way that improves the inflation-output tradeoff. However, the fact that adjusting firms choose their prices in an *optimal* way implies that, at zero inflation, a marginal increase in the inflation rate has no effect on firms' profits, and therefore it has no effect on adjustment probabilities. This *envelope* property implies that the monetary authority has no incentive to create trend inflation to influence the speed with which firms change their prices.

The second important aspect of proposition 1 is that the optimal *deviations* from zero inflation in response to technology or preference shocks are zero as well. Therefore, the occurrence of these exogenous disturbances to preferences or technology does not justify temporary departures from strict price stability.[11] The intuition for this result is as follows. There are four potential inefficiencies in the present model, related to: (1) the level and volatility of price dispersion; (2) the volatility of the average markup; (3) the waste of resources due to menu costs; and (4) the level of the average markup due to monopolistic competition. Distortions (1) through (3) are directly related to the friction in price setting, and–absent idiosyncratic shocks to desired prices–a policy of

[10] The "New Keynesian Phillips curve" is the structural relationship between inflation (current and expected) and output that arises in the standard New Keynesian model. Here, the optimal price decision (equation 7) and the relationship between inflation and the optimal relative price (equation 10) can be combined into a dynamic relationship between inflation and real marginal costs, where the latter can also be expressed in terms of aggregate output by using equations (1), (8), and (9). The resulting dynamic relationship between inflation and output may be interpreted as a "New Keynesian Phillips curve." Notice that the endogenous price adjustment frequencies, λ_{jt}, affect both the intercept and the slope of that curve.

[11] Benigno and Woodford (2005) reach the same conclusion about the standard model with Calvo pricing. While they derive their result for a linear-quadratic approximation to the actual optimal monetary policy problem, our finding is based on the exact non-linear welfare function and equilibrium conditions.

strict price stability eliminates all three. It does so by replicating the flexible-price equilibrium and eliminating the incentives for price adjustment. Inefficiency (4) is a static markup distortion due to monopolistic competition. As we have just seen, the optimal plan does not involve a correction of this inefficiency because it is outweighed by the gains of committing to zero inflation and achieving the minimum possible price dispersion in the long run, independently of the price-setting policies followed by firms. The aforementioned envelope property, by which a marginal increase in inflation leaves price adjustment frequencies unaffected, continues to hold as the economy is hit by aggregate shocks.

3.2 An illustration with two cohorts

While the appendix provides the proof of the optimality of zero inflation in the full-blown model, it is illustrative to formalize the above intuitions with a simplified version of the model. In particular, we consider the case of $J = 2$ cohorts, such that firms that adjust their nominal price today may or may not adjust in the following period, but adjust with certainty two periods after the last price change. To further simplify, we assume functional forms $u(C_t) = \log(C_t)$ and $x(N_t; \chi_t) = \chi_t N_t$, such that the real wage is $w_t = x'(N_t; \chi_t) / u'(C_t) = \chi_t C_t$. As in proposition 1, we also assume away government spending, $G_t = 0$, such that $C_t = Y_t$. To simplify the notation, let $\psi_t \equiv \psi_{1t}$ and $\lambda_t \equiv \lambda_{1t}$ denote the measure and adjustment probability of firms in vintage 1. The measure of firms in vintage 2 is then $\psi_{2t} = 1 - \psi_t$, and the law of motion of ψ_t is simply $\psi_t = 1 - (1 - \lambda_{t-1}) \psi_{t-1}$. Let also $v_t \equiv v_{1t}$ denote the value of firms in vintage 1. Finally, we define $\tilde{v}_{0t} \equiv v_{0t}/Y_t$ and $\tilde{v}_t \equiv v_t/Y_t$, such that $(v_{0t} - v_t) / w_t = (\tilde{v}_{0t} - \tilde{v}_t) / \chi$. Taking all these elements, the central bank maximizes the following Lagrangian:

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \log(Y_t) - \chi_t \frac{Y_t \Delta_t}{z_t} - \chi_t \psi_t \int_0^{(\tilde{v}_{0t} - \tilde{v}_t)/\chi} \kappa g(\kappa)\, d\kappa$$

$$+\phi_t^{p^*} \left[p_t^* \left(1 + \beta (1 - \lambda_{t+1}) \pi_{t+1}^{\epsilon-1}\right) - \frac{\epsilon}{\epsilon - 1} \left(\frac{\chi_t Y_t}{z_t} + \beta (1 - \lambda_{t+1}) \pi_{t+1}^{\epsilon} \frac{\chi_t Y_{t+1}}{z_{t+1}} \right) \right]$$

$$+\phi_t^{\pi} \left[(p_t^*)^{1-\epsilon} (\lambda_t \psi_t + 1 - \psi_t) + \left(\frac{p_{t-1}^*}{\pi_t} \right)^{1-\epsilon} (1 - \lambda_t) \psi_t - 1 \right]$$

$$+\phi_t^{\Delta} \left[(p_t^*)^{-\epsilon} (\lambda_t \psi_t + 1 - \psi_t) + \left(\frac{p_{t-1}^*}{\pi_t} \right)^{-\epsilon} (1 - \lambda_t) \psi_t - \Delta_t \right]$$

$$+\phi_t^{\lambda} \left[\lambda_t - G\left(\frac{\tilde{v}_{0t} - \tilde{v}_t}{\chi_t} \right) \right] + \phi_t^{\psi} \left[\psi_t + (1 - \lambda_{t-1}) \psi_{t-1} \right]$$

$$+\phi_t^{v_0} \left[\left(p_t^* - \frac{\chi_t Y_t}{z_t} \right) (p_t^*)^{-\epsilon} - \tilde{v}_{0t} + \beta \left(\lambda_{t+1} \tilde{v}_{0,t+1} + (1 - \lambda_{t+1}) \tilde{v}_{t+1} - \chi_t \int_0^{(\tilde{v}_{0,t+1} - \tilde{v}_{t+1})\ /\chi} \kappa g(\kappa)\, d\kappa \right) \right]$$

$$+\phi_t^{v} \left[\left(\frac{p_{t-1}^*}{\pi_t} - \frac{\chi_t Y_t}{z_t} \right) \left(\frac{p_{t-1}^*}{\pi_t} \right)^{-\epsilon} - \tilde{v}_t + \beta \tilde{v}_{0,t+1} \right] \Bigg\}.$$

For the present analysis, it suffices to differentiate the Lagrangian with respect to inflation and the optimal relative price for a particular state at time t. While the derivative of the Lagrangian with respect to π_t captures the direct marginal effect of inflation on welfare, the derivative with respect to p_t^* captures its indirect effect through its structural relationship with the optimal relative price. That relationship is given by the equation multiplied by ϕ_t^{π} in the Lagrangian. Indeed, if we use the latter equation to solve for the optimal relative price as a function of current and past inflation, and then use the resulting expression to substitute for p_t^* in the optimal price decision (the equation multiplied by $\phi_t^{p^*}$), we obtain a dynamic relationship between inflation and aggregate activity. The latter may be interpreted as a "New Keynesian Phillips curve." The derivatives with respect to π_t and p_t^* are given by

$$\begin{aligned} \frac{\partial \mathcal{L}_0}{\partial \pi_t} &= \phi_{t-1}^{p^*} \left[\frac{p_{t-1}^*}{\pi_t} (\epsilon - 1) - \frac{\epsilon}{\epsilon - 1} \frac{\chi_t Y_t}{z_t} \epsilon \right] \pi_t^{\epsilon-1} (1 - \lambda_t) \\ &\quad + \left[\phi_t^{\pi} (\epsilon - 1) \frac{p_{t-1}^*}{\pi_t} + \phi_t^{\Delta} \epsilon \right] \left(p_{t-1}^* \right)^{-\epsilon} \pi_t^{\epsilon-1} (1 - \lambda_t) \psi_t \\ &\quad + \phi_t^{v} \left[(\epsilon - 1) \frac{p_{t-1}^*}{\pi_t} - \epsilon \frac{\chi_t Y_t}{z_t} \right] \left(p_{t-1}^* \right)^{-\epsilon} \pi_t^{\epsilon-1}, \end{aligned} \tag{15}$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}_0}{\partial p_t^*} &= \phi_t^{p^*}\left[1+\beta\left(1-\lambda_{t+1}\right)\pi_{t+1}^{\epsilon-1}\right]-\left[\phi_t^{\pi}\left(\epsilon-1\right)p_t^*+\phi_t^{\Delta}\epsilon\right]\left(p_t^*\right)^{-\epsilon-1}\left(\lambda_t\psi_t+1-\psi_t\right) \\
&\quad -\beta E_t\left[\phi_{t+1}^{\pi}\left(\epsilon-1\right)\frac{p_t^*}{\pi_{t+1}}+\phi_{t+1}^{\Delta}\epsilon\right]\left(p_t^*\right)^{-\epsilon-1}\pi_{t+1}^{\epsilon}\left(1-\lambda_{t+1}\right)\psi_{t+1} \\
&\quad +\phi_t^{v_0}\left[\epsilon\frac{\chi_t Y_t}{z_t}-\left(\epsilon-1\right)p_t^*\right]\left(p_t^*\right)^{-\epsilon-1}+\beta E_t\phi_{t+1}^{v}\left[\epsilon\frac{\chi_t Y_{t+1}}{z_{t+1}}-\left(\epsilon-1\right)\frac{p_t^*}{\pi_{t+1}}\right]\left(p_t^*\right)^{-\epsilon-1}\pi_{t+1}^{\epsilon},
\end{aligned} \tag{16}
$$

respectively.[12] We now conjecture that the central bank commits to follow a policy of zero net inflation, or $\pi_t = 1$. It is straightforward to show that under such a policy the economy converges to an equilibrium in which $p_t^* = \Delta_t = 1$. That is, both firm vintages have the same relative price, and price dispersion is eliminated. Thus, both vintages end up having the same value, $v_{0t} = v_t$, which in turn implies $\lambda_t = G(0) \equiv \bar{\lambda} > 0$. The vintage distribution converges to $\psi_t = 1/\left(2-\bar{\lambda}\right) \equiv \bar{\psi}$. Finally, the real marginal cost equals the inverse of the monopolistic mark-up, $\chi_t Y_t / z_t = \left(\epsilon - 1\right)/\epsilon$, implying that output equals its flexible-price level of section 2.5.1 at all times.

Imposing the latter conjecture in expressions (15) and (16), we obtain

$$
\frac{\partial \mathcal{L}_0}{\partial \pi_t} = -\phi_{t-1}^{p^*}\left(1-\bar{\lambda}\right)+\left[\phi_t^{\pi}\left(\epsilon-1\right)+\phi_t^{\Delta}\epsilon\right]\left(1-\bar{\lambda}\right)\bar{\psi}, \tag{17}
$$

$$
\frac{\partial \mathcal{L}_0}{\partial p_t^*} = \phi_t^{p^*}\left[1+\beta\left(1-\bar{\lambda}\right)\right]-\left[\phi_t^{\pi}\left(\epsilon-1\right)+\phi_t^{\Delta}\epsilon\right]\bar{\psi}-\beta\left(1-\bar{\lambda}\right)E_t\left[\phi_{t+1}^{\pi}\left(\epsilon-1\right)+\phi_{t+1}^{\Delta}\epsilon\right]\bar{\psi}, \tag{18}
$$

where we have also used the fact that $\bar{\lambda}\bar{\psi} + 1 - \bar{\psi} = \bar{\psi}$. The first effect to notice is that, under our conjecture, all terms involving the Lagrange multipliers $\phi_t^{v_0}$ and ϕ_t^{v} in expressions (15) and (16) have disappeared. Such terms capture the marginal welfare effect of both variables through their effect on the value of both firm cohorts (v_{0t}, v_t). Therefore, once the economy has converged to the timeless perspective regime with zero inflation, a marginal deviation of inflation from zero has no effect on the gains from adjustment, and hence it has no effect on the adjustment frequency either. This is the "envelope property" that we referred to before.

In equation (17), the term involving $\phi_{t-1}^{p^*}$ captures the marginal welfare effect from an increase in time $(t-1)$ *expectations* of inflation at time t, whereas the term involving $\phi_t^{p^*}$ in equation (18) reflects the marginal welfare effect from an increase in the optimal relative price (and thus in inflation) at time t. We show in the Appendix that, in the full-blown model, the multiplier $\phi_t^{p^*}$ converges to a constant value $\bar{\phi}^{p^*}$ in the timeless perspective regime, which is also true in this simplified version. Using this in (18), setting the resulting expression equal to zero (as required by

[12] Both derivatives have been rescaled by β^t times the probability of reaching the particular state at time t conditional on the state at time 0.

the first-order optimality condition), and solving for ϕ_t^π, we obtain

$$\phi_t^\pi = \left(\bar{\phi}^{p^*}/\bar{\psi} - \phi_t^\Delta \epsilon\right)/(\epsilon - 1).$$

Using this to substitute for ϕ_t^π in (17), the latter becomes

$$\begin{aligned}\frac{\partial \mathcal{L}_0}{\partial \pi_t} &= \left[\bar{\phi}^{p^*}/\bar{\psi} - \phi_t^\Delta \epsilon + \phi_t^\Delta \epsilon\right]\left(1-\bar{\lambda}\right)\bar{\psi} - \bar{\phi}^{p^*}\left(1-\bar{\lambda}\right) \\ &= \phi_t^\Delta (\epsilon - \epsilon)\left(1-\bar{\lambda}\right)\bar{\psi} + \bar{\phi}^{p^*}(1-1)\left(1-\bar{\lambda}\right) \\ &= 0 + 0 = 0.\end{aligned} \tag{19}$$

Therefore, once the economy has converged to the timeless perspective regime with zero inflation, the central bank has no incentive to create positive or negative inflation at the margin, because the potential welfare costs cancel out the potential gains. The term involving ϕ_t^Δ in (19) captures the marginal welfare effect of inflation through its effect on price dispersion, which disappears under the timeless perspective regime with zero inflation. Finally, the term involving $\bar{\phi}^{p^*}$ is the difference between the positive marginal effect stemming from a movement along the NKPC, $\bar{\phi}^{p^*}\left(1-\bar{\lambda}\right)$, and the negative marginal effect due to the shift in the NKPC, $-\bar{\phi}^{p^*}\left(1-\bar{\lambda}\right)$. Under the zero inflation policy, both effects exactly cancel each other out.

The specific example above is intended to formalize the main intuition; more generally, the optimality of zero inflation from the timeless perspective holds for *any* number of cohorts and for standard (isoelastic) preferences, as shown in the Appendix.

4 Optimal policy with positive government expenditure

The previous section derived the optimal policy under the assumption that government expenditure is zero. We now briefly analyze the more general case with positive government expenditure. In this case, we no longer have a closed-form analytical solution, so we illustrate the results by simulation. We show the optimal dynamic responses of several key variables to two types of shocks: aggregate productivity and government consumption. Our main finding is that, under a first- or second-order approximation to the general equilibrium dynamics of the model, the optimal deviations of inflation from zero are negligible. Thus, the optimal stabilization policy is basically equivalent to strict inflation targeting, and all real variables follow closely their flexible-price counterparts. We also find that the responses are virtually identical to the ones obtained in the Calvo model.

4.1 Calibration

To produce impulse responses, we must first choose functional forms and assign values to the model's parameters. We take most of the parameters from Golosov and Lucas (2007). In particular, $u(C_t) = C_t^{1-\gamma}/(1-\gamma)$ with $\gamma = 2$, and $x(N_t) = \chi N_t^{1+\varphi}/(1+\varphi)$ with $\chi = 6$ and $\varphi = 1$. The discount factor is $\beta = 1.04^{-1/4}$ and the elasticity of substitution among product varieties is $\epsilon = 7$.

We further assume that the cumulative distribution function of menu costs takes the form

$$G(\kappa) = \frac{\xi + \kappa}{\alpha + \kappa},$$

where both ξ and α are positive parameters. Therefore, from equation (6) the fraction of vintage-j firms that adjust their price in a given period equals

$$\lambda_{jt} = G\left(\frac{v_{0t} - v_{jt}}{w_t}\right) = \frac{\xi + (v_{0t} - v_{jt})/w_t}{\alpha + (v_{0t} - v_{jt})/w_t}.$$

As in Costain and Nakov (2011), this function is increasing in the gain from adjustment $v_{0t} - v_{jt}$ and is bounded above by 1. Unlike Costain and Nakov (2011), the function is bounded below not by 0 but by $\xi/\alpha > 0$. We make this technical assumption to ensure a unique stationary distribution of firms over the (finite number of) price vintages in the case of zero inflation. Any arbitrarily small ξ would work and so we pick the value 10^{-10}. We then set $\alpha = 0.0006$ so that, under a policy targeting 2% annual inflation (broadly consistent with the average observed rate in the United States since the mid-1980s), the model produces an average frequency of price changes of once every three quarters (broadly consistent with the micro evidence found, for example, by Nakamura and Steinsson, 2008). With these settings, the model implies virtually zero probability of adjustment when the gain from adjustment is zero. Finally, we set the maximum price duration to $J = 24$ quarters, a number that is much greater than any observed price duration in recent U.S. evidence.

Figure 1 shows the adjustment hazard function and the distribution of firms by price vintage with 2% trend inflation. In the left panel, the adjustment probability increases rapidly with price age, reaching 90% after 10 quarters. As shown in the right panel, this implies that virtually no price survives more than eight quarters.

We focus on two types of shocks. One is an aggregate productivity shock with persistence $\rho_z = 0.95$ and the other is a government expenditure shock with persistence $\rho_g = 0.9$. Government expenditure is calibrated so that it accounts for roughly 17% of GDP in steady state, consistent with U.S. postwar experience.

4.2 Impulse responses under the optimal policy

We use a first-order Taylor expansion to approximate the equilibrium dynamics of our model. Figure 2 plots the responses of several variables of interest to two independent shocks: a 1% improvement in aggregate productivity, and a 1% increase in the level of government spending. Characteristically, four variables – the optimal reset price, inflation, price dispersion (shown in the last row of the figure), and the output gap, defined as the ratio between actual output and its flexible price counterpart (and shown in the third panel on the top row), remain constant in response to each of the shocks. This is precisely what happens in response to the same shocks in the Calvo model (not shown due to the overlap, but available upon request). Moreover, the responses of the interest rate, consumption, hours worked, and wages, all coincide with their counterparts in the Calvo model. Hence, the central bank's incentives to deviate from zero inflation to reduce monopolistic distortions are virtually nonexistent in response to the two real shocks.

In passing, we note that a second-order accurate solution of the model yields virtually identical impulse responses, both under Calvo and under stochastic menu costs, at least for small aggregate shocks.[13] We thus find that the simple linear Calvo framework offers a very good approximation to the behavior of a cashless state-dependent pricing economy under the optimal monetary policy from the timeless perspective, even though the two economies behave quite differently under suboptimal policies.

5 Conclusion

We have shown that the main lessons for optimal monetary policy derived in the canonical Calvo model carry over to a more general setup in which firms' likelihood of adjusting prices depends on the state of the economy. In particular, the optimal long-run rate of inflation is zero, and the optimal dynamic policy is strict inflation targeting. This finding means that the central bank should not use inflation to try to offset the static distortion arising from monopolistic competition.

We show that, under conditions typically assumed in the literature, the probability of adjustment remains constant even if pricing is state-dependent, provided that monetary policy is set optimally. Thus, when the sufficient conditions are met, any difference between time-dependent and state-dependent pricing vanishes under the optimal policy. These results lend support to more informal statements about the suitability of the Calvo model for studying optimal monetary policy despite its apparent conflict with the Lucas (1976) critique.

Our analysis is a step toward a fuller model that would include firm-level shocks not only to the price adjustment costs, but also to desired prices, for example, due to idiosyncratic productivity

[13] We use 24 vintages when approximating the solution to first order, and 8 vintages when approximating it to second order. When plotted, the two sets of impulse responses are indistinguishable to the naked eye.

shocks.[14] In this extended model, monetary policy would not be able to replicate the flexible-price equilibrium "for free," because firms with different productivities would want to set different prices. Instead, deviations from price stability would affect the balance between price increases and price decreases, with potential welfare gains coming from this rebalancing. The magnitude of such a welfare effect of inflation is an intriguing question, which we leave for future research.

[14] A simple extension with firm-level shocks to desired prices is to assume that such shocks happen with a constant probability. If the shocks to desired prices are so large that adjustment to them brings gains exceeding some maximum menu cost, then prices would be flexible with respect to the micro-level shocks, but sticky with respect to aggregate shocks. By construction, in this environment, our analysis from section 3 would remain true.

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Appendix

In this appendix, we obtain the solution to the optimal monetary policy problem from the timeless perspective. The central bank maximizes the Lagrangian given by expression (14) in the main text. The first-order conditions are as follows (all expressions are equal to zero):

$$\begin{aligned}
&u'(C_t) + \sum_{j=0}^{J-1} \phi_{t-j}^{p^*} \left[\frac{p^*_{t-j}}{\pi^{acc}_{jt}} \left[u'(C_t) + Y_t u''(C_t) \right] - \frac{\epsilon}{\epsilon-1} \frac{x'(N_t;\chi_t)}{z_t} \right] \theta_{jt} (\pi^{acc}_{jt})^{\epsilon} - \phi_t^N \frac{\Delta_t}{z_t} \\
&+ \sum_{j=0}^{J-1} \phi_t^{v_j} \left[\frac{p^*_{t-j}}{\pi^{acc}_{jt}} \left[u'(C_t) + u''(C_t) Y_t \right] - \frac{x'(N_t;\chi_t)}{z_t} \right] \left(\frac{p^*_{t-j}}{\pi^{acc}_{jt}} \right)^{-\epsilon}, \qquad (Y_t)
\end{aligned}$$

$$\begin{aligned}
&\phi_t^{p^*} E_t \sum_{j=0}^{J-1} \beta^j \theta_{j,t+j} (\pi^{acc}_{j,t+j})^{\epsilon-1} Y_{t+j} u'(C_{t+j}) - \left[\phi_t^{\pi} (\epsilon-1) p^*_t + \phi_t^{\Delta} \epsilon \right] (p^*_t)^{-\epsilon-1} \sum_{j=1}^{J} \lambda_{jt} \psi_{jt} \\
&- E_t \sum_{j=1}^{J-1} \beta^j \left[\phi^{\pi}_{t+j} (\epsilon-1) \frac{p^*_t}{\pi^{acc}_{j,t+j}} + \phi^{\Delta}_{t+j} \epsilon \right] (p^*_t)^{-\epsilon-1} (\pi^{acc}_{j,t+j})^{\epsilon} (1-\lambda_{j,t+j}) \psi_{j,t+j} \\
&+ E_t \sum_{j=0}^{J-1} \beta^j \phi^{v_j}_{t+j} \left[\epsilon \frac{x'(N_{t+j};\chi_{t+j})}{z_{t+j} u'(C_{t+j})} - (\epsilon-1) \frac{p^*_t}{\pi^{acc}_{j,t+j}} \right] (p^*_t)^{-\epsilon-1} (\pi^{acc}_{j,t+j})^{\epsilon} Y_{t+j} u'(C_{t+j}), \qquad (p^*_t)
\end{aligned}$$

$$\begin{aligned}
&\phi^{p^*}_{t-j} \left[\frac{p^*_{t-j}}{\pi^{acc}_{jt}} (\epsilon-1) - \frac{\epsilon}{\epsilon-1} \epsilon \frac{x'(N_t;\chi_t)}{z_t u'(C_t)} \right] \theta_{jt} (\pi^{acc}_{jt})^{\epsilon-1} Y_t u'(C_t) \\
&+ \left[\phi_t^{\pi} \frac{p^*_{t-j}}{\pi^{acc}_{jt}} (\epsilon-1) + \phi_t^{\Delta} \epsilon \right] (p^*_{t-j})^{-\epsilon} (\pi^{acc}_{jt})^{\epsilon-1} (1-\lambda_{jt}) \psi_{jt} \\
&+ \phi_t^{v_j} \left[\frac{p^*_{t-j}}{\pi^{acc}_{jt}} (\epsilon-1) - \frac{x'(N_t;\chi_t)}{z_t u'(C_t)} \epsilon \right] (p^*_{t-j})^{-\epsilon} (\pi^{acc}_{jt})^{\epsilon-1} Y_t u'(C_t) \\
&+ \phi_t^{\pi^{acc}_j} - \beta E_t \phi_{t+1}^{\pi^{acc}_{j+1}} \pi_t, \qquad (\pi^{acc}_{j=1,\dots,J-2,\ t})
\end{aligned}$$

$$\begin{aligned}
&\phi^{p^*}_{t-(J-1)} \left[\frac{p^*_{t-(J-1)}}{\pi^{acc}_{J-1,t}} (\epsilon-1) - \frac{\epsilon}{\epsilon-1} \epsilon \frac{x'(N_t;\chi_t)}{z_t u'(C_t)} \right] \theta_{J-1,t} (\pi^{acc}_{J-1,t})^{\epsilon-1} Y_t u'(C_t) \\
&+ \left[\phi_t^{\pi} \frac{p^*_{t-(J-1)}}{\pi^{acc}_{J-1,t}} (\epsilon-1) + \phi_t^{\Delta} \epsilon \right] (p^*_{t-(J-1)})^{-\epsilon} (\pi^{acc}_{J-1,t})^{\epsilon-1} (1-\lambda_{J-1,t}) \psi_{J-1,t} \\
&+ \phi_t^{v_j} \left[\frac{p^*_{t-(J-1)}}{\pi^{acc}_{J-1,t}} (\epsilon-1) - \frac{x'(N_t;\chi_t)}{z_t u'(C_t)} \epsilon \right] (p^*_{t-(J-1)})^{-\epsilon} \left(\pi^{acc}_{J-1,t} \right)^{\epsilon-1} Y_t u'(C_t) + \phi_t^{\pi^{acc}_{J-1}}, (\pi^{acc}_{J-1,\ t})
\end{aligned}$$

$$\phi^{p^*}_{t-j} \left[\frac{p^*_{t-j}}{\pi^{acc}_{jt}} - \frac{\epsilon}{\epsilon-1} \frac{x'(N_t;\chi_t)}{z_t u'(C_t)} \right] (\pi^{acc}_{jt})^{\epsilon} Y_t u'(C_t) + \phi_t^{\theta_j} - \beta E_t \phi_{t+1}^{\theta_{j+1}} (1-\lambda_{j+1,t+1}), \qquad (\theta_{j=1,\dots,J-2,\ t})$$

$$\phi_{t-(J-1)}^{p^*}\left[\frac{p_{t-(J-1)}^*}{\pi_{J-1,t}^{acc}}-\frac{\epsilon}{\epsilon-1}\frac{x'\left(N_t;\chi_t\right)}{z_t u'\left(C_t\right)}\right](\pi_{J-1,t}^{acc})^{\epsilon}Y_t u'\left(C_t\right)+\phi_t^{\theta_{J-1}}, \tag{$\theta_{J-1,\,t}$}$$

$$\begin{aligned}&\phi_t^{\pi}\left[\left(p_t^*\right)^{1-\epsilon}-\left(\frac{p_{t-j}^*}{\pi_{jt}^{acc}}\right)^{1-\epsilon}\right]\psi_{jt}+\phi_t^{\Delta}\left[\left(p_t^*\right)^{-\epsilon}-\left(\frac{p_{t-j}^*}{\pi_{jt}^{acc}}\right)^{-\epsilon}\right]\psi_{jt}\\&+\phi_t^{\lambda_j}+\beta E_t\phi_{t+1}^{\psi_{j+1}}\psi_{jt}+\phi_{t-1}^{v_{j-1}}\left(\tilde{v}_{0,t}-\tilde{v}_{jt}\right)+\phi_t^{\theta_j}\theta_{j-1,t-1},\end{aligned} \tag{$\lambda_{j=1,\ldots,J-1,\,t}$}$$

$$\begin{aligned}&-\phi_t^N\int_0^{(\tilde{v}_{0t}-\tilde{v}_{1t})/x'(N_t;\chi_t)}\kappa g\left(\kappa\right)d\kappa+\phi_t^{\pi}\left[\left(p_t^*\right)^{1-\epsilon}\lambda_{1t}+\left(\frac{p_{t-1}^*}{\pi_{1t}^{acc}}\right)^{1-\epsilon}\left(1-\lambda_{1t}\right)\right]\\&+\phi_t^{\Delta}\left[\left(p_t^*\right)^{-\epsilon}\lambda_{1t}+\left(\frac{p_{t-1}^*}{\pi_{1t}^{acc}}\right)^{-\epsilon}\left(1-\lambda_{1t}\right)\right]-\beta E_t\phi_{t+1}^{\psi_2}\left(1-\lambda_{1t}\right)+\phi_t^{\psi_1},\end{aligned} \tag{$\psi_{1,t}$}$$

$$\begin{aligned}&-\phi_t^N\int_0^{(\tilde{v}_{0t}-\tilde{v}_{jt})/x'(N_t;\chi_t)}\kappa g\left(\kappa\right)d\kappa+\phi_t^{\pi}\left[\left(p_t^*\right)^{1-\epsilon}\lambda_{jt}+\left(\frac{p_{t-j}^*}{\pi_{jt}^{acc}}\right)^{1-\epsilon}\left(1-\lambda_{jt}\right)\right]\\&+\phi_t^{\Delta}\left[\left(p_t^*\right)^{-\epsilon}\lambda_{jt}+\left(\frac{p_{t-j}^*}{\pi_{jt}^{acc}}\right)^{-\epsilon}\left(1-\lambda_{jt}\right)\right]\\&+\phi_t^{\psi_j}-\beta E_t\phi_{t+1}^{\psi_{j+1}}\left(1-\lambda_{jt}\right)+\phi_t^{\psi_1},\end{aligned} \tag{$\psi_{j=2,\ldots,J-1,\,t}$}$$

$$\phi_t^{\pi}\left(p_t^*\right)^{1-\epsilon}+\phi_t^{\Delta}\left(p_t^*\right)^{-\epsilon}+\phi_t^{\psi_J}+\phi_t^{\psi_1}, \tag{$\psi_{J,t}$}$$

$$-\phi_t^N\sum_{j=1}^{J-1}\frac{\psi_{jt}}{x'\left(N_t;\chi_t\right)}L_{jt}g\left(L_{jt}\right)-\sum_{j=1}^{J-1}\phi_t^{\lambda_j}g\left(L_{jt}\right)\frac{1}{x'\left(N_t;\chi_t\right)}-\phi_t^{v_0}+\sum_{j=0}^{J-2}\phi_{t-1}^{v_j}\left[\lambda_{j+1,t}-L_{j+1,t}g\left(L_{j+1,t}\right)\right]+\phi_{t-1}^{v_{J-1}}, \tag{$\tilde{v}_{0t}$}$$

$$\phi_t^N\psi_{jt}\frac{1}{x'\left(N_t;\chi_t\right)}L_{jt}g\left(L_{jt}\right)+\phi_t^{\lambda_j}g\left(L_{jt}\right)\frac{1}{x'\left(N_t;\chi_t\right)}-\phi_t^{v_j}+\phi_{t-1}^{v_{j-1}}\left[1-\lambda_{jt}+L_{jt}g\left(L_{jt}\right)\right], \tag{$\tilde{v}_{j=1,\ldots,J-1,\,t}$}$$

$$\begin{aligned}&-x'\left(N_t;\chi_t\right)-\sum_{j=0}^{J-1}\phi_{t-j}^{p^*}\frac{\epsilon}{\epsilon-1}\theta_{jt}(\pi_{jt}^{acc})^{\epsilon}Y_t\frac{x''\left(N_t;\chi_t\right)}{z_t}+\phi_t^N\left[1+\sum_{j=1}^{J-1}\psi_{jt}\frac{x''\left(N_t;\chi_t\right)}{x'\left(N_t;\chi_t\right)}\left(L_{jt}\right)^2g\left(L_{jt}\right)\right]\\&+\sum_{j=1}^{J-1}\phi_t^{\lambda_j}g\left(L_{jt}\right)L_{jt}\frac{x''\left(N_t;\chi_t\right)}{x'\left(N_t;\chi_t\right)}-\sum_{j=0}^{J-1}\phi_t^{v_j}\frac{x''\left(N_t;\chi_t\right)}{z_t}\left(\frac{p_{t-j}^*}{\pi_{jt}^{acc}}\right)^{-\epsilon}Y_t\\&+\sum_{j=0}^{J-2}\phi_{t-1}^{v_j}x''\left(N_t;\chi_t\right)\left[\left(L_{j+1,t}\right)^2g\left(L_{j+1,t}\right)-\int_0^{L_{j+1,t}}\kappa g\left(\kappa\right)d\kappa\right],\end{aligned} \tag{N_t}$$

$$-\phi_t^N\frac{Y_t}{z_t}-\phi_t^{\Delta}, \tag{Δ_t}$$

$$-\phi_t^{\pi_1^{acc}} - \sum_{j=2}^{J-1} \phi_t^{\pi_j^{acc}} \pi_{j-1,t-1}^{acc}, \qquad (\pi_t)$$

where we have defined the adjustment gain $L_{jt} \equiv (v_{0t} - v_{jt})/w_t = (\tilde{v}_{0t} - \tilde{v}_{jt})/x'(N_t;\chi_t)$ for compactness. We now conjecture that the timeless perspective optimal policy involves zero net inflation at all times, $\pi_t = 1$. Under such a policy, in the timeless perspective regime (that is, after all transitional dynamics have disappeared) the economy converges to the following equilibrium:

$$\pi_t = p_t^* = \Delta_t = \frac{\epsilon}{\epsilon - 1} \frac{x'(N_t;\chi_t)}{z_t u'(C_t)} = 1 = \pi_{jt}^{acc}, \quad j = 1, ..., J-1$$

$$v_{0t} = v_{jt} \Rightarrow L_{jt} = 0, \quad j = 1, ..., J-1$$

$$\lambda_{jt} = G(0) \equiv \bar{\lambda} > 0 \Rightarrow \theta_{jt} = \left(1 - \bar{\lambda}\right)^j, \quad j = 1, ..., J-1$$

$$\psi_{jt} = \frac{\left(1 - \bar{\lambda}\right)^{j-1}}{\sum_{k=0}^{J-1} \left(1 - \bar{\lambda}\right)^k} \equiv \bar{\psi}_j,$$

$$N_t = \frac{Y_t}{z_t} = \frac{C_t + G_t}{z_t}$$

for all t. Thus, all firms end up having the same relative prices. Price dispersion is eliminated; the average price markup is constant at the level $\epsilon/(\epsilon - 1)$, such that output, employment and consumption equal their flexible-price levels of section 2.5.1 at all times; adjustment gains are zero and the vintage distribution converges to a stationary distribution. Imposing our conjecture in the first-order conditions, we obtain

$$0 = 1 + \frac{Y_t u''(C_t)}{u'(C_t)} \sum_{j=0}^{J-1} \phi_{t-j}^{p^*} \left(1 - \bar{\lambda}\right)^j - \frac{\phi_t^N}{z_t u'(C_t)} + \sum_{j=0}^{J-1} \phi_t^{v_j} \left[1 + \frac{u''(C_t) Y_t}{u'(C_t)} - \frac{\epsilon - 1}{\epsilon}\right], \qquad (20)$$

$$\begin{aligned} 0 \;\; = \;\; & \phi_t^{p^*} E_t \sum_{j=0}^{J-1} \beta^j \left(1 - \bar{\lambda}\right)^j Y_{t+j} u'(C_{t+j}) - \left[\phi_t^{\pi}(\epsilon - 1) + \phi_t^{\Delta} \epsilon\right] \left(\bar{\lambda} \sum_{j=1}^{J-1} \bar{\psi}_j + \bar{\psi}_J\right) \\ & - E_t \sum_{j=1}^{J-1} \beta^j \left[\phi_{t+j}^{\pi}(\epsilon - 1) + \phi_{t+j}^{\Delta} \epsilon\right] \left(1 - \bar{\lambda}\right) \bar{\psi}_j, \qquad (21) \end{aligned}$$

$$0 = -\phi_{t-j}^{p^*} \left(1 - \bar{\lambda}\right)^j Y_t u'(C_t) + \left[\phi_t^{\pi}(\epsilon - 1) + \phi_t^{\Delta} \epsilon\right] \left(1 - \bar{\lambda}\right) \bar{\psi}_j + \phi_t^{\pi_j^{acc}} - \beta E_t \phi_{t+1}^{\pi_{j+1}^{acc}}, \qquad (22)$$

$$0 = -\phi_{t-(J-1)}^{p^*} \left(1 - \bar{\lambda}\right)^{J-1} Y_t u'(C_t) + \left[\phi_t^{\pi}(\epsilon - 1) + \phi_t^{\Delta} \epsilon\right] \left(1 - \bar{\lambda}\right) \bar{\psi}_{J-1} + \phi_t^{\pi_{J-1}^{acc}}, \qquad (23)$$

$$0 = \phi_t^{\theta_j} - \beta E_t \phi_{t+1}^{\theta_{j+1}} \left(1 - \bar{\lambda}\right), \quad j = 1, ..., J-2, \qquad (24)$$

$$0 = \phi_t^{\theta_{J-1}}, \qquad (25)$$

$$0 = \phi_t^{\lambda_j} + \beta E_t \phi_{t+1}^{\psi_{j+1}} \bar{\psi}_j + \phi_t^{\theta_j} \left(1 - \bar{\lambda}\right)^{j-1}, \quad j = 1, ..., J-1, \qquad (26)$$

$$0 = \phi_t^{\pi} + \phi_t^{\Delta} - \beta\left(1-\bar{\lambda}\right) E_t \phi_{t+1}^{\psi_2} + \phi_t^{\psi_1}, \tag{27}$$

$$0 = \phi_t^{\pi} + \phi_t^{\Delta} + \phi_t^{\psi_j} - \beta\left(1-\bar{\lambda}\right) E_t \phi_{t+1}^{\psi_{j+1}} + \phi_t^{\psi_1}, \quad j = 2, ..., J-1, \tag{28}$$

$$0 = \phi_t^{\pi} + \phi_t^{\Delta} + \phi_t^{\psi_J} + \phi_t^{\psi_1}, \tag{29}$$

$$0 = -\sum_{j=1}^{J-1} \phi_t^{\lambda_j} \frac{g(0)}{x'(N_t;\chi_t)} - \phi_t^{v_0} + \bar{\lambda} \sum_{j=0}^{J-2} \phi_{t-1}^{v_j} + \phi_{t-1}^{v_{J-1}}, \tag{30}$$

$$0 = \phi_t^{\lambda_j} \frac{g(0)}{x'(N_t;\chi_t)} - \phi_t^{v_j} + \left(1-\bar{\lambda}\right) \phi_{t-1}^{v_{j-1}}, \tag{31}$$

$$0 = -1 - \sum_{j=0}^{J-1} \phi_{t-j}^{p^*} \frac{\epsilon}{\epsilon-1} \left(1-\bar{\lambda}\right)^j \frac{N_t x''(N_t;\chi_t)}{x'(N_t;\chi_t)} + \frac{\phi_t^N}{x'(N_t;\chi_t)} - \sum_{j=0}^{J-1} \phi_t^{v_j} \frac{x''(N_t;\chi_t) N_t}{x'(N_t;\chi_t)}, \tag{32}$$

$$0 = -\phi_t^N \frac{Y_t}{z_t} - \phi_t^{\Delta}, \tag{33}$$

$$0 = -\phi_t^{\pi_1^{acc}} - \textstyle\sum_{j=2}^{J-1} \phi_t^{\pi_j^{acc}}. \tag{34}$$

We now use equations (20) to (33) to solve for the Lagrange multipliers. From (25) and (24), it follows immediately that

$$\phi_t^{\theta_j} = 0, \quad j = 1, ..., J-1. \tag{35}$$

Equations (27) to (29) allow us to solve for the $\phi_t^{\psi_j}$ multipliers, obtaining

$$\phi_t^{\psi_1} = -\left(\phi_t^{\pi} + \phi_t^{\Delta}\right),$$

$$\phi_t^{\psi_j} = 0, \quad j = 2, ..., J. \tag{36}$$

Using (35) and (36) in equations (26), we obtain

$$\phi_t^{\lambda_j} = 0, \quad j = 1, ..., J-1. \tag{37}$$

Using the latter, equations (30) and (31) can be expressed compactly as $\boldsymbol{\phi}_t^v = \mathbf{A}\boldsymbol{\phi}_{t-1}^v$, where $\boldsymbol{\phi}_t^v = [\phi_t^{v_0}, \phi_t^{v_1}, ..., \phi_t^{v_{J-1}}]'$ and

$$\underset{J\times J}{\mathbf{A}} = \begin{bmatrix} \bar{\lambda} & \bar{\lambda} & \bar{\lambda} & ... & \bar{\lambda} & \bar{\lambda} & 1 \\ 1-\bar{\lambda} & 0 & 0 & ... & 0 & 0 & 0 \\ 0 & 1-\bar{\lambda} & 0 & ... & 0 & 0 & 0 \\ ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & 1-\bar{\lambda} & 0 & 0 \\ 0 & 0 & 0 & .. & 0 & 1-\bar{\lambda} & 0 \end{bmatrix}.$$

The matrix $\mathbf{A}$ has $J-1$ eigenvalues with modulus equal to $1-\bar{\lambda}<1$ and one unit eigenvalue.[15] The system is thus stable, and the elements in $\boldsymbol{\phi}_t^v$ converge to finite values that depend on initial conditions. Therefore, in the timeless perspective regime, in which all transitional dynamics have disappeared, the multipliers $\phi_t^{v_j}$ converge to constant values $\bar{\phi}^{v_j}$, $j=0,...,J-1$. We then use (32) to solve for ϕ_t^N, obtaining

$$\phi_t^N = x'(N_t;\chi_t)\left[1+\frac{\epsilon}{\epsilon-1}\frac{N_t x''(N_t;\chi_t)}{x'(N_t;\chi_t)}\sum_{j=0}^{J-1}\left(1-\bar{\lambda}\right)^j\phi_{t-j}^{p^*}+\frac{x''(N_t;\chi_t)N_t}{x'(N_t;\chi_t)}\sum_{j=0}^{J-1}\bar{\phi}^{v_j}\right].$$

Using the latter in (20), we obtain

$$\left[\frac{Y_t u''(C_t)}{(-)u'(C_t)}+\frac{N_t x''(N_t;\chi_t)}{x'(N_t;\chi_t)}\right]\sum_{j=0}^{J-1}\left(1-\bar{\lambda}\right)^j\phi_{t-j}^{p^*}=\frac{1}{\epsilon}+\left[\frac{1}{\epsilon}-\frac{u''(C_t)Y_t}{(-)u'(C_t)}-\frac{\epsilon-1}{\epsilon}\frac{x''(N_t;\chi_t)N_t}{x'(N_t;\chi_t)}\right]\sum_{j=0}^{J-1}\bar{\phi}^{v_j},$$

where we have used the fact that, under our conjecture, $x'(N_t;\chi_t)/[z_t u'(C_t)]=(\epsilon-1)/\epsilon$. At this point, we assume away government spending, $G_t=0$, such that $Y_t=C_t$. We also assume that functional forms for preferences are of the constant elasticity type. Let $\sigma\equiv(-)C_t u''(C_t)/u'(C_t)$ and $\varphi\equiv N_t x''(N_t;\chi_t)/x'(N_t;\chi_t)$ denote the constant elasticities of marginal consumption utility and marginal labor disutility, respectively. Then we have

$$\sum_{j=0}^{J-1}\left(1-\bar{\lambda}\right)^j\phi_{t-j}^{p^*}=\frac{1/\epsilon}{\sigma+\varphi}+\frac{1/\epsilon-\sigma-\varphi(\epsilon-1)/\epsilon}{\sigma+\varphi}\sum_{j=0}^{J-1}\bar{\phi}^{v_j}\equiv\Xi.$$

It can be shown that all $J-1$ roots of the characteristic polynomial $\sum_{j=0}^{J-1}\left(1-\bar{\lambda}\right)^j x^{J-1-j}$ have modulus equal to $1-\bar{\lambda}<1$, hence they all lie inside the unit circle. Therefore, in the timeless perspective regime, the multiplier $\phi_t^{p^*}$ converges to the constant value $\bar{\phi}^{p^*}\equiv\Xi/\sum_{j=0}^{J-1}\left(1-\bar{\lambda}\right)^j$. Using this in equation (21), together with $\bar{\lambda}\sum_{j=1}^{J-1}\bar{\psi}_j+\bar{\psi}_J=\bar{\psi}_1$ and $\bar{\psi}_j=\left(1-\bar{\lambda}\right)^{j-1}\bar{\psi}_1$, the latter equation can be expressed as

$$\begin{aligned}0 &= E_t\sum_{j=0}^{J-1}\beta^j\left(1-\bar{\lambda}\right)^j\left\{\bar{\phi}^{p^*}Y_{t+j}u'(C_{t+j})-\left[\phi_{t+j}^{\pi}(\epsilon-1)+\phi_{t+j}^{\Delta}\epsilon\right]\bar{\psi}_1\right\}\\ &= E_t\sum_{j=0}^{J-1}\beta^j\left(1-\bar{\lambda}\right)^j\Sigma_{t+j}, \qquad (38)\end{aligned}$$

where we have defined $\Sigma_t\equiv\bar{\phi}^{p^*}Y_t u'(C_t)-\left[\phi_t^{\pi}(\epsilon-1)+\phi_t^{\Delta}\epsilon\right]\bar{\psi}_1$. All $J-1$ roots of the polynomial $\sum_{j=0}^{J-1}\beta^j\left(1-\bar{\lambda}\right)^j x^{J-1-j}$ have modulus equal to $\beta\left(1-\bar{\lambda}\right)<1$ and are thus inside the unit circle.

[15] Every column of A sums to unity, which implies that unity is an eigenvalue of A (Hamilton, 1994, p. 681), but A is also a Leslie matrix, hence it has only one positive and dominant eigenvalue (Poole, 2006, p. 328). Hence, all other eigenvalues must lie inside the unit circle.

Therefore, equation (38) has a unique solution given by $\Sigma_t = 0$, or equivalently

$$\bar{\phi}^{p^*} Y_t u'(C_t) - \left[\phi_t^{\pi}(\epsilon - 1) + \phi_t^{\Delta}\epsilon\right]\bar{\psi}_1 = 0, \tag{39}$$

which pins down the multiplier ϕ_t^{π} as a function of the variables $Y_t u'(C_t)$ and ϕ_t^{Δ}. The latter multiplier is in turn determined by equation (33).

Equation (23) can be solved for $\phi_t^{\pi^{acc}_{J-1}}$, obtaining

$$\phi_t^{\pi^{acc}_{J-1}} = \left(1 - \bar{\lambda}\right)^{J-1}\left\{\bar{\phi}^{p^*} Y_t u'(C_t) - \left[\phi_t^{\pi}(\epsilon - 1) + \phi_t^{\Delta}\epsilon\right]\bar{\psi}_1\right\} = 0,$$

where we have used $\bar{\psi}_{J-1} = \left(1 - \bar{\lambda}\right)^{J-2}\bar{\psi}_1$ and where the second equality follows from (39). Using $E_t\phi_{t+1}^{\pi^{acc}_{J-1}} = 0$ and $\bar{\psi}_{J-2} = \left(1 - \bar{\lambda}\right)^{J-3}\bar{\psi}_1$ in equation (22) for $j = J-2$, the latter implies $\phi_t^{\pi^{acc}_{J-2}} = 0$. Operating in the same fashion, equations (22) for $j = 1, ..., J-3$ imply that $\phi_t^{\pi^{acc}_j} = 0$ for $j = 1, ..., J-3$.

It only remains to verify that equation (34) holds given the solution of the Lagrange multipliers. This is obvious, as we have already shown that $\phi_t^{\pi^{acc}_j} = 0$ for $j = 1, ..., J-1$.

Fig.1: Price adjustment probability and firm distribution by vintage

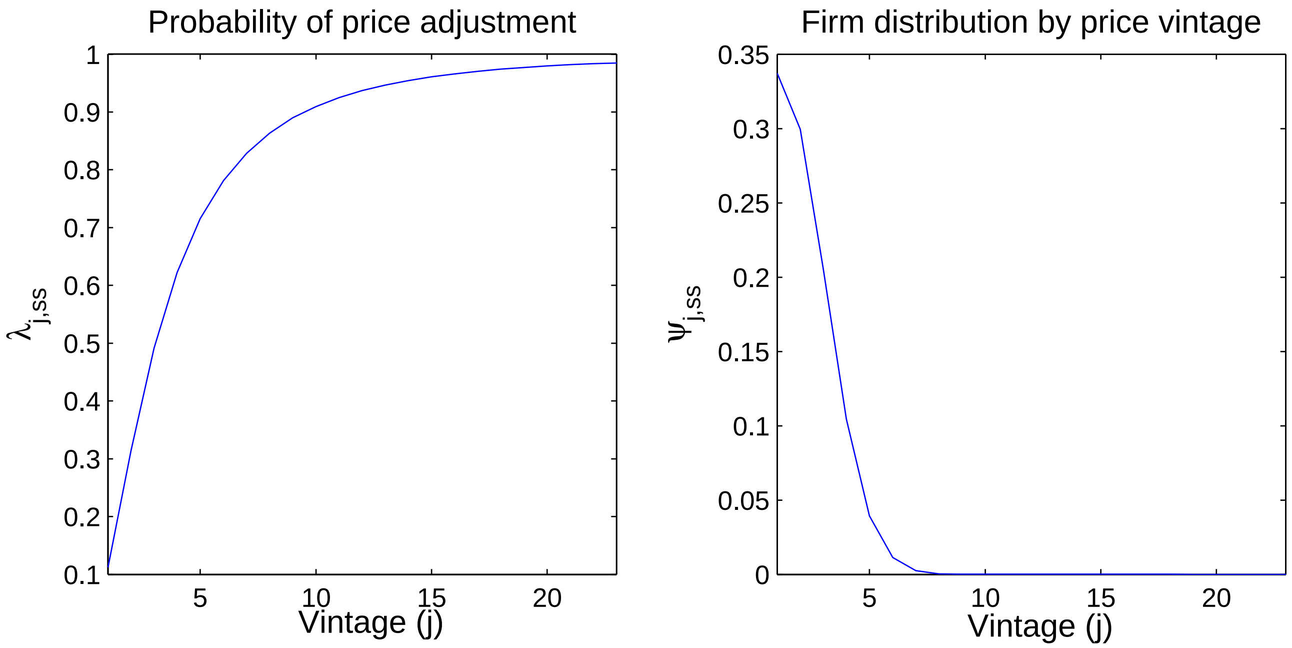

Fig.2: Responses to a technology and a government spending shock