**Finance and Economics Discussion Series**
**Divisions of Research & Statistics and Monetary Affairs**
**Federal Reserve Board, Washington, D.C.**

# Stock Return Predictability and Variance Risk Premia: Statistical Inference and International Evidence

**Tim Bollerslev, James Marrone, Lai Xu, and Hao Zhou**

**2011-52**

# Stock Return Predictability and Variance Risk Premia: Statistical Inference and International Evidence*

Tim Bollerslev†, James Marrone‡, Lai Xu§, and Hao Zhou¶

This Version: December 2011

**Abstract**

Recent empirical evidence suggests that the variance risk premium, or the difference between risk-neutral and statistical expectations of the future return variation, predicts aggregate stock market returns, with the predictability especially strong at the 2-4 month horizons. We provide extensive Monte Carlo simulation evidence that statistical finite sample biases in the overlapping return regressions underlying these findings can not "explain" this apparent predictability. Further corroborating the existing empirical evidence, we show that the patterns in the predictability across different return horizons estimated from country specific regressions for France, Germany, Japan, Switzerland and the U.K. are remarkably similar to the pattern previously documented for the U.S. Defining a "global" variance risk premium, we uncover even stronger predictability and almost identical cross-country patterns through the use of panel regressions that effectively restrict the compensation for world-wide variance risk to be the same across countries. Our findings are broadly consistent with the implications from a stylized two-country general equilibrium model explicitly incorporating the effects of world-wide time-varying economic uncertainty.

***JEL classification:*** C12, C22, G12, G13.
***Keywords:*** Variance risk premium; return predictability; over-lapping return regressions; international stock market returns; global variance risk.

---

*The analysis and conclusions set forth are those of the authors and do not indicate concurrence by other members of the research staff or the Board of Governors. We would like to thank Qianqiu Liu, Andrew Patton, George Tauchen, Guofu Zhou, and seminar participants at the 2011 China International Conference in Finance (CICF) in Wuhan, the 2011 NBER-NSF Time Series Conference at Michigan State University, the 2011 Inquire Europe Autumn Seminar, Notre Dame University, and the Duke Financial Econometrics Lunch Group for their helpful comments. We would also like to acknowledge the Best Paper Award from CICF. Bollerslev's research was supported by a grant from the NSF to the NBER, and CREATES funded by the Danish National Research Foundation.

†Department of Economics, Duke University, Durham, NC 27708, USA, and NBER and CREATES, boller@duke.edu, 919-660-1846.

‡Department of Economics, University of Chicago, Chicago, IL 60637, USA, jmar@uchicago.edu, 773-702-9016.

§Department of Economics, Duke University, Durham, NC 27708, USA, lai.xu@duke.edu, 919-257-0059.

¶Division of Research and Statistics, Federal Reserve Board, Mail Stop 91, Washington DC 20551 USA, hao.zhou@frb.gov, 202-452-3360.

# Stock Return Predictability and Variance Risk Premia: Statistical Inference and International Evidence

This Version: October 11, 2011


**Abstract**

Recent empirical evidence suggests that the variance risk premium, or the difference between risk-neutral and statistical expectations of the future return variation, predicts aggregate stock market returns, with the predictability especially strong at the 2-4 month horizons. We provide extensive Monte Carlo simulation evidence that statistical finite sample biases in the overlapping return regressions underlying these findings can not "explain" this apparent predictability. Further corroborating the existing empirical evidence, we show that the patterns in the predictability across different return horizons estimated from country specific regressions for France, Germany, Japan, Switzerland and the U.K. are remarkably similar to the pattern previously documented for the U.S. Defining a "global" variance risk premium, we uncover even stronger predictability and almost identical cross-country patterns through the use of panel regressions that effectively restrict the compensation for world-wide variance risk to be the same across countries. Our findings are broadly consistent with the implications from a stylized two-country general equilibrium model explicitly incorporating the effects of world-wide time-varying economic uncertainty.



***JEL classification:*** C12, C22, G12, G13.

***Keywords:*** Variance risk premium; return predictability; over-lapping return regressions; international stock market returns; global variance risk.

# 1 Introduction

A number of recent studies have suggested that aggregate U.S. stock market return is predictable over horizons ranging up to a few quarters based on the difference between options-implied and actual realized variation measures, or the so-called variance risk premium (see, e.g., Bollerslev, Tauchen, and Zhou, 2009; Drechsler and Yaron, 2011; Gabaix, 2011; Kelly, 2011; Zhou, 2010; Zhou and Zhu, 2009, among others). These findings of apparent predictability over relatively short quarterly horizons have potentially far reaching implications for many issues in asset pricing finance. They are also distinctly different from the longer-run multi-year return predictability patterns that have been studied extensively in the existing literature, in which the predictability is typically associated with more traditional valuation measures such as dividend yields, P/E ratios, or consumption-wealth ratios (see, e.g., Fama and French, 1988; Campbell and Shiller, 1988b; Lettau and Ludvigson, 2001, among others). Motivated by these observations, the main goal of the present paper is to further examine the robustness and the scope of these striking new empirical findings.

Our investigations are essentially twofold. First, to assess the validity of the statistical inference procedures underlying the empirical findings, we report the results from an extensive Monte Carlo simulation exercise designed to closely mimic the dynamic dependencies inherent in daily returns and variance risk premia. Our results clearly suggest that statistical biases can not "explain" the documented return predictability patterns. At the same time, the results also suggest that the use of finer sampled observations, say daily as opposed to monthly data as employed in the above cited studies, provides limited additional power to detect the predictability inherent in the variance risk premium.

Second, in a separate effort to expand on and corroborate the existing empirical evidence pertaining to monthly U.S. returns, we extend the same basic ideas and regressions to several other countries. In so doing, we also define a "global" variance risk premium. We show that this simple aggregate measure of world-wide economic uncertainty results in even stronger predictability for all of the countries in the sample. We also show that these new empirical findings are broadly consistent with the implications from a stylized two-country general equilibrium model that explicitly incorporates the effect of time-varying economic uncertainty across countries.

The finite sample properties of overlapping long-horizon return regressions have been studied extensively in the existing literature. Boudoukh, Richardson, and Whitelaw (2008), for instance, have recently shown that even in the absence of any increase in the true predictability, the values of the $R^2$'s in regressions involving highly persistent predictor variables and overlapping returns will by construction increase roughly proportional to the return horizon and the length of the overlap.[1] In line with the procedures adapted in the existing literature, we will focus on the Newey and West (1987) and Hodrick (1992) type $t$-statistics. Both of these are robust asymptotically to heteroskedasticity and serial correlation in the residuals from the estimated regressions. Our simulation design is based on an empirically realistic bivariate VAR-GARCH-DCC model for the joint daily return and variance risk premium dynamics. We find both of the $t$-statistics to be reasonably well behaved, albeit slightly over-sized under the null hypothesis of no predictability. We also find that the Newey-West based $t$-statistics result in marginally more powerful (size adjusted) tests under the alternative. Moreover, directly in line with the results in the existing literature on long-horizon return predictability, the quantiles in the finite sample distribution of the $R^2$'s from the regressions are spuriously increasing with the return horizon under the null of no predictability.[2] At the same time, the $R^2$'s implied by the daily VAR-GARCH-DCC model exhibit a distinct hump shape in the degree of predictability that closely mimics the pattern actually observed in U.S. return regressions.

Guided by the Monte Carlo simulations, we rely on the Newey-West based $t$-statistics and monthly return regressions to summarize our new international evidence. Due to data availability and the required liquidity of options markets, we restrict our attention to the six major financial markets of France, Germany, Japan, Switzerland, the U.K., and the U.S. Our empirical result shows that the country specific regressions based on regressing each country's

[1] Closely related issues pertaining to the use of persistent predictor variables have also been studied by, e.g., Stambaugh (1999), Ferson, Sarkissian, and Simin (2003), Baker, Taliaferro, and Wurgler (2006), Campbell and Yogo (2006), Ang and Bekaert (2007), and Goyal and Welch (2008), among others.

[2] Although the variance risk premium is not especially persistent at the monthly frequency typically employed in the literature, its first order autocorrelation is still in excess of 0.97 at the daily level.

return on its own variance risk premium result in similar hump-shaped regression coefficients and $R^2$'s for all of the six countries. However, the degree of predictability afforded by the country specific variance risk premia and the statistical significance of the results generally are not as strong as the previously reported results for the U.S. market.

These results naturally point to the possibility of world-wide variance risk, as opposed to the country specific variance risk premia, being priced. To investigate this idea, we construct a "global" variance risk premium, defined as a simple market capitalization weighted average of the individual country variance risk premia. Restricting the effect on this "global" variance risk premium to be the same across countries in a panel return regression results in much stronger findings for all of the countries, with a uniform peak in the degree of predictability at the four month horizons. Moreover, the degree of predictability afforded by this "global" variance risk premium easily exceeds that of the implied and realized variation measures when included in isolation. It also clearly dominates that of other traditional predictor variables that have been shown to work well over longer annual horizons, including the P/E ratio. While this new international evidence indirectly corroborates the previous findings based exclusively on U.S. data reported in the studies cited above, importantly the results also point to the existence of even stronger predictability through the use of alternative definitions of world-wide variance risk.[3]

Our new empirical findings are, of course, related to the large existing literature on international stock return predictability (see, e.g., Harvey, 1991; Bekaert and Hodrick, 1992; Campbell and Hamao, 1992; Ferson and Harvey, 1993, among others). However, the focus of this literature has traditionally been on longer-run multi-year return predictability. By contrast, our results pertaining to the "global" variance risk premium concern much shorter-run within year predictability, and are essentially "orthogonal" to the findings reported in

[3]These results are also indirectly in line with those reported in a few other recent studies pertaining to other markets. In particular, in concurrent independent work, Londono (2010) finds that the U.S. variance risk premium predicts several foreign stock market returns. In a slightly different context, Mueller, Vedolin, and Zhou (2011) argue that the U.S. variance risk premium predicts bond risk premia, beyond the predictability afforded by forward rates, while Buraschi, Trojani, and Vedolin (2010) and Zhou (2010) show that the variance risk premium also helps predict credit spreads, over and above the typical interest rate predictor variables.

the existing literature.[4] At the same time, however, the new empirical results are generally in line with the calibrations from a simple theoretical two-country model that explicitly incorporates the equilibrium effects of time-varying economic uncertainty across countries.

The rest of the paper is organized as follows. Section 2 presents our Monte Carlo based simulation evidence pertaining to the statistical inference procedures underlying the existing empirical findings. Section 3 discusses our new international evidence and the results for our "global" variance risk premia measure, along with our equilibrium model based calibrations. Section 4 concludes.

## 2 General Setup and Monte Carlo Simulations

The key empirical findings reported in Bollerslev, Tauchen, and Zhou (2009) (BTZ2009, henceforth), and the subsequent studies cited above, are based on simple OLS regressions of the returns on the aggregate market portfolio over monthly and longer return horizons on a measure of the one-month variance risk premium. In particular, let $r_{t,t+\tau}$ and $VRP_t$ denote the continuously compounded return from time $t$ to time $t+\tau$ and the variance risk premium at time $t$, respectively. Defining the unit time interval to be one trading day, the multi-period return regressions in BTZ2009 may then be expressed as special cases of,

$$\frac{1}{h}\sum_{j=1}^{h} r_{t+(j-1)s,t+js} \;=\; a_s(h) \;+\; b_s(h)VRP_t \;+\; u_{t,t+hs} \tag{1}$$

for $s=20$ (monthly) and return horizon $hs$, where $t=1,s+1,2s+1,...,T-hs$ refer to the specific observations used in the regression. Of course, the use of finer sampling frequencies, say $s=5$ (weekly) or $s=1$ (daily), may give rise to more powerful inference, and we will investigate that below.

Meanwhile, it is well known that in the context of overlapping return observations, the regression in (1) can result in spuriously large and highly misleading regression $R^2$'s, say $R_s^2(h)$, as the horizon $h$ increases; see, e.g., the discussion and many references in Campbell,

[4] Other recent studies highlighting short-run international predictability include Rapach, Strauss, and Zhou (2010) based on lagged U.S. returns, Ang and Bekaert (2007) and Hjalmarsson (2010) based on short-term interest rates, and Bakshi, Panayotov, and Skoulakis (2011) based on the Baltic Dry Index.

Lo, and MacKinlay (1997). Similarly, the standard errors for the OLS estimates designed to take account of the serial correlation in $u^s_{t+hs,t}$ based on the Bartlett kernel advocated by Newey and West (1987) (NW, henceforth), and the modification proposed by Hodrick (1992) (HD, henceforth), can also both result in $t$-statistics for testing hypotheses about $a_s(h)$ and $b_s(h)$ that are poorly approximated by a standard normal distribution. Most of the existing analyses pertaining to these and other related finite sample biases, however, have been calibrated to situations with a highly persistent predictor variable, as traditionally used in long-horizon return regressions. Even though the variance risk premium is fairly persistent at the daily frequency, it is much less so at the monthly level, and as such one might naturally expect the finite sample biases to be less severe in this situation.[5] Our Monte Carlo simulations discussed in the next section confirm this conjecture in an empirically realistic setting designed to closely mimic the joint dependencies in actual daily returns and variance risk premia.

## 2.1 Simulation Design

The model underlying our simulations is based on daily S&P500 composite index returns (obtained from CRSP). The corresponding daily observations on the variance risk premium are defined as $VRP_t = IV_t - RV_{t-20,t}$, where we rely on the square of the new VIX index (obtained from the CBOE) to quantify the implied variation $IV_t$, and the summation of current and previous 20 trading days daily realized variances (obtained from the Oxford-Man Institute's Realized Volatility Library) together with the squared overnight returns to quantify the total realized variation over the previous month $RV_{t-20,t}$.[6] The span of the data runs from February 1, 1996 to December 31, 2007, for a total of 2,954 daily observations.

After some experimentation, we arrived at the following bivariate VAR(1)-GARCH$(1,1)$-

[5]The first order autocorrelation coefficient for the monthly U.S. variance risk premium analyzed in the empirical section below equals 0.50, and it is even lower for all of the other countries included in our analysis. By comparison, the first order autocorrelations for monthly dividend yields and P/E ratios, and other variables typically employed in the long-horizon regression literature, are around 0.95-0.99.

[6]This directly mirrors the definition of the variance risk premium employed in BTZ2009. Forward looking measures of $VRP_t$ that align $IV_t$ with a measure of the expected volatility $E_t(RV_{t,t+20})$ have also been used in the literature. However, this requires additional modeling assumptions for calculating $E_t(RV_{t,t+20})$, whereas the $VRP_t$ used here has the obvious advantage of being directly observable at time $t$.

DCC model (see Engle, 2002, for additional details on the DCC model) for the two daily time series,

$$\begin{aligned}
r_{t-1,t} &= \underset{(0.001)}{-1.958\text{e-}5} - \underset{(0.016)}{0.009} r_{t-2,t-1} + \underset{(0.010)}{0.025} VRP_{t-1} + \epsilon_{t,r} \\
VRP_t &= \underset{(0.001)}{3.759\text{e-}5} + \underset{(0.017)}{0.033} r_{t-2,t-1} + \underset{(0.010)}{0.972} VRP_{t-1} + \epsilon_{t,vrp} \\
\sigma^2_{t,r} &= \underset{(1.68\text{e-}6)}{1.280\text{e-}6} + \underset{(0.004)}{0.071} \epsilon^2_{t-1,r} + \underset{(0.008)}{0.920} \sigma^2_{t-1,r} \\
\sigma^2_{t,vrp} &= \underset{(7.59\text{e-}6)}{2.038\text{e-}7} + \underset{(0.004)}{0.133} \epsilon^2_{t-1,vrp} + \underset{(0.028)}{0.871} \sigma^2_{t-1,vrp} \\
Q_t &= \begin{pmatrix} \underset{(0.036)}{0.997} & \underset{(0.040)}{-0.754} \\ \underset{(0.040)}{-0.754} & \underset{(0.060)}{1.023} \end{pmatrix} + \underset{(0.002)}{0.011} \eta_{t-1}\eta'_{t-1} + \underset{(0.004)}{0.979} Q_{t-1} \\
R_t &= diag\{Q_t\}^{-1} Q_t diag\{Q_t\}^{-1},
\end{aligned}$$

where $\eta_t \equiv (\frac{\epsilon_{t,r}}{\sigma_{t,r}}, \frac{\epsilon_{t,vrp}}{\sigma_{t,vrp}})'$, and $E_{t-1}(\eta_t) = 0$ and $E_{t-1}(\eta_t\eta'_t) = R_t$ by assumption. The specific parameter values refer to Quasi Maximum Likelihood Estimates (QMLE) obtained under the auxiliary assumption of conditional normality, with robust standard errors following Bollerslev and Wooldridge (1992) in parentheses. With the exception of the lagged daily returns, most of the dynamic coefficients are highly significant at conventional levels.

The model implies a strong negative (on average) correlation between the innovations to the return and VRP equations. This, of course, is consistent with the well documented "leverage" effect; see, e.g., Bollerslev, Sizova, and Tauchen (2011) and the many references therein. At the same time, as is evident from the equation for $Q_t$, and the corresponding plot in the top panel in Figure 1, the value of the conditional correlation clearly varies over time, reaching a low of close to -0.85 toward the end of the sample. The bottom three panels in Figure 1 indicate that the distribution of the estimated standardized residuals from the model (i.e., $\widehat{c\eta}_t \equiv \widehat{F}_t^{-1}\widehat{\eta}_t$, where $\widehat{F}_t \cdot \widehat{F}'_t = \widehat{R}_t$) are well behaved and centered at zero, with variances close to unity, albeit not normally distributed.[7] All in all, however, the model provides a reasonably good fit to the joint dynamic dependencies inherent in the two daily

[7]The sample means for $\widehat{c\eta}_{t,1}$ and $\widehat{c\eta}_{t,2}$ equal -0.044 and 0.088, the standard deviations equal 0.999 and 1.007, while the skewness and kurtosis equal -0.469 and 0.894, and 4.913 and 7.860, respectively. Further diagnostic checks also reveal that while the residuals from the return equation appear close to serially uncorrelated, there is some evidence for neglected longer-run serial dependencies in the equation for the variance risk premium.

series.

As such, we will use this relatively simple-to-implement model as our basic data generating process for the Monte Carlo simulations, and our analysis of the finite sample properties of the NW and HD $t$-statistics, and $R_s^2(h)$'s from the overlapping return regressions in equation (1).[8] Our simulated finite sample distributions will be based on a total of 2,000 bootstrapped replications from the model. We will look at sample frequencies of $s = 1$ ("daily"), $s = 5$ ("weekly") and $s = 20$ ("monthly"), and return horizons $hs$ ranging up to 240 "days," or 12 "months." The number of observations for each of the simulated samples is fixed at $T = 2,954$ "days" (or 598 "weeks," or 149 "months"), corresponding to the length of the actual sample used in the estimation of the VAR-GARCH-DCC model above. We begin with a discussion of the size and power properties of the two $t$-statistics.

## 2.2 Size and Power

Our characterization of the distributions under the null hypothesis of no return predictability is based on restricting the coefficients associated with $r_{t-2,t-1}$ and $VRP_{t-1}$ in the return equation to be identically equal to zero, leaving all of the other coefficients at their estimated values. Table 1 reports the resulting simulated the 95th percentiles of the $t^{NW}$ and $t^{HD}$ test statistics, along with the regression $R^2$'s. Directly in line with the evidence in the existing literature, both of the $t$-statistics exhibit non-trivial size distortions relative to the nominal one-side 95-percent critical value of 1.645. Also, the distortions tend to increase with the return horizon $h$. Moreover, consistent with the results reported in Hodrick (1992), the biases for the NW based standard error calculations generally exceed those for the HD standard errors, and markedly more so the longer the return horizon.

To more directly illustrate the results, we plot in the three left panels in Figure 2 the simulated 95-percent critical values for $t^{NW}$ (dashed lines) and $t^{HD}$ (solid lines) for $s = 1, 5, 20$. We also include in the figure the $t$-statistics obtained by running these same regressions

[8] The bandwidth in the Bartlett kernel employed in our implementation of the NW standard errors is set to $m = [h + 4 * ((T - hs)/100)^{2/9}]$, where $[\cdot]$ refers to the integer value. We also experimented with the reverse regression technique suggested by Hodrick (1992) for testing $b^s(h) = 0$. The results, available upon request, were very similar to the ones for the HD $t$-statistic reported below.

on the actual daily, weekly and monthly data over the February 1996 through December 2007 sample period used in calibrating the simulated model. As the figure shows, the actual $t^{NW}$-statistics systematically exceeds the simulated critical values for return horizons in the range of 2 to 3 months. This is true regardless of whether the regressions are based on daily, weekly, or monthly data. Meanwhile, the $t^{HD}$-statistics generally do not exceed the simulated critical values and accordingly do not support the idea of return predictability.

In order to better understand this discrepancy in the conclusions drawn from the two tests, we report in Table 2 the power of the tests to detect predictability implied by the unrestricted VAR-GARCH-DCC model. To facilitate comparisons we only report the size-adjusted power for a 5-percent test. Not surprisingly, the power of both tests decrease with the return horizon. At the same time, the power of the $t^{NW}$ test systematically exceed that of the $t^{HD}$ test for return horizons less than a year, and the differences appear most pronounced at the 2-4 month horizons.

These differences are also evident in the three right panels in Figure 2, which plot the relevant power curves. Comparing the simulations across the three different panels in the table and the figure also point to fairly small loses in terms of power when decreasing the sampling frequency of the data used in the regressions from $s = 1$ ("daily") to $s = 5$ ("weekly") to $s = 20$ ("monthly").

Guided by these findings we will base our subsequent empirical investigations on the most commonly used monthly return regressions and NW-based standard errors, recognizing that the finite sample distributions of the $t^{NW}$-statistics tend to be slightly upward biased under the null of no predictability.

## 2.3 $R^2$

In addition to the $t$-statistics associated with the $b_s(h)$ coefficients, the $R_s^2(h)$'s from the return regressions are often used to assess the strength of the relationship and the effectiveness of the predictor variable across different horizons. Of course, as previously noted above, it is well known that the biases exhibited by the $t$-statistics in the context of long-horizon return regressions carry over to the $R_s^2(h)$'s, and that these need to be carefully interpreted in the context of persistent predictor variables (see, e.g., the aforementioned study by Boudoukh, Richardson, and Whitelaw, 2008, for a recent analysis, along with the many references therein).

The corresponding columns in Table 1 show that, while less dramatic than the biases over multi-year return horizons, the $R_s^2(h)$'s may still be quite different from zero under the null of no predictability in the present setting. In particular, the 95th percentiles are around 5-6 percent at the 2-4 months horizon for all of the three sampling frequencies $s = 1, 5, 20$.

Further to this effect, we show in the top panel in Figure 3 select quantiles in the simulated distribution of the $R_1^2(h)$'s that obtain in the absence of any predictability. Consistent with the findings in the extant literature pertaining to monthly observations and longer return horizons, all of the quantiles increase monotonically with the return horizon, and this increase is especially marked for the higher percentiles. Intuitively as the horizon increases, the overlapping return regressions become closer to a spurious type regression.

In addition to the simulated quantiles, we also include in the same figure the $R_1^2(h)$'s obtained from the actual return regressions based on the same daily data used in estimating the VAR-GARCH-DCC model. Comparing the actual $R_1^2(h)$'s to the simulated percentiles again suggest that the degree of predictability is most significant at the intermediate 2-4 months horizon. This, of course, is directly in line with the inference based on the $t$-statistics discussed in the previous section, and the prior empirical evidence reported in BTZ2009.

The hump-shaped pattern in the actual $R_1^2(h)$'s also closely mimics the patterns in the simulated quantiles for the estimated VAR-GARCH-DCC model depicted in the bottom panel in Figure 3. Interestingly, this striking similarity with an apparent peak in the degree of predictability at the intermediate 2-4 months horizon arises in spite of the fact that the simulated model involves only first-order dynamics in the equations that describe the daily conditional means.

To help understand this result, consider the VAR(1) corresponding to the conditional mean dependencies in the Monte Carlo simulation design,

$$\begin{aligned} r_{t-1,t} &= a_1 + b_1 r_{t-2,t-1} + c_1 VRP_{t-1} + \epsilon_{r,t} \ , \\ VRP_t &= a_2 + b_2 r_{t-2,t-1} + c_2 VRP_{t-1} + \epsilon_{vrp,t} \ . \end{aligned}$$

Following Campbell (2001), it is possible to show that the population regression coefficients and $R^2$'s from the overlapping return regressions in (1) may be expressed as,[9]

$$\begin{aligned} b(h) = &\left(c_1 \frac{1-c_2^h}{1-c_2}\right) + \left(b_1 + c_1 b_2 \frac{1-c_2^{h-1}}{1-c_2}\right) \frac{Cov(r_{t-1,t}, VRP_t)}{Var(VRP_t)} \\ &+ b_1 b(h-1) + c_1 b_2 [b(h-2) + c_2 b(h-3) + \cdots + c_2^{h-3} b(1)] \ , \end{aligned}$$

$$R^2(h) = \frac{b(h)^2}{h} \frac{Var(VRP_t)}{[h^{-1} Var(\sum_{j=1}^{h} r_{t-1+j,t+j})]} \ .$$

Hence, the strength of the predictability over different horizons $h$ is primarily determined by the interaction between the short-run predictability, or $Cov(r_{t-1,t}, VRP_t)$ and $c_1$, and the own persistence of the $VRP_t$ predictor variable and $c_2$.

To illustrate this, the solid lines in each of the four panels in Figure 4 show the $R^2(h)$'s implied by the unrestricted VAR(1) coefficient estimates used in the simulations. Indirectly confirming the satisfactory fit of the model, the theoretically implied population $R^2(h)$'s are generally close to the $R^2(h)$'s actually estimated from the sample regressions depicted by the star-dashed line in the previous Figure 3. Meanwhile, marginally decreasing the value of each of the VAR(1) coefficients, $b_1$, $c_1$, $b_2$ and $c_2$, by ten percent, results in quite different $R^2(h)$'s, as shown by the dashed lines in Figure 4. In particular, the decrease in $c_2$ has by far the largest effect. Moreover, the value of $c_2$, and the own persistence of $VRP_t$, is intimately linked to the location of the maximum in the hump shaped predictability pattern.[10]

Taken as a whole, our Monte Carlo simulations and the new regression results based on daily U.S. returns discussed above clearly support the variance risk premium as a powerful predictor at the 2-4 month horizons. At the same time, the overlapping nature of the return

[9] We have omitted the implicit dependence on the sampling frequency $s = 1$ for notational simplicity. Further details concerning these derivations are available upon request.

[10] These same ideas also underlie the economic mechanisms and risk-return trade-offs across different return horizons analyzed within the stylized equilibrium model setting in Bollerslev, Sizova, and Tauchen (2011).

regressions tend to attenuate the strength of the predictability somewhat. Hence, in an effort to further corroborate the existing empirical evidence pertaining exclusively to the U.S. market and data prior to the 2008 financial crisis, we next turn to a discussion of our new empirical findings involving more recent data and several other countries.

# 3 International Evidence

Motivated by the Monte Carlo simulation results in the previous section, we will rely exclusively on the common benchmark monthly sampling frequency, along with the traditional NW-based standard errors and $t^{NW}$-statistics, keeping in mind the finite sample biases documented above. We will restrict our analysis to France, Germany, Japan, Switzerland, the U.K., and the U.S., all of which have highly liquid options markets and readily available model-free implied variances for their respective aggregate market indexes (see Siriopoulos and Fassas, 2009, for a recent summary of the model-free and parametric options implied volatility indexes available for different countries). We begin with a brief discussion of the relevant data.

## 3.1 Data and Summary Statistics

Our monthly aggregate market index returns are based on daily data for the French CAC 40 (obtained from Euronext), the German DAX 30 (obtained from Deutsche Börse), the Japanese Nikkei 225, the Swiss SMI, and the U.K. FTSE 100 (all obtained from Datastream), and the U.S. S&P 500 (obtained from Standard & Poor's). We use the sum of the daily squared returns over a month to construct end-of-month realized variances $RV_t^i$ for each of the countries. The corresponding end-of-month model-free implied volatilities $(IV_t^i)^{1/2}$ for the S&P 500 (VIX) were obtained from the CBOE, the CAC (VCAC) from Euronext, the DAX (VDAX) from Deutsche Börse, while those for the FTSE (VFTSE) and the SMI (VSMI) were both obtained from Datastream. Our data for the Japanese volatility index (VXJ) is obtained directly from the Center for the Study of Finance and Insurance at Osaka University (see Nishina, Maghrebi, and Kim, 2006, for a more detailed discussion of the VXJ index). The sample period for each of the series extends from January 2000 to December 2010, and as such also allows for an out-of-sample validation of the existing empirical evidence for the U.S. based exclusively on data prior to the recent financial crisis.[11]

In accordance with the empirical analysis in the previous section, the variance risk premium for each of the individual countries is simply defined by $VRP_t^i \equiv IV_t^i - RV_{t-20,t}^i$. The resulting time series plots in Figure 5 clearly show the dramatic impact of the financial crisis, and the exceptionally large volatility risk premia observed in the Fall of 2008 for all of the countries. Interestingly, the premium for the DAX, and to a lesser extent the SMI, were almost as large and negative as in 2001-2002.

The standard set of summary statistics reported in Table 3 also show a remarkable coherence in the distributions of the variance risk premia and monthly excess returns for each of the countries.[12] In particular, looking at Panel A the average excess returns are all negative, ranging from a high of -2.15 for Switzerland to a low of -6.52 for France, reflective of the often-called "lost decade." Of course, the corresponding standard deviations all point to considerable variations in the returns around their sample means.

The variance risk premia are all positive on average, ranging from a low of 4.13 for France to a high of 13.26 for Japan on a percentage-squared monthly basis. "Selling" volatility has been highly profitable on average over the last decade. Meanwhile, consistent with the visual impressions from Figure 5, all of the premia are significantly negatively skewed and exhibit large excess kurtosis. Even though implied and realized variances are both strongly serially correlated for all of the countries, the variance risk premia are generally not very persistent with the maximum first order serial correlation for the S&P 500 just 0.50. Turning to Panels B and C, the sample cross-country correlations are all fairly high, and with the exceptions of those for the Nikkei, the correlations for the returns all exceed 0.75, while those for the variance risk premia are in excess of 0.70.

The similarity in the summary statistics in Table 3 and the time series plots in Figure

[11]The beginning of the sample coincides with the back-dated initial date of the NYSE Euronext volatility indices.

[12]The riskfree rates used in the construction of the excess returns were obtained from the Federal Reserve Board and Eurocurrency via Datastream. The use of excess returns, as opposed to raw returns, has almost no effect on the results from the return predictability regressions reported below.

5 across the different countries, naturally suggests that the same predictive relationship between the multi-period U.S. returns and variance risk premium may hold true for the other countries as well. The results discussed in the next subsection corroborate this conjecture.

### 3.2 Country Specific Variance Risk Premia Regressions

In parallel to the general multi-period return regression defined in equation (1), our monthly return regressions for each of the individual countries may be expressed as,

$$h^{-1} r^i_{t,t+h} \; = \; a^i(h) \; + \; b^i(h) VRP^i_t \; + \; u^i_{t,t+h} \; , \tag{2}$$

where $r^i_{t,t+h}$ and $VRP^i_t$ refer to the $h = 1, 2, ..., 12$ month excess return and variance risk premium for country $i$, respectively.[13] The resulting regressions results for each of the six countries are reported in Table 4.

The actual estimates for $b^i(h)$ and the corresponding $t^{NW}$-statistics obviously differ somewhat across the countries. However, with the exception of France and the U.S., the estimated coefficients all show the same general pattern starting out fairly low and insignificant at the shortest one-month horizon, rising to their largest values at 3-5 months, and then gradually declining thereafter for longer return horizons. These similarities are also evident in Figure 6, which displays the regression coefficients for the variance risk premia along with the conventional 95-percent confidence bands based on two NW standard errors.[14]

These general patterns in the estimated values of $b^i(h)$ naturally translate into very similar patterns in the corresponding regression $R^2(h)$'s. In particular, looking at the plots in Figure 7, all of the $R^2(h)$'s exhibit an almost identical hump-shaped pattern with the degree of predictability maximized at the 4 months horizon. Of course, the actual values of the $R^2(h)$'s again vary somewhat across the different indices, achieving a maximum of only 0.96 percent for the Nikkei 225 compared to 14.18 percent for the S&P 500.[15]

[13] We omit the $s = 20$ monthly subscript on the regressors and regression coefficients for notational simplicity.

[14] Of course, the results from the Monte Carlo simulations reported in Table 1 indicate that the two standard error bands are likely somewhat conservative and need to be interpreted accordingly.

[15] Interestingly, this value of $R^2(4)$ for the U.S. exceeds that obtained with monthly data through the end of 2007, reported in BTZ2009 and Drechsler and Yaron (2011), as well as the corresponding daily results discussed in Section 2 above.

Taken as whole, the results in Table 4 and the significance of the country specific VRP's as predictor variables help to underscore the significance of the existing results based exclusively on the U.S. data. The similarities in the patterns obtained across countries also suggest that even stronger results may be available by pooling the regressions and entertaining the notion of a common "global" variance risk premium. We explore these ideas next.

### 3.3 Global Variance Risk Premium and Panel Regressions

Our definition of a "global" variance risk premium is based on a simple capitalization weighted average of the country specific variance risk premia,

$$VRP_t^{global} \equiv \sum_{i=1}^{6} w_t^i VRP_t^i ,$$

where $i = 1, 2, ..., 6$ refer to each of the six countries included in our analysis.[16] The end-of-month market capitalizations used in defining the weights $w_t^i$ are obtained from Thomson Reuters Institutional Brokers' Estimate System (I/B/E/S) via Datastream. The plot of the weights in Figure 8 shows that the U.S. market accounts for more than sixty percent through most of the sample period, with Japan a distant second. This relatively large weight assigned to the U.S. market in our definition of the "global" VRP index is also evident from the aforementioned summary statistics in Table 3.

The results for the regressions obtained by replacing the country specific $VRP_t^i$'s in equation (2) with the new $VRP_t^{global}$ index,

$$h^{-1} r_{t,t+h}^i = a^i(h) + b^i(h) VRP_t^{global} + u_{t,t+h}^i , \tag{3}$$

are reported in Table 5, along with the corresponding $t^{NW}$-statistics. Comparing the results to the ones for the country specific regressions in Table 4, reveals even stronger commonalities and uniform patterns across countries. The "global" VRP index serves as a highly significant predictor variable for all of the different country indexes, with $t^{NW}$-statistics in excess of 5.0 at the 4 months horizon. Meanwhile, increasing $h$, the $VRP_t^{global}$ predictor variable always

[16]This parallels the idea used in Harvey (1991) in the estimation of the world price of covariance risk.

becomes insignificant over the longer 9 and 12 months return horizons reported in the last two columns of the table.

These striking cross country similarities are also immediately evident from the plots of the estimated regression coefficients and the two NW-based standard error bands in Figure 9. Not only do the individual country estimates for the $b^i(h)$'s look very similar, the confidence bands also become much tighter compared to the country specific regressions discussed above. Further along these lines, Figure 10 shows the general patterns in the predictability, as measured by the $R^2(h)$'s, to be very similarly shaped for the different countries, with uniform peaks at the 4 months return horizon.[17]

Going one step further, we next restrict the coefficients for the "global" variance risk premium to be the same across countries,

$$h^{-1} r^i_{t,t+h} \;=\; a(h) \;+\; b(h) VRP^{global}_t \;+\; u^i_{t,t+h} \;, \tag{4}$$

as a way to further enhance the efficiency of the estimates. The corresponding panel regression estimates for the $b(h)$'s together with the NW-based $t$-statistics are reported in Table 6 (for additional details on the calculations, see, e.g., Petersen, 2009).[18] As the table clearly shows, the use of panel regressions do indeed result in more accurate estimates, and a highly significant $t^{NW}$-statistics of 11.21 at the 4-months horizon. Similarly, the average panel regression $R^2(h)$'s across the six countries gradually rise from around one percent at the one-month horizon to a large 7.46 percent for the four-month returns, tapering off to zero for the longer 9-12 month return horizons.

These key empirical findings are succinctly summarized in Figure 11, which plots the panel regression estimates for $b(h)$ based on the country specific and "global" VRP measures along with two NW standard error bands (top two panels), and the corresponding

[17]The large weight assigned to the U.S. in our construction of the "global" risk premium means that fairly similar results are obtained by replacing the new $VRP^{global}_t$ in the regressions in equation (3) with $VRP^{S\&P500}_t$. These additional results are available upon request. Comparable empirical results based on the U.S. variance risk premium have also recently been reported in concurrent independent work by Londono (2010), who ascribes the predictability to informational frictions along the lines of Rapach, Strauss, and Zhou (2010).

[18]We also experimented with the two-way cluster analysis in Cameron, Gelbach, and Miller (2011), resulting in qualitatively very similar findings.

panel regression $R^2(h)$'s (bottom two panels). The $VRP^{global}$-based regressions (depicted in the right two panels) obviously result in sharper coefficient estimates and stronger average predictability across the six countries, compared to the individual country $VRP^i$ regression (depicted in the left two panels).

The panel regression $R^2(h)$'s, of course, mask important cross-country differences in the degree of predictability. We therefore also show in Figure 12 the country specific implied $R^2(h)$'s obtained by evaluating the regressions in equation (3) at the more precisely estimated common $\hat{a}(h)$ and $\hat{b}(h)$ obtain from the panel regressions in equation (4). Interestingly, comparing Figure 12 to the earlier Figure 10 for the individual country regressions, it is clear that the added precision afforded by restricting the $a^i(h)$ and $b^i(h)$ coefficients to be the same across countries sacrifices very little in terms of the implied predictability.

To assess the robustness of these impressive empirical findings, the next panel in Table 6 reports the results obtained by including a capitalization weighted average of the country specific P/E ratios as an additional regressor. Consistent with the results for the U.S. market in isolation reported in BTZ2009, the "global" P/E ratio adds nothing to the predictability afforded by $VRP^{global}$ within the one-year horizons reported in the table, leaving all of the estimates for $b(h)$ and the $R^2(h)$'s the same to within the second decimal place. The predictability of the "global" variance risk premium is effectively orthogonal to that documented in the existing literature based on more traditional macro-finance variables, such as the P/E ratio, dividend yields, and consumption-wealth ratios, which are typically only significant over longer multi-year return horizons (see, e.g., the classic studies by Fama and French, 1988; Campbell and Shiller, 1988b; Lettau and Ludvigson, 2001).[19]

To further highlight the predictive gains afforded by the use of our "global" VRP as opposed to the own country VRP's, the last two panels in Table 6 report the results obtained by including each individual country's premium in a panel regression,

$$h^{-1} r^i_{t,t+h} \;=\; a(h) \;+\; b(h) VRP^i_t \;+\; u^i_{t,t+h}\;. \tag{5}$$

[19] Further corroborating the results for the U.S. market in BTZ2009, we also found that including the implied "global" variance or the realized "global" variance together with the "global" variance risk premium resulted in mostly insignificant coefficient estimates. These additional results are available upon request.

While the results still point to overall efficiency gains from the panel regression setting relative to the individual country specific regressions in Table 4, the magnitude of the return predictability is obviously much lower than for $VRP^{global}$. The "global" variance risk premium is clearly a much better predictor of the future returns than the individual country specific premia. Also, including the country specific P/E ratios in the same panel regression setting again results in insignificant coefficient estimates, while the $t^{NW}$-statistics for the variance risk premia remain highly significant at the intermediate 2-6 month horizons.

To help better understand the economic mechanisms underlying these new empirical findings, we next present a stylized two-country equilibrium model. This relatively simple model provides a possible rationale for why the estimated "global" VRP regression coefficients are fairly similar across countries, and why the $R^2(h)$'s for the panel regressions depicted in Figure 11 are generally larger for the "global" VRP than for the "local" VRP's, except for the U.S.

## 3.4 Global Variance Risk in Equilibrium

Our two-country model is based on a direct extension of the "long-run risk" model in BTZ2009.[20] Specifically, denoting the geometric growth rate of consumption in country $i$ by $g^i_{t+1} \equiv \log(C^i_{t+1}/C^i_t)$, we will assume that

$$g^i_{t+1} = \mu_g + \sigma_{g_i,t} z_{g_i,t+1}, \tag{6}$$

$$\sigma^2_{g_i,t+1} = \alpha_\sigma \varphi_{q,i} + \nu_\sigma \sigma^2_{g_i,t} + \varphi_{q,i}\sqrt{q_t} z_{\sigma,t+1}, \tag{7}$$

$$q_{t+1} = \alpha_q + \nu_q q_t + \varphi_q \sqrt{q_t} z_{q,t+1}, \tag{8}$$

where $\mu_g$ denotes mean growth rate, assumed to be constant and the same for the two countries, $\sigma^2_{g_i,t}$ refers to the conditional variance of consumption growth for each of the countries, and $q_t$ represents time-varying volatility-of-volatility, or aggregate world-wide economic uncertainty. In parallel to existing "long-run risk" models, we will assume that $z_{\sigma,t+1}$ and $z_{q,t+1}$ are independent *i.i.d.* N(0,1) process, and jointly independent of the two consumption

[20] Even though the model explicitly excludes predictability in consumption growth, following the terminology of Bansal and Yaron (2004), we will refer to the basic setup as a "long-run risk" model.

growth shocks, $z_{g_i,t+1}$. For simplicity, we will fix the dynamic variance parameters $\alpha_\sigma$ and $\nu_\sigma$ to be the same across the two countries.[21] The system is normalized by fixing $\varphi_{q,1}$ at unity. The scaling of the mean and volatility parameters for the second country by $\varphi_{q,2}$, in turn ensures that the two variance processes move proportional to each other. To complete the specification, we assume that the conditional covariance between $z_{g_i,t+1}$ and $z_{g_j,t+1}$ is determined by the process

$$cv_{t+1,ij} = \alpha_{cv} + \nu_{cv} cv_{t,ij} + \varphi_{cv,ij}\sqrt{q_t} z_{\sigma,t+1}. \tag{9}$$

This trivially implies time-varying conditional correlations, unless the parameters are identical across the covariance and two variance processes.[22]

We assume that the two international equity markets are fully integrated. We further assume the existence of a global representative agent with a claim on the world aggregate consumption, defined as the per capita weighted average consumption in each of the two countries, say $C_t^{global}$. Moreover, this agent is endowed with Epstein-Zin-Weil recursive preferences of the form

$$U_t = [(1-\delta)(C_t^{global})^{\frac{1-\gamma}{\theta}} + \delta(E_t[U_{t+1}^{1-\gamma}])^{\frac{1}{\theta}}]^{\frac{\theta}{1-\gamma}}. \tag{10}$$

In the specific calibration reported on below, we follow Bansal and Yaron (2004) and BTZ2009 in fixing the discount rate at $\delta$=0.997, the risk aversion parameter at $\gamma$=10, and the intertemporal elasticity of substitution at $\varphi$=1.5.

The parameters for the consumption dynamics are calibrated to mimic the U.S. as country "1", and the U.K. as country "2". In particular, following BTZ2009 we fix the base parameters for the U.S. at $\mu_g$=0.0015, $\nu_\sigma$=0.979, $\alpha_\sigma$=$0.0078^2(1-\nu_\sigma)$, $\nu_q$=0.80, $\alpha_q$=1.0*10e-6, and $\varphi_q$=0.001, respectively. For simplicity, we treat the weights used in the calculation of "global" consumption as constant and equal to $\omega_{US}$=0.855 and $\omega_{UK}$=0.145, corresponding to the consumption shares at the end of the sample.

[21]Bansal and Shaliastovich (2010) employs a similar assumption for the variance dynamics in their related two-country model. By contrast, the two-country model in Londono (2010) involves separate shocks for the "leader" and "follower" countries, but assumes that all of the parameters driving the $\sigma_{g_i,t}$'s and the country specific $q_{i,t}$'s are the same across the two countries.

[22]The consumption data discussed below strongly supports the notion of time-varying covariances (and correlations). By contrast, the aforementioned two-country model in Bansal and Shaliastovich (2010) postulates constant cross-country conditional covariances.

The relevance of allowing for time-varying cross-country covariances is highlighted by Figure 13, which plots exponentially weighted moving average estimates of U.S. variances, and U.S.-U.K. covariances and correlations from 1951 to 2009.[23] Although the U.S.-U.K. consumption covariances clearly changes through time, the process is not as persistent as the process for the U.S. variance depicted in the top panel. We consequently set $\nu_{cv}$=0.85 and $\varphi_{cv,US,UK} = \varphi_{q,UK}^{1/2}\varphi_{q,US}^{1/2}$. Finally, we set the parameter $\varphi_{q,UK}^{1/2}$=$(2.5)^{1/2} \approx 1.581$ to reflect the generally higher variability of UK consumption growth.

Turning to the actual calibration results, the top two panels in Figure 14 show the implied regression coefficients for the "local" (dashed lines) and "global" (solid lines) VRP regressions for each of the two countries, while the bottom two panels show the implied $R^2$'s from the "local" (dashed lines) and "global" (solid lines) VRP panel regressions.[24] The model-implied regressions in the figure generally match the qualitative features in the actual international return regressions quite well.

First, the implied slope coefficients for $VRP^{global}$ in the two individual country regressions tend to be close across all horizons. For instance, at the four-month horizon, the model implied slope coefficients equal 0.34 and 0.33 for the U.S. and U.K., respectively, both of which are well within two standard errors of their corresponding estimates reported in Table 5. Of course, these numbers are also very close to the estimate of 0.32 for the six-country panel regression in Table 6.

Second, the exposure to the "local" VRP is systematically lower than the exposure to the "global" VRP for the smaller country in the model (U.K.), directly mirroring the empirical results. Conversely, for the larger country (U.S.), the "local" VRP gives rise to marginally higher slope coefficients than the "global" VRP within the model, again directly mirroring the actual empirical results. Specifically, focusing again on the four-month horizon, the slope

[23]The exponential weighted moving averages depicted in the figure are based on annual real total consumption expenditures from the Penn World database, and a smoothing parameter equal to $\lambda$=$1-(1-0.06)^4$. Specifically, for the U.S. variance $\sigma^2_{US,t+1}$=$(1-\lambda)\sigma^2_{US,t} + \lambda(g_t^{US} - \widehat{\mu_{US}})^2$, and the U.S.-U.K. covariance $cv_{t+1}$=$(1-\lambda)cv_t + \lambda(g_t^{US} - \widehat{\mu_{US}})(g_t^{UK} - \widehat{\mu_{UK}})$, with the correlation defined accordingly.

[24]The calibrated model also implies equity premiums for the U.S. and U.K. of 6.85 percent and 6.02 percent, respectively, along with a world-wide risk free rate of 0.96 percent. Additional technical details concerning the solution of the model, together with explicit formulas for the regression coefficients and $R^2$'s depicted in the figure, are relegated to Appendix A.

coefficient implied by the model equals 0.17 for the U.K. compared to 0.15 for the actual U.K. regression. In comparison, the model implied slope coefficient for the U.S. equals 0.40, compared to 0.36 for the actual "local" U.S. regression.

Third, looking at the $R^2$'s from the corresponding panel regressions in the bottom two panels of Figure 14, both of the plots exhibit a hump shaped pattern with an apparent peak at the 2-4 month horizons. This overall shape closely matches that for the actual six-country panel regressions depicted in the bottom two panels in Figure 11. Of course, the values of the $R^2$'s from the theoretical model are somewhat muted compared to the six-country panel regressions $R^2$'s. Importantly, however, the model implied panel regression $R^2$'s based on $VRP^{global}$ uniformly dominate the "local" VRP panel regression $R^2$'s. Again, these theoretical implications directly mirror the empirical results for the six-country panel regressions in Figure 11. Intuitively, the "global" VRP effectively isolates the aggregate world-wide economic uncertainty that is being priced in both markets, in turn providing better overall predictions for the future returns than the "local" VRP's.[25]

In a sum, while the qualitative implications form our stylized equilibrium model are generally in line with the international predictability patterns documented in the data, some of the quantitative implications from the model fall short in explaining the magnitude of the effects. However, we purposely kept the model relatively simple, involving only two independent volatility shocks. It is certainly possible that by extending the basic model setup to include additional sources of covariance, or correlation, risks, a full-fledged risk-based explanation for the new international evidence may be feasible.

# 4 Conclusion

A number of recent studies have argued that the aggregate U.S. stock market return is predictable over relatively short 2-4 month horizons by the difference between options implied and actual realized variances, or the so-called variance risk premium. We provide extensive

[25]We also experimented with other calibrations and model specifications. In particular, restricting the covariance to be proportional to the U.S. variance $cv_{t,us,uk}=\sqrt{\varphi_{q_{uk}}}\sigma^2_{g_{us}}\rho$, and fixing the implied constant conditional correlation at $\rho$=0.18 as in Bansal and Shaliastovich (2010), result in dramatically lower $R^2$'s (less than 0.03 percent across all return horizons) for $VRP^{global}$.

Monte Carlo simulation evidence that this newly documented predictability is not due to finite sample biases in the statistical inference procedures, and that the hump-shape in the degree of predictability with a maximum at the 2-4 month horizons is entirely consistent with the implications from an empirically realistic bivariate daily time series model for the returns and variance risk premia.

Further corroborating the existing empirical evidence for the U.S. market, we show that the same basic predictive relationship between future returns and current variance risk premia holds true for a set of five other countries, although the magnitude of the predictability and the statistical significance of the own country variance risk premia tend to be somewhat muted relative to those for the U.S. Meanwhile, regressing the individual country returns on a capitalization weighted "global" variance risk premium, results in almost identical shapes in the degree of predictability across horizons and uniformly larger $t$-statistics for all of the countries in the sample. Further restricting the regression coefficients and the compensation for the "global" variance risk to be the same across countries, we find even stronger results and highly significant test statistics, with the degree of predictability maximized at the four month horizon. By contrast, the predictability documented in the existing literature based on more traditional macro-finance variables are generally only significant over longer multi-year return horizons.

These new empirical findings naturally raise the question of why the "global" variance risk premium works so well as a predictor variable, and why the predictability is restricted to within-year horizons. Building on the equilibrium based model in Bollerslev, Tauchen, and Zhou (2009), we argue that the "global" variance risk premium may be seen as a proxy for world-wide aggregate economic uncertainty. We also show why this "global" variance risk premium may serve as a more effective predictor variable for future international equity returns than the own country's individual variance risk premium.

Alternatively, following the analysis in Bekaert, Engstrom, and Xing (2009), the variance risk premium may be interpreted as a measure of aggregate risk aversion in world financial markets, or a summary measure of disagreements in beliefs across international market participants, as discussed in Buraschi, Trojani, and Vedolin (2010). All of these competing explanations are likely at work to some degree, and we leave it for future research to more clearly sort out the extent to which each of these competing explanations best accounts for the strong international return predictability embodied in the "global" variance risk premium documented here.

# A Two-Country Equilibrium Model Solution

Following Epstein and Zin (1989), the logarithm of the world unique intertemporal marginal rate of substitution, $m_{t+1}$=log($M_{t+1}$), must satisfy,

$$m_{t+1} = \theta \log(\delta) - \theta\psi^{-1} g_{t+1} + (\theta - 1) r_{t+1}, \tag{A.1}$$

where $r_{t+1}$ refers to the time $t$ to $t+1$ logarithmic return on the "global" consumption asset, and $g_{t+1}$ denotes the corresponding "global" consumption growth rate.[26] Further, utilizing the standard Campbell and Shiller (1988a) log-linearization technique, the "world" and country specific returns may be expressed as,

$$r_{t+1} = k_0 + k_1 w_{t+1} - w_t + g_{t+1}, \tag{A.2}$$

$$r^i_{t+1} = k_{i,0} + k_{i,1} w^i_{t+1} - w^i_t + g^i_{t+1}, \tag{A.3}$$

where $w_t$ and $w^i_t$ denote the logarithmic price-consumption ratios for the "world" and the two individual countries, respectively.[27] Following the standard approach in the "long-run risk" literature, we proceed by conjecturing solutions to $w_t$ and $w^i_t$ of the form,[28]

$$w_{t+1} = A_0 + \sum A_{\sigma_j} \sigma^2_{g_j,t+1} + A_q q_{t+1} + A_{cv,ij} cv_{t+1,ij}, \tag{A.4}$$

[26] For notational simplicity, here and throughout the Appendix, we omit the "global" superscript on the relevant variables.

[27] For the calibration exercise discussed in the main text we set $k_1 = k_{US,1} = k_{UK,1}$=0.9. The constants $k_0$ and $k_{i,0}$ only enter the expressions for $A_0$ and $A_{i,0}$ below, which are not actually needed for the calculations of the regression coefficients, $R^2$'s, and equity premia.

[28] In the following, unless explicitly noted, all of the summations are over the two countries, running from $j = 1$ to 2.

$$w^i_{t+1} = A_{i,0} + \sum A_{i,\sigma_j}\sigma^2_{g_j,t+1} + A_{i,q}q_{t+1} + A_{i,cv,ij}cv_{t+1,ij}. \tag{A.5}$$

Combining the equations for $r^i_{t+1}$ and $w^i_{t+1}$ above, with equation (6) for $g^i_{t+1}$ in the main text, the equilibrium return for country $i$ may alternatively be expressed as,

$$r^i_{t+1} = c_{i,r} + \sum_{l=1}^{2} A_{r_i,g_l}\sigma^2_{g_l,t} + A_{r_i,q}q_t + A_{r_i,cv,ij}cv_{t,ij} + \sqrt{q_t}k_{i,1}[A_{i,\varphi}z_{\sigma,t+1} + A_{i,q}\varphi_q z_{q,t+1}] + \sigma_{g_i,t}z_{g_i,t+1}, \tag{A.6}$$

where $c_{i,r} = -\log(\delta) + \varphi^{-1}\mu_g$, $A_{r_i,g_j} = A_{i,\sigma_j}(k_{i,1}\nu_\sigma - 1)$, $A_{r_i,q} = A_{i,q}(k_{i,1}\nu_q - 1)$, $A_{r_i,cv,ij} = A_{i,cv,ij}(k_{i,1}\nu_{cv} - 1)$, and $A_{i,\varphi} = \sum A_{i,\sigma_j}\varphi_{q_j} + A_{i,cv,ij}\varphi_{cv,ij}$.

Next, utilizing the standard no-arbitrage condition $E_t(\exp(r_{t+1} + m_{t+1})) = 1$, the parameters for the "world" in equation (A.4) may be solved as,[29]

$$\begin{aligned}
A_0 &= \frac{\log(\delta) + (1-\varphi^{-1})\mu_g + k_0 + k_1[\sum A_{\sigma_j}\alpha_\sigma\varphi_{q,j} + A_q\alpha_q + A_{cv,ij}\alpha_{cv}]}{1-k_1},\\
A_{cv,ij} &= \frac{(\gamma-1)^2\omega_i\omega_j}{\theta(1-k_1\nu_{cv})},\\
A_{\sigma_j} &= \frac{(\gamma-1)^2\omega_j^2}{2\theta(1-k_1\nu_\sigma)},\\
A_q &= \theta^{-1}\varphi_q^{-2}k_1^{-2}((1-k_1\nu_q) - [(1-k_1\nu_q)^2 - \theta^2k_1^4\varphi_q^2(\sum A_{\sigma_j}\varphi_{q_j} + A_{cv,ij}\varphi_{cv,ij})^2]^{1/2}).
\end{aligned}$$

Similarly, the parameters for the individual countries in equation (A.5) may be solved as,[30]

$$\begin{aligned}
A_{i,0} &= \frac{\log(\delta) + (1-\varphi^{-1})\mu_g + k_{i,0} + k_{i,1}[\sum A_{i,\sigma_j}\alpha_\sigma\varphi_{q,j} + A_{i,q}\alpha_q + A_{i,cv,ij}\alpha_{cv,ij}]}{1-k_{i,1}},\\
A_{i,cv,ij} &= A_{cv,ij} + \frac{(2\gamma-1)\omega_i\omega_j - \gamma\omega_j}{(1-k_1\nu_{cv})},\\
A_{i,\sigma_j} &= (1-\theta)A_{\sigma_j}\frac{1-k_1\nu_\sigma}{1-k_{i,1}\nu_\sigma} + \frac{\gamma^2\omega_j^2 + I_{i=j}(-2\gamma\omega_j + 1)}{2(1-k_{i,1}\nu_\sigma)},\\
A_{i,q} &= \frac{k_1}{k_{i,1}}(1-\theta)A_q + \frac{(1-k_{i,1}\nu_q)}{\varphi_q^2k_{i,1}^2}\\
&\quad - \varphi_q^{-2}k_{i,1}^{-2}((1-k_{i,1}\nu_q)^2 - \theta^2k_{i,1}^2\varphi_q^2\{k_1^2(\sum A_{\sigma_j}\varphi_{q_j} + A_{cv,ij}\varphi_{cv,ij})^2 - 2(\theta-1)A_q(1-\frac{k_1}{k_{i,1}})\\
&\quad + \frac{2}{\theta^2}[0.5(\sum A_{i,\sigma_j}\varphi_{q_j} + A_{i,cv,ij}\varphi_{cv,ij})^2k_{i,1}^2 + (0.5-\theta)k_1^2(\sum A_{\sigma_j} + A_{cv,ij}\varphi_{cv,ij})^2\\
&\quad + (\theta-1)k_1k_{i,1}(\sum A_{\sigma_j}\varphi_{q_j} + A_{cv,ij}\varphi_{cv,ij})(\sum A_{i,\sigma_j}\varphi_{q_j} + A_{i,cv,ij}\varphi_{cv,ij}]\})^{1/2}.
\end{aligned}$$

[29]Note, the aforementioned restrictions that $\gamma > 1$ and $\varphi > 1$, readily imply that the impact coefficient associated with the volatility and correlation state variables are negative; i.e. $A_{cv,ij} < 0$, $A_{\sigma_j} < 0$, and $A_q < 0$.

[30]Intuitively, the larger the covariance for the "small" country, the more risky the country. Also, in general, the more volatile the consumption of country $i$, the less risky is country $j$.

Going one step further and building on the derivations in BTZ2009, the two country specific VRP's may be approximated as,

$$VRP_t^i \approx (\theta - 1)k_1 A_{vrp_i,q} q_t, \tag{A.7}$$

where $A_{vrp_i,q} = k_{i,1}^2 A_q \varphi_q^2 (A_{i,\varphi}^2 + A_{i,q}^2 \varphi_q^2) + A_\varphi \varphi_{q,i}$.

Based on these expressions, it is now possible to derive the slope coefficients from regressing country $i$'s return on country $j$'s VRP,

$$\beta_{i,j}(h) = \frac{A_{r_i,q}\frac{1-\nu_q^h}{1-\nu_q}}{h(\theta-1)k_1 A_{vrp_j,q}}, \tag{A.8}$$

as well as the slope coefficient from regressing country $i$'s return on the global VRP,

$$\beta_i(h) = \frac{A_{r_i,q}\frac{1-\nu_q^h}{1-\nu_q}}{h(\theta-1)k_1 A_{vrp,q}}. \tag{A.9}$$

The final expressions for the "global" and "local" panel regressions discussed in the main text may be derived analogously. In particular, it is possible to show that

$$R^2_{global}(h) = \left(\frac{1}{2}\sum \beta_j(h)\right)^2 \frac{2(\theta-1)^2 k_1^2 A_{vrp,q}^2 Var(q_t)}{\sum Var(\sum_{m=1}^h r_{t+m}^j)}, \tag{A.10}$$

and

$$R^2_{local}(h) = \left(\sum Var^2(VRP_t^j)\right)^{-2} \left(\sum \beta_{j,j}(h) Var^2(VRP_t^j)\right)^2 \frac{\sum Var(VRP_t^j)}{\sum Var(\sum_{m=1}^h r_{t+m}^j)}, \tag{A.11}$$

where $Var(VRP_t^j) = (\theta-1)^2 k_1^2 A_{vrp_j,q}^2 Var(q_t)$,

$$Var(\sum_{m=1}^{h} r_{t+m}^i) = hVar(r_{t+1}^i) + 2\sum_{s=1}^{h-1}(h-s)(A_{i,s} + B_{i,s}),$$

$$\begin{aligned} Var(r_{t+1}^i) = {} & A_{r_i,q}^2 Var(q_t) + (A_{r_i,g_i}^2 + A_{r_i,g_j}^2 \frac{\varphi_{q,j}^2}{\varphi_{q,i}^2} + 2A_{r_i,g_i}A_{r_i,g_j}\frac{\varphi_{q,j}}{\varphi_{q,i}})Var(\sigma_{g_i,t}^2) + A_{r_i,cv,ij}^2 Var(cv_{t,ij}) \\ & + 2\sum_{l=1}^{2} A_{r_i,g_l} A_{r_i,cv,ij} Cov(\sigma_{g_l,t}^2, cv_{t,ij}) + k_{i,1}^2[(A_{i,\varphi}^2 + A_{i,q}^2\varphi_q^2)E(q_t)] + E(\sigma_{g_i,t}^2), \end{aligned}$$

$$A_{i,s} = A^2_{r_i,q}\nu^s_q Var(q_t) + \nu^s_\sigma(A^2_{r_i,g_i} + A^2_{r_i,g_j}\frac{\varphi^2_{q,j}}{\varphi^2_{q,i}} + 2A_{r_i,g_i}A_{r_i,g_j}\frac{\varphi_{q,j}}{\varphi_{q,i}})Var(\sigma^2_{g_i,t})$$
$$+ A^2_{r_i,cv,ij}\nu^s_{cv}Var(cv_{t,ij}) + \sum_{l=1}^{2} A_{r_i,g_l}A_{r_i,cv,ij}(\nu^s_{cv} + \nu^s_\sigma)Cov(\sigma^2_{g_l,t}, cv_{t,ij}),$$

$$B_{i,s} = k_{i,1}E(q_t)(\nu^{s-1}_\sigma \sum_{l=1}^{2} A_{r_i,g_l}A_{i,\varphi}\varphi_{q_l} + \nu^{s-1}_q A_{r_i,q}A_{i,q}\varphi^2_q + \nu^{s-1}_{cv}A_{r_i,cv,ij}A_{i,\varphi}\varphi_{cv,ij}),$$

and the model-implied moments entering the above expressions are given by,

$$E(q_t) = \frac{\alpha_q}{1-\nu_q},\ E(\sigma^2_{g_i,t}) = \frac{\alpha_\sigma\varphi_{q,i}}{1-\nu_\sigma},\ E(\sigma^2_{g,t}) = \frac{\alpha_\sigma\sum\omega_i\varphi_{q,i}}{1-\nu_\sigma},\ E(cv_{t,ij}) = \frac{\alpha_{cv}}{1-\nu_{cv}},$$

$$Var(q_t) = \frac{\varphi^2_q E(q_t)}{1-\nu^2_q},\ Var(\sigma^2_{g_i,t}) = \frac{\varphi^2_{q_i}E(q_t)}{1-\nu^2_\sigma},\ Var(cv_{t,ij}) = \frac{\varphi^2_{cv,ij}E(q_t)}{1-\nu^2_{cv}},$$

$$Cov(\sigma^2_{g_i,t}, \sigma^2_{g_j,t}) = \frac{\varphi_{q,i}\varphi_{q,j}E(q_t)}{1-\nu^2_\sigma},\ Cov(\sigma^2_{g_l,t}, cv_{t,ij}) = \frac{\varphi_{q_l}\varphi_{cv,ij}E(q_t)}{1-\nu_\sigma\nu_{cv,ij}}.$$

# References


Ang, Andrew and Geert Bekaert (2007), "Stock Return Predictability: Is it There?" *Review of Financial Studies*, vol. 20, 651–707.

Baker, Malcolm, Ryan Taliaferro, and Jeffrey Wurgler (2006), "Predicting Returns with Managerial Decision Variables: Is There a Small-Sample Bias?" *Journal of Finance*, vol. 61, 1711–1730.

Bakshi, Gurdip, George Panayotov, and Georgios Skoulakis (2011), "The Baltic Dry Index as a Predictor of Global Stock Returns, Commodity Returns, and Global Economic Activity," Working Paper, Smith School of Business, University of Maryland.

Bansal, Ravi and Ivan Shaliastovich (2010), "A Long-Run Risks Explanation of Predictability Puzzles in Bond and Currency Markets," Working Paper, Fuqua School of Business, Duke University, and The Wharton School, University of Pennsylvania.

Bansal, Ravi and Amir Yaron (2004), "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles," *The Journal of Finance*, vol. 59, 1481–1509.

Bekaert, Geert, Eric Engstrom, and Yuhang Xing (2009), "Risk, Uncertainty and Asset Prices," *Journal of Financial Economics*, vol. 91, 59–82.

Bekaert, Geert and Robert J. Hodrick (1992), "Characterizing Predictable Components in Excess Returns on Equity and Foreign Exchange Markets," *Journal of Finance*, vol. 47, 467–509.

Bollerslev, Tim, Natalia Sizova, and George Tauchen (2011), "Volatility in Equilibrium: Asymmetries and Dynamics Dependencies," *Review of Finance*, forthcoming.

Bollerslev, Tim, George Tauchen, and Hao Zhou (2009), "Expected Stock Returns and Variance Risk Premia," *Review of Financial Studies*, vol. 22, 4463–4492.

Bollerslev, Tim and Jeffrey M. Wooldridge (1992), "Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time-Varying Covariances," *Econometric Reviews*, vol. 11, 143–172.

Boudoukh, Jacob, Matthew Richardson, and Robert F. Whitelaw (2008), "The Myth of Long-Horizon Predictability," *Review of Financial Studies*, vol. 21, 1577–1605.

Buraschi, Andrea, Fabio Trojani, and Andrea Vedolin (2010), "Economic Uncertainty, Disagreement, and Credit Markets," Working Paper, London School of Economics.

Cameron, A. Colin, Jonah B. Gelbach, and Douglas L. Miller (2011), "Robust Inference with Multiway Clustering," *Journal of Business and Economic Statistics*, vol. 29, 238–249.

Campbell, John Y. (2001), "Why Long Horizons? A Study of Power Against Persistent Alternatives," *Journal of Empirical Finance*, vol. 8, 459–491.

Campbell, John Y. and Yasushi Hamao (1992), "Predicting Stock Returns in the United States and Japan: A Study of Long-Term Capital Market Integration," *Journal of Finance*, vol. 47, 63–49.

Campbell, John Y., Andrew W. Lo, and A. Craig MacKinlay (1997), *The Econometrics of Financial Markets*, Princeton University Press, Princeton, NJ.

Campbell, John Y. and Robert J. Shiller (1988a), "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors," *Review of Financial Studies*, vol. 1, 195–228.

Campbell, John Y. and Robert J. Shiller (1988b), "Stock Prices, Earnings, and Expected Dividends," *Journal of Finance*, vol. 43, 661–676.

Campbell, John Y. and Motohiro Yogo (2006), "Efficient Tests of Stock Return Predictability," *Journal of Financial Economics*, vol. 81, 27–60.

Drechsler, Itamar and Amir Yaron (2011), "What's Vol Got to Do With It," *Review of Financial Studies*, vol. 24, 1–45.

Engle, Robert F. (2002), "Dynamic Conditional Correlation: A Simple Class of Multivariate GARCH Models," *Journal of Business and Economic Statistics*, vol. 20, 339–350.

Epstein, Larry and Stanley Zin (1989), "Substitution, Risk Aversion and the Intertemporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, vol. 57, 937–969.

Fama, Eugene F. and Kenneth R. French (1988), "Dividend Yields and Expected Stock Returns," *Journal of Financial Economics*, vol. 22, 3–25.

Ferson, Wayne E. and Campbell R. Harvey (1993), "The Risk and Predictability of International Equity Returns," *Review of Financial Studies*, vol. 6, 527–566.

Ferson, Wayne E., Sergei Sarkissian, and Timothy T. Simin (2003), "Spurious Regressions in Financial Economics?" *Journal of Finance*, vol. 58, 1393–1414.

Gabaix, Xavier (2011), "Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance," Working Paper, Stern School of Business, New York University.

Goyal, Amit and Ivo Welch (2008), "A Comprehensive Look at the Empirical Performance of Equity Premium Prediction," *Review of Financial Studies*, vol. 21, 1455–1508.

Harvey, Campbell R. (1991), "The World Price of Covariance Risk," *Journal of Finance*, vol. 46, 111–157.

Hjalmarsson, Erik (2010), "Predicting Global Stock Returns," *Journal of Financial and Quantitative Analysis*, vol. 45, 49–80.

Hodrick, Robert J. (1992), "Dividend Yields and Expected Stock Returns: Alternative Procedures for Inference and Measurement," *Review of Financial Studies*, vol. 5, 357–386.

Kelly, Bryan T. (2011), "Tail Risk and Asset Prices," Working Paper, Booth School of Business, University of Chicago.

Lettau, Martin and Sydney Ludvigson (2001), "Consumption, Aggregate Wealth, and Expected Stock Returns," *Journal of Finance*, vol. 56, 815–849.

Londono, Juan-Miguel (2010), "The Variance Risk Premium Around the World," Working Paper, Department of Finance, Tilburg University, The Netherlands.

Mueller, Philippe, Andrea Vedolin, and Hao Zhou (2011), "Short-Run Bond Risk Premia," Working Paper, London School of Economics, and Federal Reserve Board, Washington D.C.

Newey, Whitney K. and Kenneth D. West (1987), "A Simple Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, vol. 55, 703–708.

Nishina, Kazuhiko, Nabil Maghrebi, and Moo-Sung Kim (2006), "Stock Market Volatility and the Forecasting Accuracy of Implied Volatility Indices," Working Paper, Graduate School of Economics and School of International Public Policy, Osaka University, Japan.

Petersen, Mitchell A. (2009), "Estimating Standard Errors in Finance Panel Data Sets: Comparing Approaches," *Review of Financial Studies*, vol. 22, 435–480.

Rapach, David E., Jack K. Strauss, and Guofu Zhou (2010), "International Stock Return Predictability: What is the Role of United States?" Working Paper, Olin School of Business, Washington University in St. Louis.

Siriopoulos, Costas and Athanasios Fassas (2009), "Implied Volatility Indices - A Review," Working Paper, Department of Business Administration, University of Patras, Greece.

Stambaugh, Robert F. (1999), "Predictive Regressions," *Journal of Financial Economics*, vol. 54, 375–421.

Zhou, Guofu and Yingzi Zhu (2009), "A Long-Run Risks Model with Long- and Short-Run Volatilities: Explaining Predictability and Volatility Risk Premium," Working Paper, Olin School of Business, Washington University.

Zhou, Hao (2010), "Variance Risk Premia, Asset Predictability Puzzles, and Macroeconomic Uncertainty," Working Paper, Federal Reserve Board, Washington D.C.

## Table 1 Simulated Size and $R^2$

The table reports the simulated 95-percentiles in the finite sample distributions of $t^{NW}$ and $t^{HD}$ for testing the hypothesis that $b_s(h) = 0$ based on the return predictability regression in equation (1), along with the adjusted $R^2$ from the regression. The data are generated from the VAR-GARCH-DCC model discussed in the main text, restricting the coefficients in the conditional mean equation for the returns to be equal to zero. The "daily" return regressions are based on 2,954 observations, while the "weekly" and "monthly" regressions involve 598 and 149 observations, respectively. All of the simulations are based on a total of 2,000 replications.

| $h$ | $t^{NW}$ | $t^{HD}$ | $adj.R^2$ | $h$ | $t^{NW}$ | $t^{HD}$ | $adj.R^2$ | $h$ | $t^{NW}$ | $t^{HD}$ | $adj.R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Daily | | | | Weekly | | | | Monthly | | | |
| 20 | 2.445 | 2.182 | 3.036 | 4 | 2.434 | 2.212 | 2.971 | 1 | 2.260 | 2.276 | 3.017 |
| 40 | 2.602 | 2.112 | 4.804 | 8 | 2.600 | 2.078 | 4.800 | 2 | 2.520 | 2.187 | 4.837 |
| 60 | 2.815 | 2.085 | 5.756 | 12 | 2.805 | 2.016 | 5.790 | 3 | 2.788 | 2.083 | 5.774 |
| 80 | 2.969 | 2.107 | 6.324 | 16 | 2.967 | 2.031 | 6.345 | 4 | 2.941 | 2.106 | 6.315 |
| 120 | 3.054 | 2.152 | 7.639 | 24 | 3.095 | 2.098 | 7.615 | 6 | 3.220 | 2.124 | 7.502 |
| 180 | 3.289 | 2.171 | 8.594 | 36 | 3.322 | 2.113 | 8.482 | 9 | 3.314 | 2.163 | 8.192 |
| 240 | 3.400 | 2.229 | 8.951 | 48 | 3.476 | 2.146 | 8.682 | 12 | 3.509 | 2.186 | 8.679 |

## Table 2 Simulated Power

The table reports the simulated power of the size-adjusted 5-percent $t^{NW}$ and $t^{HD}$ statistics for testing the null hypothesis of no predictability and $b_s(h) = 0$ in the return regression in equation (1). The data are generated from the VAR-GARCH-DCC model discussed in the main text. The "daily" return regressions are based on 2,954 observations, while the "weekly" and "monthly" regressions involve 598 and 149 observations, respectively. All of the simulations are based on a total of 2,000 replications.

| $h$ | $pw^{NW}$ | $pw^{HD}$ | $h$ | $pw^{NW}$ | $pw^{HD}$ | $h$ | $pw^{NW}$ | $pw^{HD}$ |
|---|---|---|---|---|---|---|---|---|
| Daily | | | Weekly | | | Monthly | | |
| 20 | 0.919 | 0.873 | 4 | 0.910 | 0.852 | 1 | 0.886 | 0.807 |
| 40 | 0.888 | 0.808 | 8 | 0.877 | 0.799 | 2 | 0.845 | 0.762 |
| 60 | 0.804 | 0.725 | 12 | 0.793 | 0.729 | 3 | 0.768 | 0.711 |
| 80 | 0.710 | 0.630 | 16 | 0.709 | 0.639 | 4 | 0.686 | 0.627 |
| 120 | 0.567 | 0.474 | 24 | 0.553 | 0.481 | 6 | 0.507 | 0.497 |
| 180 | 0.375 | 0.352 | 36 | 0.371 | 0.352 | 9 | 0.368 | 0.350 |
| 240 | 0.288 | 0.283 | 48 | 0.285 | 0.289 | 12 | 0.277 | 0.302 |

## Table 3 Summary Statistics

The monthly excess returns are in annualized percentage form. The variance risk premia are in monthly percentage-squared form. The “global” index of variance risk premium are defined in the main text. The sample period extends from January 2000 to December 2010.

### Panel A: Excess Returns and Variance Risk Premia

| | CAC 40 | | DAX 30 | | FTSE 100 | | Nikkei 225 | | SMI | | S&P 500 | | Global Index |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $r_t - r_{f,t}$ | $\text{VRP}_t$ | $r_t - r_{f,t}$ | $\text{VRP}_t$ | $r_t - r_{f,t}$ | $\text{VRP}_t$ | $r_t - r_{f,t}$ | $\text{VRP}_t$ | $r_t - r_{f,t}$ | $\text{VRP}_t$ | $r_t - r_{f,t}$ | $\text{VRP}_t$ | $\text{VRP}_t$ |
| Mean | -6.52 | 4.13 | -2.76 | 6.01 | -4.82 | 8.16 | -6.12 | 13.26 | -2.15 | 5.36 | -3.70 | 7.69 | 8.15 |
| Std. Dev | 67.72 | 41.45 | 82.53 | 32.74 | 52.87 | 32.63 | 73.08 | 35.26 | 52.04 | 29.03 | 57.82 | 34.08 | 32.14 |
| Skewness | -0.62 | -4.97 | -0.85 | -2.90 | -0.68 | -5.35 | -0.79 | -5.23 | -0.70 | -4.21 | -0.67 | -5.06 | -5.62 |
| Kurtosis | 3.74 | 43.09 | 5.45 | 16.67 | 3.56 | 47.38 | 4.67 | 52.11 | 3.57 | 31.83 | 3.93 | 39.41 | 49.01 |
| AR(1) | 0.11 | 0.27 | 0.08 | 0.07 | 0.07 | 0.34 | 0.15 | 0.18 | 0.27 | 0.12 | 0.16 | 0.50 | 0.46 |

### Panel B: Correlations for Excess Returns

| | CAC 40 | DAX | FTSE 100 | Nikkei 225 | SMI | S&P 500 |
|---|---|---|---|---|---|---|
| CAC 40 | 1.00 | 0.94 | 0.89 | 0.59 | 0.84 | 0.85 |
| DAX | | 1.00 | 0.83 | 0.56 | 0.81 | 0.82 |
| FTSE 100 | | | 1.00 | 0.62 | 0.80 | 0.87 |
| Nikkei 225 | | | | 1.00 | 0.56 | 0.64 |
| SMI | | | | | 1.00 | 0.77 |
| S&P 500 | | | | | | 1.00 |

### Panel C: Correlations for Variance Risk Premia

| | CAC 40 | DAX | FTSE 100 | Nikkei 225 | SMI | S&P 500 | Global |
|---|---|---|---|---|---|---|---|
| CAC 40 | 1.00 | 0.83 | 0.89 | 0.72 | 0.83 | 0.85 | 0.90 |
| DAX | | 1.00 | 0.78 | 0.59 | 0.88 | 0.71 | 0.77 |
| FTSE 100 | | | 1.00 | 0.78 | 0.86 | 0.90 | 0.94 |
| Nikkei 225 | | | | 1.00 | 0.69 | 0.72 | 0.81 |
| SMI | | | | | 1.00 | 0.74 | 0.81 |
| S&P 500 | | | | | | 1.00 | 0.99 |
| Global | | | | | | | 1.00 |

## Table 4 Country Specific Regressions

The results are based on the monthly regression in equation (2). $t^{NW}$-statistics are reported in parentheses. The sample period extends from January 2000 to December 2010.

| Index | Monthly Horizon | 1 | 2 | 3 | 4 | 5 | 6 | 9 | 12 |
|---|---|---|---|---|---|---|---|---|---|
| CAC 40 | Constant | -7.41 | -8.07 | -8.13 | -8.30 | -8.35 | -8.30 | -8.52 | -8.39 |
| | | (-1.04) | (-1.16) | (-1.18) | (-1.20) | (-1.19) | (-1.18) | (-1.14) | (-1.10) |
| | $\text{VRP}_t^i$ | 0.22 | 0.22 | 0.22 | 0.21 | 0.16 | 0.11 | 0.06 | 0.04 |
| | | (2.71) | (3.59) | (3.30) | (3.79) | (3.86) | (3.01) | (1.37) | (0.88) |
| | Adj. $R^2$ | 0.99 | 2.61 | 3.89 | 4.88 | 3.19 | 1.36 | -0.03 | -0.32 |
| DAX 30 | Constant | -2.72 | -4.58 | -4.78 | -5.28 | -5.29 | -4.76 | -4.63 | -4.80 |
| | | (-0.30) | (-0.54) | (-0.58) | (-0.66) | (-0.66) | (-0.58) | (-0.54) | (-0.56) |
| | $\text{VRP}_t^i$ | -0.01 | 0.19 | 0.19 | 0.24 | 0.22 | 0.13 | 0.07 | 0.10 |
| | | (-0.06) | (1.10) | (1.51) | (2.04) | (2.58) | (1.55) | (1.44) | (2.25) |
| | Adj. $R^2$ | -0.77 | 0.34 | 0.76 | 2.40 | 2.42 | 0.39 | -0.30 | 0.52 |
| FTSE 100 | Constant | -4.67 | -5.50 | -6.28 | -6.56 | -6.51 | -6.38 | -6.08 | -5.86 |
| | | (-0.82) | (-0.99) | (-1.19) | (-1.25) | (-1.23) | (-1.20) | (-1.12) | (-1.09) |
| | $\text{VRP}_t^i$ | -0.02 | 0.05 | 0.13 | 0.15 | 0.13 | 0.10 | 0.02 | -0.01 |
| | | (-0.20) | (0.91) | (3.02) | (3.86) | (4.14) | (2.22) | (0.59) | (-0.29) |
| | Adj. $R^2$ | -0.76 | -0.61 | 0.93 | 2.55 | 1.84 | 0.94 | -0.72 | -0.82 |
| Nikkei 225 | Constant | -6.00 | -6.75 | -8.63 | -8.79 | -7.97 | -7.57 | -6.10 | -5.67 |
| | | (-0.80) | (-0.90) | (-1.15) | (-1.20) | (-1.10) | (-1.04) | (-0.79) | (-0.73) |
| | $\text{VRP}_t^i$ | -0.01 | 0.03 | 0.14 | 0.16 | 0.10 | 0.08 | 0.00 | 0.01 |
| | | (-0.09) | (0.27) | (1.46) | (1.80) | (1.22) | (0.91) | (0.04) | (0.36) |
| | Adj. $R^2$ | -0.77 | -0.75 | 0.29 | 0.95 | 0.11 | -0.19 | -0.83 | -0.81 |
| SMI | Constant | -2.39 | -3.07 | -3.24 | -3.81 | -3.91 | -3.87 | -3.77 | -3.76 |
| | | (-0.36) | (-0.47) | (-0.51) | (-0.62) | (-0.64) | (-0.63) | (-0.59) | (-0.57) |
| | $\text{VRP}_t^i$ | 0.04 | 0.15 | 0.15 | 0.24 | 0.22 | 0.18 | 0.10 | 0.09 |
| | | (0.37) | (1.76) | (1.49) | (2.37) | (3.24) | (2.88) | (2.43) | (3.42) |
| | Adj. $R^2$ | -0.71 | 0.40 | 0.73 | 3.55 | 3.53 | 2.59 | 0.71 | 0.78 |
| S&P 500 | Constant | -6.93 | -6.88 | -7.16 | -7.09 | -6.65 | -6.08 | -5.31 | -5.02 |
| | | (-1.30) | (-1.29) | (-1.38) | (-1.34) | (-1.23) | (-1.11) | (-0.95) | (-0.94) |
| | $\text{VRP}_t^i$ | 0.42 | 0.40 | 0.39 | 0.36 | 0.28 | 0.18 | 0.04 | 0.00 |
| | | (5.11) | (5.29) | (8.43) | (8.80) | (6.52) | (3.83) | (0.90) | (0.13) |
| | Adj. $R^2$ | 5.40 | 8.72 | 13.13 | 14.18 | 9.40 | 4.06 | -0.54 | -0.84 |

## Table 5 "Global" Variance Risk Premium Regressions

The results are based on the monthly regression in equation (3). $t^{NW}$-statistics are reported in parentheses. The sample period extends from January 2000 to December 2010.

| Index | Monthly Horizon | 1 | 2 | 3 | 4 | 5 | 6 | 9 | 12 |
|---|---|---|---|---|---|---|---|---|---|
| | Constant | -8.51 | -10.00 | -9.96 | -10.09 | -9.79 | -9.16 | -8.58 | -8.28 |
| | | (-1.15) | (-1.39) | (-1.43) | (-1.44) | (-1.39) | (-1.29) | (-1.15) | (-1.09) |
| CAC 40 | $\text{VRP}_t^{global}$ | 0.24 | 0.35 | 0.33 | 0.33 | 0.26 | 0.16 | 0.04 | 0.01 |
| | | (2.44) | (4.52) | (6.21) | (8.21) | (6.95) | (3.58) | (0.96) | (0.22) |
| | Adj. $R^2$ | 0.58 | 4.22 | 5.99 | 7.43 | 5.27 | 1.87 | -0.59 | -0.83 |
| | Constant | -4.72 | -6.44 | -6.66 | -6.96 | -6.55 | -5.55 | -4.56 | -4.39 |
| | | (-0.54) | (-0.75) | (-0.81) | (-0.85) | (-0.79) | (-0.67) | (-0.53) | (-0.51) |
| DAX 30 | $\text{VRP}_t^{global}$ | 0.24 | 0.37 | 0.38 | 0.39 | 0.33 | 0.20 | 0.05 | 0.02 |
| | | (2.43) | (2.61) | (6.26) | (7.90) | (4.07) | (2.32) | (0.72) | (0.31) |
| | Adj. $R^2$ | 0.10 | 3.20 | 5.10 | 7.21 | 5.85 | 1.91 | -0.63 | -0.82 |
| | Constant | -5.87 | -6.52 | -7.00 | -7.03 | -6.89 | -6.58 | -6.08 | -5.83 |
| | | (-1.09) | (-1.23) | (-1.38) | (-1.38) | (-1.33) | (-1.26) | (-1.13) | (-1.09) |
| FTSE 100 | $\text{VRP}_t^{global}$ | 0.13 | 0.18 | 0.22 | 0.22 | 0.18 | 0.13 | 0.02 | -0.01 |
| | | (1.78) | (1.85) | (3.44) | (5.07) | (3.79) | (2.21) | (0.56) | (-0.38) |
| | Adj. $R^2$ | -0.16 | 1.40 | 4.23 | 5.55 | 4.30 | 2.02 | -0.70 | -0.78 |
| | Constant | -7.31 | -7.70 | -8.72 | -8.66 | -8.08 | -7.49 | -6.00 | -5.45 |
| | | (-0.94) | (-1.03) | (-1.19) | (-1.20) | (-1.12) | (-1.04) | (-0.79) | (-0.71) |
| Nikkei 225 | $\text{VRP}_t^{global}$ | 0.15 | 0.16 | 0.24 | 0.25 | 0.19 | 0.12 | -0.01 | -0.01 |
| | | (1.13) | (2.51) | (4.44) | (5.05) | (3.15) | (1.67) | (-0.17) | (-0.19) |
| | Adj. $R^2$ | -0.36 | 0.11 | 1.97 | 2.81 | 1.77 | 0.47 | -0.81 | -0.84 |
| | Constant | -3.58 | -4.53 | -4.76 | -5.08 | -4.89 | -4.50 | -3.65 | -3.45 |
| | | (-0.55) | (-0.71) | (-0.77) | (-0.82) | (-0.79) | (-0.73) | (-0.57) | (-0.53) |
| SMI | $\text{VRP}_t^{global}$ | 0.17 | 0.28 | 0.29 | 0.32 | 0.27 | 0.20 | 0.05 | 0.02 |
| | | (1.51) | (5.78) | (5.32) | (7.70) | (8.39) | (5.62) | (1.52) | (0.59) |
| | Adj. $R^2$ | 0.40 | 4.10 | 6.10 | 8.92 | 7.54 | 4.47 | -0.34 | -0.76 |
| | Constant | -7.03 | -6.95 | -7.41 | -7.40 | -6.88 | -6.29 | -5.49 | -5.20 |
| | | (-1.32) | (-1.32) | (-1.46) | (-1.43) | (-1.30) | (-1.17) | (-1.00) | (-0.98) |
| S&P 500 | $\text{VRP}_t^{global}$ | 0.41 | 0.38 | 0.40 | 0.38 | 0.29 | 0.20 | 0.06 | 0.03 |
| | | (4.98) | (5.01) | (6.93) | (7.88) | (5.70) | (3.40) | (1.24) | (0.68) |
| | Adj. $R^2$ | 4.43 | 7.05 | 12.04 | 13.69 | 9.04 | 4.31 | -0.16 | -0.62 |

## Table 6 Panel Regressions

The results are based on on the monthly "global" and county-specific panel regressions in equations (4) and (5), respectively. NW-based $t$-statistics are reported in parentheses. The sample period extends from January 2000 to December 2010.

| | "Global" Regressors | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Horizon (mos) | 1 | 2 | 3 | 4 | 5 | 6 | 9 | 12 |
| Constant | -6.17 | -7.02 | -7.42 | -7.54 | -7.18 | -6.60 | -5.73 | -5.44 |
| | (-2.15) | (-2.51) | (-2.74) | (-2.78) | (-2.64) | (-2.42) | (-2.12) | (-2.07) |
| $\text{VRP}_t^{global}$ | 0.22 | 0.29 | 0.31 | 0.31 | 0.25 | 0.17 | 0.04 | 0.01 |
| | (4.66) | (6.60) | (9.70) | (11.21) | (9.43) | (6.04) | (1.41) | (0.41) |
| Adj. $R^2$ | 1.08 | 3.43 | 5.85 | 7.46 | 5.72 | 2.81 | 0.04 | -0.12 |
| Constant | -14.90 | -10.73 | -2.25 | 4.05 | 4.41 | 2.03 | -8.12 | -13.83 |
| | (-0.63) | (-0.47) | (-0.11) | (0.20) | (0.23) | (0.11) | (-0.46) | (-0.83) |
| $\text{VRP}_t^{global}$ | 0.22 | 0.28 | 0.32 | 0.32 | 0.26 | 0.18 | 0.03 | 0.00 |
| | (4.45) | (5.34) | (9.34) | (10.81) | (8.17) | (5.28) | (1.18) | (0.03) |
| $\log(P_t/E_t)^{global}$ | 2.70 | 1.14 | -1.60 | -3.57 | -3.57 | -2.66 | 0.73 | 2.59 |
| | (0.37) | (0.16) | (-0.25) | (-0.59) | (-0.62) | (-0.47) | (0.14) | (0.53) |
| Adj. $R^2$ | 0.99 | 3.32 | 5.75 | 7.51 | 5.80 | 2.81 | -0.08 | -0.06 |
| | Country-Specific Regressors | | | | | | | |
| Horizon (mos) | 1 | 2 | 3 | 4 | 5 | 6 | 9 | 12 |
| Constant | -5.23 | -6.00 | -6.43 | -6.68 | -6.50 | -6.18 | -5.79 | -5.64 |
| | (-1.80) | (-2.12) | (-2.35) | (-2.47) | (-2.39) | (-2.27) | (-2.13) | (-2.12) |
| $\text{VRP}_t^i$ | 0.12 | 0.18 | 0.20 | 0.22 | 0.18 | 0.12 | 0.05 | 0.04 |
| | (1.85) | (3.31) | (4.86) | (5.95) | (5.82) | (4.42) | (2.00) | (1.68) |
| Adj. $R^2$ | 0.27 | 1.40 | 2.82 | 4.30 | 3.27 | 1.73 | 0.22 | 0.19 |
| Constant | 11.57 | 11.54 | 11.83 | 11.92 | 10.79 | 9.20 | 5.48 | 2.19 |
| | (0.96) | (0.96) | (1.00) | (0.99) | (0.89) | (0.75) | (0.45) | (0.19) |
| $\text{VRP}_t^i$ | 0.13 | 0.19 | 0.22 | 0.24 | 0.20 | 0.14 | 0.06 | 0.05 |
| | (2.07) | (3.52) | (5.33) | (6.38) | (6.09) | (4.59) | (2.30) | (1.94) |
| $\log(P_t^i/E_t^i)$ | -5.55 | -5.80 | -6.03 | -6.13 | -5.70 | -5.06 | -3.70 | -2.57 |
| | (-1.46) | (-1.50) | (-1.59) | (-1.59) | (-1.47) | (-1.29) | (-0.96) | (-0.69) |
| Adj. $R^2$ | 0.51 | 1.99 | 3.79 | 5.59 | 4.58 | 2.90 | 1.04 | 0.63 |

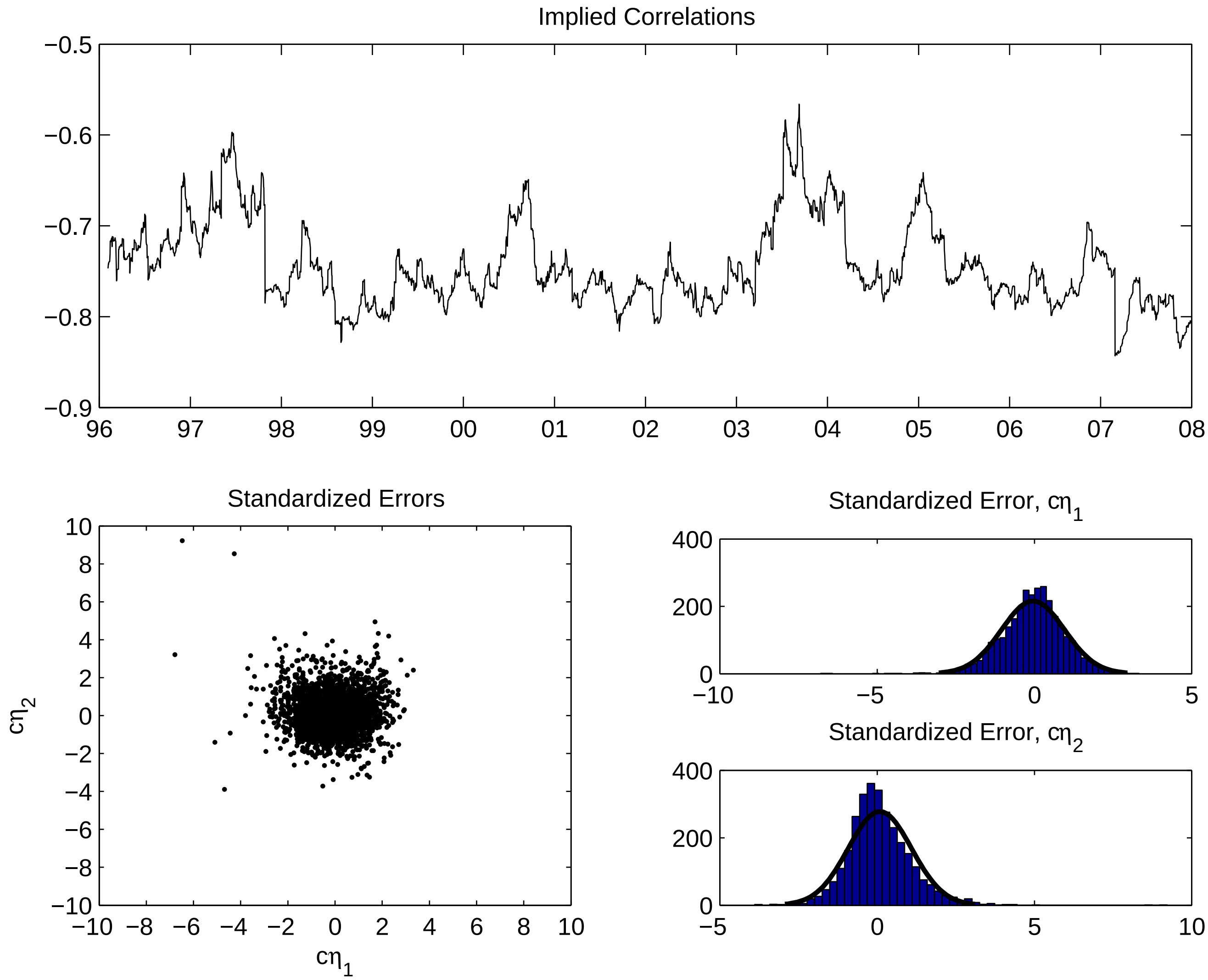


**Figure 1 Estimated VAR-GARCH-DCC Model**

The first panel plots the daily conditional correlations between the returns and the variance risk premium implied by the estimated VAR(1)-GARCH(1,1)-DCC model described in the main text. The lower left and right two panels provide a scatterplot and histograms, respectively, for the standardized residuals from the estimated model, $\widehat{c\eta}_t$. The daily sample used in estimating the model spans the period from February 1, 1996 to December 31, 2007, for a total of 2,954 daily observations.

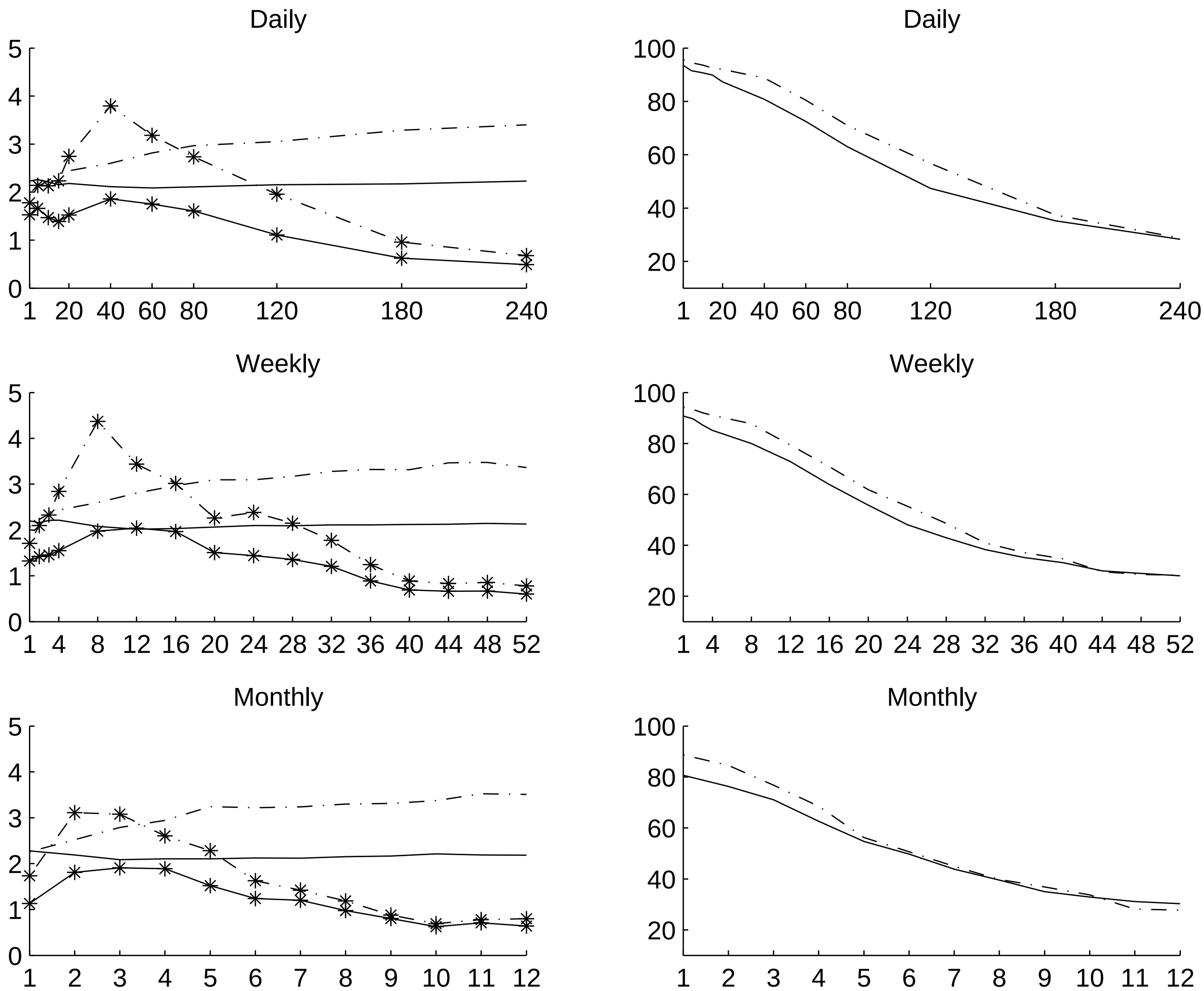


**Figure 2 Simulated Size and Power**

The upper left panel reports the 95-percentiles in the finite-sample distributions of the $t^{NW}$ (dash line) and $t^{HD}$ (solid line) based on simulated "daily" data from the restricted VAR-GARCH-DCC model under the null of no predictability. The dashed and solid star lines refer to the corresponding $t$-statistics for actual daily U.S. S&P 500 returns spanning February 1, 1996 to December 31, 2007. The middle and bottom two left panels give the results for the simulated "weekly" and "monthly" data, together with the results based on the actual weekly and monthly S&P 500 returns. The right three panels give simulated "daily," "weekly" and "monthly" percentage power based on the unrestricted VAR-GARCH-DCC model and the size-adjusted 5-percent $t^{NW}$ (dashed line) and $t^{HD}$ (solid line) statistics.

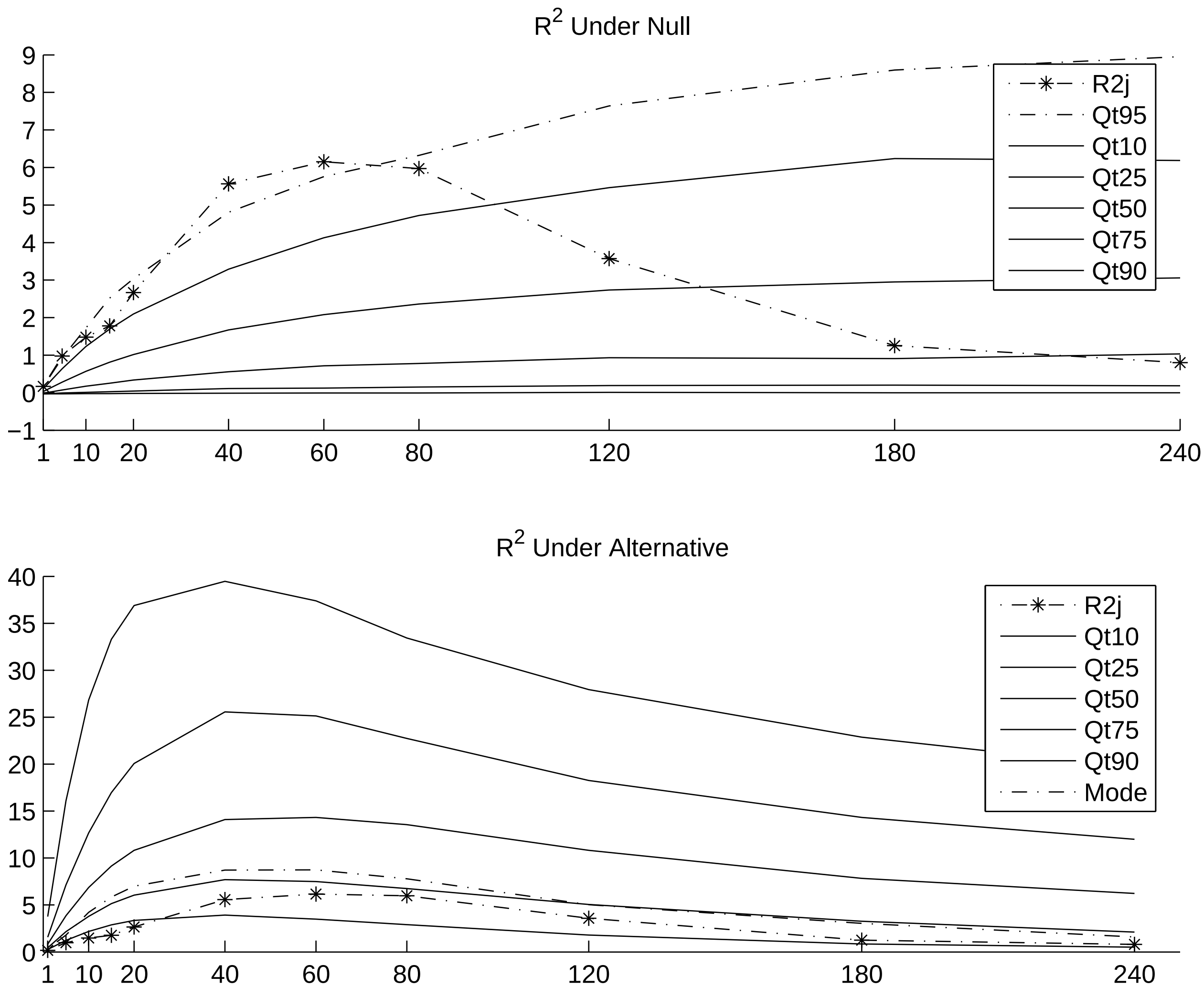


**Figure 3 Simulated $R^2$**

The top panel in the figure plots the quantiles in the finite-sample distribution of the $R^2$ from the return regression in equation (1) and simulated "daily" date from the restricted VAR-GARCH-DCC model under the null of no predictability. The star dashed line refer to the corresponding $R^2$'s in actual daily U.S. S&P 500 returns spanning February 1, 1996 to December 31, 2007. The bottom panel reports the quantiles in the simulated finite-sample distribution based on the unrestricted VAR-GARCH-DCC model.

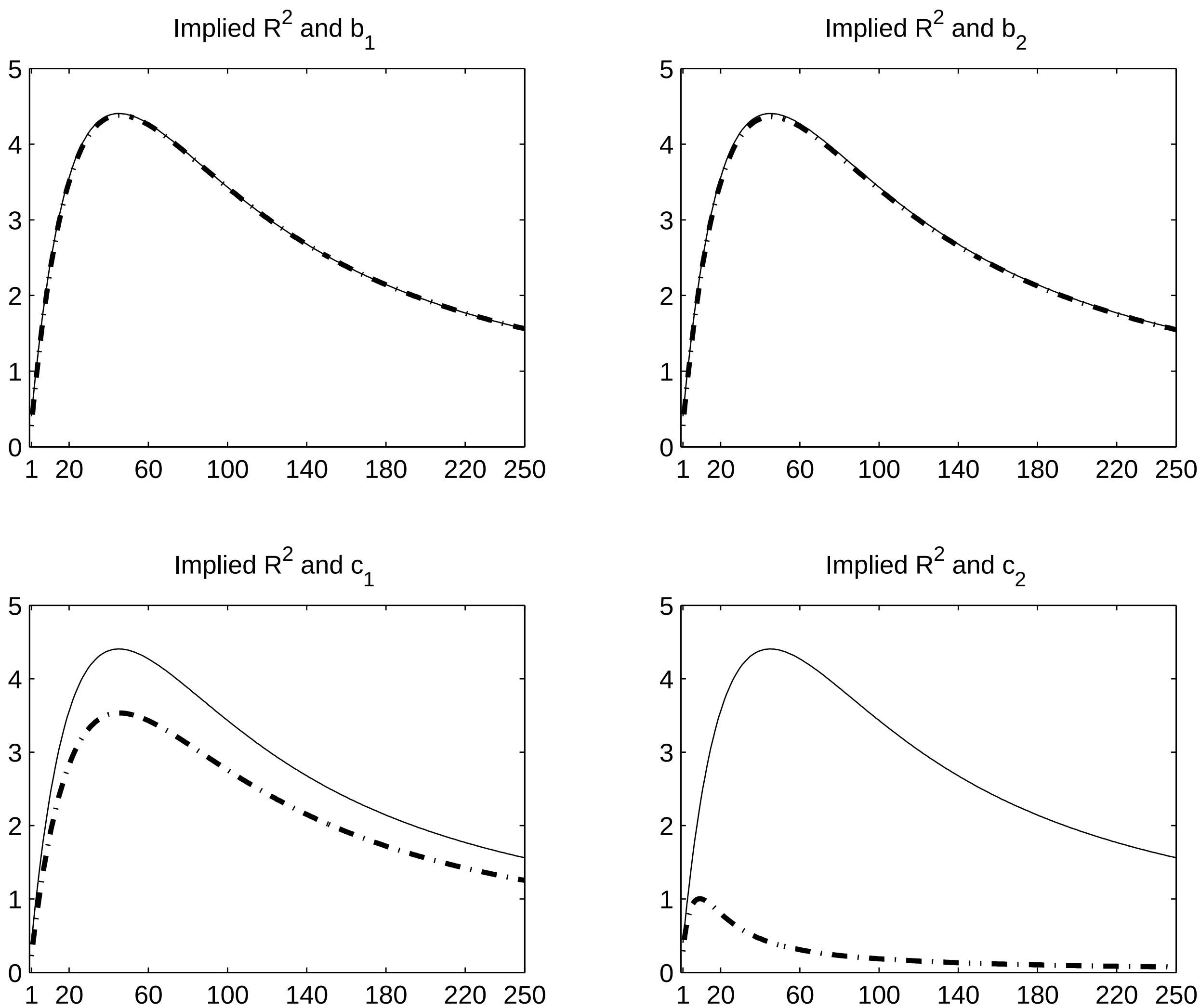


**Figure 4 Implied $R^2$**

The solid lines in each of the four panels show the $R^2(h)$'s implied by the formula in Section 2.3 in the main text and the estimated unrestricted VAR-GARCH-DCC model. The dashed lines in each of the four panels show the implied $R^2(h)$'s for a 10-percent decrease in the values of the $b_1$, $b_2$, $c_1$, and $c_2$ VAR coefficients, respectively.

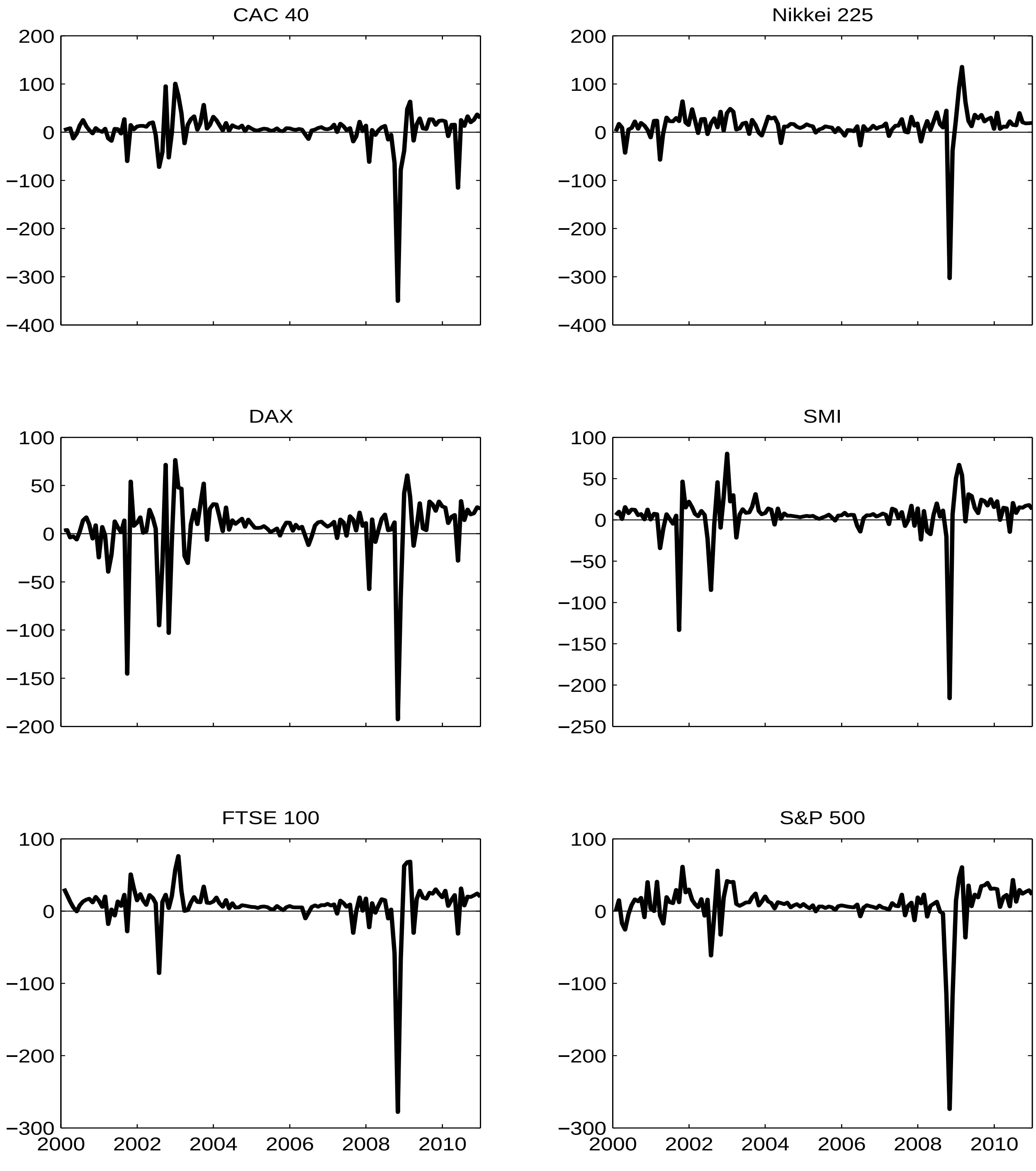


**Figure 5 Variance Risk Premia**

The figure shows the monthly variance risk premia $VRP_t^i$ for France (CAC 40), Japan (Nikkei 225), Germany (DAX 30), Switzerland (SMI 20), the U.K. (FTSE 100), and the U.S. (S&P 500). The risk premia are constructed by subtracting the actual realized variation from the model-free options implied variation. The sample period spans January 2000 to December 2010.

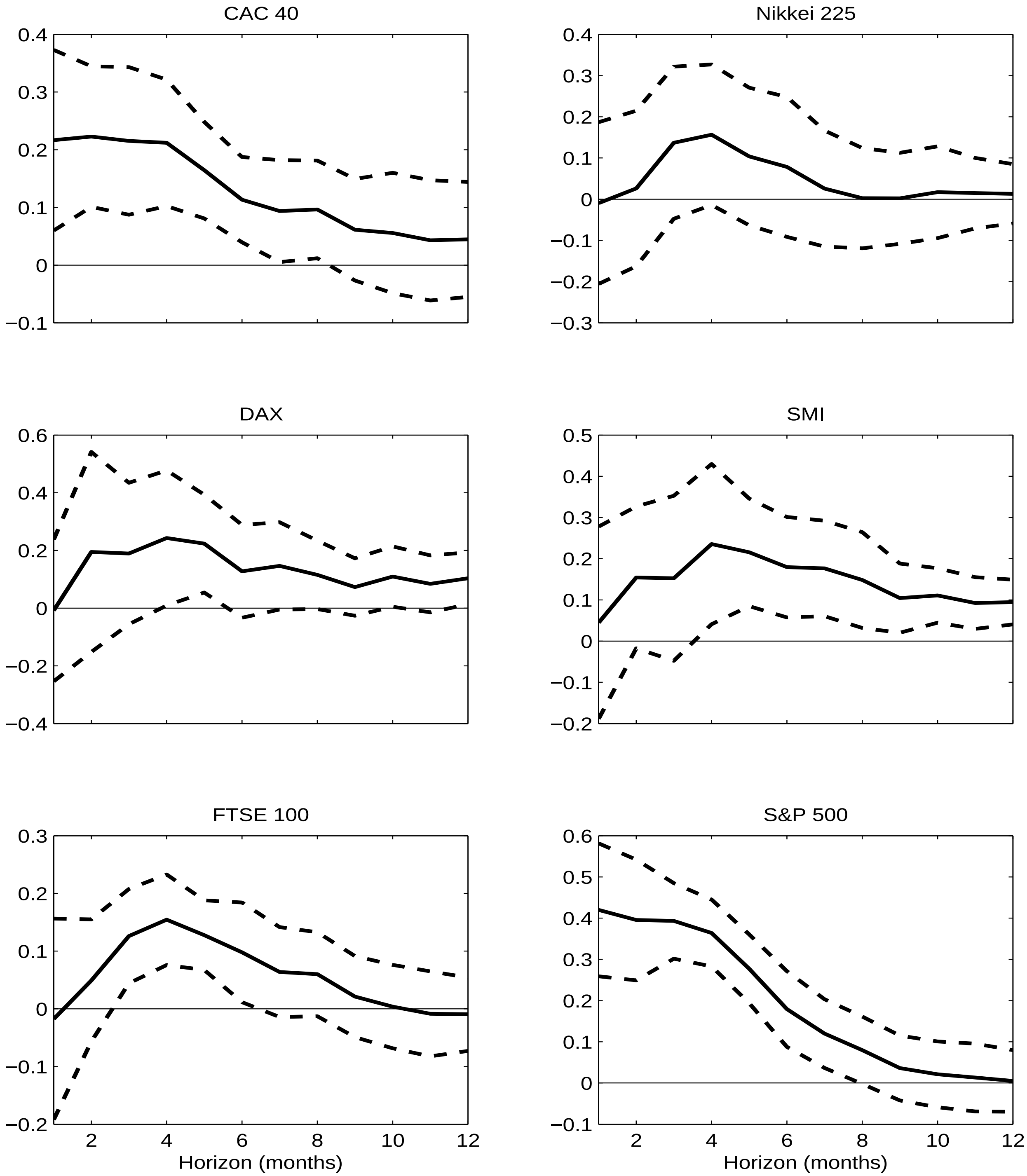


**Figure 6 Country Specific Regression Coefficients**

The figure shows the estimated regression coefficients for $VRP_t^i$ for each of the country specific return regressions reported in Table 4, together with two NW-based standard error bands. The regressions are based on monthly data from January 2000 to December 2010.

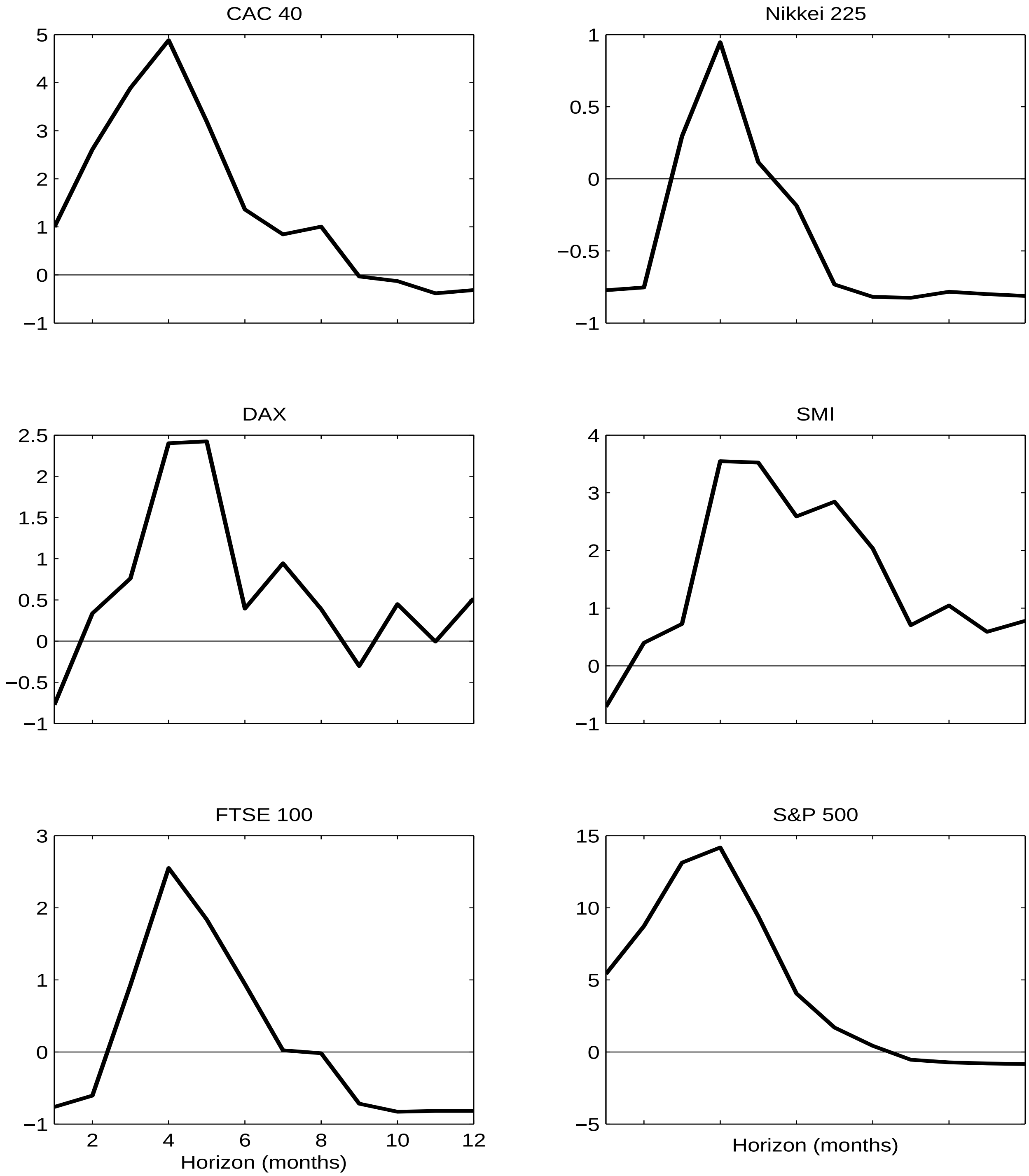


**Figure 7 Country Specific Regression $R^2$'s**

The figure shows the adjusted $R^2(h)$'s for the country specific return regressions reported in Table 4. The regressions are based on monthly data from January 2000 to December 2010.

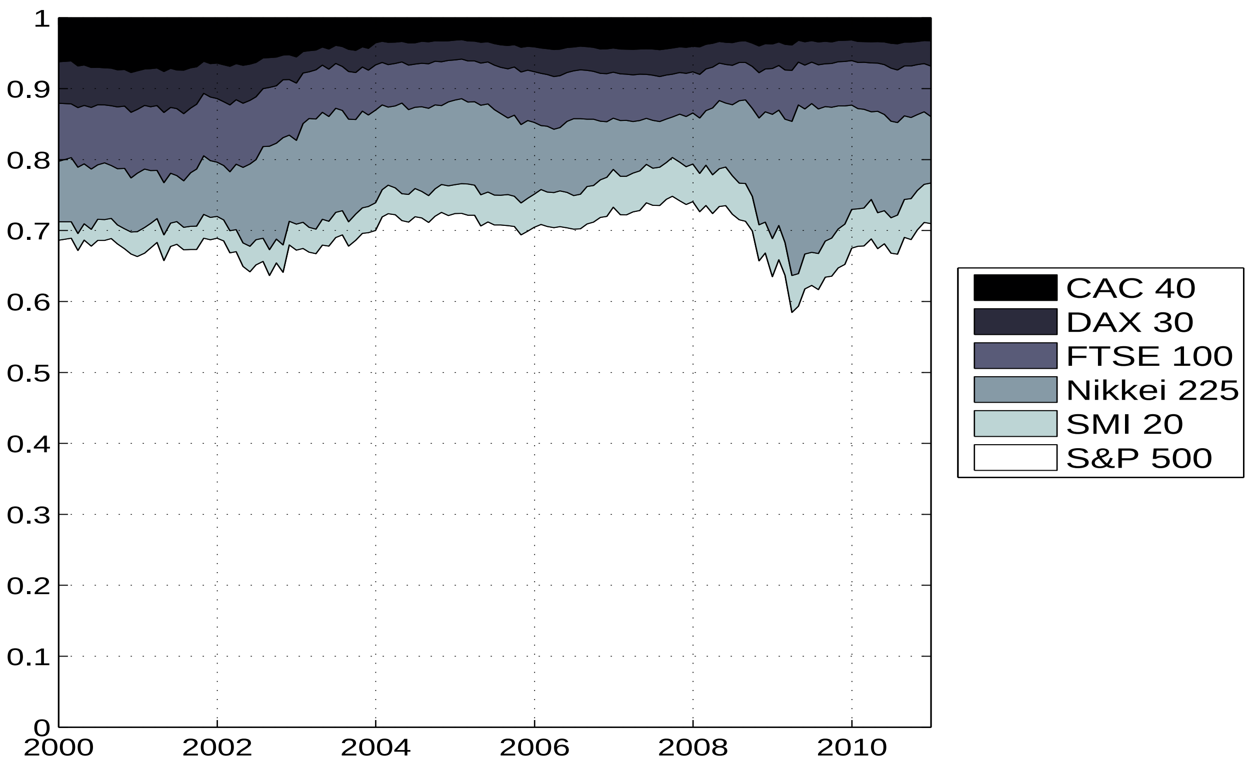


**Figure 8 Market Capitalization**

The figure shows the relative market capitalization by aggregate index for France (CAC 40), Germany (DAX 30), the U.K. (FTSE 100), Japan (Nikkei 225), Switzerland (SMI 20), and the U.S. (S&P 500).

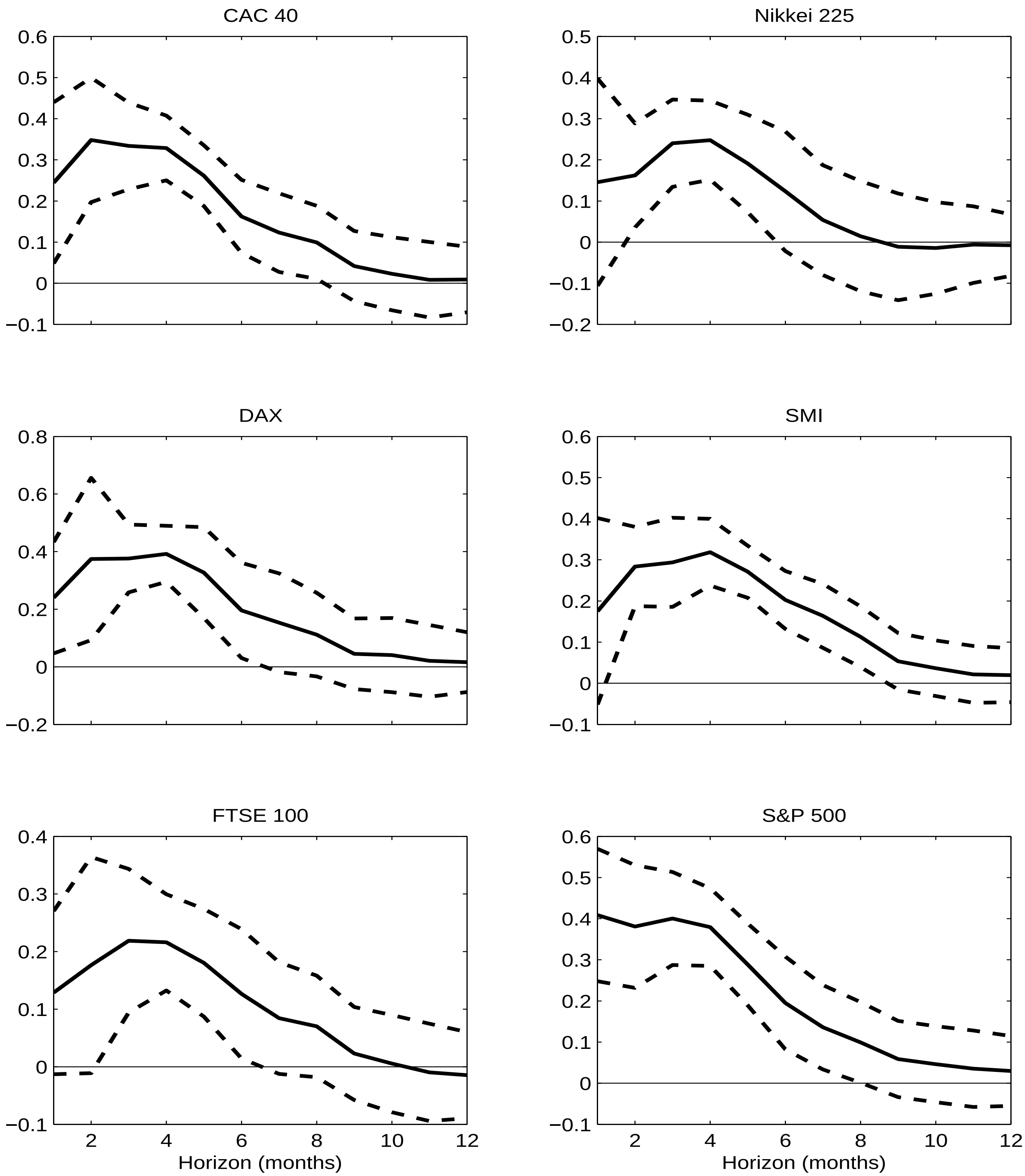


**Figure 9 "Global" VRP Regression Coefficients**

The figure shows the coefficient estimates for $VRP_t^{global}$ from the return regressions reported in Table 5, together with two NW-based standard error bands. The regressions are based on monthly data from January 2000 to December 2010.

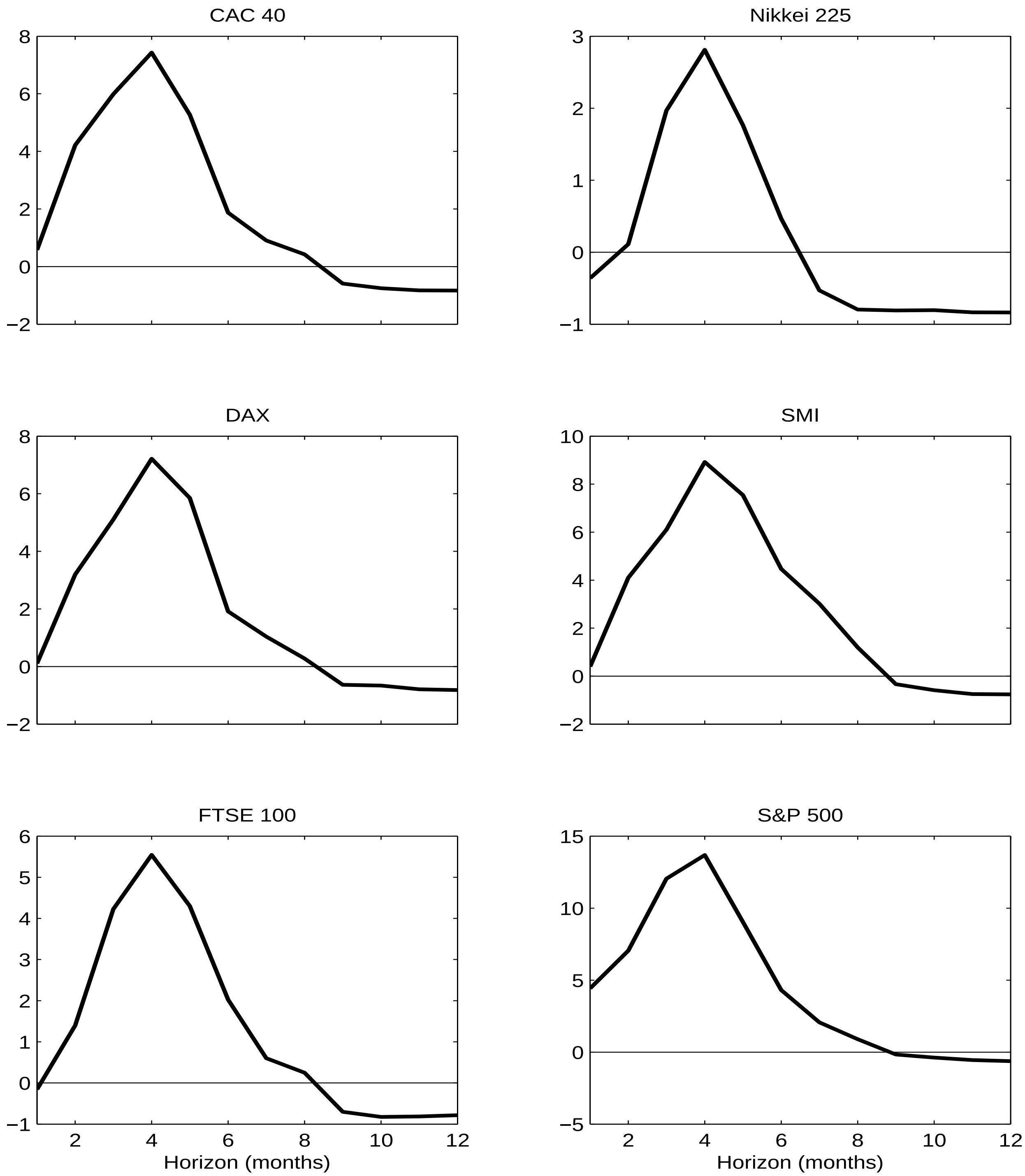


**Figure 10 "Global" VRP Regression $R^2$'s**

The figure shows the adjusted $R^2(h)$'s from regressing the individual country returns on $VRP_t^{global}$ reported in Table 5. The regressions are based on monthly data from January 2000 to December 2010.

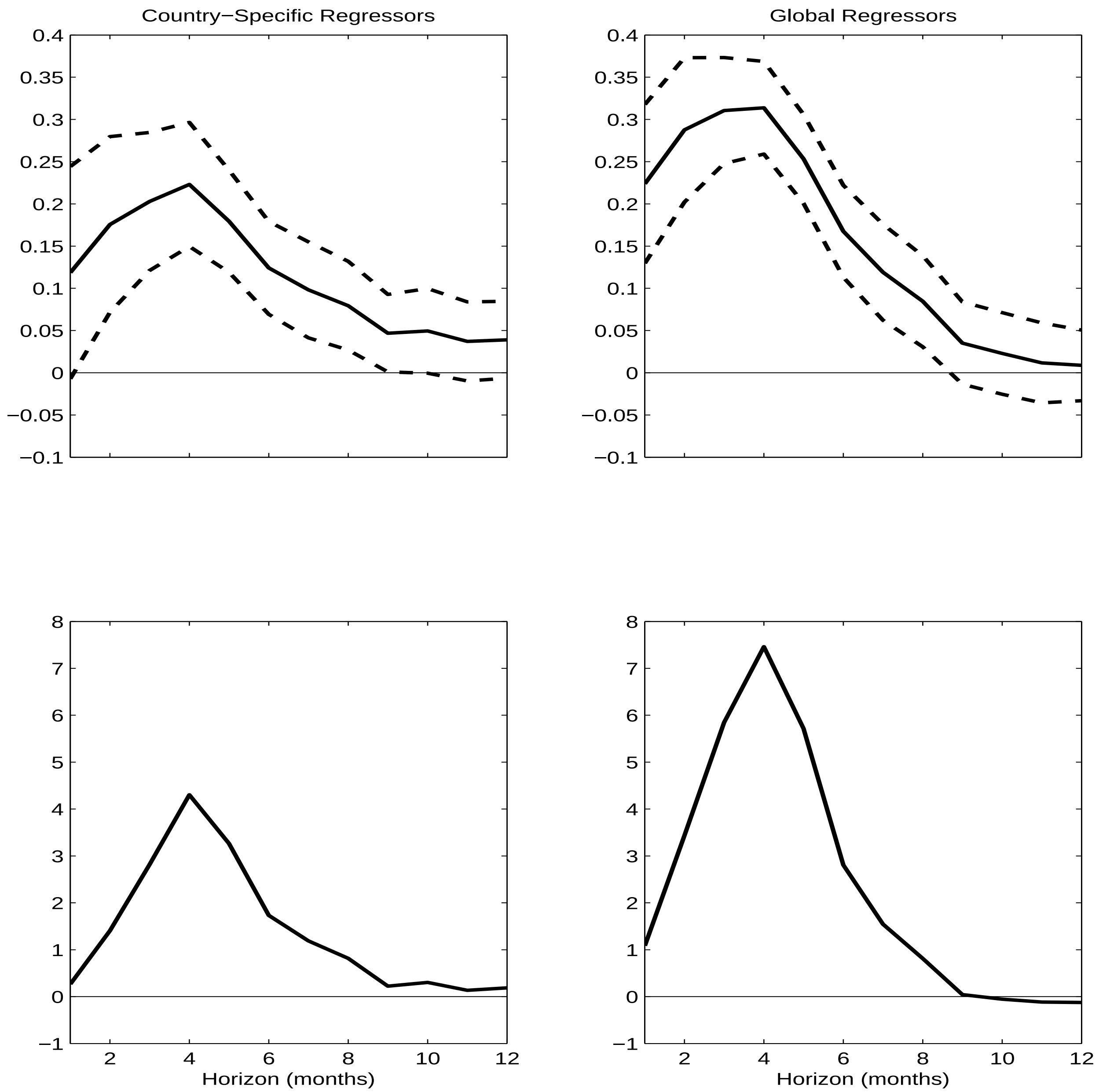


**Figure 11 Panel Regression Coefficients and $R^2$'s**

The top two panels show the estimated panel regression coefficients from regressing the returns on the individual country variance risk premia $VRP_t^i$ and the "global" variance risk premium $VRP_t^{global}$, respectively, reported in Table 6, together with two NW-based standard error bands. The bottom two panels show the $R^2(h)$'s from the same two panel regressions. The regressions are based on monthly data from January 2000 through December 2010.

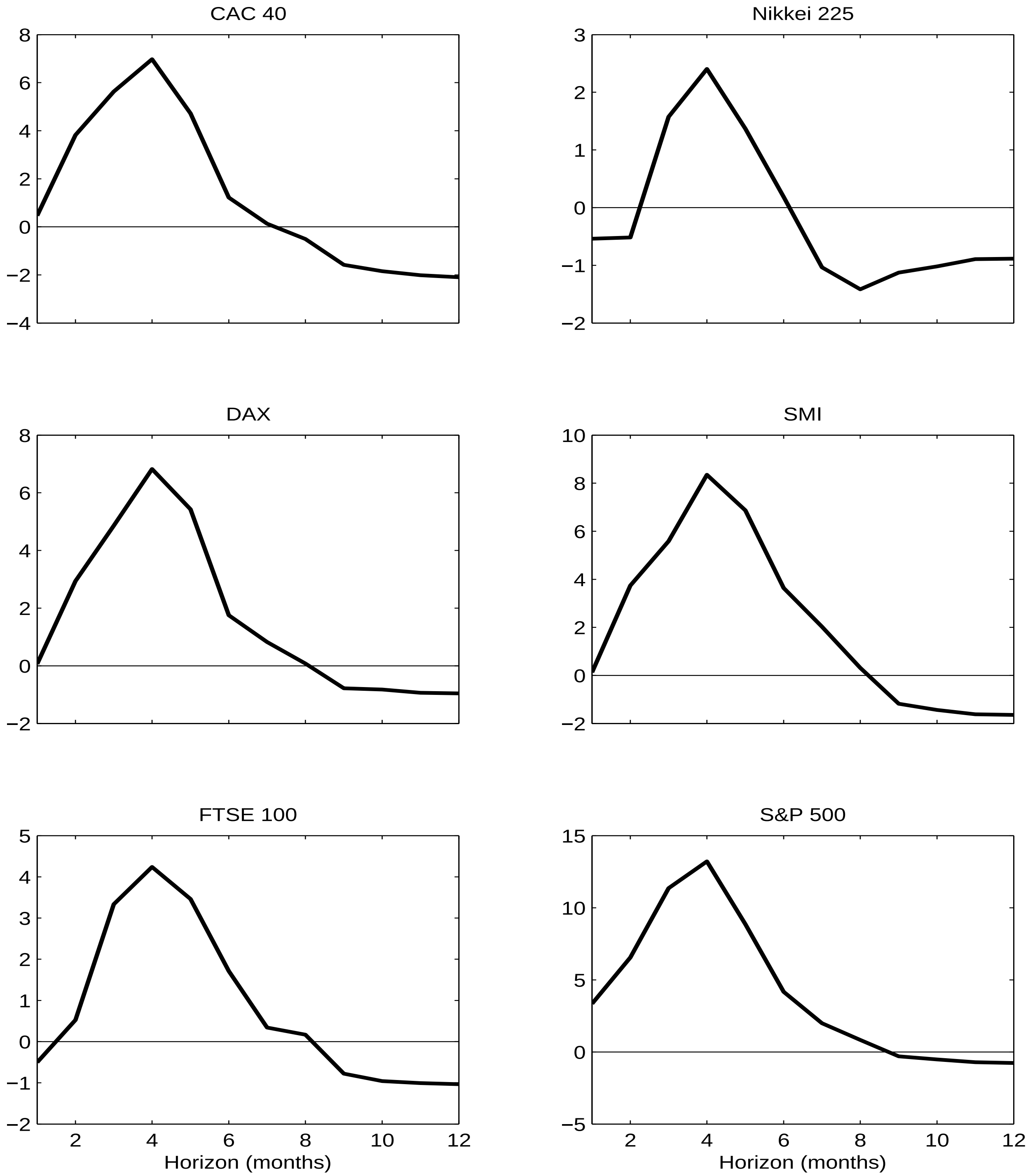


**Figure 12 "Global" VRP Panel Regression $R^2$'s**

The figure shows the adjusted $R^2(h)$'s implied by the $VRP_t^{global}$ panel regressions reported in the top panel in Table 5. The regressions are based on monthly data from January 2000 to December 2010.

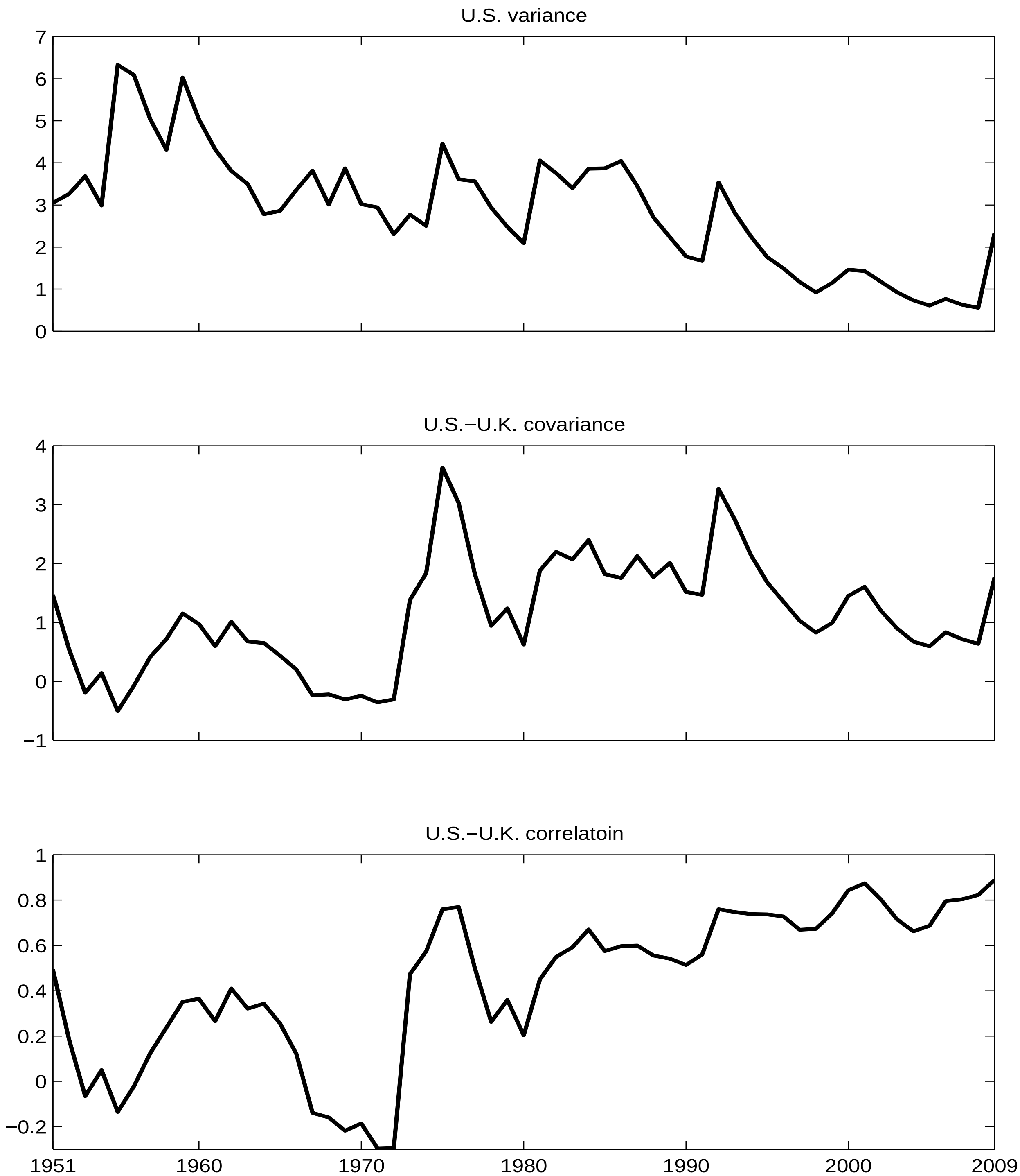


**Figure 13 Consumption Growth Variances, Covariances, and Correlations**
The figure shows exponentially weighted moving average estimates for U.S. consumption growth variances, and covariances and correlations with U.K. consumption growth. The estimates are based on annual total real consumption expenditures from 1951 to 2009, and a exponential smoothing parameter of $\lambda = 1 - (1 - 0.06)^4$. The variances and covariances are both scaled by a factor of $10^4$.

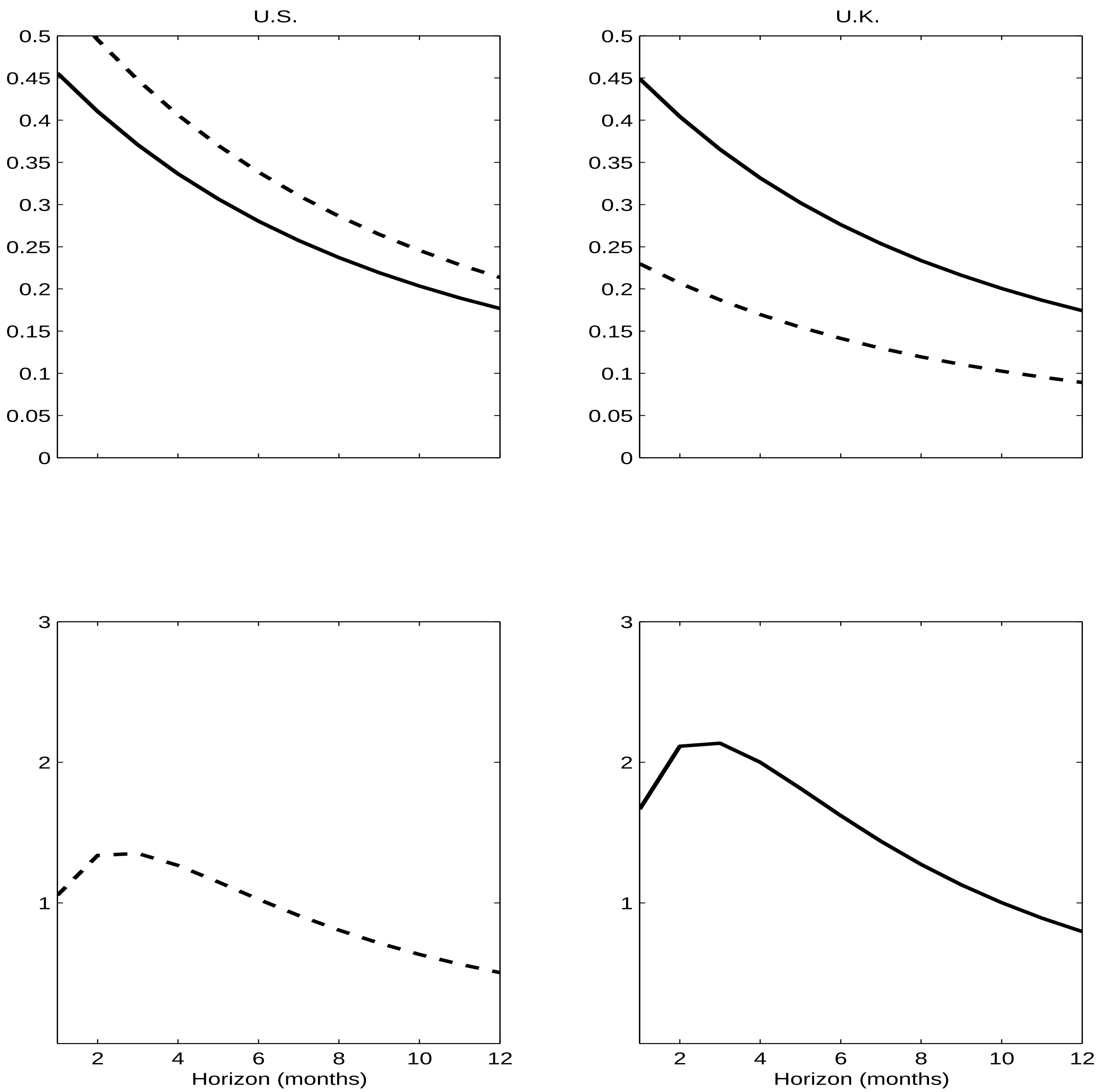


**Figure 14 Equilibrium VRP Regression Coefficients and $R^2$'s**

The figure shows the implications from the calibrated stylized two-country general equilibrium model. The upper two panels plot the slope coefficients in the country specific regressions for each of the two countries based on the "local" VRP's (dashed lines) and "global" VRP (solid lines). The lower two panels show the implied panel regression $R^2$'s based on the "local" (dashed line) and "global" (solid line) VRPs, respectively.