**Finance and Economics Discussion Series**
**Divisions of Research & Statistics and Monetary Affairs**
**Federal Reserve Board, Washington, D.C.**

# The Interplay Between Student Loans and Credit Card Debt: Implications for Default in the Great Recession

**Felicia Ionescu and Marius Ionescu**

**2014-14**

# The Interplay Between Student Loans and Credit Card Debt: Implications for Default in the Great Recession*

Felicia Ionescu
*Federal Reserve Board*

Marius Ionescu
*Colgate University*

February 3, 2014

**Abstract**

We theoretically and quantitatively analyze the interactions between two different forms of unsecured credit and their implications for default behavior of young U.S. households. One type of credit mimics credit cards in the United States and the default option resembles a bankruptcy filing under Chapter 7; the other type of credit mimics student loans in the United States and the default option resembles Chapter 13 of the U.S. Bankruptcy Code. In the credit card market a financial intermediary offers a menu of interest rates based on individual default risk, which account for borrowing and repayment behavior in both markets. In the student loan market, the government sets the interest rate and chooses a wage garnishment to pay for the cost associated with default.

We prove the existence of a steady-state equilibrium and characterize the circumstances under which a household defaults on each of these loans. We demonstrate that the institutional differences between the two markets make borrowers prefer to default on student loans rather than on credit card debt. Our quantitative analysis shows that the increase in student loan debt together with the expansion of the credit card market fully explains the increase in the default rate for student loans in recent *normal* years (2004-2007). Worse labor outcomes for young borrowers during the Great Recession (2008-2009) significantly amplified student loan default, whereas credit card market contraction during this period helped reduce this effect. At the same time, the accumulation of student loan debt did not affect much the default risk in the credit card market during *normal* times, but significantly increased it during the Great Recession. An income contingent repayment plan for student loans completely eliminates the default risk in the credit card market and induces important redistribution effects. This policy is beneficial (in a welfare improving sense) during the Great Recession but not during *normal* times.

JEL Codes: D11; D91; G33; H81; I28;

Keywords: Default, Bankruptcy, Student Loans, Credit Cards, Great Recession

*The views expressed herein are those of the authors and do not necessarily reflect those of the Federal Reserve Board or its staff. The authors thank Simona Hannon, Juan-Carlos Hatchondo, Wenli Li, Leo Martinez, Makoto Nakajima, Michael Palumbo, Pierre-Daniel Sarte, Nicole Simpson, and Rob Valletta for comments and participants at the Society for Economic Dynamics, Midwest Macroeconomics Meeting, Society of Advancements in Economic Theory, the NASM, the NBER, Household Finance group, and seminar participants at Bank of Canada, Colgate University, FRB of Richmond, Federal Reserve Board, University of Alberta. Special thanks to Kartik Athreya, Satyajit Chatterjee, Dirk Krueger, and Victor Rios-Rull for helpful suggestions and to Gage Love for excellent research assistance.
Emails: felicia.ionescu@frb.gov; mionescu@colgate.edu.

# 1 Introduction

Student loan debt has steadily increased in the last two decades, reaching 1.2 trillion dollars in 2012. In June 2010, total student loan debt surpassed total credit card debt for the first time (see Figure 1 in Section 2). Currently, 70 percent of individuals who enroll in college take out student loans; the graduates of 2013 are the most indebted in history, with an average debt load of $27,300 (College Board (2013)). At the same time, the two-year basis cohort default rate (CDR) for Federal student loans steadily declined from 22.4 percent in 1990 to 4.6 percent in 2005 and has increased ever since, reaching record highs in the last decade (at 10 percent for FY2011).[1]

The accumulation of student loan debt alone cannot explain the recent increase in student loan default rates of young U.S. households. A second market is needed to understand this behavior: the majority of individuals with student loan debt (66 percent in 2004-2007) also have credit card debt, according to our findings from the Survey of Consumer Finances (SCF). Credit card usage is common among college students, with approximately 84 percent of the student population having at least one credit card in 2008 (Sallie Mae (2009)). While both of these loans represent important components of young households' portfolios in the United States, the financial arrangements in the two markets are very different, in particular with respect to the roles played by bankruptcy arrangements and default pricing. Furthermore, credit terms on credit card accounts have worsened in recent years, adversely affecting households' capability to diversify risk but also limiting the young borrowers' indebtedness.

We propose a theory about the interactions between student loans and credit card loans in the United States and their impact on default incentives of young U.S. households. As we argue in this paper, this interaction between different bankruptcy arrangements induces significant trade-offs in default incentives in the two markets. Understanding these trade-offs is particularly important in the light of recent trends in borrowing and default behavior. Data show that young U.S. households (of which a large percentage have both college and credit card debt) now have the second highest rate of bankruptcy (just after those aged 35 to 44). Furthermore, the bankruptcy rate among 25- to 34-year-olds increased between 1991 and 2001, indicating that this generation is more likely to file for bankruptcy as young adults than were young boomers at the same age.[2] Moreover, student loans have a higher default rate than credit card loans or any other type of loan, including car loans and home loans.[3]

[1]The 2-year CDR is computed as the percentage of borrowers who enter repayment in a fiscal year and default by the end of the next fiscal year. Trends in the 2-year CDR are presented in Figure 2 in the Appendix.

[2]Source: www.creditcards.com/.

[3]According to a survey conducted by the FRB New York, the national student loan delinquency rate 60+ days in 2010 is 10.4 percent compared to only 5.6 percent for the mortgage delinquency rate 90+ days, 1.9 percent for bank card delinquency rate and 1.3 percent for auto loans delinquency rate. Based on an analysis of the Presidents FY2011 budget, in FY2009 the total defaulted loans outstanding are around $45 billion.

These trends are concerning, considering the large risks that young borrowers face: first, the college dropout rate has increased significantly in the past decade (from 38 percent to 50 percent for the cohorts that enrolled in college in 1995 and 2003, respectively).[4] Furthermore, the unemployment rate among young workers with a college education has jumped up significantly during the Great Recession: 8 percent of young college graduates and 14.1 percent of young workers with some college education were unemployed in 2010 (Bureau of Labor Statistics). In addition, in order to begin repaying their student loan debt, many college graduates resort to underemployment outside their fields of study, especially after the Great Recession, a move that may have long-term deleterious financial effects.[5]

The combination of high indebtedness and high income risk in the Great Recession implies that borrowers are more likely to default on at least one of their loans. A few questions arise immediately: First, which default option do young borrowers find more attractive and why? In particular, is the current environment conducive to higher default incentives in the student loan market? Second, absent the Great Recession, how much of the increase in default on student loans is explained by trends in the student loan market and how much by trends in the credit card market? Lastly, how much does the Great Recession amplify default incentives?

In order to address the proposed issues, we develop a general equilibrium economy that mimics features of student and credit card loans. Infinitely lived agents differ in student loan debt and income levels. Agents face uncertainty in income and may save/borrow and, as in practice, borrowing terms are individual specific. Central to the model is the decision of young college-educated individuals to repay or default on their credit card and student loans. Consequences of defaulting on student and credit card loans differ in several important ways: for student loans, they include a wage garnishment, while for credit card loans, they induce exclusion from borrowing for several periods. More importantly, credit card loans can be discharged in bankruptcy (under Chapter 7), whereas student loans cannot be discharged (borrowers need to reorganize and repay under Chapter 13). Borrowing and default behavior in both markets determine the individual default risk. This risk, in turn, determines the loan terms agents face on their credit card accounts, including loan prices. In contrast, the interest rate in the student loan market does not account for the risk that some borrowers may default.

In the theoretical part of the paper, we first characterize the default behavior and show how it varies with households' characteristics and behavior in both markets. Then we demonstrate the existence of *cross-market effects* and their implications for default behavior. This represents the main contribution of our paper, a contribution which is two-fold:

[4]We define the dropout rate as the fraction of students who enroll in college and do not obtain a bachelor degree 6 years after they enroll. Numbers are based on the BPS 1995 and 2003 data.

[5]Research argues that young college-educated individuals graduating during the Great Recession earn 15 to 20 percent less on average relative to those who graduated before the Great Recession Kahn (2010).

1) Our theory delivers the result that in equilibrium, credit card loan prices depend not only on the size of the credit card loan (as in Chatterjee, Corbae, Nakajima, and Rios-Rull (2007)), but also on the size of the loan and the default status in the student loan market. This is a direct consequence of the result that the probability of default on any credit card loan decreases with the amount of debt owed in the student loan market. Also, this default probability is higher for an individual with a default flag in the student loan market relative to an individual without a default flag. To our knowledge, these results are new in the literature and provide a rationale for pricing credit card loans based on behavior in all credit markets in which individuals participate.

2) In any steady-state equilibrium, we find a combination of student loan and credit card debt for which the agent defaults on at least one type of her loans. Moreover, we find that for larger levels of student loans or credit card debt than the levels in this combination, default occurs for student loans. This result is novel because it shows that while a high student loan debt is necessary to induce default on student loans, this effect is amplified by indebtedness in the credit card market. This arises from the differences in bankruptcy arrangements in the two markets: the financially constrained borrower finds it optimal to default on student loans (even though she cannot discharge her debt) in order to be able to access the credit card market. Since defaulting on student loans causes a limited effect on her credit card market participation (shortly-lived exclusion and higher costs of loans in the credit card market), this borrower prefers the default penalty in the student loan market over defaulting in the credit card market, an action which would trigger long-term exclusion from the credit card market.

In the quantitative part of our paper, we parametrize the model to match statistics regarding student loan debt, credit card debt, and income of young borrowers with student loans (as delivered by the SCF 2004-2007). There are several sets of results.

First, our findings reveal large gaps in credit card rates across individuals with different levels of student loan debt and default status in the student loan market. This result strengthens our theory and emphasizes the quantitative importance of correctly pricing credit card debt based on behavior in other credit markets.

Second, we find that individuals with no credit card debt have lower default rates on student loans than individuals with credit card debt. Furthermore, individuals with low levels of credit card debt and low levels of student loan debt do not default on credit card debt, but they do default on their student loans. For them, the benefit of discharging their credit card debt is small compared to the large cost associated with default (exclusion from borrowing). Individuals with large levels of credit card and student loan debt are more likely to default on student loans.

Third, we determine combinations of levels of student loans and credit card debt above which borrowers are more likely to default, a result which complements our main theoretical result. Our findings suggest that having debt in the credit card market amplifies the incentive to default on student loans.

Fourth, an interesting result is that conditional on participating in the credit card market, individuals with medium levels of student loan debt or with low income levels (and large levels of student loans) use credit card debt to reduce their default on student loans. On the one hand, participating in the credit card market pushes borrowers towards increased default on their student loans, while on the other hand, taking on credit card debt helps student loan borrowers smooth consumption and pay their student loan debt, in particular when their student loan debt burdens are large. At the same time, given the importance of student loan borrowing and default behavior in credit card loan pricing, individuals with high levels of credit card debt are mostly "good risk" borrowers, i.e. individuals with low levels of student loan debt. Overall, these three effects induce a hump-shaped profile of student loan default on credit card debt. Similarly, we find a hump-shaped profile of student loan default on income. Individuals with medium levels of income default the most on their student loan debt but not as frequently on their credit card debt.

Next, we use our theory to understand how the interaction between the two credit markets affects default behavior in recent *normal* times (2004-2007) and in the Great Recession. Specifically, we quantify how much of the recent increase in default rates for student loans is due to an increase in student loan debt and how much is explained by changes in the credit card market. We find that the expansion of both markets in normal times fully explains the increase in student loan default from 5 percent to 6.7 percent during 2004-2007, with 88 percent of the increase in default coming from the increase in student loan debt (by 20.7 percent) during this period. At the same time, a decline of 19 percent in income levels of young borrowers during the Great Recession accounts for a significant portion of the increase in student loan default (to 9 percent in 2010), whereas the changes in the credit card market have no effects on aggregate default rate. Specifically, while a lower risk-free rate (by 1.5 percent during 2007-2010) transfers risk from the credit card market to the student loan market and increases student loan default, a higher transaction cost during this period has the opposite effect. Overall, these two effects offset each other, resulting in a negligible combined effect on default incentives.

Lastly, we explore the policy implications of our model and study the impact of an income contingent repayment plan on student loans.[6] We find that this plan completely eliminates the default risk in the credit card market and induces high levels of dischargeability of student loans. Overall, the policy induces an increase in welfare of 2.86 percent in a Great Recession environment but has a negative, although small, effect on welfare in normal times (0.14 percent).[7] The elimination of risk in the Great Recession environment more than outweighs the welfare cost associated

[6]This plan assumes payments of 20 percent of discretionary income and loan forgiveness after 25 years. Details are presented in Section 4.4.

[7]The Great Recession environment in the paper supposes worse income outcomes, higher transaction costs in the credit card market and a lower risk-free rate in the economy.

with high dischargeability and thus with high taxation in the economy. Results show important redistributional effects: poor borrowers with large levels of student loans benefit from the policy, while medium income borrowers with low and medium levels of debt are hurt by it. Medium earners are precisely the group who default the most under the standard repayment plan. Under income contingent repayment plans, these borrowers repay most of the student loan debt without discharging and also pay higher taxes to pay for bailing out delinquent borrowers. In contrast, poor borrowers with large levels of student loans are most likely to discharge their student loan debt under income contingent repayment plans, whereas in the absence of this repayment plan they are most likely to discharge their credit card debt. Our findings are particularly important in the current market conditions in which, due to a significant increase in college costs, students borrow more than ever in both the student loan and the credit card markets, and at the same time, they face worse job outcomes and more severe terms on their credit card accounts.

## 1.1 Related literature

Our paper is related to two strands of existing literature: credit card debt default and student loans default. The first strand includes important contributions by Athreya, Tam, and Young (2009), Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), Chatterjee, Corbae, and Rios-Rull (2010), and Livshits, MacGee, and Tertilt (2007). The first two studies explicitly model a menu of credit levels and interest rates offered by credit suppliers with the focus on default under Chapter 7 within the credit card market. Chatterjee, Corbae, and Rios-Rull (2010) provide a theory that explores the importance of credit scores for consumer credit based on a limited information environment. Livshits, MacGee, and Tertilt (2007) quantitatively compare liquidation in the United States to reorganization in Germany in a life-cycle model with incomplete markets, earnings and expense uncertainty.

In the student loan literature, there are several papers closely related to the current study, including research by Ionescu (2010), Ionescu and Simpson (2010), and Lochner and Monge (2010). These papers incorporate the option to default on student loans when analyzing various government policies. Of these studies, the only one that accounts for the role of individual default risk in pricing loans is Ionescu and Simpson (2010), who recognize the importance of this risk in the context of the private student loan market. Their model, however, is silent with respect to the role of credit risk for credit cards or for the allocation of consumer credit because the study is restricted to the analysis of the student loan market. Ionescu (2010) models both dischargeability and non-dischargeability of loans, but only in the context of the student loan market. Furthermore, as in Livshits, MacGee, and Tertilt (2007), Ionescu (2010) studies various bankruptcy rules in distinct environments that mimic different periods in the student loan program (in Livshits, MacGee, and Tertilt (2007) in different countries) rather than modeling them as alternative insurance mechanisms available to borrowers.

Our paper builds on this body of work and improves on the modeling of insurance options available to borrowers with student loans and credit card debt. On a methodological level, our paper is related to Chatterjee, Corbae, Nakajima, and Rios-Rull (2007). As in their paper, we model a menu of prices for credit card loans based on the individual risk of default. In Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), individual probabilities of default are linked to the size of the credit card loan. We take a step further in this direction and condition individual default probabilities not only on the size of the credit card loan, but also on the default status and the amount owed on student loans. All three components determine credit card loan pricing in our model. We argue that this is an important feature to account for in models of consumer default. Furthermore, we allow interest rates to respond to changes in default incentives induced by different bankruptcy arrangements in the two markets. To our knowledge, we are the first to embed such trade-offs into a quantitative dynamic theory of unsecured credit default. But capturing these trade-offs induced by multiple default decisions with different consequences poses obvious technical challenges. We provide mathematical tools to address these issues.

To this end, the novelty of our work consists in providing a theory about interactions between credit markets with different financial arrangements and their role in amplifying consumer default for student loans. Previous research analyzed these two markets separately, mainly focusing on credit card debt. Our paper attempts to bridge this gap. Our results are not specific to the interpretation for student loans and credit cards and speak to consumer default in any environments that feature differences in financial market arrangements and thus induce a trade-off in default incentives. In this respect our paper is related to Chatterjee, Corbae, and Rios-Rull (2008), who provide a theory of unsecured credit based on the interaction between unsecured credit and insurance markets. Also related to our paper is research by Mitman (2012), who develops a general-equilibrium model of housing and default to jointly analyze the effects of bankruptcy and foreclosure policies. However, our research is different from Mitman (2012) in several important ways: our paper focuses on the interplay between two types of unsecured credit that feature dischargeability and non-dischargeability of loans. In addition, we study how this interaction between two credit markets with different bankruptcy arrangements changes during normal times and during the Great Recession.[8]

The paper is organized as follows. In Section 2, we describe important facts about student loans and credit card terms. We develop the model in Section 3 and present the theoretical results in Section 4. We calibrate the economy to match important features of the markets for student

[8] In related empirical work, Edelberg (2006) studies the evolution of credit card and student loan markets and finds that there has been an increase in the cross-sectional variance of interest rates charged to consumers, which is largely due to movements in credit card loans: the premium spread for credit card loans more than doubled, but education loan and other consumer loan premiums are statistically unchanged.

and credit card loans and present quantitative results in Section 5. Section 6 concludes.

# 2 Data facts

This section contains two sets of facts: 1) facts related to default behavior and loan pricing and 2) facts related to trends in the student loan, credit card, and labor markets for young college-educated individuals during 2004-2010. We build an economy that is consistent with the first set of facts and use the second set of facts to guide our experiments, which explain recent trends in default rates for student loans.

## 2.1 Default behavior and risk pricing

1. High student loan debt increases the likelihood of default for student loans (see Dynarsky (1994) and Ionescu (2008)).

2. High credit card debt increases the likelihood of default on credit card debt (see Athreya, Tam, and Young (2009) and Chatterjee, Corbae, Nakajima, and Rios-Rull (2007)).

3. Low income increases the likelihood of default on credit card debt (see Sullivan, Warren, and Westbrook (2001)).

4. Individuals with high credit risk receive higher interest rates on their credit card debt (Chatterjee, Corbae, and Rios-Rull (2010)).

## 2.2 Trends in the student loan and credit card market

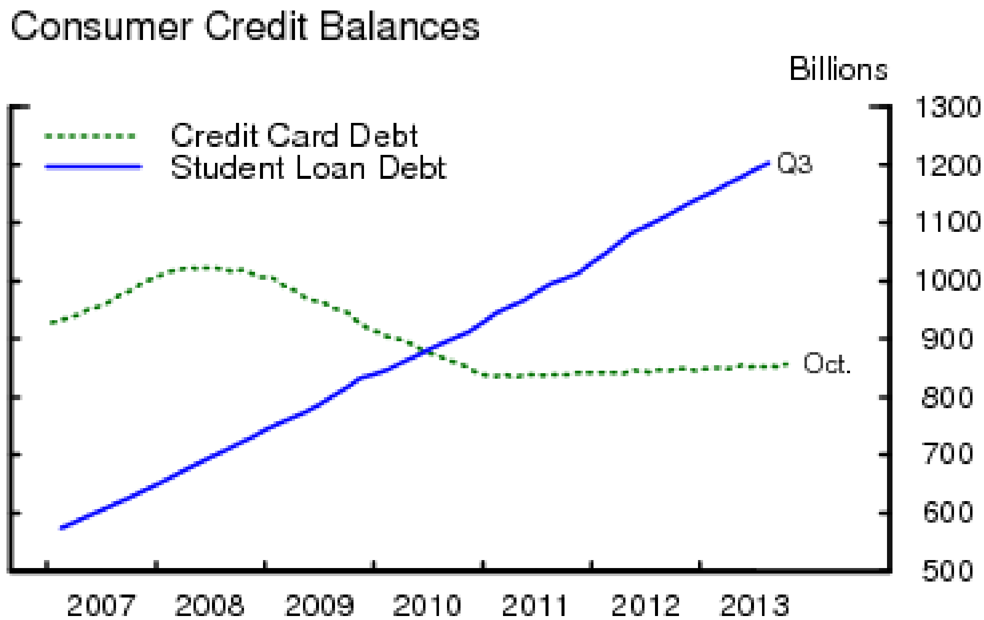
Findings documented in this section are primarily based on the SCF data for young borrowers aged 20-30 years old who have some college education (with or without a college degree), who are no longer enrolled in college and who took out student loans to finance their college education.[9] We construct these samples using the SCF 2004, the SCF 2007 and SCF 2010. The sample sizes are 466, 430, and 675, respectively. We also use Equifax data, in which similarly constructed samples consists of 15,000 observations, on average, for the years 2004-2010.[10]

[9]While older individuals also participate in the two credit markets studied in the paper, default behavior is a concern for young individuals. Therefore, we focus on young individuals in the current study.

[10]Even though Equifax data contain more observations and delinquent behavior in the two markets, there are important shortcomings of this data set for the current study. Equifax contains no information related to income and terms in the credit card market, both of which are essential for the current analysis. It also does not contain information on education, and student loan levels are lower, on average, than those in SCF and those reported by the Department of Education (DoE) and College Board. Therefore, we will benchmark our quantitative results against the SCF data.

1. Student loan debt borrowed by young U.S. households increased significantly in the recent years, passing credit card debt for the first time in 2010 (See Figure 1).[11]

Figure 1: Trends in student loans and credit card debt




Source: Federal Reserve Board (G.19)

2. According to our samples from the SCF data, the amount of student loans increased by almost 21 percent in both normal times (2004-2007) and in the Great Recession (2007-2010).[12]

3. At the same time, the unemployment rate went up from 4.3% before the Great Recession (2004-2007) to 7.6% (2010) and labor income went down 19 percent, on average.[13]

4. Young borrowers with student loans use credit cards at very high rates: 71 percent of young U.S. households have at least one credit card and 93 percent of credit card users have positive balances.[14]

[11]According to the Federal Reserve releases, U.S. households owed \$826.5 billion in revolving credit (98 percent of revolving credit is credit card debt) and they owed \$829.785 billion in student loans — both federal and private — in 2010. The accumulation of student loan debt is partially due to the 40 percent increase in the cost of college in the past decade and partially due to paying down credit card debt.

[12]We use more years for the Great Recession than in the actual definition (2008-2009) to allow for the effects of the economic downturn to be properly reflected in borrowing and repayment behavior, in particular given the lag in unemployment.

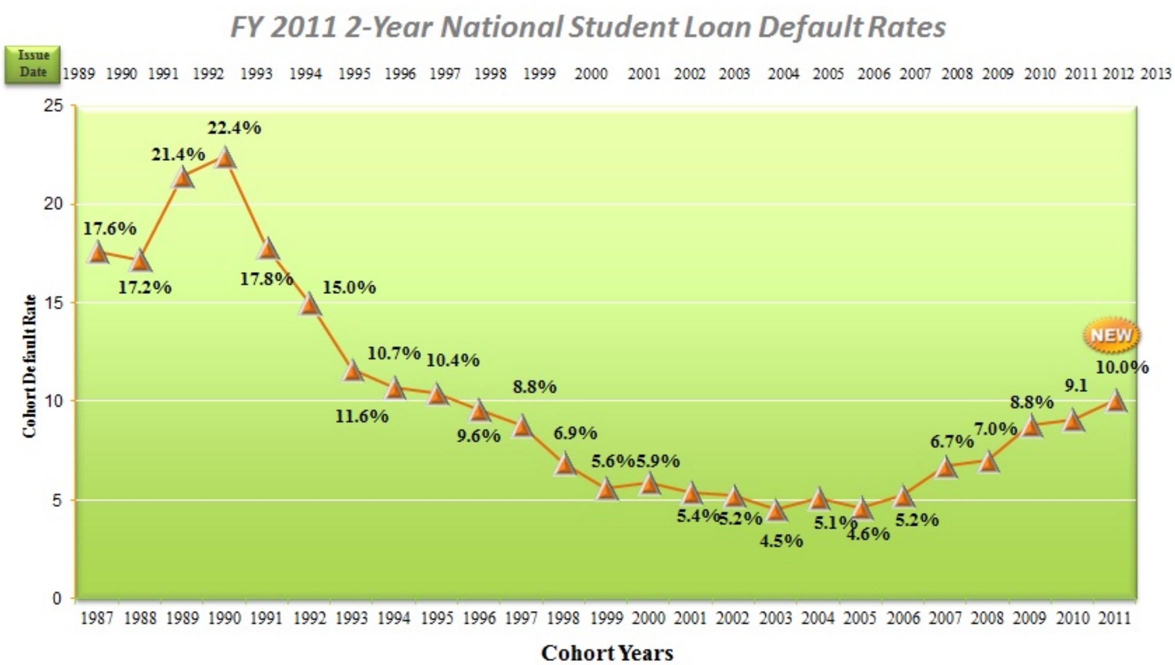
[13]The unemployment numbers compare to those in the CPS of 4.8 percent and 7.3 percent as reported by Farber and Valletta (2013).

[14]These numbers are larger for young college-educated individuals with residential debt.

5. Terms on credit card accounts of young borrowers have changed: the credit card limit increased by 30 percent from 2004 to 2007 but decreased by about the same percentage during the Great Recession (2007-2010). The average amount lent by credit card issuers to young borrowers declined by 31.5 percent from 2007 to 2010.[15]

6. According to statistics from the DoE, the national two-year basis CDR on student loans increased from 5 percent in 2004-2006 to 10 percent in 2011 (see Figure 2).[16]

Figure 2: Trends in default rates




Source: Department of Education

[15]In general, credit card terms deteriorated in the past several years: credit card providers have levied some of the largest increases in interest rates, fees and minimum payments. For instance, JPMorgan Chase, the biggest credit card provider, raised the minimum payment on outstanding balances from 2 percent to 5 percent for some customers and raised its balance-transfer fee from 3 percent to 5 percent – the highest rate among the large consumer banks (June 30 Bloomberg article). Citigroup has reportedly raised rates on outstanding balances by nearly 3 percentage points to an average of 24 percent for 13 million to 15 million cardholders (July 1 2009 Financial Times article).

[16]These trends are consistent with our findings in the Equifax data that show that delinquency rates for student loans went up from 5.1 percent in 2004 to 9.3 percent in 2010. At the same time, delinquency rates for credit card debt did not change much, with 1.6 percent in 2004 and 1.4 percent in 2010. Our samples are constructed in a similar way to those in the SCF and consist of 15,000 observations, on average. We define delinquency as being delinquent at least two quarters in a year.

# 3 Model

## 3.1 Legal environment

Consumers who participate in the student loan and credit card markets, namely, young college educated individuals with student loans, are small, risk-averse, price takers. They differ in levels of student loan debt, $d$ and income, $y$. They are endowed with a line of credit, which they may use for transactions and consumption smoothing. They choose to repay or default on their student loans as well as on their credit card debt. Both types of loans are not secured by any tangible assets, but eligibility conditions are very different and default has different consequences in each market.

### 3.1.1 Credit cards

Bankruptcy for credit cards in the model resembles Chapter 7 "total liquidation" bankruptcy. The model captures the fact that in practice, credit card issuers use consumer repayment and borrowing behavior on all types of loans to assess the likelihood that any single borrower will default (as reflected by FICO scores). Loan prices and credit limits imposed by credit card issuers are set to account for the individual default risk and are tailored to each credit account.

Consider a household that starts the period with some credit card debt, $b_t$. Depending on the household decision to declare bankruptcy as well as on the household borrowing behavior, the following things happen:

1. If a household files for bankruptcy, $\lambda_b = 1$, then the household unsecured debt is discharged and liabilities are set to 0.

2. The household cannot save during the period when default occurs, which is a simple way of modeling that U.S. bankruptcy law does not permit those invoking bankruptcy to simultaneously accumulate assets.

3. The household begins the next period with a record of default on credit cards. Let $f_t \in F = \{0, 1\}$ denote the default flag for a household in period $t$, where $f_t = 1$ indicates in period $t$ a record of default and $f_t = 0$ denotes the absence of such a record. Thus a household who defaults on credit in period $t$ starts period $t + 1$ with $f_{t+1} = 1$.

4. A household who starts the period with a default flag cannot borrow and the default flag can be erased with a probability $p_f$.

5. In contrast, a household who starts the period with $f_t = 0$ is allowed to borrow and save according to individual credit terms: credit rates assigned to households by credit lenders

vary with individual characteristics. This feature is important to allow for capturing default risk pricing in equilibrium.

This formulation captures the idea that there is restricted market participation for borrowers who have defaulted in the credit card market relative to borrowers who have not. It also implies more stringent credit terms for consumers who take on more credit card debt, i.e. precisely the type of borrowers who are more constrained in their capability to repay their loans. In addition, creditors take into account borrowing behavior in the other type of market, i.e. the student loan amount owed, $d_t$ as well as the default status for student loans, $h_t$. These features are consistent with the fact that credit card issuers reward good repayment behavior and penalize bad repayment behavior, taking into account this behavior in all markets in which borrowers participate. Finally, we assume that defaulters on credit cards are not completely in autarky, which is consistent with evidence. In U.S. consumer credit markets, households retain a storage technology after bankruptcy, namely, the ability to save. We assume that without loss of generality, defaulters cannot borrow. In practice, borrowers who have defaulted in the past several years are still able to obtain credit at worse terms. In our model, allowing them a small negative amount or 0 does not have an effect on the results.

### 3.1.2 Student loans

Government-guaranteed student loans are conditioned on financial need, not credit ratings. Agents are eligible to borrow up to the full college cost minus expected family contributions. Once borrowers are out of college, they enter a standard 10-year repayment plan with fixed payments. The interest rate on student loans does not incorporate the risk that some borrowers might exercise the option to default. The interest rate is set by the government. Several default penalties implemented in the student loan program might bear part of the default risk. Bankruptcy for student loans in the model resembles Chapter 13 "reorganization" bankruptcy, which requires the reorganization and repayment of defaulted loans. Under the current Federal Student Loan Program (FSLP), students who participate cannot discharge on their student loans except in extreme circumstances. Consequently, default on student loans in the model at period $t$ (denoted by $\lambda_d = 1$) simply means a delay in repayment that triggers the following consequences:

1. There is no debt repayment in period $t$. However, the student loan debt is not discharged. The defaulter must repay the amount owed for payment in period $t + 1$.[17]

[17] Borrowers are considered in default on student loans if they do not make any payments within 270 days in the case of a loan repayable in monthly installments or 330 days in the case of a loan repayable in less frequent installments. Loan forgiveness is very limited. It is granted only in the case that constant payments are made for 25 years or in the case that repayment causes undue hardship. As a practical matter, it is very difficult to demonstrate undue hardship unless the defaulter is physically unable to work. Partial dischargeability occurs in less than 1 percent of the default cases.

2. The defaulter is not allowed to borrow or save in period $t$, which is in line with the fact that credit bureaus are notified when default occurs and thus access to the credit card market is restricted. Also, as in the case of the credit card market, this feature captures the fact that U.S. bankruptcy law does not permit those invoking bankruptcy to simultaneously accumulate assets.

3. A fraction $\gamma$ of the defaulter's wages is garnished starting in period $t+1$. Once the defaulter rehabilitates her student loan, the wage garnishment is interrupted. This penalty captures the default risk for student loans in the model.[18]

4. The household begins the next period with a record of default on student loans. Let $h_t \in H = \{0,1\}$ denote the default flag for a household in period $t$, where $h_t = 1$ indicates a record of default and $h_t = 0$ denotes the absence of such a record. Thus a household who defaults in period $t$ starts period $t+1$ with $h_{t+1} = 1$.

5. A household that begins period $t$ with a record of default must pay the debt owed in period $t$, $d_t$. The default flag is erased with probability $p_h$.[19]

6. There are no consequences on credit card market participation during the periods after a default on student loan occurs. However, there are consequences on the pricing of credit card loans from defaulting on student loans, as mentioned above. This assumption is justified by the fact that in practice, student loan default is reported to credit bureaus and so creditors can observe the default status immediately after default occurs. However, immediate repayment and rehabilitation of the defaulted loan will result in the removal of the default status reported by the loan holder to the national credit bureaus. In practice, the majority of defaulters rehabilitate their loans. Therefore, they are still able to access the credit card market (on worse terms, as explained above).

## 3.2 Preferences and endowments

At any point in time the economy is composed of a continuum of infinitely lived households with unit mass.[20] Agents differ in student loan payment levels, $d \in D = \{d_{\min}, ..., d_{\max}\}$ and income

[18] This penalty can be as high as 15 percent of the defaulter's wages. In addition, consequences in practice include seizure of federal tax refunds, possible holds on transcripts and ineligibility for future student loans. In the model we encapsulate all of these into the wage garnishment penalty.

[19] The household cannot default the following period after default occurs. As mentioned before, less than 1 percent of borrowers repeat default, given that the U.S. government seizes tax refunds in the case that the defaulter does not rehabilitate her loan soon after default occurs. This penalty is severe enough to induce immediate repayment after default.

[20] The use of infinitely lived households is justified by the fact that we focus on the cohort default rate for young borrowers, which means that age distributions are not crucial for analyzing default rates in the current study. The

levels, $y \in Y = [y_{\min}, y_{\max}]$. There is a constant probability $(1-\rho)$ that households will die at the end of each period. Households that do not survive are replaced by newborns who have not defaulted on student loans $(h = 0)$ or credit cards $(f = 0)$, have zero assets $(b = 0)$, and with labor income and student loan debt drawn independently from the probability measure space $(Y \times D, \mathcal{B}(Y \times D), \psi)$ where $\mathcal{B}(\cdot)$ denotes the Borel sigma algebra and $\psi = \psi_y \times \psi_d$ denotes the joint probability measure. Surviving households independently draw their labor income at time $t$ from a stochastic process. The amount that the household needs to pay on her student loan is the same.[21] Household characteristics are then defined on the measurable space $(Y \times D, \mathcal{B}(Y \times D))$. The transition function is given by $\Phi(y_{t+1})\delta_{d_t}(d_{t+1})$, where $\Phi(y_t)$ is an i.i.d. process and $\delta_d$ is the probability measure supported at $d$.

This assumption ensures that even for the worst possible realization of income, the amount owed on student loans each period does not exceed the per period income.[22]

The preferences of the households are given by the expected value of a discounted lifetime utility, which consists of:

$$E_0 \sum_{t=0}^{\infty} (\rho\beta)^t U(c_t) \tag{1}$$

where $c_t$ represents the consumption of the agent during period $t$, $\beta \in (0,1)$ is the discount factor, and $\rho \in (0,1)$ is the survival probability.

**Assumption 1.** *The utility function $U(\cdot)$ is increasing, concave, and twice differentiable. It also satisfies the Inada condition:* $\lim_{c \to 0^+} U(c) = -\infty$ *and* $\lim_{c \to 0^+} U'(c) = \infty$.

## 3.3 Market arrangements

There are several similarities as well as important differences between the credit card market and the market for student loans.

### 3.3.1 Credit cards

The market for privately issued unsecured credit in the United States is characterized by a large, competitive marketplace in which price-taking lenders issue credit through the purchase of securities backed by repayments from those who borrow. These transactions are intermediated principally by credit card issuers. Given a default option and consequences on the credit record from default behavior, the market arrangement departs from the conventional modeling of borrowing and lending. As in Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), our model handles

use of a continuum of households is natural, given the size of the credit market.

[21] Federal student loan payments are fixed and computed based on a fixed interest rate and duration of the loan.

[22] This assumption is made for expositional purposes and is not crucial for the results. In fact, all results go through if this assumption is relaxed. Details are provided in the Appendix.

the competitive pricing of default risk, a risk that varies with household characteristics.[23] In this dimension, our model departs from Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) in several important ways: the default risk is based on the borrowing behavior in both markets, i.e. it depends on the size of the loan on credit cards, $b_t$ as well as the amount of student loans owed, $d_t$. In addition, it depends on the default status on student loans, $h_t$. Competitive default pricing is achieved by allowing prices to vary with all three elements. This modeling feature is novel in the literature and is meant to capture the fact that in practice, the price of the loan depends on past repayment and borrowing behavior in all the markets in which borrowers participate. Unsecured credit card lenders use this behavior (which, in practice, is captured in a credit score) as a signal for household credit risks and thus their probability of default. They tailor loan prices to individual default risk, not only to individual loan sizes. Obviously, in the case of a default flag on credit cards, no loan is provided.

A household can borrow or save by purchasing a single one-period pure discount bond with a face value in a finite set $B \subset \mathbb{R}$. The set $B = \{b_{\min}, \ldots, b_{\max}\}$ contains 0 and positive and negative elements. Let $N_B$ be the cardinality of this set. Individuals with $f_t = 1$ (which is a result of defaulting on credit cards in one of the previous periods) are limited in their market participation, $b_{t+1} \geq 0$.[24]

A purchase of a discount bond in period $t$ with a non-negative face value $b_{t+1}$ means that the household has entered into a contract where it will receive $b_{t+1} \geq 0$ units of the consumption good in period $t+1$. The purchase of a discount bond with a negative face value $b_{t+1}$ means that the household receives $q_{d_t,h_t,b_{t+1}}(-b_{t+1})$ units of the period-t consumption good and promises to deliver, conditional on not declaring bankruptcy, $-b_{t+1} > 0$ units of the consumption good in period $t+1$; if it declares bankruptcy, the household delivers nothing. The total number of credit indexes is $N_B \times N_D \times N_H$. Let the entire set of $N_B \times N_D \times N_H$ prices in period $t$ be denoted by the vector $q_t \in \mathbb{R}^{N_B \times N_D \times N_H}$. We restrict $q_t$ to lie in a compact set $Q \equiv [0, q_{\max}]^{N_B \times N_D \times N_H}$ where $0 < q_{\max} < 1$.

### 3.3.2 Student loans

Student loans represent a different form of unsecured credit. First, loans are primarily provided by the government (either direct or indirect and guaranteed through the FSLP), and do not share

[23] Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) handle the competitive pricing of default risk by expanding the "asset space" and treating unsecured loans of different sizes for different types of households (of different characteristics) as distinct financial assets.

[24] Note that households are liquidity constrained in the model. The existence of such constraints in credit card markets has been documented by Gross and Souleles (2002). Overall credit availability has not decreased along with bankruptcy rates over the past several years before the Great Recession, so aggregate response of credit supply to changing default has not been that large (see Athreya (2002)).

the features of a competitive market.[25] Unlike credit cards, the interest rate on student loans, $r_g$ is set by the government and does not reflect the risk of default in the student loan market.[26] However, the penalties for default capture some of this risk. In particular, the wage garnishment is adjusted to cover default. More generally, loan terms are based on financial need, not on default risk. Second, taking out student loans is a decision made during college years. Once households are out of college, they need to repay their loans in equal rounds over a determined period of time subject to the fixed interest rate. We model college-loan-bound households that are out of school and need to repay $d$ per period; there is no borrowing decision for student loans.[27] Third, defaulters cannot discharge their debt. Recall that in the case that the household has a default flag ($h = 1$), a wage garnishment is imposed and she keeps repaying the amount owed during the following periods after default occurs.

We define the state space of credit characteristics of the households by $\mathcal{S} = B \times F \times H$ to represent the asset position, the credit card, and student loan default flags. Let $N_{\mathcal{S}} = N_B \times 2 \times 2$ be the cardinality of this set.

To this end, an important note is that the assumption that all debt that young borrowers access is unsecured is made for a specific purpose and is not restrictive. The model is designed to represent the section of households who have student loans and credit card debt. As argued, these borrowers rely on credit cards to smooth consumption and have little or no collateral debt.

## 3.4 Decision problems

The timing of events in any period is: (i) idiosyncratic shocks, $y_t$ are drawn for survivors and newborns and student loan debt is drawn for newborns; (ii) households' decisions take place: they choose to default/repay on both credit card and student loans, make borrowing/savings and consumption decisions, and default flags for the next period are determined. We focus on steady state equilibria where $q_t = q$.

### 3.4.1 Households

We present the households' decision problem in a recursive formulation where any period $t$ variable $x_t$ is denoted by $x$ and its period $t+1$ value by $x^{'}$.

[25]Recently, students have started to use pure private student loans not guaranteed by the government. This new market is a hybrid between government loans and credit cards, featuring characteristics of both markets. However, this new market is still small and concerns about the national default rates are specific to student loans in the government program, because default rates for pure private loans are of much lower magnitudes (for details see Ionescu and Simpson (2010)). Therefore, we focus on Federal student loans in the current study.

[26]Interest rates on Federal student loans are set in statute (after the Higher Education Reconciliation Act of 2005 was passed). Details are provided in Section 5.

[27]While returning to school and borrowing another round of loans is a possibility, this decision is beyond the scope of the paper.

Each period, given their student loan debt, $d$, current income, $y$, and beginning-of-period assets, $b$, households must choose consumption, $c$ and asset holdings to carry forward into the next period, $b'$. In addition, agents may decide to repay/default on their student loans, $\lambda_d \in \{0, 1\}$ and credit card loans, $\lambda_b \in \{0, 1\}$. As described before, these decisions have different consequences: while default on student loans implies a wage garnishment $\gamma$ and no effect on market participation (although it may deteriorate terms on credit card accounts), default on credit card payments triggers exclusion from borrowing for several periods and has no effect on income.

The household's current budget correspondence, $B_{b,f,h}(d, y; q)$ depends on the exogenously given income, $y$, student loan debt, $d$, beginning of period asset position, $b$, credit card default record, $f$, student loan default record, $h$, and the prices in the credit card market, $q$. It consists of elements of the form $(c, b', h', f', \lambda_d, \lambda_b) \in (0, \infty) \times B \times H \times F \times \{0, 1\} \times \{0, 1\}$ such that

$$c + q_{d,h,b'} b' \leq y(1 - g) - t + b(1 - \lambda_b) - d(1 - \lambda_d),$$

and such that the following cases hold:

1. If a household with income $y$ and student loan debt $d$ has a good student loan record, $h = 0$, and a good credit card record, $f = 0$, then we have the following: $\lambda_d \in \{0, 1\}$ and $\lambda_b \in \{0, 1\}$ if $b < 0$ and $\lambda_b = 0$ if $b \geq 0$. In the case where $\lambda_d = 1$ or $\lambda_b = 1$ then $b' = 0$ and in the case where $\lambda_d = \lambda_b = 0$ then $b' \in B$. Also $g = 0$, $h' = \lambda_d$, $f' = \lambda_d$. The household can choose to pay off both loans ($\lambda_b = \lambda_d = 0$), in which case the household can borrow freely on the credit card market. If the household chooses to exercise its default option on either of the loans ($\lambda_d = 1$ or $\lambda_b = 1$), then the household cannot borrow or accumulate assets. Since $h = 0$, there is no income garnishment ($g = 0$).

2. If a household with income $y$ and student loan debt $d$ has a good student loan record, $h = 0$, and a bad credit card record, $f = 1$, then $\lambda_b = 0$, $\lambda_d \in \{0, 1\}$, $b' \geq 0$, $g = 0$, $h' = \lambda_d$, $f' = 1$. In this case, there is no repayment on credit card debt; the household chooses to pay or default on the student loan debt. The household cannot borrow and the credit card record will stay 1.

3. If a household with income $y$ and student loan debt $d$ has a bad student loan record, $h = 1$, and a good credit card record, $f = 0$, then $\lambda_b \in \{0, 1\}$ if $b < 0$ and $\lambda_b = 0$ if $b \geq 0$, $\lambda_d = 0$, $g = \gamma$, $f' = \lambda_b$, and $h' = 1$. The household pays back the credit card debt (if net liabilities, $b < 0$) or defaults, pays the student loan and has its income garnished by a factor of $\gamma$. The student record will stay 1. As in case 1, $b' \in B$ if $\lambda_b = 0$ and $b' = 0$ if $\lambda_b = 1$.

4. If a household with income $y$ and student loan debt $d$ has a bad student loan record, $h = 1$, and a bad credit card record, $f = 1$, then $\lambda_d = \lambda_b = 0$, $b' \geq 0$, $g = \gamma$, $f' = 1$, $h' = 1$. The household cannot borrow in the credit card market, pays the student loan, and has her income garnished.

There are several important observations: 1) we account for the fact that the budget constraint may be empty; in particular ,if the household is deep in debt, earnings are low, and new loans are expensive, then the household may not be able to afford non-negative consumption. The implication of this is that involuntary default may occur; and 2) Repeated default on student loans occurs on a limited basis (i.e. when $B_{b,f,1}(d,y;q) = \emptyset$) and is followed by partial dischargeability, an assumption that is in line with the data. All households pay taxes $t$.

**Assumption 2.** *We assume that consuming $y_{min}$ today and starting with zero assets, $b = 0$ and a bad credit card record, $f = 1$ and student loan default record, $h = 1$ with garnished wages (i.e. the worst utility with a feasible action) gives a better utility than consuming zero today and starting next period with maximum savings, $b_{max}$ and a good credit card record, $f = 0$ and student loan default record, $h = 0$ (i.e. the best utility with an unfeasible action).*

Let $v(d,y;q)(b,f,h)$ or $v_{b,f,h}(d,y;q)$ denote the expected lifetime utility of a household that starts with student loan debt $d$ and earnings $y$, has asset $b$, credit card default record $f$, and student loan default record $h$, and faces prices $q$. Then $v$ is in the set $\mathcal{V}$ of all continuous functions $v : D \times Y \times Q \to \mathbb{R}^{N_S}$. The household's optimization problem can be described in terms of an operator $(Tv)(d,y;q)(b,f,h)$ which yields the maximum lifetime utility achievable if the household's future lifetime utility is assessed according to a given function $v(d,y;q)(b,f,h)$.

**Definition 1.** For $v \in \mathcal{V}$, let $(Tv)(d,y;q)(b,f,h)$ be defined as follows:

1. For $h = 0$ and $f = 0$

$$(Tv)(d,y;q)(b,f,h) \quad = \max_{(c,b',h',f',\lambda_d,\lambda_b) \in B_{b,f,h}(d,y;q)} U(c) - \tau_b \lambda_b + \beta\rho \int v_{b',f',h'}(d,y';q)\Phi(dy')$$

where $\tau_d$ and $\tau_b$ are utility costs that the household incurs in case of default in the student loan market ($\tau_d$) and in the credit card market ($\tau_b$).

2. For $h = 0$ and $f = 1$ (in which case $\lambda_b = 0$ and $f' = 1$ with probability $1 - p_f$ and $f' = 0$ with probability $p_f$)

$$\begin{aligned}(Tv)(d,y;q)(b,f,h) \quad &= \max_{B_{b,f,h}(d,y;q)} \quad \Big\{ U(c) - \tau_d \lambda_d + (1-p_f)\beta\rho \int v_{b',1,h'}(d,y';q)\Phi(dy') \\ &\quad + p_f \beta\rho \int v_{b',0,h'}(d,y';q)\Phi(dy') \Big\}.\end{aligned}$$

3. For $h = 1$ and $f = 0$ (in which case $\lambda_d = 0$ and $h' = 1$ with probability $1 - p_h$ and $h' = 0$

with probability $p_h$)

$$(Tv)(d,y;q)(b,f,h) = \max\Bigg\{ \max_{B_{b,f,h}(d,y;q)} \Bigg\{ U(c) - \tau_b\lambda_b + (1-p_h)\beta\rho \int v_{b',f',1}(d,y';q)\Phi(dy') + p_h\beta\rho \int v_{b',f',0}(d,y';q)\Phi(dy') \Bigg\}, U(y) - \tau_b + \beta\rho \int v_{0,1,1}(d,y';q)\Phi(dy') \Bigg\}.$$

4. For $h = 1$ and $f = 1$

$$(Tv)(d,y;q)(b,f,h) = \max\Bigg\{ \max_{B_{b,f,h}(d,y;q)} \Bigg\{ U(c) + (1-p_f)(1-p_h)\beta\rho \int v_{b',1,1}(d,y';q)\Phi(dy') + (1-p_f)p_h\beta\rho \int v_{b',1,0}(d,y';q)\Phi(dy') + p_f(1-p_h)\beta\rho \int v_{b',0,1}(d,y';q)\Phi(dy') \\ p_f p_h\beta\rho \int v_{b',0,0}(d,y';q)\Phi(dy') \Bigg\}, U(y) + \beta\rho \int v_{0,1,1}(d,y';q)\Phi(dy') \Bigg\}.$$

The first part of this definition says that a household with good student loan and credit card default records may choose to default on either type of loan, on both or on none of them. For all these cases to be feasible, we need to have that the budget sets conditional on not defaulting on student loans or on credit card debt are non-empty. In the case that at least one of these sets is empty, then the attached option is automatically not available. In the case that both default and no default options deliver the same utility, the household may choose either. Finally, recall that in the case that the household chooses to repay her student loans or her credit card debt, she may also choose borrowing and savings, and in the case that she decides to default on either of these loans there is no choice on assets position.

The second part of the definition says that if the household has a good student loan default record and a default flag on credit cards, she will only have the choice to default/repay on student loans since she does not have any credit card debt. Recall that as long as the household carries the default flag in the credit card market, she cannot borrow.

The last two parts represent cases for a household with a bad student loan default record. In these last cases, defaulting on student loans is not an option. In part three, the household has the choice to default on her credit card loan. As before, this is only an option if the associated budget set is non-empty. In the case that all of these sets are empty, then default involuntarily occurs. We assume that when involuntarily default happens it will occur on both markets (this is captured in the second term of the maximization problem).[28]

In part four, however, there is no choice to default given that $f = 1$ and $h = 1$. Thus, the household simply solves a consumption/savings decision if the budget set conditional on not defaulting on either loan is non-empty. Otherwise, we assume that default involuntarily occurs. In this case, this happens only in the student loan market since there is no credit card debt.

There are two additional observations: First, in all the cases in which default occurs on credit card debt, the household incurs a utility cost, which is denoted by $\tau_b$. Consistent with modeling of consumer default in the literature, these utility costs are meant to capture the stigma following default as well as the attorney and collection fees associated with default.[29] Second, involuntary default happens when borrowers with very low income realizations and high indebtedness have no choice but default. Note that this case occurs repeatedly in the student loan market, i.e. for a household with default flag, $h = 1$. Under these circumstances we assume that the household may discharge her student loan and there is no wage garnishment. This feature captures the fact that in practice, a small proportion of households partially discharge their student loan debt.

### 3.4.2 Financial intermediaries

The (representative) financial intermediary has access to an international credit market where it can borrow or lend at the risk-free interest rate $r \geq 0$. The intermediary operates in a competitive market, takes prices as given, and chooses the number of loans $\xi_{d_t,h_t,b_{t+1}}$ for all type $(d_t, h_t, b_{t+1})$ contracts for each $t$ to maximize the present discounted value of current and future cash flows $\sum_{t=0}^{\infty}(1+r)^{-t}\pi_t$, given that $\xi_{d_{-1},h_{-1},b_0} = 0$. The period $t$ cash flow is given by

$$\pi_t = \rho \sum_{d_{t-1},h_{t-1}} \sum_{b_t \in B} (1 - p^b_{d_{t-1},h_{t-1},b_t})\xi_{d_{t-1},h_{t-1},b_t}(-b_t) - \sum_{d_t,h_t} \sum_{b_{t+1} \in B} \xi_{d_t,h_t,b_{t+1}}(-b_{t+1})q_{d_t,h_t,b_{t+1}} \tag{2}$$

where $p^b_{d_t,h_t,b_{t+1}}$ is the probability that a contract of type $(d_t, h_t, b_{t+1})$ where $b_{t+1} < 0$ experiences default; if $b_{t+1} > 0$, automatically $p^b_{d_t,h_t,b_{t+1}} = 0$. These calculations take into account the survival probability $\rho$.

If a solution to the financial intermediary's problem exists, then optimization implies $q_{d_t,h_t,b_{t+1}} \leq \frac{\rho}{(1+r)}(1 - p^b_{d_t,h_t,b_{t+1}})$ if $b_{t+1} < 0$ and $q_{d_t,h_t,b_{t+1}} \geq \frac{\rho}{(1+r)}$ if $b_{t+1} \geq 0$. If any optimal $\xi_{d_t,h_t,b_{t+1}}$ is nonzero then the associate conditions hold with equality.

[28]This assumption is made such that default is not biased towards one of the two markets.

[29]See Athreya, Tam, and Young (2009), Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), and Livshits, MacGee, and Tertilt (2007).

### 3.4.3 Government

The only purpose of the government in this model is to operate the student loan program. The government needs to collect all student loans. The cost to the government is the total amount of college loans plus the interest rate subsidized in college.[30] Denote by $L$ this loan price. We compute the per period payment on student loans, $d$ as the coupon payment of a student loan with its face value equals to its price (a debt instrument priced at par) and infinite maturity (console). Thus the coupon rate equals its yield rate, $r_g$. In practice, this represents the government interest rate on student loans. When no default occurs, the present value of coupon payments from all borrowers (revenue) is equal to the price of all the loans made (cost), i.e. the government balances its budget.

However, since default is a possibility, the government's budget constraint may not hold. In this case the government revenue from a household in state $b$ with credit card default status $f$, income $y$ and student loan debt $d$ is given by $(1 - p_d^d)d$ where $p_d^d$ is the probability that a contract of type $d$ experiences default for student loans. The government will choose taxes, $t$ to recover the losses incurred when default for student loans arises. The budget constraint is then given by

$$\int d\psi_d(dd) = \int [(1 - p_d^d) d\psi_d(dd) + \int t d\mu$$

Taxes are lump-sum and equally distributed in the economy. They are chosen such that the budget constraint balances. We turn now to the definition of equilibrium and characterize the equilibrium in the economy.

## 3.5 Steady-state equilibrium

In this section we define a steady state equilibrium, prove its existence, and characterize the properties of the price schedule for individuals with different default risks.

**Definition 2.** A steady-state competitive equilibrium is a set of non-negative price vector $q^* = (q^*_{d,h,b'})$, non-negative credit card loan default frequency vector $p^{b*} = (p^{b*}_{d,h,b'})$, a non-negative student loan default frequency $p_d^*$, taxes, $t^*$, a vector of non-trivial credit card loan measures, $\xi^* = (\xi_{d,h,b'*})$, decision rules $b'^*(y,d,f,b,h,q^*)$, $\lambda_b^*(y,d,f,b,h,q^*)$, $\lambda_d^*(y,d,f,b,h,q^*)$, $c^*(y,d,f,b,h,q^*)$, and a probability measure $\mu^*$ such that:

1. $b'^*(y,d,f,b,h,q)$, $\lambda_b^*(y,d,f,b,h,q)$, $\lambda_d^*(y,d,f,b,h,q)$, $c^*(y,d,f,b,h,q)$ solve the household's optimization problem;

[30] The government pays for the interest accumulated during college for subsidized loans but does not pay interest for unsubsidized loans. For simplicity and ease of comparability, we assume that all student loans were subsidized. Lucas and Moore (2007) find that there is little difference between subsidized and unsubsidized Stafford loans.

2. $t^*$ solves the government's budget constraint;

3. $p_d^{d*} = \int \lambda_d^*(y, d, f, b, h) d\mu^*(dy, d, df, db, dh)$ (government consistency);

4. $\xi^*$ solves the intermediary's optimization problem;

5. $p^{b*}_{d,h,b'} = \int \lambda_b^*(y', d, 0, b', h'^*)\Phi(dy')H^*(h, dh')$ for $b' < 0$ and $p^{b*}_{d,h,b'} = 0$ for $b' \geq 0$ (intermediary consistency);

6. $\xi^*_{d,h,b'} = \int \mathbf{1}_{\{b'^*(y,d,f,b,h,q^*)=b'\}}\mu^*(dy, d, df, db, h)$ (market clearing conditions (for each type $(d, h, b')$));

7. $\mu^* = \mu_{q^*}$ where $\mu_{q^*} = \Gamma_{q^*}\mu_{q^*}$ ($\mu^*$ is an invariant probability measure).

The computation of equilibrium in incomplete markets models has been made standard by a series of papers including (Aiyagari, 1994) and (Huggett, 1993) and have been extensively used in recent papers with the one by (Chatterjee, Corbae, Nakajima, and Rios-Rull, 2007) being the most related to the current study. However, our computation is more involved than previous work because of the dimensionality of the state vector, the non-trivial market clearing conditions, which include a menu of loan prices, the condition for the government balancing budget as well as the interaction between the two types of credit.

Next, we proceed as following: we provide a first set of results which contains the existence and uniqueness of the household's problem and the existence of the invariant distribution. The second set of results contains the characterization of both default decisions in terms of household characteristics and market arrangements. The last set of results contains the existence of the equilibrium and the characterization of prices. We prove the existence of cross-market effects and characterize how financial arrangements in one market affect default behavior in the other market. All proofs are provided in the Appendix.

# 4 Results

**Existence and uniqueness of a recursive solution to the household's problem**

**Theorem 1.** *There exists a unique $v^* \in \mathcal{V}$ such that $v^* = Tv^*$ and*

1. *$v^*$ is increasing in $y$ and $b$.*

2. *Default decreases $v^*$.*

3. *The optimal policy correspondence implied by $Tv^*$ is compact-valued, upper-hemicontinuous.*

4. *Default is strictly preferable to zero consumption and optimal consumption is always positive.*

Since $Tv^*$ is a compact-valued upper-hemicontinuous correspondence, Theorem 7.6 in Lucas, Stokey, and Prescott (1989) (Measurable Selection Theorem) implies that there are measurable policy functions, $c^*(d, y, ; q)(b, f, h)$, $b^*(d, y; q)(b, f, h)$, $\lambda_b^*(d, y; q)(b, f, h)$ and $\lambda_d^*(d, y; q)(b, f, h)$. These measurable functions determine a transition matrix for $f$ and $f'$, namely $F^*_{y,d,b,h,q} : F \times F \to [0, 1]$:

$$F^*_{y,d,b,h,q}(f, f' = 1) = \begin{cases} 1 & \text{if } \lambda_b^* = 1 \\ 1 - p_f & \text{if } \lambda_b^* = 0 \text{ and } f = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$F^*_{y,d,b,h,q}(f, f' = 0) = \begin{cases} 0 & \text{if } \lambda_b^* = 1 \\ p_f & \text{if } \lambda_b^* = 0 \text{ and } f = 1 \\ 1 & \text{otherwise} \end{cases}$$

The policy functions determine a transition matrix for the student loan default record, $H^*_{y,d,b,f,q} : H \times H \to [0, 1]$ which gives the student loan record for the next period, $h'$:

$$H^*_{y,d,b,f,q}(h, h' = 1) = \begin{cases} 1 & \text{if } \lambda_d^* = 1 \text{ and } h = 0 \\ 1 - p_h & \text{if } h = 1 \\ 0 & \text{otherwise,} \end{cases}$$

$$H^*_{y,d,b,f,q}(h, h' = 0) = \begin{cases} 0 & \text{if } \lambda_d^* = 1 \text{ and } h = 0 \\ p_h & \text{if } h = 1 \\ 1 & \text{otherwise.} \end{cases}$$

**Existence of invariant distribution**

Let $X = Y \times D \times B \times F \times H$ be the space of household characteristics. In the following we will write $F_q^*(y, d, b, h, f, f') := F^*_{y,d,b,h,q}(f, f')$ and $H_q^*(y, d, b, f, h, h') := H^*_{y,d,b,f,q}(h, h')$. Then the transition function for the surviving households' state variable $TS_q^* : X \times \mathcal{B}(X) \to [0, 1]$ is given by

$$TS_q^*(y, d, b, f, h, Z) = \int_{Z_y \times Z_d \times Z_f \times Z_h} \mathbf{1}_{\{b^* \in Z_b\}} F_q^*(y, d, b, h, f, df') H_q^*(y, d, b, f, h, dh') \Phi(dy') \delta_d(d')$$

where $Z = Z_y \times Z_d \times Z_b \times Z_f \times Z_h$ and $\mathbf{1}$ is the indicator function. The households that die are replaced with newborns. The transition function for the newborn's initial conditions, $TN_q^*$ :

$X \times \mathcal{B}(X) \to [0,1]$ is given by

$$TN_q^*(y,d,b,f,h,Z) = \int_{Z_y \times Z_d} \mathbf{1}_{\{(b',h',f')=(0,0,0)\}} \Psi(dy', dd')$$

Combining the two transitions, we can define the transition function for the economy, ${T^*}_q : X \times \mathcal{B}(X) \to [0,1]$ by

$${T^*}_q(y,d,b,f,h,Z) = \rho TS_q(y,d,b,f,h,Z) + (1-\rho) TN_q(y,d,b,f,h,Z)$$

Given the transition function $T_q^*$, we can describe the evolution of the distribution of households $\mu$ across their state variables $(y,d,b,f,h)$ for any given prices $q$. Specifically, let $\mathcal{M}(x)$ be the space of probability measures on $X$. Define the operator $\Gamma_q : \mathcal{M}(x) \to \mathcal{M}(x)$:

$$(\Gamma_q \mu)(Z) = \int T_q^*((y,d,b,f,h),Z) d\mu(y,d,b,f,h).$$

**Theorem 2.** *For any $q \in Q$ and any measurable selection from the optimal policy correspondence there exists a unique $\mu_q \in \mathcal{M}(x)$ such that $\Gamma_q \mu_q = \mu_q$.*

## 4.1 Characterization of the default decisions

We first determine the set for which default occurs for student loans (including involuntary default with partial dischargeability), the set for which default occurs for credit card debt, as well as the set for which default occurs for both of these two loans. Let $D_{b,f,1}^{SL}(q)$ be the set for which involuntary default on student loans and partial dischargeability occurs. This set is defined as combinations of earnings, $y$, and student loan amount, $d$, for which $B_{b,f,1}(d,y;q) = \emptyset$ in the case $h=1$. For $h=0$ let $D_{b,f,0}^{SL}(d;q)$ be the set of earnings for which the value of defaulting on student loans exceeds the value of not defaulting on student loans. Similarly, let $D_{b,0,h}^{CC}(d;q)$ be the set of earnings for which the value of defaulting on credit card debt exceeds the value of not defaulting on credit card debt in the case $f=0$. Finally, let $D_{b,0,0}^{Both}(d;q)$ be the set of earnings for which default on both types of loans occurs with $h=0$ and $f=0$. Note that the last two sets are defined only in the case $f=0$, since for $f=1$ there is no credit card debt to default on.

Theorem 3 characterizes the sets when default on student loans occurs (voluntarily or involuntarily). Theorem 4 characterizes the sets when default occurs on credit card debt and Theorem 5 presents the set for which default occurs for both types of loans.

**Theorem 3.** *Let $q \in Q$, $b \in B$. If $h=1$ and the set $D_{b,f,1}^{SL}(q)$ is nonempty, then $D_{b,f,1}^{SL}(q)$ is closed and convex. In particular, the sets $D_{b,f,1}^{SL}(d;q)$ are closed intervals for all $d$. If $h=0$ and the set $D_{b,f,0}^{SL}(d;q)$ is nonempty, then $D_{b,f,0}^{SL}(d;q)$ is a closed interval for all $d$.*

**Theorem 4.** *Let $q \in Q$, $(b, 0, h) \in \mathcal{S}$. If $D^{CC}_{b,0,h}(d; q)$ is nonempty then it is a closed interval for all $d$.*

**Theorem 5.** *Let $q \in Q$, $(b, 0, 0) \in \mathcal{S}$. If the set $D^{Both}_{b,0,0}(d; q)$ is nonempty then it is a closed interval for all $d$.*

Next, we determine how the set of default on credit card debt varies with the credit card debt, the student loan debt, and the default status on student loans of the individual. Specifically, Theorem 6 shows that the set of default on credit card debt expands with the amount of debt for credit cards. This result was first demonstrated in Chatterjee, Corbae, Nakajima, and Rios-Rull (2007).

**Theorem 6.** *For any price $q \in Q$, $d \in D$, $f \in F$, and $h \in H$, the sets $D^{CC}_{b,f,h}(d; q)$ expand when $b$ decreases.*

In addition, we show two new results in the literature: 1) the set of default on credit card loans only shrinks when the student loan amount increases and the set of default on both credit card and student loans expands when the student loan amount increases. These findings imply that individuals with lower levels of student loans are more likely to default only on credit card debt and individuals with higher levels of student loans are more likely to default on both credit card and student loan debt (Theorem 7); and 2) the set of default on credit card loans is larger when $h = 1$ relative to the case in which $h = 0$. This result implies that individuals with a default record on student loans are more likely to default on their credit card debt (Theorem 8).

**Theorem 7.** *For any price $q \in Q$, $b \in B$, $f \in F$, and $h \in H$, the sets $D^{CC}_{b,f,h}(d; q)$ shrink and $D^{Both}_{b,f,h}(d; q)$ expand when $d$ increases.*

**Theorem 8.** *For any price $q \in Q$, $b \in B$, $d \in D$, and $f \in F$, the set $D^{CC}_{b,f,0}(d; q) \subset D^{CC}_{b,f,1}(d; q)$.*

This last set of theorems shows the importance of accounting for borrowing and default behavior in the student loan market when determining the risk of default on credit card debt. These elements will be considered in the decision of the financial intermediary, which we explain next.

## 4.2 Existence of equilibrium and characterization

**Theorem 9. Existence** *A steady-state competitive equilibrium exists.*

In equilibrium, the credit card loan price vector has the property that all possible face-value loans (household deposits) bear the risk-free rate and negative face-value loans (household borrowings) bear a rate that reflects the risk-free rate and a premium that accounts for the default probability. This probability depends on the characteristics of the student loan markets, such as loan amount and default status, as well as the size of the credit card loan. This result is delivered by the free entry condition of the financial intermediary which implies that cross-subsidization across loans made to individuals of different characteristics in the student loan market is not possible. Each $(d, h)$ market clears in equilibrium and it is not possible for an intermediary to charge more than the cost of funds for individuals with very low risk in order to offset losses on loans made to high risk individuals. Positive profits in some contracts would offset the losses in others, and so intermediaries could enter the market for those profitable loans. We turn now to characterizing the equilibrium price schedule.

**Theorem 10. Characterization of equilibrium prices** *In any steady-state equilibrium, the following is true:*

1. *For any $b' \geq 0$, $q^*_{d,h,b'} = \rho/(1+r)$ for all $d \in D$ and $h \in H$.*
2. *If the grids of $D$ and $B$ are sufficiently fine, and $h = 0$, there are $\underline{d} > 0$ and $\underline{b}' < 0$ such that $q^*_{d,h,b'} = \rho/(1+r)$ for all $d < \underline{d}$ and $b' > \underline{b}'$.*
3. *If the set of income levels for which the household is indifferent between defaulting on credit card debt and any other available option is of measure zero, then $d_1 < d_2$ implies $q^*_{d_1,h,b'} > q^*_{d_2,h,b'}$ for any $h \in H$ and $b' \in B$.*
4. *If the set of income levels for which the household is indifferent between defaulting on credit card debt and any other available option is of measure zero, then $q^*_{d,h=1,b'} < q^*_{d,h=0,b'}$ for any $d \in D$ and $b' \in B$.*

Theorem 10 demonstrates that firms charge the risk-free interest rate on deposits (property 1) and on small loan sizes made to individuals with no default record on student loans and small enough levels of student loans (property 2). Property 3 shows that individuals with lower levels of student loans are assigned higher loan prices. The last property shows that individuals with a default record on student loans pay higher prices than individuals with no default record for any loan size, $b'$ and for any amount of student loans they owe, $d$.

## 4.3 The interplay between the two markets

Since the novel feature in this paper is the interaction between different types of unsecured credit markets and its effects on default decisions, we show how the default decision varies not only with the loan amount in the respective market, but also with the loan amount in the other market. We already established that the default probability on credit card loans increases in the amount of student loans. In this section we demonstrate that a borrower with high enough loans will prefer defaulting on her student loans rather than on her credit card debt. Theorem 11 shows that we can find a combination of credit card debt and student loan debt which induces a borrower to default. Furthermore, if the amounts owed to student loans and credit card accounts are higher than the two values in this combination, then the borrower will choose to default on student loans rather than on credit card debt.

**Theorem 11.** *If the grid of $D$ is fine enough, then we can find $d_1 \in D$ and $b_1 \in B$ such that the agent defaults. Moreover, we can find $d_2 \geq d_1$ and $b_2 \leq b_1$ such that the agent defaults on student loans.*

The intuition behind this result is that with high enough debt levels, consumption is very small in the case that the agent does not default at all. Consequently, she finds it optimal to default. In the case that the student loan amount and credit card debt are large, defaulting on student loans is optimal since the option of defaulting on credit card debt triggers limited market participation. Defaulting on credit card debt is too costly compared with the benefit of discharging one's debt. When borrowers find themselves in financial hardship and have to default, they choose to default on student loans. They delay their repayments on student loans at the expense of having their wage garnished in the future. But this penalty is less severe compared to being excluded from borrowing for several periods. These are precisely the types of borrowers who most need the credit card market to help them smooth out consumption.

To conclude, our theory produces several facts consistent with reality (presented in Section 2): First, the incentive to default on student loans increases in student loan debt burden (debt-to-income ratio), i.e. default on student loans is more likely to occur for individuals with low levels of earnings and high levels of student loan debt. Second, the incentive to default on credit card debt increases in credit card debt, which is consistent with findings in (Chatterjee, Corbae, Nakajima, and Rios-Rull, 2007).

Our theory is innovative because it shows that a household with a high amount of student loans or with a record of default on student loans is more likely to default on credit card debt. This result emphasizes the importance of accounting for other markets in which the individual participates when studying default on credit card debt. Finally, we show that while a high student loan debt burden is necessary to induce default on student loans, this effect is amplified by high indebtedness in the credit card market. The financial arrangements in the two markets, and in particular the differences in bankruptcy rules and default consequences between the two types of credit, certainly play an important role in shifting default incentives. In the next section we quantify the role each of these two types of credit played in the increase in student loan default rates in recent years.

# 5 Quantitative analysis

## 5.1 Mapping the model to the data

There are four sets of parameters that we calibrate: 1) standard parameters, such as the discount factor and the coefficient of risk aversion; 2) parameters for the initial distribution of student loan debt and income; 3) parameters specific to student loan markets such as default consequences and interest rates on student loans; and 4) parameters specific to credit card markets. Our approach includes a combination of setting some parameters to values that are standard in the literature, calibrating some parameters directly to data, and jointly estimating the parameters that we do not observe in the data by matching moments for several observable implications of the model.

Our model is representative for college-educated individuals who are out of college and have student loans. We calibrate the model to 2004-2007 and use the Survey of Consumer Finances in 2004 and 2007 for moments in the distribution of income, student loan, and credit card debt. The sample consists of young households (aged 20-30 years old) with college education and student loan debt. The age group is specifically chosen to include college dropouts and recent graduates. All individuals are out of college and in the labor force. The sample sizes are 466 and 430, respectively. All numbers in the paper are provided in 2004 dollars.[31]

The model period is one year and the coefficient of risk aversion chosen ($\sigma = 2$) is in the range of estimates suggested by Auerbach and Kotlikoff (1987) and Prescott (1986). The discount factor ($\beta = 0.96$) is also standard in the literature. We set the interest rate on student loans $r_g = 0.068$ as the most representative rate for student loans.[32] The annual risk-free rate is set equal to $r_f = 0.04$, which is the average return on capital reported by McGrattan and Prescott (2000). Table 1 presents the basic parameters of the model. We set the transaction cost in the credit card market to 0.053 following Evans and Schmalensee (1999). We estimate the survival probability $\rho = 0.975$ to match average years of life to 40.[33] The probabilities to keep default flags in the two markets are set to $1 - p_f = 0.9$ for credit card debt and $1 - p_h = 0.5$ for student loan debt to match average years of punishments, ten for the credit card market and two for the student loan market. The first is consistent with estimates in the literature (see Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) and Livshits, MacGee, and Tertilt (2007)) and the second is consistent with regulations from the DoE. Specifically, it takes one period before borrowers restructure and reorganize and another period before completing loan rehabilitation. Borrowers must make 10 consecutive payments to

[31] We use both SCF 2004 and SCF 2007 for the benchmark calibration rather than only one of the two surveys to better capture the borrowing and default behavior before the Great Recession.

[32] The interest rate for Federal student loans was set to 6.8 percent in 2006 and it remained to this level for unsubsidized loans. The rate further decreased for new undergraduate subsidized loans after July 1, 2008. Before 2006 the rate was variable, ranging from 2.4 to 8.25 percent. For details see DoE (2014).

[33] Since our agents are 27 years old, this calibration matches a lifetime expectancy of 67 years old.

Table 1: Parameter Values

| Parameter | Name | Value | Target/Source |
|---|---|---|---|
| $\sigma$ | Coef of risk aversion | 2.00 | standard |
| $\beta$ | Discount factor | 0.96 | standard |
| $r_g$ | Interest on student loans | 0.068 | Dept. of Education |
| $r_f$ | Risk-free rate | 0.04 | Avg rate 2004-2007 (FRB-G19) |
| $\phi$ | Transaction cost | 0.053 | Evans and Schmalensee (1999) |
| $p_f$ | Prob to keep CC default flag | 0.9 | Avg years of punishment=10 |
| $p_h$ | Prob to keep SL default flag | 0.5 | Avg years of punishment=2 |
| $\rho$ | Survival probability | 0.975 | Avg years of life=40 |
| $\gamma$ | Wage garnishment if SL default | 0.028 | Default rate on SL =5% |
| $\tau_p$ | Utility loss from CC default | 19.5 | CC debt/income ratio=0.057 |

rehabilitate. We assume that the default flag is immediately removed after rehabilitation. We estimate the wage garnishment ($\gamma$) and the utility loss from defaulting on credit card loans ($\tau_p$) to match the two year cohort default rate for student loans of 5 percent during 2004-2006 (see Figure 2 in section 2.2) and the credit card debt to income ratio in our sample from SCF.[34]

We use the joint distribution of student loan debt and income for young households as delivered by the SCF 2004 and SCF 2007. The mean of income is \$51,510 and the standard deviation \$41,688. The mean amount of student loan debt owed per period is \$2,741 and the standard deviation \$2,400. We assume a log normal distribution with parameters $(\mu_y, \sigma_y, \mu_d, \sigma_d, \rho_{yd}) = (0.3212, 0.2633, 0.0174, 0.0153, 0.2633)$ on $[0, 1] \times [0, 0.12]$.[35] We pick the grid for assets consistent with the distribution of credit card debt in the SCF 2004-2007, for which the mean and standard deviations are \$2,979 and \$4,934, respectively.

## 5.2 Results: Benchmark economy

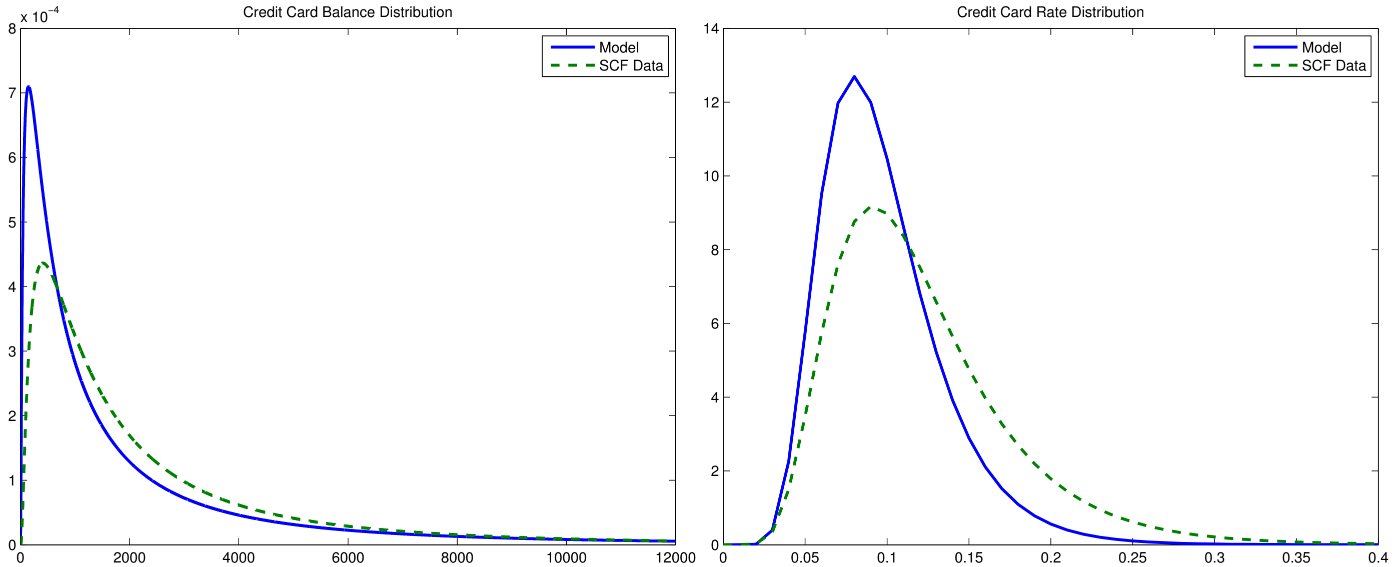
### 5.2.1 Model versus data

The model does a good job of matching debt burdens in the two markets for borrowers in the SCF 2004-2007. It delivers a credit card debt burden of 0.056 and a student loan debt burden of 0.054. The data counterparts are 0.058 and 0.054, respectively. The model predicts that 18 percent of individuals have negative assets (without including student loans). The data counterpart is 34

[34] Our estimate is in line with the data where the garnishment can be anywhere from 0 to 15 percent. Also, as in practice, wage garnishments do not apply if income levels are below a minimum threshold below which the borrower experiences financial hardship.

[35] We normalize \$163,598=1. This represents the maximum level of income which is equal to mean of income plus 3 times the standard deviation of income.

Figure 3: Distributions of credit card debt and interest rate
Model versus data




percent.[36] Also, the model replicates the distribution of credit card debt and credit card interest rate quite well , as evident in Figure 3. The model delivers an average credit card debt of $2,990 and an average credit card interest rate of 9.8 percent. The data counterparts are $2,979 and 12 percent, respectively. The interest rate in the model is lower compared to the credit card rate in the data since the interest rate in the model represents the effective rate at which borrowers pay, whereas in the data borrowers pay the high rate only in the case that they roll over their debt.

The default rate on credit card debt is 0.3 percent, which is in the range used in the literature (see Athreya, Tam, and Young (2009)). Lastly, taxes to cover defaulters in the economy are insignificant (3.615e-004 percent of income, on average).

### 5.2.2 Credit card default and pricing

We study default behavior in the two markets across individual characteristics (student loan amount, $d$, credit card debt, $b$, and income, $y$). Table 2 shows these findings across individuals with high levels of $d$, $b$, $y$ (defined as the top 50th percentile) versus individuals with low levels of $d$, $b$, $y$ (defined as the bottom 50th percentile) and Figure 4 shows credit card default rates by deciles of student loan and credit card debt.

[36]This measure is computed using total unsecured debt (but excluding student loans) minus financial assets, defined as the sum of checking and savings accounts, money market deposit accounts, money market mutual funds, value of certificates of deposit, and the value of savings and bonds.

Figure 4: Credit card default rates by debt deciles

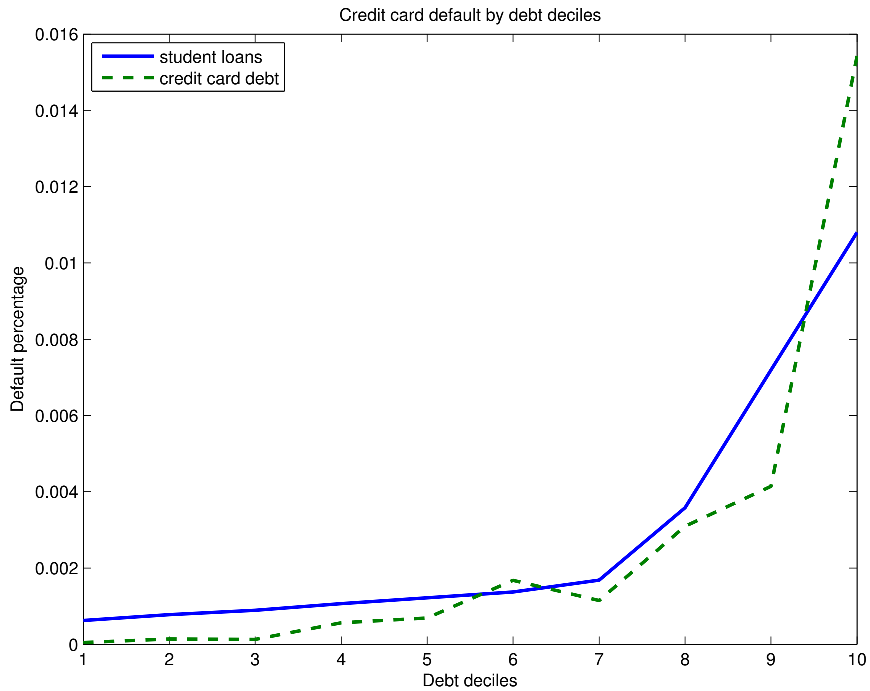



Table 2: Default rates across individual characteristics

| | Characteristic | Low | High |
|---|---|---|---|
| Default SL | SL debt | 0.68% | 9.08% |
| | CC debt | 6.35% | 5.16% |
| | Income | 8.27% | 1.71% |
| Default CC | SL debt | 0.28% | 0.84% |
| | CC debt | 0.45% | 0.55% |
| | Income | 1.1% | 0.0% |

Note: Low means the bottom 50th percentile and high means the top 50th percentile

Our results are consistent with data facts presented in Section 2.1. Default rates on credit card debt are higher for individuals with high levels of *both* types of loans. Individuals with high levels of credit card debt are more likely to default on their credit card debt (consistent with fact number 2 in Section 2.1). In addition, having high levels of student loans makes borrowers more likely to default on credit card loans. Recall that our theory predicts that high levels of student loans decrease the incentive to default only on credit card debt but increase the incentive to default on both types of loans. Quantitatively, the second effect dominates. In our model, defaulters on credit card loans default on both their college loans and their credit card debt. As evidenced in Figure 4, default on credit card debt is more sensitive to the debt in the credit card market, but student loan amounts have important effects on the incentive to default on credit card debt. Lastly, the likelihood of default on credit card debt is higher for individuals with low income relative to that of individuals with high income. This finding is consistent with fact 3 in Section 2.1. However, for the most part, the literature on unsecured default delivers the opposite result. The intuition behind this previous result in the literature is that agents with relatively low income levels stand to lose more from defaulting on their credit card debt relative to individuals with high income levels, for whom the penalties associated with default are less costly in relative terms. In our model, however, individuals also possess other types of loans, i.e. student loans with different default consequences; individuals in our model make a joint default decision. It turns out that this interaction is key in delivering the default probability in the credit market to decrease in income. This finding shows the importance of accounting for other types of loans when analyzing default behavior, a feature that is absent in previous models of consumer default. Details of the interaction between the two markets together with the importance of income for default are discussed in Section 5.2.4.

Consistent with our results on the individual probability of default for credit cards, the model delivers a pricing scheme of credit card loans based on individual default risk as proxied by the size of the loan, the amount owed in the student loan market, and the default status in the student loan market. Recall that our theoretical results show that the interest rate on credit card debt increases in both amounts of loans and is higher for individuals with a default flag on student loans. These results are consistent with fact number 4 in Section 2.1. Table 3 summarizes our quantitative results regarding credit card loan pricing across these individual characteristics.

Table 3: Credit card interest rates across individual characteristics

| Characteristic | Low | High |
|---|---|---|
| CC debt | 9.35% | 10.1% |
| SL debt | 9.44% | 10.3% |
| Income | 9.8% | 9.3% |

First, agents with high levels of credit card debt (top 50th percentile) have a credit card rate of 10.1 percent and agents with low credit card debt (bottom 50th percentile) have a credit card rate of 9.35 percent. Second, agents with high levels of student loans receive a credit card rate of 10.3 percent and agents with low levels of student loans receive a credit card rate of 9.44 percent. The wedge in the interest rates accounts for the gap in the probabilities of default between these two groups (presented in Table 2). Finally, defaulters on student loans ($h = 1$) have a credit card rate of 10 percent and nondefaulters on student loans ($h = 0$) have a credit card rate of 9.7 percent. This last result is driven by the fact that defaulters on student loans have a higher likelihood of default on credit card debt relative to non-defaulters in the student loan market. There are two main reasons behind this behavior: first, defaulters on student loans do not have the option to default on their student loans, so if they must default they do so in the credit card market; and second, in addition to being asked to repay their student loans, individuals with a default record on student loans also have part of their earnings garnished. We conclude that the amount of student loan debt and the default status on student loans represent important components of credit card loan pricing. These three findings represent the quantitative counterpart of our theoretical results in Theorem 10. In addition, our quantitative analysis predicts that agents with low income receive higher rates, on average, than agents with high income, as Table 3 shows. This is a direct implication of the differences in default rates across income groups presented in Table 2.

#### 5.2.3 Student loan default

As Table 2 shows, default rates on student loans are larger for individuals with high amounts owed to the student loan program relative to those with low amounts of student loans. The gap between the default rates for the two groups is significant. Similarly, the default rates for individuals with low income levels are higher than those for individuals with high levels of income, and the difference between the two groups is significant. Overall, the default probability for student loans is higher for individuals with relatively high student loan debt burdens in the student loan market, a fact consistent with the data (fact number 1 in Section 2.1). At the same time, individuals with credit card debt have higher default rates for student loans (5.8 percent) relative to individuals with no credit card debt (4.8 percent). However, an interesting finding is that conditional on having credit card debt, the model delivers that individuals with relatively low levels of credit card debt have a default rate of 6.3 percent, whereas individuals with have high levels of credit card debt have a default rate of 5.2 percent. We further investigate this issue. Figure 5 shows default rates on student loans across deciles of credit card and student loan debt. We find that while default on student loans increases in student loan debt, it is hump-shaped in credit card debt. This result could be interpreted in two ways: individuals use their credit card debt to repay student loans or individuals with high credit card debt levels are individuals with low risk, on average. We analyze this issue in more detail in the next section, which focuses on the interaction between the two markets.

#### 5.2.4 The interplay between the two markets

We turn now to the interaction between the two markets and its effect on default behavior, the main focus of the paper. Recall from Theorem 11 that in any steady-state equilibrium, we can find a combination of student loans and credit card debt such that individuals default. Furthermore, if loan amounts in both markets are larger than these two levels of debt, then default occurs first on student loans. Our quantitative analysis in this subsection complements this theoretical result.

First, recall that in our model, everyone who defaults on credit card debt also defaults on

Figure 5: Student loan default rates by debt deciles

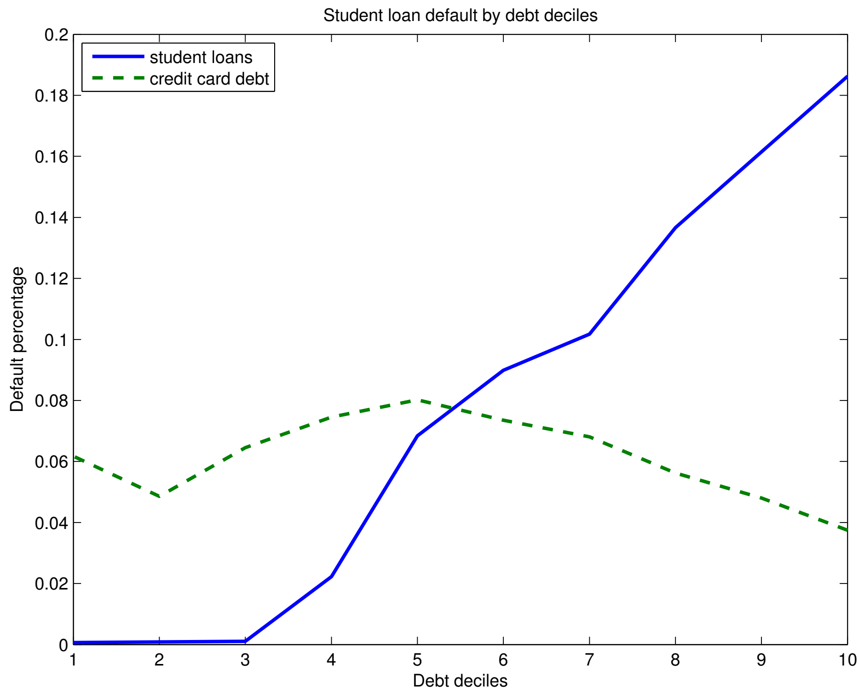



Table 4: Default rates across debt levels in the two markets

| | $b \geq 0$ | $b < 0$ Low | $b < 0$ High |
|---|---|---|---|
| $d$ Low | | | |
| Default SL | 0.65% | 0.85% | 0.76% |
| Default CC | NA | 0.00% | 0.23% |
| $d$ High | | | |
| Default SL | 8.37% | 14.8% | 12.7% |
| Default CC | NA | 0.12% | 1.1% |

student loan debt. There is no borrower who strictly prefers defaulting on credit card debt to defaulting on student loans. Table 4 shows our findings regarding default behavior across groups of student loan and credit card debt. We divide individuals in two groups based on the amount owed to the student loan program, $d$ (low and high defined as before) and in three groups based on the credit card debt, $b$: one group with positive assets and two groups with negative assets (low and high defined as before).

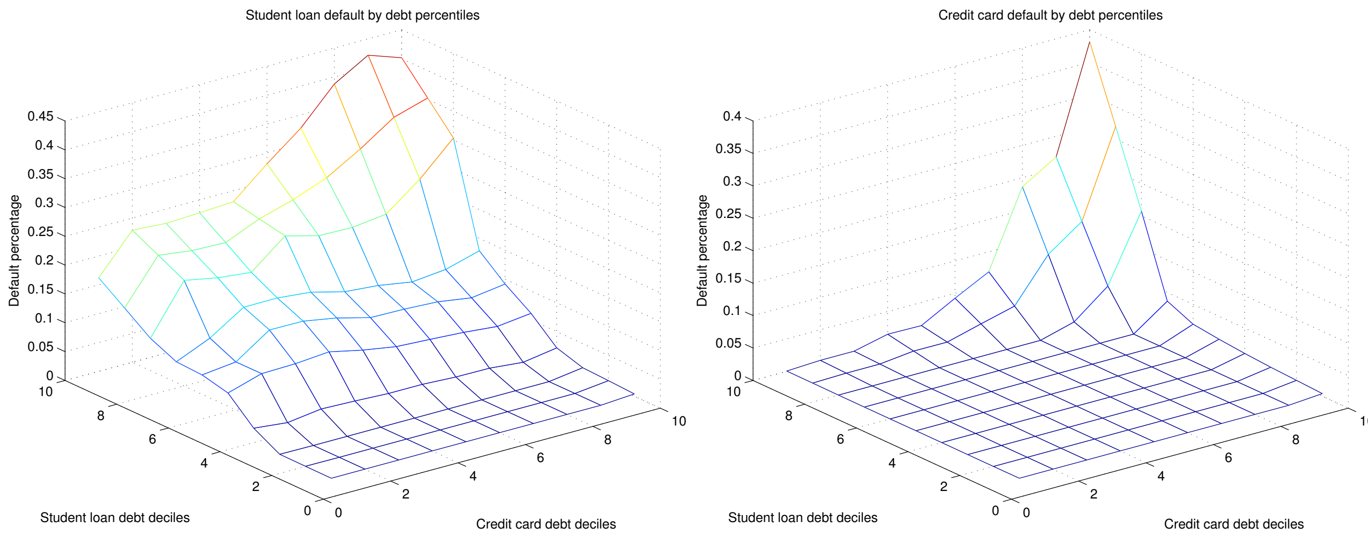
We find that individuals with no credit card debt have lower default rates on student loans than individuals with credit card debt, regardless of the amount owed in the student loan market. Second, conditional on having low levels of student loan debt, individuals with low levels of credit card debt do not default on their credit card debt, but rather default on their student loans (if they must default). The benefit of discharging their credit card debt upon default is too small compared to the large cost of being excluded from borrowing. At the same time, the penalties associated with default in the student loan market are not contingent on their credit card debt. Similarly, conditional on having high levels of student loan debt, individuals with high levels of credit card debt have a higher likelihood of defaulting on their credit card debt. Third, the gap between default rates by student loan amounts is higher for individuals with low levels of credit card debt relative to individuals with high levels of credit card debt.

These findings confirm our conjecture that while both types of debt increase incentives to default in both credit markets, some individuals may substitute credit card debt for student loan debt, in particular individuals with high levels of student loans. But these individuals with high levels of student loans represent high risk for the credit card market and therefore receive worse terms on their credit card accounts. More expensive credit card debt together with the need to access the credit card market may increase incentives to default on student loans. We further examine which individuals can use the credit card market to pay off student loan debt and which ones are defaulting even more on their student loans because of more (and expensive) credit. We determine combinations of student loans and credit card debt levels such that above these levels of debt in the two markets, the incentives to default on student loans increase rapidly and no one strictly prefers to default on their credit card debt. This is the quantitative counterpart of our main theoretical result (Theorem 11), which showed that there exists a combination of student loans ($d_1 \in D$) and credit card debt ($b_1 \in B$) such that above this threshold $d_1$ individuals may default first on their student loans. We determine such $(d_1, b_1)$ combinations next. In addition, our quantitative analysis establishes that under these thresholds $d_1$ and $b_1$, students may be able to use the credit card market to pay off their student loan debt. These findings are evidenced in Figure 6, which illustrates the default rates in the two markets conditional on both types of debt.

Note in the left panel of Figure 6 that for a borrower in the 10th decile of student loans, there is a sharp increase in student loan default once the borrower has more credit card debt than in the 5th decile. Similarly, for a borrower in the 9th (8th) decile of student loans, there is a rapid increase in student loan default once the borrower has more debt than in the 6th (8th) decile of credit card debt. Below these levels of credit card debt, however, default on student loans is quite flat across deciles of credit card debt. These findings imply that before hitting a critical credit card debt level, individuals are able to use the credit card market to keep their student loan default rate low. Once they borrow more than this threshold level, their default on student loans is amplified by their credit card debt. This threshold of credit card debt (or critical point) is decreasing with student loan debt, in part because the interest rates on credit card loans increase with student loan levels.

An interesting result is that for borrowers with intermediate levels of student loan debt (5th and 6th deciles) default on student loans is hump-shaped in credit card debt levels. This result

Figure 6: Default rates conditional on both types of debt




suggests that these borrowers may use credit card debt to pay off their student loans. Their student loan levels are high enough for them to need to borrow in the credit card market, but not high enough to induce high default incentives; at the same time, terms on credit card accounts for these individuals are good enough for them to be able to use the credit card market to keep student loan default rates low. For individuals with very low levels of student loan debt, however, default on student loans is flat across deciles of credit card debt. Their incentive to default on student loans is very low and credit card debt does not affect this decision. The combination of these factors delivers the hump-shaped student loan default pattern across levels of credit card debt (Figure 5). This pattern is a result of a composition effect in addition to a strategic default effect. Borrowers with high levels of credit card debt are mostly low risk individuals with low levels of student loans. They receive lower interest rates and have higher levels of credit card debt in equilibrium. Finally, credit card default increases with *both* levels of debt (right panel in Figure 6). As expected, a lower credit card level is needed to trigger default on credit card debt for individuals with high levels of student loans relative to individuals with low levels of student loans. Consistent with our theory, all defaulters on credit card debt also default on their student loans.

We conclude that, on average, having debt in one of the two markets amplifies the incentives to default in the other market. However, while student loan debt increases credit card default regardless of loan amount, debt in the credit card market amplifies the incentive to default on student loans only for certain combinations of debt. More importantly, some individuals may use the credit card market to reduce their default on student loans. On the one hand, participating in the credit card market and at worse terms pushes borrowers towards more default on their student loans. On the other hand, taking on credit card debt helps student loan borrowers smooth consumption

Figure 7: Student loan default rates conditional on both types of debt and income

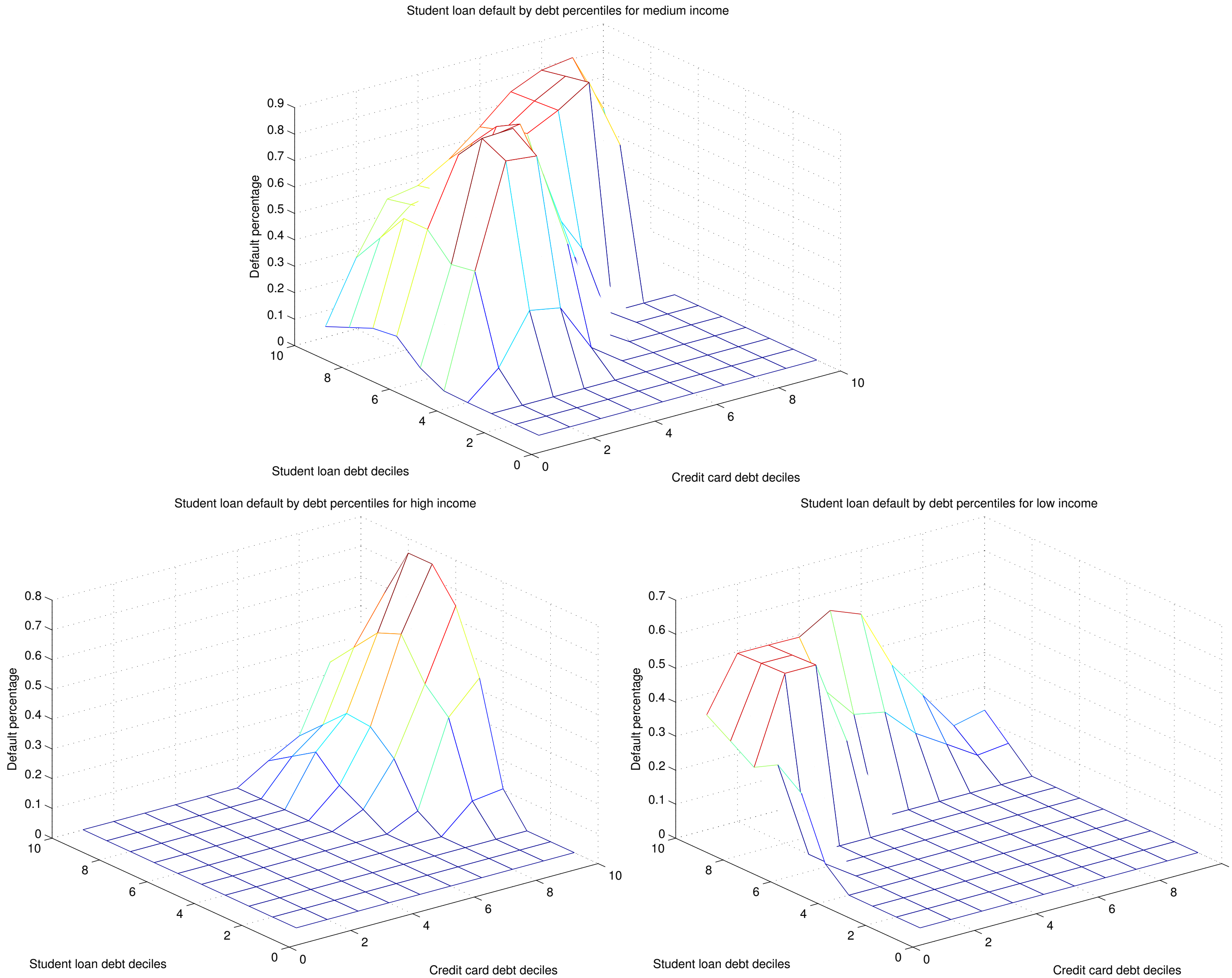
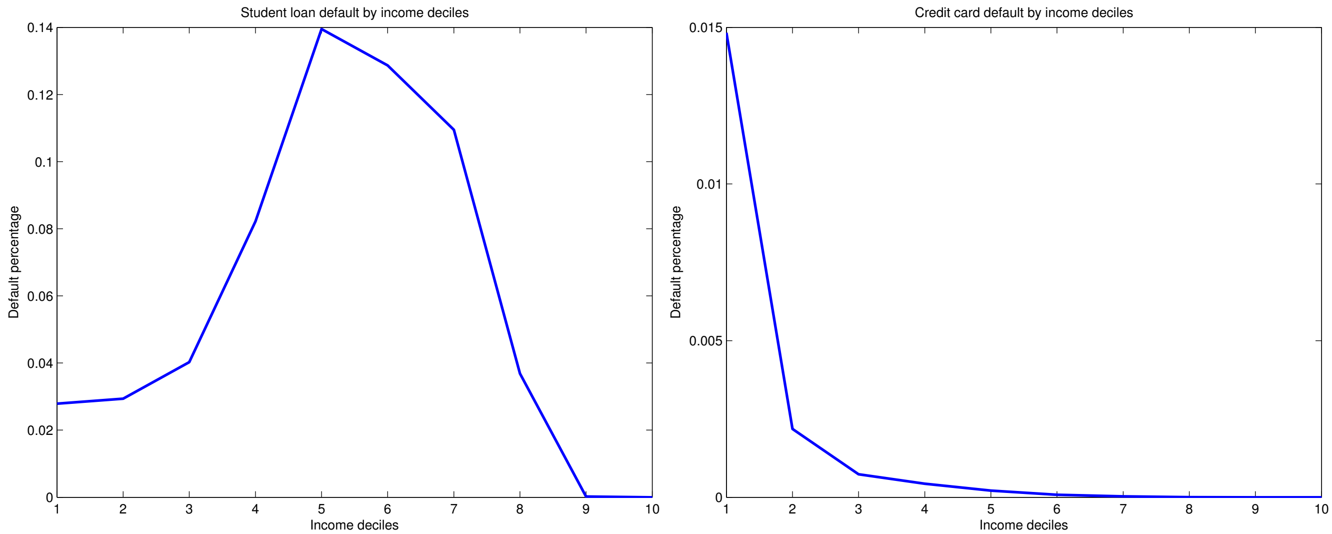


Figure 8: Default rates by income deciles




and pay their student loan debt. Certainly these channels work differently for individuals with different levels of income. We further investigate this issue and present our findings in Figure 7. First, we find that individuals with medium levels of income (top panel) have higher default rates on student loans than individuals with low or high levels of income; in addition, having credit card debt further increases default on student loans for most borrowers in this income group. Second, individuals with high income levels (middle panel) have lower default rates on student loan debt. As expected, they need larger amounts for both types of loans to default and their incentives to default on student loans are amplified by having more credit card debt. Third, an interesting result is that for individuals with low levels of income (bottom panel), incentives to default on student loans are not amplified by credit card debt. On the contrary, poor individuals with large levels of student loans seem to primarily use credit card debt to lower default on student loans. Notice the decline in default rates for student loans across deciles of credit card debt for top deciles of student loans, shown in the bottom panel.

Overall, individuals with medium levels of income default the most on their student loans (Figure 8, left panel). Those with high levels of income are not financially constrained and the wage garnishment punishment is too costly for them to warrant default on their student loans, while individuals with low levels of income would rather use the credit card market to pay off their student loans. Some of these low income individuals may also default on their credit card debt (Figure 8, right panel). We conjecture that various terms and changes in the credit market affect the default behavior in the student loan market differently across income groups, especially during the Great Recession, when credit card terms worsened and income was negatively affected. We analyze these issues in the next section.

## 5.3 Analysis: Recent trends in the two markets

In this section, we analyze how the interaction between student loan and credit card markets affects default behavior in *normal* times (2004-2007) and in the Great Recession (2007-2010). Recall from Section 2.2 that for both periods, student loans increased steadily (about 21 percent in a three year period in both normal times and in the Great Recession). However, the credit card market expanded during normal times and contracted during the Great Recession. Specifically, the credit card limit increased by 30 percent during normal times and declined by about the same percentage during the Great Recession; also, transaction costs and fees increased during the Great Recession. At the same time, the risk-free rate declined by 1.5 percent from 2007 to 2010, on average affecting interest rates in the credit card market. In addition, while the income of young borrowers did not change much during 2004-2007, it declined significantly (by 19 percent on average) during the Great Recession. Lastly, recall that the national default rate for student loans increased by 1.7 percentage points during *normal* times (from 5 percent in 2004-2006 to 6.7 percent in 2007) and further increased by more than two percentage points during the Great Recession (to 8.95 percent in 2009-2010).[37]

We conduct several experiments to understand how each of these changes affected default behavior. Specifically, in Section 5.3.1 we 1) quantify the share of the increase in student loan default rates that can be explained by the increase in student loans alone; 2) quantify the effects of the expansion of the credit card market on student loan default rates in normal times and 3) quantify the effects of the Great Recession on student loan default. In Section 5.3.2 we first quantify the share of the increase in student loan default rates during the Great Recession that can be explained by worse labor outcomes for college-educated individuals and the share that can be explained by the changes in the credit card market during the Great Recession. Finally, we disentangle the effects of each channel in the credit card market and study whether there is an amplification effect on student loan default from the Great Recession.

### 5.3.1 Debt and default: Normal times versus the Great Recession

Table 5 shows our findings for the first set of experiments: the first column represents the benchmark economy, while the second column shows the results from experiment 1 (E1-d only), which assumes an increase in student loan debt by 20.7% on average, relative to the benchmark economy (fact 2 in Section 2.2). Columns three and four show the results for experiment 2, normal times (E2-Normal) and experiment 3, Great Recession (E3-GR). In experiment 2 we suppose an increase

[37] Recall that we calibrate the benchmark economy to match the average default rate for 2004-2006 (5 percent) rather than for the single year 2004 (5.1 percent) to better capture the default behavior before the Great Recession. Similarly, for the Great Recession experiment we consider a default rate of 8.95 percent, the average rate for 2009-2010 rather than a single year after the Great Recession.

in credit card limits by 30 percent (fact 5 in Section 2.2) in addition to the increase in student loan debt in experiment 1. To put discipline in this exercise we model the expansion in the credit card market via a decrease in transaction cost, which is exogenous in the economy. We obtain a transaction cost of 3.4 percent (consistent with the number used in Athreya (2008)) compared to 5.3 percent in the benchmark economy. In experiment 3, Great Recession, we suppose a decline in income by 19 percent, on average, together with a decline in the risk-free rate of 1.5 percent and a decline in the credit card limit by 30 percent. The decline in income of 19 percent is obtained using the distribution of income in SCF 2010 together with unemployment rates, duration, and eligibility from CPS 2008-2009 (fact 3 in Section 2.2). The decline in credit card limit is modeled via an increase in the transaction cost. We know that transaction costs and fees increased during the Great Recession, but there is no estimate in the literature, in particular for the group of interest in our paper. Similarly to experiment 2, to put discipline on this exercise we find the transaction cost that delivers a 30 percent decline in credit card limits (fact 5 in Section 2.2). We obtain a transaction cost of 6.8 percent (compared to 5.3 percent in the benchmark economy).

Table 5: Summary of results: normal times versus the Great Recession

| | Baseline | E1-d only | E2-Normal | E3-GR |
|---|---|---|---|---|
| SL default | 5.00% | 6.50% | 6.80% | 8.98% |
| CC default | 0.30% | 0.40% | 0.30% | 1.22% |
| Perc with neg assets | 18.00% | 16.00% | 20.00% | 17.00% |
| CC interest rate | 9.80% | 9.83% | 7.90% | 11.20% |
| CC balance | $2,920 | $2,541 | $3,239 | $1,963 |
| Tax rel to bench | - | 1.41% | 1.52% | 1.71% |

Results show that the expansion of both markets fully accounts for the increase in student loan default during normal times, with most of the increase due to the increase in student loan debt (88 percent). The expansion of credit card debt for young borrowers contributes to the increase in default on student loans during this period, although the effect is small. On the one hand, more people are borrowing, and having credit card debt increases the incentive to default on student loan debt. On the other hand, the average level of credit card debt is higher, but the average interest rate on credit cards is lower. This fact, in turn, dampens the effect of credit card debt on default incentives. Recall that individuals with high levels of student loans who are more likely to default on student loan debt borrow lower amounts of credit card debt, on average. At the same time, for individuals with medium and low levels of student loans, high credit card debt does not amplify the incentives to default on student loan debt. The accumulation of student loan debt induces a higher risk in the credit card market, but the effect is small in equilibrium. The credit card market contracts and the average credit card rate increases slightly as a result of relatively riskier borrowers in the credit card market (i.e. borrowers with higher levels of student loan debt and student loan default flags).

From experiment 3, we observe that during the Great Recession, default rates on both student loans and credit card debt increased significantly. Fewer borrowers access the credit card market and they borrow less, on average, relative to the benchmark economy. To account for the extra risk, the interest rate increases significantly relative to the benchmark economy. There are several forces at play: young borrowers have worse labor outcomes, and at the same time there is a higher transaction cost but also a lower risk-free rate in the economy. These three channels may have opposite effects on default rates. In the next section we disentangle these effects.

## 5.4 Great Recession channels: Implications for default

Table 6 presents our results regarding the effects of various channels during the Great Recession. The first column (experiment E3a-y only) supposes only a decline of 19 percent relative to the benchmark economy, while the second column (experiment E3b-d and y) supposes a decline of 19 percent relative to the benchmark economy in addition to an increase in student loan debt by 20.7 percent. In experiments E3c and E3d we disentangle the effects coming from the credit card market. We consider the same changes as in experiment E3b together with a decline in the credit card limit by 30 percent (E3c) or a decline in the risk-free rate of 1.5 percent (E3d).

Table 6: Summary of results: Great Recession channels

| | E3a-y only | E3b-d and y | E3c-GR | E3d-GR |
|---|---|---|---|---|
| SL default | 7.1% | 8.98% | 8.68% | 9.29% |
| CC default | 0.84% | 1.26% | 1.35% | 1.1% |
| Perc with neg assets | 17.7% | 15.5% | 14.3% | 19.5% |
| CC balance | $2,513 | $1,992 | $1,701 | $2,409 |
| CC interest rate | 10.6% | 11.24% | 12.9% | 9.5% |
| Tax rel to bench | 1.24% | 1.71% | 1.65% | 1.81% |

Our findings show that the decline in income alone induces a significant increase in default rates in both markets, relative to the benchmark economy. This effect on default rates is larger than the effect induced by an increase in student loan debt in experiment 1. Consequently, the interest rate in the economy is much higher, on average, than in experiment 1. However, the credit card market does not shrink as much as in experiment 1. About the same percentage of individuals as in the benchmark economy take credit card debt given worse income levels, on average, but they borrow at higher rates, resulting from the fact that there is more default in both markets. Recall that the credit card default risk and pricing also depend on the default status for student loans. Experiment 3b shows the results for the cumulative effect of a decline in income and an increase in student loan levels. Note that there is an amplification effect for default behavior in both markets. The combination of lower income levels and higher student loan levels induces higher default rates than simply adding the two effects together. Taking on more student loan debt when post-college job prospects are worse adds extra risk. Consequently, the credit card market shrinks significantly, with only 15.5 percent of individuals borrowing, and the interest rate increases to 11.24 percent.

An interesting result is that the effects on default rates delivered in experiment 3b are the same as those delivered in experiment 3, in which all three channels in the Great Recession are accounted for. This result suggests that most of the risks in the two credit markets are induced by the combination of lower income and higher student loan amounts for young borrowers. When a higher transaction cost and a lower risk-free interest rate are added, there is not much change in terms of borrowing and default behavior on credit card loans and default behavior on student loans. However, as experiments 3c and 3d show, an increase in the transaction cost in column 3 (to 6.8 percent) and a decline in the risk-free interest rate in column 4 (by 1.5 percentage points) impact borrowing and default behavior but with opposite signs. An increase in transaction costs delivers an increase in credit card default but a decrease in student loan default (by 0.3 percentage points). Having more expensive credit card loans makes borrowers borrow less and lower amounts, on average, which in turn lowers their incentives to default on student loans (for the same amount of student loan debt). This effect of a higher transaction cost induces further tightening of the credit card market and higher interest rates. In contrast, the effect coming from a lower risk-free rate relaxes the credit card market; it induces lower interest rates, more borrowers and lower default rates in the credit card market. This change, however, induces a substantial increase in student loan defaults (from 8.98 percent to 9.29 percent). This suggests that a decrease in the risk-free interest rate in the economy induces a transfer of risk from the credit card market to the student loan market, whereas the opposite is true when transaction costs increase in the economy. Overall, the two effects combined allow for more borrowing in the credit card market (17 percent compared to 15.5 percent in experiment 3b) and induce a slightly lower default rate for credit card loans.[38]

We conclude that the accumulation of debt in the student loan market increased the risk of default in the credit card market, and in particular in the Great Recession when young borrowers faced worse labor income outcomes. At the same time, the expansion of the credit card market induced more default on student loans. A change that relaxes the credit card market during the Great Recession transfers risk from the credit card market into the student loan market, significantly increasing student loan default, whereas the opposite is true when the credit card market tightens. In the former case, borrowers receive lower prices on the same loan sizes, whereas

[38] Taxes in the economy are larger as a result of higher student loan default rates. The increase in taxation is 71 percent relative to the benchmark economy, but the magnitude is small (6.18e-004 of income, on average). Recall that the only role of taxation in this economy is to cover default.

in the latter they receive higher rates on the same loan sizes. More or less expensive credit card debt affects borrowing behavior in the credit card market and consequently affects default behavior in the student loan market.

### 5.5 Policy analysis: Income contingent repayment

There are currently four versions of student loan repayment plans based on income.[39] All of these plans assume loan payments as a percentage of discretionary income. Borrowers who earn less than 150 percent of the poverty line have a loan payment of zero (or $5 depending on the income plan type).[40] Borrowers who have an income higher than this threshold pay a fraction of their income (between 10-25 percent, depending on the income plan type). The income contingent repayment plan (ICR) provides more flexibility in eligibility criteria and therefore is used in the current experiment. According to the ICR, borrowers pay 20 percent of discretionary income and any remaining debt after 25 years in repayment is forgiven, including both principal and interest. When the ICR was introduced in 2010 (The Health Care and Education Reconciliation Act of 2010), it resulted in a lot of discussions among policy makers, in particular regarding its cost.

We analyze the quantitative implications of the ICR in both normal times and in the Great Recession. Our analysis takes into account the fact that the amount of student loans discharged is recovered through taxes. Note that our welfare calculations represent an upper bound since we ignore the fact that in reality, other versions of income repayment plans already existed.[41]

We conduct two experiments: we introduce the ICR in the benchmark economy and then in the Great Recession economy with relatively higher levels of student loans, lower levels of income, and a tight credit card market. We find that dischargeability is high in both experiments and therefore taxes are high when the ICR is introduced: 21 percent of borrowers do not fully repay their student loans when the ICR is introduced in the benchmark economy and 28 percent do not repay the full amount in the Great Recession. With higher amounts to pay and worse income, on average, more borrowers cannot finish their payments under the ICR during the Great Recession. This effect induces a decline in welfare. At the same time, the ICR completely eliminates the risk in the credit card market. The credit card default rate is 0 in both experiments. This effect induces an increase in welfare. Given a relatively higher risk in the credit card market in the Great Recession

[39] The four plans are income-contingent, income-sensitive, income based repayment, and pay as you go.

[40] This threshold is $14,148 (in 2004 constant dollars) for a single borrower. We use the value for a single borrower given that our model is representative for U.S. households aged 20-30 years old.

[41] At the same time, we abstract from the fact that the policy encourages as many as 5.8 million borrowers with both federally guaranteed student loans and direct loans to move their guaranteed loans into the Direct Loan program. These "split borrowers" have to make loan payments to two different entities. Moving these loans into the Direct Loan program will save the government money because the government will get all of the interest from the loans, instead of just some of the interest. This secondary effect of the policy, if effective, would considerably lower that cost on taxpayers.

than in the benchmark economy, this last effect is more important, quantitatively, when the ICR is introduced in the Great Recession economy. More people are borrowing in the credit card market and at lower rates. Participation in the credit card market increases to 30 percent when the ICR is introduced in the benchmark economy and to 45 percent when it is introduced in the Great Recession. Overall, we find that the introduction of the ICR in the benchmark (normal) economy induces a small decrease in welfare (by 0.14 percent), but it induces a significant improvement in welfare when introduced in the Great Recession economy (by 2.86 percent).[42]

We find that the ICR induces important redistributional effects (see Table 7). In the Great Recession for instance experiment poor borrowers (bottom quartile of income) gain more than 10 percent and those within the top quartile of student loans gain more than 20 percent. Poor borrowers with high levels of student loans benefit the most from discharging their loans after 25 years of repayment under the ICR. At the same time, the other groups lose from the ICR implementation, given that now they have to pay higher taxes to pay for delinquent borrowers. As expected, welfare changes are monotonous in student loan levels, with individuals in the bottom quartile losing the most. However, that is not the case by income groups: middle earners (quartiles 2 and 3 of income) lose the most from the policy.[43] They lose about 2 to 2.32 percent while borrowers in the top income quartile only lose 0.35 percent. Middle earners repay most of their student loans under the ICR without discharging; at the same time, they do not benefit from paying their loans faster (as opposed to rich individuals), and they pay higher taxes. They do not have the option to delay their repayment via default either. Recall that middle earners default the most under standard 10-year repayment. The same pattern across income and student loan groups emerges when the ICR is introduced during normal years, although the effects are smaller (Table 7). To conclude, while the ICR improves welfare when it is introduced during the Great Recession, it induces a decline, although small, when it is introduced during normal times. The income contingent repayment policy induces significant redistribution effects, with poor individuals with large levels of student loan debt benefiting from the policy and middle income individuals with low and medium levels of student loan debt being hurt the most.

[42]The welfare results and risk elimination in the credit card market are robust to alternative specifications of the ICR, which assume various combinations of the fraction of income used for repayment and the length of repayment before forgives occurs. In particular, one can also set the fraction and lenght of repayment to deliver zero default in the student loan market as well.

[43]It is a common feature of proportional schemes to be burdensome on the middle class. However, the magnitudes of the effects are significant, in particular in the Great Recession.

Table 7: Welfare changes from ICR

| Quartile | Q1 | Q2 | Q3 | Q4 |
|---|---|---|---|---|
| Great Recession | | | | |
| Income | +10.73% | -2% | -2.32% | -0.35% |
| Student loan | -6.27% | -5.07% | -3.62% | +21.8% |
| Normal | | | | |
| Income | +0.61% | -0.25% | -0.39% | +0.07% |
| Student loan | -0.29% | -0.19% | -0.14% | +3.1% |

# 6 Conclusion

We developed a quantitative theory of unsecured credit and default behavior of young U.S. households based on the interplay between two forms of unsecured credit, and we analyzed the implications of this interaction for default incentives. Our theory is motivated by facts related to borrowing and repayment behavior of young U.S. households with college and credit card debt, and in particular by recent trends in default rates for student loans. Specifically, different financial market arrangements and in particular, different bankruptcy rules in these two markets alter incentives to default.

We built a general equilibrium economy that mimics features of student and credit card loans. In particular, our model accounts for 1) bankruptcy arrangement differences between the two types of loans and 2) differences in pricing default risk in the two markets. Our theory is consistent with observed borrowing and default behavior of young U.S. households: incentives to default on student loans increase in student loan debt and incentives to default on credit card debt increase in credit card debt.

In addition, our model produces four new results in the literature. First, the likelihood to default on credit card debt increases with the amount of student loans. Second, individuals with a default flag in the student loan market have higher default probabilities in the credit card market than individuals who have not defaulted on their student loans. Third, in the quantitative part of our paper we show that individuals with high levels of income are less likely to default in both the student loan and the credit card markets relative to individuals with low levels of income. While this result is intuitive and consistent with empirical evidence, it is not a straightforward result from models of unsecured credit. The fact that individuals in our model also have other types of loans produces this result. Lastly, having more credit card debt induces higher incentives to default in the student loan market. These four results reveal the importance of accounting for interactions between different financial markets in which individuals participate when one analyzes default behavior for unsecured credit.

Our main contribution is that we demonstrate that differences in market arrangements can lead to amplification of default in the student loan market. Our main theoretical result shows that a borrower with high enough student loan debt and credit card debt chooses to default in the student loan market rather than in the credit card market. We further explore this issue in the quantitative part of the paper and show that while an increase in student loan debt is necessary to deliver an increase in the default rate on student loans, this effect is amplified by the expansion of the credit card market in normal times. An interesting finding is that once poor individuals (bottom quartile of income) access the credit card market, they can actually use it to reduce their default incentives on student loans. Good credit card terms for these individuals are essential. Overall, individuals with medium levels of income (quartiles 2 and 3 of income) default the most on their student loans. We find that the decline in income levels of young borrowers during the Great Recession significantly increased the risk in both the student loan and credit card market. At the same time, changes in the credit card market during the Great Recession did not much affect the default behavior: a decrease in the risk-free interest rate that relaxes credit card markets during the Great Recession transfers risk from the credit card market into the student loan market, significantly increasing student loan default, but the opposite is true when the credit card market tightens (transaction costs increase). Overall, the two effects cancel each other.

Lastly, we explore the policy implications of our model and study the impact of income contingent repayment plans on student loans. We find that the proposal induces a welfare gain of 2.86 percent when it is available in a Great Recession economy where individuals face worse job outcomes and tight credit markets. However, the policy has a (small) negative welfare effect when it is available in normal times. The policy induces significant redistributional effects, with poor borrowers with large levels of student loans benefiting the most and middle earners losing the most. Middle earners are precisely the group who choose to default the most under the standard repayment scheme. Our findings suggest that an income contingent repayment scheme is important in the current market conditions when, due to a significant increase in college costs, students borrow more than ever in both the student loan and the credit card markets, and at the same time, they face stringent terms on their credit card accounts and worse job outcomes.

# References


Aiyagari, S. (1994): "Uninsured idiosyncratic risk and aggregate saving," *The Quarterly Journal of Economics*, 109(3), 659–684.

Athreya, K. (2002): "Welfare implications of the Bankruptcy Reform Act of 1999," *Journal of Monetary Economics*, 49(8), 1567–1595.

Athreya, K., X. Tam, and E. Young (2009): "Unsecured credit markets are not insurance markets," *Journal of Monetary Economics*, 56(1), 83–103.

Athreya, K. B. (2008): "Default, insurance, and debt over the life-cycle," *Journal of Monetary Economics*, 55(4), 752–774.

Auerbach, A. J., and L. J. Kotlikoff (1987): *Dynamic fiscal policy*, vol. 11. Cambridge University Press Cambridge.

Chatterjee, S., D. Corbae, M. Nakajima, and J. Rios-Rull (2007): "A Quantitative theory of Unsecured Consumer Credit With Risk of Default," *Econometrica*, 75(6), 1525–1589.

Chatterjee, S., D. Corbae, and J. Rios-Rull (2008): "A finite-life private-information theory of unsecured consumer debt," *Journal of Economic Theory*, 142(1), 149–177.

——— (2010): "Credit scoring and competitive pricing of default risk," Working paper.

College Board (2013): *Trends in student aid*. College Board Publication.

DoE (2014): "Student Loans Overview: Fiscal Year 2014 Budget Proposal," .

Dynarsky, M. (1994): "Who defaults on student loans? Findings for the National Postsecondary Student Aid Study," *Economic of Education Review*, 13(1), 55–68.

Edelberg, W. (2006): "Risk-based pricing of interest rates for consumer loans," *Journal of Monetary Economics*, 53(8), 2283–2298.

Evans, D., and R. Schmalensee (1999): *Paying with plastic: the digital revolution in buying and borrowing*. The MIT Press.

Farber, H. S., and R. G. Valletta (2013): "Do extended unemployment benefits lengthen unemployment spells? evidence from recent cycles in the US labor market," Discussion paper, National Bureau of Economic Research.

Gross, D., and N. Souleles (2002): "An empirical analysis of personal bankruptcy and delinquency," *Review of Financial Studies*, 15(1), 319–347.

Huggett, M. (1993): "The risk-free rate in heterogeneous-agent incomplete-insurance economies," *Journal of economic Dynamics and Control*, 17(5-6), 953–969.

Ionescu, F. (2008): "Consolidation of Student Loan Repayments and Default Rates Across Income Groups," *The B.E. Journal of Macroeconomics*, 8(1), Topics.

——— (2010): "Risky Human Capital and Alternative Bankruptcy Regimes for Student Loans," Working paper.

Ionescu, F., and N. Simpson (2010): "Credit Scores and College Investment," Working paper.

Kahn, L. B. (2010): "The long-term labor market consequences of graduating from college in a bad economy," *Labour Economics*, 17(2), 303–316.

Livshits, I., J. MacGee, and M. Tertilt (2007): "Consumer bankruptcy: A fresh start," *The American Economic Review*, 97(1), 402–418.

Lochner, L., and N. Monge (2010): "The Nature of Credit Constraints and Human Capital," *American Economic Review*, Forthcoming.

Lucas, D., and D. Moore (2007): "The student loan consolidation option: An analysis of an exotic financial derivative," Discussion paper, CBO Working Paper.

Lucas, R., N. Stokey, and E. Prescott (1989): "Recursive methods in economic dynamics," *Cambridge MA*.

McGrattan, E., and E. Prescott (2000): "Is the Stock Market Overvalued?," *Federal Reserve Bank of Minneapolis Manuscript*.

Mitman, K. (2012): "Macroeconomic effects of bankruptcy & foreclosure policies," .

Prescott, E. . (1986): "Theory ahead of business cycle measurement," *Real business cycles: a reader*.

Reed, M., and B. Simon (1972): *Methods of modern mathematical physics. I. Functional analysis*. Academic Press, New York.

Rudin, W. (1987): *Real and complex analysis*. McGraw-Hill Book Co., New York, third edn.

Sallie Mae (2009): "How Undergraduate Students Use Credit Cards: Sallie Mae's National Study of Usage Rates and Trends," Discussion paper.

Sullivan, T., E. Warren, and J. Westbrook (2001): *The fragile middle class: Americans in debt*. Yale Univ Pr.

# A Appendix

# Proofs of theorems

## A.1 Proofs of Theorems 1 and 2

Let $c_{\min} = y_{\min}(1-\gamma)$ and $c_{\max} = y_{\max} + b_{\max} - b_{\min}$. Then, if $c$ is the consumption in any of the cases in the definition of $T$, we have that $U(c_{min}) \leq U(c) \leq U(c_{\max})$ and that $c_{\min}$ is a feasible consumption. Recall that $\mathcal{S} = B \times F \times H$ is a finite set and let $N_{\mathcal{S}}$ be the cardinality of $\mathcal{S}$.

**Definition A1.** Define $\mathcal{V}$ to be the set of continuous functions $v : D \times Y \times Q \to \mathbb{R}^{N_{\mathcal{S}}}$ such that

1. For all $(b, f, h) \in \mathcal{S}$ and $(d, y, q) \in D \times Y \times Q$

$$\frac{U(c_{\min})}{1-\beta\rho} \leq v(d, y, q)(b, f, h) \leq \frac{U(c_{\max})}{1-\beta\rho}. \tag{3}$$

2. $v$ is increasing in $b$ and $y$.

3. $v$ is decreasing in $f$: $v(d, y, q)(b, 0, h) \geq v(d, y, q)(b, 1, h)$ for all $d, y, q, b, h$.

Let $\left(C(D \times Y \times Q; \mathbb{R}^{N_{\mathcal{S}}}), \|\cdot\|\right)$ denote the space of continuous functions $v : D \times Y \times Q \to \mathbb{R}^{N_{\mathcal{S}}}$ endowed with the supremum norm

$$\|v\| = \max_{(d,y,q)} \|v(d, y, q)\|,$$

where the norm of a vector $w = (w(b, f, h)) \in \mathbb{R}^{N_{\mathcal{S}}}$ is

$$\|w\| = \max_{(b,f,h)\in S} |w(b, f, h)|.$$

Then $\mathcal{V}$ is a subset of $C(D \times Y \times Q; \mathbb{R}^{N_{\mathcal{S}}})$. Define also $C(D \times Y \times Q \times \mathcal{S})$ to be the set of continuous real valued functions $v : D \times Y \times Q \times \mathcal{S} \to \mathbb{R}$ with the norm

$$\|v\| = \max_{(d,y,q,b,f,h)} |v(d, y, q, b, f, h)|.$$

In the first lemma we show that the two spaces of functions that we defined above are interchangeable.

**Lemma A1.** *The map* $V : C(D \times Y \times Q; \mathbb{R}^{N_{\mathcal{S}}}) \to C(D \times Y \times Q \times \mathcal{S})$ *defined by*

$$V(v)(d, y, q, b, f, h) = v(d, y, q)(b, f, h)$$

*is a surjective isomorphism.*

*Proof.* We prove first that if $v \in C(D\times Y\times Q;\mathbb{R}^{N_{\mathcal{S}}})$ then $V(v)$ is continuous. Let $(d_n, y_n, q_n, b_n, f_n, h_n)_{n\in\mathbb{N}}$ be a sequence that converges to $(d, y, q, b, f, h)$ and let $\varepsilon > 0$. Since $\mathcal{S}$ is a finite set it follows that there is some $N_1 \geq 1$ such that $b_n = b$, $f_n = f$, and $h_n = h$ for all $n \geq N_1$. Since $v$ is continuous then there is $N_2 \geq 1$ such that if $n \geq N_2$ then

$$\|v(d_n, y_n, q_n) - v(d, y, q)\| < \varepsilon.$$

Thus $|v(d_n, y_n, q_n)(b, f, h) - v(d, y, q)(b, f, h)| < \varepsilon$ for all $n \geq N := \max\{N_1, N_2\}$. Therefore

$$|V(v)(d_n, y_n, q_n, b_n, f_n, h_n) - V(v)(d, y, q, b, f, h)| < \varepsilon \text{ for all } n \geq N$$

and $V(v)$ is continuous. It is clear from the definition of the norms that $\|V(v)\| = \|v\|$ for all $v \in C(D \times Y \times Q; \mathbb{R}^{N_{\mathcal{S}}})$. Thus $V$ is an isomorphism. Finally, if $w \in C(D \times Y \times Q \times \mathcal{S})$ then one can define $v \in C(D \times Y \times Q; \mathbb{R}^{N_{\mathcal{S}}})$ by

$$v(d, y, q)(b, f, h) = w(d, y, q, b, f, h).$$

Then $T(v) = w$ and $T$ is surjective. □

In the following we are going to tacitly view $\mathcal{V}$ either as a subset of $C(D \times Y \times Q; \mathbb{R}^{N_{\mathcal{S}}})$ or as a subset of $C(D \times Y \times Q \times \mathcal{S})$ via $V(\mathcal{V})$. For example, we are going to prove in the following lemma that $(\mathcal{V}, \|\cdot\|)$ is a complete metric space by showing that it('s image under $V$) is a closed subspace of $C(D \times Y \times Q \times \mathcal{S})$, which is a complete metric space.

**Lemma A2.** *$(\mathcal{V}, \|\cdot\|)$ is a complete metric space.*

*Proof.* We are going to show that $\mathcal{V}$ is a closed subspace of $C(D\times Y\times Q\times\mathcal{S})$. Notice first that $\mathcal{V}$ is nonempty because any constant function that satisfies (3) is in $\mathcal{V}$. Let now $\{v_n\}_{n\in\mathbb{N}}$ be a sequence of functions in $\mathcal{V}$ that converge to a function $v$. Then, since $C(D\times Y\times Q\times\mathcal{S})$ is complete, it follows that $v$ is continuous. Since inequalities are preserved by taking limits it follows immediately that $v$ satisfies the conditions of Definition A1, because each $v_n$ satisfies those conditions. Therefore $v \in \mathcal{V}$ and, thus, $(\mathcal{V}, \|\cdot\|)$ is a closed subspace of $C(D \times Y \times Q \times \mathcal{S})$ and, hence, a complete metric space. □

**Lemma A3.** *The operator $T$ defined on $C(D \times Y \times Q; \mathbb{R}^{N_{\mathcal{S}}})$ maps $\mathcal{V}$ into $\mathcal{V}$ and its restriction to $\mathcal{V}$ is a contraction with factor $\beta\rho$.*

*Proof.* We will show first that if $v \in \mathcal{V}$ then $Tv \in \mathcal{V}$. Since $v \in \mathcal{V}$ we have that

$$\frac{U(c_{\min})}{1-\beta\gamma} \leq v(d, y', q)(b', f', h') \leq \frac{U(c_{\max})}{1-\beta\gamma}$$

for all $(d, y', q) \in D \times Y \times Q$ and $(b', f', h') \in \mathcal{S}$. Integrating with respect to $y'$ we obtain that

$$\frac{U(c_{\min})}{1-\beta\gamma} \leq \int v_{(b',f',h')}(d, y'; q)\Phi(dy') \leq \frac{U(c_{\max})}{1-\beta\rho},$$

because $\int \Phi(dy') = 1$. Since $U(c_{\min}) \leq U(c) \leq U(c_{\max})$ for all $c$ appearing in the definition of $T$, it follows that

$$U(c) + \beta\rho \int v_{(b',f',h')}(d, y'; q)\Phi(dy') \leq U(c_{\max}) + \frac{\beta\rho U(c_{\max})}{1-\beta\rho} = \frac{U(c_{\max})}{1-\beta\rho},$$

and, similarly

$$\frac{U(c_{\min})}{1-\beta\rho} \leq U(c) + \beta\rho \int v_{(b',f',h')}(d, y'; q)\Phi(dy').$$

Thus the condition (3) of Definition A1 is satisfied. To prove that $Tv$ is increasing in $b$ and $y$ and decreasing in $f$, note that the sets $B_{b,f,h}(d, y, ; q)$ are increasing with respect to $b$ and $y$, and decreasing with respect to $f$. These facts coupled with the same properties for $v$ (which are preserved by the integration with respect to $y'$) imply that $Tv$ satisfies the remaining conditions from Definition A1, with the exception of the continuity, which we prove next.

Since $B, F, H$ and $D$ are finite spaces, it suffices to show that $Tv$ is continuous with respect to $y$ and $q$. Since $Q$ is compact and $v$ is uniformly continuous with respect to $q$, it follows by a simple $\varepsilon - \delta$ argument that the integral is continuous with respect $q$. Since $U(\cdot)$ is continuous with respect to $c$ and $c$ is continuous with respect to $d$ and $y$, it follows that $T(v)$ is continuous.

Finally we prove that $T$ is a contraction with factor $\beta\rho$ by showing that $T$ satisfies Blackwell's conditions. For simplicity, we are going to view $\mathcal{V}$ one more time as a subset of $C(D \times Y \times Q \times \mathcal{S})$. Let $v, w \in \mathcal{V}$ such that $v(d, y, q, b, f, h) \leq w(d, y, q, b, f, h)$ for all $(d, y, q, b, f, h) \in D \times Y \times Q \times \mathcal{S}$. Then

$$\beta\rho \int v_{(b',f',h')}(d, y'; q)\Phi(dy') \leq \beta\rho \int w_{(b',f',h')}(d, y'; q)\Phi(dy')$$

for all $(d, y, q, b', f', h')$. This implies that $Tv \leq Tw$. Next, if $v \in \mathcal{V}$ and $a$ is a constant it follows that

$$\beta\rho \int \big(v_{(b',f',h')}(d, y'; q) + a\big)\Phi(dy') = \beta\rho \int v_{(b',f',h')}(d, y'; q)\Phi(dy') + \beta\rho a.$$

Thus $T(v + a) = Tv + \beta\rho a$. Therefore $T$ is a contraction with factor $\beta\rho$. □

**Theorem 1.** *There exists a unique $v^* \in \mathcal{V}$ such that $v^* = Tv^*$ and*

*1. $v^*$ is increasing in $y$ and $b$.*

*2. Default decreases $v^*$.*

*3. The optimal policy correspondence implied by $Tv^*$ is compact-valued, upper hemi-continuous.*

*4. Default is strictly preferable to zero consumption and optimal consumption is always positive.*

*Proof.* The first two parts follows from Definition A1 and Lemmas A2 and A3. The last part follows from our assumptions on $U$. So we need only to prove the third part of the theorem. The optimal policy correspondence is

$$\Xi_{(d,y,q,b,f,h)} = \left\{(c, b', h', f', \lambda_d, \lambda_b) \in B_{b,f,h}(d, y; q) \text{ that attain } v^*_{b,f,h}(d, y, q)\right\}.$$

For simplicity of our notation we will write $x = (d, y, q, b, f, h)$. For a fixed $x$ we need to show that if $\Xi_x$ is nonempty then it is compact. First notice that

$$\Xi_x \subset [c_{\min}, c_{\max}] \times B \times H \times F \times \{0, 1\} \times \{0, 1\}$$

and, thus, it is a bounded set. We need to prove that it is closed. Let $\{(c_n, b'_n, h'_n, f'_n, \lambda^n_d, \lambda^n_b)\}_{n\in\mathbb{N}}$ be a sequence in $\Xi_x$ that converges to some

$$(c, b', h', f', \lambda_d, \lambda_b) \in [c_{\min}, c_{\max}] \times B \times H \times F \times \{0, 1\} \times \{0, 1\}.$$

Since $B, F$, and $\{0, 1\}$ are finite sets it follows that there is some $N \geq 1$ such that $b'_n = b'$, $h'_n = h'$, $f'_n = f'$, $\lambda^n_d = \lambda_d$, and $\lambda^n_b = \lambda_b$ for all $n \geq N$. Define

$$\phi(c) = U(c) + \beta\rho \int v_{(b',f',h')}(d, y'; q)\Phi(dy').$$

Then $\phi$ is continuous and, since $\phi(c_n) = v^*_{(b,f,h)}(d, y; q)$ for all $n \geq 1$, we have that

$$\phi(c) = \lim_{n\to\infty} \phi(c_n) = v^*_{(b,f,h)}(d, y; q).$$

Thus $(c, b', h', f', \lambda_d, \lambda_b) \in \Xi_x$ and $\Xi_x$ is a closed and, hence, compact set.

To prove that $\Xi$ is upper hemi-continuous consider $x = (d, y, q, b, f, h) \in D \times Y \times Q \times S$ and let $\{x_n\} \in D \times Y \times Q \times \mathcal{S}$, $x_n = (d_n, y_n, q_n, b_n, f_n, h_n)$ be a sequence that converges to $x$. Since $D$, $B$, $F$, and $H$ are finite sets it follows that there is $N \geq 1$ such that if $n \geq N$ then $d_n = d$, $b_n = b$, $f_n = f$, and $h_n = h$. Let $z_n = (c_n, b'_n, h'_n, f'_n, \lambda^n_d, \lambda^n_b) \in \Xi_{x_n}$ for all $n \geq N$. We need to find a convergent subsequence of $\{z_n\}$ whose limit point is in $\Xi_x$. Since $B$, $H$, $F$, and $\{0, 1\}$ are finite sets we can find a subsequence $\{z_{n_k}\}$ such that $b'_{n_k} = b'$, $h'_{n_k} = h'$, $f'_{n_k} = f'$, $\lambda^{n_k}_d = \lambda_d$, $\lambda^{n_k}_b = \lambda_b$ for some $b' \in B$, $h' \in H$, $f' \in F$, $\lambda_d, \lambda_b \in \{0, 1\}$. Since $\{c_{n_k}\} \subset [c_{\min}, c_{\max}]$ which is a compact interval, there must be a convergent subsequence, which we still label $c_{n_k}$ for simplicity. Let $c = \lim_{k\to\infty} c_{n_k}$ and let $z_{n_k} = (c_{n_k}, b', h', f', \lambda_d, \lambda_b)$ for all $k$. Then $\{z_{n_k}\}$ is a subsequence of $\{z_n\}$ such that

$$\lim_{k\to\infty} z_{n_k} = z := (c, b', h', f', \lambda_d, \lambda_b).$$

Moreover, since

$$\phi(c_{n_k}) = v^*_{b,f,h}(d_{n_k}, y_{n_k}; q_{n_k}) \text{ for all } k$$

and since $\phi$ and $v^*$ are continuous functions it follows that

$$\phi(c) = \lim_{k\to\infty} \phi(c_{n_k}) = \lim_{k\to\infty} v^*_{b,f,h}(d_{n_k}, y_{n_k}; q_{n_k}) = v^*_{b,f,h}(d, y; q).$$

Thus $z \in \Xi_x$ and $\Xi$ is an upper hemi-continuous correspondence. □

**Theorem 2.** *For any $q \in Q$ and any measurable selection from the optimal policy correspondence there exists a unique $\mu_q \in \mathcal{M}(x)$ such that $\Gamma_q \mu_q = \mu_q$.*

*Proof.* The Measurable Selection Theorem implies that there exists an optimal policy rule that is measurable in $X \times \mathcal{B}(X)$ and, thus, $T^*_q$ is well defined. We show first that $T^*_q$ satisfies Doeblin's condition. It suffices to prove that $TN^*_q$ satisfies Doeblin's condition (see Exercise 11.4g of Stockey, Lucas, Prescott (1989)). If we let $\varphi(Z) = TN^*_q(y, d, b, f, h, Z)$ for any $(y, d, b, f, h) \in X$ it follows that if $\varepsilon < 1/2$ and $\varphi(Z) < \varepsilon$ then $1 - \varepsilon > 1/2$ and

$$TN^*_q(y, d, b, f, h, Z) < \varepsilon < \frac{1}{2} < 1 - \varepsilon$$

for all $(y, d, b, f, h) \in X$. Thus Doeblin's condition is satisfied.

Next, notice that if $\varphi(Z) > 0$ then $TN^*_q(y, d, b, f, h, Z) > 0$ and, thus,

$$T^*_q(y, d, b, f, h, Z) = \rho TS^*_q(y, d, b, f, h, Z) + (1 - \rho) TN^*_q(y, d, b, f, h, Z) > 0.$$

Then Theorem 11.10 of Stockey, Lucas, Prescott (1989) implies the conclusion of the theorem. □

## A.2 Proofs of Theorems 3-8

Let $(b, f, h) \in \mathcal{S}$ and $q \in Q$ be fixed. Before proving the theorem we will introduce some notation which will ease the writing of our proofs. For $y \in Y$, $d \in D$ we define the following maps:

$$\psi_{nodef}(y, d)(c, b', f', h', \lambda_d = 0, \lambda_b = 0) := U(c) + \beta\rho \int v_{b',f',h'}(d, y'; q)\Phi(dy')$$

for all $(c,b',f',h',0,0) \in B_{b,f,h}(d,y;q)$;

$$\psi_{sl}(y,d)(c,b',f',h',\lambda_d=1,\lambda_b=0) = U(c) + \beta\rho \int v_{b',f',1}(d,y';q)\Phi(dy')$$

for all $(c,b',f',h',1,0) \in B_{b,f,h}(d,y;q)$;

$$\psi_{cc}(y,d)(c,b',f',h',\lambda_d=0,\lambda_b=1) = U(c) + \beta\rho \int v_{b',1,h'}(d,y';q)\Phi(dy')$$

for all $(c,b',f',h',0,1) \in B_{b,f,h}(d,y;q)$; and

$$\psi_{both}(y,d)(c,b',f',h',\lambda_d=1,\lambda_b=1) = U(c) + \beta\rho \int v_{0,1,1}(d,y';q)\Phi(dy')$$

for all $(c,b',f',h',0,1) \in B_{b,f,h}(d,y;q)$. Note that these functions are continuous in $y$ and $d$. Also, these functions depend on $b$, $f$, and $q$. Also, we will write $\omega_{b,f,h}(q,d)$ for the expected utility of an household that starts next period with $(b,f,h,q,d)$.

**Theorem 3.** *Let $q \in Q$, $f \in F$, $b \in B(f)$. If $h=1$ and the set $D^{SL}_{b,f,1}(q)$ is nonempty, then $D^{SL}_{b,f,1}(q)$ is closed and convex. In particular the sets $D^{SL}_{b,f,1}(d;q)$* are closed intervals for all $d$. If $h=0$ and *the set $D^{SL}_{b,f,0}(d;q)$ is nonempty,* then $D^{SL}_{b,f,0}(d;q)$ is a closed interval for all $d$.

*Proof.* If $h=1$ then $D^{SL}_{b,f,1}(q)$ is the combinations of earnings $y$ and student loan amount $d$ for which $B_{b,f,1}(d,y;q)=\emptyset$. Then they satisfy the inequality $y(1-\gamma)+b(1-\lambda_b)-d-q_{b',d,h}b' \leq 0$ for all $\lambda_b \in \{0,1\}$ and $b' \in B$ . Thus $D^{SL}_{b,f,1}(q)$ is closed. Moreover, if $(y_1,d_1)$ and $(y_2,d_2)$ are elements in $D^{SL}_{b,f,1}(q)$ then if $(y,d)=t(y_1,d_1)+(1-t)(y_2,d_2)$ with $t \in (0,1)$ it follows easily that

$$y(1-\gamma)+b(1-\lambda_b)-d-q_{b',d,h}b' \leq 0$$

and, thus, $(y,d) \in D^{SL}_{b,f,1}(q)$. So $D^{SL}_{b,f,1}(q)$ is convex.

Assume now that $h=0$ and let $d \in D$ be fixed. Let $y_1$ and $y_2$ with $y_1 < y_2$ be in $D^{SL}_{b,f,0}(d;q)$. Therefore

$$\begin{aligned}\psi_{sl}(y_i,d)(c_i^*,b_i'^*,f_i'^*,h_i'^*,1,0) \geq \max \quad & \{\psi_{nodef}(y_i,d)(c,b',f',h',0,0), \\ & \psi_{cc}(y_i,d)(c,b',h',0,1), \\ & \psi_{both}(y,d)(c,b',h',1,1)\}\end{aligned} \tag{4}$$

for all $(c,b',f',h',0,0),(c,b',f',h',0,1),(c,b',f',h',1,1) \in B_{b,f,0}(d,y_i;q)$, $i=1,2$. Let $y \in (y_1,y_2)$ and assume, by contradiction, that $y \notin D^{SL}_{b,f,0}(d;q)$. Assume, without loss of generality, that the agent chooses not to default on either market, i.e.

$$\psi_{sl}(y,d)(c,b',f',h',1,0) < \psi_{nodef}(y,d)(c^*,b'^*,f'^*,h'^*,0,0), \tag{5}$$

for all $(c,b',f',h',1,0) \in B_{b,f,0}(d,y;q)$, where $(c^*,b'^*,f'^*,h'^*,0,0) \in B_{b,f,0}(d,y;q)$ is the optimal choice for the maximization problem. Let $\bar{c}_1 = c^* - (y - y_1)$. If $\bar{c}_1 \leq 0$ then $\bar{c}_1 < y_1 + b$ and thus

$$c^* = \bar{c}_1 + (y - y_1) < y_1 + b + (y - y_1) = y + b. \tag{6}$$

If $\bar{c}_1 > 0$ we have that $(\bar{c}_1, b'^*, f'^*, h'^*, 0, 0) \in B_{b,f,0}(d,y_1;q)$ and, thus,

$$\psi_{sl}(y_1,d)(c_1^*,b_1'^*,f_1'^*,h_i'^*,1,0) \geq \psi_{nodef}(y_1,d)(\bar{c},b'^*,f'^*,h'^*,0,0).$$

Therefore

$$U(y_1+b) + \beta\rho \int v_{b_1'^*,f_1'^*,1}(d,y';q)\Phi(dy') \geq U(\bar{c}_1) + \beta\rho \int v_{b'^*,f'^*,0}(d,y';q)\Phi(dy'), \tag{7}$$

Subtracting (7) from (5) we have that

$$U(y+b) - U(y_1+b) < U(c^*) - U(\bar{c}_1).$$

Since $(y+b) - (y_1+b) = y - y_1 = c^* - \bar{c}_1$ and $U$ is strictly concave it follows that $c^* < y + b$.

Consider now $\bar{c}_2 = c^* + (y_2 - y)$. Then $(\bar{c}_2, b'^*, f'^*, h'^*, 0, 0) \in B_{b,f,0}(d,y_2;q)$ and thus

$$U(y_2+b) + \beta\rho \int v_{b_2'^*,f_2'^*,1}(d,y';q)\Phi(dy') \geq U(\bar{c}_2) + \beta\rho \int v_{b^*,f^*,0}(d,y';q)\Phi(dy'). \tag{8}$$

Using inequalities (5), and (8) we obtain that

$$U(y_2+b) - U(y+b) > U(\bar{c}_2) - U(c^*).$$

Thus $c^* > y + b$, and we obtain a contradiction with $c^* < y + b$. Therefore $y \in D^{SL}_{b,f,0}(d;q)$ and, thus, $D^{SL}_{b,f,0}(d;q)$ is an interval. It is also a closed set because the maps $\psi_{sl}$, $\psi_{both}$, $\psi_{cc}$, and $\psi_{nodef}$ are continuous with respect to $y$. Thus, $D^{SL}_{b,f,0}(d;q)$ is a closed interval. □

**Theorem 4.** *Let $q \in Q$, $(b,f,0) \in \mathcal{S}$. If $D^{CC}_{b,f,0}(d;q)$ is nonempty then it is a closed interval for all $d$.*

*Proof.* If $b \geq 0$ then $D^{CC}_{b,f,0}(d;q)$ is empty. If $b < 0$ the proof of the theorem is very similar with the proof of Theorem 3 and we will omit it. □

**Theorem 5.** *Let $q \in Q$, $(b, f, 0) \in \mathcal{S}$. If the set $D^{Both}_{b,f,0}(d; q)$* is nonempty then it i*s a closed interval* for all $d$.

*Proof.* If $b \geq 0$ then the set $D^{Both}_{b,f,0}(d; q)$ is empty. For $b < 0$ the proof is similar with the proof of Theorem 3. □

**Theorem 6.** *For any price $q \in Q$, $d \in D$, $f \in F$, and $h \in H$, the sets $D^{CC}_{b,f,h}(d; q)$ expand when $b$ decreases.*

*Proof.* Let $b_1 > b_2$. Then

$$
\begin{aligned}
\big\{(c, b', f', h', 0, 1) \in B_{b_1,f,h}(d, y; q)\big\} &= \big\{(c, b', f', h', 0, 1) \in B_{b_2,f,h}(d, y; q)\big\},\\
\big\{(c, b', f', h', 1, 1) \in B_{b_1,f,h}(d, y; q)\big\} &= \big\{(c, b', f', h', 1, 1) \in B_{b_2,f,h}(d, y; q)\big\},\\
\big\{(c, b', f', h', 0, 0) \in B_{b_1,f,h}(d, y; q)\big\} &\supseteq \big\{(c, b', f', h', 0, 0) \in B_{b_2,f,h}(d, y; q)\big\},\\
\big\{(c, b', f', h', 1, 0) \in B_{b_1,f,h}(d, y; q)\big\} &\supseteq \big\{(c, b', f', h', 1, 0) \in B_{b_2,f,h}(d, y; q)\big\}.
\end{aligned}
$$

Thus, if for $b_1$,

$$
\begin{aligned}
\psi_{cc}(y, d)(c^*, b'^*, f'^*, h'^*, 0, 1) \geq \max \ &\big\{\psi_{nodef}(y, d)(c, b', f', h', 0, 0),\\
&\psi_{sl}(y, d)(c, b', h', 1, 0),\\
&\psi_{both}(y, d)(c, b', h', 1, 1)\big\},
\end{aligned}
$$

it follows that the same inequality will hold for $b_2$ as well. Therefore, $D^{CC}_{b_1,f,h}(d; q) \subseteq D^{CC}_{b_2,f,h}(d; q)$. □

**Theorem 7.** *For any price $q \in Q$, $b \in B$, $f \in F$, and $h \in H$, the sets $D^{CC}_{b,f,h}(d; q)$ shrink and $D^{Both}_{b,f,h}(d; q)$ expand when $d$ increases.*

*Proof.* Let $d_1 < d_2$. Then

$$
\begin{aligned}
\big\{(c, b', f', h', 0, 1) \in B_{b,f,h}(d_1, y; q)\big\} &\supseteq \big\{(c, b', f', h', 0, 1) \in B_{b,f,h}(d_2, y; q)\big\},\\
\big\{(c, b', f', h', 1, 1) \in B_{b,f,h}(d_1, y; q)\big\} &= \big\{(c, b', f', h', 1, 1) \in B_{b,f,h}(d_2, y; q)\big\},\\
\big\{(c, b', f', h', 0, 0) \in B_{b,f,h}(d_1, y; q)\big\} &\supseteq \big\{(c, b', f', h', 0, 0) \in B_{b,f,h}(d_2, y; q)\big\},\\
\big\{(c, b', f', h', 1, 0) \in B_{b,f,h}(d_1, y; q)\big\} &= \big\{(c, b', f', h', 1, 0) \in B_{b,f,h}(d_2, y; q)\big\}.
\end{aligned}
$$

Thus, if

$$
\begin{aligned}
\psi_{both}(y, d_1)(c^*, b'^*, f'^*, h'^*, 1, 1) \geq \max \ &\big\{\psi_{nodef}(y, d_1)(c, b', f', h', 0, 0),\\
&\psi_{sl}(y, d_1)(c, b', h', 1, 0),\\
&\psi_{cc}(y, d_1)(c, b', h', 0, 1)\big\},
\end{aligned}
$$

it follows that the same inequality holds for $d_2$. Therefore, $D^{Both}_{b,f,h}(d_1;q) \subseteq D^{Both}_{b,f,h}(d_2;q)$. On the other hand, if

$$\begin{aligned} \psi_{cc}(y,d_1)(c^*,b'^*,f'^*,h'^*,0,1) \geq \max \ & \big\{\psi_{nodef}(y,d_1)(c,b',f',h',0,0), \\ & \psi_{sl}(y,d_1)(c,b',h',1,0), \\ & \psi_{both}(y,d_1)(c,b',h',1,1)\big\}, \end{aligned}$$

the inequalities can reverse for $d_2$. Therefore $D^{CC}_{b,f,h}(d_1;q) \supseteq D^{CC}_{b,f,h}(d_2;q)$. □

**Theorem 8.** *For any price $q \in Q$, $b \in B$, $d \in D$, and $f \in F$, the set $D^{CC}_{b,f,0}(d;q) \subset D^{CC}_{b,f,1}(d;q)$.*

*Proof.* Let $y \in Y$. For $h = 1$ we have that

$$\big\{(c,b',f',h',1,1) \in B_{b,f,1}(d,y;q)\big\} = \emptyset$$

and

$$\big\{(c,b',f',h',1,0) \in B_{b,f,1}(d,y;q)\big\} = \emptyset.$$

Therefore, if for $f = 0$ we have that

$$\begin{aligned} \psi_{cc}(y,d_1)(c^*,b'^*,f'^*,h'^*,0,1) \geq \max \ & \big\{\psi_{nodef}(y,d_1)(c,b',f',h',0,0), \\ & \psi_{sl}(y,d_1)(c,b',h',1,0), \\ & \psi_{both}(y,d_1)(c,b',h',1,1)\big\}, \end{aligned}$$

then the same inequalities hold for $f = 1$. □

## A.3 Proofs of Theorems 9 and 10

**Theorem 9. Existence** *A steady-state competitive equilibrium exists.*

*We see that once $q^*$ is known, then all the other components of the equilibrium are given by the formulas in Definition 2. We can rewrite part 5 of the Definition as*

$$\begin{aligned} q^*_{d,h,b'} &= \frac{\rho}{1+r}(1 - p^b_{d,h,b'}) \\ &= \frac{\rho}{1+r}\left(1 - \int \lambda^*_b(y',d,0,b',h',q^*)\phi(dy')H^*(h,dh')\right), \end{aligned}$$

*where $\lambda^*_b$ and $f'^*$ are measurable selections guaranteed by Theorem 1, and $H^*$ is the transition matrix provided by Theorem 1. Thus $q^*$ is a fixed point of the map $T : [0, q_{\max}]^{N_D \times N_H \times N_B} \mapsto$*

$[0, q_{\max}]^{N_D \times N_H \times N_B}$

$$T(q)(d,h,b') = \frac{\rho}{1+r}\left(1 - \int \lambda_b^*(y',d,0,b',h',q)\phi(dy')H^*(h,dh')\right). \tag{9}$$

*Since* $Q := [0, q_{\max}]^{N_D \times N_H \times N_B}$ *is a compact convex subset of* $\mathbb{R}^{N_D \times N_H \times N_B}$ *we can apply the Schauder theorem (Theorem V.19 of Reed and Simon (1972)) if we prove that the map*

$$q \mapsto \int \lambda_b^*(y',d,0,b',h',q)\phi(dy')H^*(h,dh')$$

*is continuous.*

*Before starting the proof we remark that the above map is well defined because even though apriori the transition matrix* $H^*$ *depends on* $(y,d,b,f,q)$*, in fact, knowing the pair* $(h,b')$ *completely determines* $H^*(h,dh')$ *when* $b' < 0$*. If* $b' < 0$ *then* $f = 0$*,* $\lambda_d^* = 0$*. Thus* $H^*(0,0) = 1$*,* $H^*(0,1) = 0$*,* $H^*(1,0) = p_h$ *and* $H^*(1,1) = 1 - p_h$*. Also, if* $b' \geq 0$ *then* $p^b_{d,h,b'} = 0$ *by definition.*

*We begin by showing that the sets of discontinuities of* $\lambda_b^*(\cdot,q)$ *and* $b^*(\cdot,q)$ *,* $q \in Q$*, and* $\lambda_b^*(x,\cdot)$ *and* $b^*(x,\cdot)$*,* $x \in X$*, have measure* 0*. This will follow from the following lemmas. Let us begin by noticing that the sets of discontinuities of these functions are contained in the sets of indifference.*

*We fix* $b \in B$*,* $f \in F$*,* $h \in H$*,* $d \in D$*, and* $q \in Q$ *and we will suppress the dependence of functions on these variables. That is, we study the behavior with respect to* $y$*. Since* $B, F, H$*, and* $D$ *are finite sets this will suffice to prove the continuity of* $\lambda_b^*(\cdot,q)$*. The first step is to study in more detail the maximization problem on the no default path. Recall that*

$$\psi_{nodef}(y,d)(c,b',f',h',0,0) = U(c) + \beta\rho \int v_{b',0,0}(d,y';q)\Phi(dy')$$

*for all* $(c,b',0,0,0,0) \in B_{b,f,h}(d,y;q)$*. For* $y \in Y$ *we write* $b'(y)$ *for the the values of* $b'$ *that maximize* $\psi_{nodef}$*. Recall that* $b,f,h,d$*, and* $q$ *are fixed and that* $b'(y)$ *can be a correspondence. Since* $t$ *is a lump sum tax that is paid by every agent in the economy, it does not affect the choices. For simplicity we assume that* $t = 0$ *in the following.*

**Lemma A4.** *Let* $b \in B$*,* $f \in F$*,* $h \in H$*,* $d \in D$*, and* $q \in Q$ *be fixed. Then for any* $y_0 \in Y$ *there is* $\varepsilon > 0$ *such that the following holds:*

1. *If* $b'(y_0)$ *is a single valued then* $b'$ *is constant and single valued on* $(y_0 - \varepsilon, y_0 + \varepsilon)$*.*

2. *If* $b'(y_0)$ *is multi-valued then either* $b'(y)$ *is single valued on* $(y_0 - \varepsilon, y_0 + \varepsilon) \setminus \{y_0\}$ *and there is* $\bar{b} \in b'(y_0)$ *such that* $b'(y) = \bar{b}$ *for all* $y \in (y_0 - \varepsilon, y_0 + \varepsilon) \setminus \{y_0\}$*, or* $b'(y) = b'(y_0)$ *for all* $y \in (y_0 - \varepsilon, y_0 + \varepsilon)$ *.*

*Proof.* If $b'(y_0)$ is single valued, then

$$U(y_0 + b - d - q_{d,h,b'(y_0)} b'(y_0)) + \beta\rho \int v_{b'(y_0),0,0,}(d, y'; q)\Phi(dy') \quad > \quad U(y_0 + b - d - q_{d,h,b'} b') + \beta\rho \int v_{b',0,0}(d, y'; q)\Phi(dy'), \tag{10}$$

for all $b' \in B \setminus \{b'(y_0)\}$ (the right hand side is $-\infty$ if $(c, b', 0, 0, 0, 0) \notin B_{b,f,h}(y_0, d; q)$, where, here, $c = y_0 + b - d - q_{d,h,b'} b')$. Then, since $B(f)$ is finite and $U$ is continuous with respect to $y$, we can find $\varepsilon > 0$ such that if $|y - y_0| < \varepsilon$ then

$$U(y + b - d - q_{d,h,b'(y_0)} b'(y_0)) + \beta\rho \int v_{b'(y_0),0,0}(d, y'; q)\Phi(dy') \quad > \quad U(y + b - d - q_{d,h,b'} b') + \beta\rho \int v_{b',0,0}(d, y'; q)\Phi(dy'), \tag{11}$$

for all $b \in B(f) \setminus \{b'(y_0)\}$. Thus $b'(y) = b'(y_0)$ for all $|y - y_0| < \varepsilon$.

Suppose now that $b'(y_0)$ is multi-valued. WLOG, assume that $b'(y_0)$ consists of two elements $b'_1$ and $b'_2$ (we can assume this since $B$ is finite). Then

$$U(y_0 + b - d - q_{d,h,b'_1} b'_1) + \beta\rho \int v_{b'_1,0,0}(d, y'; q)\Phi(dy') \quad = \quad U(y_0 + b - d - q_{d,h,b'_2} b'_2) + \beta\rho \int v_{b'_2,0,0}(d, y'; q)\Phi(dy')$$

and they both satisfy inequality (10) for all $b' \in B \setminus \{b'_1, b'_2\}$. There is $\varepsilon > 0$ such that if $|y - y_0| < \varepsilon$, then (11) is satisfied for both $b'_1$ and $b'_2$. We need to compare, thus, $U(y + b - d - q_{d,h,b'_1} b'_1) + \beta\rho \int v_{b'_1,0,0}(d, y'; q)\Phi(dy')$ and $U(y + b - d - q_{d,h,b'_2} b'_2) + \beta\rho \int v_{b'_2,0,0}(d, y'; q)\Phi(dy')$. If $q_{d,h,b'_1} b'_1 = q_{d,h,b'_2} b'_2$, then it follows that $\int v_{b'_1,0,0}(d, y'; q)\Phi(dy') = \int v_{b'_2,0,0}(d, y'; q)\Phi(dy')$. Therefore

$$U(y + b - d - q_{d,h,b'_1} b'_1) + \beta\rho \int v_{b'_1,0,0}(d, y'; q)\Phi(dy') \quad = \quad U(y + b - d - q_{d,h,b'_2} b'_2) + \beta\rho \int v_{b'_2,0,0}(d, y'; q)\Phi(dy')$$

for all $y$. Thus $b'(y) = b'(y_0)$ for all $y \in (y_0 - \varepsilon, y_0 + \varepsilon)$. Suppose now that $q_{d,h,b'_!} b'_1 < q_{d,h,b'_2} b'_2$. Then

$$s_0 := y_0 + b - d - q_{d,h,b'_1} b'_1 > y_0 + b - d - q_{d,h,b'_2} b'_2 =: t_0.$$

Assume that $\varepsilon$ is so that $t_0 + \varepsilon < s_0 - \varepsilon$. Then, if $|y - y_0| < \varepsilon$ we have that $t_0 < y + b - d - q_{d,h,b'_1} b'_1 =: s_1$,

$t_1 := y + b - d - q_{d,h,b'_2} b'_2 < s_0$, and $t_1 < s_1$. Then we have

$$\begin{aligned} U(t_1) + \beta\rho \int v_{b'_2,0,0}(d, y'; q)\Phi(dy') &= U(t_1) - U(t_0) + U(t_0) + \beta\rho \int v_{b'_2,0,0}(d, y'; q)\Phi(dy') \\ &= U(t_1) - U(t_0) + U(s_0) + \beta\rho \int v_{b'_1,0,0}(d, y'; q)\Phi(dy') \\ &= U(t_1) - U(t_0) + U(s_0) - U(s_1) \\ &+ U(s_1) + \beta\rho \int v_{b'_1,f',0}(d, y'; q)\Phi(dy'). \end{aligned}$$

Since $U$ is strictly concave, $t_0 < s_0$, $t_0 < s_1$, $t_1 < s_1$, $t_1 < s_0$, and $t_1 - t_0 = s_1 - s_0 = y - y_0$, it follows that $U(t_1) - U(t_0) > U(s_1) - U(s_0)$. Thus

$$U(t_1) + \beta\rho \int v_{b'_2,f',0}(d, y'; q)\Phi(dy') > U(s_1) + \beta\rho \int v_{b'_1,f',0}(d, y'; q)\Phi(dy')$$

and $b'_2$ is *the only* solution to the maximization problem. Therefore $b'$ is single valued and equals $b'_2$ on $(y_0 - \varepsilon, y_0 + \varepsilon) \setminus \{y_0\}$. The case $q_{d,h,b'} b'_1 > q_{d,h,b'_2} b'_2$ is similar. □

**Lemma A5.** *Let $b \in B$, $f \in F$, $h \in H$, $d \in D$, and $q \in Q$ be fixed. Suppose that $y_1$ is a point of indifference between not defaulting and defaulting on student loans. Then, if $\varepsilon$ is small enough, either there is no other point $y$ of indifference with $|y - y_1| < \varepsilon$ or all $y \in (y_1 - \varepsilon, y_1 + \varepsilon)$ are points of indifference.*

*Proof.* Let $\varepsilon > 0$ be such that for all $y \in Y$ with $|y - y_1| < \varepsilon$ we have that $b'(y) = b'(y_1) =: b'$. We can find such an $\varepsilon$ by Lemma (A4): if $b'(y_1)$ is single-valued, then this is the first part of the lemma; if $b'(y_1)$ is multi-valued, the second part of the lemma implies that we can pick $\bar{b} \in b'(y_1)$ such that $\bar{b} \in b'(y)$ or $b'(y) = \bar{b}$ for all $y \in (y_1 - \varepsilon, y_1 + \varepsilon)$. We will consider $b'(y) = \bar{b}$ in both cases (note that this choice does not alter the measurability of $b'^*$). Assume first that $d \neq q_{d,h,b'} b'$, which implies that $c_1 \neq y_1 + b$, and assume, by contradiction, that $y_2$ is another point of indifference and the distance between $y_1$ and $y_2$ is smaller than $\varepsilon$. Then

$$U(c_1) + \beta\rho \int v_{b',0,0}(d, y'; q)\Phi(dy') = U(y_1 + b) + \beta\rho \int v_{0,0,1}(d, y'; q)\Phi(dy')$$

and

$$U(c_2) + \beta\rho \int v_{b',0,0}(d, y'; q)\Phi(dy') = U(y_2 + b) + \beta\rho \int v_{0,0,1}(d, y'; q)\Phi(dy').$$

Therefore $U(c_1) - U(c_2) = U(y_1 + b) - U(y_2 + b)$. However, we have that

$$c_1 - c_2 = y_1 - y_2 = (y_1 + b) - (y_2 + b).$$

This is a contradiction with $U$ being strictly concave. If $d = q_{d,h,b'}b'$ then $c_1 = y_1 + b$, and, hence, $c = y + b$ for all $y$, then all points $y$ with $|y - y_1| < \varepsilon$ are indifference points. □

*The above lemma holds also for for all types of indifference. Thus, since $Y$ is compact, if we fix $d$ and $q$, there are only a finite number of earning levels that are discontinuity points for $\lambda_d^*, \lambda_b^*$, and $b^*$.*

**Lemma A6.** *The set of pairs $\{y, d\}$ that are points of discontinuity for $\lambda_d^*, \lambda_b^*$, and $b^*$ has measure* 0.

*Proof.* Lemma A5 implies that we can change the maps in a Borel way so that for each $d \in D$ the set of $y \in Y$ for which these maps are discontinuous is finite. The conclusion follows now since $D$ is finite. □

*Proof. of Theorem 9* Let $\{q_n\}_{n\in\mathbb{N}} \subset Q$ be a sequence that converges to $q$. We will show that $\lim_{n\to\infty} \lambda_b^*(y, d, f, b, h, q_n) = \lambda_b^*(y, d, f, b, h, q)$ almost everywhere. Since the sequence $\{q_n\}$ is countable, by Lemma A5 we can find a set $E \subset X$ of measure 0 that contains all the points of indifference for the prices $q_n$, $n \in \mathbb{N}$, and $q$. Let $(y, d, f, b, h) \in X \setminus E$ be fixed. Since $v_{b,f,h}(d, y; \cdot)$ is continuous and $Q$ is a compact space it follows that $v_{b,f,h}(d, y; \cdot)$ is uniformly continuous. Therefore, since $B$ is finite, there is $\delta > 0$ such that if $\|q' - q''\| < \delta$ and

$$\begin{aligned}\psi_{nodef}(q')(c^*, b^*, f', h', 0, 0) > \max\Big\{ &\max_{(c,b',f',h',1,0)} \psi_{sl}(q')(c, b', f', h', 1, 0), \\ &\max_{(c,b',f',h',0,1)} \psi_{cc}(q')(c, b', f', h', 0, 1), \\ &\max_{(c,b',f',h',1,1)} \psi_{both}(q')(c, b', f', h', 1, 1)\Big\}\end{aligned}$$

then the same inequality holds for $q''$. In the inequality above we suppressed the dependence on $(y, d, f, b, h)$ to simplify the notation. Thus, if $\lambda_b^*(y, d, f, b, h, q') = 0$ and $\lambda_d^*(y, d, f, b, h, q') = 0$ then $\lambda_b^*(y, d, f, b, h, q'') = 0$ and $\lambda_d^*(y, d, f, b, h, q'') = 0$. Similar statements hold for all possible combinations of values of $\lambda_b^*$ and $\lambda_d^*$. Therefore, by shrinking $\delta$ if necessary, we have that if $\|q' - q''\| < \delta$ then $\lambda_b^*(y, d, f, b, h, q') = \lambda_b^*(y, d, f, b, h, q'')$. This implies that $\lim_{n\to\infty} \lambda_b^*(y, d, f, b, h, q_n) = \lambda_b^*(y, d, f, b, h, q)$ for all $(y, d, f, b, h, q) \in X \setminus E$. Finally, since $|\lambda_b^*(y, d, f, b, h, q)| \leq 1$ and $X$ is a compact space, the Lebesgue's Dominated Convergence Theorem (see, for example, (Rudin, 1987, Theorem 1.34)) implies that

$$\lim_{n\to\infty} \int \lambda_b^*(y', d, f', b', h', q_n)\Phi(dy)H(h, dh') = \int \lambda_b^*(y', d, f', b', h', q_n)\Phi(dy)H(h, dh').$$

Thus the map $T$ defined in (9) is continuous and, hence, has a fixed point. □

**Theorem 10.** *In any steady-state equilibrium the following is true:*

1. *For any* $b^{'} \geq 0$, $q^*_{d,h,b'} = \rho/(1+r)$ *for all* $d \in D$ *and* $h \in H$.

2. *If the grids of* $D$ *and* $B$ *are sufficiently fine, and* $h = 0$ *there are* $\underline{d} > 0$ *and* $\underline{b}' < 0$ *such that* $q^*_{d,h,b'} = \rho/(1+r)$ *for all* $d < \underline{d}$ *and* $b^{'} > \underline{b}'$.

3. *If the set of income levels for which the household is indifferent between defaulting on credit card debt and any other available option is of measure zero, then* $d_1 < d_2$ *implies* $q^*_{d_1,h,b'} > q^*_{d_2,h,b'}$ *for any* $h \in H$ *and* $b' \in B$.

4. *If the set of income levels for which the household is indifferent between defaulting on credit card debt and any other available option is of measure zero, then* $q^*_{d,h=1,b'} > q^*_{d,h=0,b'}$ *for any* $d \in D$ *and* $b' \in B$.

*Proof.* The first part follows from part 5) of the definition of an equilibrium.

For the second part, assume that there are $b_1 < 0$ and $\underline{d} > 0$ such that $y + b_1 - d_1 > 0$ for all $y \in Y$ and consider any household with $b_1 < b < 0$ and $0 < d < \underline{d}$. In particular the household must have a clean default flag on the credit card market and on the student loan market. If an household with debt $b < 0$ defaults only on the credit card market then its utility is

$$u(y-d) - \tau_b + \beta\rho \int u\big(y' - d - q^*_{b'^*(d,y';q)(b,0,0),d,0} b'^*(d,y';q)(b,0,0)\big)\Phi(dy') \\ + (\beta\rho)^2 \int (1-p_f)\omega_{b'^*(d,y';q)(b,0,0),1,0}(q^*,d) + p_f \omega_{b'^*(d,y';q)(b,0,0),0,0}(q^*,d)\Phi(dy').$$

On the other hand, one feasible action of the household is to not default on any market, pay off the debt and save in the following period $b'^*(d,y';q)(b,0,0)$. The utility from this course of action is

$$u(y+b-d) + \beta\rho \int u\big(y' - d - q^*_{b'^*(d,y';q)(b,0,0),d,0} b'^*(d,y';q)(b,0,0)\big)\Phi(dy') \\ + (\beta\rho)^2 \int \omega_{b'^*(d,y';q)(b,0,0),0,0}(q^*,d)\Phi(dy').$$

Then property 3) of Definition A1 implies that the utility gain by not defaulting is at least

$$u(y+b-d) - u(y-d) + \tau_b.$$

Assuming that the grid of $B$ is sufficiently fine so that we can find $\underline{b} > b_1$ such that the above expression is positive for all $b > \underline{b}$ and $d < \underline{d}$ the conclusion follows. The proof for the case when the household defaults on both markets is similar.

Assuming that the set of income levels for which the household is indifferent between defaulting on credit card debt and any other available option, Theorem 7 implies that if $d_1 < d_2$ then $p^{b*}_{d_1,h,b'} \leq p^{b*}_{d_2,h,b'}$ for any $h \in H$ and $b' \in B$. The third part of the theorem follows. One can similarly prove the last part of the theorem. $\square$

## A.4 Proof of Theorem 11

**Theorem 11.** *If the grids of $D$ and $B$ are fine enough, then we can find $d_1 \in D$ and $b_1 \in B$ such that the agent defaults. Moreover, we can find $d_2 \geq d_1$ and $b_2 \leq b_1$ such that the agent defaults on student loans.*

*Proof.* Suppose that $D$ is fine enough so that we can find $d_1 > 0$ such that given $A > 1$ to be specified below we have that $|u'(y - d_1)| \geq A$ for all $y \in Y$ such that $y > d_1$. Since $q_{\max} < 1$ then we can find $b_1 < 0$ such that $b - q_{\max} b' < 0$ for all $b' \in B$. The utility from defaulting on the credit card for $b_1$ is

$$u(y - d_1) - \tau_b + \beta\rho\omega_{0,1,0}(q^*, d_1)$$

and the utility from not defaulting on either path is

$$u(y + b_1 - d_1 - q_{b'^*(d,y;q)(b,f,h),d_1,h} b'^*(d, y; q)(b, f, h)) + \beta\rho\omega_{b'^*(d,y;q)(b,f,h),d_1,h}(q^*, d_1).$$

Using the mean value theorem we can find $c'$ such that $y + b_1 - d_1 - c_{b'^*(d,y;q)(b,f,h),d_1,h} b'^*(d, y; q)(b, f, h) < c' < y - d_1$ and

$$u(y-d_1) - u(y+b_1-d_1-q_{b'^*(d,y;q)(b,f,h),d_1,h} b'^*(d, y; q)(b, f, h)) = u'(c')(b_1-q_{b'^*(d,y;q)(b,f,h),d_1,h} b'^*(d, y; q)(b, f, h)).$$

In particular, $|u'(c')| > A$. We chose $A$ such that

$$A(q_{b'} b' - b_1) > \tau_b + \beta\rho(\omega_{b'^*(d,y;q)(b,f,h),d_1,h}(q^*, d_1) - \omega_{0,1,0}(q^*, d_1)),$$

for all $b' \in B$. It follows that the utility from defaulting on credit card is higher than the utility of not defaulting at all.

Suppose now that the grids of $D$ and $B$ are fine enough so that we can find $d_2$ and $b'_2$ such that $u(y + b'_2) - u(y - d_2) - \tau_d + \tau_b$ is zero or as close to zero as we want. That is, the agent's current utility from defaulting on student loans or credit card are basically the same. Then, if an agent chooses to default on the credit card market today, in the next period her utility will be

$$u(y' - d_2 - q_{d_2,0,b^{"*}_{CC}} b^{"*}_{CC}) + \beta\rho\big((1 - p_f)\omega_{b^{"*}_{CC},0,1}(d_2, q^*) + p_f \omega_{b^{"*}_{CC},0,0}(d_2, q^*)\big),$$

where $b^{"*}_{CC} \geq 0$. If the agent chooses to default on student loans, she can chose to borrow $b^{"}_2 < 0$ such that $y'(1-\gamma) - d_2 - q_{b^{"}_2} b^{"}_2 > y' - d_2 - q_{d_2,0,b^{"*}_{CC}} b^{"*}_{CC}$ and $|u'(y'(1-\gamma) - d_2 - q_{b^{"}_2} b^{"}_2)| > B$, where $B$ is so that

$$\begin{aligned} u'(c')(-\gamma y' - q_{b^{"}_2} b^{"}_2 + q_{d_2,0,b^{"*}_{CC}} b^{"*}_{CC}) &\geq (1-p_h)\omega_{b^{"}_2,0,1}(d_2, q^*) + p_h \omega_{b^{"}_2,0,0}(d_2, q^*) \\ &- \big((1-p_f)\omega_{b^{"*}_{CC},0,1}(d_2, q^*) + p_f \omega_{b^{"*}_{CC},0,0}(d_2, q^*)\big). \end{aligned}$$

Thus, if $b_2 = \min\{b'_2, b''_2\}$ it follows that the agent chooses to default on student loans. □