Finance and Economics Discussion Series
Divisions of Research & Statistics and Monetary Affairs
Federal Reserve Board, Washington, D.C.

The Welfare Costs of Skill-Mismatch Employment

David M. Arseneau and Brendan Epstein

2014-042

The Welfare Costs of Skill-Mismatch Employment*

David M. Arseneau[†]
Federal Reserve Board

Brendan Epstein[‡]
Federal Reserve Board

First Draft: April, 2014
This Draft: June 5, 2014

Abstract

Skill-mismatch employment occurs when high-skilled individuals accept employment in jobs for which they are over-qualified. These employment relationships can be beneficial because they allow high-skilled individuals to more rapidly transition out of unemployment. They come at the cost, however, in the form of lower wage compensation. Moreover, an externality arises as high-skilled individuals do not take into account the affect that their search activity in the market for low-tech jobs has on low-skilled individuals. This paper presents a tractable general equilibrium model featuring mismatch employment and on-the-job search to articulate these tradeoffs. We derive a set of efficiency conditions that describe the labor market distortions associated with these two model features and illustrate how they alter the standard notion of the labor wedges inherent in general equilibrium search models. Finally, we calibrate the model to U.S. data and show that the distortions associated with mismatch employment are largely distributional and can be quantitatively large.

Keywords: Job-to-job transitions, labor market frictions, skill premium
JEL Classification: E24, J31, J64

*The views expressed here are solely those of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.

[†]email address: david.m.arseneau@frb.gov.
[‡]email address: brendan.epstein@frb.gov

1 Introduction

Skill-mismatch employment occurs when the skill set of an individual is not well-aligned with the requirements of the job they were hired to perform. Empirical evidence suggests that this phenomenon has become more prevalent in recent years as labor market slackness resulting from the global recession has increased competition for employment, making it more likely that high-skilled individuals would be willing to settle for lower paying and lower quality jobs.[1] The growth of mismatch employment has important policy implications. Oreopoulos, van Wachter, and Heisz (2012) argue that over-qualification can lead to long lasting scarring effects for those that find themselves in skill-mismatch employment. In addition, there may be externalities that arise as the search behavior of high-skilled job seekers crowds out that of the low-skilled. This crowding out could prolong the recovery of the labor market, particularly for low-skilled individuals.

In this paper, we develop a general equilibrium model to better understand the labor market distortions associated with skill-mismatch. Our model builds on the seminal work of Mortensen and Pissarides (1994) and Pissarides (1999) by introducing two-sided heterogeneity whereby low- and high-skilled job seekers search for employment in two separate labor markets for low- or high-tech jobs, respectively. Low-skilled individuals are only qualified for low-tech jobs. Skill-mismatch is defined as a situation in which a high-skilled job seeker accepts a position with a low-tech firm. As in Dolado, Jansen, and Jimeno (2009), such an outcome leads to "permanent" mismatch if on-the-job (OTJ) search is not possible. On the other hand, mismatch is "transitory" when OTJ search is possible and leads to job-to-job (JTJ) transitions by high-skilled workers out of mismatch employment and into a higher paying job in the high-tech industry.

Within this framework, our model captures the trade-off that high-skilled individuals face between accepting a lower quality job in order to move out of unemployment more quickly, but doing so at the cost of having to accept a lower wage in a job for which they are over-qualified. Moreover, the model reveals an externality associated with increased competition for low-tech jobs that crowds out the search activity of low-skilled individuals. This crowding out externality is distinct from the standard congestion externality that arises from inefficient division of the match surplus.

The paper makes two main contributions. First, we derive a set of efficiency conditions that fully characterize the distortions generated by both permanent and transitory mismatch. Previously, Arseneau and Chugh (2012) showed that general equilibrium efficiency in an economy where the labor market is characterized by search and matching frictions is described by a set of static and dynamic conditions for efficiency in the labor market. Those authors derived a set of search-based labor wedges to illustrate how the standard congestion externality as well as, separately,

[1] See, for example, World Economic Forum Global Agenda Council on Employment (2014) and Estevão and Tsounta (2011).

the presence of unemployment benefits can distort these two margins. This paper extends those earlier results to a more general setting with mismatch and OTJ search. In our more general setting labor market efficiency is described by a set of two static and three dynamic efficiency conditions. We show that permanent mismatch distorts the labor market even in absence of the standard congestion externality and unemployment benefits and that this distortion is amplified through the introduction of OTJ search.

Specifically, our theoretical results show that the dynamic margin for mismatch job creation is always distorted in the private equilibrium. Provided mismatch is permanent, this distortion only spills over to the low-tech labor market. Intuitively, the reason is because high-skilled individuals do not internalize the fact that their participation in the market for low-tech jobs makes it more difficult for low-skilled job seekers to successfully find employment. Transitory mismatch amplifies this distortion as OTJ search spreads the inefficiency across nearly all aspects of the labor market, additionally affecting both the static and dynamic efficiency conditions for high-tech job creation. Finally, we show that reintroducing congestion externalities and unemployment benefits cause these well-understood distortions to interact with permanent and transitory mismatch in a complicated way.

The second main contribution is to measure the quantitative magnitude of these various distortions in a carefully calibrated version of the model. We make use of data from the Bureau of Labor Statistics (BLS) on educational attainment to calibrate worker heterogeneity and BLS data on employment and wages by occupation to calibrate firm heterogeneity. Our calibration is consistent with a wide set of empirical labor market facts both at the aggregate as well as the disaggregated level. For example, among other things, it captures an empirically realistic skill premium in the wage distribution and it endogenously gives rise to a fraction of employed individuals actively engaged in on-the-job search that is in line with empirical estimates by Fallick and Fleischman (2005).

Our quantitative results show that the welfare effects of mismatch are purely distributional. Permanent mismatch generates welfare gains on the order of 0.2 percent of steady state consumption for the high-skilled household and these gains come at the expense of the low-skilled households. Introducing OTJ search amplifies the welfare effects for both types of households, but the amplification of the welfare costs is particularly pronounced for the low-skilled household. Our results show the transitory component of mismatch doubles the welfare gains for high-skilled households to just under $\frac{1}{2}$ percent of steady state consumption and raises the welfare costs for low-skilled households nearly seven-fold to roughly 1.4 percent. From a policy perspective, one conclusion to take from this is that the concern regarding mismatch primary manifests as a transitory issue, as opposed to a longer-lasting structural labor market issue. We also illustrate that mismatch has only a small influence on wage inequality and may, in fact, compress the skill premium.

In terms of related literature, our paper builds on a strand of the labor search and matching literature that studies the impact of OTJ search on wages, unemployment, and vacancies.[2] More narrowly, our focus on skill mismatch with two-sided heterogeneity ties our paper to Albrecht and Vroman (2002), Gautier (2002), Dolado, Jansen, and Jimeno (2009), Khalifa (2010), and Chassamboulli (2011), which are representative of a literature that studies the impact of two-sided heterogeneity on differences in wages, employment levels, and the persistence of unemployment rates across skill groups.[3] Our model is similar to Dolado, Jansen, and Jimeno (2009) in many respects, but the focal point of our analysis is different because we are interested in the efficiency properties of mismatch employment. This focus on efficiency leads us to introduce three modeling features that are not jointly present in previous research: (1.) a general equilibrium framework; with (2.) endogenous labor force participation on the part of households; and (3.) directed search on the part of both households and firms.

Another closely related paper is Gautier, Tuelings, and van Vuuren (2010) who also study efficiency in a model with mismatch and OTJ search. However, their analysis is limited to a partial equilibrium model of the labor market. In contrast, the general equilibrium setting in our paper is crucial for a complete accounting of both the static and dynamic distortions associated with mismatch as demonstrated in previous work by Arseneau and Chugh (2008, 2012). That said, our general theoretical results should be viewed as complimentary to Gautier, Tuelings, and van Vuuren (2010) in that we both identify mismatch as a source of inefficiency in the private economy, even in absence of a congestion externality and/or unemployment benefits.

The remainder of the paper is organized as follows. The next section presents the model and describes the competitive search equilibrium. The socially efficient outcome is described in Section 3. The private equilibrium is compared to the socially efficient equilibrium in Section 4, allowing us to define a set of static and dynamic labor market wedges that characterize the distortion generated by mismatch and, separately, on-the-job search. With this understanding in mind, Section 5 uses a calibrated version of the model to produce a quantitative measure of the welfare costs of mismatch in the U.S. economy. Finally, Section 6 concludes.

2 The Model

The model can be thought of loosely as an extension of Dolado, Jansen, and Jimeno (2009) to a general equilibrium setting. That said, as mentioned earlier, we introduce a number of additional

[2] See, for example, Pissarides (1994), Shimer (2003, 2006), Nagypal (2005), and Moscarini (2005), among others.

[3] Krause and Lubik (2006) and Pries (2008) study similar issues with one-sided heterogeneity, while Epstein (2012) considers the effect of two-sided heterogeneity for the propagation of shocks.

modeling features including endogenous labor force participation on the part of households and directed search on the part of both households and firms.

In addition, we use the "instantaneous hiring" view of transitions between search unemployment and employment. Under this timing convention job destruction takes place at the beginning of the period. Then, after observing the period t productivity shocks, households and firms allocate search activity and matches are formed in the frictional labor markets. Finally, production takes place making full use of newly formed matches; as a result, the measurement of unemployment has to take into account the possibility that a searching individual can successfully find a match and be productive within the period. We adopt this timing convention because it allows our analytical results on efficiency—a key contribution of the paper—to be directly comparable to the results reported previously in Arseneau and Chugh (2012).

2.1 Households

The economy is inhabited by a unit mass of individuals, a fraction κ of which are low-skilled and the remaining fraction $1-\kappa$ are high-skilled. Low-skilled individuals are only qualified for performing low-tech jobs, while high-skilled individuals can perform both high- and low-tech jobs. Mismatch occurs when a high-skill individual is matched with a low-tech job, as the surplus arising from this match type is lower than that arising from a high-skill individual with a high-tech job. Because of this surplus differential, in the event of mismatch there is an incentive for OTJ search directed toward the high-skill sector.

Individuals are aggregated into two separate households, differentiated by type. For the sake of convenience, we assume there is aggregate risk sharing across individuals both within and between households.[4] Each household decides how much to consume, the number of state-contingent bonds to hold, and the mass of household members who participate in the labor force. Participants in labor force activity are either employed or actively searching for jobs (unemployed). Employed individuals receive a wage and unemployed individuals receive a constant unemployment flow benefit. Individuals that are outside of the labor force enjoy the utility value of leisure.

2.1.1 Low-Skilled Households

The mass of low-skill individuals participating in the labor force is given by $lfp_t^L = n_t^L + (1-f_t^L)s_t^L$, where: n_t^L denotes the mass of low-skill individuals working in low-tech jobs; s_t^L denotes the mass of low-skill individuals searching in the market for low-tech jobs; and f_t^L is the endogenous probability that an individual searching in the market for low-tech jobs finds a match (discussed below). As

[4] Because of its tractability, this approach has been common in search-theoretic genral equilibrium models of the labor market since Merz (1995) and Andolfatto (1996).

alluded to above, due to the timing of the model the mass of unemployed individuals at the end of the period is given by $u_t^L = (1 - f_t^L)s_t^L$ in order to net out successful search within the period. Leisure obtained by a low-skill household is given by $l_t^L = \kappa - lfp_t^L$.

The low-skilled household chooses sequences of consumption, denoted c_t^L, state-contingent bond holdings, B_t^L, and search activity, s_t^L, to achieve a desired low-tech employment stock, n_t^L, in order to maximize discounted lifetime utility:

$$\max \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(u(c_t^L) - h\left(lfp_t^L\right)\right),$$

where: $\mathbb{E}_t$ is the expectations operator; $\beta \in (0,1)$ is the exogenously determined subjective discount factor; u is utility from consumption, with $u' > 0$ and $u'' < 0$; h is utility from leisure, with $h^{L\prime} > 0$ and $h^{L\prime\prime} < 0$.[5]

Low-skilled households face the following budget constraint

$$c_t^L + B_t^L = w_t^L n_t^L + \chi^L(1 - f_t^L)s_t^L + R_t B_{t-1}^L + \kappa\left(\Pi_t^L + \Pi_t^H\right),$$

where: w_t^L is the wage received by a low-skilled individual employed in a low-tech job; χ^L is an exogenously determined unemployment benefit paid to actively searching low-skilled workers; the real state-contingent bond pays an interest rate of R_t; Π_t^L and Π_t^H denote the profits of intermediate low- and high-tech goods producing firms (discussed below) paid to the household in the form of a dividend. We assume that low-skilled households receive a dividend from ownership in proportion to their share of the total population.

In addition to the budget constraint, the household also faces a constraint on the perceived law of motion for the stock of employment, n_t^L, given by

$$n_t^L = (1 - \rho^L)n_{t-1}^L + f_t^L s_t^L,$$

which simply says that the number of low-skilled workers employed in low-tech jobs today is equal the number employment relationships that existed yesterday, net of those that terminate exogenously with probability ρ^L, plus new inflow. The new inflow is equal to the probability that a searching low-tech individual finds a job in the market for low-tech employment, f_t^L, times the number of searching individuals, s_t^L.

The job finding probability, f_t^L, is equal to the ratio of matches to job seekers in the low-tech sector. Matches in the low-tech sector, $m_t^L = m^L(s_t^L + s_t^M, v_t^L)$, are increasing and concave in v_t^L, which denotes vacancies posted by low-tech firms (discussed below), and the total number of

[5] Matches between low-skill workers and high-tech jobs are not productive, so given that search is directed low-skill households will never choose to devote search activity to high-tech jobs. For expositional simplicity, we omit this choice.

individuals searching for low-tech jobs. Total searchers is the sum of low-skill searchers, s_t^L, and unemployed high-skill individuals searching for low-tech jobs, denoted s_t^M (also discussed below). Finally, define $\theta_t^L = v_t^L / \left(s_t^L + s_t^M\right)$ as market tightness in the low-tech sector. It is clear that high-skilled individuals engaged in search for mismatch employment crowd out low-skill search in the sense that $\partial\theta_t^L/\partial s_t^M < 0$. This is the basis for the externality we study in the paper.

The first order conditions for c_t^L and B_t^L can be manipulated into a standard bond Euler equation

$$1 = \mathbb{E}_t \left\{ \frac{\beta u_{c,t+1}^L}{u_{c,t}^L} R_{t+1} \right\}, \tag{1}$$

which defines the stochastic discount factor for pricing the one-period, risk-free government bond, $\Xi_{t+1|t} \equiv \beta u_{c,t+1}^L / u_{c,t}^L$.

We can also use the first order conditions on s_t^L and n_t^L to obtain the optimal labor-force participation condition for low-skilled individuals:

$$\frac{h_t^{L\prime}}{u_{c,t}^L} = f_t^L \left[w_t^L + (1-\rho^L)\mathbb{E}_t \Xi_{t+1|t} \left\{ \frac{1-f_{t+1}^L}{f_{t+1}^L} \left(\frac{h_{t+1}^{L\prime}}{u_{c,t+1}^L} - \chi^L \right) \right\} \right] + (1-f_t^L)\chi^L, \tag{2}$$

which says that the low-skilled household will search for low-tech employment up until the point at which the probability-weighted cost of doing so—the disutility of search effort net of the outside option, χ^L—is exactly offset by the probability weighted expected benefit of getting a low-tech job. The expected benefit of low-tech employment is the wage plus the continuation value of forming a low-tech employment relationship.

2.1.2 High-Skilled Households

The mass of high-skill individuals participating in the labor force is given by $lfp_t^H + lfp_t^M$, where: $lfp_t^H = n_t^H + (1-f_t^H)s_t^H$; n_t^H denotes high-skill individuals working in high-tech jobs; s_t^H denotes high-skill individuals searching for high-tech jobs; and f_t^H is the probability a searching individual finds a match in the high-tech market (discussed below). Similarly, $lfp_t^M = n_t^M + (1-f_t^L)s_t^M$, where n_t^M denotes high-skill individuals working in low-tech jobs; and s_t^M denotes high-skill individuals searching for low-tech jobs. Due to the timing of the model, only unsuccessful searchers in the market for high- and low-tech jobs, $(1-f_t^H)s_t^H$ and $(1-f_t^L)s_t^M$, respectively, are considered unemployed at the end of the period. It follows that the mass of unemployed high-skill individuals is $u_t^H = (1-f_t^L)s_t^M + \left(1-f_t^H\right)s_t^H$. Leisure obtained by a high-skill household is $l_t^L = 1 - \kappa - lfp_t^H - lfp_t^M$.

High-skilled households choose sequences of consumption, c_t^H, state-contingent bond holdings, B_t^H, and search activity in both the market for low- and high-tech jobs, s_t^M and s_t^H, respectively, in order to achieve a desired stock of mismatch and high-tech employment, n_t^M and n_t^H, respectively.

Specifically, high-skilled households maximize discounted lifetime utility:

$$\max \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(u(c_t^H) - h\left(lfp_t^H, lfp_t^M \right) \right)$$

subject to a budget constraint

$$c_t^H + B_t^H = w_t^H n_t^H + w_t^M n_t^M + \chi^H \left[(1 - f_t^L) s_t^M + (1 - f_t^H) s_t^H \right] + R_t B_{t-1}^H + (1 - \kappa) \left(\Pi_t^L + \Pi_t^H \right)$$

and perceived laws of motion for the stocks of mismatch and high-tech employment

$$n_t^M = \left(1 - \pi f_t^H\right)(1 - \rho^L) n_{t-1}^M + f_t^L s_t^M,$$

and

$$n_t^H = (1 - \rho^H) n_{t-1}^H + f_t^H s_t^H + \pi f_t^H (1 - \rho^L) n_{t-1}^M,$$

where: in the budget constraint w_t^H and w_t^M are the wages received by high-skilled individuals in high-tech and mismatch jobs, respectively; χ^H is an unemployment benefit paid to actively searching high-skilled workers; and, in the laws of motion for high-tech and mismatch employment, $\pi \in (0,1)$ denotes the search efficiency of an OTJ searcher relative to that of an unemployed individual. Search on-the-job is as efficient as search from a state of unemployment when $\pi = 1$; in contrast, $\pi = 0$ shuts down OTJ search entirely.

Any high-skilled individual engaged in OTJ search will accept a higher paying job in the high-tech sector, which occurs with probability πf_t^H. So, in terms of allocations, the primary effect of OTJ search is to increase the outflows from mismatch employment as well as the inflows into high-tech employment. The additional outflow is given by $\pi f_t^H (1 - \rho^L) n_{t-1}^M$, or the stock of yesterday's mismatch jobs that were not exogenously destroyed but were successful in matching with a high-tech firm (note that given the timing of the model, the job-to-job transition implies that the successful OTJ searcher becomes immediately productive as a high-tech worker). This outflow simply becomes a new inflow into high-tech employment through job-to-job transition as shown by the last term to the right of the equals sign in the law of motion for n_t^H above.

As with the market for low-tech jobs, the job finding probability f_t^H is equal to the ratio of matches to job seekers in the high-tech sector. Matches in the high-tech sector, $m_t^H = m^H(s_t^H + \pi(1 - \rho^L) n_t^M, v_t^H)$, are increasing and concave in v_t^H, which denotes vacancies posted by high-tech firms, and the effective mass of individuals searching for high-tech jobs, $s_t^H + \pi(1 - \rho^L) n_t^M$, which captures both high-skill unemployed individuals and OTJ searchers. Define $\theta_t^H = v_t^H / \left(s_t^H + \pi \left(1 - \rho^L\right) n_{t-1}^M \right)$ as market tightness in the market for high-tech jobs.

The first-order conditions over c_t^H and B_t^H can be combined to yield a standard consumption Euler equation[6]

[6] Note that $\beta u_{c,t+1}^L / u_{c,t}^L = \beta u_{c,t+1}^H / u_{c,t}^H = \Xi_{t+1|t}$.

$$1 = \mathbb{E}_t \left\{ \frac{\beta u^H_{c,t+1}}{u^H_{c,t}} R_{t+1} \right\} \quad (3)$$

Using this relationship in the first order condition for n^H_t, we can write the optimal participation condition in the market for high tech employment as

$$\frac{h^{H\prime}_t}{u^H_{c,t}} = f^H_t \left[w^H_t + (1-\rho^H)\mathbb{E}_t \Xi_{t+1|t} \left\{ \frac{1-f^H_{t+1}}{f^H_{t+1}} \left(\frac{h^{H\prime}_{t+1}}{u^H_{c,t+1}} - \chi^H \right) \right\} \right] + (1-f^H_t)\chi^H, \quad (4)$$

where $h^{H\prime}_t$ is the derivative of the subutility of the high-skilled household over participation in the market for high-tech employment. The equation which has a similar interpretation to equation (2) above.

Finally, the condition governing optimal participation for high-skilled individuals in the market for low-tech jobs can be written as

$$\frac{h^{M\prime}_t}{u^H_{c,t}} = f^L_t \left[w^M_t + (1-\rho^L)\mathbb{E}_t \Xi_{t+1|t} \left\{ \left[\frac{1-f^L_{t+1}}{f^L_{t+1}} - \pi \left(1 - \frac{f^H_{t+1}}{f^L_{t+1}} \right) \right] \left(\frac{h^{M\prime}_{t+1}}{u^H_{c,t+1}} - \chi^H \right) \right\} \right] + \left(1-f^L_t\right) \chi^H \quad (5)$$

where $h^{M\prime}_t$ is the derivative of the subutility of the high-skilled household over participation in the market for mismatch employment. When OTJ search is shut down, so that $\pi = 0$, the interpretation of this equation is identical to that of equations (2) and (4). For $\pi > 0$, the continuation value of a mismatch job is adjusted owing to the possibility that successful OTJ search may shorten the duration of a mismatch employment relationship. The size of this adjustment is increasing in the relative ease with which a match can be made in the high-tech sector, f^H_{t+1}/f^L_{t+1}.

2.2 Production

The production side of the economy is divided into a final goods sector and an intermediate goods sector. We describe each stage of production, in turn, below.

2.2.1 Final Goods Production

The representative final goods producer purchases both low- and high-tech intermediate inputs (denoted y^L_t and y^H_t, respectively) and then aggregates both into a final good using the technology $Z_t F(y^L_t, y^H_t)$, where Z is total factor productivity and F is increasing and concave in each of its arguments. This final good is then sold to households in a perfectly competitive market for final consumption. The final goods producer chooses intermediate inputs to solve the following problem:

$$\max \; \mathbb{E}_t \sum_{t=0}^{\infty} \Xi_{t+1|t} \left[Z_t F(y^L_t, y^H_t) - p^L_t y^L_t - p^H_t y^H_t \right]$$

where: p_t^L and p_t^H, respectively, are the prices of the low- and high-tech intermediate inputs relative to the final good. The demand for each intermediate input equates the marginal product to the price, so that $Z_t F_{L,t} = p_t^L$ for the low-tech good and $Z_t F_{H,t} = p_t^H$ for the high-tech good.

2.2.2 Intermediate Goods Production

At the intermediate goods level, both low- and high-tech firms use labor to produce an intermediate input which is then sold to the final goods producer in a perfectly competitive market. Regardless of firm type, the intermediate goods producer must engage in costly search and matching in order to find a worker before production can take place. In order to make a match, the low-tech intermediate goods producing firm needs to pay a fixed flow cost, γ^L, to post a vacancy for an open position in the low-tech market, and the high-tech intermediate goods producing firm needs to pay a fixed flow cost, γ^H, in order to post a vacancy for an open position in the high-tech market.

Low-tech Firms For a given low-tech vacancy, the low-tech firm can hire either a low- or a high-skilled worker. The low-tech firm uses these two labor inputs to produce its intermediate good according to the production technology, $y_t^L = Z_t^L g^L(n_t^L, n_t^M)$, where Z_t^L is a technology parameter that is specific to low-tech production and g is increasing and concave in each of its arguments.

The low-tech firm chooses the desired stock of low-skill employees, n_t^L, the desired stock of high-skill employees, n_t^M, and vacancies, v_t^L, to solve the following profit maximization problem:

$$\max \ \mathbb{E}_t \sum_t \Xi_{t+1|t} \left[p_t^L Z_t^L g^L(n_t^L, n_t^M) - w_t^L n_t^L - w_t^M n_t^M - \gamma^L v_t^L \right],$$

subject to the firm's perceived laws of motion for low-skill and mismatch employment stocks, respectively

$$n_t^L = (1-\rho^L) n_{t-1}^L + \eta_t^L q_t^L v_t^L$$

and

$$n_t^M = \left(1 - \pi f_t^H\right)(1-\rho^L) n_{t-1}^M + \left(1 - \eta_t^L\right) q_t^L v_t^L,$$

where q_t^L is the probability that a given vacancy posted in the market for low-tech jobs is successful in finding a worker, regardless of whether the worker is low- or high-skill. In particular, q^L is equal to the ratio of matches to vacancies in the low-tech sector. Furthermore, the fraction of low-skill workers in the total pool of individuals searching for low-skill jobs is given by $\eta_t^L = s_t^L/(s_t^L + s_t^M)$, so the probability that a low-tech vacancy turns into an employment match with a low-skill worker is $\eta_t q_t^L$. Similarly, the probability that a low-tech vacancy turns into a mismatch employment relationship with a high-skill worker is $(1-\eta_t)\, q_t^L$. Also, note that the perceived law of motion for mismatch employment takes into account the fact that the low-tech firm will lose high-skill workers who are successful in OTJ search with probability πf_t^H.

Total employment in the low-tech sector is given by $N_t^L = n_t^L + n_t^M$ so that the sectoral mismatch rate is given by n_t^M / N_t^L. In addition, total aggregate employment is $N_t = n_t^L + n_t^M + n_t^H$. Thus, the aggregate mismatch rate is n_t^M / N_t. Furthermore, the average wage in the low-tech sector is $W_t^L = (w_t^L n_t^L + w_t^M n_t^M)/N_t^L$.

The first order condition on v_t^L gives

$$\frac{\gamma}{q_t^L} = \eta_t^L \mathbf{J_t^L} + \left(1 - \eta_t^L\right) \mathbf{J_t^M} \tag{6}$$

which says that the low-tech firm posts vacancies up until the point at which the cost, γ^L, is exactly offset by the expected gain from making a match. The expected gain is the probability that a match is made in the low-tech market, q_t^L, times a probability weighted average of the value of a match with a low-tech worker, $\eta_t^L \mathbf{J_t^L}$, and a high-tech worker, $\left(1 - \eta_t^L\right) \mathbf{J_t^M}$ where $\mathbf{J_t^L}$ and $\mathbf{J_t^M}$ are defined by the Lagrangian multipliers on the perceived laws of motion for low-tech and mismatch employment, respectively.

The first-order conditions for n_t^L and n_t^M give expressions for the value to the firm of both types of matches, respectively. We have

$$\mathbf{J_t^L} = p_t^L Z_t^L g_{n_t^L}^L - w_t^L + \left(1 - \rho^L\right) \mathbb{E}_t \left\{ \Xi_{t+1|t} \mathbf{J_{t+1}^L} \right\} \tag{7}$$

and

$$\mathbf{J_t^M} = p_t^L Z_t^L g_{n_t^M}^L - w_t^M + \left(1 - \rho^L\right) \mathbb{E}_t \left\{ \Xi_{t+1|t} \left(1 - \pi f_{t+1}^H\right) \mathbf{J_{t+1}^M} \right\} \tag{8}$$

Equation (7) equates the value of a low-skilled employee working in the low-tech job to the marginal revenue net of the wage. In addition, there is also a benefit to forming a match that comes from the continuation value of establishing an employment relationship. Equation (8) is interpreted in a similar way with the exception that OTJ search effectively lowers the potential benefit of making a match by reducing the continuation value. Intuitively, because high-skilled workers will always chose to leave a low-tech job for a higher wage in the high-tech sector conditional on being successful in OTJ search, which happens with probability f_t^H, there is an increase in the outflow from mismatch employment. By increasing the outflow, OTJ search lowers the value of a match because the benefits accrue to the low-tech firm over a shorter duration.

High-tech Firms High-tech firms only employ high-skilled workers because those workers are the only ones qualified to do the job. Production is given by the following: $y_t^H = Z_t^H g^H(n_t^H)$, where Z_t^H is a technology parameter that is specific to high-tech production and g is increasing and concave. The high-tech firm chooses the stock of high-skill employees and vacancies to solves the following profit maximization problem:

$$\max \; \mathbb{E}_t \sum_t \Xi_{t+1|t} \left[p_t^H Z_t^H g^H(n_t^H) - w_t^H n_t^H - \gamma^H v_t^H \right],$$

subject to the perceived law of motion for high-tech employment:

$$n_t^H = (1-\rho^H) n_{t-1}^H + q_t^H v_t^H,$$

where q_t^H is the probability that a given vacancy posted in the market for high-tech jobs is successful in finding a worker. In particular, q_t^H is equal to the ratio of matches in the high-tech sector to vacancies in the high-tech sector. It clearly follows that the average wage in the high-tech sector is simply w_t^H.

The first order condition on v_t^H gives

$$\frac{\gamma^H}{q_t^H} = \mathbf{J_t^H} \tag{9}$$

where $\mathbf{J_t^H}$ is the Lagrangian multiplier on the perceived laws of motion for high-tech employment.

The high-tech firm's first order conditions for n_t^H gives the following job creation condition

$$\mathbf{J_t^H} = p_t^H Z_t^H g_{n_t^H}^H - w_t^H + \left(1-\rho^H\right) \mathbb{E}_t \left\{ \Xi_{t+1|t} \mathbf{J_{t+1}^H} \right\} \tag{10}$$

which has a similar interpretation as the job creation conditions above. Note that we could express equations (9) and (10) as a single efficiency condition so that $\gamma^H / q_t^H = p_t^H Z_t^H g_{n_t^H}^H - w_t^H + \left(1-\rho^H\right) \mathbb{E}_t \left\{ \Xi_{t+1|t} \left(\gamma^H / q_{t+1}^H \right) \right\}$. In contrast, we cannot derive a similar equation directly for the low-tech firm because, as shown in equation (6), the first order condition on v_t^L creates a direct link between the value of low-skilled and mismatched workers in low-tech jobs.

2.3 The Labor Market

In order to close the model, we need to address matching and wage determination in each of the two labor markets.

2.3.1 Matching

Labor market matches are formed according to a constant returns matching technology in both the market for low- and high-tech jobs. Aggregate employment of low-skilled workers employed in low-tech jobs evolves according to

$$n_t^L = (1-\rho^L) n_{t-1}^L + \eta_t^L m_t^L. \tag{11}$$

As noted above, η_t^L is the probability that a match in the market for low-tech employment is formed with a low-skill worker so that $\eta_t^L = s_t^L/(s_t^L + s_t^H)$ is endogenously determined by the search activity of low- and high-skilled individuals.

The law of motion for mismatch employment is given by

$$n_t^M = (1-\rho^L)n_{t-1}^M - \eta_t^H m_t^H + (1-\eta_t^L)m_t^L, \tag{12}$$

where $\eta_t^H = \pi\left(1-\rho^L\right)n_{t-1}^M/(s_t^H + \pi\left(1-\rho^L\right)n_{t-1}^M)$ is the probability that a given match made in the high-tech labor market is made with an OTJ searcher.

Finally, the law of motion for high-tech jobs is

$$n_t^H = (1-\rho^H)n_{t-1}^H + m_t^H. \tag{13}$$

2.3.2 Wage Determination

Wages are determined by Nash bargaining over the match surplus.[7] We assume that bargaining does not involve commitment to the future path of wages. Let $\psi^i \in (0,1)$ for $i \in \{L, H\}$ denote the bargaining power of workers. For the sake of brevity we present only the wage that solves the bargaining problem, leaving the details—including a full derivation of the fundamental value functions used in the bargaining problem itself—to Appendix A.1.3. and A.2.3.

The wage for a low-skilled worker employed in a low-tech job is given by

$$w_t^L = \psi^L p_t^L Z_t^L g_{n_t^L}^L + \left(1-\psi^L\right)\chi^L + \psi^L(1-\rho^L)\mathbb{E}_t\left\{\Xi_{t+1|t} f_{t+1}^L \mathbf{J_{t+1}^L}\right\}, \tag{14}$$

where, as discussed above, the value of a low-skilled worker employed in low-tech production is denoted by $\mathbf{J_t^L}$. The wage paid by low-tech firms to low-skilled workers is a weighted average of the marginal revenue product of labor plus the continuation value of the match, both of which accrue to the low-tech firm, and the outside option to the worker given by the unemployment benefit.

The wage paid to a high-skilled worker by the low-tech intermediate goods producer is complicated by the possibility of OTJ search. The mismatch wage is given by the expression

$$\begin{aligned} w_t^M &= \psi^H p_t^L Z_t^L g_{n_t^M}^L + \left(1-\psi^H\right)\chi^H \\ &+\psi^H(1-\rho^L)\mathbb{E}_t\left\{\Xi_{t+1|t}\left[\left(1-\pi f_{t+1}^H\right)f_{t+1}^L \mathbf{J_{t+1}^M} - \pi\left(1-f_{t+1}^H\right)f_{t+1}^H \mathbf{J_{t+1}^H}\right]\right\}, \end{aligned} \tag{15}$$

[7] Though the assumption of Nash bargaining will be relevant for the quantitive exercises we conduct, it does not matter for the main points we want to make regarding efficiency in Section 4. The reason is because when we evaluate how mismatch and OTJ search influence efficiency, we will do so under the assumption that the match surplus is split efficiently. There are a number of ways to implement efficient surplus splits—including wage posting in a competitive search equilibrium—that satisfy this criterion, suggesting there is nothing special about Nash bargaining in driving our results.

where the value of a high-skilled worker employed in low-tech production is denoted by $\mathbf{J_t^M}$ (also defined above). The wage expression takes a generally similar form as equation 14, but there is one key difference. The continuation value for mismatch employment, and hence the mismatch wage, takes into account the effect of OTJ search through two separate channels. First, the continuation value of mismatch employment, $f_{t+1}^L \mathbf{J_{t+1}^M}$, must be adjusted downward to address the fact that these employment relationships have a shorter expected duration owing to OTJ search. This is captured by the term $\left(1 - \pi f_{t+1}^H\right) f_{t+1}^L \mathbf{J_{t+1}^M}$ inside the expectations operator. Second, the worker is willing to accept a lower wage in order to have an opportunity to move to high-tech employment and eventually obtain the value $f_{t+1}^H \mathbf{J_{t+1}^H}$ through OTJ search. This is captured by the term $\pi \left(1 - f_{t+1}^H\right) f_{t+1}^H \mathbf{J_{t+1}^H}$ inside the expectations operator. Both of these adjustments have a depressing effect on the wage and are both driven entirely by OTJ search. In the absence of OTJ search ($\pi = 0$), the continuation value reduces to $\psi^H (1 - \rho^L) \mathbb{E}_t \left\{\Xi_{t+1|t} f_{t+1}^L \mathbf{J_{t+1}^M}\right\}$ and the mismatch wage takes a similar form as equation (14).

Finally, the wage for a high-skilled worker employed in a high-tech job is given by:

$$w_t^H = \psi^H p_t^H Z_t^H g_{n_t^H}^H + \left(1 - \psi^H\right) \chi^H + \psi^H (1 - \rho^H) \mathbb{E}_t \left[\Xi_{t+1|t} \left(f_{t+1}^H \mathbf{J_{t+1}^H}\right)\right], \qquad (16)$$

The interpretation is identical to the wage for low-tech employment. Note that because free entry into vacancy postings drives $\mathbf{J_{t+1}^H} = \gamma^H / q_t^H$, the continuation value can also be expressed as $\mathbb{E}_t \left[\Xi_{t+1|t} \left(f_{t+1}^H / q_t^H\right) \gamma^H\right]$.

2.4 Competitive Search Equilibrium

Given the exogenous processes for technology, $\{Z_t, Z_t^L, Z_t^H\}$, the equilibrium of the system is a sequence of allocations and prices $\{c_t^L$, c_t^H, n_t^L, n_t^M, n_t^H, s_t^L, s_t^M, s_t^H, v_t^L, v_t^H, $\mathbf{J_t^L}$, $\mathbf{J_t^M}$, $\mathbf{J_t^H}$, w_t^L, w_t^M, w_t^H, p_t^L, $p_t^H\}$ that solves the optimality conditions for: low-skilled households, summarized be equations (1) through (2); high-skilled households, summarized be equations (3) through (5); demand for the low- and high-tech intermediate input, given by $F_{L,t} = p_t^L / Z_t$ and $F_{H,t} = p_t^H / Z_t$, respectively; low-tech intermediate goods producers, summarized by equations (6) through (8); high-tech intermediate goods producers, summarized by equation (10). We also have the laws of motion for respective employment stocks, equations (11) through (13); and the wage expressions, equations (14) through (16).

In addition, we have the economy-wide resource constraint

$$Y_t = c_t^L + c_t^H + \gamma^L v_t^L + \gamma^H v_t^H \qquad (17)$$

All told, the system is 18 equations in 18 unknowns.

3 Social Efficiency

We define social efficiency as an equally-weighted sum of the utility of low- and high-skilled households. With this definition, the efficient allocations $\{c_t^L,\ c_t^H,\ n_t^L,\ n_t^M,\ n_t^H,\ s_t^L,\ s_t^M,\ s_t^H,\ v_t^H,\ v_t^L,\ \eta_t^L,\ \eta_t^H\}$ are characterized by a set of 12 equations that include: equalization of the marginal rate of consumption for low- and high-skilled individuals, a set of two static labor market efficiency conditions; a set of three dynamic labor market efficiency conditions; the economy-wide resource constraint; a set of three laws of motion for the respective employment stocks; and, finally, two equations defining η_t^L and η_t^H, respectively. Details for the solution to the social planner's problem are provided in Appendix B. For the sake of brevity, we concentrate only on the set of static and dynamic efficiency conditions that summarize the labor market.

The static efficiency condition for overall search activity directed toward the market for low-tech employment is given by

$$\eta_t^L \frac{h_t^{L\prime}}{u_{c,t}^L} + \left(1-\eta_t^L\right)\frac{h_t^{M\prime}}{u_{c,t}^H} = \frac{m_{s,t}^L}{m_{v,t}^L}\gamma^L \tag{18}$$

where $m_{s,t}^L$ and $m_{v,t}^L$ denote the derivative of the low-tech matching function with respect to search unemployment and vacancies, respectively. This expression equates a weighted average of the static marginal rates of substitution (MRS) between consumption and leisure for low- and high-skilled individuals (on the left hand side) to the static marginal rate of transformation (MRT) of a unit of leisure into a unit of the final consumption good through the low-tech intermediate input (on the right hand side).[8]

Intuitively, within the period there are two distinct ways for the social planner to transform a unit of leisure into the final consumption good through the production of the low-tech intermediate good. The first is through the participation of low-skilled individuals, where the effectiveness of a unit of search in the matching pool for low-tech jobs is governed by the probability, η_t^L. This unit of search is transformed into productive labor through the matching function (captured by the right hand side), which is then ultimately used in production. Alternatively, the planner can achieve the same outcome through high-skilled individuals, transforming an effective unit of search into mismatch employment with probability $1-\eta_t^L$. Mismatch employment is governed by $1-\eta_t^L$ and links the MRS between consumption and leisure for low- and high-skilled individuals in the socially efficient equilibrium.

For high-tech employment, the static efficiency condition is

$$\left(1-\eta_t^H f_t^H\right)\frac{h_t^{H\prime}}{u_{c,t}^H} + \eta_t^H f_t^H \frac{h_t^{M\prime}}{u_{c,t}^H} = \gamma^H \frac{m_{s,t}^H}{m_{v,t}^H} + \eta_t^H f_t^H \left(\frac{\gamma^L}{m_{v,t}^L} + \Gamma_t^M\right) \tag{19}$$

[8] See Arseneau and Chugh (2012) for a more detailed description of how to interpret both the static and dynamic efficiency conditions in a general equilibrium labor search model and, in particular, how to think about the marginal rate of transformation in this class of models.

where we define $\Gamma_t^M \equiv \frac{\eta_t^L}{f_t^L}\left(\frac{h_t^{M\prime}}{u_{c,t}^H} - \frac{h_t^{L\prime}}{u_{c,t}^L}\right)$. The interpretation of equation 19 is broadly similar to that of equation 18 but is complicated by the role of OTJ search. The left hand side is a weighted average of the MRS for high-skilled individuals where the weight is given by $\eta_t^H f_t^H$. In absence of OTJ search, so that $\eta_t^H = 0$, the expression simplifies to $h_t^{H\prime}/u_{c,t}^H = \gamma^H m_{s,t}^H / m_{v,t}^H$, which equates the MRS between consumption and participation in the market for high-tech jobs to the MRT of a unit of leisure into a unit of the final consumption good through the high-tech intermediate input. The opportunity for OTJ search through mismatch employment ($\eta_t^H > 0$) opens up another channel through which the the planner can transform a unit of leisure of the high-skilled individual into a the high-tech intermediate output. In the socially efficient equilibrium, equation 19 ensures that the planner is indifferent between the two approaches.

The efficient equilibrium is also characterized by a set of dynamic efficiency conditions for each of the three employment stocks. The dynamic efficiency condition for the creation of low-tech jobs staffed by low-skill workers is given by

$$\frac{\gamma^L}{m_{v,t}^L} = Y_{1,t} + \Gamma_t^L + \left(1 - \rho^L\right) \mathbb{E}_t \left\{ \frac{\beta u_{c,t+1}^L}{u_{c,t}^L} \left(\frac{\gamma^L}{m_{v,t+1}^L} - \Gamma_{t+1}^L - \frac{h_{t+1}^{L\prime}}{u_{c,t+1}^L} \right) \right\} \tag{20}$$

where; $Y_{1,t}$ is the derivative of the aggregate production function with respect to low-skilled labor; and we define $\Gamma_t^L \equiv \frac{1-\eta_t^L}{f_t^L}\left(\frac{h_t^{M\prime}}{u_{c,t}^H} - \frac{h_t^{L\prime}}{u_{c,t}^L}\right)$. This condition ensures that the social cost of generating a low-tech job staffed by a low-skilled worker is exactly offset by the discounted expected benefit. The left hand side is the cost of generating an additional low-tech job staffed by a low-skilled worker; equivalently, it is the cost of posting the low-tech vacancy (normalized by the number of new matches generated by an additional vacancy posting) net of the potential benefit (cost) owing to being able to staff the job with a low-skill individual that has a lower (higher) MRS between consumption and leisure, $\Gamma_t^L > (<) 0$. The right hand side is the social gain from forming an additional low-skill match, which is the marginal product of low-skilled labor plus the discounted future value of the employment relationship over its expected duration. The continutation value of a low-tech match can be thought of as the savings associated with not having to form a new match (because the match already exists and, hence, neither party has to undertake costly search in order to produce) net of the stream of disutility associated with the household having to work in the job to keep the employment relationship going.

Equation 20, as well as equations 21 and 22 below, can be re-expressed in the following form

$$1 = \mathbb{E}_t \left\{ \frac{\beta u_{c,t+1}^L}{u_{c,t}^L} \frac{\left(1-\rho^L\right)\left(\frac{\gamma^L}{m_{v,t+1}^L} - \Gamma_{t+1}^L - \frac{h_{t+1}^{L\prime}}{u_{c,t+1}^L}\right)}{\frac{\gamma^L}{m_{v,t}^L} - \Gamma_t^L - Y_{1,t}} \right\},$$

which can be interpreted as an asset pricing equation. The first term in brackets is the stochastic discount factor while the second can be thought of as the socially efficient return to low-tech job creation. Writing the expression this way is informative because it allows us to think of the efficiency condition as the ratio of the intertemporal marginal rate of transformation–the second term in brackets) to the intertemporal marginal rate of substitution (the first term in brackets).

Similarly, the dynamic efficiency condition for the creation of mismatched jobs (low-tech jobs staffed by high-skilled workers) is given by

$$\frac{\gamma^L}{m_{v,t}^L} = Y_{2,t} - \Gamma_t^M + \left(1-\rho^L\right)\mathbb{E}_t\left\{\frac{\beta u_{c,t+1}^H}{u_{c,t}^H}\left(\left(1-\pi f_{t+1}^H\right)\left(\frac{\gamma^L}{m_{v,t+1}^L}+\Gamma_{t+1}^M\right)+\pi\left(1-f_{t+1}^H\right)\frac{h_t^{M\prime}}{u_{c,t+1}^H}\right)\right\} \tag{21}$$

where $Y_{2,t}$ is the derivative of the aggregate production function with respect to mismatched labor. The interpretation is similar to that of equation 20, but there are two things worth pointing out. First, the cost of generating an additional mismatch employment relationship incorporates the cost (savings) associated with staffing a low-tech job with a high-skilled individual with a higher (lower) MRS between consumption and leisure, $\Gamma_t^M > (<) 0$. Put another way, in comparing equations 20 and 21, given that the production of the low-tech intermediate good can be done by either low- or high-skilled individuals, it is more costly to produce using the agent with the higher utility valuation of leisure. The other important difference is that when $\pi > 0$, OTJ search implies that the expected duration of a mismatch employment relationship is shorter than an employment relationship with a low-skilled worker. As soon as a high-skill individual employed in low-tech production finds a job in the high-tech sector, he/she will leave for the better opportunity.

Finally, the dynamic efficiency condition for the creation of high-tech jobs is given by

$$\frac{\gamma^H}{m_{v,t}^H} = Y_{3,t} - \eta_t^H\Gamma_t^H + \left(1-\rho^H\right)\mathbb{E}_t\left\{\frac{\beta u_{c,t+1}^H}{u_{c,t}^H}\left(\frac{\gamma^H}{m_{v,t+1}^H}+\eta_{t+1}^H\Gamma_{t+1}^H-\frac{h_{t+1}^{H\prime}}{u_{c,t+1}^H}\right)\right\} \tag{22}$$

where; $Y_{3,t}$ is the derivative of the aggregate production function with respect to high-skilled labor; and we define $\Gamma_t^H \equiv \frac{\gamma^L}{m_{v,t}^L} + \Gamma_t^M + \frac{1}{m_{s,t}^L}\left(\frac{h_t^{H\prime}}{u_{c,t}^H} - \frac{h_t^{M\prime}}{u_{c,t}^H}\right)$. The term to the left of the equal sign captures the cost associated with creating a high-tech job, which derives from two sources. The $\gamma^H/m_{v,t}^H$ term captures the cost of posting a vacancy directly to the high-tech market and the second term in the square brackets captures the cost associated with creating a high-tech job via OTJ search. As above, the benefit is the marginal product of high-tech labor plus the continuation value of a high-tech job. Notice that shutting down OTJ search, so that $\eta_t^H = 0$, means that both the static and the dynamic social efficiency conditions for the high-tech market are largely independent of developments in the low-tech market.

4 Characterizing the Distortion

In this section, we demonstrate the conditions under which the competitive search equilibrium does or does not coincide with the socially efficient equilibrium. Our approach is to manipulate the conditions that describe the private competitive search equilibrium into a set of efficiency conditions that take a similar form as what was presented in the previous section on social efficiency. To the degree that the private and socially optimal efficiency conditions do not perfectly coincide, we use the difference between the two to define a wedge that summarizes the distortion to that particular margin. Details of all derivations are given in Appendix C. Our focus is on the distortionary effects of labor market mismatch and OTJ search.

4.1 Static Distortions

The static labor efficiency condition for low-tech jobs in the private equilibrium is derived by dividing the low-tech firm's job creation condition for jobs staffed by low-skilled employees, given by equation (7), by the low-skill household's optimal search condition, given by equation (2). We then exploit the Nash sharing rule and the optimal posting condition for low-tech vacancies to simplify the resulting expression to:

$$\eta_t^L \frac{h_t^{L\prime}}{u_{c,t}^L} + \left(1-\eta_t^L\right) \frac{h_t^{M\prime}}{u_{c,t}^H} =$$
$$\frac{\psi^L}{1-\psi^L} \frac{f_t^L}{q_t^L}\gamma^L + \left(1-\eta_t^L\right) f_t^L \left[1 - \frac{\psi^L\left(1-\psi^H\right)}{\left(1-\psi^L\right)\psi^H}\right] \left(\frac{h_t^{M\prime} - u_{c,t}^H \chi^H}{f_t^L u_{c,t}^H}\right) + \eta_t^L \chi^L + \left(1-\eta_t^L\right)\chi^H.$$

Comparing this expression to the corresponding static socially efficient condition for low-tech job creation, equation (18), we see that the left hand sides are equal, but the right hand sides are potentially different. We can define an expression for $\Omega_{Static,t}^L$ that, when multiplied by the right hand side of equation (18), gives the expression above. The resulting static labor wedge for the low-tech job market is defined as

$$\Omega_{Static,t}^L = \frac{\frac{m_{s,t}^L}{m_{v,t}^L}\gamma^L}{\frac{\psi^L}{1-\psi^L}\frac{f_t^L}{q_t^L}\gamma^L + \left(1-\eta_t^L\right)\left[1-\frac{\psi^L\left(1-\psi^H\right)}{\left(1-\psi^L\right)\psi^H}\right]\left(\frac{h_t^{M\prime}}{u_{c,t}^H} - \chi^H\right) + \eta_t^L\chi^L + \left(1-\eta_t^L\right)\chi^H} \tag{23}$$

We follow a similar strategy to get an expression for the static labor efficiency condition for high-tech jobs in the private equilibrium, which simplifies to

$$\frac{h_t^{H\prime}}{u_{c,t}^H} = \frac{\psi^H}{1-\psi^H}\frac{f_t^H}{q_t^H}\gamma + \chi^H$$

As above, we isolate $h_t^{H\prime}/u_{c,t}^H$ on the left hand side of the socially efficient condition, equation (19), and define the static wedge for high-tech employment, $\Omega_{Static,t}^H$ that results in the expression above. The expression for the static high-tech wedge is

$$\Omega_{Static,t}^H = \frac{\gamma^H \frac{m_{s,t}^H}{m_{v,t}^H} + \eta_t^H f_t^H \left(\frac{\gamma^L}{m_{v,t}^L} + \Gamma_t^M - \frac{h_t^{M\prime}}{u_{c,t}^H} \right)}{\left(1 - \eta_t^H f_t^H\right) \left(\frac{\psi^H}{1-\psi^H} \gamma^H \frac{f_t^H}{q_t^H} + \chi^H \right)} \tag{24}$$

4.2 Dynamic Distortions

We derive the dynamic distortion for low-tech jobs staffed by low-skilled individuals by substituting the corresponding Nash wage into the low-tech firm's job creation condition and, where necessary, apply the optimal low-tech vacancy posting condition. The resulting expression can then be expressed as a ratio to equation (20) from the social planning problem. Some additional algebra allows us to write the dynamic distortion for low-tech job creation as follows

$$\Omega_{Dynamic,t}^L = \frac{Y_{1,t} + \Gamma_t^L + \left(1 - \rho^L\right) \mathbb{E}_t \Xi_{t+1} \left[\frac{\gamma^L}{m_{2,t+1}^L} - \Gamma_{t+1}^L - \frac{h_{t+1}^{L\prime}}{u_{c,t+1}^L} \right]}{\frac{1-\psi^L}{1-\xi^L} \eta_t^L \left(Y_{1,t} - \chi^L\right) + \Lambda_t^L + \left(1 - \rho^L\right) \mathbb{E}_t \Xi_{t+1|t} \left\{ \left[\frac{\gamma^L}{m_{v,t+1}^L} - \Lambda_{t+1}^L \right] \frac{\eta_t^L}{\eta_{t+1}^L} \left(1 - \frac{\psi^L}{\xi^L} m_{s,t+1}^L\right) \right\}} \tag{25}$$

where we define $\Lambda_t^L \equiv \frac{1-\eta_t^L}{1-\xi^L} \frac{1-\psi^H}{\psi^H} \left(\frac{h_t^{M\prime} - u_{c,t}^H \chi^H}{f_t^L u_{c,t}^H} \right)$ from the private equilibrium. The dynamic wedge for low-tech employment measures the gap between the discounted expected return to investing in low-tech job creation in the private versus socially efficient equilibrium. In addition to potential congestion externalities and the unemployment benefit, one key driver of this gap comes from the fact that with mismatch ($\eta_t^L < 1$) the social planner internalizes the fact that participation of high-skilled individuals in the market for low-tech jobs crowds out the search activity of low-skilled individuals. This spillover is captured by the link between the MRS of low- and high-skilled individuals in the term $\Gamma_t^L = \left(\left(1 - \eta_t^L\right) / f_t^L\right) \left(h_t^{M\prime}/u_{c,t}^H - h_t^{L\prime}/u_{c,t}^L\right)$ in the numerator. In contrast, this spillover is not internalized in the private equilibrium, where high-skilled individuals only consider their own MRS net of the outside option of unemployment benefits, as evidenced by the term Λ_t^L in the denominator.

A similar derivation allows us to define the dynamic distortion for mismatch as follows

$$\Omega^M_{Dynamic,t} = \frac{\begin{pmatrix} Y_{2,t} - \Gamma^M_t \\ + \left(1-\rho^L\right) \mathbb{E}_t \Xi_{t+1} \left\{ \left(1 - \pi f^H_{t+1}\right) \left[\frac{\gamma^L}{m^L_{v,t+1}} + \Gamma^M_{t+1} \right] + \pi \left(1 - f^H_{t+1}\right) \frac{h^{H\prime}_{t+1}}{u^H_{c,t+1}} \right\} \end{pmatrix}}{\begin{pmatrix} \frac{1-\psi^L}{1-\xi^L} \left(1-\eta^L_t\right) \left(Y_{2,t} - \chi^H\right) + \Lambda^M_t \\ + \left(1-\eta^L_t\right)\left(1-\rho^L\right) \mathbb{E}_t \Xi_{t+1|t} \left[\left(1 - \pi f^H_{t+1}\right) \Upsilon^1_{t+1} + \pi \left(1 - f^H_{t+1}\right) \Upsilon^2_{t+1} \right] \end{pmatrix}} \tag{26}$$

where: $\Lambda^M_t \equiv \frac{\eta^L_t}{1-\xi^L} \frac{1-\psi^L}{\psi^L} \left(\frac{h^{L\prime}_t - u^L_{c,t}\chi^L}{f^L_t u^L_{c,t}} \right)$ from the private equilibrium; and we define $\Upsilon^1_t \equiv \left(\frac{\gamma^L}{m^L_{v,t+1}} - \Lambda^M_{t+1} \right) \frac{1-\psi^H f^L_{t+1}}{1-\eta^L_{t+1}}$ and $\Upsilon^2_{t+1} \equiv \frac{1-\psi^H}{1-\xi^L} \left(\frac{h^{H\prime}_{t+1}}{u^H_{c,t+1}} - \chi^H \right)$. As above, abstracting from potential congestion externalities and the unemployment benefit, a key determinant of the gap stems from the fact that the planner internalizes the spillover to low-skilled individuals that arises from high-skilled participation in the low-tech job market. Private individuals ignore this externality. The resulting gap can be seen by comparing Γ^M_t, which shows up in the numerator from the social planning solution, and Λ^M_t, which shows up in the denominator from the private equilibrium.

Finally, the dynamic distortion for high-tech job creation can be written as

$$\Omega^H_{Dynamic,t} = \frac{Y_{3,t} - \eta^H_t \Gamma^H_{t+1} + \left(1-\rho^H\right) \mathbb{E}_t \Xi_{t+1} \left\{ \frac{\gamma^H}{m^H_{v,t+1}} - \frac{h^{H\prime}_{t+1}}{u^H_{c,t+1}} + \eta^H_{t+1}\Gamma^H_{t+1} \right\}}{\frac{1-\psi^H}{1-\xi^H}\left(Y_{3,t} - \chi^H\right) + \left(1-\rho^H\right) \mathbb{E}_t \Xi_{t+1|t} \left[\frac{\gamma^H}{m^H_{v,t+1}} \left(1 - \frac{\psi^H}{\xi^h} m^H_{s,t+1} \right) \right]}. \tag{27}$$

When $\eta^H_t > 0$, high-tech jobs can be created either via direct search in the market for high-tech jobs or indirectly through OTJ search. Exploiting this later channel has implications for the welfare of low-skilled individuals because it involves mismatch employment. This spillover is internalized in the socially efficient equilibrium—hence the appearance of the Γ^H_t term in the numerator—but is neglected in the private equilibrium.

4.3 Four Special Cases

We present three special cases which, taken together, provide a complete characterization of the effect of mismatch and on-the-job search—both separately and together—on the standard search-based labor wedges derived in Arseneau and Chugh (2012). Throughout this subsection, we assume the matching functions are Cobb-Douglas with the elasticity of matches with respect to household search denoted ξ^L and ξ^H, respectively. Unless otherwise noted, the Hosios condition is assumed to hold in the markets for both low- and high-tech jobs, so that $\xi^L = \psi^L$ and $\xi^H = \psi^H$, and there are no unemployment benefits, $\chi^L = \chi^H = 0$. We make these assumption because it zeroes out both the congestion externality generated by inefficient surplus splits and the distortion created by unemployment benefits. These distortions are well understood, so by zeroing them out we can focus attention directly on inefficiencies related to mismatch and OTJ search.

4.3.1 Mismatch ($0 < \eta_t^L < 1$), No OTJ Search ($\pi = \eta_t^H = 0$)

When we allow for permanent mismatch by shutting down OTJ search, the static labor wedge for low-tech employment reduces to

$$\Omega^L_{Static,t} = \frac{\frac{m^L_{s,t}}{m^L_{v,t}}\gamma^L}{\frac{\psi^L}{1-\psi^L}\frac{f^L_t}{q^L_t}\gamma^L + \left(1-\eta^L_t\right)\left[1 - \frac{\psi^L\left(1-\psi^H\right)}{\left(1-\psi^L\right)\psi^H}\right]\left(\frac{h^{M\prime}_t}{u^H_{c,t}} - \chi^H\right)}.$$

Closer inspection of the denominator shows that any distortion in this static margin is driven by asymmetries in the parameterization of bargaining power across the two labor markets. Indeed, as long as $\psi^L = \psi^H$ the second term in the denominator drops out and the fact that the Cobb-Douglas matching function has the property that $\frac{m^L_{s,t}}{m^L_{v,t}} = \frac{\xi^L}{1-\xi^L}\theta^L_t = \frac{\psi^L}{1-\psi^L}\frac{f^L_t}{q^L_t}$ when $\xi^L = \psi^L$ implies $\Omega^L_{Static,t} = 1$. In contrast, when $\xi^L \neq \psi^L$, permanent mismatch introduces a distortion into the static margin for low-tech employment.

For the static wedge for high-tech employment, the static efficiency conditions in the private equilibrium coincide with those in the socially efficient equilibrium; accordingly, the static labor market wedges disappear so that

$$\Omega^H_{Static,t} = 1$$

Similarly, shutting down OTJ search eliminates the wedge for high-tech job creation, so that

$$\Omega^H_{Dynamic,t} = 1$$

In contrast, for both low-tech and mismatch job creation the search-based wedges reduce to the following, respectively:

$$\Omega^L_{Dynamic,t} = \frac{Y_{1,t} + \Gamma^L_t + \left(1-\rho^L\right)\mathbb{E}_t\Xi_{t+1}\left[\frac{\gamma^L}{m^L_{v,t+1}} - \Gamma^L_{t+1} - \frac{h^{L\prime}_{t+1}}{u^L_{c,t+1}}\right]}{\eta^L_t Y_{1,t} + \Lambda^L_t + (1-\rho^L)\mathbb{E}_t\Xi_{t+1|t}\left\{\left[\frac{\gamma^L}{m^L_{v,t+1}} - \Lambda^L_{t+1}\right]\frac{\eta^L_t}{\eta^L_{t+1}}\left(1 - m^L_{s,t+1}\right)\right\}}$$

and

$$\Omega^M_{Dynamic,t} = \frac{Y_{2,t} - \Gamma^M_t + \left(1-\rho^L\right)\mathbb{E}_t\Xi_{t+1}\left[\frac{\gamma^L}{m^L_{v,t+1}} + \Gamma^M_{t+1}\right]}{\left(1-\eta^L_t\right)Y_{2,t} + \Lambda^M_t + \left(1-\eta^L_t\right)(1-\rho^L)\mathbb{E}_t\Xi_{t+1|t}\left[\Upsilon^1_{t+1}\right]}$$

In summary, mismatch employment generates a distortion that manifests primarily in the dynamic margins for both low-tech and mismatch job creation. The static margin for low-tech employment is distorted only to the degree that there is an asymmetry in bargaining power across the two markets. Notice also that the search-based labor wedges presented in this special case without OTJ search are considerably more simple than the more general wedges presented in the previous subsection (even when we shut down the congestion externality and unemployment benefits). This highlights the role of OTJ search in propagating the mismatch distortion: when high-skilled individuals engage in OTJ search from mismatch employment, the fundamental distortion created by mismatch extends to all aspects of the frictional labor market.

4.3.2 No Mismatch ($\eta_t^L = 1$), No OTJ Search ($\pi = \eta_t^H = 0$)

Shutting down mismatch entirely reduces the model to a two-sector model with completely segmented labor markets. In this case, under our assumption of the Hosios condition and no unemployment benefits, the static wedges reduce to

$$\Omega^L_{Static,t} = \Omega^H_{Static,t} = 1$$

This implies $h^{L\prime}_{t+1}/u^L_{c,t+1} = \left(m^L_{s,t}/m^L_{v,t}\right)\gamma^L$. We can substitute this in with the fact that $\eta^L_t = 1$ implies $\Gamma^L_t = \Lambda^L_t = 0 \ \forall\ t$ to show that the dynamic wedges collapse to

$$\Omega^L_{Dynamic,t} = \Omega^H_{Dynamic,t} = 1$$

and, of course, because there is no mismatch assumed in this special case the concept of $\Omega^M_{Dynamic,t}$ is meaningless. This result implies that in absence of mismatch the labor market efficiency conditions $i \in (H, L)$ boil down to

$$\frac{h^i_{lfp,t}}{u^i_{c,t}} = \frac{m^i_{s,t}}{m^i_{v,t}}\gamma$$

and

$$\gamma/m^i_{v,t} = Y_{i,t} + \mathbb{E}_t\Xi_{t+1}\left\{(1-\rho)\left(\gamma/m^i_{v,t+1}\left(1 - m^i_{s,t+1}\right)\right)\right\}$$

In other words, under the Hosios parameterization and zero unemployment benefits, shutting down both mismatch and OTJ search results in a private search equilibrium that is socially efficient. Indeed, both the static and dynamic efficiency conditions are identical to those presented in Arseneau and Chugh (2012) for the one sector general equilibrium labor search model. In this sense, our paper illustrates how the efficiency results presented in that earlier paper extend to a more general economy characterized by mismatch and OTJ search.

4.3.3 Congestion Externality ($\psi \neq \xi$) and Unemployment Benefits ($\chi > 0$)

With mismatch and OTJ search shut down, we reintroduce both the congestion externality and unemployment benefits under the assumption that $\psi^L = \psi^H$. In this case, the static and dynamic distortions for $i \in (H, L)$ collapse to

$$\Omega^i_{Static,t} = \left[\frac{(1-\xi)\,\psi}{\xi\,(1-\psi)} + \frac{1-\xi}{\xi}\frac{1}{\gamma\theta^i_t}\chi^i\right]$$

and

$$\Omega^i_{Dynamic,t} = \frac{Y_{i,t} + \left(1-\rho^i\right)\mathbb{E}_t\Xi_{t+1}\left\{\frac{\gamma^i}{m^i_{v,t+1}}\left(1-m^i_{s,t+1}\right)\right\}}{\frac{1-\psi^i}{1-\xi^i}\left(Y_{i,t}-\chi^i\right) + \left(1-\rho^i\right)\mathbb{E}_t\Xi_{t+1|t}\left[\frac{\gamma^i}{m^i_{v,t+1}}\left(1-\frac{\psi^i}{\xi^i}m^i_{s,t+1}\right)\right]}$$

Note that the above two wedges are derived under the assumption that the matching function is Cobb-Douglas with an elasticity parameter of ξ^i. Imposing the functional form makes it clear that either deviations from the Hosios condition ($\psi \neq \xi$) or positive unemployment benefits ($\chi > 0$) are sufficient to introduce a distortion to the competitive search equilibrium.

Taken together the preceding three special cases demonstrate that mismatch generates a distortion that is independent from more standard distortions owing to congestion externalities and/or unemployment benefits. Allowing for OTJ search amplifies the welfare effects generated by mismatch employment. That said, the complicated expressions for the static and dynamic distortions presented in Sections 4.1 and 4.2 clearly illustrate that all of these distortions interact in a complicated way in general equilibrium.

4.3.4 Frictionless Labor Markets

Lastly, it is useful to illustrate that in absence of search frictions the model collapses to a standard two-sector RBC model. To see this consider that we can shut down the long lived nature of employment relationships by making matches last only one period, so that $\rho = 1$. In this case, the dynamic efficiency conditions given by equations 20 and 22 reduce to a simple static relationship, $\gamma/m^i_{v,t} = Y_{i,t}$ for $i \in (H, L)$. Plugging this relationship into equations 18 and 19 gives $h^i_{lfp,t}/u^i_{c,t} = m^i_{s,t}Y_{i,t}$. Finally, in absence of search frictions effort expended by the household in the labor market is trivially translated one-for-one into new "matches" (though, to be clear, the concept of a labor market match is meaningless in absence of frictions), so that $m^i_{s,t} = 1$.

We retrieve the following expression

$$\frac{h^i_t}{u^i_{c,t}} = Y_{i,t}$$

which is the familiar efficiency condition at the heart of the (two sector) RBC model.

5 Quantitative Results

We calibrate our model to U.S. labor market data and use it to conduct some simple experiments to gauge the size of the welfare effects of mismatch employment. The calibration is described in the next subsection before turning to the main quantitative results of the paper.

5.1 Calibration

Our calibration, which is summarized in Table 1, is at monthly frequency and uses data on educational attainment to calibrate worker heterogeneity and data on employment by occupation to calibrate firm heterogeneity. We also make use of aggregate labor market data where applicable. All data are publicly available from the Bureau of Labor Statistics (BLS).

We take the empirical counterpart to our low- and high-tech sectors to be routine and nonroutine occupations, respectively, as per standard BLS occupational classifications.[9] With this dichotomy in mind, we use the BLS occupational outlook handbook to obtain educational attainment requirements for entry-level positions by occupation. Roughly 82 percent of nonroutine jobs require at least some post-secondary education, while only 14 percent of routine jobs require at least some post-secondary education. Accordingly, in our model high-skill workers are those with at least some post-secondary education and low-skill workers as those with at most a high school degree. Data from the BLS shows that about one-half the U.S. population has at most a high school degree. Accordingly, we set the model economy's fraction of low-skill individuals, κ, equal to 0.5.

With regard to preferences, because we assume that the time period is equal to one month we set the discount factor $\beta = 0.996$, which is consistent with an annual interest rate of 5 percent. We assume a standard functional form for the sub-utility of over consumption for both low- and high-skilled individuals:

$$u(c_t^i) = \frac{1}{1-\sigma} c_t^{i\ 1-\sigma} \text{ for } i \in (H, L).$$

and set $\sigma = 1$ so that $u(c_t^i) = \ln c_t^i$ for $i \in (H, L)$.

The sub-utilities over labor force activity for low- and high-skilled individuals, respectively are given by:

$$h(lfp_t^L) = \frac{\phi^L}{1+1/\varepsilon} \left(n_t^L + (1-f_t^L)s_t^L\right)^{1+1/\varepsilon}$$

[9] Specifically, routine occupations include: (1.) sales and related occupations; (2.) office and administrative support occupations; (3.) farming, fishing, and forestry occupations; (4.) construction and extraction occupations; (5.) instalation, maintenance, and repair ocupations; (6.) production occupations; and (7.) transportation and material moving occupations. Nonroutine occupations include: (1.) management, business, and financial occupations; (2.) professional and related occupations; and (3.) service occupations.

and

$$h\left(lfp_t^H\right) + h\left(lfp_t^M\right) = \frac{\phi^H}{1+1/\varepsilon}\left(n_t^H + (1-f_t^H)s_t^H\right)^{1+1/\varepsilon} + \frac{\phi^M}{1+1/\varepsilon}\left(n_t^M + (1-f_t^L)s_t^M\right)^{1+1/\varepsilon}.$$

In calibrating preferences over labor force activity, quadratic labor disutility (so that $\varepsilon = 1$) implies that the model's aggregate labor force participation rate is highly inelastic with respect to output per worker, which is in line with the data.[10] We use both aggregate and disaggregate labor force participation data from the BLS to calibrate the scaling parameters for the disutility of participation in the low- and high-skill labor markets, respectively. The average participation rate of individuals with at least some post-secondary education (high skill from the vantage point of our model) is 1.33 times as high as the participation rate of individuals with at most a high school education (low skill from the vantage point of our model). Also, the average labor force participation rate in the US is 0.631. We calibrate the scaling parameters ϕ^L and ϕ^H, to target these participation-rate data. The scaling parameter for the disutility of mismatch employment for high-skilled individuals, ϕ^M, is calibrated to target a steady-state ratio of total employment in high-tech to low-tech jobs of $n^H/N^L = 1.11$. This number corresponds to the average ratio of total employment in nonroutine occupations to total employment in routine occupations in the U.S.

For production, we assume that output of final goods is a CES aggregate of the low- and high-tech intermediate good, so that

$$Y_t = Z_t\left(\varrho^H\left(y_t^H\right)^{\omega_F} + \left(1-\varrho^H\right)\left(y_t^L\right)^{\omega_F}\right)^{1/\omega_F},$$

where: Z_t is aggregate productivity; $\varrho^H \in (0,1)$ is the share of the high-tech intermediate input in final goods production; and ω_F governs the degree of substitutability between the high- and low-tech goods in final goods production. In turn, $y_t^H = Z_t^H n_t^H$, where Z_t^H is high-tech productivity. Production of the low tech good is determined by the CES aggregator of low-skill and mismatch employment relationships

$$Y_t^L = Z_t^L\left(\varrho^L\left(y_t^L\right)^{\omega_L} + \left(1-\varrho^L\right)\left(y_t^M\right)^{\omega_L}\right)^{1/\omega_L},$$

where $\varrho^L \in (0,1)$ is the share of low-tech input and ω_L governs the substitutability of low and mismatch inputs. Finally, we have that $y_t^L = z_t^L n_t^L$, and $y_t^M = z_t^M n_t^M$ where z_t^L, z_t^M, and Z_t^L all denote input-specific productivities. The steady state values of Z^H, Z^L, z^L, and z^M are normalized to one. In contrast, the value of Z is chosen to normalize steady state aggregate output that $Y = 1$.

[10] Although our analysis does not focus on dynamics, our assessment of this elasticity comes from using quarterly data on real GDP from the Bureau of Economic Analysis and data on aggregate employment and the aggregate labor force participation rate from the BLS. We detrend the natural logarithm of output per worker and the participation rate using a Hodrick-Prescott filter with smoothing parameter equal to 1600, and find that the coefficient on a simple OLS regression of the detrended participation data on output per worker is -0.111 with a standard error of 0.025.

The remainder of the production parameters are either chosen based on the existing literature or calibrated to match empirically observed wage differentials.

Krusell, Ohanian, Rios-Rull, and Violante (2000) find an elasticity of substitution between skilled and unskilled inputs equal to 0.4. This value is broadly in line with several research surveyed in Hammermesh (1993). Thus, ω_F is set to 0.4 so that high- and low-tech inputs are imperfect substitutes in final goods production. In turn, we assume $\omega_L = 1$ so that low-skilled and mismatch workers are perfect substitutes in the production of the low-tech intermediate input. To calibrate the share parameter in the low-tech intermediate goods aggregator, ϱ^L, we set the equilibrium mismatch wage 15 percent above the low-skill wage based on Sicherman (1991). (We assume that a 4-year education differential is a reasonable characterization of the educational difference between high- and low-skill workers in the model economy.). For the share parameter in the final goods aggregator, ϱ^H, we draw on occupational wage data from the BLS. The employment-weighted median wages of individuals employed in nonroutine occupations is 1.35 times that of median wages of individuals employed in routine occupations. Accordingly, we choose ϱ^H to achieve an average steady-state employment-weighted wage ratio of individuals employed in high-tech jobs to individuals employed in low-tech jobs of $w^H/W^L = 1.35$.

Turning to the labor market, we assume that both the low- and high-tech job markets are characterized by a standard Cobb-Douglas matching function

$$m_t^i = A^i \left(s_t^i\right)^{\xi^i} \left(v_t^i\right)^{1-\xi^i}, \text{ for } i \in \{L, H\}$$

where A^i is matching efficiency and ξ^i gauges the elasticity of the matching function with respect to search activity. We set $\xi^i = 0.5$ for $i \in \{L, H\}$, which is broadly in line with research surveyed in Petrongolo and Pissarides (2001). The matching efficiency parameters, A^L and A^H, are jointly calibrated to hit empirical targets that we obtain from both aggregate and sector-specific data on job finding probabilities. Starting with the aggregate data and following the methodology in Elsby, Michaels, and Solon (2009) and Shimer (2012), monthly data on unemployment since 1951 reveal that the probability that an average unemployed individual finds a job within a month is 0.431. Thus, one calibrating target for the two matching efficiency parameters is the steady-state value $\frac{(1-\eta^H)m^H+m^L}{s^L+s^M+s^H} = 0.431$. Moving to the sector-specific data, we follow a similar methodology using data on the total number of unemployed individuals who were last employed in routine and nonroutine occupations. Under the assumption that an individual's last occupation is roughly indicative of their skill level, we find that since 2000 the average job-finding probability of individuals last employed in routine occupations is 0.99 times that of individuals last employed in nonroutine occupations. This gives us our second calibrating target for the matching efficiency parameters, which is a steady-state value $\frac{m^L/\left(s^L+s^M\right)}{(1-\eta^H)m^H/s^H} = 0.99$.

The exogeous job destruction probabilities, ρ^L and ρ^H, are calibrated using BLS data on aggregate and occupation-specific unemployment rates. These data show the average US unemployment rate since 1951 is 0.058, so in our model we pin down one of the job destruction rates by targeting the steady-state ratio $(u^L + u^H)/(lfp^L + lfp^H) = 0.058$. In addition, these data also show that the average unemployment rate of individuals last employed in nonroutine occupations is about 1.62 times as high as that of individuals last employed in nonroutine occupations. So, the calibrating target that pins down the second job destruction rate is the steady-state ratio $\frac{u^L}{lfp^L}/\frac{u^H}{lfp^H} = 1.62$.

We assume symmetry in the vacancy posting costs, $\gamma^H = \gamma^L$, and calibrate these costs to target the ratio of aggregate vacancies to aggregate unemployment: $\frac{v^L+v^H}{(1-f^L)s^L+(1-f^H)s^H} = 0.68$. We arrive at this number by using data on aggregate job openings from the BLS Job Openings and Labor Turnover Survey since 2000 (when first available) combined with the Conference Board's Help-Wanted Index from 1951 through 2000. Taken together with time series for aggregate unemployment, these data imply that in the US the average post-war period ratio of vacancies to unemployment is 0.68.

We also assume symmetry in bargaining power, so that $\psi^H = \psi^L = 0.5$. This parameterization has the virtue that, in our model, $\psi^H = \psi^L = \xi^H = \xi^L$ delivers both an efficient split of match surplus (see Hosios (1990)) as well as cross-market efficiency under permanent mismatch. Per Shimer (2005), unemployment benefits are set to deliver a 40 percent replacement rate of wages. In particular, the low-skill unemployment benefit, χ^L, is set to deliver a 40 percent replacement rate of the steady-state wage of low-skill workers. Hence, $\chi^L = 0.4w^L$. We target the high-skill unemployment benefit, χ^H, analogously so that it delivers a 40 percent replacement rate of employment-weighted steady-state average wages of high-skill workers. Therefore, $\chi^H = 0.4\frac{w^H n^H + w^M n^M}{n^H + n^M}$.

Finally, we calibrate the value for the on-the-job search efficiency parameter, π, following Nagypal (2005) who finds that the ratio of job-to-job transitions to unemployment-to-employment transitions is between 2.57 and 3.07 for individuals with at least some post-secondary education. We take the midpoint of this range as a reference point and calibrate π so that

$$\frac{\pi f^H \left(1 - u^H/lfp^H\right)}{\left((f^L s^M + f^H s^H)/(s^M + s^H)\right)\left(u^H/lfp^H\right)} = 2.82.$$

For instance, then, given an unemployment rate of 5 percent a ratio of 2.82 implies that the relative transition rates are about 14 percent.

5.2 Main Results

Table 2 presents the main results in the baseline economy for the private (Panel A) and socially efficient equilibrium (Panel B). The solution to the planning problem has the planner endogenously choosing positive mismatch (as opposed to no equilibrium mismatch as in a so-called *ex post seg-*

mentation equilibrium in the language of Albrecht and Vroman (2002)), but no OTJ search. In the private equilibrium, one interesting result that arises endogenously from the calibration is that the aggregate mismatch rate, n^M/N is about 5 percent. This is very much in line with empirical results in Fallick and Fleischman (2004) who report a fraction of all employed individual actively engaged in OTJ search equal to about 0.045.

The welfare costs, measured as the percent of additional consumption that would be required to give to (or to take away from) each household to make them as well off in the private equilibrium as they are in the socially efficient equilibrium, are shown in lines 1 and 2 of the table. The welfare cost in the baseline calibration falls primarily on low-skill individuals—on the order of $1\frac{1}{2}$ percent of steady state consumption. In contrast, the costs imposed on the high-skilled household are fairly modest at under 10 basis points. The aggregate welfare cost is simply a weighted average of the costs for the low- and high-skill households.

The remainder of the table presents the set of allocations in each of the two equilibria. The allocations make clear that the welfare costs in the baseline economy stem from inefficiently high labor force participation for both households. In short, firms post an inefficiently low number of vacancies in the private equilibrium and households devote an inefficiently high amount of search effort in order to find a job. The result is a tighter labor market, which lowers job finding probabilities for both low- and high-tech jobs and, in turn, pushes unemployment rates above their socially optimal level. On net, the increase in search activity more than offsets the lower level of employment, resulting in participation rates that are inefficiently high for both low- and high-skilled households alike.

5.2.1 Isolating the Welfare Effects of Mismatch

The results presented in Table 2 include positive unemployment benefits and mismatch, both of which are distortionary. In order to isolate the welfare effects of skill-mismatch employment, it is useful to strip these distortions out of the model.

Table 3 parses the total welfare effects by holding all other parameters in the model constant and showing the incremental distortionary effects caused by permanent and temporary mismatch and, separately, the unemployment benefit. For reference, Panel D restates the total welfare costs reported in Table 2, and summing across Panels A through C in Table 3 add up to the total welfare costs reported in Panel D.

The distortionary effects of permanent mismatch are isolated in Panel A by shutting down both the unemployment benefit and OTJ search in the baseline economy. For high-skilled households, permanent mismatch creates welfare gains on the order of a $\frac{1}{4}$ of a percentage point of steady state consumption; these welfare gains comes entirely at the expense of low-skilled households.

Intuitively, the inability to engage in OTJ search makes mismatch employment more costly from the point of view of a high-skilled individual simply because it entails accepting a lower wage over a longer expected duration of the job. Thus, firms must be willing to pay a higher wage in order to entice high-skilled workers into accepting mismatch jobs. Even with the higher wage, activity in the market for mismatch jobs (both search as well as employment) remains suboptimally low and, as a result, a greater burden of production of the low-tech intermediate good shifts to low-skill workers. This negative spillover results from the fact that high-skill agents do not internalize the affect that their search activity in the market for low-tech jobs has on low-skilled workers.

Panel B isolates the distortion associated with the temporary nature of mismatch by re-introducing OTJ search into the economy in Panel A. The principle effect of OTJ search is to increase the flows out of mismatch employment. In doing so, this lowers the cost of mismatch employment to the high-skilled household simply because it shortens the expected length of time that the mismatched worker needs to accept a lower wage before potentially moving to a higher paying job in the high-tech sector. As a result, the high-skilled household reallocates search activity away from the market for high-tech jobs toward the market for low-tech jobs. The increase in search activity for low-tech jobs notwithstanding, sharp outflows through job-to-job transitions from successful OTJ search cause an overall decline in the stock of mismatch jobs. This decline taken together with the drop in search activity for high-skilled jobs cause labor force participation for high-skilled households to fall. Hence, the ability to engage in OTJ search generates welfare gains that are similar in magnitude to the gains from permanent mismatch. From the perspective of the low-skilled household, inefficiently low mismatch employment shifts an even greater burden of production onto low-skilled workers, who now need to devote even more search activity to an otherwise more competitive market for low-tech employment. This carries a significant welfare cost, nearly 1.2 percentage points of the steady state consumption of the low-skilled household. In this sense, we can say that OTJ search tends to amplify the welfare effects of mismatch for the low-skilled household.

Finally, Panel C shows the incremental welfare costs when we add back in unemployment benefits, taking us back to the baseline economy. Reintroducing the unemployment benefit generates welfare costs for both types of households. For both, the welfare costs stem from the fact that the unemployment benefit raises the outside option of workers and, in doing so, it drives up the bargained wage. Households respond by devoting more effort to search in order to benefit from the increase in compensation, while the declining capital value of a job leads firms to post fewer vacancies. All told, the market for both low- and high-skilled labor tightens, making it harder for workers to find jobs and significantly increasing the unemployment rate.

All told, for high-skilled households skill-mismatch employment is welfare enhancing and the resulting welfare gains offset roughly 80% of the costs associated with the unemployment benefit. The high-skilled household benefits in roughly equal proportion from both permanent and temporary mismatch. These welfare gains come at the expense of low-skilled households where skill-mismatch employment accounts for roughly 90% of the welfare costs in the baseline economy with the costs associated with the unemployment benefit explaining the remainder. The negative spillovers associated with the temporary nature of mismatch are considerably more costly (roughly six times higher) than the costs associated with permanent mismatch.

5.2.2 Mismatch and Wage-Inequality

Table 4 summarizes the effect of mismatch on two measures of wage inequality in the model. The first measures the skill premium as the ratio of the average wage for high-skilled over the average wage for low-skilled individuals. The second measures the within educational group occupational premium as the ratio of the average wage for high-skilled individuals to the mismatch wage. These two measures of wage inequality are identical to those examined in Dolado, Jansen, and Jimeno (2009).

The middle three columns reveal the incremental effect of the three distortions on each measure of wage inequality. Permanent mismatch has essentially no effect on wages, and hence wage inequality, at all. In the case of transitory mismatch, the sharply lower wage required to compensate the low-tech firm for the shorter expected duration of mismatch employment given the possibility of OTJ search results in a modest decline in the skill premium—by nearly $1\frac{1}{2}$ percent relative to the socially efficient equilibrium—and an increase in the within educational group occupational premium by nearly 10 percent. The response of the skill premium in our model is qualitatively different from Dolado, Jansen, and Jimeno (2009), who find that transitory mismatch raises the skill premium. The difference likely owes to the endogenous labor force participation margin in our model which neutralizes the response of the high-tech wage (whereas the high-tech wage increases sharply in Dolado, Jansen, and Jimeno (2009) where the participation margin is exogenous in their partial equilibrium setup). Finally, the unemployment benefit compresses both measures of wage inequality owing to the greater responsiveness of the low-skill and mismatch wage to the unemployment benefit.

5.3 The Relative Size of the Mismatch Distortion

Our theoretical results show a complicated interaction between the distortions generated by mismatch, the unemployment benefit, and the size of the congestion externality.

Figure 1, which is divided into four panels, explores this interaction. The top two panels present welfare calculations for values of the replacement rate for unemployment benefits varying between

0 and 0.8 (the baseline assumption is 0.4). In all cases, we assume the Hosios condition holds in both of the segmented labor markets ($\psi^i = \xi^i$) in order to isolate the interaction of the mismatch distortion with the distortion generated by unemployment benefits.

The top left panel considers the case in which mismatch is permanent. For reference, the results that isolate the affect of permanent mismatch (reported in Panel A of Table 3) occur at a 0% replacement rate and the baseline results (reported in Table 2) occur at a 40% replacement rate. The plot shows that there are small distributional effects associated with permanent mismatch (in the sense that high-skilled households gain at the expense of the low-skilled) in absence of the unemployment benefit. But, for sufficiently high levels of the replacement rate these distributional effects are overwhelmed by the overall welfare costs associated with the unemployment benefit.

The top right panel conducts the same exercise for the case in which mismatch is transitory. Comparing the top right to the top left panel for the case with no unemployment benefits clearly illustrates the amplification of the welfare costs of transitory mismatch. The distributional effects—the spread between the dotted and dashed lines—are more pronounced and the overall costs to low-skilled individuals are much higher. That said, qualitatively the story is similar in that the distortion created by the unemployment benefit eventually dominates the welfare effects of mismatch regardless of whether it is permanent or transitory.

The lower two panels conduct a similar exercise focusing on the strength of the congestion externality by varying worker's bargaining power between 0.2 and 0.8 (the baseline assumption is $\psi^i = \xi^i = 0.5$). In all cases, we assume that there are no unemployment benefits, $\chi^i = 0$, in order to isolate the interaction of mismatch and the congestion externality.

The bottom left panel considers the case in which mismatch is permanent. The welfare gains for both low- and high-skilled households zero out *in the neighborhood* of $\psi^i = \xi^i = 0.5$ reflecting, in part, the fact that there is no distortion from the unemployment benefit. That said, the welfare costs are not exactly zero at $\psi^i = \xi^i = 0.5$ due to the distortion generated by permanent mismatch (see Panel A from Table 3). The parabolic shape of the welfare curve illustrates that the welfare costs of the congestion externality are significant as we move farther away from the Hosios parameterization in either direction. Notice also that the distributional impact is sensitive to worker bargaining power. Higher bargaining power leads to larger welfare costs for low-skilled relative to high-skilled households; conversely, lower worker bargaining power leads to larger gains for high-skilled households. Finally, the bottom right panel conducts the same exercise for the case in which mismatch is transitory. At $\psi^i = \xi^i = 0.5$, the spread between the welfare costs to low-skilled individuals and the welfare gains to high-skilled individuals again reflects the amplification effect of transitory relative to permanent mismatch. The figure shows that the distributional aspect of the welfare costs of transitory mismatch is preserved in the range of $0.4 < \psi^i < 0.65$ but outside that range the welfare costs associated with the congestion externality dominate.

6 Conclusion

This paper analyzes the welfare costs of mismatch employment. The first main contribution is to derive a set of efficiency conditions that provide a complete characterization of the distortions generated by permanent and transitory mismatch. The second main contribution is to measure the quantitative magnitude of these distortions in a carefully calibrated version of the model that matches a number of aspects of the occupational and skill-based heterogeneity found in U.S. labor markets. Our quantitative results show that the welfare effects of mismatch are purely distribution in the sense that high-skilled individuals gain at the expense of the low-skilled. These distributional effects are most pronounced when mismatch is transitory as OTJ search acts to amplify both the welfare gains that accrue to high-skilled individuals and the welfare costs that accrue to low-skilled households.

There are a number of possible avenues for extending our analysis. Due to consumption risk sharing, our welfare effects are largely driven by differences in labor market outcomes for low- and high-skill households. It would clearly be interesting to see how our results change when relaxing this assumption. Our results point to different magnitudes for the welfare effects of permanent versus temporary mismatch. To this end, an interesting extension might be to re-examine our welfare results in a model that allows for lock-in to mismatch employment due to skill deterioration in the spirit of Pissarides (1994). Finally, although we have defined and measured the distortions associated with mismatch, we have not examined the design of optimal labor market policy to address these distortions. We leave these extensions for future research.

References

[1] Albrecht, J., and S. Vroman, (2002), "A Matching Model with Endogenous Skill Requirements," *International Economic Review*, 43, 283-305.

[2] Andolfatto, D, (1996), "Business Cycles and Labor-Market Search," *American Economic Review*, 86, 112-132.

[3] Arseneau, D.M. and S. K. Chugh, (2008), "Optimal Fiscal and Monetary Policy with Costly Wage Bargaining," *Journal of Monetary Economics,* 55, 1401-1414.

[4] Arseneau, D.M. and S. K. Chugh, (2012), "Tax Smoothing in Frictional Labor Markets," *Journal of Political Economy* 120(5), 926-985.

[5] Chassambouli, A. (2011). "Cyclical Upgrading of Labor and Employment Differences Across Skill Groups," *The BE Journal of Macroeconomics,* 11(1), art. 14.

[6] Davis S. J., R. J. Faberman, and J. Haltiwanger (2006), "The Flow Approach to Labor Markets: New Data Sources and Micro-Macro Links," *Journal of Economic Perspectives*, 20, 3-26.

[7] Dolado, J.J., M. Jansen, and J.F. Jimeno, (2009), "On-the-job Search in a Matching Model with Heterogeneous Jobs and Workers," *The Economic Journal* 119, 200-228.

[8] Elsby, M., R. Michaels, and G. Solon (2009), "The Ins and Outs of Cyclical Unemployment," *American Economic Journal: Macroeconomics*, 1(1), 84-110.

[9] Epstein, B. (2012). "Heterogeneous Workers, Optimal Job Seeking, and Aggregate Labor Market Dynamics." *Board of Governors of the Federal Reserve System International Finance Discussion Papers*, (1053).

[10] Estevão, Marcello, and Evridiki Tsounta. 2011. "Has the Great Recession Raised U.S. Structural Unemployment?" *IMF Working Paper* 11/105, May.

[11] Fallick, B. and C. Fleischman (2004), "Employer-to-Employer Flows in the U.S. Labor Market: The Complete Picture of Gross Worker Flows", *Finance and Economics Discussion Series* #2004-34. Federal Reserve Board, Washington D.C.

[12] Gautier, P.A., (2002), "Unemployment and Search Externalities in a Model with Heterogeneous Jobs and Workers," *Economica,* 69. 21-40.

[13] Gautier, P.A., C.N. Tuelings, and A. Van Vuuren, (2010), "On-the-Job Search, Mismatch, and Efficiency," *Review of Economic Studies,* 77. 245-272.

[14] Hagedorn, M. and I. Manovskii, (2008), “The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited,” *American Economic Review*, 98, 1692-1706.

[15] Hammermesh, D. S., (1993), *Labour Demand* Princeton: Princeton University Press.

[16] Hosios, A. (1990), “On the Efficiency of Matching and Related Models of Search and Unemployment,” *Review of Economic Studies*, 57(2), 279-298.

[17] Khalifa, S. (2010), “Job Competition, Crowding Out and Unemployment Fluctuations,” *Macroeconomic Dynamics*, 16(1), 1-34

[18] Krause, M. and T. Lubik (2006) “The Cyclical Upgrading of Labor and On-the-Job Search,” *Labour Economics*, 13(4), 459-477.

[19] Krusell, P., Ohanian, L. E., Ríos-Rull, J. V., and G.L. Violante (2000). “Capital-skill complementarity and inequality: A macroeconomic analysis,” *Econometrica,* 68(5), 1029-1053.

[20] McLaughlin, K.J., and M. Bils (2001). “Interindustry mobility and the cyclical upgrading of labor,” *Journal of Labor Economics*, 19(1), 94-135.

[21] Merz, M., (1995), “Search in the Labor Market and the Real Business Cycle,” *Journal of Monetary Economics*, 36, 269-300.

[22] Mortensen, D., and Christopher Pissarides. 1994. "Job Creation and Job Destruction in the Theory of Unemployment." The Review of Economic Studies, 61(3): 397-415.

[23] Moscarini, G. (2005), “Job Matching and the Wage Distribution,” *Econometrica*, 73(2), 481-516.

[24] Nagypal, E., (2005), “On the Extent of Job-to-Job Transitions,” Northwestern University Mimeo.

[25] Petrongolo, B. and C. Pissarides, (2001), “Looking into the Black Box: A Survey of the Matching Function,” *Journal of Economic Literature*, 39, 390-431.

[26] Pissarides, C. (1994), “Search Unemployment with On-the-job Search,” *Review of Economic Studies,* 61(3), 457-475.

[27] Pissarides, Christopher. 2000. Equilibrium Unemployment Theory. Cambridge, MA: MIT Press.

[28] Pries, M. (2008), “Worker Heterogeneity and Labor Market Volatility in Matching Models,” *Review of Economic Dynamics,* 11(3), 664-678.

[29] Sicherman, N. (1991), "Overeducation in the Labor Market," *Journal of Labor Economics*, 9, 101-112.

[30] Shimer, R, (2003), "Dynamics in a model of on-the-job search," University of Chicago, mimeo.

[31] Shimer, R. (2005), "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," American Economic Review 95(1), 25-49.

[32] Shimer, R. (2006), "On-the-Job Search and Strategic Bargaining," *European Economic Review*, 50(4), 811-830.

[33] Shimer, R. (2012), "Reassessing the Ins and Outs of Unemployment," *Review of Economic Dynamics,* 15(2), 127-148.

[34] World Economic Forum. Global Agenda Council on Employment (2014), "Matching Skills and Labour Market Needs," Davos, Switzerland. Available on-line at: http://www3.weforum.org/docs/GAC/2014/WEF_GAC_Employment_ MatchingSkillsLabourMarket_Report_2014pdf.

Table 1. Baseline parameterization (monthly frequency)

Parameter	Value
Preferences	
Discount factor, β	0.996
Utility curvature, σ	1
Participation disutility exponent, ε	1
Low-skill participation disutility scaling, ϕ^L	9.79
Mismatch participation disutility scaling, ϕ^M	89.22
High-skill participation disutility scaling, ϕ^H	11.42
Production	
Aggregate technology, Z	4.75
Sectoral technologies, $Z^H = Z^L = z^M = z^L$	1
High-skill share in final goods, ϱ^H	0.505
Mismatch share in low-tech production, ϱ^L	0.547
Final goods input substitutability, ω_F	0.4
Low-tech input substitutability, ω_L	1
Labor Market	
Fraction of low-skill population, κ	0.5
Vacancy flow costs, $\gamma^H = \gamma^L$	2.33
Low-tech job destruction probability, ρ^L	0.061
High-tech job destruction probability, ρ^H	0.035
Low-tech matching efficiency, A^L	0.769
High-tech matching efficiency, A^H	0.652
Matching function elasticity, $\xi^L = \xi^H$	0.5
Worker bargaining power, $\psi^H = \psi^L$	0.5
Low-skill unemployment benefits, χ^L	0.524
High-skill unemployment benefits, χ^H	0.709
On-the-job search efficiency, π	0.136

Table 2: Allocations in Baseline Economy

Variable	A. Private Equilibrium		B. Socially Efficient Equilibrium	
Welfare Costs				
1. Hh welfare (L, H)	1.523	0.086		
2. Agg welfare	0.805			
Aggregates				
3. c^L, c^H	0.471		0.469	
4. LFP rates (L, H)	0.542	0.719	0.527	0.716
Labor Market Variables				
5. n^L, n^H	0.251	0.313	0.252	0.314
n^M	0.030		0.034	
6. s^L, s^H	0.036	0.021	0.027	0.019
s^M	0.008		0.004	
7. v^L, v^H	0.014	0.011	0.017	0.014
8. θ^L, θ^H	0.314	0.446	0.534	0.742
9. f^L, f^H	0.431	0.436	0.563	0.562
10. U rates (L, H)	0.075	0.046	0.045	0.028

Table 3: Incremental Welfare Effects of the Three Distortions*

	A. Permanent Mismatch		B. Transitory Mismatch		C. Unemployment Benefits		D. Total Welfare Costs	
1. Hh welfare (L, H)	0.186	−0.184	1.182	−0.214	0.156	0.484	1.523	0.086
2. Agg welfare	0.001		0.484		0.320		0.805	

* Welfare costs (gains) are calculated as percent of steady state consumption required to give to (take away from) each household (low- and high-skilled, seperately) in the private equilibrium to make them as well off as in the socially efficient equilibrium. Aggregate welfare costs are simply an equally weighted sum of the costs to low- and high-skilled households. Positive numbers indicate welfare costs and negative numbers indicate gains.

Table 4: Mismatch and Wage Inequality

		Change Owing to:			
	A. Socially Eff. Equilibrium*	B. Permanent Mismatch	C. Transitory Mismatch	D. Unemployment Benefit	E. Baseline Economy
1. Skill Premium, (W^H/w^L)	1.378	˜0	−0.019	−0.007	1.351
2. Within Education Occup. Premium, (W^H/w^M)	1.139	˜0	0.109	−0.074	1.175

* Measures of wage inequality in the socially efficient equilibrium are backed out using the socially efficient allocations and the Nash wage expression. Also, note that $W^H = \left(n^H w^H + n^M w^M\right)/(n^H + n^M)$.

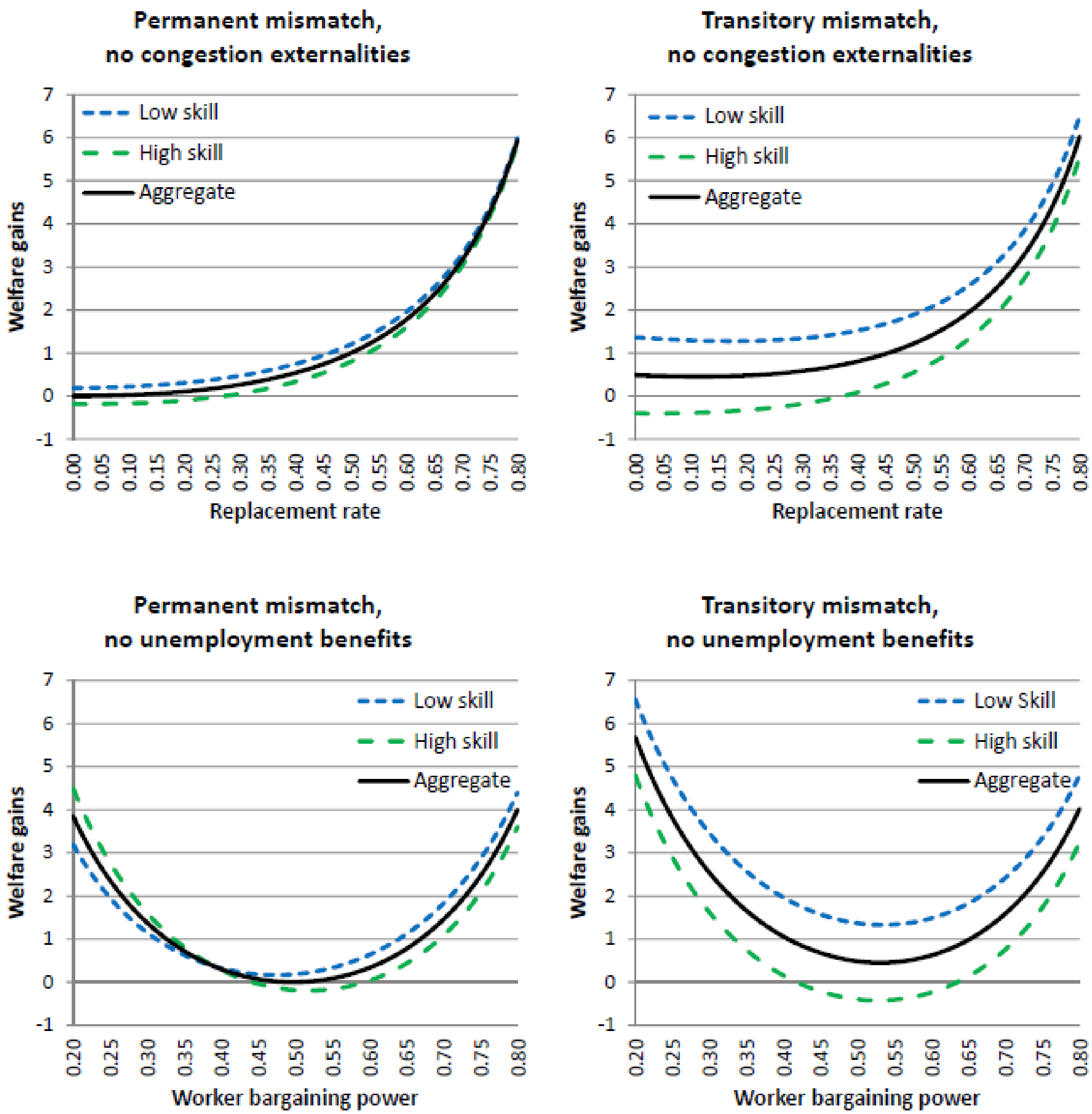

Figure 1: Size of mismatch distortion relative to unemployment benefit and congestion externality.