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A Two Country Model of Financial Capital Movements as Stock Adjustments with Emphasis on the Effects of Central Bank Policy

by

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I. Introduction

Over the past decade theoretical and empirical analyses of financial asset holding have relied increasingly upon the portfolio balance approach. According to this approach the desired holdings of each financial asset by a well defined group of wealth holders depend upon the levels of the interest rates on the asset in question and on alternative assets, the size of the portfolio of the group of asset holders under consideration, and perhaps other variables. Changes in desired asset holdings depend upon changes in these variables.

In an open economy in which residents hold both domestic and foreign financial assets changes in desired holdings of financial assets result in capital flows. Previous authors who have applied the portfolio balance approach to the analysis of capital flows have divided such changes in desired holdings into two components.[1/] The part of

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1/ See for example Bryant and Hendershott (1970), Miller and Whitman (1970a, and 1970b), Willett and Forte (1969), and Grubel (1968).

changes in desired asset holdings due to changes in one or more interest rates or other variables with the size of the portfolio held constant has been called the stock-shift component. The part of changes in desired asset holdings due to growth in the size of the portfolio with interest rates and other variables held constant has been called the continuing-flow effect.[1/]

In this paper we focus upon the short run portfolio balancing behavior of wealth holders. We assume that the size of wealth holders' portfolios are fixed and examine in detail the stock-shift component of capital flows.[2/] This emphasis seems useful for two reasons. First, casual observation and what empirical evidence we have suggest that the stock-shift component of capital flows is very important quantitatively.[3/] Secondly, many observers have become increasingly convinced

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1/ A slight modification of the formulation in Bryant and Hendershott (1970) allows us to write their expression for the desired holding of a financial asset designated as F*, as follows:

$$F^* = g(R,0)S,$$

where R represents a vector of interest rates, 0 is a vector of other relevant variables, and S is a measure of portfolio size. The change in desired holdings when the right hand side variables change is,

$$dF^* = Sdg(R,0) + g(R,0)dS.$$

The stock shift effect is $Sdg(R,0)$, and the continuing flow effect is $g(R,0)dS$.

2/ The size of wealth holders' portfolios is assumed to be fixed except in Chapter VI where the effects of a devaluation are considered. A devaluation directly affects the size of wealth holders' portfolios.

3/ See, for example, Miller and Whitman (1970a and 1970b).

that rapid movements of large amounts of financial capital across national boundaries, which are probably best viewed as being made up mainly of stock-shift changes in desired asset holdings, severely limit the freedom of policy makers to use the instruments at their command to achieve their policy objectives under a regime of fixed exchange rates.

Much useful effort has been expended in spelling out the implications of the portfolio balance formulation for the case of a small open economy.[1/] Our objective is to trace out the implications of the portfolio balance approach to economies which loom large in the world economy. In order to accomplish this objective we develop a model that deals explicitly with two countries which, taken together, constitute the world economy. This model is outlined in Chapter II. One important feature which distinguishes our model of the large-country case from models of the small-country case is that interest rates are jointly determined with desired asset holdings instead of being taken as exogenous.[2/] Another important feature of our model is that we are able to specify in detail the relationship between variables directly under the control of the policy authorities and both the level of interest rates and desired financial asset holdings. For this reason we believe that empirical work based on the theoretical model we consider

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1/ See, for example, Mundell (1968, Chapter 18) and McKinnon and Oates (1966).

2/ For examples of work on capital flows which takes interest rates as exogenously determined see inter alia Branson (1968), Branson and Hill (1971), Miller and Whitman (1970a and 1970b).

here would be of more use to policy makers in large countries than most of the existing empirical work on capital flows which is based on the assumption that interest rates can be taken as exogenous. 1/

In a static short-run portfolio balance model such as the one we shall employ a change in some aspect of the economic environment disturbs an existing equilibrium and sets in motion changes in interest rates, flows of financial capital between countries, and changes in the reserves of central banks. These changes persist until wealth holders in both countries have attained a new equilibrium position in which they hold their desired quantities of all financial assets. Exactly what this new equilibrium looks like depends crucially upon the behavior of the two central banks. Two aspects of central bank behavior are particularly important. We must know the policy objectives of the central banks, that is, whether or not they are trying to stabilize the money supply or the interest rate on domestic liabilities or are pursuing some other objective. Also of primary concern are the "rules of the game" under which the central banks pursue their policy objectives. We explore in detail the implications of two alternative fixed exchange regimes under various assumptions about the policy objectives of the central banks. In Chapter III we investigate an outside reserve regime, of which the gold standard is an example. In Chapter IV we turn to

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1/ Two recent empirical studies which attempt to take account of the fact that interest rates and desired asset holdings are simultaneously determined when the country being studied should be regarded as large are Miller and Whitman (1970c) and Kouri and Porter (1972).

what we call the key currency reserve regime which we believe captures some crucial aspects of more recent international monetary experience.

It has often been argued that exchange rate changes can be expected to have little or no impact on desired holdings of financial assets and therefore on the flows of financial capital captured by the capital account of the balance of payments. This argument neglects the fact that an exchange rate change results in capital gains for some wealth holders and capital losses for others. These capital gains and losses arise because of changes in the value of particular assets so that, in general, wealth holders are left with unbalanced portfolios. The implications of the wealth effects consequent upon a devaluation for interest rate levels and central bank reserve holdings are traced out in Chapter V.

For purposes of simplicity we assume throughout most of the paper that there is no commercial banking system, that is, that all money is in the form of currency or deposits at the central banks. In Chapter VI we relax this assumption in order to determine the sensitivity of our conclusions to the presence of a fractional reserve commercial banking system and in order to consider some of the implications of broadening the range of alternatives for the holding of international reserves available to central banks.

Armed with the results of previous Chapters, we attempt in Chapter VII to isolate some other factors which determine the effectiveness of a monetary policy action, such as an open market purchase, undertaken by the central bank in an open economy in attaining some

proximate objective, such as lowering the interest rate on liabilities of domestic residents. While, as we show in Chapters III through VI, the effectiveness of monetary policy depends in part on the behavior patterns of the two central banks and the prevailing type of fixed exchange rate regime, two other features of the economic environment play extremely important roles. The effectiveness of monetary policy depends in an important and relatively straightforward way on the relative economic size of the country undertaking the policy action. In addition the degree of substitution between the liabilities of domestic residents and the liabilities of foreigners in the portfolios of the world's wealth holders has a significant if less obvious impact on the effectiveness of monetary policy.

In Chapter VIII we draw some general conclusions from our analysis.

II. The Model

A. An Overview

In the world of our model there are two countries, the United States (U.S.) and the United Kingdom (U.K.). The U.S. currency is the dollar; the U.K. currency is the pound. We analyze the behavior of four groups: the United States Central Bank (USCB), the United Kingdom Central Bank (UKCB), ultimate wealth holders in the U.S., and ultimate wealth holders in the U.K. We assume that there are only four types of assets: U.S. money, U.K. money, U.S. securities, and U.K. securities.[1/]

1/ We assume initially that there are no private banking sectors in the two countries. A brief discussion of the effects of introducing a simple private fractional reserve banking system is included in Chapter VI.

The assumption that only a single type of security is issued in each country affords considerable simplicity in exposition and is not as restrictive as it might seem at first. It is well known that a bundle of securities can be treated as a simgle security if "enough" market participants view the securities in the bundle as perfect substitutes. Our results are applicable in situations in which this condition is fulfilled even if it is not literally true that only one security is issued in each country.

Our results would not be substantially affected by the introduction of non-bank financial intermediaries if these intermediaries did not hold cash and/or foreign assets (that is, if the intermediaries held only "primary" or "secondary" securities issued by domestic residents) and if "enough" market participants viewed the "secondary" securities (liabilities) issued by these intermediaries as perfect substitutes for the "primary" securities (liabilities) of the government, the non-financial corporate sector, and consumers.

Each country's money is held only by its own ultimate wealth holders,[1] but both types of securities, which are identified by the issuer's country of residence, are held in both countries. Money is held in the form of currency or deposits at the central bank. Securities are fixed in nominal value in the currency of the country in which the issuer resides and have variable interest rates.[2] Ultimate wealth holders in both countries regard the two securities in the model as imperfect substitutes.[3]

1/ The model can be adapted easily to allow each central bank to hold the money of the other country as part of its reserves. See Chapter VI.

2/ They are like call loans or savings and loan shares. This assumption makes the analysis much more straightforward, but the results would be substantially unaffected if each of the two securities had a coupon rate and a fixed maturity date or were a consol.

3/ The variety of financial assets which one can imagine being available in a two country world can obviously be arranged into several alternative simple classification schemes for purposes of analysis. Given the issues we want to consider, we have found it most advantageous to treat all the securities issued by the residents of a given country as a "single" security denominated in the currency of that country which is "different" from the "single" security issued by residents of the other country and denominated in that country's currency. The assumptions under which we could regard the bundle of securities issued by the residents of a country as a "single" security are discussed in footnote 1, on the previous page.

What might cause wealth holders to view U.S. and U.K. securities as imperfect substitutes under a fixed exchange rate system or under a flexible exchange rate system in which exchange rates are not "expected" to change? Perhaps most important is the fact that fluctuations in economic activity are not perfectly correlated across countries. In addition, actual or potential exchange restrictions may affect investors views of "foreign" securities. Also governments may be viewed as having different degrees of "responsibility." Thus even if forward exchange markets are "perfect," so that a security denominated in foreign currency can, in effect, be converted into a security denominated in domestic currency, at a small transaction cost, foreign and home securities may not be treated as perfect substitutes if the underlying sources of the value of the securities are subject to different influences.

Everywhere in this study except in Chapter V it is assumed that the exchange rate is fixed and that ultimate wealth holders in both countries expect it to remain constant. Anticipated rates of inflation are assumed to be zero in both countries, so that there is no divergence between nominal and real interest rates.[1/]

We analyze the behavior of central banks and ultimate wealth holders in the "short run". Disturbances to financial asset equilibrium cause instantaneous adjustments in interest rates and in the financial asset holdings of both ultimate wealth holders and central banks, but in our short run prices, incomes, employment levels, and capital stocks remain fixed. We also assume that wealth holders do not take into account their current savings when deciding how to allocate their existing wealth among available assets.[2/]

B. The Portfolio Balancing Behavior of Ultimate Wealth Holders

Ultimate wealth holders in the U S. base their nominal demands in terms of dollars for U.S. securities (B^d), for U.K.

1/ We could carry out our analysis completely in terms of nominal rates of interest if wealth holders anticipated inflation in either or both countries so long as the disturbances we consider do not affect the anticipated inflation rates.

2/ More precisely, our model can be viewed as a period model in which wealth holders balance their portfolios at the beginning of the period taking no account of the additional wealth they will accumulate during the period. Prices, incomes, capital stocks, and employment in the current period are affected by financial variables from previous periods but not those of the current period. A disturbance to financial asset equilibrium causes instantaneous changes in interest rates and asset holdings of both private and public wealth holders at the beginning of the period.

securities (F^d), and for U.S. money (M^d) on their existing dollar denominated nominal wealth (W). We assume that the fraction of their nominal wealth which they wish to hold in each of these three assets depends only upon the interest rate on U.S. securities (r) and the interest rate on U K. securities (r'),[1/]

1/ Nominal demands are assumed to be homogeneous of degree one in nominal wealth.

Often asset demand functions are written so as to give demands for real asset values as functions of real wealth and other variables. An example of a quite general asset demand function is the following:

$$\left(\frac{B}{Q}\right)^d = h\left(r, r', \frac{PY + rB^A + r'\Pi F^A - rB^{A'}}{Q}, \frac{W}{Q}\right),$$

where $\left(\frac{B}{Q}\right)^d$ is real asset value of U.S securities demanded, P is the price of the single U.S. good the output of which is given by Y, Q is a price index which reflects the relative weight in consumption of the single U.S and the single U.K. good, B^A represents U.S. holdings of U.S. sevurities, ΠF^A is the dollar value of U.S. holdings of U.K. securities, B'^A is the dollar value of U.K. holdings of U S. securities, and $PY + rB^A + r'\Pi F^A - rB'^A$ is the nominal income of U.S. residents (assuming no taxes or transfers). The superscript (^{A}) denotes actual holdings of an asset. If we ignore interest payments and assume that the asset demand function is homogeneous of degree one in real wealth and that asset demand is unchanged so long as the ratio of real income to real wealth remains constant, we have,

$$\left(\frac{B}{Q}\right)^d = h\left(r, r', \frac{PY}{W}\right)\frac{W}{Q}.$$

Since prices, outputs, and existing nominal wealth levels are assumed to remain constant we can subsume $\frac{PY}{W}$ into the functional form and write,

$$\left(\frac{B}{Q}\right)^d = b(r,r')\frac{W}{Q}$$

or,

$$B^d = b(r,r')W.$$

We restrict our attention to this special form of asset demand function because of its simplicity and intuitive appeal.

$$B^d = b(r, r')W \tag{1}$$

$$F^d = f(r, r')W \tag{2}$$

$$M^d = m(r, r')W. \tag{3}$$

Similarly U.K. wealth holders base their pound denominated nominal demands for U.S securities ($B^{d'}$), for U.K. securities ($F^{d'}$), and for U.K. money ($N^{d'}$) on their existing pound denominated nominal wealth (W'). Their demand functions are assumed to have the same general form as those of U.S. wealth holders,

$$B^{d'} = b'(r, r')\ W' \tag{4}$$

$$F^{d'} = f'(r, r')\ W' \tag{5}$$

$$N^{d'} = n'(r, r')\ W' \tag{6}$$

The (') superscript denotes the value of a variable for the U.K.

Since we assume that ultimate wealth holders do not take account of their current savings when deciding how to hold their existing wealth, disturbances to financial asset equilibrium result only in attempts by wealth holders to change the composition of the assets they hold. The behavior of each country's wealth holders in allocating their fixed wealth is subject to a <u>balance sheet constraint</u>. For ultimate wealth holders in, say the U.S., wealth is defined to be the sum of the nominal values of the securities and money they currently hold. This sum must be <u>identically</u> equal to the sum of the nominal demands by U.S. wealth holders for all assets. They cannot, after all, rationally desire to allocate more wealth than they possess. Thus b, f, and m must sum to one. The balance sheet constraint also implies

that if, as the result of an interest rate change, wealth holders want more of one asset they must want to reduce their holdings of one or both of the other two assets by the same amount. More technically, the sum of the partial effects on the three asset demands of a change in either of the two interest rates must be zero. U.K. ultimate wealth holders face a similar balance sheet constraint in allocating their fixed wealth. All this means that under our assumption of fixed portfolio size in the short run only two of the three asset demand functions are independent in each country.[1/] We will use the implications of the balance sheet constraints for ultimate wealth holders in both countries repeatedly in what follows.

1/ From the U.S. balance sheet constraint we have,

$$W \equiv B^A + \Pi F^A + M^A \equiv B^d + F^d + M^d,$$

where W has a fixed dollar value by assumption. Π is the dollar price of pounds. From the U.K. balance sheet constraint we have,

$$W' \equiv \frac{B^{A'}}{\Pi} + F^{A'} + N^{A'} \equiv B^{d'} + F^{d'} + N^{d'}$$

where W' has a fixed pound value. From the first of these two identities and equations (1) through (3) we have,

$$M^d \equiv m(r, r')\, W \equiv [1 - b(r, r') - f(r, r')]\, W.$$

A similar relationship holds for the U.K. Using the two identities and equations (1) through (6) we obtain,

$$b_r + f_r + m_r \equiv b_{r'} + f_{r'} + m_{r'} \equiv b'_r + f'_r + n'_r \equiv b'_{r'} + f'_{r'} + n'_{r'} \equiv 0,$$

where b_r is the partial derivative of b(r, r') with respect to r, etc.

We assume that the two securities and home country money are strict gross substitutes in the portfolios of ultimate wealth holders in each of the two countries. This assumption means that if the interest rate on a given security rises, desired holdings of that security increase while the desired holdings of the other security and money decline. Of course, we know from the balance sheet constraint that the sum of the declines in the desired holdings of the other security and money must equal the increase in desired holding of the security the interest rate on which has risen.[1/]

C. The Balance Sheets and Behavior of the Central Banks

Consider first the U.S. Central Bank (USCB). Its only liability is the domestic money stock (currency and/or deposits at the USCB) denoted by M^s; its assets are its holdings of U.S. securities (B^c), the dollar value of its international reserve assets (σR), and, if necessary, a dummy asset (S) which makes the balance sheet balance. The USCB balance sheet identity is given by,

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1/ More formally, the effects of a rise in the interest rate on U.S. securities on the portfolio of U.S. wealth holders can be summarized as follows

$$b_r > 0,\ f_r < 0,\ m_r < 0,$$

$$b_r + f_r + m_r \equiv 0.$$

Similar relationships hold for the effect of an increase in r on U.K. portfolios and the effect of an increase in r' on both U S. and U K. portfolios.

$$M^S \equiv B^C + \sigma R + S. \tag{7}$$

σ is the dollar price of reserve assets. For the UKCB we have,

$$N^{S'} \equiv F^{C'} + \sigma' R' + S', \tag{8}$$

where $N^{S'}$ is the U.K. money stock, $F^{C'}$ is the UKCB holdings of U.K. securities, σ' is the pound price of reserve assets so that $\sigma' R'$ is pound value of U.K. holdings of international reserves, and S' is a dummy asset which may be needed to balance the UKCB balance sheet.[1/] σ and σ' are assumed to remain constant at a value of one throughout this study except in Chapter V where the effects of devaluation are considered. Since $\pi = \frac{\sigma}{\sigma'}$, it remains constant at a value of one except in Chapter V. S and S' are assumed to remain constant throughout the study except in Chapter V when they are assumed to change so as to offset the initial effects of changes in σ and σ' on the asset side of the USCB and UKCB balance sheets.

Although a full discussion of the behavior of the central banks must be deferred until we have finished laying out the model, some brief remarks here can provide a sense of direction. Central banks may respond "passively" to disturbances to asset equilibrium, or they may initiate such disturbances. How they choose to respond or what changes they choose to initiate will be reflected in which

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1/ We assume that the USCB does not hold U.K. securities or U.K. money. In Chapter IV we examine the implications of a reserve currency system where the UKCB holds reserves in the form of U.S. securities and deposits at the USCB. In Chapter VI we make some comments on the implications of the UKCB holding reserves in the form of deposits at U.S. commercial banks.

items in their balance sheets they allow to or make change and which they keep constant. Our analysis of the effects of central bank behavior is based upon translating central bank responses and initiatives into changes in central bank balance sheets and spelling out the implications of these changes for the supplies of the various financial assets available to the public.

D. The Market Equilibrium Conditions

We are now prepared to consider the equilibrium conditions for the markets in the four assets in the model. We turn first to the market for U.S. securities. The total supply of U.S. securities ($\bar{B}$)[1/] minus the holdings of the USCB must equal the demand for these securities by ultimate wealth holders in the two countries,

$$\bar{B} - B^c = b(r, r')W + \pi\, b'(r, r')W' \tag{9}$$

Similarly the total supply of foreign securities minus the holdings of the UKCB, both in dollar terms, must equal the dollar denominated demand for these securities by ultimate wealth holders,

$$\pi\,(\bar{F} - F^{c'}) = f(r, r')W + \pi\, f'(r, r')W'.^{2/} \tag{10}$$

1/ The total supply of U.S. securities ($\bar{B}$) is equal to the sum of cumulated U.S. government deficits and the value of net claims to the income of the capital stock of the U.S. non-financial corporate sector. $\bar{B}$ might also include consumer debt, but we assume that obligations of ultimate wealth holders to one another are netted out. $\bar{F}$ has a similar interpretation.

2/ Instead of including in $\bar{B}$ and $\bar{F}$ all securities issued in the two countries it may be useful to restrict attention to some subset of securities. A significant part of short-run financial capital movements seems to be made up of changes in holdings of short-term (liquid) assets. If people quickly balance money and short-term security holdings according to rates of return independently of their holdings of long-term (illiquid) securities, then for some purposes defining W and W' to include only short-term securities and money may be useful. Our treatment is theoretical, and we make no attempt to prejudge the question of what is the best empirical definition of allocatable wealth in any particular application.

Using the central bank balance sheet identities we can write the money market equilibrium conditions for the two countries as follows

$$B^c + \sigma R + S = m(r, r')W \tag{11}$$

$$F'^c + \sigma'R' + S' = n'(r, r')W' \tag{12}$$

These four market equilibrium conditions are not independent. Adding the left hand sides of 9, 10, 11 and Π times 12 we obtain the total dollar value for all assets available to ultimate wealth holders, which can be expressed as $\bar{B} + \Pi \bar{F} + \sigma R + \Pi \sigma'R' + S + \Pi S'$. Adding the right hand sides of 9, 10, 11, and Π times 12 and taking account of the fact that $b + f + m \equiv b' + f' + n' \equiv 1$ we obtain world wealth in dollar terms or $W + \Pi W'$. $\bar{B} + \Pi \bar{F} + \sigma R + \Pi \sigma'R' + S + \Pi S'$ is identically equal to $W + \Pi W'$, so only three of the four market equilibrium conditions are independent. 1/

1/ In the text the symbols W and W' are used to refer to the wealth levels of the residents of the U.S. and the U.K. respectively. It is assumed that the residents of each of the two countries regard the government securities they hold as part of their wealth or net worth; that is, they do not take account of the fact that they will be taxed to pay the interest on government securities when they calculate their net worth. Under this assumption an open market operation which is simply an exchange of securities for money does not affect the net worth of the public.

This treatment affords considerable simplicity, but under some reasonable assumptions our analysis is not substantially affected if an alternative treatment is adopted. Suppose private wealth holders in their role as taxpayers treat government securities as their own liabilities. It is then useful to distinguish between the allocatable assets of wealth holders which include their holdings of government securities gross of discounted tax liabilities and the net worth of wealth holders which is net of discounted tax liabilities. Under this assumption open market operations would alter the net worth of the public but not its allocatable assets. Now let us reinterpret W and W' as referring to the allocatable assets of the public. If we assume that in the short run the demand for money and securities (including government securities) depends upon the level of allocatable wealth

(Footnote continued on following page)

Three independent market equilibrium conditions are sufficient to determine three endogenous variables. Different sets of three variables are taken to be endogenous at different stages in the study depending upon the situation which is being analyzed. In one important case, for example, the two interest rates (r and r') and the stock of reserves held by the USCB (R) are regarded as being endogenously determined and all the variables are considered to be given exogenously. As the analysis proceeds it should be clear from the situation being analyzed which three variables should be regarded as endogenous.

E. A Geometric Presentation of the Model

For most of what we do in the following chapters a geometric presentation of the model will be sufficient though we will revert to algebra in some of the later Chapters. We introduce the geometric presentation by using it to describe the equilibrium of the model under a fixed exchange rate system (Π, σ, and σ' constant). In Figure 1 we plot four schedules which show the pairs of r and r' which are compatible with equilibrium in each of the four financial

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1/ Continued from previous page

and is independent of the level of net worth then our analysis is unaffected. This assumption seem quite reasonable in the context of a short run model. However, if wealth holders base their desires to accumulate on some net worth target then the results of models meant to refer to the medium or long run will be materially affected by the assumptions made regarding the way in which wealth holders regard their tax liabilities.

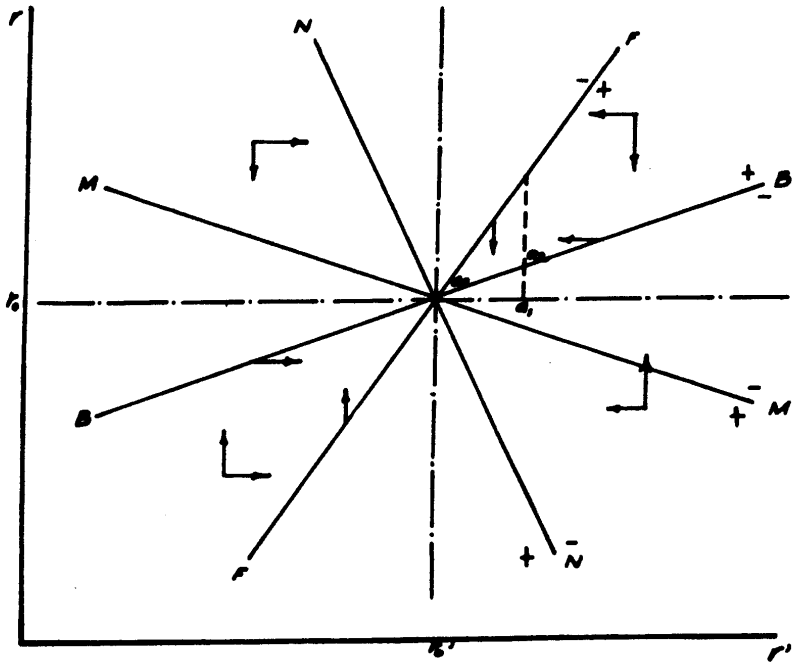

FIGURE 1

asset markets. The curve labeled BB gives the combinations of r and r' for which the private demand for securities issued in the U.S. is equal to the fixed supply of securities issued in the U S. minus the holdings of U.S. securities by the USCB. For a given supply of U S. securities available for U.K. and U.S. residents to hold, an increase in the interest rate on U.S. securities must be accompanied by an increase in the interest rate on U.K. securities if private wealth holders are to continue to be content to hold the available supply of U.S. securities. This is true because an increase in r causes an excess demand for U.S. securities so that r' must rise

in order to cut demand back until it matches the fixed supply of U.S. securities to ultimate wealth holders.[1/] To the right of the BB curve there is an excess supply of U.S. securities, and to the left of BB there is excess demand for U.S. securities. A region which contains interest rate pairs that imply excess supply in a market is indicated by a minus sign near the schedule for that market in the relevant region of the diagram; a plus sign in a sector near a schedule for a given market indicates that interest rate combinations in that sector imply excess demand. An excess supply of U.S. securities while excess demand for U.S. securities tends to force down the U.S. rate. The vertical arrows in Figure 1 represent the direction of pressure on r.

The curve labeled FF in Figure 1 represents the combinations of r and r' which issue that the demand for securities issued in the U.K. is equal to the available supply given a fixed total supply and fixed holdings of U.K. securities by the UKCB. The FF curve is positively sloped; an increase in r is required to offset the excess

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1/ Totally differentiating equation (9) with $\bar{B}$, B^c, π, W, and W' fixed and solving for the desired slope we have,

$$\left.\frac{dr}{dr'}\right|_{BB} = -\frac{b_{r'}\, W + \pi b'_{r'} W'}{b_r\, W + \pi b'_r\, W'}.$$

Given our assumption that the two securities are strict gross substitutes we have $b_{r'}, b'_{r'} < 0$ and $b_r, b'_r > 0$ so that $\left.\frac{dr}{dr'}\right|_{BB} > 0$.

demand caused by an increase in r' if private wealth holders are to continue to be satisfied holding the available supply of U.K. securities.[1/] The area to the right of the FF curve is an area of excess demand for U.K. securities; r' is too high given the level of r, and there is downward pressure on r'. For combinations of r and r' lying to the left of FF there is an excess supply of U.K. securities, and at any of these combinations upward pressure is exerted on r'. The horizontal arrows in Figure 1 represent the direction of pressure on the interest rate on U.K. securities.[2/]

The assumption that the three assets held by wealth holders in each of the two countries are strict gross substitutes insures that the slope of the FF curve must be greater than the slope of the BB curve. If r' is increased while r is held constant, there

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1/ Totally differentiating equation (10) with $\bar{F}$, $F^{c'}$, π, W, and W' held fixed and solving for the desired slope we have,

$$\left.\frac{dr}{dr'}\right|_{FF} = - \frac{f_{r'}W + \pi f'_{r'} W'}{f_r W + \pi f'_r W'}$$

The assumption that the two securities are strict gross substitutes insures that $\left.\frac{dr}{dr'}\right|_{FF} > 0$.

2/ See the Appendix for a more detailed discussion of adjustment behavior.

will be an excess demand for U.K. securities and an excess supply of U.S. securities as at point a_1 in Figure 1. Also, since it is assumed that money holdings are strict gross substitutes for securities, the demand for money in each country must have fallen. At the new, higher r' and constant r there is an excess demand for U.K. securities and an excess supply of U S. securities, U.K. money, and U.S. money. If now r is increased holding r' at its new higher level then the excess demand for U.K. securities and the excess supply of U.S. securities must decline. Also, the excess supply of the two kinds of money must increase since money is a strict gross substitute for U.S. and U.K. securities in both countries. As r is raised both the FF and BB curves must be crossed eventually. Suppose that the FF curve is met first, that is, that the FF curve is flatter than the BB curve. When the FF curve is encountered, the world excess demand for U.K. securities is zero, but there must still be an excess supply of U.S. securities since the BB curve has not been crossed. However, this is inconsistent with the balance sheet constraints for wealth holders. The excess supply for all assets must sum to zero since, at a point in time, people can only reallocate their existing wealth. There cannot be an excess supply of U.S. securities and both kinds of money when the excess demand for U.K. securities is zero. Therefore the FF curve must be steeper than the BB curve as shown in Figure 1.[1/] At point a_2 in Figure 1, the excess demand for U.K. securities matches the excess supply of U.S. and U.K. money.

1/ Footnote on following page.

Combinations of r and r' that equate the demand for and supply of U.S money, given a fixed U.S. money supply available for the public to hold, are plotted as the MM curve in Figure 1. The MM curve is negatively sloped since an increase in either r or r' reduces the demand for U.S. money. If the U.S. money market

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Footnote from previous page.
1/ Using the expressions derived in the two preceding footnotes we have,

$$\left.\frac{dr}{dr'}\right|_{FF} - \left.\frac{dr}{dr'}\right|_{BB} = -\frac{f_r'W + \Pi\, f'_{r'}\, W'}{f_r\, W + \Pi\, f'_r\ W'} + \frac{b_r'W + \Pi\, b'_{r'}\, W'}{b_r\, W + \Pi\, b'_r\ W'}.$$

Using the results of footnote 1, p. 12, this difference can be rewritten as,

$$\left.\frac{dr}{dr'}\right|_{FF} - \left.\frac{dr}{dr'}\right|_{BB} = -\frac{(m_r' + b_r')W + \Pi\,(n'_{r'} + b'_{r'})W'}{(m_r + b_r)W + \Pi\,(n'_r + b'_r)W'}$$

$$+ \frac{b_r'W + \Pi\, b'_{r'}W'}{b_r\, W + \Pi\, b'_r\, W'}$$

Collecting terms we have,

$$\left.\frac{dr}{dr'}\right|_{FF} - \left.\frac{dr}{dr'}\right|_{BB} =$$

$$\frac{(m_r b_r' - m_r' b_r)W^2 + (m_r b'_{r'} + n'_r b_r' - m_r' b'_r - n'_{r'} b_r)\Pi WW' + (n'_r b'_{r'} - n'_{r'} b'_r)\Pi^2 W'^2}{[(m_r + b_r)W + \Pi\,(n'_r + b'_r)W'][b_r W + \Pi\, b'_r W']}.$$

The numerator of this expression is positive given our assumption that the three assets held by wealth holders are strict gross substitutes. Using the results of footnote 1, p. 12, the first term in the denominator can be rewritten as $-(f_r W + \Pi\, f'_r W')$. Using the strict gross substitute assumption again we can determine that the denominator is positive. Thus the FF curve has a larger positive slope than the BB curve.

is to remain in equilibrium when interest rates change then r and r' must move in opposite directions.[1/] The area to the right of MM is a sector of excess supply of home country money, and the area to the left is a region of excess demand.

Combinations of r and r' that equate the demand for and supply of U.K. money given a fixed U.K. money supply available for the public to hold, are plotted as the NN curve in Figure 1. The NN curve is negatively sloped since an increase in either r or r' reduces the demand for U.K. money. If the U.K. money market is to remain in equilibrium when interest rates change, then r and r' must move in opposite directions.[2/] The area to the right of NN is a sector of excess supply of U.K. money, and the area to the left is a region of excess demand.

1/ Totally differentiating equation (11) with B^c, σR, and S held constant and solving for the required slope we have,

$$\left.\frac{dr}{dr'}\right|_{MM} = -\frac{m_{r'}}{m_r} .$$

Our strict gross substitutes assumption implies that this expression is negative.

2/ Totally differentiating equation (12) with $F^{c'}$, $\sigma' R'$, and S' held constant and solving for the required slope we have,

$$\left.\frac{dr}{dr'}\right|_{NN} = -\frac{n'_{r'}}{n'_r} .$$

The strict gross substitutes assumption implies that this expression is negative.

In Figure 1 the NN curve is drawn with a steeper (more negative) slope than the MM curve. The slope of the NN curve is more negative than the slope of the MM curve if and only if the ratio of the response of U.K. money demand to the rate on U.K. securities to the response of U.K. money demand to the rate on U.S. securities is greater than the ratio of U.S. money demand responses to these same rates.[1/] We assume that this condition is fulfilled. This assumption is actually rather weak. It will certainly be met if the absolute value of the response of U.S. money demand to the U.S. interest rate is greater than the absolute value of the response of U.S. money demand to the U.K. interest rate and if the absolute value of the response of U.K. money demand to the U.K. rate is greater than the absolute value of the response of U.K. money demand to the U.S. interest rate.[2/] We believe that most investigators would find these two conditions quite plausible.

1/ Using the results of the preceding two footnotes we have,

$$\frac{dr}{dr'}\bigg|_{MM} - \frac{dr}{dr'}\bigg|_{NN} = -\frac{m_{r'}}{m_r} + \frac{n'_{r'}}{n'_r} .$$

This expression is positive, that is, the MM curve is flatter (less negatively sloped) if $m_{r'}n'_r < m_r n'_{r'}$.

2/ The condition for the MM curve to be flatter than the NN curve given in the previous footnote is met if $|m_r| > |m_{r'}|$ and $|n'_{r'}| > |n'_r|$.

We have argued above that the four market equilibrium conditions are not independent because of the balance sheet contraints faced by ultimate wealth holders. If three of the four markets are cleared, the fourth market must also be cleared. In geometric terms if three of the four schedules intersect at a common point in Figure 1, the fourth schedule must also pass through that point. As the analysis proceeds it will become clear that it is useful to retain all four schedules even though in a given situation three schedules are sufficient to determine the equilibrium values of the endogenous variables.

It is worth pointing out that interest rate changes above are not sufficient to insure that three independent schedules will have a common intersection point. There must be a third variable not shown explicitly on the graph which can change if we are to be sure that a common intersection point exists. One possibility, which will be explored further in what follows, is that the stock of reserves held by the USCB changes so as to guarantee that the MM schedule, and by implication the NN schedule, will pass through the point of intersection of the BB and FF schedules on the interest rate coordinates.

III. The Outside Reserve Regime

A. An Overview

In this Chapter we analyze the effects of disturbances to financial asset equilibrium in a regime under which all international reserves are held in the form of outside reserve assets. Gold and SDR's are examples of such outside reserve assets.[1/] Under an outside reserve regime the total world supply of reserves ($\bar{R}$) is fixed in the short run so that, for example, if the U.S. gains reserves as the result of some disturbance, the U.K. must lose an equal amount of reserves ($\bar{R} = R + R'$). We turn first to consider the effect of a shift in the asset demands of ultimate wealth holders toward U.K. securities and away from U.S. securities.

B. The Effect of a Shift in Asset Preferences for Securities on the Securities Markets

We can complete the first step in analyzing the effects of an exogenous shift in the asset preferences of wealth holders towards U.K. securities and away from U.S. securities on the basis of what we have learned so far. In Figure 2 the initial equilibrium pair of interest rates is given by the intersection of F_0F_0, B_0B_0, M_0M_0, and N_0N_0. If there is a shift upward in the demand for U.K. securities while the supply of these securities to the public remains unchanged, the FF curve must shift to the left, say to F_1F_1. A lower value of

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1/ In Chapter IV we study a key currency reserve regime under which the UKCB holds at least part of its reserves in the form of U.S. securities.

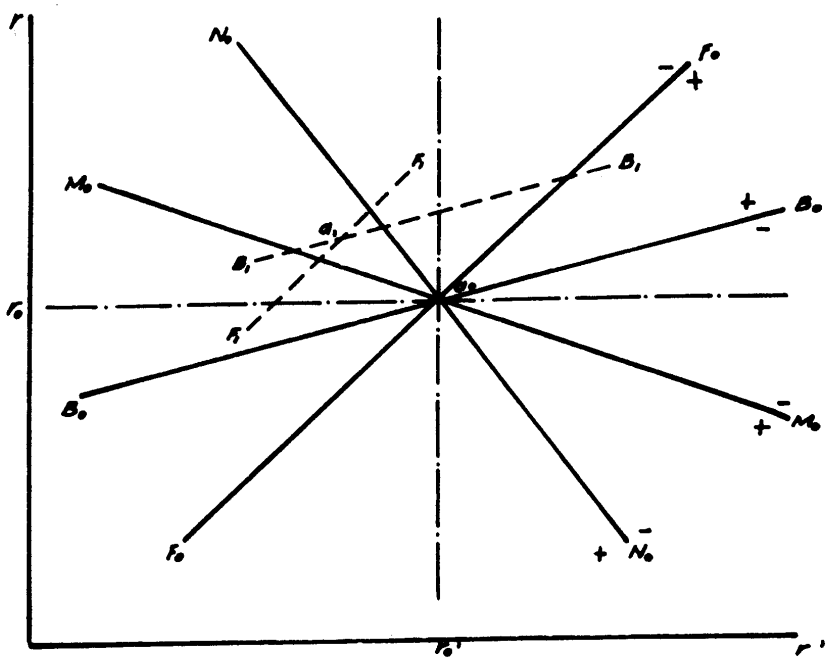

FIGURE 2

r' must be associated with each value of r after the shift in order to choke off the resulting excess demand. We assume that the upward shift in the demand for U.K. securities is matched by a downward shift of the same absolute magnitude in the demand for U.S. securities, the available supply of which is assumed to remain fixed. As a result of this decline in demand for U.S. securities, the BB curve must shift upward say to B_1B_1. For each level of r' a higher level of r is required in order to raise demand for U.S. securities by enough to offset the original decline. The markets for both of the two

securities would be cleared at the intersection of the F_1F_1 and B_1B_1 schedules.

We have drawn F_1F_1 and B_1B_1 so that they intersect in a region of excess demand for U.K. money and excess supply of U.S. money. This must be the case. Since world excess demand for the two kinds of securities is zero at the intersection of the F_1F_1 and B_1B_1 curves, the excess demand for the two kinds of money must sum to zero. If there is an excess supply of U.K.(U.S.) money, there must be an excess demand for U.S.(U.K.) money. We have argued that the assumed shift in asset demands will result in upward shifts in both the FF and BB schedules. Given our assumption regarding the relative slopes of the MM and NN schedules, F_1F_1 and B_1B_1 must intersect at an interest rate pair at which there is an excess demand for U.K. money and an excess supply of U.S. money.

If we allowed no change in any variables besides the two interest rates, full equilibrium would be impossible since both securities markets are cleared at the intersection of the F_1F_1 and B_1B_1 curves, and both money markets are cleared at the intersection of the M_0M_0 and N_0N_0 curves, but these two intersection points do not coincide. How this apparent dilemma is resolved depends crucially upon the behavior of the two central banks which we must now consider in somewhat greater detail than we have up to this point.

C. Policy Responses by the Central Banks

We consider first what we call the gold standard policy

response (GSPR). The shift in asset demands from U.S. to U.K. securities with the supplies of the two securities to the public unchanged leads to a selling off of U.S. securities which results in upward pressure on r. Sellers of U.S. securities attempting to buy pounds place upward pressure on π, the dollar price of pounds. To keep the exchange rate fixed the UKCB buys dollars which they convert into outside reserve assets at the USCB. This operation extinguishes U.S. money and creates U.K. money in equal amounts in terms of dollars. Wealth holders bid for U.K. securities causing r' to fall. This process continues until the U.S. rate has risen enough and the U.K. rate has fallen enough that wealth holders even with their altered asset preferences are willing to hold the still constant supplies of U.S. and U.K. securities.

What of the dilemma posed at the end of the last section? If both central banks employ a GSPR it is resolved by a decrease in the U.S. money stock and an increase in the U.K. money stock. In graphical terms MM and NN shift from M_0M_0 and N_0N_0 to M_1M_1 and N_1N_1 as in Figure 3. Decreases in the U.S. money stock result in upward shifts in the MM curve; for each r' there must be a higher r if demand for U.S. money is to shrink to match the reduced supply. Increases in the U.K. money stock necessitate leftward shifts in NN since for each r there must be a lower r' if demand for U.K. money is to rise to equal the increased supply. We know that the U.S. money stock

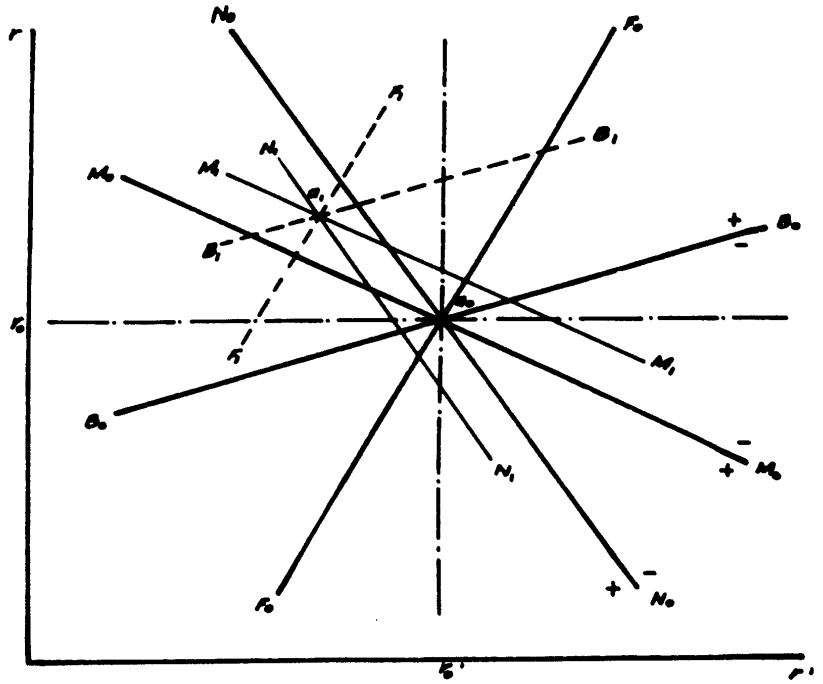

FIGURE 3

decrease which is sufficient to insure that the U.S. money market clears at the intersection of F_1F_1 and B_1B_1 is exactly the same as the U.K. money supply increase which is sufficient to make the U.K. money market clear at the same intersection point from the fact that only three of the four market equilibrium conditions are independent.

When both central banks employ a GSPR, the new equilibrium interest rate pair is the combination which clears the two securities markets given the shifts in asset demands. Some U.K. wealth holders

who held U.K. securities before the shift have willingly reduced their holdings to accomodate the new demand because r' is lower. They are induced to hold either U.S. securities sold off by dissatisfied holders because r is higher or U.K. money given the new configuration of r and r'. Some U.S. wealth holders hold more U.S. securities and fewer U.K. securities and less U.S. money at the new r, r' combination. Changes in the central banks' holdings of reserves match the adjustments in both countries' money supplies which are necessary for the reattainment of equilibrium when both central banks are pursuing a GSPR. It is important to notice that neither central bank alters its holdings of domestic securities when employing a GSPR.

One of the central banks may, however, be unwilling to allow its money supply to change by as much as would be required under a GSPR. Suppose that the UKCB believes that its money supply is "just right" for domestic purposes; then it might pursue a money supply constant policy response (MSCPR). We continue to assume that the USCB pursues a GSPR. Under these assumptions an asset demand shift in favor of U.K. securities again leads to sales of U.S. securities, an increase in r, upward pressure on π, and a decrease in the U.S. money supply with an accompanying outflow of international reserves. However, there is no increase in the U.K. money supply since the UKCB stands ready to sell as many U.K. securities from its holdings as necessary to keep $N^{s'}$ constant. In this

case the U.S. interest rate is higher, the U.K. interest rate is higher, the U.S. money supply is lower, and the U.K. money supply is lower when the new equilibrium is reached than was the case when both countries reacted according to the GSPR.

The dilemma posed at the end of the last section is resolved in a different way when the U.K. pursues a MSCPR. We know that following the demand shift FF and BB shift to F_1F_1 and B_1B_1 and that MM and NN remain in their original positions, M_0M_0 and N_0N_0. We also know that given the policy response of the UKCB, N_0N_0 will not be shifted in the new equilibrium. In addition the supply of U.S. securities does not change so that once BB has shifted to B_1B_1 it must remain there. All this means that the new equilibrium interest rate pair is given by the intersection of N_0N_0 and B_1B_1 at point a_2 in Figure 4. The drop in the U.S. money supply results in a shift of MM to M_2M_2 while the sales of U.K. securities by the UKCB required to keep the U.K. money supply unchanged imply a shift of FF from F_1F_1 to F_2F_2.

The important difference between the current case and the case in which both central banks use a GSPR arises because the supply of U.K. securities available to the public is increased by the UKCB. This increase in supply reduces to some extent the amount by which r' falls. However, the constant supply of U.S. securities must be held in equilibrium, so r must rise by more than before in order to assure that it will be held. The U.S. money stock must decline by

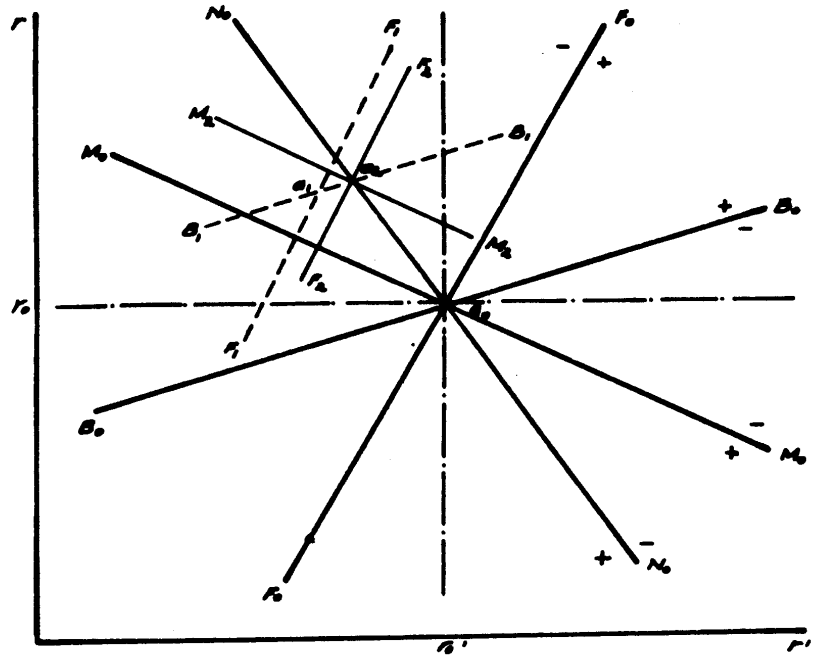

FIGURE 4

more and the U.K. money stock must rise by less than in the previous case since wealth holders as a group must be willing to hold more securities and less money in their portfolios than when the UKCB employed a gold standard policy response. The larger decline in the U.S. money stock implies a larger U.S. reserve loss than before. In contrast to the case in which both central banks pursued a GSPR the UKCB holdings of U.K. securities are reduced.

Instead of trying to stabilize the outstanding stock of its monetary liabilities the UKCB might follow an <u>interest rate constant policy response</u> (IRCPR). We continue to assume that the

USCB employs a GSPR. Following the asset demand shift, all the same pressures arise as in the two previous cases, but now, instead of keeping its holdings of U.K. securities constant, or selling enough securities to keep the U.K. money stock constant the UKCB sells enough securities to keep r' from falling at all. In this case the U.S. interest rate must rise even more than in the previous case and both money supplies must fall.[1/]

The graphical analysis in this case is slightly more difficult than it was in the two previous cases. Following the asset demand shift the relevant FF curve is F_1F_1 and the relevant BB curve is B_1B_1 in Figure 5. We know that the B_1B_1 curve must remain fixed since there is no change in the supply of U.S. securities to the public. We also know that the UKCB will increase the supply of U.K. securities to the public by whatever amount is necessary to insure that r' remains constant. The means that the new FF curve designated F_3F_3 must intersect B_1B_1 at the point a_3 where B_1B_1 crosses the dashed line which passes through the original equilibrium point and which is parallel to the r axis. The MM and NN curves must therefore shift from M_0M_0 and N_0N_0 to M_3M_3 and N_3N_3.

In this case we have the largest increase of any of the cases considered so far in the U.S. interest rate; it must rise by enough to remove the excess supply for U.S. securities since there is

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1/ Note that if the NN curve is vertical ($n'_r = 0$) then the results of a MSCPR and a IRCPR by the UKCB are the same.

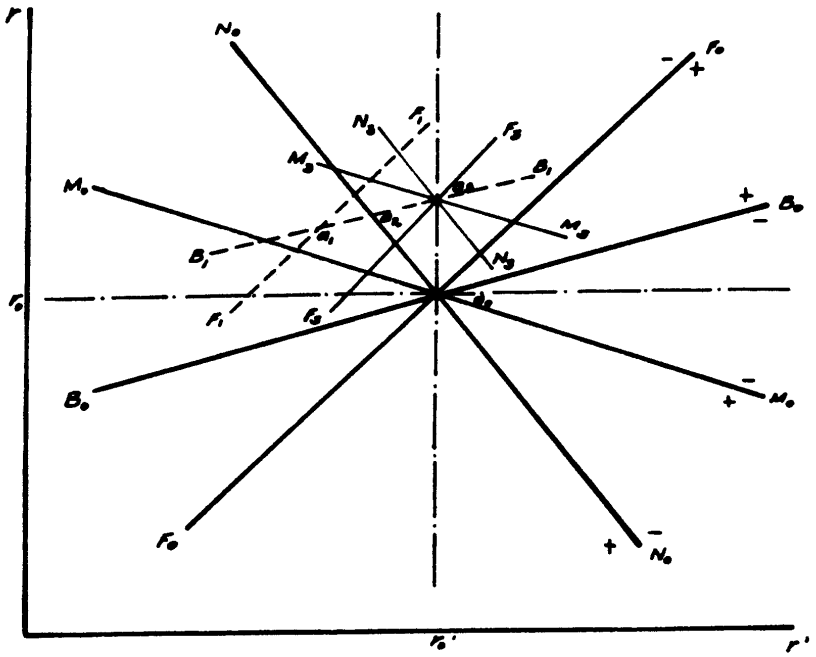

FIGURE 5

no movement in r' to help reduce the excess supply. The U.S. money stock must decline further than in any of the other cases, and the U.K. money stock must decline instead of rising because wealth holders as a group must hold an even larger amount of securities in their portfolios than they had to hold when the UKCB pursued a MSCPR.

D. Policy Responses and Target Variables

The analysis up to this point highlights the fact that, although equilibrium is reattained in each of the three cases the impacts of the asset demand shift on the two countries' money supplies

and interest rates are quite different. Suppose that before the demand shift the USCB and the UKCB each considered the level of the proximate target variable in its country, for example, the interest rate on securities issued by residents of its country, to be exactly right from the point of view of the desired values of the ultimate target variables in its country such as the rate of unemployment and the rate of inflation. In our analysis each central bank has only one policy instrument, the amount of liabilities of domestic residents that it holds. When both central banks pursue a GSPR, that is, keep their holdings of domestic liabilities constant interest rates in both countries are disturbed from their initial levels which for purposes of discussion we are taking to be their desired levels. If UKCB uses its single instrument to try to reattain the desired value of the U.K. interest rate while the USCB stubbornly sticks to a GSPR the U.S. interest rate is driven away from its desired value. If the UKCB knows that the USCB is going to continue to pursue a GSPR and if the UKCB is willing to experience some deviation of its proximate target variable from its desired value to reduce the discomfiture of the USCB, the UKCB might well choose to follow a MSCPR or even a GSPR rather than an IRCPR.

However, there is a set of policy responses by the two central banks which affords an even happier resolution to the apparent conflict created by the assumed shift in asset demands. Suppose both central banks pursue an IRCPR, that is, that they both stand ready to

buy or sell enough securities issued in their own countries to stabilize the interest rate on those securities. In this case FF and BB shift from F_1F_1 and B_1B_1 back to F_0F_0 and B_0B_0, and MM and NN stay fixed at M_0M_0 and N_0N_0. Thus the two interest rate targets can both be attained if the central banks properly employ their two policy instruments. It is interesting to note that in this case keeping interest rates constant implies keeping money stocks constant. The same equilibrium point is reached if both central banks pursue an IRCPR or if they pursue a MSCPR.

E. Another Type of Shift in Asset Demands

It is not in general true that keeping both interest rates constant implies keeping both money stocks constant. To demonstrate this contention we consider a shift in the asset demands of U.S. wealth holders toward U.K. securities and away from U.S. money.

This shift in asset demand functions given fixed asset supplies leads to shifts in MM and FF from M_0M_0 and F_0F_0 to M_1M_1 and F_1F_1 in Figure 6. If both central banks pursue an IRCPR all the schedules must intersect at the same point after adjustment is complete as they did before the disturbance. MM and FF must shift back from M_1M_1 and F_1F_1 to M_0M_0 and F_0F_0. The USCB keeps its holdings of U.S. securities unchanged and allows the U.S. money supply to decline by the full amount of the original shift in asset demands. This is also the amount of U.S. reserve losses. In order to prevent any decline in r' the UKCB must sell an amount of U.K. securities equal to the shift in asset demands. The U.K. money supply remains unchanged.

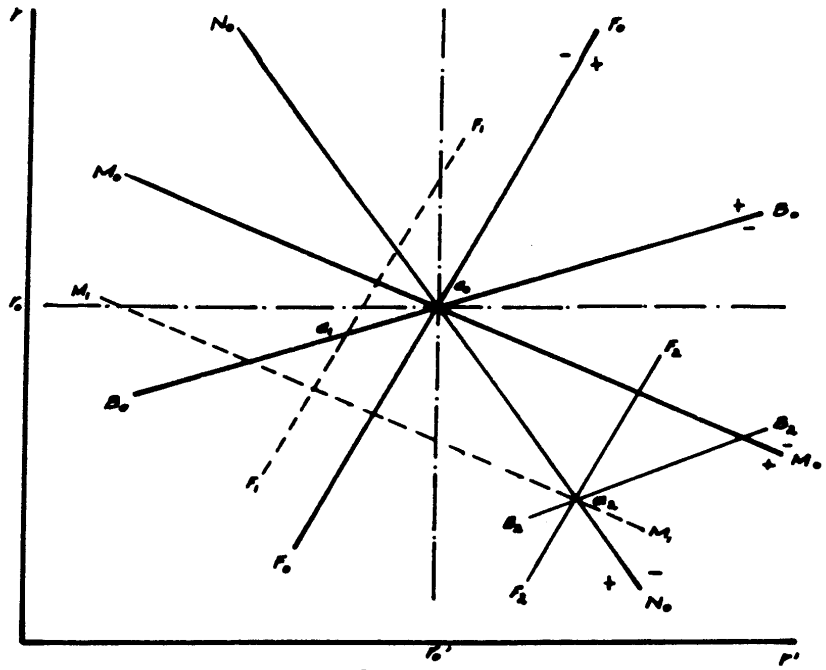

FIGURE 6

However, if both central banks pursue a MSCPR the new equilibrium interest rate pair is given by the intersection of M_1M_1 and N_0N_0. BB and FF shift to B_2B_2 and F_2F_2 respectively. The USCB buys enough U.S. securities to offset the decline in the U.S. money supply which would otherwise result from the fact that the UKCB acquires and converts into reserve assets the dollars which U.S. citizens want to sell as a result of the shift in asset demands. The UKCB sells U.K. securities equal to the amount of pounds it sells to U.S. wealth holders. The increased supply of U.K. securities and diminished supply of U.S. securities resulting from the money stock

stabilization operations of the central banks will only be held by wealth holders if r falls and r' rises even when account is taken of the increased demand for U.K. securities. The U.S. loses reserves equal to the amount of U.S. securities the USCB must purchase in order to shift the BB curve from B_0B_0 to B_2B_2.

If both the MM and NN curves shift while the BB and FF curves stay fixed,[1/] an IRCPR by both central banks implies that both money stocks must change while a MSCPR by both central banks implies that both interest rates must change.

We can draw a general conclusion regarding cases in which shifts in asset demands involve net shifts in only two schedules. Under our assumptions so far whenever shifts in asset demands involve only shifts in demands for securities stabilizing interest rates implies stabilizing money supplies in the hands of the public and vice versa but whenever shifts in asset demands involve a shift in at least one money demand function stabilizing interest rates will involve a change in the supply held by the public of at least one of the two kinds of money in the model while stabilizing money supplies in the hands of the public will involve a change in both

1/ Both MM and NN could shift with no net shift in FF or BB if, for example, U.S. wealth holders asset demands shift in favor of U.K. securities at the expense of U.S. money and U.K. wealth holders experience a simultaneous shift in asset demand in favor of U.K. money at the expense of U.K. securities.

interest rates.[1/] This conclusion is consistent with the well known proposition from the theory of economic policy that if only two policy instruments are available, as in the present case, then desired values for only two target variables can be attained when all possible shocks to the system are considered. What our conclusion suggests is that for certain kinds of shocks more than two target variables can be returned to their initial levels by the use of only two policy instruments. If the initial levels of the target variables happen to be the desired levels, the cost to the two central banks of having fewer policy instruments than potential target variables depends in part upon what kind of shocks the system experiences.

1/ We have focused our attention on the levels of the two interest rates and on the supplies of the two types of money held by the public and proceed as if the supplies of the two types of securities held by the public was a matter of indifference to the two central banks except in so far as those supplies affect money supplies and interest rates. This approach allows those readers who prefer to emphasize the importance of "rates" and those who prefer to emphasize the importance of "monetary aggregates" to relate our analysis to other available treatments of the same or similar issues. Whether or not the supplies of the two types of securities in the hands of the public are variables which should have the same claim to central banks' attention as "rates" and "monetary aggregates" is an interesting and unresolved question which we will not attempt to answer here. We only remark that giving "securities" a more "equal" role in the analysis as our model attempts to do helps to emphasize the need for asking how important the supplies of securities held by the public really are.

F. The Reserve Stock Constant Policy Response

For convenience in exposition we have been ignoring one type of possible policy response. Assume again that asset demands have shifted in favor of U.K. securities at the expense of U.S. securities and that the USCB is employing a GSPR. We know that as wealth holders try to sell off U.S. securities there is upward pressure on the U.S. rate and upward pressure on π as wealth holders convert the proceeds of their sales of U.S. securities into pounds. In each of the cases in Section C the U.S. money stock fell and the U.S. lost reserves of the same amount during the adjustment following asset demand shifts of the type being considered here. However, if the UKCB is satisfied with its holdings of international reserves, it might take steps to prevent any further accumulations of reserves; that is, it might pursue a reserve stock constant policy response (RSCPR).

In order to insure that its stock of reserves does not change the UKCB must add to the excess demand for U.K. securities. Although this result seems at first to be somewhat paradoxical, it can be easily explained. Since it is following a GSPR the USCB does nothing to directly affect the U.S. money stock. Thus if the U.S. money stock declines it is because the UKCB has had to buy dollars in the exchange market and, therefore, under our assumptions has added to its stock of outside reserves by converting the dollars at the USCB. All this means that in order to insure that

its reserve stock does not change the UKCB must see to it that U.S. citizens are content to hold the money stock which they held before the shift in asset demands.

We know from our analysis in Section B of the case in which both countries pursue a GSPR after an asset demand shift of the kind considered here that if both countries do not change their holdings of domestic liabilities, the U.S. interest rate rises, the U.K. interest rate falls, and the U.S. money stock falls. Since the USCB is assumed to be pursuing a GSPR here, the UKCB must pursue an expansionary open market operation in order to drive r' down far enough so that U.S. citizens are persuaded to hold as much money as they did before the shift in asset demands even though r rises somewhat.

The original change in asset demands shifts FF and BB from F_0F_0 and B_0B_0 to F_1F_1 and B_1B_1 as shown in Figure 7. In order to keep its stock of reserves constant the UKCB must purchase enough domestic liabilities to drive FF from F_1F_1 to F_4F_4 and NN from N_0N_0 to N_4N_4.

Wealth holders as a group must hold fewer securities. U.S. wealth holders take up part of the initial excess supply of U.S. securities and surrender some U.K. securities but hold the same amount of money as before the demand shift. U.K. wealth holders take up the rest of the initial excess supply of U.S. securities and increase their holdings of U.K. money while reducing their holdings of U.K. securities.

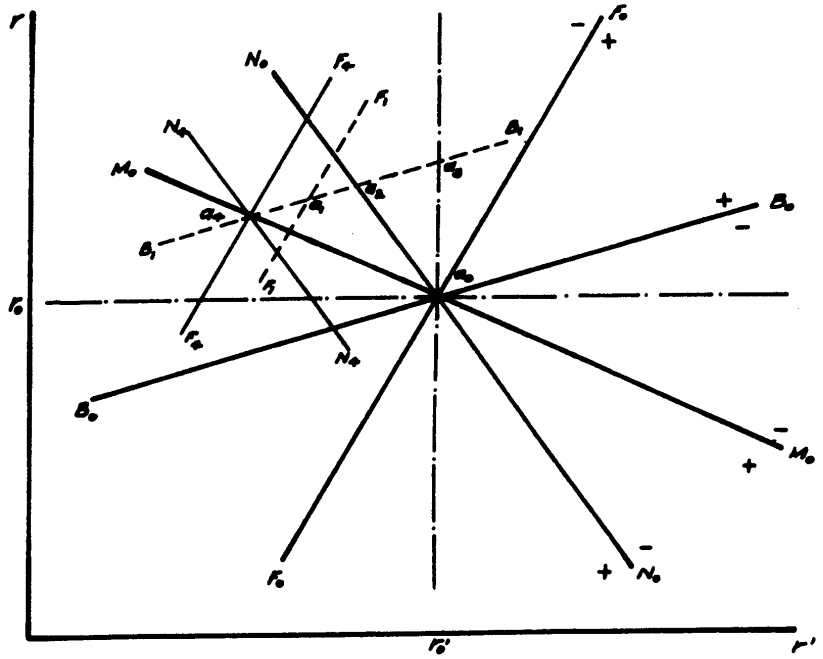

FIGURE 7

G. Policy Responses and Policy Initiatives

Up to this point we have concerned ourselves with the implications of the way in which central banks respond to disturbances to asset equilibrium which take the form of shifts in asset demands. When we treated central bank objectives, we assumed that in the initial equilibrium at least two target variables had their desired values. If some target variables do not have their desired values in the existing equilibrium or if the desired values for some target variables change for any reason, central banks may wish to undertake policy initiatives, and it is to a consideration of these initiatives that we now turn.

It is useful to consider the correspondence between policy response and policy initiatives. If it is following a GSPR, a central bank keeps its holdings of domestic assets constant and accepts passively the changes in its money supply, international reserves, and the interest rate on domestic assets which occur as the result of disturbances to asset equilibrium. We could as well have called a GSPR a domestic assets constant policy response. Suppose that a central bank has been following this kind of policy response but decides that the results of its behavior for the economy have been too contractionary. It might decide to purchase some additional domestic assets and to stabilize its holdings of domestic assets at a new higher level. This policy initiative, which we will call a gold standard policy initiative (GSPI), is just an open market purchase of a given size by the central bank. The policy initiative which corresponds to the GSPR is an open market operation of a given size. This policy initiative will have different implications for the money stock and the interest rate of the country undertaking it depending upon the policy response being pursued by the other central bank.

Using Figure 8 we can trace the implications of different U.K. policy responses to a GSPI undertaken by the U.S. As before, the initial equilibrium position is given by the intersection of the curves with the (0) subscript. If the U.S. authorities plan to increase their holdings of U.S. securities by a given amount and then to allow

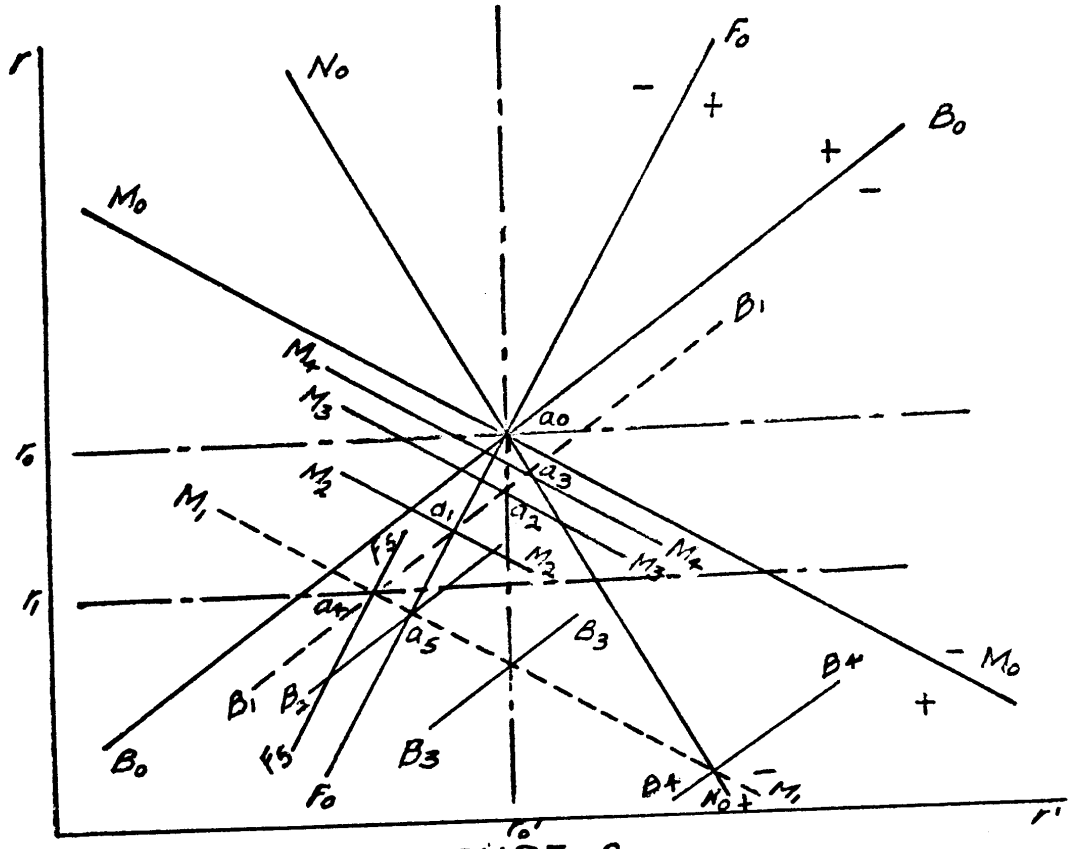

FIGURE 8

no further changes in these holdings, the BB curve must shift down, say from B_0B_0 to B_1B_1 since, for a given value of r', the public will hold a reduced supply of U.S. securities only if r is lower. Following a GSPI, the new equilibrium must be on the new BB curve. If the UKCB follows a GSPR, that is, if they neither buy nor sell U.K. securities, the new equilibrium is given by the intersection of F_0F_0 and B_1B_1 at point a_1. An IRCPR by the UKCB implies that the new equilibrium will be at point a_2. If the UKCB decides to pursue a MSCPR, the new equilibrium is at the intersection of N_0N_0 and B_1B_1 at point a_3.

In order to facilitate further exposition of these results we adopt the convenient fiction that the USCB purchases the securities it wants from U.S. wealth holders at unchanged interest rates. The USCB tells each U.S. wealth holder that he must trade some of his U.S. securities for U.S. money but that he is then free to adjust his portfolio as he likes. This fiction in no way affects the results of the model. Our assumption means that, in addition to the shift of BB from B_0B_0 to B_1B_1, MM must shift from M_0M_0 to M_1M_1. For a given r', U.S. wealth holders will voluntarily hold the increased supply of money only if r is lower. Our assumption also implies that immediately following the open market purchase U.S. wealth holders have an excess supply of money and an excess demand for securities at the original interest rate pair r_0, r'_0 [1/] whereas U.K. residents are satisfied with their holdings of money and securities at this interest rate combination. How this imbalance in the portfolios of U.S. residents is removed depends upon the policy response pursued by the UKCB.

First consider the case in which the UKCB follows a GSPR under which it passively accepts whatever changes occur in the U.K. money stock and the U.K. interest rate and keeps its holdings of domestic liabilities constant. As U.S. wealth holders reduce their excess money holdings by making net purchases of securities from U.K. residents downward pressure is exerted on both of the interest rates.

1/ More specifically U.S. wealth holders have an excess demand for U.S. securities. They are satisfied with their holdings of U.K. securities at the original interest rate pair.

and upward pressure is exerted on the dollar price of pounds. In order to keep the dollar price of pounds from rising the UKCB must buy dollars for pounds, an action which reduces the U.S. money supply and raises the U.K. money supply by equal amounts since neither central bank pursues any offsetting measures. Thus the initial excess supply of money in the U.S. is removed partly by interest rate declines which raise the U.S. demand for money and partly by net purchases of securities from U.K. residents who are willing to surrender these securities and hold more U.K. money because of the lower interest rates. The decline in the U.S. money stock from the level it reaches immediately following the open market purchase is shown by the shift of MM from M_1M_1 to M_2M_2 which passes through a_1 the point of intersection of B_1B_1 and F_0F_0. The U.S. loses reserves equal to the decline in the U.S. money supply as the UKCB converts the proceeds of its exchange rate stabilization operations into reserve assets.

Suppose now that the UKCB reacts to an open market purchase by the USCB with an IRCPR. As before there is downward pressure on both interest rates and upward pressure on π, the dollar price of pounds, as U.S. wealth holders make net purchases of securities from U.K. residents. The UKCB sells pounds to relieve the upward pressure on π, and, in contrast to the previous case, supplies U.K. securities to private wealth holders in the amount required to keep r' from falling.

This provision of securities by the UKCB shifts FF down from F_0F_0 until it passes through a_2 where B_1B_1 intersects the dashed line drawn through the original equilibrium point and parallel to the r axis. We have omitted the new FF curve. Here again the U.S. money supply falls and the U.K. money supply rises, but the reduction in the U.S. money supply must be greater in this case. Since the UKCB is stabilizing r', r need not fall as far in order for wealth holders to be satisfied with a reduced supply of U.S. securities as it did when the UKCB was pursuing a GSPR. Since neither interest rate falls as far in this case as it did before, less of the initial excess supply of money in the U.S. is removed by interest rate declines which raise the U.S. demand for money and more of the initial excess supply of money is removed by net purchases of securities from U.K. residents and the UKCB. The decline in the U.S. money stock is shown by the shift of MM from M_1M_1 to M_3M_3 which passes through point a_2. Corresponding to the larger decline in the U.S. money supply is a larger loss of reserves by the U.S. It is important to note that, in contrast to what we observed in the previous case, U.K. money holdings do not rise as much as U.S. money holdings have fallen. This must be the case since equilibrium U.K. money demand is less than before because both interest rates are higher and since the U.S. money supply declines more in this case. The UKCB, in effect, replaces a part of that fraction of the U.S. money stock which it absorbs and extinguishes with U.K. securities and the rest with U.K. money instead

of simply taking in dollars and supplying pounds. The mix of assets available to the world's wealth holders contains more securities and less money than it did immediately following the policy initiative.

While the UKCB allows some increase to occur in the U.K. money stock when it pursues an IRCPR, it acts to prevent any increase at all when it pursues a MSCPR. Forced to support π as U.S. citizens run down their excess money balances, the UKCB sells U.K. securities equal in pound value to the dollars it absorbs. This sterilization operation shifts FF down until it passes through the intersection of B_1B_1 and N_0N_0 at point a_3. The new FF curve is not shown. As we have seen, stabilizing r' involves some increase in the U.K. money stock, so preventing an increase in the money stock must involve even larger open market sales by the UKCB than were required in the previous cases. The fact that larger open market sales are undertaken implies that r' must be higher in the new equilibrium. r must still fall in order for wealth holders to be content to hold the smaller supply of U.S. securities, but not as far as in either of the two previous cases. Thus even less of the original excess supply of money in the U.S. is removed by increased demand for money and even more must find its way into securities via net purchases from U.K. residents and the UKCB than was the case when the UKCB pursued an IRCPR. The larger magnitude of the decline in the U.S. money stock in this case is shown by the shift of MM from M_1M_1 to M_4M_4, the MM curve which passes through point a_3. The corresponding reserve

loss is, of course, larger here too. The actions of the UKCB when it follows a MSCPR represent the polar opposite of its actions in the GSPR case. Instead of being content to trade U.K. money for U.S. money the UKCB takes measures which are tantamount to replacing the fraction of the U.S. money stock which it receives through the exchange market with U.K. securities. The final mix of money and securities available to wealth holders is more like the original mix before the policy initiative than in the other two cases. The equilibrium world supply of securities contains a relatively higher proportion of U.K. securities than it did in either of the other two cases or in the original position.

We know that the UKCB can pursue a fourth kind of policy response, a RSCPR. In order to keep from gaining reserves the UKCB must see to it that interest rates move so as to make U.S. residents content with the money holdings they acquire as a result of the GSPI by the USCB. If it succeeds U.S. residents will not make any net purchases of securities from U.K. residents. When the UKCB detects downward pressure on r' and upward pressure on π, it must add to the downward pressure on r' by <u>buying</u> U.K. securities so as to make U.K. securities less attractive relative to U.S. securities and U.S. money. To achieve its desired result the UKCB must buy enough securities to shift FF from F_0F_0 to F_5F_5 so that it passes through the intersection of B_1B_1 and M_1M_1 at point a_4. Since the supplies of both securities are reduced both interest rates must end up lower than

in the original position. At these lower rates U.K. residents are content to increase their money holdings by an amount equal to the open market purchases of the USCB. They must do so since U.S. residents make no net purchases or sales of securities after the original sale called for by the GSPI.

There are policy initiatives which correspond to the other types of policy responses besides the GSPR. For instance, if a central bank has been following a MSCPR, that is, if it has been sterilizing all gains and losses in international reserves, but concludes that, although this is the right rule for its behavior, it has chosen too small a value of the money supply to shelter from the impacts of disturbances, it might pursue the necessary actions to insure that the domestic money supply is stabilized at a new higher level. This money stock constant policy initiative (MSCPI), of course, involves the purchase of domestic securities in the open market, but what differentiates it from a GSPI is that a new value for the money stock rather than a new value for the central banks holdings of domestic assets is the objective. Achieving the new target value for the money stock will involve different sized open market purchases depending upon what policy response is being employed by the other central bank.

Since we explored the implications of different policy responses to a GSPI in some detail, we provide a somewhat briefer account of how different policy responses affect the results of a

MSCPI. We refer again to Figure 8. Suppose the USCB has been keeping the U.S. money stock at the level represented by M_0M_0 but decides to raise it to the level represented by M_1M_1. While in the case of a GSPI equilibrium points had to be on B_1B_1 in the case of a MSCPI possible new equilibria must be on the M_1M_1 curve. If the UKCB pursues a RSCPR, the USCB can achieve the money supply represented by M_1M_1 by an open market purchase of exactly the amount of the increase in the money supply it desires. The open market purchase shifts BB from B_0B_0 to B_1B_1 and MM from M_0M_0 to M_1M_1. U.S. residents end up making no net purchase of securities from U.K. residents because the UKCB purchases enough U.K. securities to drive interest rates down far enough that U.S. residents are content to hold the entire increase in U.S. money supply brought about by the original open market purchase by the USCB. Note that if the UKCB is pursuing a RSCPR, then a GSPI and a MSCPI of the same size have the same implications for all the variables in the model.

A GSPR by the UKCB implies that it undertakes no open market operations. As we know from above, the ultimate increase in the U.S. money stock following an open purchase by the USCB is less than the size of open market purchase when the UKCB follows a GSPR. Thus in order to obtain an increase in the U.S. money stock from the level represented by M_0M_0 to the level represented by M_1M_1 the USCB must make an open market purchase larger than the desired increase in the money stock. The relative size of the required purchase is indicated

by the fact that the BB curve must be shifted from B_0B_0 to B_2B_2 which passes through the new equilibrium point at a_5 where M_1M_1 intersects F_0F_0 and which lies below B_1B_1. The MM curve corresponding to B_2B_2 must lie below M_1M_1. Immediately following the open market purchase represented by B_2B_2 the MM curve is shifted to a position (not shown) below M_1M_1. Just after the open market purchase by the USCB there is an excess supply of money and an excess demand for securities in the U.S. This excess supply is removed partly by a decline in interest rates from r_0, r'_0 to the levels represented by point a_5 and partly by net purchases of securities by U.S. residents from U.K. residents. The net purchase of securities by U.S. residents, which are equal to U.S. reserve losses, cause MM to shift back to M_1M_1. Since the reduction in the U.S. money supply or the U.S. reserve loss is just equal to the amount by which the original open market purchase exceeded the target increase in the money stock, it can also be represented by the gap between B_1B_1 and B_2B_2.

A completely analogous method of argument can be used to show that the USCB must make even larger open market purchases to achieve the money supply represented by M_1M_1 if the UKCB pursues an IRCPR or a MSCPR. The successively larger U.S. reserve losses are represented by the gap between B_1B_1 and B_3B_3 and the gap between B_1B_1 and B_4B_4 in these two cases. These results arise because, as we have seen above, successively more of any initial increase in the U.S. money supply resulting from an open market purchase of a given size is extinguished

by net purchases of securities by U.S. residents as we move from a GSPR by the UKCB to an IRCPR and then to a MSCPR. Larger and larger open market purchases, with the accompanying increases in reserve losses, are required to raise the money supply by a given amount as we consider this sequence of policy responses.

Following the logic employed so far we can see that an interest rate constant policy initiative (IRCPI) involves the setting and maintaining of a new domestic interest rate by a central bank which has been pursuing an IRCPR. An expansionary IRCPI by one central bank will involve different sized open market purchases depending upon the policy response being followed by the other central bank. Suppose the USCB wants to lower r from r_0 to r_1 in Figure 9.[1/] Under both types of policy initiatives by the USCB considered above the equilibria which were reached after the policy initiative and various policy responses by the UKCB lay along a single line in Figure 8; for the GSPI it was B_1B_1, and for the MSCPI it was M_1M_1. In this case the equilibria for different policy responses by the UKCB lie along the dashed line drawn through point a_4 parallel to the r' axis. We have chosen r_1 because it is the interest rate which would emerge from the GSPI corresponding to B_1B_1 or the MSPI corresponding to M_1M_1 if the UKCB follows a RSCPR. It is interesting to compare the sizes of the open market purchases by the USCB which are required to achieve r_1 when the UKCB pursues other policy responses with the size of the open market purchase required when the UKCB pursues a RSCPR.

1/ Schedules and points with the same labels represent the same assumptions about policy initiatives and policy responses in Figures 8 and 9.

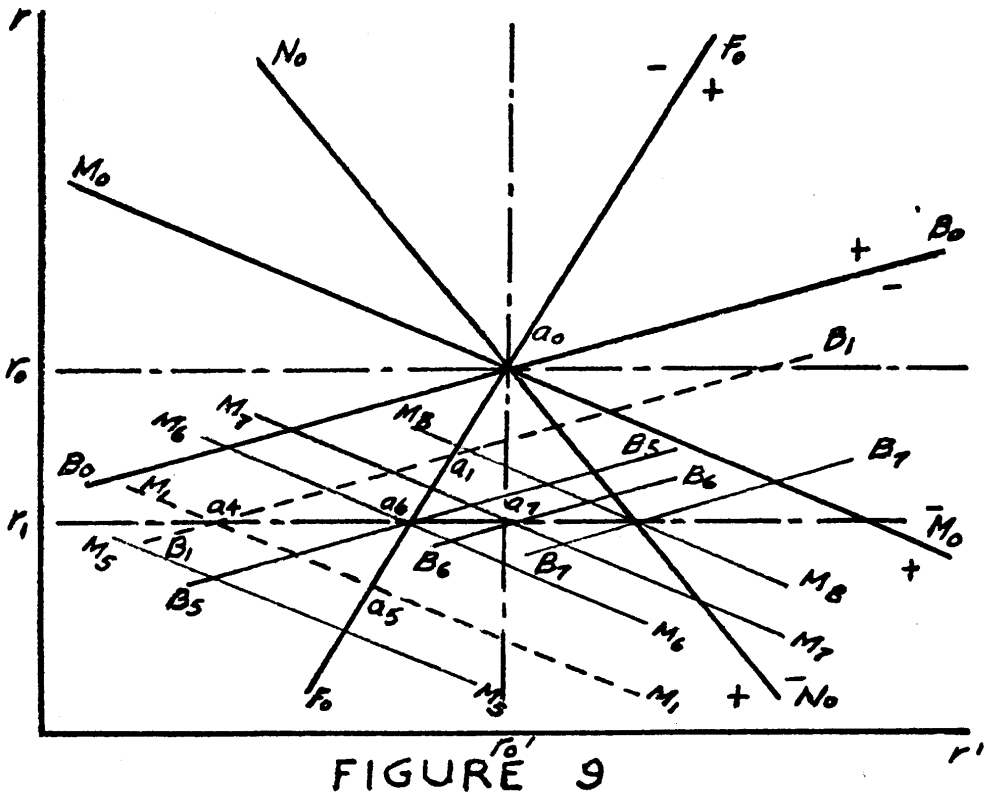

FIGURE 9

Suppose the UKCB is following a GSPR. Our earlier analysis revealed that the GSPI by the USCB associated with B_1B_1 would lead to an equilibrium at point a_1 where r is higher than r_1. The MSCPI corresponding to M_1M_1 results in a new equilibrium at point a_5 where r is lower than r_1. The open market purchase required to achieve r_1 is the one associated with B_5B_5 in Figure 9. This open market purchase is larger than the one associated with a_1 but smaller than the one associated with a_5. B_5B_5 is drawn so that it passes through a_6, the point of intersection of the dashed line drawn so that it passes point a_4 and so that it is parallel to the r' axis and F_0F_0. The MM line which results immediately

following the open market purchase is M_5M_5. Following the open market operation we have the by now familiar pattern of excess supply of money and excess demand for securities in the U.S. Point a_6 is reached as part of the excess supply for money is removed by the decline in interest rates from (r_0, r'_0), and part is removed by net purchases of securities by U.S. residents. The final MM curve is M_6M_6. The reserve loss associated with the IRCPI when the UKCB pursues a GSPR is equal to the money supply decline associated with the movement in the MM curve from M_5M_5 to M_6M_6 or, what is the same thing, the sum of the money supply decline represented by the shift from M_1M_1 to M_6M_6 and the amount of the open market purchase represented by the movement from B_1B_1 to B_5B_5.

Using the same method of argument we can show that the USCB must undertake more substantial open market purchases in order to reach r_1 if the UKCB follows an IRCPR or a MSCPR. More substational open market purchases are required in each of these two cases because the objectives of the UKCB lead it to sell U.K. securities, an action which relieves the downward pressure on r generated by an open market operation of a given size. The successively larger U.S. reserve losses are given by summing the amounts represented by the shifts from B_1B_1 to B_6B_6 and from M_1M_1 to M_7M_7 under an IRCPR and by summing the amounts corresponding to the shifts from B_1B_1 to B_7B_7 and from M_1M_1 to M_8M_8 under a MSCPR.

Corresponding to the RSCPR there is a reserve stock constant policy initiative (RSCPI). For example, a central bank which has been determined in the past to maintain a given stock of international reserves might decide that it now wants a smaller stock of international reserves.

There is an important difference between the RSCPI and other policy initiatives we have considered. If the USCB wants to undertake a GSPI, a MSCPI, or an IRCPI, it can accomplish its objective no matter which of the four types of policy response are pursued by the UKCB. Of course these initiatives may require different actions by the USCB depending upon the type of policy response being pursued by the UKCB, but the objective can be achieved. The same cannot be said for an RSCPI. When the only reserves in the system are outside assets which are fixed in supply, one central bank can only increase its holdings of reserves if the other central bank is willing to reduce its holdings. Thus a RSCPI is consistent with a GSPR, a MSCPR, or an IRCPR but is incompatible with an RSCPR.

In order to achieve a decrease in reserves the "active" central bank would undertake purchases of domestic liabilities in the open market. Exactly what the magnitude of these purchases would have to be is determined by whether the "passive" central bank is pursuing a GSPR a MSCPR, or an IRCPR. There is little need to show this in detail since a careful study of the cases already considered can reveal what is involved in a RSCPI. An example will suffice.

Refer again to Figure 8. Suppose the U.S. wants to lose reserves in the amount corresponding to the shift from M_1M_1 to M_2M_2 and it has reason to believe that the U.K. will react with a GSPR. The USCB should then pursue open market purchases large enough to shift BB to B_1B_1 and MM to M_1M_1 initially and then undertake no further actions. Our analysis above should convince the reader that smaller open market purchases would be required in order to achieve the same decline in reserves under an IRCPR or a MSCPR by the UKCB.

IV. The Key Currency Reserve Regime

A. An Overview

Up to this point we have assumed that all reserves were held in the form of outside reserve assets like gold or SDR's. However, a notable feature of the international monetary system as it has evolved in this century is that significant portions of the reserves of many countries are held in the form of claims denominated in so called "key currencies" and that a large proportion of international payments imbalances are financed by the use of these key currency assets. We call the regime under which changes in reserves take the form of changes in central bank holdings of key currency denominated assets the key currency reserve regime.

International economists have focused considerable attention on some aspects of the key currency reserve regime. It has been argued that the pressure to take corrective measures in the face of persistent payments imbalances and the burden of adjustment when it takes place are distributed assymetrically under such a regime. 1/

1/ It is widely believed that if other countries will accept U.S. liabilities instead of demanding gold or SDR's when they accumulate dollars in exchange rate support operations, the U.S. is not put under pressure to adjust, but the other countries will feel pressure to take corrective action.

Roper ( ) has argued that if adjustment is actually to take place the U.S. price level will have to fall by less and the price level in other countries will have to rise by more under the key currency reserve regime than under the outside reserve regime. Roper assumes that under an outside reserve regime each central bank allows its money supply to fall (rise) as its stock of international reserves falls (rises).

Other aspects of the key currency regime have received less attention. It is important to know how central banks facing the same disturbances and pursuing the same objectives must modify their behavior given that they are operating under the key currency regime rather than the outside reserve regime. One must turn to the financial press to find any discussion of the implications of security financing of U.S. deficits for U.S. financial variables.[1/] It is also useful to determine whether or not given distrubances result in larger changes in reserves under the key currency reserve regime then under the outside reserve regime when central banks remain passive, that is, when they neither buy nor sell domestic assets. Our model provides a natural framework within which to consider these relatively neglected problems.

A central feature of our model is the way in which we have specified the markets for securities. These markets have not been adequately treated in many of the previous discussions of models which

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1/ In the spring and summer of 1972 the impact of large foreign official purchases of U S. securities on U.S. interest rates was widely discussed in the financial press. See, for example, Journal of Commerce, April 15, 1972, p. 1. Some analysts believed that the conversions of dollar exchange to interest bearing securities helped to keep U.S. short term interest rates from rising as much as they had been expected to rise for various reasons. Some of the same analysts argued that these interest rate developments had an adverse effect on the U S. capital account.

allow for capital movements. As a result these discussions have missed one of the avenues through which changes in reserves may affect an economy. For example, in the last Chapter it was shown that, contrary to the view held by some, sterilizing the effects of reserve changes on money supplies does not in general completely insulate the economy from the effects of disturbances to equilibrium since the changes in the supplies of securities required to keep money stocks constant usually lead to a change in interest rates. A careful specification of the securities markets is, if anything, more important for studying the key currency reserve regime. Under the outside reserve regime changes in central bank holdings of reserve assets were associated with changes in national money supplies when central banks kept their holdings of domestic assets constant. However, under the key currency reserve regime where reserve changes are held in the form of key currency securities, reserve changes lead directly to a reduction in the amount of key currency securities available for the public to hold when central banks are passive. It is particularly useful when studying the key currency regime to have a model when takes explicit account of the market for U.S securities.

B. Modifications in the Model

In order to analyze the key currency regime some minor modifications in our model are required. The four market equilibrium conditions from the model presented in Chapter II with the necessary

alterations are summarized in the following equations:

$$B^S \equiv \bar{B} - B^c - B^{c'} = b(r,r')\ W + \Pi b'\ (r,r')W' \qquad (9a)$$

$$\Pi\ F^S \equiv \Pi\ (\bar{F} - F^{c'}) = f(r,r')W + \Pi\ f'(r,r')W' \qquad (10a)$$

$$M^S \equiv B^c + \sigma\ R + S = m(r,r')W + M^{c'} \qquad (11a)$$

$$\Pi\ N^S \equiv \Pi\ (F^{c'} + \sigma'R' + B^{c'} + S') = \Pi\ n'(r,r')W'. \qquad (12a)$$

The holdings of U.S. securities and U.S. money by the UKCB are represented by $B^{c'}$ and $M^{c'}$ respectively. As before, only three of the four equations are independent, a fact which can be verified by noting that adding the four equations together yields an identity. The modified equations can be represented using the same graphical framework that we have employed previously. As we shall see the set of possible new equilibrium positions following a given disturbance are the same under the key currency regime as they were under the outside reserve regime. However, the two central banks must undertake different operations to arrive at the same equilibrium position under the two alternative reserve regimes. In addition, the way in which reserves are held may affect the way in which central banks react to reserve changes. For these two reasons the equilibrium position actually reached after a given disturbance may be different under alternative regimes even though all the same positions are attainable.

C. A Speculative Shift

To illustrate how the key currency reserve regime operates we consider a type of shock to asset equilibrium which is especially pertinent given recent history, a shift in preferences away from U.S.

securities and toward U.K. securities due to a change in expectations regarding the exchange rate. As we saw in Chapter III above such a shift in preferences causes FF to move from F_0F_0 to F_1F_1 and BB to move from B_0B_0 to B_1B_1 as in Figure 10. We assume that the USCB

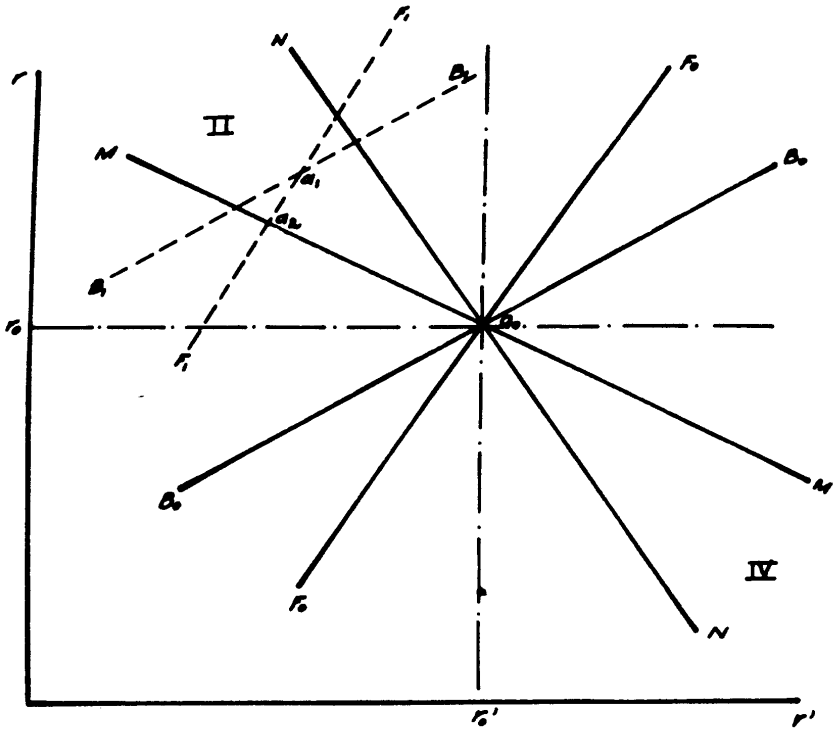

FIGURE 10

follows a GSPR; that is, it keeps its holdings of U.S. securities constant. The UKCB follows a GSPR and accumulates reserves in the form of U.S. securities. Following the shift there is an excess demand for U.K. securities and an excess supply of U.S. securities at the original interest rate pair (r_0, r_0'). As wealth holders seek to rebalance their

portfolios they exert downward pressure on r', upward pressure on r, and upward pressure on π. The UKCB buys dollars for pounds in support operations. These operations result in a rise in the U.K. money supply, but the U S money supply remains unchanged because the UKCB immediately uses the dollar proceeds of its support operations to purchase U.S. securities in the open market. When the non key currency country holds changes in reserves in the form of key currency securities, they, in effect, sterilize the key currency country's money supply, and the security purchases by the UKCB reduce the supply of U.S. securities available to private holders.[1/] r rises but by less than if the UKCB had accumulated reserves in outside assets because part of the initial excess supply of U.S. securities is removed by the UKCB purchases. r' falls but by more than it would under an outside reserve regime since with a lower r the excess demand for U.K. securities can only be removed by a larger decline in r'.

1/ Some of the recent increase in dollar-denominated, interest-bearing assets held by foreign official institutions has come in the form of special non-marketable securities. To study the effects of this form of reserve holding in the context of our model we can suppose that the UKCB converts the dollars acquired from support operations into special non-marketable assets. This action, in itself, results in no reduction in the supply of U.S. securities available to the public. If the USCB wants to keep the U.S. money supply constant, it must purchase marketable securities from the public. When non key currency countries hold reserve changes in the form of any kind of non-marketable instrument, the key currency country must actively pursue open market operations in order to sterilize its money stock.

U.K. residents are persuaded by the changes in interest rates to reduce their holdings of U.K. securities and to hold the increased U K. money stock. The increase in the U.K. money stock which is identical to the increase in U.K. reserves is represented by the shift of NN from N_0N_0 to N_1N_1 which passes through the intersection of F_1F_1 and the original MM curve at a_2. We know from above that if the U.K. had held all reserve increases in outside assets the equilibrium position would have been at a_1. Position a_2 implies a larger increase in reserves for the U.K. since NN must shift down further from N_0N_0 to reach a_2 than to reach a_1. This conclusion demonstrates the contention that the same shock may lead to a greater accumulation of reserves by the non key currency country under a key currency reserve regime than would occur under an outside reserve regime.

It is generally the case that when both central banks are passive, U.K. reserve holdings will fluctuate more for given shocks to asset equilibrium if the UKCB holds reserve changes in U.S. securities than if it holds these changes in outside assets. No matter what the form of the disturbance the BB and FF curves resulting directly from the shock must intersect between the MM and NN curves resulting directly from the shock in regions of excess demand for one type of money and excess supply of the other. These regions are designated as II and IV in Figure 10. Given this configuration the NN curve must

always shift further to reach the intersection of the new FF and MM curves than to reach the intersection of the new FF and BB curves.[1/]

The analysis above suggests that the European countries might have had to accumulate fewer reserves preceding the formal suspension of dollar convertibility in August 1972 if they had taken reserve increases in the form of outside assets. Although this course of action was impossible, since the dollar was de facto inconvertible, our model can be used to show that the size of European reserve accumulations could have been limited in exactly the same way if Europeans had held reserve increases in the form of U.S. money instead of in the form of U.S. securities. They would still have had to accumulate de facto inconvertible dollars, but the amount would have been smaller. There would, of course, have been a loss of interest income on securities.

In terms of our model the demonstration of this contention is quite straightforward. If we continue to assume that both central banks are passive the new equilibrium following the speculative shift is the same when reserve changes are held as key currency money as the equilibrium reached under the outside reserve regime. The UKCB accumulates dollars and supplies pounds in support operations but does not convert the dollars into either outside assets or U.S.

1/ It is not possible to make a general statement about the relative movements of the two interest rates between equilibria under the two different reserve holding regimes. Depending upon the type of disturbance to equilibrium the initial equilibrium pair of interest rates (r_0, r_0') may be anywhere inside the four sided figure bounded by the new MM, NN, FF, and BB curves or even outside this figure.

securities. Thus the U.S. money supply falls and the U.K. money supply rises until equilibrium is reestablished at point a_1, but this is exactly what happened under the outside reserve regime. The size of U.K. reserve gains are exactly the same when reserve changes are held as money as when reserve changes are held as outside assets.

We should not be too quick to conclude, however, that if non key currency countries want to minimize reserve fluctuations they should hold reserve changes in outside assets during periods of effective convertibility and in key currency money during periods of de facto or de jure inconvertibility. It is niave to assume that the central bank of the key currency country would remain passive as we have been assuming up until now. Turning again to the model we find that if the USCB is going to follow, for example, MSCPR regardless of how the UKCB holds its reserve changes, then if the UKCB does not sterilize the effects on the U.S. money supply of disturbances to asset equilibrium by holding reserve changes in the form of U.S. securities the USCB will do so through open market purchases. In either case the new equilibrium is at a_2. The only decision facing the UKCB is whether to hold assets paying interest or non interest bearing assets.[1/] Of course if the USCB only offsets

1/ If the U.S. monetary authorities were pursuing a money supply target in the summer of 1972, it would have made no difference for the behavior of interest rates if foreign official institutions had chosen to hold the proceeds from their support operations in the form of U.S. money rather than using them to purchase U.S. securities from the public. The USCB would have bought enough securities from the public to keep the money supply on target if foreign official institutions had not done so.

part of any decline in the U S. money stock the UKCB can reduce its reserve accumulation to some extent, at some cost in terms of interest foregone, by holding reserve increases in non interest bearing assets.

V. The Effects of a Devaluation on Financial Asset Holding

It has often been argued that since an exchange rate change has no effect on the relative attractiveness of domestic and foreign financial assets, there is no incentive for wealth holders to rearrange their portfolios of financial assets following a devaluation. Those who make this argument usually assume that wealth holders do not expect the exchange rate to change and that once the rate changes they do not expect it to change again.[1/] Under these conditions a U.S. investor compares the proceeds from one dollar placed in U.S. securities, $1+r_0$ dollars, with the proceeds from one dollar placed abroad, $\left[\frac{1}{\pi 0}\ (1+r_0')\ \pi_0\ \right]$ dollars, at time zero before the devaluation. If nothing else has changed and the exchange rate changes from π_0 to π_1, the same investor must compare $1+r_0$ dollars with $\left[\frac{1}{\pi_1}\ (1+r'_0)\pi_1\ \right]$ dollars. Since the exchange rate cancels out in both comparisons,

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1/ Of course, if a devaluation is anticipated asset demands shift away from the securities of the country which is expected to devalue. After the devaluation has occurred assets demands probably shift back if no further devaluation is expected. We consider here the less realistic case of an unanticipated devaluation in order to highlight what we believe to be a necessary modification of conventional arguments. Later we consider the case in which a shift in asset demands occurs at the same time as a devaluation.

U.S. and U.K. securities have the same relative attractiveness after the exchange rate change as before it.[1/] Thus, the argument proceeds, there should be no incentive for wealth holders to alter their portfolios.

This argument is correct as far as it goes. What it ignores is the fact that some wealth holders experience capital gains and others experience capital losses as a result of the devaluation and that these changes in wealth are realized completely in the first instance as changes in the home currency value of foreign assets. Both the losers and the gainers from the devaluation find themselves with the wrong mix of assets at the prevailing interest rate given their changed wealth and the form in which the change has accrued. Thus when the wealth effects of the devaluation are taken into account there is an incentive for wealth holders to alter their portfolios.

Consider a devaluation of the dollar in the framework of our model.[2/] U.S. citizens experience a capital gain of $F^A d\pi$ dollars

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1/ There may be other avenues in addition to those we consider below in the text through which an exchange rate change can affect the relative attractiveness of U.S. and U.K. securities. For example, if a country devalues in order to free itself from an underemployment-deficit dilemma situation investors might expect that increased economic activity resulting from the direct impact of the devaluation and a relaxation of constraints on expansionary policies would lead to a higher return on the securities of the devaluing country. Our model is not well suited for an investigation of these effects, so we abstract from them here.

2/ The devaluation may be reflected in a change in σ, the dollar price of outside reserve assets; in σ', the pound price of outside reserve

(Footnote continued on next page)

while U.K. residents suffer a capital loss of $-B^{A'}d\Pi$ pounds where Π is the dollar price of pounds, taken to be unity initially, and F^{A} and $B^{A'}$ are the initial holdings of U.K. securities by U.S. residents and the initial holdings of U.S. securities by U.K. residents respectively.[1/] The increase in wealth in the U.S. comes in the first instance as an increase in the dollar value of U.K. holdings of U.S. securities, and the decrease in wealth in the U.K. comes initially in the form of a decrease in the pound value of U.K. holdings of U.S. securities. Following the devaluation U.S. residents have an excess demand for U.S. securities and U.S. money and an excess supply of U.K. securities and U.K. residents have an

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1/ continued from previous page

reserve assets; or in both. Of course, at least one currency price of reserve assets must change. The relationship between $d\Pi$, $d\sigma$, and $d\sigma'$ is given by,

$$d\Pi = \frac{1}{\sigma'} = d\sigma - \frac{\sigma}{\sigma'^2}\, d\sigma'.$$

Changes in σ and σ' alter the domestic currency value of the initial reserve holdings of both central banks. The size of these effects, the impact effects of a devaluation on the value of reserve holdings, depends upon how much of the devaluation is reflected in σ and how much in σ'. We want to assume that these impact effects themselves cause no changes in the money supply of either country, so we assume that the two central banks simply change S and S' so as to offset the impact effects of changes in σ and σ' on the domestic currency value of their assets,

$$dS = R\, d\sigma$$
$$dS' = (\bar{R} - R)\, d\sigma'.$$

1/ We continue to assume that asset demands are homogeneous of degree one in nominal wealth and that output prices are constant.

excess demand for U.S. securities and an excess supply of U.K. securities and U.K. money at the original interest rate pair. Thus, there is a world excess demand for U.S. securities and a world excess supply of U.K. securities. In Figure 11 the pre-devaluation equilibrium

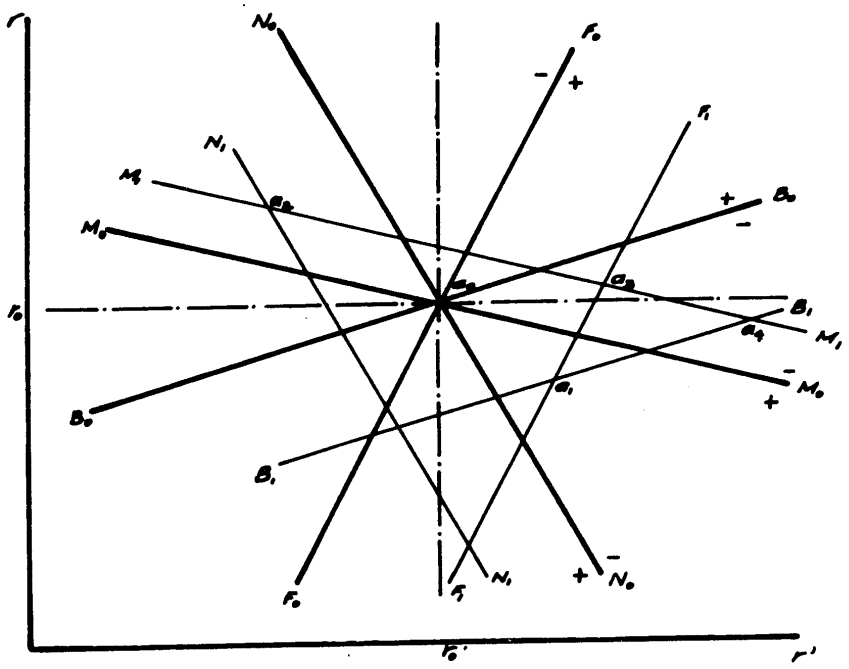

FIGURE 11

is at the intersection of the schedules with the $(_0)$ subscript. The impact effect of the devaluation is shown by the schedules labeled with the $(_1)$ subscript.[1/]

1/ For reasons discussed in detail above the new FF and BB curves must intersect between the new MM and NN curves.

As we have come to expect by now the ultimate configuration of interest rates and the required changes in reserve holdings depend upon the policy responses of the USCB and the UKCB. It is instructive to begin with the case in which both central banks pursue an IRCPR and the UKCB holds changes in its reserves as outside assets or U.S. money. If interest rates are held constant by central bank action the proportion of wealth held in each asset by residents of both countries remains constant. U.S. residents want to divest themselves of $(1-f)F^{A'}d\Pi$ in U K. securities and hold $mF^{A}d\Pi$ of the proceeds in U.S. money and $bF^{A}d\Pi$ in U.S. securities. U.K. residents want to sell $f'B^{A'}d\Pi$ additional pounds into dollars in order to buy $(1-b')B^{A'}d\Pi$ U.S. securities. If interest rates are to remain unchanged the USCB must meet the demand for U.S. securities and the UKCB must absorb the unwanted U.K. securities. U.K. reserve losses are equal to the sales of U.K. securities by U.S. citizens plus the purchases of U.S. securities by U.K. residents or $[(1-f)F^{A} + (1-b')B^{A'}]d\Pi$. Of course, the effect of the actions by central banks described above is to shift all the curves back to their original positions so that they intersect at a_0 again.

If the UKCB uses U.S. securities instead of outside assets or U.S. money to finance imbalances it will increase the supply of

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1/ For reasons discussed in detail above the new FF and BB curves must intersect between the new MM and NN curves.

U.S. securities by the full amount of the sales of U.K. securities by U.S. citizens plus the amount of the purchase of U.S. securities by U.K. residents. This increase in the supply of U.S. securities exceeds the initial excess demand for U.S. securities at the original interest rate pair because U.S. citizens are selling off U.K. securities partly in order to increase their money balances. In this case the USCB must buy U.S. securities in an amount equal to the initial excess demand for money in the U.S. at the original interest rate pair in order to keep U.S. interest rates from rising.

Returning again to the assumption that outside reserves are used to finance payments imbalances we find that if both central banks follow GSPR's, the new equilibrium is at the intersection of B_1B_1 and F_1F_1 at a_1. r must fall, and r' must rise in order to remove the initial excess demand for U.S. securities and excess supply of U.K. securities. MM and NN must shift down until they pass through a_1. The U.S. money supply increases and the U.K. money supply declines by equal amounts. The shift of reserves from the U.K. to the U.S. can be represented by either the shift in MM or the shift in NN.

Complete monetary sterilization by the two central banks when they follow MSCPR's implies that equilibrium must be at point a_2 where M_1M_1 and N_1N_1 intersect. The USCB sells enough U.S. securities and the UKCB buys enough U.K. securities to cause the U.S. interest rate to rise and the U.K interest rate to decline by the amount

necessary to remove the initial excess demand for money in the U.S. and the excess supply of money in the U.K. The loss of reserves by the UKCB is equal to the purchases of securities by the UKCB which are equal in amount to the sales of securities by the USCB and can thus be represented by the shift in either FF or BB from F_1F_1 or B_1B_1 to the position they would have to have if they were to pass through a_2.

Point a_3 is the equilibrium point if the U.S. money supply and the supply of U.K. securities reamin unchanged. This point would be reached if imbalances are financed by changes in outside reserves and the U K. pursues a GSPR while the USCB employs a MSCPR. Asset equilibrium would also be at point a_3 if the UKCB sells U.S. securities to finance its deficit and both central banks are passive. In this case the U.K. interest rate must rise in order to induce residents of both countries to hold the unchanged supply of U.K. securities. The U.S. rate may rise (as in Figure 11) or fall since either the USCB or the UKCB sells off enough U.S. securities to keep the U.S. money stock constant, and these sales may exceed or fall short of the excess demand for U.S. securities at an unchanged U.S. interest rate given the U.K. interest rate on F_1F_1 corresponding to the unchanged U.S. rate. 1/

1/ In graphical terms the question is whether or not the distance between M_0M_0 and M_1M_1 measured along a line through (r_0, r_0') and parallel to the r' axis is greater than the distance between F_0F_0

(footnote continued on following page)

It is important to note that the UKCB can avoid reserve losses due to portfolio shifts if it pursues a RSCPR to the U.S. devaluation while the U.S. follows either a GSPR, a MSCPR, or an IRCPR.[1/] Consider the case in which the U.S. follows a GSPR. If the UKCB pursues a large enough contractionary open market operation it can assure that the new equilibrium is at point a_4 where B_1B_1 and M_1M_1 intersect. Since neither the U.S. money stock nor USCB holdings of U.S. securities is changed at a_4, U.S. reserves must be unchanged.[2/] Of course, this policy response by the UKCB implies an increase in the U.K. interest rate and a decline in the U.K. money supply. Point a_4 would also be attained if the USCB employed a MSCPR while the UKCB

1/ footnote continued from previous page

and F_1F_1 measured in the same way. An algebraic analysis reveals that the answer to this question is indeterminante. The shift in MM is given by,

$$\frac{dr'}{d\Pi}\Big|^{MM}_{r=r_0} = -\frac{mF^A}{m_{r'}W},$$

and the shift in FF is given by,

$$\frac{dr'}{d\Pi}\Big|^{FF}_{r=r_0} = \frac{mF^A + [bF^A + f'B^{A'}]}{f_{r'}w + f'_{r;}w'}.$$

The expressions for both shifts are positive and the difference between them may be positive or negative.

1/ The two central banks cannot, of course, attain inconsistent reserve targets.

2/ It was explained in footnote 1 on page 70 that a devaluation has impact effects on the values of initial reserve holdings. In describing the reserve changes consequent upon a devaluation we ignore the impact effects and concentrate insteady on reserve changes which reflect changes in the two money supplies and changes in central bank holdings of securities.

pursued a RSCPR. The analysis is somewhat more difficult if the USCB employs an IRCPR, and we will not consider this case.

The graphical analysis can be used to determine the relative magnitude of UKCB reserve losses implied by some of the equilibrium positions we have considered. The technique for assessing the relative magnitude of UKCB reserve losses depends crucially upon assessing the implication of each of the equilibrium positions for either the USCB or the UKCB balance sheet. We showed that point a_4 was an equilibrium which involved no reserve loss for the UKCB by looking at the balance sheet of the USCB. Since neither the U.S. money stock nor the USCB holdings of U.S. securities has changed the U.S. must not have gained any reserves. Point a_1 involves some reserve loss by the UKCB since the U.K. money supply declines while the UKCB holdings of U.K. securities remain fixed. An even greater U.K. reserve loss is implied by point a_3 since the U.K. money supply declines further while UKCB holdings of U.K. securities are still constant. The U.K. reserve loss (U.S. reserve gain) implied by point a_3 can also be represented by the shift in BB from B_1B_1 to the position it would have if it passed through a_3 since the U S. money stock is the same at a_1 and at a_3. The same line of argument leads to the conclusion that a_2 involves a larger U.K. reserve loss than a_3 since the BB curve must shift even further to reach a_2. Thus ranking points from least to greatest U.K. reserve loss we obtain the following: $a_4 < a_1 < a_3 < a_2$. The techniques for ranking each

pair of points is to find an item on the balance sheet of one central bank or the other which remains unchanged between the two points and then compare the changes in another item between the two points; this comparison allows us to determine how the third item must have changed. 1/

The magnitude of U.S. reserve gains from the portfolio adjustments due to the wealth effects of a devaluation depends upon the policy responses of the two central bank which in turn depend upon what policies the central banks deem appropriate from a domestic point of view and how convinced the UKCB is that the U.S. should be allowed to accumulate reserves.

If a devaluation by the U.S. has been expected, asset demands may have been shifted away from U.S. securities and in favor of U.K.

1/ We have not ranked a_0 because the graphical technique is insufficient in this case. However, it can be proved algebraically that the reserve loss for point a_0 is ranked between the losses for point a_1 and a_2. It is shown in Part II of the Appendix that for any shock to asset equilibrium, of which devaluation is only one example, it is more efficient in terms of the reserve shift required for the reattainment of equilibrium for central banks to keep their holdings of securities constant and allow money supplies to change, that is, to rely completely upon what we call pure money adjustment, than for central banks to keep money supplies constant and allow their holdings of securities to adjust, that is, to rely completely upon what we call pure security adjustment. Pure money adjustment alone leads to point a_1 in Figure II while pure security adjustment alone leads to point a_2. It is also shown in Part II of the Appendix that pure money adjustment alone is more efficient and pure security adjustment alone is less efficient than any combination of the two pure adjustment mechanisms and what we call mixed money-security adjustment which lead to points which lie in the four-sided figure in Figure II the corners of which are a_1, a_3, a_2, and a_5.

securities at some point before the devaluation. After the devaluation asset demands might well shift back to their original configuration; that is, the demand for U.S. securities would rise at the expense of the demand for U.K. securities. If this shift in asset demands occurred BB and FF would shift farther to the right than the positions shown by B_1B_1 and F_1F_1 at the time of the devaluation. This increased shift means that the size of the adjustments in the quantity variables in the model required in order for equilibrium to be reattained are larger. For any given combination of policy responses by the USCB and UKCB, changes in stocks of money and securities held by the U.S. and U.K. publics, and reserves held by the central banks will be at least as large, in absolute value, as they would have been without the additional shift in asset demands. 1/

1/ Examples can be constructed to show that the same statement cannot be made for the required changes in interest rates.

VI. Fractional Reserve Commercial Banking

A. An Overview

In this Chapter some of the implications of introducing a fractional reserve commercial banking system into the model are explored. We show how our model can be modified to incorporate a relatively simple description of fractional reserve commercial banking. In order to suggest what difference fractional reserve banking makes we compare and contract the effects of a shift in asset preferences in the modified model with the effects of an identical disturbance in the model without fractional reserve banking under some alternative assumptions about central bank behavior. In the modified model we are able to allow for the possibility that the UKCB might hold some of its international reserves at the USCB or as deposits at U.S. commercial banks. At the end of the Chapter we discuss briefly the analytical equivalance between our description of fractional reserve commercial banking and what we call "fractional reserve central banking."

B. Assumptions and Modifications in the Model

In order to facilitate the analysis we make several convenient simplifying assumptions. We assume that U.S. (U.K.) commercial banks must hold bank reserves equal to a constant fraction, α (α'), of their deposit liabilities;[1/2/] Commercial bank reserves in

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1/ It is assumed that banks do not pay explicit or implicit interest on demand deposits.

2/ α and α' could be interpreted as customary reserve ratios or as sums of required and customary reserve ratios.

the U.S. (H^b) and in the U.K. ($J^{b'}$) are deposits at the USCB and the UKCB respectively. In order to avoid confusion between international reserves and bank reserves we call the latter "high powered money." The U.S. and U.K. publics are assumed to hold no high powered money. There is no currency in the model. Commercial banks in both countries hold only domestically issued securities.[1/] As is customary at this level of abstraction we assume that banks in both countries are "fully loaned up," that is, that they expand their deposit liabilities as much as possible in order to be able to hold as many interest bearing securities as possible. Thus for any given supplies of high powered money in the U.S. and the U.K., H^b and $J^{b'}$, the money supplies available to the U.S and U.K. public, M^s and N^s respectively, are given by,

$$M^s = \frac{1}{\alpha} H^b,$$

$$N^s = \frac{1}{\alpha'} J^{b'},$$

and commercial bank demand for securities in the U.S. and U.K. (B^b and $F^{b'}$ respectively) are given by,

$$B^b = \frac{1-\alpha}{\alpha} H^b$$

$$F^{b'} = \frac{1-\alpha'}{\alpha'} J^{b'}$$

The supplies of high powered money to the commercial banks in the U.S. and the U.K. are given by,

$$H^b \equiv B^c + \sigma R - H^{c'} + S$$

$$J^b \equiv B^{c'} + \sigma(\bar{R} - R) + \frac{1}{\pi}(B^{c'} + M^{c'} + H^{c'}) + S'$$

1/ This assumption does little violence to reality in the case of U.S. banks to the extent that the Voluntary Foreign Credit Restraint (VFCR) ceilings are binding.

where $H^{c'}$ represents UKCB deposits at the USCB and $M^{c'}$ represents UKCB deposits at U.S. commercial banks. All other symbols should be interpreted in the same way as they were in previous Chapters.

Given the assumptions above we can rewrite the equilibrium conditions of the model in the following way,

$$B^s \equiv \bar{B} - B^c - B^{c'} - \left(\frac{1-\alpha}{\alpha}\right)\left(B^c + \sigma R - H^{c'} + S\right) = b(r,r')W + \pi b'(r,r')W' \tag{9b}$$

$$\pi F^s \equiv \pi\left[\bar{F} - F^{c'} - \left(\frac{1-\alpha'}{\alpha'}\right)\left\{F^{c'} + \sigma'(\bar{R} - R) + \frac{1}{\pi}(B^{c'} + M^{c'} + H^{c'}) + S'\right\}\right] = f(r,r')W + \pi f'(r,r')W' \tag{10b}$$

$$M^s \equiv \frac{1}{\alpha}(B^c + \sigma R - H^{c'} + S) - M^{c'} = m(r,r')W \tag{11c}$$

$$\pi N^s \equiv \pi\left[\frac{1}{\alpha}\left\{F^{c'} + \sigma'(\bar{R} - R) + \frac{1}{\pi}(B^{c'} + M^{c'} + H^{c'}) + S'\right\}\right] = \pi n'(r,r')W' \tag{12c}$$

where, as before, B^s, F^s, M^s, and N^s represent the quantities of the four assets available for the public to hold. Since adding the four equations together yields an identity, only three of the four equations are independent. In analytical terms the important modification which arises from the introduction of commercial banking systems is that the supply of high powered money to commercial banks given by $H^b \equiv B^c + \sigma R - R - H^{c'} + S$ for the U.S. [$J^{b'} \equiv B^{c'} + \sigma(\bar{R} - R) + \frac{1}{\pi}(B^{c'} + M^{c'} + H^{c'}) + S'$ for the U.K.] enters as an argument in the U.S. (U.K.) security market equilibrium condition so that changes in $H^b(J^{b'})$ shift both the MM and BB schedules (NN and FF schedules).

Now we make an assumption which allows us to highlight the essential features of fractional reserve commercial banking in the context of a somewhat simplified framework. We assume that the UKCB always manipulates $F^{c'}$ so as to keep the supply of high powered money to U.K. commercial banks and, therefore, the supply of money in the hands of the U.K. public constant; that is, the UKCB pursues a MSCPR.[1/2/] In terms of our graphical analysis, the NN schedule will always remain fixed. This assumption allows us to analyze the effects of introducing fractional reserve commercial banking by referring only to the U.S. commercial banking system since the size of the U.K. commercial banking system never changes.

1/ Assuming that α', π, σ', $\bar{R}$, and S' are constant, we have

$$dF^{c'} = \sigma' dR - \frac{1}{\pi}(dB^{c'} + dM^{c'} + dH^{c'}).$$

2/ In this footnote we discuss a rather complicated problem and how we will deal with it in this study. Suppose the USCB lowers the reserve requirement on deposit liabilities. This action enables U.S. commercial banks to expand their deposit liabilities and to purchase more interest bearing assets. We assume in accordance with the spirit of our model which contains only two interest rates (r and r') that banks are prohibited from paying explicit or implicit interest on demand deposits and that all costs of running the banking system are constant, at least over the range of changes in bank scale under consideration. Given these assumptions the expansion of the banking system will lead to an increase in bank profits. Presumably, the commercial banks are owned by shareholders, so bank equities are a part of $\bar{B}$, the total supply of U.S. issued securities. The way in which bank shareholders react to changes in bank profits is a factor in determining the impact of changes in the scale of the banking system on the variables in the model. We have discovered no set of logically compelling assumptions regarding the valuation of bank equity. Since it is not our purpose to develop a comprehensive treatment commercial banking, we consider two polar possibilities for illustrative purposes.

Footnote continued on following page

C. A Shift in Asset Preferences

To illustrate how the system operates with fractional reserve commercial banking we take the case of a preference shift by the public away from U.S. securities and toward U.K. securities. As before, this shift in preferences moves BB and FF from B_0B_0 and F_0F_0 to B_1B_1 and F_1F_1 in Figure 12. First consider some combinations of responses by the USCB and forms of UKCB reserve holding that make the analysis essentially the same as without fractional reserve commercial banking. Recall that we are assuming that the UKCB always acts so as to keep the U.K. money supply constant so that NN always remains at N_0N_0. If the UKCB holds its increases in reserves in the form of outside reserve assets and the USCB pursues a MSCPR, then the UKCB will sell U.K. securities and the USCB will purchase U.S. securities in order to sterilize their respective

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1/ footnote continued from previous page

If, on the one hand, we assume that bank shareholders anticipate that any increase in bank profits is transitory, then the value of bank equities, and therefore $\overline{B}$, will not change by any significant amount. A lowering of the reserve requirement under this assumption leads to an excess supply of money and an excess demand for U.S. securities. This disequilibrium is resolved in the usual way.

If, on the other hand, we assume that bank shareholders anticipate that any increase in bank profits is permanent, then the value of bank equity, and therefore $\overline{B}$, rises by the amount of bank purchases of interest bearing assets. Assuming that all bank shareholders are U.S. residents, W also rises by the same amount since the amount of money available to the public increases while the amount of securities available to the public remains unchanged. Given the rise in W and our assumptions about asset demands there will be an excess supply of money, an excess demand for U.S. securities, and an excess demand for U.K. securities. The excess supply of money and the excess demand for U.S. securities will be smaller than in the previous case.

We assume throughout this Chapter except in footnote 1 on page 87 that bank shareholders assume that all changes in bank profits are transitory. This assumption implies that changes in the scale of the banking system do not affect $\overline{B}$ and W.

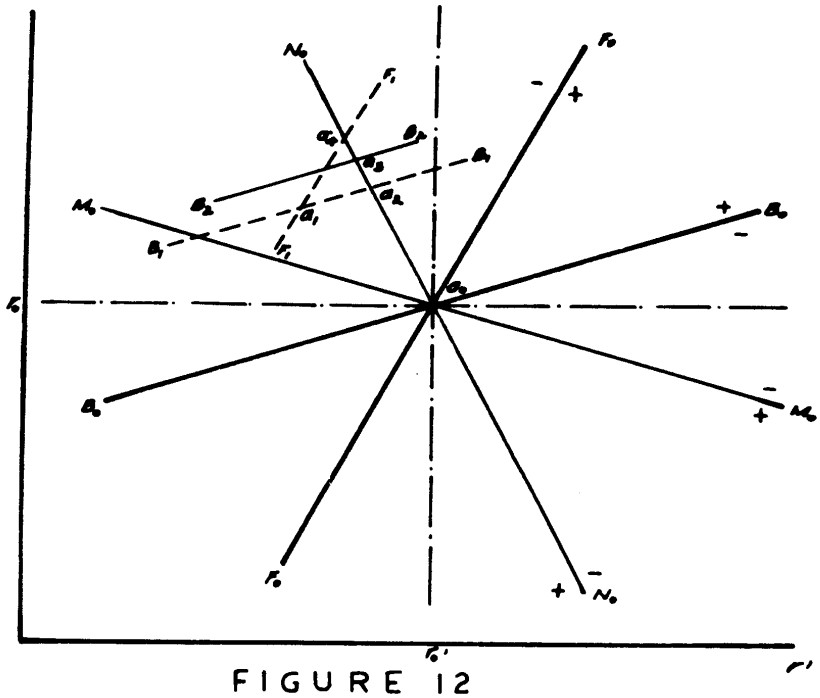

FIGURE 12

money supplies. In this situation the BB and FF curves will shift back to their original positions, and the new equilibrium will be at a_0. The same result will hold if the UKCB holds its reserves as U.S. securities and the USCB is passive. The size of the U.S. commercial banking system has not been changed in either of these cases, so fractional reserve banking does not affect the conclusions.

If, however, the UKCB holds its reserve changes in the form of outside assets and the USCB does not sterilize, then the supply of high powered money to the U.S. commercial banking system will fall as the USCB loses reserves. The drop in high powered money will, of course, necessitate a multiple contraction in deposits at U.S. commercial banks. Less obviously, the contraction of the U.S. commercial banking system causes it to sell of U.S. securities. The decline in the money supply shifts the MM curve up and to the right, while the sales of securities by the commercial banks shifts the BB curve up and to the left from B_1B_1. In addition the increase in the supply of U.K. securities due to the sterilization operations of the UKCB causes FF to shift down and to the left from F_1F_1. The MM, BB, and FF curves shift until they meet at a point, say point a_3 where B_2B_2 and F_2F_2 intersect, which must lie above a_2 and below a_4 on N_0N_0. Point a_3 must lie below point a_4, the point at which there would be no change in UKCB reserves, because reserves flow toward not away from the U.K. Since it is assumed that the UKCB always pursues a MSCPR, the relative magnitude of the reserve flow can be measured by the shift in FF from

F_1F_1 required to reach the new equilibrium position. It was shown in Chapter II that if the USCB pursues a GSPR and the UKCB pursues a MSCPR and there is no fractional reserve commercial banking system point a_2, where B_1B_1 and N_0N_0 intersect, is reached. We can conclude that although the reserve flow is in the same direction it is smaller with a fractional reserve commercial banking system than without one since FF must shift further down from a_4 to reach a_2 than to reach a_3.

The important point to notice in the case described in the previous paragraph is that the absorbtion of securities by the U.S. banking from other sectors of the world economy is positively related to the change in U.S. high powered money which, in the absence of sterilization operations by the USCB, is positively related to changes in the international reserves held by the USCB. The multiple contractio process can be conceptually separated into two parts. As reserves flow out the supply of high powered money, and therefore the money supply in the hands of the public, drops by an equal amount. Also, the reserve flow necessitates an additional (multiple) contraction which acts, in effect, like a contractionary open market operation but which emanates from the commercial banks rather than the USCB. The additional contractionary process reduces the need for reserves to flow.

What happens if the UKCB holds its increased reserves as deposits in the U.S.? First, if the UKCB holds reserves as deposits at the USCB then the amount of high powered money available to U.S. commercial banks will decline unless the USCB sterilizes by purchasing

securities. If the USCB is passive then the U.S. banking system will contract. The contraction in the U.S. banking system will release securities increasing the supply of securities to the public, and point a_3 will be reached. If the USCB instead sterilizes then the U.S. commercial banking system will not be affected by the reserve flow and position a_0 will be reattained.

If the UKCB holds reserve changes in the form of deposits at U.S. commercial banks, the supply of deposits available to the public will decline unless the USCB pursues sterilizing expansionary open market operations. If the USCB sterilizes it will purchase securities and supply high powered money so that total deposits at U.S. commercial banks go up by enough to accommodate the UKCB without reducing the amount of deposits held by the public. The securities purchased by the USCB and the U.S. commercial banks will equal the reserve flow, and the equilibrium will be at a_0. If the USCB does not sterilize the U.S. commercial banking system will not contract but the supply of deposits available to the public will decline by an amount equal to the increase in UKCB deposits and the new equilibrium will be at a_2. 1/

1/ We have indicated how the model would behave with the introduction of fractional reserve commercial banking if bank shareholders anticipate that any increases in profits in the banking system is transitory. As we have seen in footnote 2 on page 83 we could alternatively assume that shareholders anticipate that any increase in profits in the banking system is permanent. This case is more difficult to handle graphically than the case dealt with in the text because the size of private wealth holders' portfolios change when the size of the commercial

(footnote continued on following page)

D. A Note on "Fractional Reserve Central Banking"

It is interesting to note that we could have captured the essence of fractional reserve banking without ever introducing commercial banks. Suppose that we revert to the model of Chapter II with no commercial banking and postulate that the USCB always acts so as to maintain a constant ratio of international reserves to its monetary liabilities. We call such behavior "fractional reserve central banking". Under this assumption a loss of international reserves by the USCB requires it to undertake contractionary open market operations if it wishes to maintain a desired ratio of international reserves to monetary liabilities which is less than one. Now consider the model of the current Chapter. When the USCB remains passive a reserve loss leads to a reduction in high

1/ footnote continued from previous page.

banking system changes. A decrease in the size of the commercial banking system reduces private wealth since the amount of money available for the public to hold decreases, but the amount of securities (including claims on banks) available for the private non-banking sector to hold remains unchanged.

If we take the case of a preference shift toward U.K. securities and away from U.S. securities dealt with previously and assume that all U.S. bank stock is held by U.S. residents then the following will occur. In Figure 12 the B_1B_1 and F_1F_1 curves are the shifted security market curves before adjustment takes place. Assume as before that the UKCB keeps the U.K. money supply constant and that the USCB is passive. The U.S. money supply will contract as high powered money decreases and the commercial banking system reduces its deposit liabilities and sells off earning assets. Before, this process involved a leftward shift in the BB curve because of the increase in U.S. securities available for the public to hold resulting from the sales by the commercial banks. Now, instead of an increase in the supply of U.S. securities, there is a decline in the demand for U.S. securities (and U.K. securities) as the value of U.S. allocatable assets decline, shifting the BB curve leftward. The equilibrium position will be at some point on the N_0N_0 curve between a_4 and a_2.

powered money which causes the commercial banks to sell off securities. Under the circumstances specified above the behavior of the two models is exactly equivalent if the central bank's desired ratio of international reserves to monetary liabilities in the model of Chapter II is the same as the required ratio of high powered money to deposit liabilities in the model of the current Chapter. This is but one example of the more general proposition that under certain assumptions we can specify central bank behavior in such a way that the behavior of a model which does not take explicit account of commercial banks is exactly the same as the behavior of a model in which they are explicitly included. We state and prove this proposition in Part IV of the Appendix. It is important because it implies that under appropriate assumptions we can consolidate the central bank and the commercial banking system and cancel out high powered money as an intra sector item without losing sight of any of the essential features of the problem under consideration.

VIII. The Dependence of the Effects of Monetary Policy Upon the Degree of Substitutability Between Securities and Upon Relative Economic Size

A. Introduction

Up to this point we have assumed that U.S. and U.K. securities are substitutes but not perfect substitutes for one another in the portfolios of the two countries' wealth holders and that the U.S. and the U.K. are of roughly equal economic size. Recently attention has been focused upon the impact of increased capital market integration upon the effectiveness of monetary policy. One aspect of increased capital market integration is increased substitutability between domestic and foreign securities. Other analysts have considered the effects of monetary policy in models in which wealth holders in a given country regard securities issued in that country as being either completely non-substitutable or perfectly substitutable for securities issued abroad.[1/] Often these analysts have restricted their attention to the case of a single open economy which is small in economic size when compared to the rest of the world. One purpose of this Chapter is to show that familiar polar assumptions yield familiar results for the effects of monetary policy in the context of our model. A second purpose is to explore the way in

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1/ For models in which it is assumed that wealth holders regard securities issued at home and abroad as perfect substitutes see Mundell (1968, Chapter 18) and McKinnon and Oates (1966). For a model in which it is assumed that wealth holders regard securities issued abroad as completely non-substitutable for both securities issued at home and domestic money see McKinnon (1969).

which increases in the degree of substitutability between assets affect the effectiveness of monetary policy in the intermediate range in which U.S. and U.K. securities are substitutes but not perfect substitutes for one another. A third purpose is to spell out the special limitations on the effectiveness of monetary policy faced by a country which is small in economic size relative to the rest of the world even if the securities issued in the small country are not perfect substitutes for those issued in the rest of the world.

For the most part when we refer to monetary policy in this Chapter we mean an open market purchase of U.S. securities by the USCB. In our terminology the policy action we are investigating is a GSPI by the USCB. Attention is focused upon the effects of this policy initiative upon the differential between the rate on U.S. securities and the rate on U.K. securities and upon the level of the rate on U.S. securities. We usually assume that there is no attempt in either country to sterilize the effects on money supplies associated with reserve changes; that is, the GSPI by the USCB is met with a GSPR by the UKCB. We restrict our analysis in this way only to avoid an unwieldy proliferation of cases. We do not intend to take a position here, either explicitly or implicitly, as to whether the proximate target of monetary policy should be a "rate" or an "aggregate". We sketch the implications of the assumptions explored for the ability of the USCB to control the U.S.

money supply. As we shall see, the choice of the rate on U.S. securities as an illustrative proximate target for U.S. monetary policy is of little consequence since when the USCB can control this rate, it can also control the U.S. money stock, and when it cannot control this rate, it cannot control the money stock either.

B. The Effects of Monetary Policy in Some Polar Cases

First we explore the implications of some polar assumptions regarding the degree of substitutability between U.S. and U.K. securities retaining the assumption that the U.S. and the U.K. are of roughly equal economic size. We begin our discussion with the case in which wealth holders in both countries regard the two securities as completely non-substitutable. There are two important subcases which are shown in Figures 13a and 13b. Figure 13a shows a situation in which U.S. citizens do not consider U.K. securities to be substitutes for either U.S. securities or U.S. money and U.K. citizens do not consider U.S. securities to be substitutes for either U.K. securities or U.K. money. So far as financial asset holding is concerned these two economies are "closed" economies. The MM and BB schedules coincide as do the NN and FF schedules. Since in each country money and securities issued at home are the only assets considered suitable for wealth holding, the same value of the interest rate on securities issued at home must clear both the money market and the market for securities issued at home. In

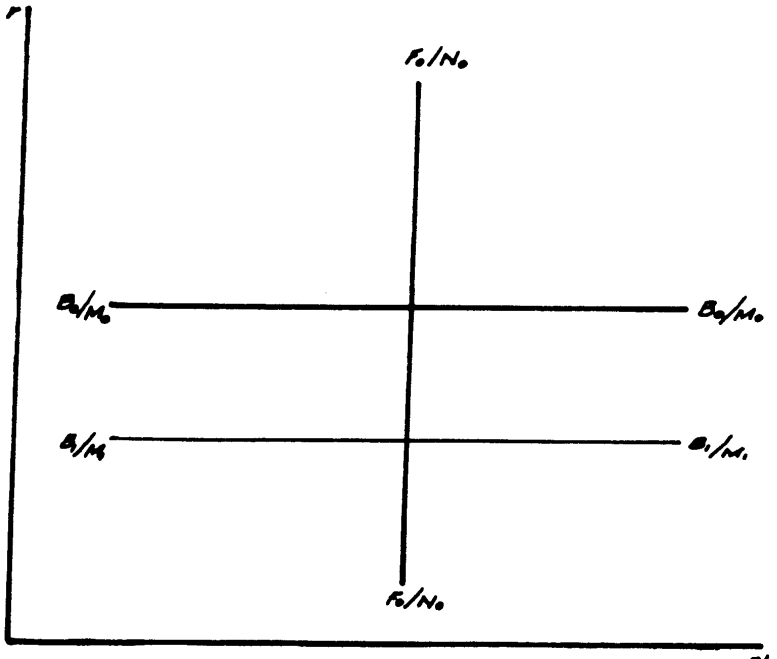

FIGURE 13a

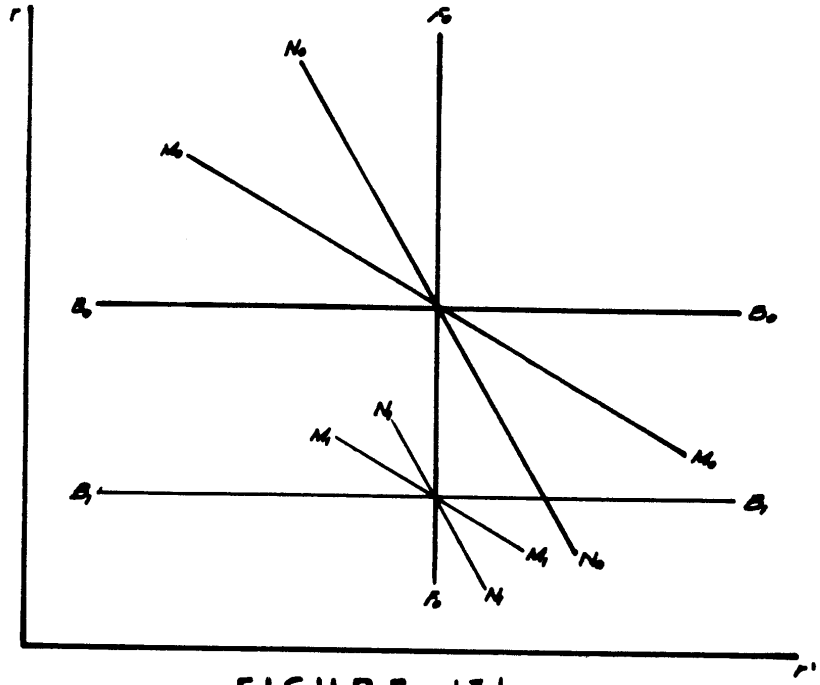

FIGURE 13b

this case an open market purchase by the USCB causes the rate on U.S. securities to fall by enough to make U.S. residents willing to reduce their holdings of U.S. securities and increase their holdings of money by the amount of the open market purchase. Of course, the differential between the U.S. and U.K. rate is reduced by the full amount of the decline in the U.S. rate.

Figure 13b portrays a situation in which U.S. and U.K. residents hold both types of securities and in which both groups of residents view both types of securities as substitutes for domestic money even though the two types of securities are regarded as being completely non-substitutable for one another. The MM (NN) schedule no longer coincides with the BB (FF) schedule. Here an open market purchase by the USCB causes the rate on U.S. securities to fall by enough to make both U.S. and U.K. residents willing to reduce their holdings of U.S. securities and increase their holdings of money by the amount of the open market purchase. Of course, if the response of U.S. money demand to the U.S. rate is the same here as in the previous case, an open market purchase of equal size causes a smaller decline in the U.S. interest rate in this case because U.K. residents surrender some U.S. securities to the USCB. For the same reason the rise in the U.S. money stock is less than before, and the USCB loses reserves. Once again the interest differential declines by the amount of the decline in the U.S. rate.

There are good arguments in favor of using each of the two cases just discussed as a base case against which to measure the

effects of increases in the degree of substitutability between the two types of securities. It seems that the base case one chooses should depend upon the question under consideration. If, on the one hand, we are interested in contrasting the effect of monetary policy in a large open economy in which wealth holders view assets issued abroad as very good substitutes for home assets with the effects of the same policy in a large closed economy, the first case is a preferable base case. If, on the other hand, our concern is focused more narrowly on the implications of increased substitutability between securities with as many other factors as possible kept constant, the second case is a better base case.

We turn now to the other polar case, the case in which U.S. and U.K. securities are virtually perfect substitutes. First we treat this case graphically. Some of our assertions about the effect on the four curves of allowing the two securities to become virtually perfect substitutes are supported by a mathematical argument later. As the degree of asset substitutability increases without limit, the slopes of the BB and FF curves approach positive one. As the two securities become virtually identical, any increase in r' must be offset by an almost identical, increase in r in order for either of the two securities markets to remain in equilibrium. The slopes of the MM and NN schedules approach negative one as the two securities

become indistinguishable.[1/] Since wealth holders in, say, the U.S. view the two securities as being almost completely alike, the responses of their money demand to the two interest rates should become very nearly the same. We can make a similar case regarding changes in the money demand responses of U.S. wealth holders. The limiting positions of the four curves are shown by the B_0B_0/F_0F_0 and M_0M_0/N_0N_0 schedules in Figure 14.

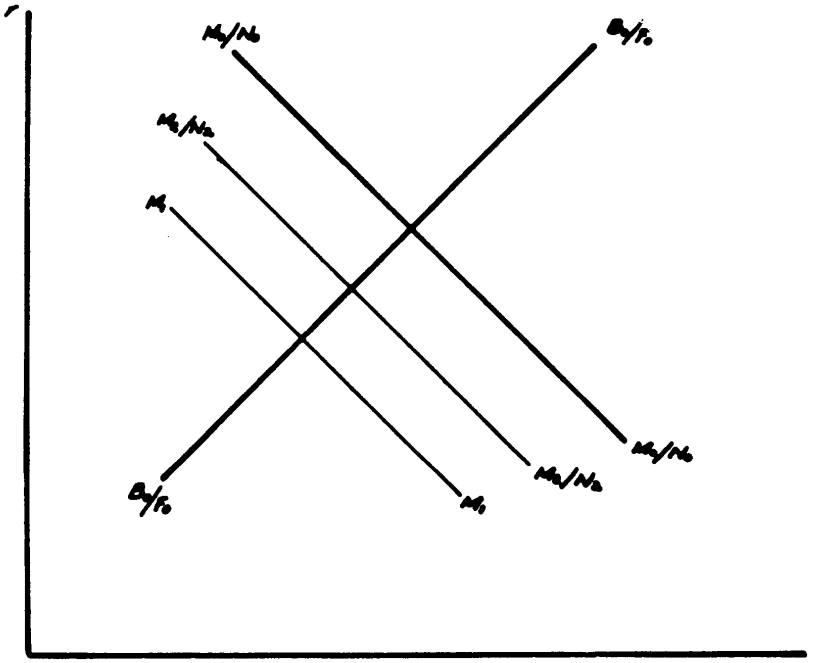

FIGURE 14

1/ The slopes of the MM and NN curves never actually take on the value of negative one for if the two securities were exactly identical in every respect a decline in r would lead residents in both countries to try to move completely out of U.S. securities into U.K. securities, but there seems to be no reason why wealth holders should want to change their money holdings. When the two securities are exactly alike the MM and NN schedules reduce to single points which must lie on the line with a slope of positive one which represents the BB and FF schedules.

Now consider an open market purchase by the USCB. One of the implications of very high degree of substitutability between securities is that a small change in either of the two interest rates causes a massive change in security demands. An open market purchase leaves the BB/FF schedule unaffected in the limiting case since it takes virtually no change in r to remove a given excess demand for U.S. securities. However, the MM schedule is shifted to M_1M_1. U.S. residents hold too much money and too few securities after the open market purchase, so they make net purchases of securities from U.K. residents. The U.S. money supply declines from the value it assumed immediately following the open market purchase but remains higher than its initial value, and the U.K. money supply rises. Graphically, the MM curve shifts up and the NN curve shifts down until they meet at M_2M_2/N_2N_2. The USCB can still lower the U.S. interest rate by making an open market purchase even when securities are virtually perfect substitutes. Of course, in the limiting case there is effectively only one world rate, so a better way of stating the result might be that the USCB can lower the world interest rate. The USCB can effect this change because a plausible sized open market purchase alters perceptibly the relative supplies of securities and the two types of money in the world so the world interest rate must change in order for equilibrium to be reestablished. The lower the sensitivity of the demands for money in the two countries to what is effectively one rate the larger the reduction in this rate

consequent upon the open market purchase. This result accords with the one obtained by other analysts.[1/]

As we would expect the differential between the U.S. and U.K. rates is unaffected by an open market purchase in the limiting case. Any change in the differential would lead wealth holders to want to make an indefinitely large shift between U.S. and U.K. securities, so no such change can occur. In graphical terms we move along a line with a slope of positive one, so that any change in r must be matched by an equal change in r' in the same direction in order for equilibrium to be maintained in the markets for securities.

C. The Impact of Increases in the Degree of Substitutability on the Effects of Monetary Policy in the General Case

Analyzing the impact of increases in the degree of substitutability between securities on the effects of money policy when securities are neither completely non-substitutable nor virtually perfect substitutes is a somewhat more difficult task than the one we have just completed. Up to this point we have assumed that U.S. securities, U.K. securities, and domestic money are strict gross substitutes in the portfolios of wealth holders in each of the two countries and that the ratio of the response of U.S. money demand to the rate on U.K. securities to the response of U.S. money demand to the rate on U S. securities is less than the ratio of U.K. money demand responses to these same rates.[2/] In order to make the problem

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1/ See, for example, the Appendix in Mundell (1968, Chapter 18).

2/ See page 24 above.

at hand more tractable, we resort to somewhat stronger assumptions. First, we assume that "own rate effects" are greater than "cross rate effects" on the demand for securities by both U.S. and U.K. wealth holders. This assumption implies, for example, that the absolute value of the effect of a one percentage point rise in the U.S. rate (own rate) on the demand for U.S. securities by U.S. residents is greater than the absolute value of the effect of a one percentage point rise in the U.K. rate (cross rate) on the demand for U.S. securities by U.S. residents. The implications for the effects of interest rate changes on other security demands are analagous.[1/] Secondly, we assume that in each country the absolute value of the response of money demand to a one per cent increase in the rate on securities issued abroad is less than the absolute value of the response of money demand to a one percentage point increase in the rate on securities issued at home.[2/] These two assumptions in conjunction with the strict gross substitutes assumption assure that the BB curve has a positive slope which is less than one, that the FF curve has a positive slope which is greater than one, that the MM curve has a negative slope which is less negative than negative one, and that the NN schedule has a negative slope which is more negative than negative one. The schedules must have the position shown in Figure 15a; cases like that shown in Figure 15b which could occur under our earlier weaker assumptions are ruled out.

1/ More explicitly we assume the following: $b_r > |b_{r'}|$, $f_{r'} > |f_r|$, $b'_r > |b'_{r'}|$, $f'_{r'} > |f'_r|$.

2/ See page 24 above.

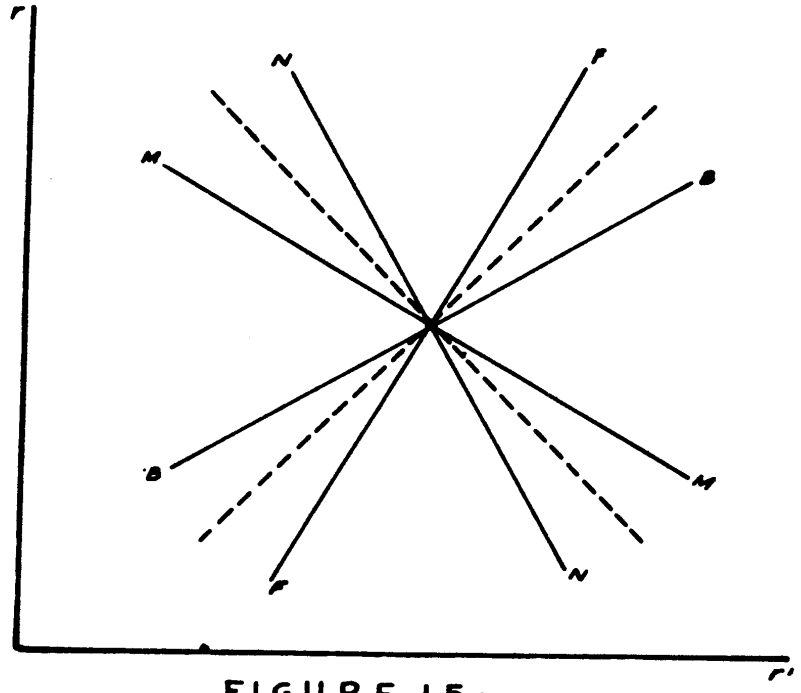

FIGURE 15a

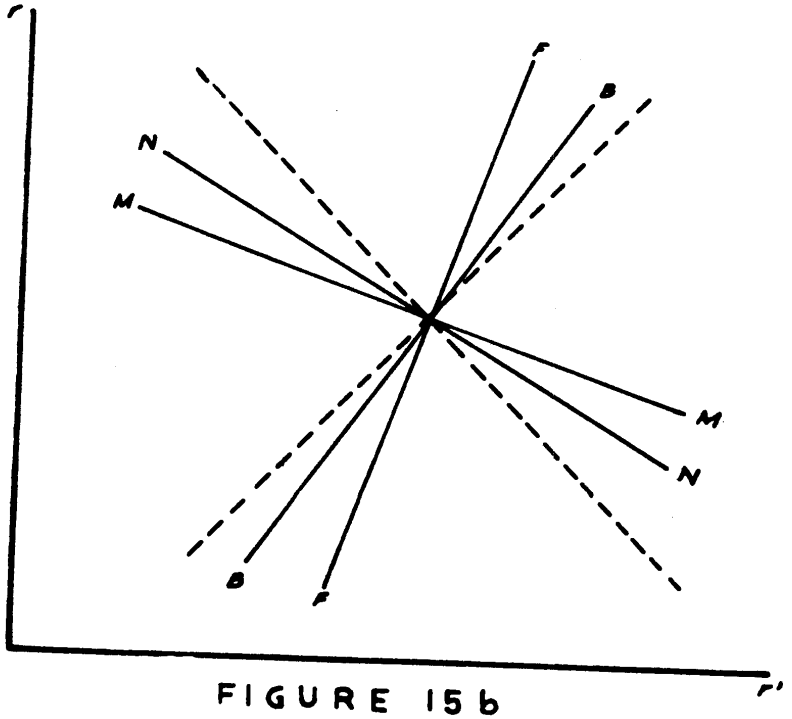

FIGURE 15b

Now we must investigate the implications of increases in substitutability between U.S. and U.K. securities for our graphical analysis. Intuition suggests that increases in the degree of substitutability between assets should cause the slope of the BB schedule to increase continuously toward positive one. It seems plausible that r should have to rise farther to remove the excess supply arising from an increase in r' the more alike the two securities appear to wealth holders in both countries and that the required rise in r should be approaching equality with the rise in r' as the two securities become virtually identical. A similar argument suggests that the slope of the FF schedule should fall continuously toward one as the degree of substitutability between assets increases. It also seems reasonable to suppose that the slope of the MM (NN) schedule should fall (rise) continuously toward negative one as the two securities become better substitutes. Since wealth holders in say, the U.S. come to view the two securities as being more and more alike, the responses of their money demand to the two interest rates should become more similar. We can make an analogous case regarding changes in the money demand responses of U.K. wealth holders. The movements we have described are shown in Figure 16. While the changes in the schedules which we have described seem reasonable, the schedules may not behave in the way we have suggested. We explore below some assumptions which assure that the schedules behave in the manner we have outlined. The reader who is satisfied with the

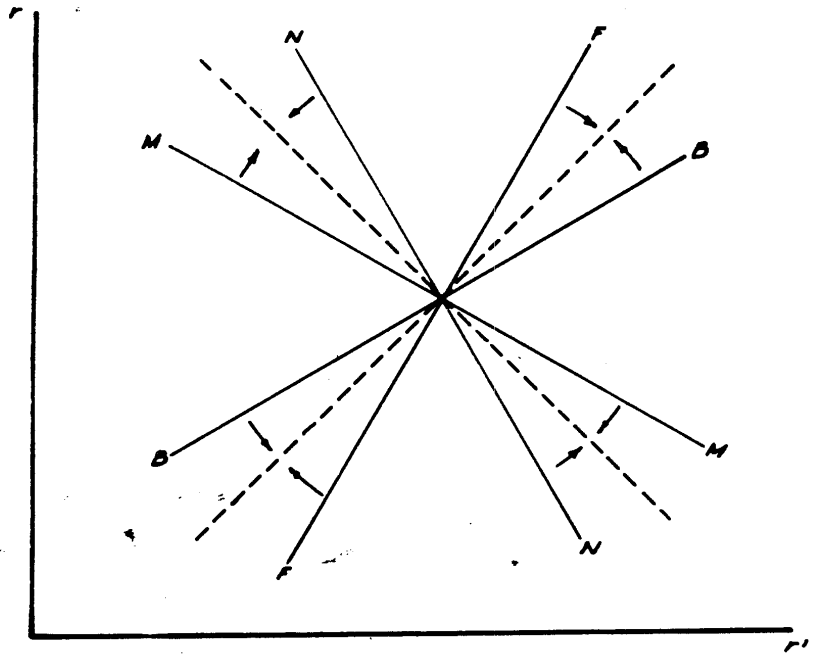

FIGURE 16

suggestive but incomplete reasoning regarding the movement of the schedules which we have given up to this point can skip to the text surrounding Figure 17.

In order to specify more exactly some conditions under which the schedules are affected in the way we have described by increased substitutability between securities we choose a convenient and what we believe to be an intuitively appealing way to represent increases in substitutability between U.S. and U.K. securities. Let s be a variable which takes on the value zero when the two assets are completely nonsubstitutable and which rises continuously and without limit as substitutability increases. We regard all of the

demand responses to interest rate changes by U.S. and U.K. residents as functions of s and write them as, for example, $b_r(s)$. We define three vectors $\alpha(s)$, $\beta(s)$, and $\gamma(s)$,

$$\alpha(s) \equiv \begin{bmatrix} b_r(s) \\ f_r'(s) \\ b'_r(s) \\ f'_{r'}(s) \end{bmatrix}, \qquad \beta(s) \equiv \begin{bmatrix} f_r(s) \\ b_r'(s) \\ f'_r(s) \\ b'_{r'}(s) \end{bmatrix}, \qquad \gamma(s) \equiv \begin{bmatrix} m_r(s) \\ m_r'(s) \\ n'_r(s) \\ n'_{r'}(s) \end{bmatrix}.$$

$\alpha(s)$ is a vector of own rate effects; $\beta(s)$ is a vector of cross rate effects; $\gamma(s)$ is a vector of money demand responses. From our earlier analysis we know that $\alpha(s) + \beta(s) + \gamma(s) \equiv 0$. When the two securities are perfectly non-substitutable for one another we have $\alpha(0) = -\gamma(0)$ and $\beta(0) = \underline{0}$.[1/] We assume that as the degree of substitutability increases the values of the our rate effects rise continuously without limit and the values of the cross rate effects decrease continuously without limit,

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1/ In the case in which the two securities are completely non-substitutable for one another and securities issued abroad are considered to be completely non-substitutable for domestic money by residents of both countries we have,

$$\alpha(0) \equiv \begin{bmatrix} b_r(0) \\ 0 \\ 0 \\ f'_{r'}(0) \end{bmatrix} = -\gamma(0) \equiv \begin{bmatrix} -m_r(0) \\ 0 \\ 0 \\ -n'_{r'}(0) \end{bmatrix}$$

$$\alpha_s > \underline{0} \ ,$$

$$\beta_s < \underline{0} \ ,$$

$$\lim_{s \to \infty} \alpha(s) \to \underline{\infty} \ ,$$

$$\lim_{s \to \infty} \beta(s) \to - \underline{\infty} \ .$$

Also we assume that the two cross rate effects on the security demands in each country become more alike, and that the difference between them approaches zero as s increases,

$$b_r's - f_{rs} \quad \begin{cases} \leq 0 \text{ if } b_r' - f_r \geq 0 \\ \geq 0 \text{ if } b_r' - f_r \leq 0 \ , \end{cases}$$

$$b'_r's - f'_{rs} \quad \begin{cases} \leq 0 \text{ if } b'_{r'} - f'_r \geq 0 \\ \geq 0 \text{ if } b'_{r'} - f'_r \leq 0 \ , \end{cases}$$

$$\lim_{s \to \infty} (b_r' - f_r) = 0 \ ,$$

$$\lim_{s \to \infty} (b'_{r'} - f'_r) = 0 \ .$$

Finally we assume that as the two securities become better substitutes the values of the interest rate responses of U.S. money demand approach one another continuously and that these effects tend toward a common value which lies between them as a limit. We make analogous assumptions for the interest rate responses of U.K. money demand, so we have,

$$m_{rs} \geq 0 \ , \quad m_r's < 0,$$

$$n'_{r's} \geq 0 \ , \ n'_{rs} < 0,$$

$$\lim_{s \to \infty} m_r(s) = \lim_{s \to \infty} m_r'(s) = k_1,$$

$$\lim_{s \to \infty} n'_{r'}(s) = \lim_{s \to \infty} n'_r(s) = k_2,$$

where $m_r \leq k_1 < m_{r'}$ and $n'_{r'} \leq k_2 < n'_r$. It could be argued that m_r and $n'_{r'}$ should remain constant and that $m'_{r'}$ and n'_r should decline to equal them if we characterize increases in substitutability as arising from changes which make wealth holders consider foreign securities to be more like domestic securities in both countries. We leave this issue open for the present.

The assumptions we have made so far regarding money demand responses are sufficient to insure that the slope of the MM curve becomes continuously more negative and approaches negative one as a limit and that the slope of the NN curve becomes continuously less negative and approaches negative one as a limit as the degree of substitutability increases.

The analysis of what happens to the BB and FF curves as substitutability increases is somewhat more complicated. As we shall see, what happens to the BB and FF curves as the interest rate responses of the security demands increase and decrease without limit depends crucially upon what happens to four ratios: $-\frac{b_{r'}}{b_r}$, $-\frac{b'_{r'}}{b'_r}$, $-\frac{f_{r'}}{f_r}$, and $-\frac{f'_{r'}}{f_r}$. Each of the ratios is the quotient of a cross effect divided by an own effect or <u>vice versa</u> for a particular security and for a single identifiable behavior group. The assumptions we have made up to this point insure that all four ratios approach positive one as s approaches infinity. Using the implications of the two balance sheet constraints and a little manipulation we can write the four sums of own effects and cross effects as follows:

$$b_r + b_{r'} \equiv -m_r + \left[-f_r + b_{r'}\right],$$

$$f_{r'} + f_r \equiv -m_{r'} + \left[-b_{r'} + f_r\right],$$

$$b'_r + b'_{r'} \equiv -n'_r + \left[-f'_r + b'_{r'}\right],$$

$$f'_{r'} + f'_r \equiv -n'_{r'} + \left[-b'_{r'} + f'_r\right].$$

We have assumed that the terms in brackets approach zero as s approaches infinity, so, given our assumptions regarding money demand responses, all of these sums approach constants as s approches infinity. Since the numerators and denominators of each of the four ratios are either increasing or decreasing without limit and since the sum of the numerator and denominator of each ratio approaches a constant, all four ratios approach positive one as s approaches infinity.

We also need to know what happens to these four ratios as s is increased in intermediate ranges, but our assumptions up to this point have no decisive implications for this situation. In our view a plausible characterization of increases in substitutability could well include the <u>added</u> assumption that $-\frac{b_{r'}}{b_r}$ and $-\frac{b'_{r'}}{b'_r}$ rise continuously toward positive one and that $-\frac{f_{r'}}{f_r}$ and $-\frac{f'_{r'}}{f'_r}$ fall continuously toward positive one. This assumption states simply that own effects and cross effects become continuously more alike except for sign as substitutability between securities increases. We presume that this assumption holds in what follows. It will be apparent that little progress can be made in the analysis of increased substitutability in intermediate ranges without it.

We are now prepared to investigate the behavior of the BB and FF schedules as the degree of substitutability is increased. The slope of the BB curve is given by,

$$\left.\frac{dr}{dr'}\right|_{BB} = -\frac{b_{r'}W + b'_{r'}W'}{b_rW + b'_rW'}.$$

This expression can be rewritten as,

$$\left.\frac{dr}{dr'}\right|_{BB} = \omega_1\left(-\frac{b_{r'}}{b_r}\right) + (1-\omega_1)\left(-\frac{b'_{r'}}{b'_r}\right),$$

where

$$\omega_1 = \frac{b_rW}{b_rW + b'_rW'};$$

that is, the slope of the BB schedule is a weighted average of the change in r required to offset a change in r' in order to keep demand for U.S. securities constant in the U.S. and the change in r required to accomplish the same effect in the U.K. Each weight is the ratio of the demand response of the country in question to a change in r divided by the total world demand response to a change in r. Since $-\frac{b_{r'}}{b_r}$ and $-\frac{b'_{r'}}{b'_r}$ approach one as s approaches infinity we have,

$$\lim_{s \to \infty}\left[\left.\frac{dr}{dr'}\right|_{BB}\right] = 1.$$

This result justifies the assertion that the slope of the BB curve approaches positive one as s approaches infinity which we made in our discussion of the polar case of perfect substitutability.

Differentiating the expression for the slope of the BB curve we have,

$$\frac{d\left[\frac{dr}{dr'}\,\middle|_{BB}\right]}{ds} = \omega_1 \frac{d\left[-\frac{b_r'}{b'_r}\right]}{ds} + (1-\omega_1)\frac{d\left[-\frac{b'_{r'}}{b'_r}\right]}{ds} + \left(\frac{b'_{r'}}{b'_r} - \frac{b_r'}{b_r}\right)\omega_{1s},$$

where

$$\omega_{1s} = \frac{b'_r b_r \, WW'}{(b_r W + b'_r W')^2}\left[\frac{b_{rs}}{b_s} - \frac{b'_{rs}}{b'_r}\right].$$

We have suggested above that intuition would lead us to believe that the expression for the change in the slope of the BB schedules as s increases should be positive. However, this need not necessarily be the case. Our earlier assumption that own rate effects and cross rate effects on the demand for U.S. securities become more alike in both countries is sufficient to insure that the first two terms on the right hand side are positive; that is, with the weights held constant the weighted average definitely rises. However, the weights do not, in general, remain constant. ω_1 rises (falls) if the per cent increase in b_r is greater (less) than the per cent increase in b'_r. The weighted average tends to rise (fall) if the weight on the larger of the two terms $\frac{b'_{r'}}{b'_r}$ and $-\frac{b_r'}{b_r}$ tends to rise (fall). We have found no convincing argument to determine the sign of the third term in the expression. What we can say, however, is that, whatever its sign, it is smaller in absolute value the more alike are the two countries either in terms of the ratios of cross effects to own effects or in terms of per cent changes in own effects and that it is zero if the two countries are exactly alike in either or both respects. Thus the possibility of a "perverse" movement in the slope of the BB

curve arises because of differences in the two countries.[1/] We proceed under the assumption that the two countries are "similar enough" that the slope of the BB curve rises continuously as s increases, that is, that if the third term is negative it is outweighed by the other two for all s. Using a similar line of argument we can establish that our assumptions are sufficient to assure that the slope of the FF curve approaches positive one as s approaches infinity and that the slope of the FF curve decreases continuously as s increases if the two countries are "similar enough" a condition which we assume to be fulfilled in what follows.[2/]

1/ A similar problem would arise if we were analyzing a closed economy with two well defined groups of wealth holders with different demand schedules for two types of securities and money. In closed-economy macro economics problems like this are said to be due to "distribution effects" which are usually assumed away. It seems somewhat less natural to assume that such distribution effects are unimportant in a two country world.

2/ The slope of the FF curve is,

$$\left.\frac{dr}{dr'}\right|_{FF} = -\frac{f_{r'}W + f'_{r'}W'}{f_r W + f'_r W'},$$

which can be rewritten as,

$$\left.\frac{dr}{dr'}\right|_{FF} = \omega_2\left(-\frac{f_{r'}}{f_r}\right) + (1-\omega_2)\left(-\frac{f'_{r'}}{f'_r}\right),$$

where

$$\omega_2 = \frac{f_r W}{f_r W + f'_r W'}.$$

The derivative of the slope of the FF curve with respect to s is,

$$\frac{d\left[\left.\frac{dr}{dr'}\right|_{FF}\right]}{ds} = \omega_2\,\frac{d\left[-\frac{f_{r'}}{f_r}\right]}{ds} + (1-\omega_2)\,\frac{d\left[-\frac{f'_{r'}}{f'_r}\right]}{ds} + \left(\frac{f'_{r'}}{f'_r} - \frac{f_{r'}}{f_r}\right)\omega_{2s},$$

where

$$\omega_{2s} = \frac{f_r f'_r\, WW'}{(f_r W + f'_r W')^2}\left(\frac{f_{rs}}{f_r} - \frac{f'_{rs}}{f'_r}\right).$$

The first two terms are positive under our earlier assumptions. The third term may be of either sign, but the whole expression is positive if the two countries are similar enough.

The groundwork for our consideration of the implications of increases in the degree of substitutability for the effects of an open market purchase by the USCB on the differential between the U.S. and U.K. interest rate and on the level of the U.S. rate is finally complete. By reference to Figure 17 we can see that in the general case an open market purchase reduces both the U.S. and U.K. interest rates. The BB schedule is shifted down from B_0B_0 to B_1B_1. Since the FF curve has a slope which is greater than one, the decline in the U.S. rate must be greater than the decline in the U.K. rate. The implications of an increase in substitutability for the effects of an open market purchase of a given size by the USCB can be seen in Figures 18a and 18b. The $B_0'B_0'$, $B_1'B_1'$, and $F_0'F_0'$ schedules represent a higher degree of substitutability between the two securities than the B_0B_0, B_1B_1, and F_0F_0 schedules. Note that for a given value of r' the distance between $B_0'B_0'$ and $B_1'B_1'$ is less than the distance between B_0B_0 and B_1B_1 in both diagrams; as substitutability increases it takes a smaller change in r to remove a given excess demand for U.S. securities. It is clear from both diagrams that an increase in substitutability implies that an open market purchase of a given size causes less of a reduction in the differential between the U.S. and U.K. rates. This result follows simply from the fact that $F_0'F_0'$ is flatter than F_0F_0. We know that r must fall to reestablish equilibrium once the USCB has made the open market purchase, but in order to maintain equilibrium in the market for U.K. securities r' must fall if r

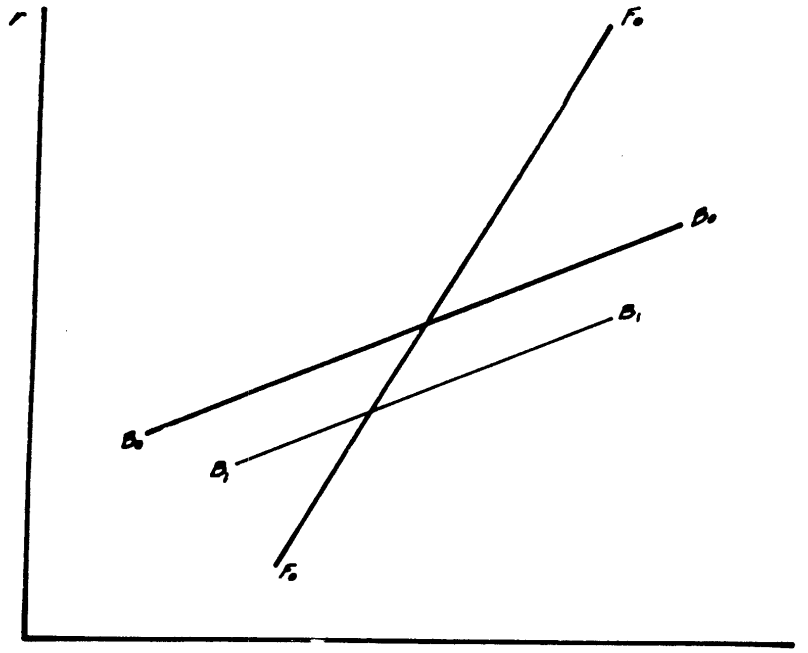

FIGURE 17

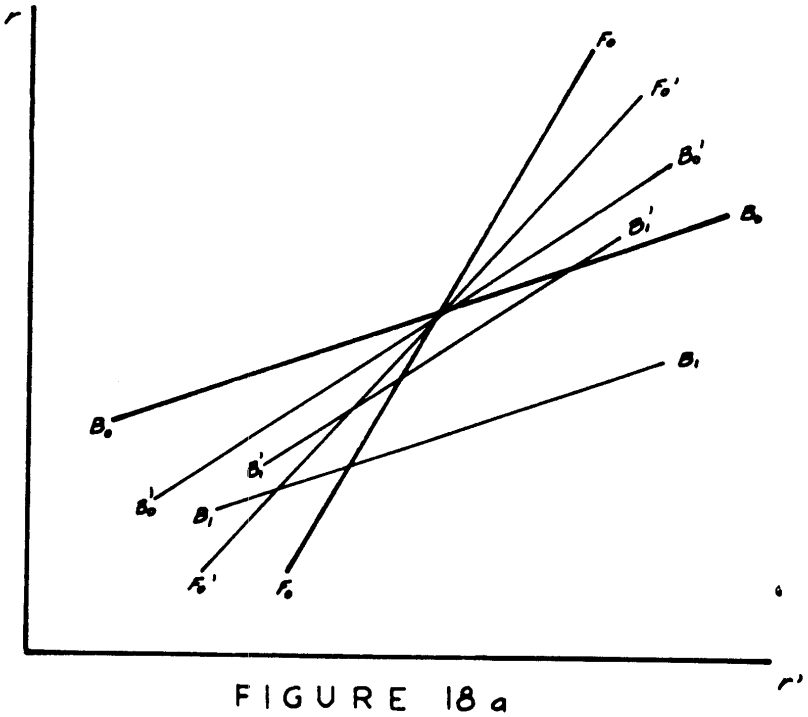

FIGURE 18 a

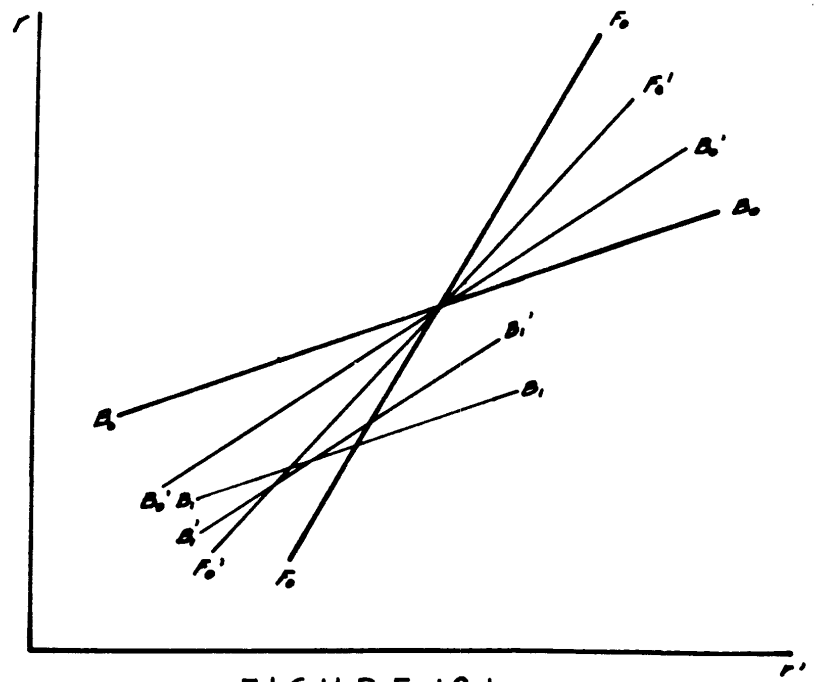

FIGURE 18b

falls. As substitutability is increased each decrease in r must be matched by a larger decrease in r' if equilibrium is to be maintained in the market for U.K. securities.

Determining the implications of increased substitutability for the effect on the level of the U.S. rate of an open market purchase by the USCB is more difficult. Although we can spell out the countervailing tendencies which lead to the ambiguity, we have been unable to arrive at a determinate result. Comparing Figures 18a and 18b we see that in the first case increased substitutability has caused an open market purchase to lead to a smaller reduction in r

and that in the second case increased substitutability has caused an open market purchase to lead to a larger reduction in r. Increased substitutability has two kinds of effects on the schedules which are important here. It causes both BB and FF to rotate toward a line with a slope of positive one, and for a given r' it causes the downward shift in the BB schedule resulting from an open market purchase by the USCB to be reduced.

We have drawn Figures 18a and 18b so that they are completely identical except that in Figure 18b the distance between $B_0'B_0'$ and $B_1'B_1'$ is greater than in Figure 18a. Clearly for a given amount of rotation of the BB and FF schedules the greater the reduction in the downward shift in the BB schedules the more likely it is that an increase in substitutability leads to a reduction in the amount by which r falls after an open market purchase. Experimentation with the graph, which we have not shown, reveals that for a given downward shift in the BB schedule the smaller the amount of rotation of either the BB or FF schedules the more likely it is that an increase in substitutability leads to a reduction in the amount by which r falls after an open market purchase.

The economic interpretation of the fact that the downward shift in the BB schedules is reduced is that smaller reductions in r are required to remove a given amount of excess demand in the market for U.S. securities for a given value of r'. The fact that both FF and BB are rotating means that a larger decrease in r' is required

to offset the effect of a given decline in r on the market for U.K. securities and that a larger drop in r is required to offset the effect of any decline in r' on the market for U.S. securities. The increased direct effect (reduced downward shift in BB) of a decline in r on the demand for U.S. securities tends to reduce the required decrease in r following the open market purchase while the increased induced effect (rotation of the FF and BB schedules) tends to raise the required decrease in r following the open market purchase. We have not been able to determine whether or not one of these tendencies dominates the other except in one very special case.[1/] Thus we must leave open the question as to whether or not an increase in substitutability between securities causes a decline in the reduction in the U.S. interest rate caused by a given sized open market purchase by the USCB when the two securities are neither completely non-substitutable nor nearly perfectly substitutable.

1/ When the two countries are exactly identical in terms of the responses of asset demands to interest rate changes, a rise in s causes $\frac{dr}{dB^c}$ to become less negative. The assumption that the responses of asset demands to interest rate changes are identical in the two countries is not compatible with our earlier assumption regarding the responses of money demands in the two countries to interest rate changes. Two restrictions required by our interpretation of what it means for countries to be identical are that $m_r = n'_r$ and $m_r' = n'_{r'}$. We have assumed above that $|m_r| > |m_r'|$, and this assumption in conjunction with our new restrictions implies $|n'_r| > |n'_{r'}|$ which violates our earlier assumptions. If we are to assume that countries are identical in the sense described here and to retain as much of the spirit of our earlier arguments regarding the interest rate responses of money demand functions as possible we must also assume $m_r = m_r'$.

Our conclusions regarding the impacts of increases in substitutability upon the effect of an open market purchase on the differential between U.S. and the U.K. rate are summarized in Figure 19. No matter which base case we begin from there is a continuous decrease in the reduction in the interest differential caused by an open market purchase as s increases.

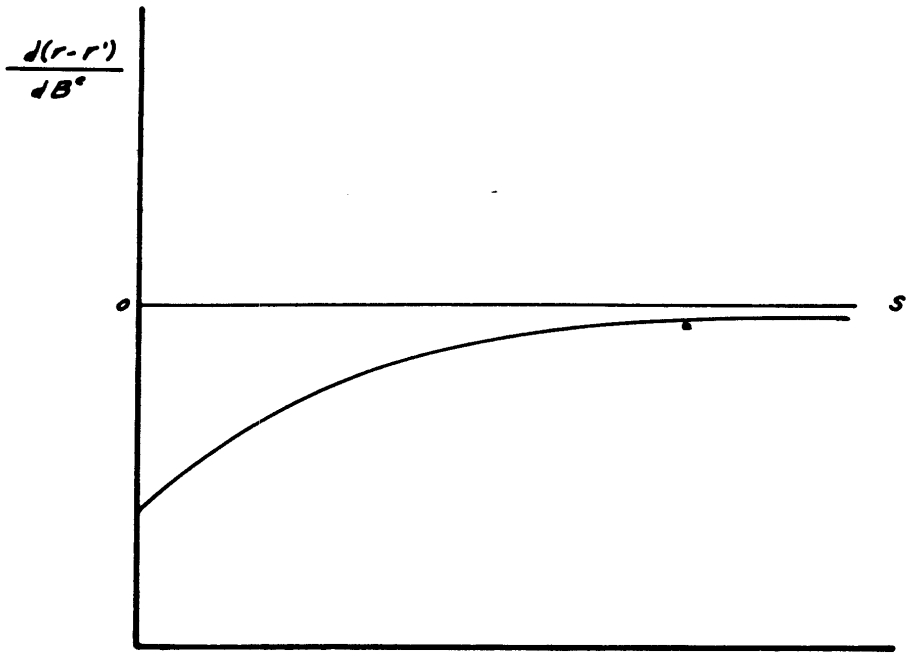

FIGURE 19

The same kind of simple summary is, of course, not possible in the case of the impact of increases in substitutability upon the effect of an open market purchase of the level of the U.S. rate, but we can say something. First, we must ask whether or not a given

open market purchase is less effective in lowering the U.S. interest rate when the two securities are virtually perfect substitutes than when the two securities are perfectly non-substitutable for one another but are both somewhat substitutable for money.[1/] When the securities are completely nonsubstitutable for one another the effect of an open market purchase on the U.S. rate is given by,

$$\lim_{s \to 0} \frac{dr}{dB^c} = -\frac{1}{b_r(0)W + b'_r(0)W'} = \frac{1}{m_r(0)W + n'_r(0)W'}.$$ [2/]

As we have seen above the U.S. rate must fall by enough to induce U.S. and U.K. residents to surrender U. S. securities to the USCB and replace them in their portfolios with money. When the two securities are virtually perfect substitutes for one another the effect of an open market purchase on the U.S. rate is given by,

$$\lim_{s \to \infty} \frac{dr}{dB^c} = \frac{1}{2}\frac{1}{k_1W + k_2W'}$$

1/ We handle the comparison of the case in which the securities are virtually perfect substitutes with the case in which they are completely non-substitutable and in which wealth holders in each country regard foreign securities as being completely non-substitutable for money in footnotes.

2/ When wealth holders in each country regard foreign securities as being completely nonsubstitutable for money initially this expression is,

$$\frac{dr}{dB^c} = -\frac{1}{b_r(0)W} = \frac{1}{m_r(0)W}.$$

where as above $m_r(0) \leq k_1 < m_r'(0)$ and $n'_{r'}(0) \leq k_2 < n'_r(0)$ and k_1 is the limiting value of both m_r and $m_{r'}$ and k_2 is the limiting value of both $n'_{r'}$ and n'_r as s approaches infinity. In this case decreases in both the U.S. and U.K. rates induce wealth holders to surrender U.S. securities and replace them with money in their portfolios. The reduction in the U.S. interest rate following an open market purchase is less when the two securities are virtually perfect substitutes if and only if,

$$|2(k_1W + k_2W')| > |m_r(0)W + n'_r(0)W'|.$$ 1/

Given the set of assumptions we have made up to this time this condition may or may not be met. We can certainly find plausible assumptions under which the condition is met. For example, if k_1 is the simple average of $m_r(0)$ and $m_r'(0)$ and k_2 is such a simple average $n'_{r'}(0)$ and $n'_r(0)$, the condition is met. $k_1(k_2)$ would be such simple average if $m_r(n'_r)$ became continuously less negative and $m_r'(n'_r)$ became continusously more negative as s increased in such a way that the sum $m_r + m_{r'}(n'_{r'} + n'_r)$ remained constant, that is, if a one percentage point rise in both rates always caused the same movement out of money into bonds taken together in both countries. It is plausible but by no means necessary that wealth holders money demand sensitivities would change in this way as they came to regard

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1/ In the case treated in the previous two footnotes the condition is,

$$|2(k_1W + k_2W')| > |m_r(0)W|.$$

the two securities as better substitutes. There are, of course, less restrictive assumptions which would lead to the fulfillment of the conditions given above, but we do not explore them here.

Even if we are prepared to presume that a given sized open market purchase is less effective in reducing r when the two securities are perfect substitutes than when they are completely non-substitutable we are still left with the fundamental ambiguity described above. We cannot establish that a given open market purchase becomes continusously less effective as the degree of substitutability is increased in the range where securities are neither virtually perfect substitutes nor completely non-substitutable. This information is summarized in Figure 20 in which three possible versions of what happens to $\frac{dr}{dB^c}$ as s is increased are shown.

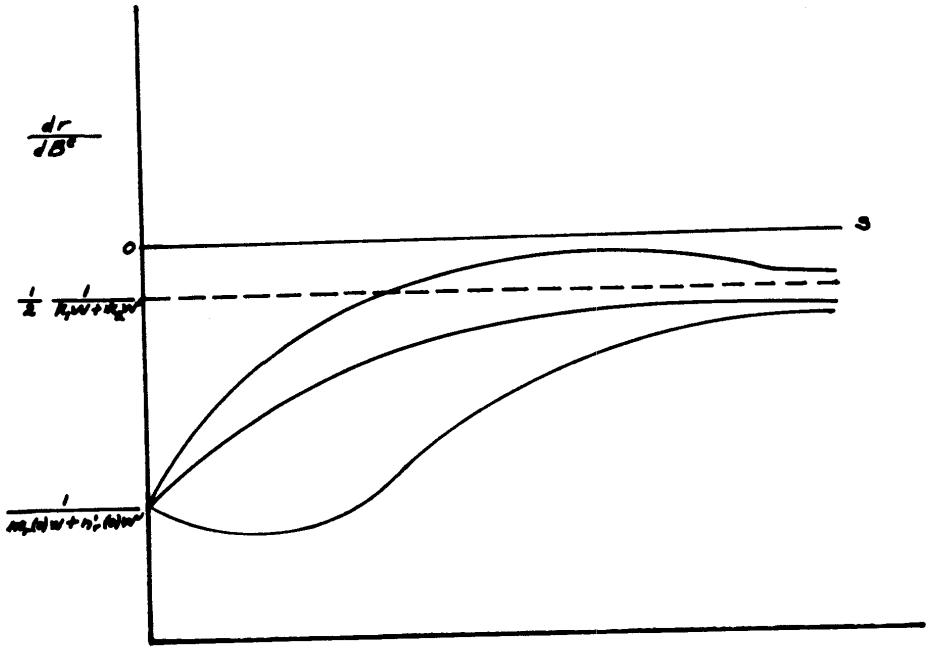

FIGURE 20

D. Relative Economic Size and the Effects of Monetary Policy

In this section we explore the implications of the assumption that the U.S. is a "small country" in the sense that its economic size as measured by nominal wealth in dollar terms is negligible when compared with the economic size of the U.K. measured in the same way. Many analysts have combined the "small country" assumption with the assumption that the securities issued in the small country are perfect substitutes for those issued in the rest of the world. We begin this section with a consideration of the case in which U.S. and U.K. securities are perfect substitutes in order to show that our model yields the familiar result that a small country can affect neither the single world interest rate nor the size of its own money supply by a plausible sized open market purchase in the limiting case.[1/] Later in the section we relax the assumption that U.S. and U.K. securities are perfect substitutes in order to demonstrate that a small country may well be unable to affect either of the two world interest rates or the size of its own money supply by a plausible sized open market purchase in this more general case.

Consider first the perfect substitutes case shown in Figure 21. As we argued in the last section an open market purchase by the USCB leaves the B_0B_0/F_0F_0 schedule unaffected and shifts the MM schedule from M_0M_0/N_0N_0 to M_1M_1. Following the open market purchase U.S. residents

1/ For another demonstration of this result see the Appendix to Mundell (1968, Chapter 18).

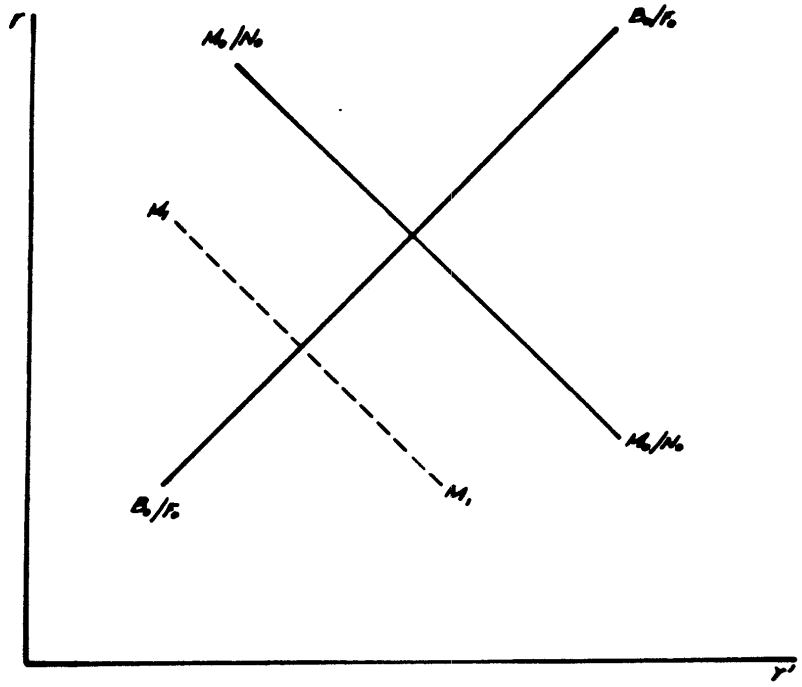

FIGURE 21

hold more money and fewer securities than they want to hold, so they make net purchases of securities from U.K. residents. The MM curve shifts up, but when the U. S. is small relative to the U.K. the NN curve does not move from M_0M_0/N_0N_0, so the final equilibrium point is the point at which M_0M_0/N_0N_0 crosses B_0B_0/F_0F_0. The world interest rate and the U.S. money stock are unchanged; U.S. reserves decrease and the U.K. money supply increases by the amount of the open market purchase. To understand this conclusion we must realize that since the U.K. is large relative to the U.S. a very small decline and in the limit no decline in the world interest rate will induce U.K. residents to reduce their holdings of securities and increase their holdings of

money by the relatively small amount necessary to satisfy the desires of U.S. residents to rebalance their portfolios following a plausible sized open market purchase by the USCB.

Our conclusion can be demonstrated formally by an analysis of the U.K. money market. The equilibrium condition for this market in the limiting case under consideration can be written as,[1/]

$$N^{S'} \equiv F^{C} + \bar{R} - R + S' = \tilde{n}'(\tilde{r})(\bar{W}-W).$$

$N^{S'}$, F^{C}, $\bar{R}$, R, S', and W have the same interpretation as in our earlier discussions. $\tilde{n}'(\tilde{r})$ gives the proportion of their wealth which U.K. citizens wish to hold in the form of money as a function of the single world interest rate $\tilde{r}$. $\bar{W}$ is the nominal value of world wealth which is assumed to remain constant, so $\bar{W}$ - W is equal to W', the nominal value of U.K. wealth. The U.S. money stock is,

$$M^{S} \equiv B^{C} + R + S.$$

Since we want to consider variations in W and W' with as many other factors as possible held constant we assume that in both countries the ratio of central bank held securities to wealth is independent of wealth so we have,

$$B^{C} = \epsilon W,$$

$$F^{C} = \epsilon' W'.$$

An open market purchase by the USCB is represented by an increase in ϵ. Substituting for B^{C} in the balance sheet identity for the USCB we

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1/ It is assumed here that the UKCB holds all its reserves in the form of outside reserves.

see that if the U.S. money stock is to remain constant following an open market purchase we must have,

$$dR = - W d\epsilon$$

The change in $\tilde{r}$ consistent with equilibrium in the U.K. money market given that the U.S. money stock remains constant after an open market purchase is found by totally differentiating the U.K. money market equilibrium condition. Substituting in the expression above for dR and rearranging we obtain,[1/]

$$\frac{d\tilde{r}}{d\epsilon}\bigg|_{M^S = \bar{M}^S} = \frac{W}{\tilde{n}'_{\tilde{r}}(\bar{W} - W)} .$$

This expression approaches zero continuously as W approaches zero. Thus a plausible-sized market purchase by the USCB results in no change in the world interest rate when the U.S. is small and U.S. and U.K. securities are perfect substitutes.

Now we turn to the case in which the U.S. is small but in which U.S. and U.K. securities are neither completely non-substitutable nor perfectly substitutable for one another in the portfolios of wealth holders. This case is shown in Figure 22. The initial effect of an open market purchase is to shift the MM schedule from M_0M_0 to M_1M_1. The open market purchase increases the U.S. money supply, and, if U.S. citizens are to be induced to hold this increase at a fixed r', there must be a decline in r. In contrast to the case in which the two countries are of roughly equal size, an open market purchase leaves the

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1/ ϵ', W, W', $\bar{R}$, S, and S' are held constant when the equilibrium condition is differentiated.

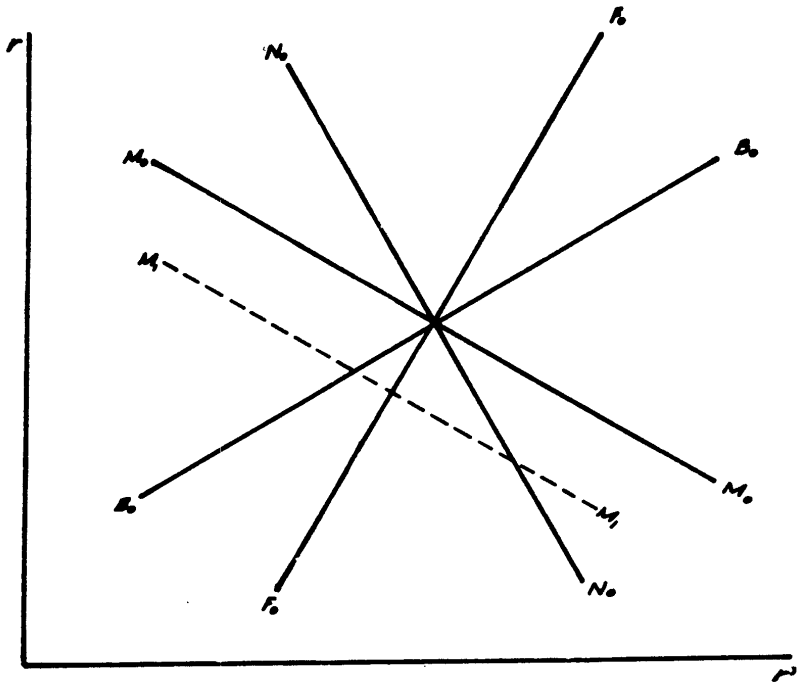

FIGURE 22

BB schedule unaffected when the U.S. is a small country. The equilibrium conditions in the market for U.S. securities is,

$$(1-\epsilon)W = b(r,r')W + b'(r,r')\ (\overline{W} - W)$$

where $\overline{W}$ has the same interpretation as in our previous discussion and ϵ is ratio of U.S. securities held by the USCB to U.S. wealth. As before we assume that ϵ is independent of the level of U.S. wealth and that open market purchases by the USCB are represented by increases in ϵ. Totally differentiating the equilibrium condition for U.S. securities and rearranging we arrive at the expression for the change in r required to offset a rise in ϵ given a fixed r',

$$\left.\frac{dr}{d\epsilon}\right|_{r'=\bar{r}'}^{BB} = -\frac{W}{b_r W + b'_r(\bar{W} - W)} .$$

This expression approaches zero as W approaches zero unless b'_r also approaches zero. Thus the BB schedule does not shift following an open market purchase by the USCB when the U.S. is relatively small unless the interest sensitivity of U.K. demand for U.S. securities approaches zero as W approaches zero.

The economic interpretation of this result is straightforward. The decrease in the supply of U.S. securities available to private wealth holders resulting from an open market purchase is $-Wd\epsilon$. The absolute value of this decrease approaches zero as the U.S. becomes of negligible size. The decrease in demand for U.S. securities by U.K. residents caused by a decline in r is $-b'_r(\bar{W}-W)$. The absolute value of this decrease remains finite so long as b'_r remains finite as W approaches zero. In the limit it takes a negligible decrease in r to induce the fall in U.K. demand for securities necessary to offset the negligible decrease in the supply of U.S. securities resulting from an open market purchase so long as b'_r remains finite.[1/] b'_r would approach zero as W approaches zero only if U.S. securities were an indispensible asset in the portfolios of U.K. wealth holders, that is, only if there were no finite r and r' pairs at which U.K. wealth holders would be prepared to hold no U.S. securities in their portfolios.

1/ Whether or not the absolute value of $\left.\frac{dr}{d\epsilon}\right|_{r=r'}^{BB}$ declines continuously as W declines is difficult to determine in general, and we leave this question open.

If U.S. securities were indispensable assets, further analysis would be required, but we will not treat this case here.

Returning to Figure 22 we see that if the BB schedule is unaffected by the open market purchase the equilibrium values of the two interest rates are unchanged. The MM curve shifts back to M_0M_0 as U.S. citizens rebalance their portfolios by making net purchases of securities from U.K. residents. The U.S. money supply returns to its pre-open-market-purchase level. The U.S. loses reserves equal to the full amount of the open market purchase. Thus the central bank in a small country is able to affect neither the interest rate on securities issued in its country nor its country's money supply by an open market purchase even if its securities are not perfect substitutes for those issued in the rest of the world provided that its securities are not regarded as indispensable assets by the world's wealth holders.

VIII. Conclusions

A. An Overview

This final Chapter is devoted to a discussion of some of the general properties of our approach rather than to a summary of the specific results obtained in earlier Chapters. The lessons of the approach for three separate areas of concern are stressed. First, we emphasize the fact that while our model can be used to study a variety of alternative policy actions under different institutional arrangements, it provides a useful framework for highlighting the similarities among policy actions. Secondly, we draw some implications from our regarding the international adjustment mechanism. Robert Mundell has argued that the mode of operation of monetary authorities in recent times may leave the world economy without an adjustment mechanism -- current world payments arrangements may constitute an "international disequilibrium system." However, in our model capital account equilibrium is reattained after a disturbance except in extreme cases. Third, we compare and contrast our portfolio balance analysis with the "monetarist" approach to balance of payments analysis, an approach which emphasizes the role of changes in the supply of and demand for money as determinants of the overall balance of payments. Our model deals exclusively with the determinants of the balance on the portfolio capital account of the balance of payments in the short run, but constructive insights can be gained from it regarding all three of these more general areas of concern.

B. The Economic Impact of Policy Actions

One of the primary purposes of analytical economics is to cut through the institutional details surrounding a particular set of policy actions and to focus attention upon the economic impact of the policies. One outcome of this type of activity is that policy actions which on the face of things seem quite different either because the identity of the institutional actors is different or because the sequence of actions taken is different can be shown to have equivalent economic impacts, that is, to be, in effect, the same policy action. A familiar example of such an application of analytical economics is the demonstration that there exists a certain pattern of trade taxes and subsidies which has the same economic effect on trade flows as an exchange rate change of a given amount.

A similar kind of result emerges from our model. There are four magnitudes under the direct control of the two central banks which influence the desired holdings of assets by private wealth holders, the supplies of the two kinds of money and of the two types of securities available for the public to hold. Of course, central banks are limited in affecting these magnitudes by their balance sheet constraints. Since we assume that markets in all four financial assets are perfect, it makes no difference in terms of economic impact which central bank alters the supplies of assets available for the public to hold.[1/] So

1/ To simplify the discussion we assume that all money is either currency or a deposit at a central bank.

long as the policy responses of the central banks are specified, a given initial shift in asset supplies, no matter how it is accomplished, results in the same change in equilibrium interest rates and asset holdings. It is also important to recognize that a given initial shift may be the result of a chain of actions containing several intermediate steps. All that is important for tracing the economic impact of the actions is the net effect of the actions on supplies of the two kinds of money and two kinds of securities to the public.

Suppose we are informed that some central bank has purchased U.S. securities with U.S. money in a transaction with the private sector. This action will shift both the MM and the BB curve down so that the intersection of the original FF and the new BB schedules lies above and to the left of the original NN and the new MM schedules. The new equilibrium position will be uniquely determined by the pattern of policy responses of the two central banks. The identity of the central bank which caused the original shift is of no consequence. If the USCB initiates such an action we call it an open market purchase while if the UKCB initiates an identical action we call it a shift in the composition of U.K. reserves. No matter what we call it, the economic impact is the same given the policy responses of the two central banks.

The same kind of argument can be used to demonstrate that two policy responses to a given initial disequilibrium which are superficially different are really the same from the point of view of economic impact. Consider an initial disequilibrium caused by a shift in wealth holders

preferences in favor of U.K. securities and away from U.S. securities resolved by central bank sales of pounds for dollars coupled with central bank use of these dollars to purchase U.S. securities. This knowledge is sufficient to determine the new equilibrium position. It makes no difference whether the UKCB performs both operations accumulating reserves in the form of U.S. securities and making no attempt to sterilize the U.K. money supply or both central banks perform the operations with the UKCB accumulating reserves in the form of outside assets and the USCB pursuing open market operations to sterilize the U.S. money supply.

C. Some Comments on Balance of Payments Adjustment Mechanisms

International economists have long been concerned with the question of whether or not there is an automatic mechanism under which balance of payments surpluses or deficits set in motion forces which lead to their elimination and with the companion question of whether or not policy making authorities can successfully frustrate such an automatic mechanism. An important contribution to the debate on these questions is Robert Mundell's model of what he called "the international disequilibrium system."[1/] Using a model of the short run IS-LM variety Mundell argued that if the monetary authorities allow deficits (surpluses) to contract (expand) the money supply, these deficits will set in motion contractionary (expansionary) forces which result in their elimination. However, if the monetary authorities sterilize the

1/ Mundell (1968, ch. 15).

effects of payments imbalances on the money supply, these imbalances can persist indefinitely. Thus a system in which monetary authorities set the value of the money supply in accordance with domestic objectives no matter what is happening to the balance of payments is a disequilibrium system from a balance of payments point of view.

What Mundell failed to account for adequately is the fact that in order to sterilize its money supply in the face of a deficit (surplus) a country must continually undertake open market purchases (sales) which reduce (increase) the quantity of domestic securities available for the public to hold. In Mundell's model this process could take place without any change in interest rates and therefore without setting in motion any forces leading to the removal of the deficit (surplus). For Mundell security supplies do not matter; only money supplies matter.[1/]

Although our short run portfolio balance model is not ideally suited for a complete analysis of the questions posed at the

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1/ In his discussion (Mundell 1968, Ch. 15) of the international disequilibrium system Mundell employs a flow specification of the capital account. He assumes that at any given level of interest rates there will be a constant and finite net capital flow per unit time. One implication of this assumption is that wealth holders must be prepared to experience increases or decreases in their security holdings at constant interest rates without altering their behavior. Wealth holders are in what might be called a "security trap" which is analogous in effect to the familiar liquidity trap. However, Mundell's money demand function is consistent with more usual portfolio balancing behavior. Wealth holders will accept more real balances at a given income only if the interest rate is lowered. It is this assumed inconsistency in wealth holders' behavior that allows Mundell to arrive at the misleading results described in the text. A similar type of inconsistency also affects some analysis of the financing of government budget deficits in the standard IS-LM framework.

beginning of this section it yields some insights. Our model describes a short run equilibrium in which there are no flows of financial capital. If we change one or more parameters in the model a new short run equilibrium is attained in which there are no flows of financial capital. In general net capital flows must occur to move from one equilibrium to another. This movement from one equilibrium on portfolio capital account to another occurs whatever the behavior of the monetary authorities in the two countries [1/] except in two extreme cases. The behavior of the monetary authorities does affect the size of the net capital flow which must occur between the two equilibrium positions. The large variety of cases considered in earlier chapters of this paper are proof enough that there are many alternative ways for equilibrium to be reestablished in our model, but here we limit our consideration to two important cases. Suppose some parameter change causes the MM and NN curve to intersect at a different interest rate pair than the

1/ In our model sterilization of the domestic money supply in both countries inhibits the adjustment process in two cases Consider s shift in the preferences of U.S. wealth holders in favor of U.S. money and away from U.S. securities. If the two types of securities are perfect substitutes so that changes in relative supplies of securities with the total supply of securities held constant do not affect interest rates, a new equilibrium cannot be reached when both countries sterilize their money supplies since the total supply of securities outstanding remains unchanged. If the MM and NN curves have the same slope, sterilization of money supplies in both countries blocks adjustment even if the two securities are not perfect substitutes since changes in interest rates brought about by the change in the relative supplies of the two kinds of securities cannot by themselves equilibrate both money markets.

one at which the BB and FF curves intersect. If on the one hand central banks accept passively whatever changes occur in their money supplies, that is, if they pursue what we have called a GSPR, the new equilibrium is given by the intersection of the BB and FF schedules and money supplies adjust so as to satisfy money demands at the interest rate pair given by this intersection. Adjustment is accomplished through what we call a pure money adjustment mechanism.[1/] If on the other hand, both central banks sterilize their money supplies, that is, if they pursue what we have called a MSCPR, the new equilibrium is given by the intersection of the MM and NN schedules and security supplies are changed through open market operations by the two central banks so that they satisfy security demands at the interest rate pair given by this intersection. Adjustment is accomplished through what we call a pure security adjustment mechanism. Combinations of these two pure adjustment mechanisms as well as what we call a mixed money-security adjustment mechanism are also possible. Whatever the exact mechanism, it is not in general possible for monetary authorities to inhibit the adjustment process.

We have shown rigorously that there is an automatic mechanism which shuts off any flow deficit or surplus on portfolio capital account. In order to obtain results which are strictly comparable to Mundell's we would have to imbed our two country portfolio balance model in a larger two country model in which outputs, price levels, trade flows, and perhaps other relevant variables are determined. Several other features of the model such as whether or not the labor supply is allowed to grow and whether or not technical progress is incorporated would also be

1/ For rigorous definitions of pure money adjustment, pure security adjustment, and mixed money-security adjustment and a discussion of some of the implications of relying on each of the different adjustment mechanisms or combinations of them see Part II of the Appendix.

important. In a portfolio balance model of an open economy in which the interest rate, output, and the capital stock are endogenous variables but in which there is no growth in the labor force and no technical progress McKinnon has found that a country cannot have a continuing balance of payments deficit (surplus) even when the monetary authorities sterilize the money supply unless the country is also experiencing a budget deficit (surplus) of the same amount. 1/ A crucial feature of McKinnon's model is that it refers to a period long enough that one of the requirements for equilibrium is that consumption expenditure equal disposable income; that is, wealth holders have their desired wealth levels. Certainly there is need for further research into the nature of the international adjustment mechanism, but it is clear that it is important to account for the portfolio balance considerations which have formed the basis of our approach.

D. The Relationship Between Changes in the Demand for and Supply of Money and Net International Movements of Financial Capital

As we have seen one cause of increases (decreases) in the money supply of an open economy is balance of payments surpluses (deficits). Some theorists have used this insight to develop a new approach to balance of payments analysis which focuses upon changes in the demand for and supply of money. This approach has been called the monetarist approach. This approach is founded on the assumption that the money market is always in equilibrium. In terms of our model the condition for equilibrium in, for example, the U.S. money market is,

1/ See McKinnon (1969)

$$M^S \equiv B^C + R + S = m(r,r',\mu)W$$

where increases in μ represent shifts in the tastes of U.S. wealth holders toward money and away from other assets. If the U.S. money market is to return to equilibrium after a change in tastes or some policy action we must have, 1/

$$dR + dB^C - m_r Wdr + m_{r'} Wdr' + m_\mu Wd\mu = 0.$$

This equation simply states that changes in supply must be matched by changes in demand. A monetarist would find it more instructive to rewrite this equation as,

$$dR = -dB^C + m_r Wdr + m_{r'} Wdr' + m_\mu Wd\mu,$$

and would interpret it as stating that the increase in the supply of money from foreign sources associated with an increase in the stock of reserves must match the excess of the increases in money demanded over the increase in the supply of money from domestic sources. In our model dR represents net capital inflow, that is, the excess of securities purchased from U.S. residents by U.K. residents over those sold by U.S. residents to U.K residents regardless of the country in which the securities were originally issued. Another way of saying the same thing is that dR represents the net change in holdings of all securities taken together by U.S. residents required to reestablish money market equilibrium.

In order for the monetarists' rewriting of the money market equilibrium condition to be transformed into a theory of the balance

1/ We assume that S is held constant.

of payments or, more accurately in the context of our model, a theory of reserve changes the monetarist must explain how dr and dr' are determined.[1/] dB^c is a policy variable under the control of the central bank, and $d\mu$ depends upon wealth holders' preferences. Now if the country under consideration is small, dr and dr' are given exogenously. Let us assume for simplicity that they are both equal to zero. Suppose in addition, we assume, as monetarists often do, that there are no shifts in the demand for money so that $d\mu$ is equal to zero. In this case reserve changes are equal to the negative of open market purchases by the central bank and we have a complete theory of the portfolio capital account for a small country.

However, as we have seen once we abandon the small country assumption dr and dr' can no longer be taken as exogenous. dR, dr, and dr' are jointly determined and their values depend upon the type

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1/ We find it useful to describe what we believe to be the essence of the monetarist approach in terms of our model which refers only to the stock shift component of financial capital flows and ignores the continuing flow component of financial capital flows as well as trade flows and some direct investment flows. We have also restricted ourselves to consideration of changes that occur between one equilibrium and another.

The monetarist approach [Johnson (1962)] is usually presented in terms of the growth rates of the variables considered. To obtain the equilibrium relationship between these growth rates the condition for equilibrium in the market for real balances is differentiated with respect to time; then various other algebraic manipulations are performed. The demand for real balances is assumed to depend upon real income as well as upon interest rates. In these more complete monetarist models the time rate of change of the stock of reserves represents something like the official settlements concept of the balance of payments.

of disturbance or policy action undertaken and upon the policy responses of the central banks. When attempting to analyze the determinants of reserve changes due to net flows of financial capital for a large country it seems to us that it is necessary to look beyond the equation of equilibrium changes for the money market to a more fully specified model of the type we have investigated in this paper.

Appendix

I. The Comparative Statics Model

The equilibrium conditions for the four markets studied in this essay when there is no commercial banking system in either country can be written as follows:

$$b(r,r',h_{11}a)W + \pi\, b'(r,r',h_{21}a)W' - (\bar{B}-B^{c}-B^{c'}) = 0 \qquad (A1)$$

$$f(r,r',h_{12}a)W + \pi\, f'(r,r',h_{22}a)W' - \pi(\bar{F}-F^{c'}) = 0 \qquad (A2)$$

$$m(r,r',h_{13}a)W - (B^{c} + \sigma R + S) = 0 \qquad (A3)$$

$$\pi n'(r,r',h_{23}a)W' - \left\{\pi[F^{c'} + \sigma'(\bar{R}-R) + S'] + B^{c'}\right\} = 0 \qquad (A4)$$

All of the symbols except the $h_{ij}a$'s are defined in the text. The $h_{ij}a$'s are shift parameters which cause changes in asset demands subject to the following restrictions:

$$b_3 = f_3 = m_3 \,, \qquad (A5)$$

$$b'_3 = f'_3 = n'_3 \,, \qquad (A6)$$

$$\sum_j h_{ij} = 0 \,, \quad i=1,2, \qquad (A7)$$

where b_3, f_3, etc., are the partial derivatives of the asset demand functions with respect to their third arguments. The h_{ij}'s are given values appropriate to the problem under consideration so that, for example, a shift in the preferences of U.S. citizens in favor of U.K. securities and against U.S. securities can be represented by setting h_{11} equal to one, h_{12} equal to minus one, and all other h_{ij}'s equal to zero and then letting a increase. Other preference shifts can be analyzed by setting the h_{ij}'s appropriately.

If it is assumed that the h_{ij}'s are held constant at whatever values are appropriate for the problem under consideration;

that $\bar{B}$,$\bar{F}$, and $\bar{R}$ are fixed; that $B^{c'}$ is held constant at zero[1/]; and that W and W' are fixed unless π changes, we can rewrite the system in the following form,

$$B(\underline{x}) = 0, \tag{A8}$$

$$F(\underline{x}) = 0, \tag{A9}$$

$$M(\underline{x}\ ,\ \sigma\ ,\ S) = 0, \tag{A10}$$

$$N(\underline{x},\ \sigma',\ S') = 0, \tag{A11}$$

where $\underline{x}$ is defined as follows:

$$\underline{x} \equiv (r,r',R,B^{c},F^{c'},\pi,a). \tag{A12}$$

As we have argued on page 16 of the text only three of the four market equilibrium conditions are independent, so we have,

$$B_{x_i} + F_{x_i} + M_{x_i} + N_{x_i} \equiv 0\ , \tag{A13}$$

where x_i is any element of the vector $\underline{x}$. It is important to note that when π changes, W and W' change so, for example,

$$B_{\pi} = b'W' + bF^{A} - \frac{1}{\pi f}\ B^{A'}\ , \tag{A14}$$

where F^{A} represents the holdings of U.K. securities by U.S. citizens in the equilibrium position which prevailed before the exchange rate change and $B^{A'}$ represents holdings of U.S. securities by U.K. citizens in the same equilibrium position.[2/] It should also be observed that

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1/ The exposition which follows could be amended to allow for a non-zero $B^{c'}$ and for changes in $B^{c'}$, but the benefits from added generality do not exceed the costs of increased complexity given the objectives of this Appendix.

2/ If this result does not seem clear, refer to the definitions of W and W' in the text.

the effect of a change in a on the excess demand for a given asset depends upon the values of the relevant h_{ij}'s so that, for example,

$$B_a = b_3 h_{11} W + b'_3 h_{21} W' \ . \tag{A15}$$

In order to analyze the comparative statics properties of the model we find the total differentials of equation (A8) through (A11). We make use of two definitions and impose two restrictions when obtaining the total differentials. The changes in B^c and $F^{c'}$ are defined as follows:

$$dB^c \equiv - \lambda \, dR + \tilde{dB}^c, \tag{A16}$$

$$dF^{c'} \equiv \lambda' dR + \tilde{dF}^{c'}. \tag{A17}$$

$-\lambda \, dR$, where $0 \leq \lambda \leq 1$, represents the open market sales undertaken by the USCB in order to sterilize the impact of reserve inflows on the U.S. money stock while $\tilde{dB}^c$ represents open market purchases unrelated to sterilization activities. Similarly, $\lambda' dR$, where $0 \leq \lambda' \leq 1$, represents open market purchases undertaken by the UKCB in order to sterilize the impact of reserve outflows on the U.K. money stock, and $\tilde{dF}^{c'}$ represents open purchases unrelated to sterilization activities. We call λ and λ' the coefficients of sterilization. Changes in S and S', the balancing items on the central bank balance sheet, are restricted to be equal in size but opposite in sign to changes in the home currency value of outside reserve assets in the relevant country due to a change in π ,

$$dS = -R d\sigma \ , \tag{A18}$$

$$dS' = -(\bar{R}-R) d\sigma' \ , \tag{A19}$$

where the relationship among $d\pi$, $d\sigma$, and $d\sigma'$ is given by,

$$d\pi = \frac{1}{\sigma'}\left[d\sigma - \frac{\sigma}{\sigma'}\, d\sigma'\right]. \qquad \text{(A20)}$$

Given the definitions and restrictions of the last paragraph we can write the total differentials of (A8) through (A11) in matrix form as shown in Table I. This is a system of four interdependent equations in seven unknowns. Omitting any one of the four equations yields a system of three independent equations in seven unknowns. All of the comparative statics results in Chapters II and IV plus some additional ones not discussed in the text can be derived by selecting the appropriate set of three variables to regard as endogenous and treating the remaining variables as exogenous and by choosing appropriate values for λ , λ' , and the h_{ij}'s which appear in the partial derivatives of the excess demand functions with respect to a.[1/]

1/ See footnote 1 on page of Appendix. If we allowed for a non-zero $B^{c'}$ and for changes in $B^{c'}$ we could derive all the results in Chapter III plus some additional ones.

Table I

$$
\begin{bmatrix}
B_r & B_{r'} & -\lambda & 1 & 0 & B_\pi & B_a \\
F_r & F_{r'} & \lambda' & 0 & 1 & F_\pi & F_a \\
M_r & M_{r'} & -(1-\lambda) & -1 & 0 & M_\pi & M_a \\
N_r & N_{r'} & (1-\lambda') & 0 & -1 & N_\pi & N_a
\end{bmatrix}
\begin{bmatrix}
dr \\ dr' \\ dR \\ d\tilde{B}^c \\ d\tilde{F}^{c'} \\ d\pi \\ da
\end{bmatrix}
= \underline{0}
$$

II. The Effect of Central Bank Sterilization Behavior on the Changes in International Reserve Holdings Required for Adjustment.

In the text we argued that in our model wealth holders can successfully adjust their portfolios following a disturbance to portfolio equilibrium whether or not central banks act to sterilize all or part of the money supply changes associated with changes in international reserves except in some extreme limiting cases. In this section we demonstrate that the larger the <u>proportion</u> of changes in the money supply due to changes in international reserves which are sterilized by <u>either</u> of the two central banks the larger the <u>absolute size</u> of the change in international reserve holdings which must occur when wealth holders adjust their portfolios following any initial disturbance. We then define what we call pure money adjustment and what we call pure security adjustment in terms of the sterilization behavior of the two central banks and show that a change in sterilization behavior which increases reliance on pure money adjustment without increasing reliance on pure security adjustment increases the efficiency of the adjustment mechanism in the sense that smaller absolute changes in international reserve holdings are required for the reattainment of equilibrium no matter what the initial disturbance.

Dropping the fourth equation from the system shown in Table I and taking dr, dr', and dR as the endogenous variables leads to the following system:

$$\begin{bmatrix} B_r & B_{r'} & -\lambda \\ F_r & F_{r'} & \lambda' \\ M_r & M_{r'} & -(1-\lambda) \end{bmatrix} \begin{bmatrix} d_r \\ d_{r'} \\ dR \end{bmatrix} = \begin{bmatrix} \underline{B_y} & \underline{dy} \\ \underline{F_y} & \underline{dy} \\ \underline{M_y} & \underline{dy} \end{bmatrix} \tag{A21}$$

$\underline{dy}$ is a 4x1 column vector and $\underline{dy'}$ is given by

$$\underline{dy'} \equiv [\tilde{dB}^c \quad \tilde{dF}^{c\,'} \quad d\pi \quad da]. \tag{A22}$$

$\underline{B_y}$, $\underline{F_y}$, and $\underline{M_y}$ are 1x4 row vectors of the negatives of the partial derivatives of B, F, and M with respect to the variables the differentials of which are the elements of $\underline{dy}$ so, for example,

$$\underline{B_y} \equiv [-1 \quad 0 \quad -B_\pi \quad -B_a]. \tag{A23}$$

Let C be the matrix of the coefficients of dr, dr', and dR.

Then we have,

$$\det C = -(1-\lambda)(B_rF_{r'}-F_rB_{r'}) + \lambda'(B_{r'}M_r-B_rM_{r'}) + \lambda(F_{r'}M_r-F_rM_{r'}) . \tag{A24}$$

Now define:

$$\Delta_{BF} \equiv B_rF_{r'} - F_rB_r > 0 \tag{A25}$$

$$\Delta_{BM} \equiv B_{r'}M_r - B_rM_{r'} > 0 \tag{A26}$$

$$\Delta_{FM} \equiv F_{r'}M_r - F_rM_{r'} < 0 \tag{A27}$$

$$\Delta_{FN} \equiv F_{r'}N_r - F_rN_{r'} < 0 \tag{A28}$$

$$\Delta_{MN} \equiv M_rN_{r'} - M_{r'}N_r > 0 \tag{A29}$$

Our assumption of gross substitutes implies the signs shown above for Δ_{BF}, Δ_{BM}, Δ_{FM}, and Δ_{FN}. Our assumption that the MM schedule is less negatively sloped than the NN schedule implies $\Delta_{MN} > 0$.

Given these definitions we can rewrite det C as,

$$\det C = -(1-\lambda)\Delta_{BF} + \lambda'\Delta_{BM} + \lambda\,\Delta_{FM} \cdot \tag{A30}$$

It is helpful to note that since,[1/]

$$\Delta_{FM} \equiv -\Delta_{BF} - \Delta_{FN} , \tag{A31}$$

we can also write,

$$\det C = -\Delta_{BF} + \lambda'\Delta_{BM} - \lambda\,\Delta_{FN} \cdot \tag{A32}$$

We can establish that det C is negative for all $0 \leq \lambda, \lambda' \leq 1$. Δ_{BF} is positive, but Δ_{BM} and $-\Delta_{FN}$ are positive, so from (A32) we can see that det C is at its least negative (most positive) when $\lambda = \lambda' = 1$. However, from (A30) we can see that if $\lambda = \lambda' = 1$ then,

$$\det C = \Delta_{BM} + \Delta_{FM} , \tag{A33}$$

and we can establish that,[2/]

$$\Delta_{MN} \equiv -\Delta_{BM} - \Delta_{FM} , \tag{A34}$$

so det C is always negative for $0 \leq \lambda , \lambda' \leq 1$.

Solving for dR we find,

$$dR = (\det C)^{-1}(-\Delta_{FM}\,\underline{By} + \Delta_{BM}\,\underline{Fy} + \Delta_{BF}\,\underline{My})\underline{dy} \equiv (\det C)^{-1}D \tag{A35}$$

where D is a scalar which can be either positive or negative. By inspecting all the terms that are involved in calculating D we can determine that D does not depend upon λ or λ'. We know from (A32) that raising either λ or λ' makes det C a smaller negative number. Thus we have proved that the larger the <u>proportion</u> of money supply changes due to changes in international reserves which are sterilized

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1/ This identity can be established by using (A13).

2/ See footnote 1.

by either central bank the larger the absolute change in holdings of international reserves required for adjustment.

The economics behind this result can be more easily understood if we multiply and divide (A34) by Δ_{BF} to obtain,

$$dR = E^{-1}G \tag{A36}$$

where,

$$E = \left[-(1-\lambda) + \lambda' \frac{\Delta_{BM}}{\Delta_{BF}} + \lambda \frac{\Delta_{FM}}{\Delta_{BF}}\right], \tag{A37}$$

$$G = D(\Delta_{BF})^{-1}, \tag{A38}$$

$$\text{sign } [E] = \text{sign } [(\det C)^{-1}], \tag{A39}$$

$$\text{sign } [G] = \text{sign } [D], \tag{A40}$$

since Δ_{BF} is positive. When the expression for dR is written in this form, G can be viewed as the net excess demand for money created by the disturbance **dy** given that interest rates are adjusted to clear both securities markets and E can be viewed as the net reduction in the excess demand for money caused by a rise in R taking into account the fact that the interest rate changes required to reequilibrate the two securities markets following a rise in R have repercussion effects on the excess demand for money. This interpretation of E is easier to explain if we rewrite it as,

$$E = \left[-(1-\lambda) + M_r\left(\frac{B_r\lambda' + F_r\lambda}{\Delta_{BF}}\right) + M_{r'}\left(-\frac{B_r\lambda' + F_r\lambda}{\Delta_{BF}}\right)\right]. \tag{A41}$$

$-(1-\lambda)$ is the direct reduction in the excess demand for money resulting from an increase in R. $(\Delta_{BF})^{-1}(B_{r'}\lambda' + F_{r'}\lambda)$ is the change in r, and $-(\Delta_{BF})^{-1}(B_r\lambda' + F_r\lambda)$ is the change in r' which are

$$\text{sgn}\left[\frac{\partial E}{\partial \lambda}\right] = \text{sgn}\left[\frac{\partial \det C}{\partial \lambda}\right] > 0 \ , \tag{A42}$$

$$\text{sgn}\left[\frac{\partial E}{\partial \lambda'}\right] = \text{sgn}\left[\frac{\partial \det C}{\partial \lambda'}\right] > 0 \ . \tag{A43}$$

We can see from (A41) that an increase in λ lowers the amount by which a rise in R directly reduces the excess demand for money. The same increase in λ also raises (lowers) the amount by which a rise in R indirectly reduces (increases) the excess demand for money. From (A42) we know that the reduction in the direct effect must more than offset the increase (decrease) in the reinforcing (offsetting) indirect effect. An increase in λ' lowers (raises) the amount by which a rise in R indirectly reduces (increases) the excess demand for money.

It is instructive to investigate how central bank sterilization behavior affects the channels through which the portfolio adjustment following an initial disturbance proceeds. Of course, changes in both interest rates are usually necessary for adjustment, but here we focus attention on the required changes in money supplies and supplies of securities in the hands of the public. From the two central bank balance sheets we have,[1/]

1/ Here we view $d\tilde{B}^c$ and $d\tilde{F}^{c\prime}$ as initial disturbances so that while these variables affect the total changes in money supplies and supplies of securities in the hands of the public between equilibria they do not affect the changes in these supplies during the adjustment process <u>following an initial disturbance</u>.

required to reequilibrate the two securities markets given the changes in security supplies which result from sterilization operations in the two countries following an increase in R. Sterilization operations following an increase in R lead to an increase in the supply of U.S. securities and a reduction in the supply of U.K. securities available for the public to hold. The increase in the supply of U.S. securities tends to drive both interest rates up, but the decrease in supply of U.K. securities tends to drive both rates down. It is helpful to consider some limiting cases. If λ is zero and λ' is positive (λ' is zero and λ is positive), then a rise in R leads to rise (fall) in r and r'. If λ is equal to λ', our assumption of gross substitutes implies that a rise in R leads to a fall in r and a rise in r'. In the general case the interest rates may both move in either direction. Of course, increases in either r or r' reinforce the direct effect of an increase R in lowering the excess demand for money while decreases in either r or r' partially offset the direct effect. We know that the net impact of the direct and indirect effects of an increase in R is always a reduction in the excess demand for money since E is always negative.

We also know that an increase in either λ or λ' lowers the amount by which a given increase in R lowers the excess demand for money since,

$$dM^S = (1-\lambda)dR, \tag{A44}$$

$$dN^S = -(1-\lambda')dR, \tag{A45}$$

$$dB^S = -\lambda dR, \tag{A46}$$

$$dF^S = \lambda' dR, \tag{A47}$$

where, as before, M^S, N^S, B^S and F^S refer to the supplies of the respective assets in the hands of the public and dR is the change in USCB reserves required for adjustment. In general the adjustment process involves changes in all four asset supplies, but we can make some useful distinctions. Part of the adjustment in asset supplies may be accomplished by a change in one country's money supply matched by an equal but opposite change in the other country's money supply. We call this form of adjustment pure money adjustment. It involves a redenomination of the world's money supply.[1/] Another part of the adjustment may be accomplished by a change in the supply of one country's securities matched by an equal but opposite change in the supply of the other country's securities. We call this form of adjustment pure security adustment. It involves a redenomination of the world's supply of securities. When both central banks remain passive ($\lambda = \lambda' = 0$) the necessary adjustment in asset supplies is

1/ In the terminology of Roper (1972) there is a change in the "composition of the world monetary base" whenever there is a "creation of new liabilities by one central bank and the destruction of liabilities by the other central bank," and a change in the composition of the world monetary base "implies a change in the distribution" of the world monetary base if it is assumed "that residents of each country hold only cash balances which are denominated in the currency of their respective countries."

accomplished completely through pure money adjustment with no change in the supply of either type of security. Part of the fixed world money supply is redenominated. When both central banks sterilize all changes in their money supplies ($\lambda = \lambda' = 1$), adjustment is accomplished completely though pure security adjustment with no change in money supplies. Part of the fixed world supply of securities is redenominated. If $0 \leq \lambda, \lambda' \leq 1$ and $\lambda = \lambda'$, then we say that adjustment is accomplished through a combination of pure money adjustment and pure security adjustment. The proportion of the total adjustment, $|dR|$, accomplished by pure money adjustment is given by $(1-\lambda) = (1-\lambda')$. The world money supply and the world supply of securities remains constant but some part of both supplies is redenominated. We can conclude that in the cases we have considered so far the greater the reliance on pure money adjustment the more efficient the adjustment mechanism in the sense that smaller changes in reserve holdings are required for adjustment since greater reliance on pure money adjustment implies equal decreases in λ and λ'.

The case in which $0 \leq \lambda, \lambda' \leq 1$ but $\lambda \neq \lambda'$ is a little more difficult. In this case part of the adjustment is pure money adjustment, and part is pure security adjustment, but a part of the adjustment is accomplished by increases (decreases) in the world money supply matched by decreases (increases) in the world supply of bonds, a process which we call <u>mixed money-security adjustment</u>. The proportion of the total adjustment, $|dR|$, accomplished by pure money adjustment is $\{\min [(1-\lambda), (1-\lambda')]\}$; the proportion of adjustment accomplished by pure security

adjustment is $\{\min [\lambda,\lambda']\}$; the proportion of adjustment accomplished by mixed money-security adjustment is $\{\max [\lambda,\lambda'] - \min [\lambda,\lambda']\}$. We can now state the general conclusion that if reliance on pure money adjustment is increased with no increase in the reliance on pure security adjustment then the adjustment mechanism is made more efficient, since an increase in the reliance on pure money adjustment requires a decrease in the larger of λ and λ' while no increase in reliance on pure security adjustment implies that the smaller of λ and λ' does not increase.[1/]

1/ When an increase in reliance on pure money adjustment is accompanied by an increase in reliance on pure security adjustment the implications for the efficiency of the adjustment mechanism can only be obtained by analyzing the effects of changes of λ and λ' in opposite directions on $(\det C)^{-1}$ in (A35).

The problem encountered here is analogous to the problem of determining whether or not a consumer is better off when we observe him consuming a new consumption bundle. If he consumes more (fewer) of all goods, he is unambiguously better (worse) off, but if he consumes more of some goods and fewer of others, we must know his utility function before we can determine whether or not his lot has improved.

In the present case if both λ and λ' fall (rise), adjustment is more (less) efficient, but if they move in opposite directions, we must know the values of the parameters in $(\det C)^{-1}$ before we can determine whether adjustment is more or less efficient.

III. Some Implications of the Application of Stability Analysis

In this section we investigate the implications of applying stability analysis of the conventional kind to the important variant of our model in which r, r', and R are the endogenous variables. We have two major points to make. First, we show that the restrictions on the slopes of the BB, FF, MM, and NN schedules employed in the text are consistent with stability under a plausible set of assumptions about how the endogenous variables change in response to disequilibria. Second, we show that certain restrictions must be placed upon the relative speeds of adjustment in the different markets if stability is to be guaranteed under our assumptions and suggest why these restrictions are required.

Let us assume that the way in which the endogenous variables change in response to disequilibria can be described by the following set of equations:

$$\frac{\dot{r}}{r} = -\alpha_1 \frac{B(\underline{x})}{B-B^c}, \quad \alpha_1 > 0, \tag{A47}$$

$$\frac{\dot{r}'}{r'} = \alpha_2 \frac{F(\underline{x})}{\pi(\bar{F}-F^{c'})}, \quad \alpha_2 > 0, \tag{A48}$$

$$\frac{\dot{R}}{R} = \alpha_3 \frac{M(\underline{x},\sigma,S)}{B^c + \sigma R + S}, \quad \alpha_3 > 0. \tag{A49}$$

r rises in response to excess supply of U.S. securities; r' rises in response to excess supply of U.K. securities; R rises in response

to excess demand for money in the U.S.[1/] We assume that the change in each endogenous variable as a percent of its level is proportional to the excess supply or demand in the market to which it responds as a percent of supply in that market. Under this assumption α_1, α_2, and α_3 are dimensionless.

We can study the local stability properties of our dynamic system by investigating the behavior of the associated linear system in the neighborhood of an equilibrium. The results of linearizing the system about the equilibrium values (r_0, r_0', R_0) using the definitions and restrictions of Part I of the Appendix are displayed in Table II.[2/3/] dr, dr', and dR represent deviations of the respective variables from (r_0, r_0', R_0).

1/ It would be just as plausible to assume that R falls in response to excess demand for money in the U.K. so that,

$$\frac{\dot{R}}{\bar{R}-R} = -\alpha_4 \frac{N(\underline{x},\sigma',S')}{\pi[F^{c'} + \sigma'(\bar{R}-R) + S']}, \quad \alpha_4 > 0. \tag{A50}$$

(A49) and (A50) can be inconsistent since for some $\underline{x}$ there is excess demand in both money markets. A similar potential inconsistency arises in other, more familiar models if all markets are fully specified. It would take us too far afield to try to specify more carefully what would happen to R when there is excess demand in both money markets. Here we restrict ourselves to reporting in footnotes the similarities and minor differences in results implied by using (A50) instead of (A49).

2/ Note that $\dot{r}_0 = \dot{r}_0' = 0$

3/ We can rewrite (A16) and (A17) as,

$$B^c - B_0^c = -\lambda(R-R_0) + \tilde{B}^c - B_0^c, \tag{A51}$$

$$F^{c'} - F_0^{c'} = \lambda'(R-R_0) + \tilde{F}^{c'} - F_0^c. \tag{A52}$$

Table II

$$\begin{bmatrix} \dot{r} \\ \dot{r}' \\ \dot{R} \end{bmatrix} = \begin{bmatrix} -\beta_1 & 0 & 0 \\ 0 & -\beta_2 & 0 \\ 0 & 0 & \beta_3 \end{bmatrix} \begin{bmatrix} B_r & B_{r'} & -\lambda \\ F_r & F_{r'} & \lambda' \\ M_r & M_{r'} & -(1-\lambda) \end{bmatrix} \begin{bmatrix} dr \\ dr' \\ dR \end{bmatrix} + \begin{bmatrix} -\beta_1 \underline{B_y}\, \underline{dy} \\ -\beta_2 \underline{F_y}\, \underline{dy} \\ \beta_3 \underline{M_y}\, \underline{dy} \end{bmatrix}$$

$\underline{B}_y$, $\underline{F}_y$, $\underline{M}_y$, and $\underline{dy}$ are defined as in Part II of the Appendix.[1/]

β_1, β_2, and β_3 are given by,

$$\beta_1 = \alpha_1 \frac{r_0}{\bar{B}-B_0^c}, > 0,$$

$$\beta_2 = \alpha_2 \frac{r_0'}{\pi_0(\bar{F}-F_0^{c'})} > 0,$$

$$\beta_3 = \alpha_3 \frac{R_0}{B_0^c + \sigma_0 R_0 + S_0} > 0.$$

The stability analysis proceeds according to the usual practice.[2/] The characteristic polynominal of the system of Table II is given by,

$$(-1)\begin{vmatrix} -\beta_1 B_r - \delta & -\beta_1 B_{r'} & \beta_1\lambda \\ -\beta_2 F_r & -\beta_2 F_{r'} - \delta & -\beta_2\lambda' \\ \beta_3 M_r & \beta_3 M_{r'} & -\beta_3(1-\lambda) - \delta \end{vmatrix} = 0, \qquad (A52)$$

which can be written in the form,

$$\delta^3 + a_1\delta^2 + a_2\delta + a_3 = 0, \qquad (A53)$$

1/ That is, $d\tilde{B}^c = \tilde{B}^c - B_0^c$, $d\tilde{F}^c = \tilde{F}^{c'} - F_0^{c'}$, $d\pi = \pi - \pi_0$, and $da = a - a_0$.

2/ See, for example, Samuelson (1965), pp. 429-439.

where,[1/2/]

$$a_1 = \beta_1 B_r + \beta_2 F_{r'} + \beta_3(1-\lambda), \tag{A57}$$

$$a_2 = \beta_1\beta_2\,\Delta_{BF} + \beta_1\beta_3\,[(1-\lambda)B_r - \lambda M_r] + \beta_2\beta_3\,[(1-\lambda)F_{r'} + \lambda' M_{r'}], \tag{A58}$$

$$a_3 = \beta_1\beta_2\beta_3\,[(1-\lambda)\Delta_{BF} - \lambda\Delta_{FM} - \lambda'\Delta_{BM}] = -\beta_1\beta_2\beta_3 \det C. \tag{A59}$$

If our comparative statics results are to be meaningful, the system which describes the responses of the endogenous variables

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1/ Δ_{BF}, Δ_{FM}, Δ_{BM} and det C are defined as in Part II of the Appendix.

2/ If we had used (A50) instead of (A49), we would have,

$$a'_1 = \beta_1 B_r + \beta_2 F_{r'} + \beta_4(1-\lambda'), \tag{A54}$$

$$a'_2 = \beta_1\beta_2\,\Delta_{BF} + \beta_1\beta_4\,[B_r(1-\lambda') + \lambda N_r] + \beta_2\beta_4\,[F_{r'}(1-\lambda') - \lambda' N_{r'}], \tag{A55}$$

$$a'_3 = \beta_1\beta_2\beta_4\,[(1-\lambda')\,\Delta_{BF} + \lambda\Delta_{FN} + \lambda'\Delta_{BN}] = \beta_1\beta_2\beta_4 \det K. \tag{A56}$$

where those symbols defined in Part II of the Appendix have the same meaning here and,

$$\beta_4 = \alpha_4\,\pi_0[F_0' + \sigma_0'(\bar{R}-R_0) + S_0'],$$

$$\Delta_{BN} = B_{r'}N_r - B_r N_{r'} > 0,$$

and det K is the determinant of the matrix K, the matrix of the coefficients dr, dr', and dR when the system made up of (A47), (A48), and (A50) is linearized.

to excess supplies and demands must be stable. Necessary and sufficient conditions for the stability of the system of Table II are given by,[1/]

$$a_1 > 0, \tag{A60}$$

$$a_3 > 0, \tag{A61}$$

$$a_1 a_2 - a_3 > 0. \tag{A62}$$

To assure that the system is stable for all plausible sterilization behavior by the two central banks, the conditions above must hold for all $0 \leq \lambda, \lambda' \leq 1$.[2/] Given our assumptions about interest rate responses and the restrictions on λ and λ' just stated, a_1 is always positive.[3/4/] From our analysis of det C in Part II of the Appendix we know that a_3 is positive for all $0 \leq \lambda, \lambda' \leq 1$ if and only if Δ_{BF} and Δ_{MN} are positive.[5/] Thus, if the system of Table II is to

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1/ See, for example, Samuelson, op. cit.

2/ As we shall see in Part IV of the Appendix it is also useful to consider cases in which $\lambda, \lambda' < 0$.

3/ a_1 is also positive if $\lambda < 0$.

4/ a_1' in footnote 2 page 155 is always positive under these same assumptions and restrictions.

5/ Δ_{MN} is defined as in Part II of the Appendix.

be stable for all $0 \leq \lambda, \lambda' \leq 1$, the relative slopes of the BB and FF and of the MM and NN curves must be as we have described them in the text. 1/ 2/

So far it has not been necessary to consider the relative magnitudes of the adjustment coefficients α_1, α_2, and α_3. However, if $a_1a_2 - a_3$ is to be positive for all $0 \leq \lambda, \lambda' \leq 1$, we must place restrictions on these coefficients. The easiest way to see this is to note that (A60), (A61), and (A62) together imply $a_2 > 0$, 3/ but from (A55) we can see that a_2 need not be positive for all values of β_1, β_2, β_3, that is, for all values of α_1, α_2, and α_3, if λ and λ' are allowed to take on all values between zero and one. To see this rewrite a_2 as,

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1/ a_3 is positive for all $\lambda, \lambda' < 0$ if and only if $\Delta_{BF} > 0$.

2/ The same conditions are necessary and sufficient to insure that a_3' in footnote 2 on page 155 is positive. a_3' is at its most negative when $\lambda = \lambda' = 1$ as can be seen from the fact that a_3' can be written as

$$a_3' = \beta_1\beta_2\beta_4 \ (\Delta_{BF} + \lambda\Delta_{FN} - \lambda'\Delta_{BM}),$$

since,

$$\Delta_{BM} = \Delta_{BF} - \Delta_{BN}.$$

Furthermore we have that,

$$a_3' = \begin{cases} \beta_1\beta_2\beta_3 \ \Delta_{BF}, & \text{if } \lambda = \lambda' = 0, \\ \beta_1\beta_2\beta_3 \ \Delta_{MN}, & \text{if } \lambda = \lambda' = 1, \end{cases}$$

since,

$$\Delta_{MN} = \Delta_{FN} + \Delta_{BN}.$$

3/ It is <u>not</u> true that $a_1 > 0$, $a_2 > 0$, and $a_3 > 0$ imply $a_1a_2 - a_3 > 0$.

$$a_2 = \beta_2\beta_3 \left\{\frac{\beta_1}{\beta_3}\Delta_{BF} + \frac{\beta_1}{\beta_2}[(1-\lambda)B_r - \lambda M_r] + [(1-\lambda)F_r' + \lambda' M_r']\right\}, \tag{A60}$$

and let β_1 approach zero, that is, let α_1 approach zero, and let λ and λ' approach one. It is clear that for values of α_1 near enough to zero and values of λ and λ' near enough to one, a_2 is negative, so $a_1a_2 - a_3$ is negative, and the system is unstable. 1/

The exact conditions on α_1, α_2, and α_3 which insure that $a_1a_2 - a_3$ is positive for all $0 \leq \lambda, \lambda' \leq 1$ are not very instructive, but the destabilizing pressures the effects of which must be limited can be described less formally. 2/ Suppose that Figure 23 represents

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1/ An analogous rewriting of a_2' yields,

$$a_2' = \beta_1\beta_4 \left\{\frac{\beta_2}{\beta_4}\Delta_{BF} + [(1-\lambda')B_r + \lambda N_r] + \frac{\beta_2}{\beta_1}[(1-\lambda')F_r' - \lambda' N_r']\right\}. \tag{A61}$$

For values of α_2 near enough to zero and values of λ and λ' near enough to one, a_2' is negative, so $a_1'a_2' - a_3'$ is negative, and the system is unstable.

2/ In this footnote we sketch the derivation of a sufficient condition for $a_1a_2 - a_3$ to be positive for all $0 \leq \lambda, \lambda' \leq 1$. We know from the analysis of Part II of the Appendix that, given β_1, β_2, and β_3 and therefore, α_1, α_2, and α_3, we have that $\alpha_3 = -\beta_1\beta_2\beta_3 \det C$ is larger the larger are λ and λ' and that it reaches its maximum value when $\lambda = \lambda' = 1$. It can also be shown that a_1 and a_2 decline with increases in λ and λ' and reach their minimum values for $\lambda = \lambda' = 1$. Thus if $a_1a_2 - a_3$ is positive for $\lambda = \lambda' = 1$, it is positive for $0 \leq \lambda, \lambda' \leq 1$. Now when $\lambda = \lambda' = 1$ we have,

$$a_1a_2 - a_3 = G_0\beta_1^2 + G_1\beta_1 + G_2, \tag{A62}$$

where

$$G_0 = \beta_2 B_r \Delta_{BF} - \beta_3 B_r M_r > 0, \tag{A63}$$

$$G_1 = \beta_2^2 F_r' \Delta_{BF} + \beta_2\beta_3 (B_r' M_r - F_r M_r') \gtrless 0, \tag{A64}$$

$$G_2 = \beta_2^2\beta_3 F_r' M_r < 0. \tag{A65}$$

(footnote continued on following page)

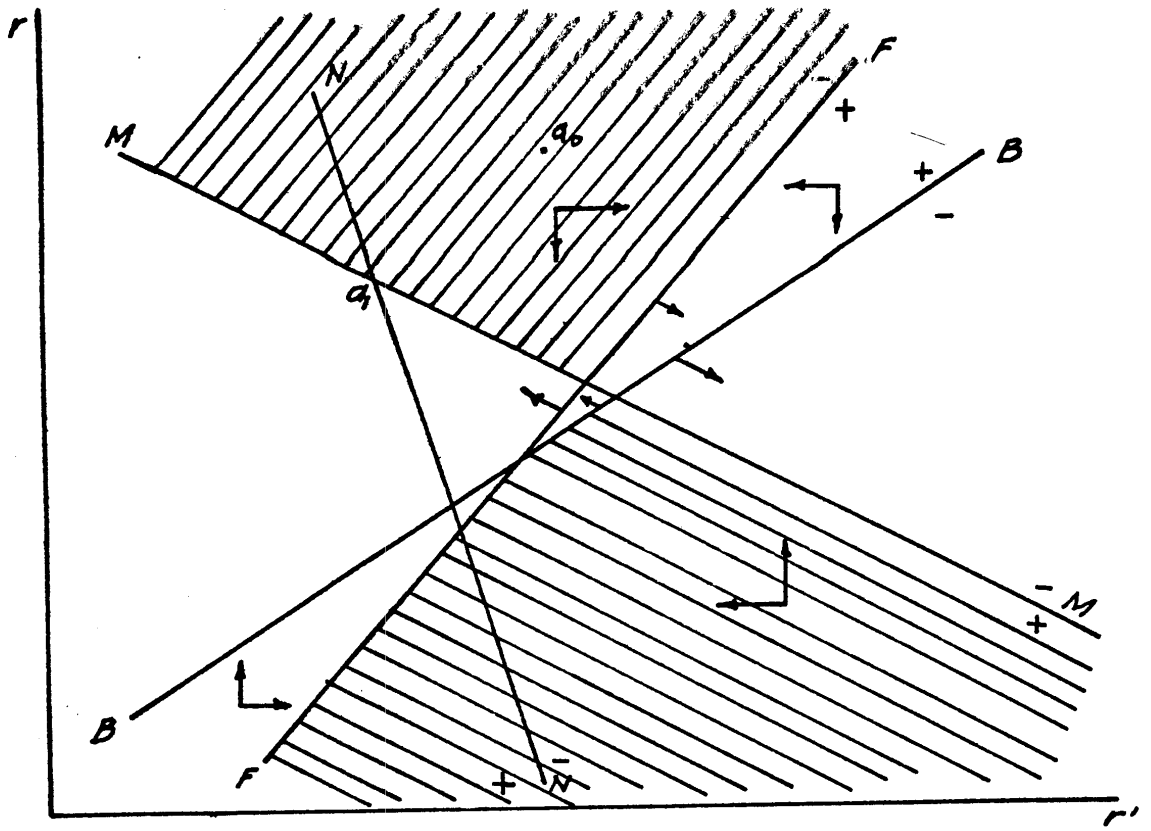

FIGURE 23

(footnote continued from previous page)

Given all the other parameters, we can solve for the positive value of β_1 which makes $a_1a_2 - a_3$ equal to zero,

$$\overset{*}{\beta}_1 = -\frac{1}{2}\frac{G_1}{G_0} + \sqrt{\frac{1}{4}\left(\frac{G_1}{G_0}\right)^2 - \frac{G_2}{G_0}} \quad . \tag{A66}$$

α_1 greater than $\left(\frac{\bar{B}-B_0^c}{r_0}\right)\overset{*}{\beta}_1$ is a necessary and sufficient condition for $a_1a_2 - a_3$ to be positive for $\lambda = \lambda' = 1$ and is a sufficient condition for $a_1a_2 - a_3$ to be positive for all $0 \leq \lambda, \lambda' \leq 1$. If the speed of adjustment in the market for U.S. securities is "large enough" relative to a function of the speeds of adjustment in the other two markets which increases in value with an increase in either of the other two speeds of adjustment, then $a_1a_2 - a_3$ is positive. Since we are assuming that all the asset markets adjust instantaneously, what we really require is that α_1, α_2, and α_3 increase without limit in such a way that α_1 is always greater than $\left(\frac{\bar{B} - B_0^c}{r_0}\right)\overset{*}{\beta}_1$.

A similar line of argument can be used to sketch a derivation of a sufficient condition for $a_1'a_2' - a_3'$ to be positive for all $0 \leq \lambda, \lambda' \leq 1$, but we do not pursue this line of argument here.

the configuration of the BB, FF, MM, and NN schedules after an initial disturbance. Let us assume that the equilibrium point before the shock was at a_0 and that $\lambda = \lambda' = 1$ so that the new equilibrium point is at a_1. Horizontal arrows pointing left or right represent the pressure on r' in each of the four regions marked off by the BB and FF schedules. Vertical arrows pointing up and down represent the pressure on r in each of the same four regions. At a_0, for example there is an excess supply of U.K. securities and an excess demand for U.S. securities, so there is upward pressure on r' and downward pressure on r. Diagonal arrows pointing southeast and northwest represent the effects of sterilization operations on the BB and FF schedules. For all interest rate pairs above (below) the MM schedule there is an excess supply of (demand for) U.S. money, and the U. S. loses (accumulates) reserves. These losses (accumulations) lead to a decrease (increase)in the supply of U.S. securities and an increase (decrease) in the supply of U.K. securities due to sterilization operations of the two central banks.[1/] At a_0, for example, there is an excess supply of U.S. money, and the sterilization of U.S. reserve losses implies that BB anf FF move to the southeast. As long as the prevailing interest rate pair remains in the shaded region above MM and FF, r' rises, r falls, and the U.S. continues to lose reserves. Unless some restrictions are placed on α_1, α_2, and α_3, the prevailing interest rate pair can become "trapped" in this region, and the process can

1/ If we replace (A49) by (A50), BB and FF move northwest if the prevailing interest rate pair lies above NN and southeast if the prevailing interest rate pair lies below NN.

continue without limit. Without attempting to be precise[1/] we can conclude that the prevailing interest rate pair will remain in the shaded region above MM and FF if the response of r to excess demand for U.S. securities, α_1, is small relative to the response of r' to excess supply for U K. securities, α_2, and the response of U.S. citizens to their excess supply of money, α_3, which determines the size of U.S. reserve losses and sterilization operations by the two central banks. Increases in r tend to equilibrate the market for U.S. securities and retard the southeastward movement of BB and FF, but increases in r' while tending to equilibrate the market for U.K. securities accentuate the southeastward movement of BB and FF. Decreases in R due to U.S. citizens attempts to get rid of their excess supply of money accentuate the southeastward movement of BB and FF. If α_1 is "too small"[2/] relative to α_2 and α_3 the shaded region above MM and FF expands rapidly enough so that it always contains the prevailing interest rate pair, and the system is unstable.

This same line of argument with minor modifications can be used to suggest how a value of α_1 which is "too small" can result in the prevailing interest rate pair becoming "trapped" in the shaded region below MM, BB, and FF. In this case r' falls, r rises, and the U.S. accumulates reserves without limit.

We have seen above that for $\lambda = \lambda' = 1$ and a value of α_1 which is "small enough" we can have $a_1a_2 - a_3$ negative in which

1/ See footnote 1 on page 158.

2/ See footnote 1 on page 158.

case the characteristic equation of the system must have one positive root or a pair of complex roots with a positive real part, and the system is unstable. In terms of Figure 23 the prevailing interest rate pair eventually becomes "trapped" in one of the two shaded regions.

When $0 \leq \lambda,\lambda' < 1$, similar destabilizing forces are at work. The discussion above could be modified to take account of this more complicated case. As above, the size of α_1 relative to the size of α_2 and α_3 would be important in determining whether or not instability could arise.

However, this essay is concerned with the comparative statics properties of the model. Since comparative statics analysis is only meaningful when the system attains a new equilibrium after an initial disturbance, we assume that α_1 is "large enough" relative to α_2 and α_3 to assure that the system is stable for $0 \leq \lambda,\lambda' \leq 1$.[1/]

IV. The Comparative Statics Model With Fractional Reserve Commercial Banking

The purpose of this part of the Appendix is to show that, under certain assumptions, conclusions based on the model of Part I of the Appendix, when properly interpreted, are identical to conclusions based on a model which takes explicit account of fractional reserve commercial banking. The equilibrium conditions

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1/ See footnote 1 on page 158 for a condition sufficient to insure stability.

for the four markets studied in this essay when fractional reserve banking is explicitly introduced in both countries can be written as follows:

$$b(r, r', h_{11}a)W + \pi b'(r,r', h_{21}a)W' - [\bar{B} - B^c - B^{c'} - \left(\frac{1-\alpha}{\alpha}\right)(B^c + \sigma R - H^{c'} + S)] = 0 \quad \text{(A67)}$$

$$f(r,r', h_{12}a)W + \pi f'(r,r',h_{22}a)W' - \pi\left[\bar{F} - F^{c'} - \left(\frac{1-\alpha'}{\alpha'}\right)\left\{F^{c'} + \sigma'(\bar{R} - R) + \frac{1}{\pi}(B^{c'} + M^{c'} + H^{c'}) + S'\right\}\right] = 0 \quad \text{(A68)}$$

$$m(r,r',h_{13}a)W - \left[\frac{1}{\alpha}(B^c + \sigma R - H^{c'} + S) - M^{c'}\right] = 0 \quad \text{(A69)}$$

$$\pi n'(r,r',h_{23}a)W' - \pi\left[\frac{1}{\alpha'}\left\{F^{c'} + \sigma'(\bar{R} - R) + \frac{1}{\pi}(B^{c'} + M^{c'} + H^{c'}) + S'\right\}\right] = 0 \quad \text{(A70)}$$

These equations are the same as those on page 82 of the text except for the $h_{ij}a$'s which are shift parameters which have the same interpretation as in Part I of the Appendix. If it is assumed that the h_{ij}'s are held constant at whatever values are appropriate for the problem under consideration; that $\bar{B}$, $\bar{F}$, $\bar{R}$, α, and α' are fixed; that $B^{c'}$, $H^{c'}$ and $M^{c'}$ are held constant at zero; and that W and W' are fixed unless π changes we can rewrite the system in the following form:

$$\hat{B}(\underline{x}) = 0, \quad \text{(A71)}$$

$$\hat{F}(\underline{x}) = 0, \quad \text{(A72)}$$

$$\hat{M}(\underline{x},\sigma,S) = 0, \quad \text{(A73)}$$

$$\hat{N}(\underline{x},\sigma',S') = 0. \quad \text{(A74)}$$

Although $\underline{x}$ is defined as above, we repeat the definition here for convenience,

$$\underline{x} \equiv (r, r', R, B^c, F^{c'}, \pi, a). \tag{A75}$$

As we have argued on page 82 of the text only three of the four market equilibrium conditions are independent, so we have,

$$\hat{B}_{x_i} + \hat{F}_{xi} + \hat{M}_{xi} + \hat{N}_{xi} \equiv 0, \tag{A76}$$

where x_i is any element of the vector $\underline{x}$. As in Part I of the Appendix, W and W' change when π changes so, for example,

$$\hat{B}_{\pi} = b'W' + bF^A - \frac{1}{\pi} fB^{A'}, \tag{A77}$$

where F^A and $B^{A'}$ are defined as above. The effect of a change in a on the excess demand for a given asset depends upon the value of the relevant h_{ij}'s.

We now obtain the total differentials of (A71) through (A74) making use of two definitions and imposing two restrictions. The changes in B^c and $F^{c'}$ are defined as follows:

$$dB^c \equiv -\hat{\lambda}\, dR + d\hat{B}^c, \tag{A78}$$

$$dF^{c'} \equiv \hat{\lambda}'\, dR + d\hat{F}^{c'}. \tag{A79}$$

$-\hat{\lambda}dR$, where $0 \leq \hat{\lambda} \leq 1$ represents the open market sales undertaken by the USCB to sterilize the impact of reserve inflows on U.S. high powered money while $d\hat{B}^c$ represents open market purchases unrelated to sterilization activities. Similarly, $\hat{\lambda}'dR$, where $0 \leq \hat{\lambda}' \leq 1$, represents open market purchases undertaken by the UKCB to sterilize the impact of reserve flows on the U.K. money stock, and $d\hat{F}^{c'}$ represents open market purchases unrelated to sterilization activities. $\hat{\lambda}$ and $\hat{\lambda}'$ are the coefficients of sterilization for high powered money.

As before changes in S and S', the balancing items on the central bank balance sheet, are restricted to be equal in size but opposite in sign to changes in the currency value of outside reserve assets in the relevant country due to a change in π,

$$dS = - Rd\sigma \tag{A80}$$

$$dS' = - (\bar{R} - R)\, d\sigma', \tag{A81}$$

where the relationship among $d\pi$, $d\sigma$, and $d\sigma'$ is given by,

$$d\pi = \frac{1}{\sigma'}\left[d\sigma - \frac{\sigma}{\sigma'}\, d\sigma'\right]. \tag{A82}$$

Given the definitions and restrictions of the last paragraph we can write the total differentials of (A71) through (A74) in matrix form as shown in Table III. The system of Table III is identical to the system of Table I except for the third column of the coefficient matrix and the two entries $\frac{1}{\alpha}\, d\hat{B}^c$ and $\frac{1}{\alpha'}\, d\hat{F}^c$ in the vector of differentials. Another way of saying the same thing is that the partial derivatives of $\hat{B}$, $\hat{F}$, $\hat{M}$, and $\hat{N}$ with respect to r, r', π, and a are equal to the partial derivatives of B, F, M, and N from Part I with respect to the same variables.

The term $[\hat{\lambda} - (1-\hat{\lambda})\, \frac{1-\alpha}{\alpha}]$ represents the change in the supply of U.S. securities available for the public to hold as a proportion of the increase in international reserves held by the USCB. The supply of securities available to the public rises by $\hat{\lambda} dR$ because of open market sales by the USCB undertaken in order to sterilize part of the effect of dR on <u>high powered money</u>. The supply of securities available to the public falls by $(1-\hat{\lambda})\, \frac{1-\alpha}{\alpha}\, dR$ because the stock of

Table III

$$\begin{bmatrix} B_r & B_{r'} & -[\hat{\lambda} - (1-\hat{\lambda})\frac{1-\alpha}{\alpha}] & 1 & 0 & B_\pi & B_a \\ F_r & F_{r'} & \hat{\lambda}' - (1-\hat{\lambda}')\frac{1-\alpha'}{\alpha'} & 0 & 1 & F_\pi & F_a \\ M_r & M_{r'} & -(1-\hat{\lambda})\frac{1}{\alpha} & -1 & 0 & M_\pi & M_a \\ N_r & N_{r'} & (1-\hat{\lambda}')\frac{1}{\alpha} & 0 & -1 & N_\pi & N_a \end{bmatrix} \begin{bmatrix} dr \\ dr' \\ dR \\ \frac{1}{\alpha}d\hat{B}^c \\ \frac{1}{\alpha'}d\hat{F}^{c'} \\ d\pi \\ da \end{bmatrix} = \underline{0}$$

high powered money expands by $(1-\hat{\lambda})dR$, and, as a result, the U.S. commercial banking system is able to increase its holdings of securities by $(1-\hat{\lambda})\frac{1-\alpha}{\alpha}dR$. The expansion of the U.S. commercial banking system works to offset the effect of sterilization operations on the supply of U.S. securities available to the public. The net effect of the open market sales and the expansion of the U.S. commercial banking system may be either a rise or fall in the supply of U.S. securities available to the public. The term $[\hat{\lambda}-(1-\hat{\lambda}')\frac{1-\alpha'}{\alpha'}]$ has an analogous interpretation. It should be clear that,

$$(1-\hat{\lambda})\frac{1}{\alpha}dR = 1 - [\hat{\lambda} - (1-\hat{\lambda})\frac{1-\alpha}{\alpha}]\ dR$$

is the increase in the U.S. money supply in the hands of the public due to an increase in international reserves held by the USCB and that $(1-\hat{\lambda}')\frac{1}{\alpha'}$, has an analogous interpretation.

Now if we let,

$$\lambda = [\hat{\lambda} - (1-\hat{\lambda})\frac{1-\alpha}{\alpha}],$$

$$\lambda' = [\hat{\lambda}' - (1-\hat{\lambda}')\frac{1-\alpha'}{\alpha'}],$$

where $-\infty < \lambda, \lambda' \leq 1$, the coefficient matrix of the current model is completely identical to the coefficient matrix of Part I except that the permissible values of λ and λ' have been expanded to include negative values.

In the current model an open market by the USCB not associated with sterilization operations of $d\hat{B}^c$ makes possible an

expansion of the money supply by $\frac{1}{\alpha} d\hat{B}^c$ and leads to a total reduction in the supply of securities available to the public by $\frac{1}{\alpha} d\hat{B}^c$. $\frac{1}{\alpha'} d\hat{F}^c$ has an analogous interpretation. If we let,

$$d\tilde{B}^c = \frac{1}{\alpha} d\hat{B}^c,$$

$$d\tilde{F}^{c'} = \frac{1}{\alpha'} d\hat{F}^c,$$

the vector of differentials for the current model is completely identical to the vector of differentials in the model of Part I.

All this means that under the assumptions implied by (A71) through (A74) which are stated fully in Chapter VI and under the additional restrictions outlined at the beginning of Part IV of the Appendix conclusions based on the model of Part I are not affected by the presence of fractional reserve commercial banking except for the relatively minor consideration that negative values of λ and λ' must be considered.

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