K.7 (#727 in RFD Series)

INTERNATIONAL FINANCE DISCUSSION PAPERS

PROFITABLE SPECULATION, PRICE STABILITY, AND WELFARE

by

Stephen W. Salant

Discussion Paper No. 54, November 7, 1974

Division of International Finance

Board of Governors of the Federal Reserve System

Profitable Speculation, Price Stability, and Welfare

by Stephen Salant

In 1953, Milton Friedman asserted that profitable speculation increases price stability: "People who argue that speculation can be destabilizing seldom realize that this is largely equivalent to saying that speculators lose money, since speculation can be destabilizing in general only if speculators sell when the currency is low in price and buy when it is high."

This remark, repeated in 1971, has generated research by Telser, Farrell, Kemp, Schimmler, and others. To analyze Friedman's assertion they had to give a precise interpretation to his words. By "speculation", they took him to mean inter-temporal arbitrage under certainty. By "stability of prices", they interpreted him to mean the sum of squared deviations from the average price. His assertion, as interpreted by them, is: with profitable, inter-temporal carry-overs under certainty, the sum of squared deviations from the average price is smaller than with no carry-overs.

Farrell, Kemp, and Schimmler analyzed this propositon, allowing the speculator to pursue any profitable strategy.1/ Telser, in contrast, required his speculator to choose the profit-maximizing, monopolistic strategy. Farrell, Kemp, and Schimmler showed that the Friedman proposition is false unless there are only two periods or linear demand curves with the same slope. Telser showed that optimal speculation does reduce price variability. However, since he used linear demand curves of the same slope, his result was guaranteed without invoking profit-maximization. The impression gained by some from Telser's article2/ is that by requiring that the monopolistic speculator behave optimally, the Friedman proposition can be saved. It cannot.

1/ The speculator is constrained to strategies which leave him with no inventory or debt at the completion of the game.

2/ See, for example, the first complete paragraph on p. 68 of Flexible Exchange Rates (revised edition), by Egon Sohmen.

The point of this paper is to show that the Friedman proposition, as interpreted by previous researchers, is both wrong and uninteresting. Profitable speculation may increase price variability relative to the situation with no carry-overs. Presumably, the proposition that speculation reduces price variability was advanced in defense of speculation. This seems, however, to be the wrong defense. Profitable speculation improves welfare no matter what its consequence for price stability. The case on behalf of speculation is strictly analogous to the argument that trade is in a nation's interest. Whether the price vector with trade is more variable than the price vector with no trade is irrelevant.

Price instability in the sense of Telser-Kemp-Farrell-Schimmler does not mean that the model (or the world it portrays) will explode. It does not even mean that consumers are exposed to greater uncertainty, since there is no randomness in the model. It concerns an unimportant characteristic of the prices at which commodities are purchased. As positive economics, the Friedman proposition is wrong. As normative economics, it addresses a question of no significance.

I have not seen any examination of the Friedman propositon under uncertainty. Constructing a simple, general-equilibrium, carry-over model with stochastic endowments is not difficult.[3/] From such a model, we can obtain the endogenous, stochastic process of prices which clears markets over time. This process can be compared to the stochastic process of prices that would emerge without speculation. In a T-period model, we would have one joint probability distribution for T variables which would arise without carry-overs and another joint distribution which would arise with optimal carry-overs. The problem is not in solving such a model but in knowing how to compare these two joint distributions. What does someone mean when he says that the prices generated by one joint distribution are more unstable than the prices generated by the other? I cannot resolve this question. Once this question is answered, the comparison of the two distributions can be made.

3/ See Robert Gustafson: "Carry-over Levels for Grains" (Dept. of Agriculture, 1958); Paul Samuelson: "Stochastic Speculative Price" (Vol. 3 of Collected Works, 1971); Stephen Salant: "The Telser Model Under Uncertainty" (unpublished); Robert Townsend: "Price Fixing Schemes and Optimal Buffer Stock Policies" (unpublished); and Michael Mussa's unpublished work on the subject.

In the first section of this paper, I place the conventional carry-over model in a general equilibrium context. This makes the analysis more familiar and leads easily to welfare comparisons. In the next two sections I show that the restrictions that speculation be profitable or optimal are inadequate to salvage the Friedman proposition. Section IV explores the welfare gains from trading with the speculator. I show that consumers gain even if the speculator increases price variability. In the fifth section, I consider extensions to a stochastic environment. By considering a two-period, carry-over model with uncertainty in only one period, I avoid the question of defining "price instability" in a multi-period model under uncertainty. I find that speculation expected to be profitable raises the welfare of the consumer but will (if the demand curve is concave) increase the variance of the random price.

I. The Micro-Underpinnings of the Usual Speculation Model

The analyses of Farrell, Telser, Schimmler and others begin by assuming that the speculator faces a set of downward-sloping excess demand curves for the commodity which the speculator can transport over time. The excess demand for the commodity in period i is assumed to depend only on the "price" in that period. In the absence of intervention by the speculator, the price which eliminates excess demand will emerge. A sale by the speculator would reduce the price, generating excess demand by consumers equal to his sale; conversely, a purchase would raise the price, generating excess supply equal to his purchase. I would like, briefly, to specify the conventional model for studying inter-temporal choice and then limit it so that it is consistent with the usual assumptions of the speculation literature.[4/]

In general, each consumer receives endowments of the two goods in each of T periods and is unable to carry either good from one period to the next. Calling the two goods "soybeans" (X) and the "background good" (B) we can picture each consumer as having his own preferences over all possible bundles of the 2T goods:

4/ The same modifications pertain to the exhaustible resource and random walk literature.

$U(X_1,B_1,X_2,B_2, \ldots\ldots, X_T,B_T)$. If an auctioneer called out 2T-1 relative prices, each consumer would choose the best bundle which he could acquire without spending more than the income from his endowments, valued at the auctioneer's prices. For each price vector each consumer would have demands for the 2T goods. These could be added across individuals to yield aggregate demand for each good. By deducting the aggregate endowments of each the goods, we would obtain 2T excess demand curves. Each would, in general, depend on 2T-1 relative prices. The auctioneer would choose the price vector so that the aggregate demand for each of the 2T goods equalled the aggregate endowment (so that excess demand is eliminated). An individual in this model can make exchanges over time. I can gain soybeans later by selling someone a (possible different) quantity of soybeans now. However, society as a whole cannot -- without the "speculator" -- increase aggregate soybean consumption later by reducing it earlier.

For any price vector specified by the auctioneer, there are 2T excess demands. In general, each excess demand curve will depend on 2T-1 relative prices. In the speculation literature, however, each excess demand curve depends on a single price. We can achieve this result only if we assume that each consumer ranks bundles by a utility function of the form $U(X_1, B_1, X_2, B_2, \ldots\ldots, X_T, B_T) = \sum_{i=1}^{T} f_i(X_i) + \alpha \sum_{i=1}^{T} B_i$.5/ Then the optimal feasible choice by the consumer satisfies the following conditions (provided the solution is interior):

$$f_i'(X_i) = \lambda q_i^X \qquad i = 1,T$$

$$\alpha = \lambda q_i^B \qquad i = 1,T$$

$\sum_{i=1}^{T} q_i^X(X_i - \overline{X}_i) + \sum_{i=1}^{T} q_i^B(B_i - \overline{B}_i) = 0$, where q_i^x is the price of commodity X in period i, X_i is the consumption of that good in that period, $\overline{X}_i$ is the endowment in that period and λ is an undetermined multiplier.

The second line indicates that, since the consumer is indifferent between consumption of the same quantity of the background good at any time, the price of the background

5/ Each consumer could, of course, rank bundles in the same way by using a monotonic transformation of this utility function. Different consumers would have different $f_i(\cdot)$ functions and different constants. In the exhaustible resource literature, an interest rate is introduced by assuming time-preference in the consumption of the background good.

good (q_i^B) is driven to equality in all periods: q^B. Hence, we can simplify the equation and can solve them for consumption of soybeans in each period (X_i) and the total consumption of the background good (ΣB_i):

$$\frac{f_i'(X_i)}{\alpha} = \frac{q_i^X}{q_B} \qquad i = 1,T$$

$$\sum_{i=1}^{T} \frac{q_i^X}{q_B}(X_i - \overline{X}_i) + \sum_{i=1}^{T}(B_i - \overline{B}_i) = 0$$

The demand for i^{th} period soybeans depends only on the i^{th} period price (relative to the background good): the desired result. We denote this i^{th} period relative price as P_i. There are two related consequences of the Marshallian assumption of a constant marginal utility good. First, all increases in income are spent on the background good; doubling the endowments will not, at fixed prices, alter the desired consumption of soybeans in any period. Second, the consumer's decision about consumption in the i^{th} period would the same no matter what relative price was called in the future; hence, uncertainty about subsequent relative prices will not affect the current, optimal decision of the consumer. Each of these two consequences will be utilized later.

By assuming consumer preferences of the above form, we create a general-equilibrium context for the speculation models where excess demand in one period is assumed to depend on the single (relative) price then prevailing.

Speculators, in this model, provide a service: they reallocate the aggregate endowments of soybeans in different periods. Following the literature, I assume away transport costs and interest charges. The speculator is assumed able to reallocate soybeans by buying in one period, storing the commodity, and selling it in another. Denoting a speculative sale in the i^{th} period by S_i (a negative value indicates a purchase), we assume the speculator is able to adopt any strategy ($S_1, S_2, \ldots\ldots, S_T$) which leaves him without debt or inventory at the end of the T^{th} period ($\sum_{i=1}^{T} S_i = 0$).

He neither injects nor withdraws soybeans from the system.[6/] The monopolist sets prices in such a way that he creates offsetting excess supplies and demands for soybeans in different periods. He buys the excess in periods of over-supply and sells it in periods of excess demand in exchange for the background good. If the resulting real value of the excess demands for soybeans is positive ($\Sigma P_i S_i > 0$), we know from Walras' Law that consumers will have supplied the speculator with a net amount of the background good of equal magnitude. The speculator takes this as profit from his carry-over service.[7/] We now consider how his carry-overs affect price variability.

II. Profitable Speculation and Price Variability

Denote the excess demand curves faced by the speculator as $E_i = g_i(P_i)$, where $g_i' < 0$. The i^{th} price is determined by the equilibrium conditions $S_i = E_i = g_i(P_i)$. The profits of the speculator are:

$$\Pi = \sum_{i=1}^{T} S_i P_i^s$$

and the variability of price is defined as:

$$V_s = \frac{1}{T} \sum_{i=1}^{T} (P_i^s - \sum_{j=1}^{T} \frac{P_j^s}{T})^2$$

The variance of prices (P_i^o) without speculation (V_{ns}) is defined analogously.

6/ In making these assumptions, I follow the literature I am amplifying. However, it should be noted that there are several problems with these assumptions. First, the speculator is assumed able to sell soybeans that he has not previously purchased and do not get from other participants in the model. This problem can be circumvented by assuming the speculator begins the game with a large inventory and is required to end with the same inventory or by adding stock-out constraints and utilizing Kuhn-Tucker conditions. The latter procedure introduces intractable non-linearities when the model is extended to a multi-period, stochastic framework. Since this paper consists largely of counter-examples constructed so that the stock-out constraint is not violated, the problem can be ignored.

7/ It is also a poor implicit assumption that the speculator has no desire for the soybeans he transports: his demand for soybeans is zero. We could avoid this by giving the speculator a utility function. Then two speculators faced with the same prices would behave in the same way if their behavior as producers were "separable" from their behavior as consumers. Under certainty, separability depends on the existence of a market term of trade for selling the transported good over time. Under certainty, however, separability depends on the existence of a futures market.

The proposition that $\pi > 0$ implies $V_s < V_{ns}$ is false, as the following example illustrates. Assume the excess demand curves are downward sloping lines of different slopes and intercepts. That is, $E_i = F_i - a_i P_i$. In constructing our example, we are free to choose the slopes, intercepts and any feasible strategy $\{S_i\}$. Can we find a case where profitable speculation increases dispersion?

The following table summarizes numerical data of the example:

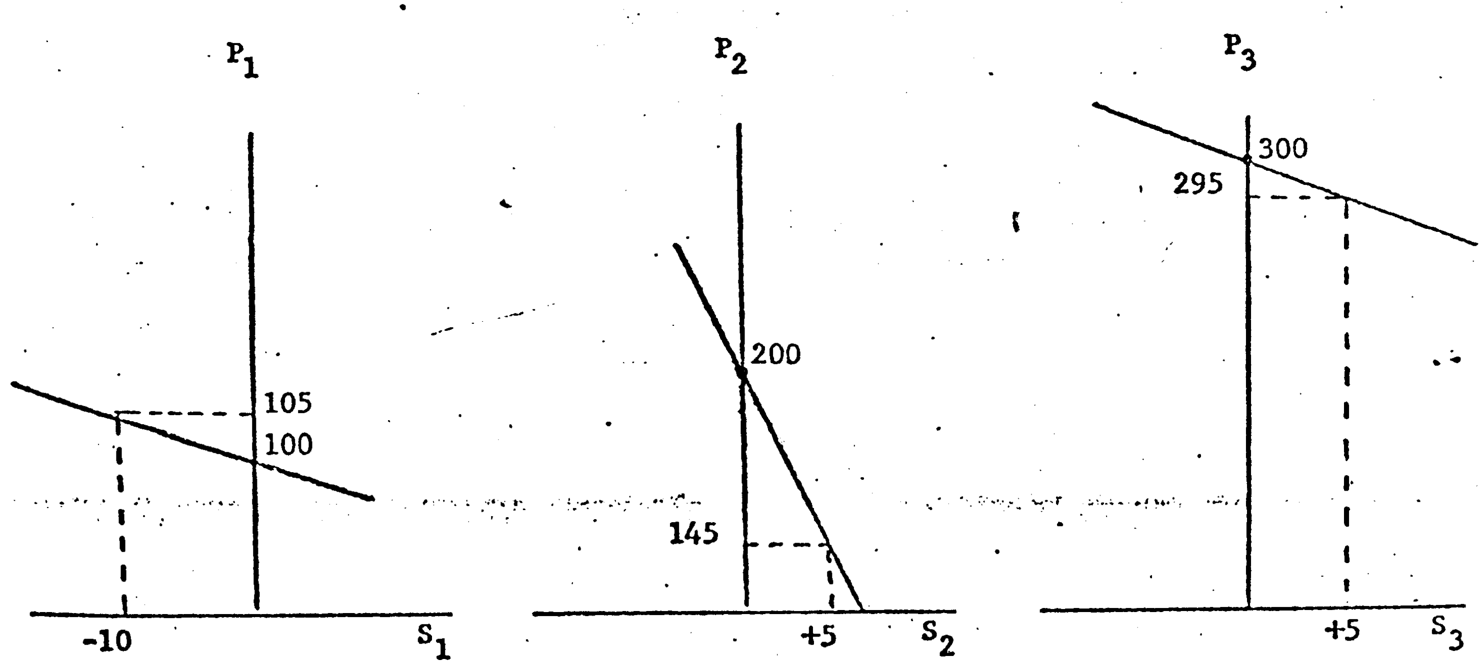
	Period 1	Period 2	Period 3
S_i	-10	+5	+5
P_i^s	\$105	\$145	\$295
P_i^o	\$100	\$200	\$300

$\pi = \$1150 > 0.$

$\Delta V = V_s - V_{ns} \cong 6688 - 6666 = 22 > 0.$

The speculator buys 10 units at the beginning, selling 5 at higher prices in each of the next two periods. Profitable speculation has increased the variance by 22.

Graphically, we may portray the situation as follows:



Period 1 Period 2 Period 3

All three demand curves slope down, the slopes being -1/2, -11, and -1, respectively. The vertical intercepts are respectively 100, 200, 300.

The general approach here is to make the outer demand curves flat and with very different vertical intercepts. This guarantees that speculation causes profits and little change in two prices. By making the middle curve steep, we can get the movement in the remaining price we need to increase the variance.

It should be obvious that Friedman's proposition is true for the uninteresting case of two periods. Then, for any downward sloping excess demand curves, profitability implies re[illegible]. To make profits, the speculator must buy low and sell high. The lower price rises and the higher one falls. The variance of two numbers falls as they move toward each other; this idea, reflected in the opening quote from Friedman, does not generalize to more than two periods.

III. Profit-Maximizing, Monopolistic Speculation and Price Variability

We restrict our ability to generate counter-examples to the Friedman proposition if we require the speculator to choose not merely a profitable strategy but the one which maximizes monopoly profits. Telser considered the case of a profit-maximizing monopolistic speculator facing linear excess demand curves of the same slope. He found that optimal speculation reduced price variability to one fourth the amount that would have occurred without carry-overs. In this section I simplify his analysis and extend it to the case of linear excess demand curves of different slopes. In this case, imposing optimality does rescue the Friedman proposition: speculation quarters price-variability as in the case examined by Telser. However, as I illustrate at the end of the section, even the imposition of optimality cannot, in general, salvage the proposition that speculation reduces price variability.

In the case of linear demand curves, imposing optimality rescues Friedman's result. The monopolistic speculator faces demand curves $E_i = F_i - a_i P_i$; if he sells S_i, he drives the price down to where $E_i = S_i$. The price will then be $\frac{F_i - S_i}{a_i}$. The speculator wishes to maximize $\sum_{i=1}^{T} S_i\{\frac{F_i - S_i}{a_i}\}$ by choosing $\{S_i\}$; he is constrained to set $\Sigma S_i = 0.$[8/] For the $\{S_i\}$ to be feasible, they must satisfy the constraint; for them to be optimal, the marginal revenue in each market must be equated. Calling λ the common marginal revenue, we obtain the necessary conditions

$$\frac{F_i - 2S_i}{a_i} = \lambda \qquad i = 1,T$$

$$\sum_{i=1}^{T} S_i = 0.$$

These T + 1 conditions determine the T + 1 unknowns: $\lambda, S_1, \ldots\ldots, S_T$. The second order conditions are satisfied since, in each period, marginal revenue declines with increased sales.

Summing the first T equations and substituting the last, we can solve for λ

$$\Sigma F_i - 2\Sigma S_i = \lambda\, \Sigma a_i.$$

$$\frac{\Sigma F_i}{\Sigma a_i} = \lambda.$$

The optimal sale in the i^{th} market is, therefore,

$$\frac{1}{2}\left\{F_i - \frac{a_i}{\Sigma a_j}\,\Sigma F_j\right\} = S_i.$$

8/ At first glance, the requirement that the speculator conclude his transactions in the same position as he began seems unsatisfying. As observers, we might have difficul evaluating the speculator's profits if he had unsold inventories; but that is our probl not his. However, the constraint may be viewed differently. Suppose the speculator maximized profits over a long horizon. As observers, we might choose to study his prof and price variability over any sub-interval where we found his purchases and sales cancelled out. If his entire strategy were optimal, his behavior in each sub-interval wou likewise be optimal. In this view, T would be endogenous. Since we are only interest in his behavior during periods where he neither accumulates nor decumulates soybeans, i is legitimate to introduce the constraint.

The price that will result from the sale S_i is

$$P_i = \frac{F_i - S_i}{a_i} = \frac{F_i}{a_i} - \frac{S_i}{a_i} = \frac{F_i}{2a_i} + \frac{1}{2\Sigma a_j}\,\Sigma F_j.$$

The average price in the T markets resulting from optimal speculation is:

$$\frac{\Sigma P_i}{T} = \left\{\frac{1}{T}\,\Sigma\left(\frac{F_i}{2a_i}\right) + \frac{\Sigma F_j}{2\Sigma a_j}\right\}.$$

The difference between the i^{th} price and this mean is:

$$P_i - \frac{\Sigma P_i}{T} = \frac{F_i}{2a_i} - \frac{1}{2T}\,\Sigma\left(\frac{F_j}{a_j}\right).$$

Hence, the variance of prices with profit maximizing speculation is:

$$V_s = \frac{1}{T}\,\Sigma\left\{\frac{F_i}{2a_i} - \frac{1}{2T}\,\Sigma\left(\frac{F_j}{a_j}\right)\right\}^2.$$

This variance is one quarter as large as the variability that would occur without the speculator. Then the i^{th} price would be:

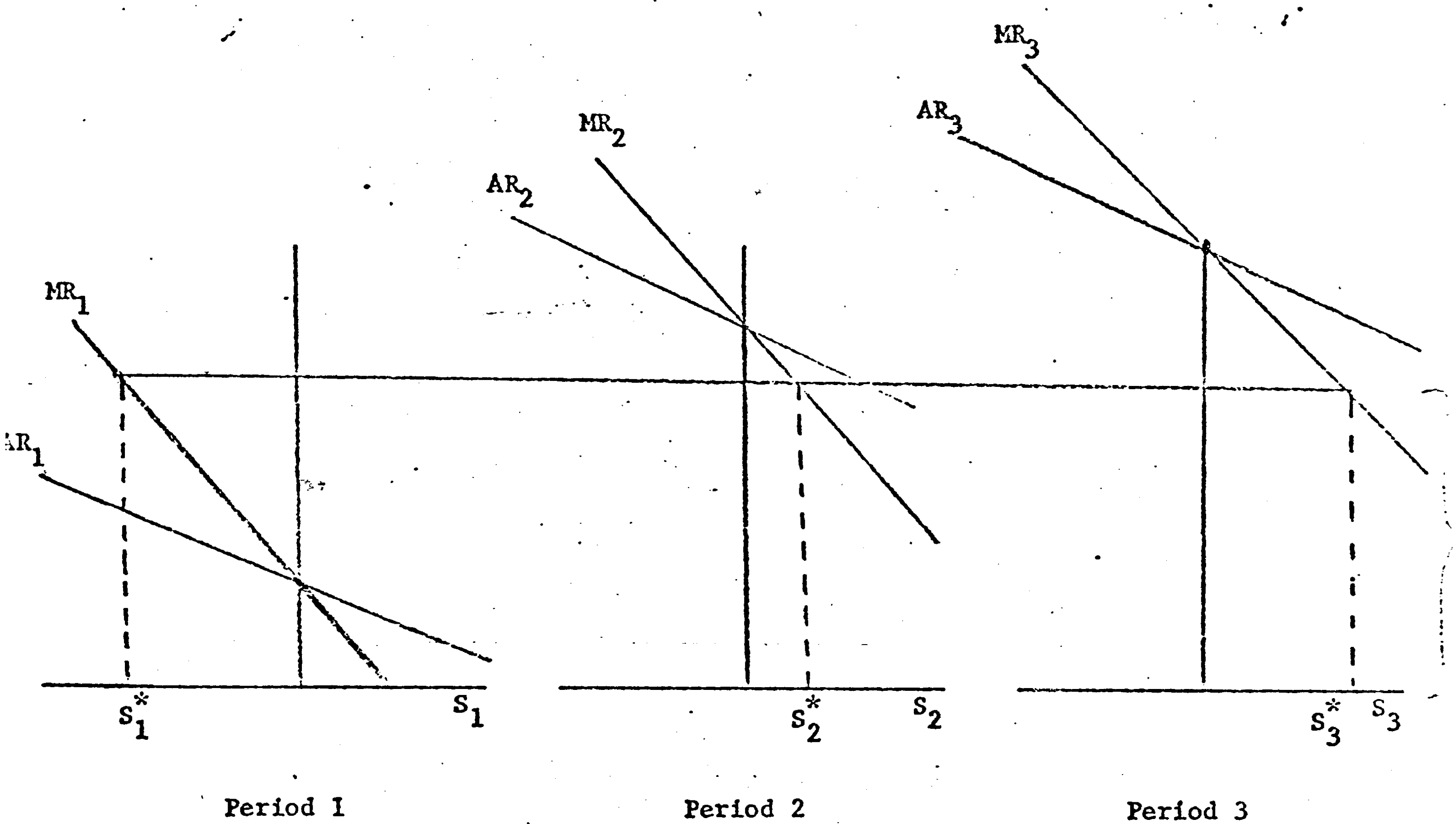
$$P_i^o = F_i/a_i.$$

The average price would be $\frac{\Sigma P_i^o}{T} = \frac{1}{T}\,\Sigma\left(\frac{F_i}{a_i}\right)$. The variability of prices without the speculator would be:

$$V_{ns} = \frac{1}{T}\sum_{i=1}^{T}\left[\frac{F_i}{a_i} - \frac{\Sigma(F_j/a_j)}{T}\right]^2 = 4V_s.$$

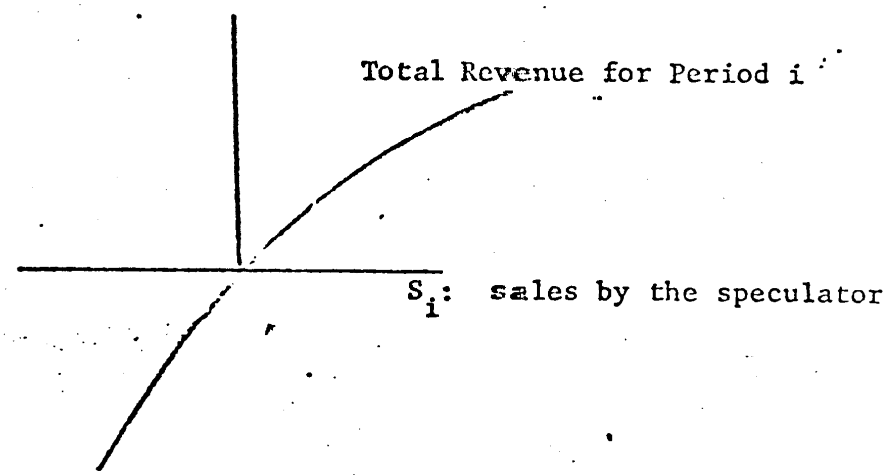
The foregoing model reduces to Telser's if all demand curves have the same slope $(a_i = a)$.

To consider optimal speculation graphically, we must sketch in the marginal curves:



We can consider $\{S_i\}$ equating the marginal revenues by drawing a horizontal line with height equal to some common marginal revenue (λ) and finding the S_i in each market which will produce it. We then must check that our solution is feasible ($\Sigma S_i = 0$). If not, the height of our horizontal line must be adjusted. Once a feasible solution equating the marginal revenues is found, the prices determined by the optimal strategy may be read from the excess demand curves. The prices which would occur without speculation are the vertical intercepts of the excess demand curves.

For each period, the total revenue function associated with the average and marginal curves above will have the following shape:



Sales generate revenue; purchases require expenditure (negative revenue). Hence, the revenue function for each period will be positive for positive S_i values and negative for negative values. If each period's revenue function is strictly concave, their sum will be strictly concave. It follows that there will be a unique, profit-maximizing strategy for the monopolist. This strategy can be identified as the only feasible one equating marginal revenues across time.

Because each revenue function slopes upward, passes through the origin, and is concave, the average revenue exceeds marginal revenue for positive S_i while the reverse is true for negative S_i.

With this in mind, we can illustrate a case where profit-maximizing, monopolistic speculation increases price variability. The Telser result does not generalize, once the assumption of linear excess demand curves is dropped.

Imagine we had a concave revenue function (R_1) for the first period with the following characteristics:

$$R_1'(0) = 100$$

$$R_1'(-10) = 140$$

$$R_1(-10) = -1050$$

These values could be generated by an upward sloping, concave function passing through the origin; we need not ~~specify the function analytically~~.

A different concave revenue function (R_2) could generate the following data for the second period:

$$R_2'(0) = 200$$

$$R_2'(5) = 140$$

$$R_2(5) = 725$$

The revenue function (R_3) associated with the third period might have the following characteristics:

$$R_3'(0) = 300$$

$$R_3'(5) = 140$$

$$R_3(5) = 1475$$

A profit-maximizing, monopolistic speculator would choose to purchase ten units in the first period and to sell five units in the second and third period. The marginal revenue in each period resulting from this strategy would then be equal (to 140) and the constraint would be satisfied. Since each revenue function is concave, the strategy would generate the highest profits ($R_1 + R_2 + R_3 = 1150$).

In the absence of speculation, the prices for the three periods would be 100, 200, and 300. With speculation the prices become 105, 145, and 295 $\left(P_i^s = \frac{R_i(S_i)}{S_i}\right)$.

We have seen (p. 7) that the second set of prices is more variable by (22). Hence, not even the imposition of optimality can salvage the proposition that speculation reduces price variability.

In general, without speculation, we have a set of different prices which we may arrange in order of increasing magnitude (to simplify the notation, assume the arrangement by size is the same as the ordering over time):

$$P_1^o < P_2^o < P_3^o \ldots < P_T^o.$$

The common marginal revenue (λ) which will occur in each market when the speculator optimizes must be below the highest price and above the lowest (otherwise he would buy or sell in all markets, violating the constraint). In all markets where the initial price (P_i^o) is smaller than λ, he buys (raising the marginal revenue in that market to λ); in all markets where the initial price exceeds λ, he sells (lowering the marginal revenue to λ).

Hence, we can insert λ in our series: $P_1^o < P_2^o < \ldots \lambda < \ldots P_T^o$. The speculator buys in markets to the left of λ and sells in markets to the right. All prices below λ rise toward it and all prices above it fall toward it. The dispersion <u>from λ</u> decreases. That is about all imposing optimality and assuming concave profit functions buys us. It cannot rescue the Friedman proposition about dispersion <u>from the average price</u>.

IV. "The Gains From Trade Once Again," Once Again

We have compared the price vector that would occur with and without carry-overs and have found that, in general, nothing can be said about which one will be more variable. But why should we care? In other certainty models we do not care about the variability of prices but about the allocation of commodities which accompanies these prices. We should then ask: Would the community be better off with the bundle of goods available to it with no speculator or with the bundle available to it with the speculator? Consider the community of consumers as the "home country" and the speculator as the trading partner. We can now re-phrase our question: Should the home country prefer trade to autarky?

Re-phrased in this manner, the question has already been answered. If the home country consists of one ~~resident, trade is never harmful~~. Trade gives the lone resident the opportunity to consume bundles not available to him under autarky without removing the option of consuming his endowment. If the lone resident chooses to trade with the speculator, he must prefer the bundle he acquires from the speculator to his own endowment since consuming the latter is also feasible.

If the home country contains more than one resident, we must consider distribution problems. Like trade with another country, dealing with the speculator can harm some while helping others. A resident with all his endowments in a period of high prices under autarky may well be injured by trade, since carry-overs would reduce the value of his endowment.

However, as Samuelson has shown for the case of international trade, all residents could be made better off by trade if, prior to trade, endowments were properly re-distributed. Because of our Marshallian assumption of a constant marginal-utility good, pre-trade re-distribution would not affect the excess demand curves faced by the speculator. Hence, his monopolistic pricing strategy would not change. No matter how endowments were re-distributed prior to trade, the community would acquire the same bundle of commodities from the speculator. If re-distribution occurred after trade instead of prior to it, the community would acquire that bundle of goods and each resident could then be given the same allocation as when re-distribution occurred prior to trade. Hence, for our

Marshallian case, the timing of the re-distribution needed to make everyone better off is unimportant.

Consider two residents, X and Y. Suppose we leave man Y at the utility level achieved under autarky. Subject to this, how well off can we make man X by selling society's aggregate endowments to the speculator (at the prices he sets) in exchange for a bundle to be consumed by our residents. If nothing is sold, we can achieve the autarkic level for man X. But, almost always, we can do better. The best we can do will be to equate the marginal rates of substitution to each other and to the "foreign rate of transformation" (the prices set by the speculator). This solution will be one where society's endowments are exchanged for a different bundle of equal value and the new bundle distributed between our two residents so as to make X <u>better</u> off and Y equally well-off compared to autarky. The market, combined with re-distribution, can reproduce this allocation when trade is opened even if the re-distribution occurs <u>after</u> trade with the speculator.[9/]

We have seen repeatedly that trade with the speculator may increase the variability of prices relative to autarky. However, in every case, the bundle acquired by the community through trade makes it better off regardless of the larger price variability. If there is a single resident in the community, he is made better off. If there are many residents, the bundle acquired can be re-distributed after trade to everyone's advantage. Hence, the comparison of price variability with and without carry-overs seems to me unimportant and misleading.

9/ If we drop the Marshallian assumption, each excess demand curve would depend on many relative prices instead of one, as conventionally assumed. In this case, we could still analyze a carry-over model. The community would gain from trade if re-distribution occurred prior to trade.

V. Carry-Overs Under Uncertainty

In this section, I consider speculation under uncertainty. The randomness results from stochastic endowments. Without carry-overs, we can derive a joint probability distribution of prices. With carry-overs, we can derive a different joint distribution. The price realizations which emerge in each setting depend on the endowment realization since the optimal reactions of all participants are endogenized. Once price variability is defined, we can easily compare the two situations. However, to my knowledge, no one has stated what he means by price variability in a multi-period, stochastic framework.

Suppose the random price vector has T components: $(P_1, \ldots\ldots, P_T)$. We might define variability to mean $E\left[\sum_i (P_i - \frac{\Sigma P_j}{T})^2\right]$, the expected value of variability in the sense used in the certainty literature. We might define it to be $\Sigma Var(P_i)$, the sum of the unconditional price variances.

In the certainty case, competitive speculation leads to an equalization of prices over time. Variability, as defined in the certainty case, vanishes (trivially) if speculation is perfectly competitive. By analogy, perhaps we should choose our definition under uncertainty so that perfectly competitive speculation eliminates variability entirely. With competitive speculation, prices follow a martingale (a generalized random walk): $E(P_{t+i} | \text{realizations through } t) = P_t$, $i = 1,2 \ldots$. The price expected to prevail at any time in the future--conditional on all realizations up through the current period--is equal to the current price. If we chose as our definition of variability $\sum_{i=1} \left[E(P_{t+i} | \text{realizations through } t) - P_t\right]^2$, competitive speculation would reduce variability to zero. For any definition, we could ask whether any kind of profitable speculation reduces variability. I doubt it would.[10] However, it would appear foolish for me to select some strange definition of variability and then use it to present a counter-example to the Friedman proposition.

10/ One reason for doubt is that, as the endowment randomness vanishes in the limit, the uncertainty case reduces to the certainty case and some of these definitions reduce to the concept of variability we used to question the Friedman proposition in the certainty case.

Instead, I will present a very simple model with only one random price. A natural definition of variability would be the variance of that price. Using this definition, we will see that carry-overs expected to be profitable improve welfare, but increase price variability.

To begin, consider the planning problem of a single resident with abundant, known current endowments of two goods $(\overline{X},\overline{B})$ and random future endowments $(\overline{X}^r,\overline{B}^r;\ \overline{X}^s,\overline{B}^s)$ which depend on the unknown weather (rain or shine). Our resident can carry some of his initial endowments, K_x, K_b, into the period of uncertainty to augment his smaller, random endowments then. Each selection of K_x, K_y provides him with a lottery with known current consumption and random future consumption:

$$(\overline{X} - K_x,\ \overline{B} - K_b,\ [\overline{X}^r + K_x,\ \overline{B}^r + K_b;\ \overline{X}^s + K_x,\ \overline{B}^s + K_b])$$

The consumer can rank these lotteries and choose the best. Assume his preferences are of the form:

$$U(K_x) = f(X-K_x) + \alpha(B-K_b) + \pi[g(X^r+K_x) + \alpha(B^r+K_b)] + (1-\pi)[g(X^s+K_x) + \alpha(B^s+K_s)],$$

where $f(\cdot)$ and $g(\cdot)$ are concave, increasing functions and π is the objective probability of rain.

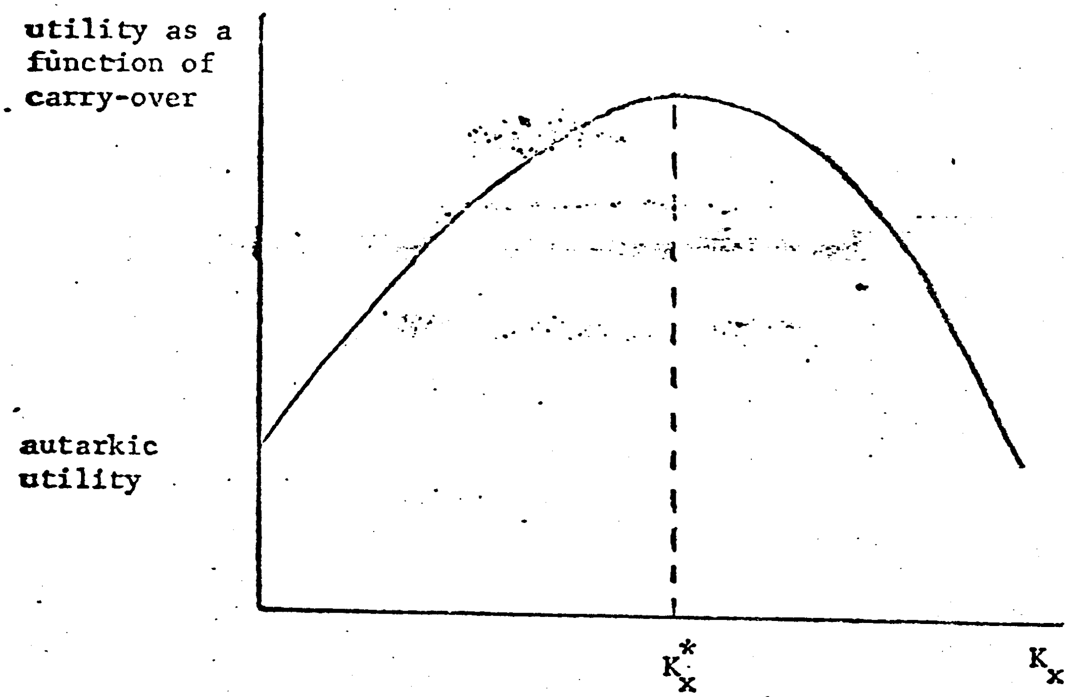
Optimally, the consumer should pick K_x and K_b to satisfy the following equations:

$$f'(\overline{X} - K_x) = \pi\ g'(\overline{X}^r + K_x) + (1-\pi)\ g'(\overline{X}^s + K_x)$$

$$\alpha = \pi\alpha + (1-\pi)\alpha$$

The first equation defines the optimal amount of soybeans to carry over. The second indicates that the consumer is indifferent about carry-overs of the background good.

The consumer chooses to carry enough soybeans to make his marginal utility from current consumption equal to his expected marginal utility from future consumption. Since the background good carry-over does not matter, we can portray utility as a function of the soybeans transported:



How would the market solve our planning problem if the speculator--not the consumer--had carry-over facilities. The consumer would have a current demand for soybeans (x) which would depend on its price relative to the background good:

$$\frac{f'(x)}{\alpha} = P_1 .$$

Next period, the consumer demand would depend on the price then:

$$\frac{g'(x)}{\alpha} = P_2 .$$

By acquiring K_x soybeans, transporting them to the next period and selling them, the speculator can generate the current price

$$P_1 = \frac{f'(\bar{x} - K_x)}{\alpha}$$

and the future prices which depend on the weather realization:

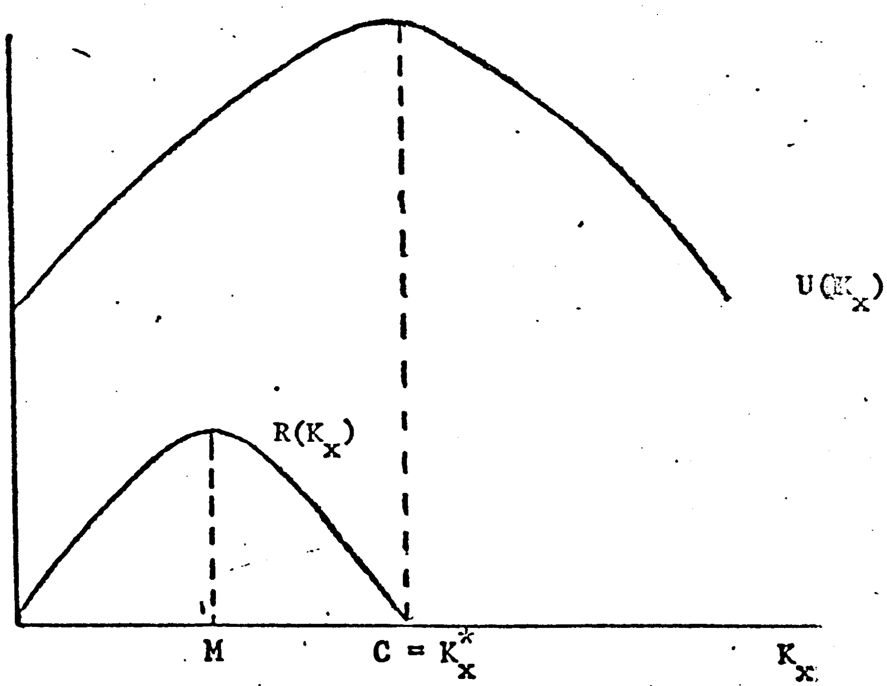
$$P_2^r = \frac{g'(\bar{x}^r + K_x)}{\alpha}, \text{ with probability } \pi$$

$$P_2^s = \frac{g'(\bar{x}^s + K_x)}{\alpha}, \text{ with probability } (1-\pi).$$

The expected profits of the speculator will be:

$$R(K_x) = [\pi P_2^r(K_x) + (1-\pi)P_2^s(K_x) - P_1(K_x)]K_x.$$

On the following graph, I show the expected profit of the speculator and the expected utility of the consumer for each carry-over decision.



A competitive sector of speculators would drive away profits in the scramble to get them. The current price would be driven up and the expected future price would be driven down to a point where they were equal. The competitive carry-over, denoted C, results in

$$P_1 = EP_2 = \Pi P_2^r + (1-\Pi)P_2^s$$

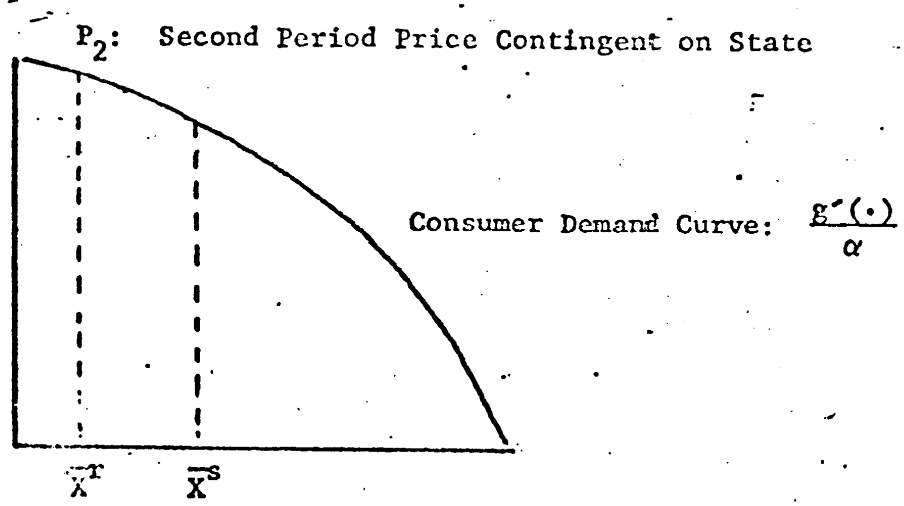
or

$$\frac{f'(\bar{x} - K_x)}{\alpha} = \frac{\Pi g'(\bar{x}^r + K_x)}{\alpha} + \frac{(1-\Pi)g'(\bar{x}^s + K_x)}{\alpha}$$

Hence, the competitive solution maximizes consumer welfare. A monopolistic speculator would equate current marginal revenue to the marginal revenue he expects next period. He would carry less to maintain a gap between his buying price and expected selling price.

In our example, any positive carry-over which makes profit improves welfare. We now must examine the effect on price variance.

Since the constant carry-over is added to the consumer's random endowment, the variance of consumption in the second period is not affected. The variance of the random price, however, will be affected, unless the demand curve happens to be linear. If $g'''(\cdot)$ is negative, the demand curve will be concave. The variance of the random price will then increase with larger carry-overs. The following graph shows a concave consumer demand curve for the second period:



with zero carry-over, the price that will emerge with the endowments in state r or state s may be read from the graph above $\overline{x}^r$ and $\overline{x}^s$. If the speculator carried over K_x units, we would read the prices above $\overline{x}^r + K_x$ and $\overline{x}^s + K_x$. Since the slope becomes increasingly steep, the difference between the two state dependent prices widens as the fixed gap between the state dependent endowments is moved to the right. Hence, any carry-over increases price variance; however, the consumer benefits from trading with the speculator whose profitable carry-over causes the prices to be more uncertain.

VI. Concluding Remarks About Speculation

In this paper, we have taken a further look at a question examined by other researchers. Using the carry-over definition of speculation, we have seen that profitable speculation does not reduce price variability, but that people gain from trading with the speculator anyway. The Friedman proposition, as interpreted by others, is wrong.

I do not know if he intended "price stability" to be defined as we have done. Nor do I know if the comparison between the regimes of autarky and profitable speculation is the one he intended. In 1971, he repeated the substance of his earlier assertion without modifying it in light of the considerable research which had been published on the subject:

> "It is worth noting that, in general, speculation can destabilize exchange rates only if speculators buy spot to hold when prices are high and sell spot out of inventories when prices are low. In that case, speculative transactions do make the swing in rates wider--but also speculators lose money. The belief that speculation is destabilizing is therefore largely equivalent to the belief that speculators on the whole lose money ... ."

This paper simplifies and extends the body of literature interpreting his proposition in a uniform way.

There are now many general-equilibrium models where all agents behave optimally over time and states. It is easy to distinguish optimal from sub-optimal behavior.

Defining which behavior is "speculative", however, is more difficult. Many different activities have been labelled speculation: playing the futures market, utilizing new information to advantage, carry-overs etc.

Why we need to define the concept at all is a legitimate question. Without a definition, we can still predict how each optimizer in our models will react to any exogenous change.

My major reason for studying speculation is to evaluate the basis for the widespread hatred of speculators among the general public. The definition of speculation we choose should, in my opinion, correspond to the behavior people abhor. We should then examine the social merits of this behavior within our models and, eventually, within the real world. Perhaps the public means something quite different from carry-overs by the term speculation. Or perhaps the public believes that, although speculators provide a service, they are monopolistic in the real world and should be made competitive.

Speculation, within the model we have considered, provides a service which is easy to overlook. The speculators appear to do nothing: they buy soybeans from the community and sell the same amount back--at a profit. People may resent buying back from the speculator in times of scarcity what they sold him cheaply on an earlier occasion. The carry-over service of the speculator might be ignored. If so, this analysis should be useful in clarifying the carry-over function of the speculator and the irrelevance of price stability.

Bibliography

Farrell, M.J., "Profitable Speculation," Economica, 33 (1966), 183-193.

Friedman, M., Essays in Positive Economics (Chicago: University of Chicago Press, 1953).

____________, "The Need for Futures Markets in Currencies," in "The Futures Market in Foreign Currencies," Chicago Mercantile Exchange (1971), p. 11.

Gustafson, Robert, "Carry-Over Levels for Grains," Technical Bulletin No. 1178, U.S. Dept. of Agriculture (1958).

Kemp, M.C., "Speculation, Profitability, and Price Stability," R.E. Stat., 45 (1963), 185-189.

Samuelson, P.A., "Stochastic Speculative Price," Proceedings of the National Academy of Sciences, Vol. 68, No. 2, 335-7, February, 1971.

____________, "Intertemporal Price Equilibrium: A Prologue to the Theory of Speculation," Waltwirtschafliches, Archiv, (Dec. 1957).

____________, "The Gains from International Trade Once Again," Economic Journal, December 1962.

Schimmler, J., "Speculation, Profitability, and Price-Stability - A Formal Approach," R.E. Stat., February 1973, 110-114.

Sohmen, Flexible Exchange Rates (Revised Edition), (Chicago: University of Chicago Press, 1969), p. 68.

Telser, L.G., "A Theory of Speculation Relating Profitability and Stability," R.E. Stat., 41 (1959), 295-301.

Townsend, R., "Price Fixing Schemes and Optimal Buffer Stock Policies," (unpublished).