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Simultaneous Determination of the U.S. Balance of Payments and Exchange Rates - An Exploratory Report

by

Richard Berner, Peter Clark, Howard Howe,
Sung Kwack and Guy Stevens

(Quantitative Studies Section)

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I. Overview and Summary

The Quantitative Studies Section has as its collective research goal the formulation and estimation of a quarterly model of the U.S. balance of payments in which certain features will play a leading part:

- exchange rates (or an effective exchange rate) are endogenously determined;
- exchange market intervention policy is explicit;
- links to the domestic economy are specified in both real and monetary terms; and
- monetary policy instruments and their impacts on the balance of payments and the exchange rates, direct and indirect, are explicit.

The model is to be used for both forecasting and policy simulations. The focus of these applications will be on the impact of domestic policies and of foreign economic developments on the components of the U.S. balance of payments and on U.S. domestic economic activity.

A basic problem in attaining this goal lies in taking explicit account of the simultaneous determination of the U.S. balance of payments and the exchange rates which affect that balance. Throughout the ensuing discussion we will often refer to the exchange-rate determination aspect of the problem; we always take it to be understood that the exchange rates are to be determined simultaneously with the other endogenous variables in the system.

Although we devote considerable attention to exchange rate determination in this sense, we do not wish to imply that exchange rates in themselves are of great ultimate significance. On the contrary, exchange rates are important not for their own sake but because of the effect they have on the U.S. balance of payments and therefore on the domestic economy. It is because exchange rate changes may have important consequences for U.S. income, prices, interest rates, etc., and because in a world of managed floating, trade and capital flows are simultaneously determined with exchange rates, that we single out exchange rates for special attention.

Exchange rates can be determined by first specifying equations for the supply and demand for the foreign currencies of interest and then setting the excess demand for each currency equal to zero. Here the demand and supply of foreign currencies are generated by the trade and the capital accounts. All three approaches to exchange rate determination in this paper--(1) an aggregate exchange rate, (2) reduced forms in exchange rates, and (3) a structural system--specify behavioral relationships for the trade and capital accounts.[1]

Modeling the world, however, is not our goal; we are interested in modeling the interaction of the U.S. with the rest of the world, with emphasis on the U.S. This national perspective led us to consider ways in which we could avoid estimating structural equations for the balance of payments of many foreign countries. The first simplification which

[1] Alternatively, following the monetarist approach, the exchange rates can be determined by setting the excess demands for monies equal to zero. The problems of integrating this macroeconomic approach with a disaggregated model of the U.S. balance of payments (which separates out the current and capital accounts) have not been solved; we will not pursue this approach at this stage.

we explored was the possibility of reducing the number of endogenous exchange rates that enter into the determination of the U.S. balance of payments. One such method of reduction is to aggregate all the bilateral rates into one weighted-average or "effective" exchange rate. At least two types of effective exchange rates are discussed at length in Part II of this paper.

The simplest approach is to postulate that the same effective exchange rate appears as a determinant in all equations that are affected by any exchange rates. It is shown in Part II and in the paper by Janet Yellen that this assumption cannot be justified theoretically and that it is not possible to determine a priori the degree to which such a procedure introduces error in simulating and forecasting.[2]

Alternatively, a single effective exchange rate can be determined by summing over all the equations in the balance-of-payments sector and using the equilibrium condition that the balance of payments equals zero. It is illuminating to study the properties of such a rate, because it clarifies our interest in predicting or explaining exchange rates. The equilibrium weighted-average exchange rate determined by this last procedure is one for the balance of payments as a whole; as such it cannot be used to predict the magnitude of individual components of the balance of payments, such as the trade balance or capital account. However, without determining the trade balance, we cannot determine the direct impact of the foreign sector on GNP; similarly, without describing capital

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[2]Janet Yellen, "The Theory of Effective Exchange-Rate Measurement," September 12, 1974.

flows we cannot determine the impact of the foreign sector on U.S. interest rates and monetary aggregates, thus losing the most important indirect effects of international factors on GNP, unemployment, and prices. Viewed in the light of these limitations, being able to predict this exchange rate is of little value, and therefore we do not intend to pursue it further.

In Part III of this paper we explore a second approach for simplifying the determination of exchange rates. This involves the use of reduced form equations. We examined this approach in some detail because if it were feasible, our modeling effort would be simplified because we could avoid having to estimate structural equations for the balance of payments of every other (i.e., non-U.S.) country. With this approach, all we would have to estimate in an n-country model would be (1) n-1 reduced form equations for n-1 independent exchange rates, and (2) structural balance of payments equations for one country only, namely, the United States. This procedure would in principle be simpler than estimating full balance of payments models for n-1 countries, and then from knowledge of the structural parameters and the values of the exogenous variables, solving for the n-1 exchange rates.

To explore the feasibility of deriving reduced-form equations for exchange rates, we set up a highly simplified three-country model with bilateral trade and capital flows. In this model, described in detail in Part III, exports depend on foreign countries' income and the prices of the output of each country. Interest rates are assumed fixed, and capital flows are a function of the difference between the current and the expected future spot exchange rates. Since domestic incomes and prices are assumed to be exogenous, the only endogenous variables are the two independent exchange rates.

Solving the system yields two equations in the two unknown exchange rates. However, because trade and capital flows in one currency have to be converted into another currency, and because cross rates have to be taken into account, these equations are highly non-linear. In fact, they turn out to be simultaneous cubic equations in the two exchange rates. This means that it is impossible to obtain a general closed-form expression for the reduced form equations for the exchange rates.

To remedy this difficulty we tried a number of simplifications. First, we considered two linear approximations: first differences and a first order Taylor series. These linearizations were not, however, very helpful because they implied that the reduced-form equations include literally hundreds of terms involving different combinations of the exogenous variables. Second, we modified the functional form of the trade equations, in one case by re-defining the coefficient of a price term to eliminate an exchange rate in the denominator, and in the other case by specifying that the trade account is a predetermined variable, i.e., is not a function of the current exchange rate. These modifications substantially reduced the non-linearities (especially in the second case), but they did not eliminate the problem. Therefore linear approximations would still be required to solve the two balance of payments equations in order to obtain the reduced form equations for the two exchange rates.

Our experience with this simple model has convinced us that the reduced form approach to the determination of exchange rates would involve a considerable approximation of the structure underlying the behavioral relationships. This means that the relationship between the exchange rates generated by these reduced forms might not be consistent with the trade and capital flows that are predicted by structural equations for the U.S. balance of payments. The approximation involved in the reduced form approach was therefore one reason why we rejected it.

Another reason for rejecting reduced forms has to do with our desire to explore the implications of alternative intervention strategies in the model. A change in intervention behavior represents a change in a structural parameter. Such a change could in principle alter a large number of the reduced form coefficients. This means that it is very difficult, if not impossible, to consider different rules for official intervention in foreign exchange markets when reduced forms are used to describe the behavior of exchange rates.

In view of the difficulties inherent in the effective exchange rate and reduced form approaches, we have found it necessary to adopt an explicit structural approach to exchange rate determination. In an n-country framework, this will involve estimating structural balance of payments models for n-1 countries. Knowing the structural parameters of these balance of payments equations and the values of the exogenous variables, the balance of payment equations can be solved for the n-1 independent exchange rates using methods described in Part IV of this paper. In contrast to the reduced form approach, the entire system is represented by n-1 balance of payments equations, rather than n-1 reduced form equations for exchange rates and structural balance of payments equations for one country: namely, the U.S.

Specifying the country models structurally has an advantage in terms of ease of estimation, since all restrictions are imposed directly and simple closed forms are used. Structural parameters are generally believed to be more stable over time than are those of reduced forms. Furthermore, their economic meaning makes it possible to evaluate the estimation results. Reduced form coefficients frequently do not have unambiguous meanings based on a priori knowledge. Hence, forecasting and policy simulations should be "easier" in a structural model, in the sense that changes in structure may change only a few parameters, whereas these shifts would change all reduced form coefficients.

By a structural model involving many countries, we mean a relatively large model of the U.S. and relatively small models of each of a few major countries. In Part IV we explain why we plan to start with four non-U.S. countries: Canada, Germany, Japan and the United Kingdom. However, we do not seek detailed models of these countries. We have no interest in the impact of world economic activity on the German economy, for example, except insofar as it has an influence on the U.S. The strategy is therefore to proceed in stages from rudimentary models to more complex, detailed specifications. Some of the basic steps that are part of our strategy are described in Part IV of this memo.

II. The Option of Using a Single Weighted-Average (Effective) Exchange Rate

The determination of all the independent bilateral exchange rates facing the United States requires, as discussed in Part I, either the use of reduced forms or the modeling of the balance-of-payments sector for every country whose exchange rate is included in the model. Since either approach would require a considerable expenditure of time and effort, the question immediately becomes one of how to simplify in order to make the problem manageable.

One of the most appealing paths, because of its simplicity, is to explore the possibility of using a single "effective" or weighted-average exchange rate in place of the multiplicity of bilateral rates. In particular, if it could be shown that effects of exchange-rate changes on _each_ flow entering the balance of payments could be represented by changes in the _same_ effective exchange rate, then the use of this effective exchange rate would obviate the need to model the balance of payments of countries other than the United States. With only one unknown exchange rate (the effective one), the condition that the U.S. balance of payments equals zero is sufficient to determine this weighted-average exchange rate.

A. Failings of Effective Exchange-Rate Approach

The first problem with this alluring alternative is that only under rather strong conditions can we, _in a given equation_, represent the effects of n-1 exchange rates by one composite rate. Second, even if the first problem is overcome, it is in general not the case that the same composite rate would be justified for use in _all_ the equations making up the balance-of-payments sector; such a failure is fatal.

The first problem will not be addressed in detail here. Rather, we shall assume that the U.S. balance-of-payments sector can be approximated by a system linear in exchange rates; in such a system the existence of an appropriate weighted-average exchange rate is assumed for any given equation.[3] As an example, consider an equation of the form:[4,5]

$$X = a_1Y + a_2r_{12} + a_3r_{13} + a_4r_{14} ,$$

where: X is some flow entering the balance of payments;

Y is an exogenous variable such as GNP;

r_{12} is the exchange rate between country 1 (the U.S.A.) and some country 2, and so on.

If we know or estimate a_2, a_3, and a_4, we can form the composite, weighted-average or effective rate, $R_1 = a_2r_{12} + a_3r_{13} + a_4r_{14}$, and express the equation above as $X = a_1Y + R_1$. For the above effective rate, R_1, the weights do not add up to unity; however, the weights can be scaled to achieve this without causing any problems. Thus, by assuming a linear system, we guarantee the existence of an appropriate effective exchange rate for any given equation.

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[3] Conditions necessary and sufficient for aggregating variables into a composite index are discussed in detail in H.A.J. Green, Aggregation in Economic Analysis, Part II.

[4] Of course, virtually all commonly used balance-of-payments equations are aggregative in nature. As such their validity depends on the correctness of the underlying aggregation procedure or estimation procedure.

[5] This equation is presented for illustrative purposes only; as is discussed in detail in Part III, one would expect most balance-of-payments equations to be nonlinear in exchange rates.

However, the second problem will still, in most cases, prove insuperable: the composite exchange rates defined for different equations will generally be different. If they are, we will not be able to solve for the equilibrium values of these composite rates without modeling the balance-of-payments sectors of some or all of the other countries in the world. Thus, for example, the composite exchange rate for a second equation will be, say, $R_2 = b_2 r_{12} + b_3 r_{13} + b_4 r_{14}$. It is easy to show that in general we cannot determine either R_1 and R_2 from the U.S. equilibrium condition. In common sense terms, we have two unknowns and only one equation. It turns out that we can determine both rates only in the case where the weights on the two are equal or proportional.

This conclusion, we might add, is completely consistent with the position taken by Janet Yellen in her study of effective exchange rates.[6] She shows there that the proper weights for an effective exchange rate differ, depending on which dependent variable we select for attention. Moreover, Yellen shows that the proper weights are proportional to the elasticity of the dependent variable with respect to the exchange rate--the a's and the b's in the above examples.

In summary, the theoretical case for solving the fundamental simplification problem by defining a single effective rate to be used in every balance-of-payments equation is a weak one. Even in a linear system this expedient works only if the effects of a particular exchange rate on one balance-of-payments flow are proportional to its effects on every other flow.[7]

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[6]"The Theory of Effective Exchange-Rate Measurement."

[7]Naturally, if all exchange rates move together, then it can be shown that any weights will work. But if that premise is true, there really is only one independent rate in the system, i.e., there was no problem in the first place.

B. Every Set of Balance-of-Payments (Linear) Equations Does Have a Unique Effective Exchange Rate. But What Is It Good For?

The equilibrium condition in a flexible rate system, that exchange rates be set so that the supply and demand for foreign exchange are equal for every currency, does give us a way to determine a unique weighted-average exchange rate for the dollar. Further, if all other foreign variables are exogenous in the model (e.g., foreign prices and GNP's), then this effective exchange rate can be determined without knowledge of the balance-of-payments sectors of any foreign country. The trick is that, unlike the approaches discussed above, all the equations in the U.S. balance-of-payments sector are used to determine this effective exchange rate.

Suppose for illustrative purposes that we have a simplified set of U.S. balance-of-payments equations for the dollar values of imports (V_M), exports (V_X) and net capital flows (V_C):

$$
\begin{aligned}
V_M &= a_1Z_1 + c_{12}r_{12} + c_{13}r_{13} + c_{14}r_{14} \\
V_X &= a_2Z_2 + c_{22}r_{12} + c_{23}r_{13} + c_{24}r_{14} \qquad (1) \\
V_C &= a_3Z_3 + c_{32}r_{12} + c_{33}r_{13} + c_{34}r_{14}
\end{aligned}
$$

where: r_{12}, r_{13}, r_{14} = the exchange rate between the U.S. (country 1) and countries 2, 3 and 4. The units are dollars per unit of foreign currency.

Z_i = the set of exogenous variables affecting the flow in question--i.e., in the case of V_M, U.S. income and prices, and foreign prices, etc.

The equilibrium condition for a flexible rate system is that the exchange rates adjust such that the balance of payments equals zero:

i.e., $V_M + V_X + V_C = 0$. This implies for our illustrative system (1) that:

(2) $$(c_{12}+c_{22}+c_{32})r_{12} + (c_{13}+c_{23}+c_{33})r_{13} + (c_{14}+c_{24}+c_{34})r_{14} = -a_1Z_1 - a_2Z_2 - a_3Z_3$$

The expression of the right-hand side of the equation is a function of known (exogenous Z's) variables and known (or estimated) coefficients; hence, it is known for every time period. On the left-hand side of the equation we have only a linear function of unknown bilateral exchange rates; however, since the right-hand side is known, we do know what this linear function must be equal to. This linear function is just an effective exchange rate, and its value is determined by the U.S. balance-of-payments equations alone. Further, if we have estimated the individual balance-of-payments equations in system (1), or the composite equation (2), we have estimates of the weights $(c_{1i}+c_{2i}+c_{3i})$.

The next question is: So what? The answer depends on what are the policy-maker's interests.

First, if we are primarily interested in a properly defined effective exchange rate, this is a good one to choose. We can track and forecast its changes as a function of our estimates of changes in the exogenous variables, the Z's (which of course include government policies such as intervention behavior). We can simulate how proposed changes in any government policy will change this effective exchange rate.

Consider, however, what the effective exchange will not do. Since it is defined for the balance of payments as a whole and not any individual equation, this exchange rate cannot in general be used to predict what any given account in the balance of payments will turn out to be. Thus we cannot predict separately what imports, exports, and the trade balance will be. This is a fundamental problem because it is crucial to know the trade balance in order to predict (or simulate) how the foreign sector affects national income and product. The trade balance, after all, is the primary direct effect of the foreign sector on GNP. Further, if we wish to model both the direct and indirect effects of the foreign sector on such basic goal variables as GNP, the unemployment rate, and price changes, we must predict such things as asset demands by foreigners (which affects interest rates) and the change in foreign asset holdings by the central bank (intervention behavior).

Looking at the balance-of-payments sector in this light has convinced us that it is not the exchange rate, or any exchange rate, that is of fundamental interest. Our ultimate goals are to measure the impact of international factors, broadly defined, on U.S. GNP, unemployment and the price level. To do this latter, an effective exchange rate is useful only insofar as it allows us to predict the changes in key lines of the balance of payments such as the trade balance. As we saw above, the effective exchange rates for different equations are usually different; and once we need more than a single effective exchange rate we cannot avoid an explicit treatment of individual bilateral exchange rates.

C. A Caveat on Approximations

There may be reasons why a single weighted-average exchange rate could work--for example, if all bilateral rates tend to move together.

Moreover, other reasons might lead to a more sophisticated rationale for a model based on a single effective rate. There are theoretical and empirical reasons to expect that the trade balance is fairly insensitive to short-run changes in exchange rates. If so, the trade balance might be a function only of lagged exchange rates, and the capital account alone could be used to define an effective exchange rate. A shortcoming of this approach is that we could not predict or simulate beyond one quarter from the present; in order to estimate the trade balance more than one quarter in the future, we would need either predictions of the individual bilateral rates or the effective rate applicable to the trade balance. For the reasons detailed in the last section, neither of these would be available.

III. The Reduced Form Approach to Exchange Rate Determination In A Multi-Country Framework

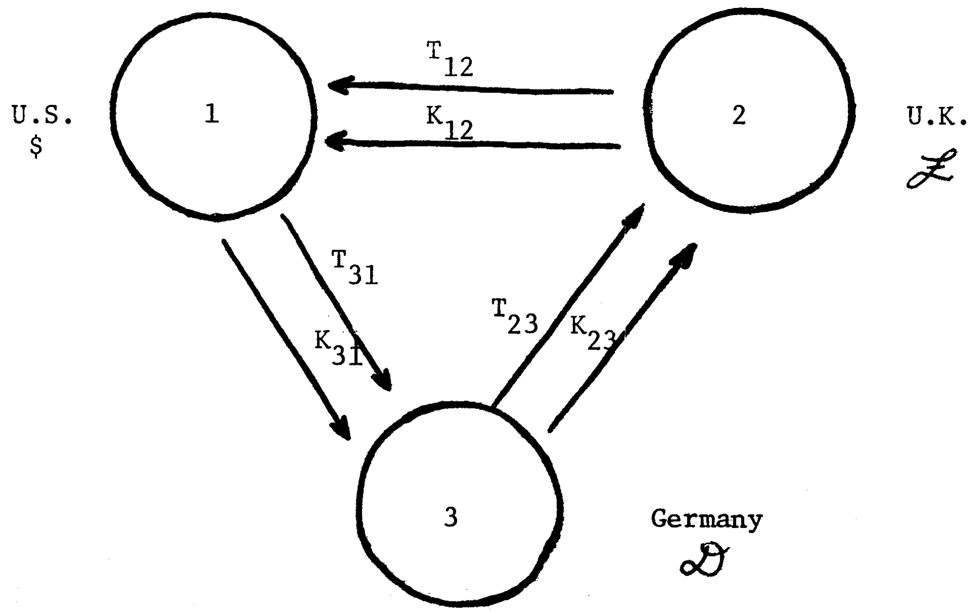
In Part II we explored one possible simplification in exchange rate determination, namely, the use of a single weighted-average exchange rate. It was shown that since we have to separate out the current account and the capital account, this approach is not feasible; for our modeling purposes it is in fact necessary to determine the individual, i.e., bilateral, exchange rates.

In this part of the paper we examine a reduced form approach to the determination of individual exchange rates. If this approach were feasible, it would simplify our modeling efforts for the balance of payments of every other (i.e., non-U.S.) country. It turns out that for our purposes it is not practical to estimate individual exchange rates as reduced form expressions of all the exogenous variables in the model. Therefore in the following section, Part IV, we describe a structural approach for incorporating flexible exchange rates in a U.S. balance-of-payments model.

We begin this section by setting up a highly simplified three-country system of bilateral trade and capital flows where prices and incomes are treated as exogenous. (If this reduced form approach were to prove feasible, then the model could be expanded to endogenize prices, incomes, etc. Structural equations to determine these variables would be substituted into the balance-of-payments equations described above and thus become part of the reduced forms for the exchange rates.) Our objective is to derive reduced form expressions for the two independent exchange rates from explicit structural equations for trade and capital flows. Such an explicit derivation from structural equations is necessary because only by this means can we insure that the reduced-form equations will yield exchange rates that are consistent with the structural model of the U.S. balance of payments that we will be constructing.

Using this approach entails that the U.S. balance of payments would be solved for recursively. We would first use these reduced form equations to generate the n-1 independent exchange rates from the exogenous variables of all _n_ countries. These exchange rates would then be fed into the structural equations that comprise the U.S. balance of payments. The entire system is therefore made up of n-1 exchange rate equations and one balance-of-payments equation.

A. Bilateral Flows of Three Countries



Notation.

T_{ij}: Trade balance between country i and country j. $T_{ij} > 0$ is interpreted as i having positive balance with respect to j. Net demand for i's currency and net supply of j's currency on trade account.

X_{ij}: Value of Exports from country i to country j valued in i's currency. (Exporters are paid in own-currency.)

M_{ij}: Value of Imports of country i from country j valued in i's currency.

r_{ij}: Exchange rate between country i and country j; units of i currency per unit of j currency. $r_{ij} = 1/r_{ji}$.

Then, $$M_{ij} = r_{ij} X_{ji}$$

and $$T_{ij} = X_{ij} - M_{ij}$$

$$T_{ij} = X_{ij} - r_{ij} X_{ji}.$$

K_{ij}: Net capital flow between country i and country j. $K_{ij} > 0$ represents net demand for i's currency and net supply of j's currency on capital account expressed in units of i's currency. Then $K_{ij} = - r_{ij} K_{ji}$.

B_i: Balance of payments of country i in own-currency. Sum of bilateral trade and capital accounts.

$$B_i \equiv \sum_{\substack{j=1 \\ j \neq i}}^{n} T_{ij} + \sum_{\substack{j=1 \\ j \neq i}}^{n} K_{ij}$$

(1) $$B_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} X_{ij} - \sum_{\substack{j=1 \\ j \neq i}}^{n} r_{ij} X_{ji} + \sum_{\substack{j=1 \\ j \neq i}}^{n} K_{ij}$$

Relationship among exchange rates.

Arbitrage in currency spot markets insures that

(2) $$r_{jk} = r_{ik} / r_{ij} .$$

If the prices of n-1 currencies in terms of the n^{th} currency are known, then it is possible to derive the relative prices of any of the n-1 currencies from (2).

Interdependence among balance of payments equations.

By Walras' Law, if n-1 currency markets are in equilibrium, the n^{th} market must also be in equilibrium. Alternatively, if B_1 through B_{n-1} equal zero, then B_n equals zero. There are n-1 independent balance of payments equations.

Strategy

Assume domestic variables (income, prices, interest rates, etc.) are exogenous. Develop structural equations for bilateral exports and bilateral capital flows. Substitute these structural equations into the n-1 independent balance of payments definitions (1) to solve for n-1 independent exchange rates. Cross-rates can be developed from the arbitrage condition (2).

B. Structural Equations

Import quantity is postulated as a linear function of domestic income and foreign and own price levels. Foreign price levels are expressed in own-currency terms through appropriate exchange rates.

Balance of payments of country 1.

$$B_1 = T_{12} + T_{13} + K_{12} + K_{13}$$

(3) $$B_1 = X_{12} + X_{13} - r_{12} X_{21} - r_{13} X_{31} + K_{12} + K_{13}$$

Balance of payments of country 2.

$$B_2 = T_{21} + T_{23} + K_{21} + K_{23}$$

(4) $$B_2 = X_{21} + X_{23} - r_{21} X_{12} - r_{23} X_{32} + K_{21} + K_{23}$$

Export equations.

$$q_{12} = f\left(Y_2, P_2, \frac{P_1}{r_{12}}, r_{23} P_3\right)$$

Make quantity function homogeneous of degree zero in prices and nominal income. Normalize on P_1 because P_1 will be used later to obtain export value equation.

$$q_{12} = g\left(\frac{Y_2}{P_1}, \frac{P_2}{P_1}, \frac{1}{r_{12}}, r_{23} \frac{P_3}{P_1}\right)$$

For simplicity, assume g is linear in arguments.

$$q_{12} = \alpha_0 + \alpha_1 \frac{Y_2}{P_1} + \alpha_2 \frac{P_2}{P_1} + \alpha_3 \frac{1}{r_{12}} + \alpha_4 r_{23} \frac{P_3}{P_1}$$

Multiply by U.S. export price to obtain dollar value of U.S. exports to U.K.

$$X_{12} = P_1 q_{12} = \alpha_0 P_1 + \alpha_1 Y_2 + \alpha_2 \frac{P_1}{r_{12}} + \alpha_3 P_2 + \alpha_4 r_{23} P_3$$

Then, export value equations for the remaining flows of B_1 follows directly:

$$X_{13} = P_1 q_{13} = \beta_0 P_1 + \beta_1 Y_3 + \beta_2 \frac{P_1}{r_{13}} + \beta_3 P_3 + \beta_4 \frac{P_2}{r_{23}}$$

$$X_{21} = P_2 q_{21} = \gamma_0 P_2 + \gamma_1 Y_1 + \gamma_2 r_{12} P_2 + \gamma_3 P_1 + \gamma_4 r_{13} P_3$$

$$X_{31} = P_3 q_{31} = \delta_0 P_3 + \delta_1 Y_1 + \delta_2 r_{13} P_3 + \delta_3 P_1 + \delta_4 r_{12} P_2$$

Capital Flow equations.

Net capital flows are functions of expected exchange rate changes and interest rates. At this point, interest rates are suppressed for simplicity. Expectations regarding exchange rate levels are assumed to be regressive. That is, if an exchange rate is now higher than the expected exchange rate, r^*, so that $(r-r^*) > 0$, then the exchange rate is expected to fall back to r^* in the future. Provisionally, r^* is taken as exogenous; eventually r^* should be explained endogenously.

$$K_{12} = a_0 + a_1 (r_{12} - r^*_{12}) + a_2 (r_{13} - r^*_{13}) + a_3 (r_{23} - r^*_{23})$$

$$K_{13} = b_0 + b_1 (r_{12} - r^*_{12}) + b_2 (r_{13} - r^*_{13}) + b_3 (r_{23} - r^*_{23})$$

Balance of payments of country 1.

Substitute structural expressions into (3).

$$(5) \quad B_1 = \alpha_0 P_1 + \alpha_1 Y_2 + \alpha_3 P_2 + \alpha_2 \frac{P_1}{r_{12}} + \alpha_4 r_{23} P_3$$

$$+ \beta_0 P_1 + \beta_1 Y_3 + \beta_4 \frac{P_2}{r_{23}} + \beta_2 \frac{P_1}{r_{13}} + \beta_3 P_3$$

$$- r_{12} (\gamma_0 P_2 + \gamma_1 Y_1 + \gamma_3 P_1) - \gamma_2 P_2 r_{12}^2 - \gamma_4 P_3 r_{13} r_{12}$$

$$- r_{13} (\delta_0 P_3 + \delta_1 Y_1 + \delta_3 P_1) - \delta_4 P_2 r_{12} r_{13} - \delta_2 P_3 r_{13}^2$$

$$+ (a_0 - a_1 r_{12}^* - a_2 r_{13}^* - a_3 r_{23}^*) + a_1 r_{12} + a_2 r_{13} + a_3 r_{23}$$

$$+ (b_0 - b_1 r_{12}^* - b_2 r_{13}^* - b_3 r_{23}^*) + b_1 r_{12} + b_2 r_{13} + b_3 r_{23}$$

Since the objective is to obtain reduced form equations for the exchange rates, regard r_{12} and r_{13} as the arguments in BOP equation (5). Group terms on r_{12} and r_{13} and consolidate into

$$(6) \quad B_1 = C_0 + C_1 r_{12} + C_2 r_{13} + C_3 r_{23} + C_4 \frac{1}{r_{12}}$$

$$+ C_5 \frac{1}{r_{23}} + C_6 \frac{1}{r_{13}} + C_7 r_{12}^2 + C_8 r_{12} r_{13} + C_9 r_{13}^2 ,$$

where the C's are functions of the parameters and exogenous variables in the model.

Employ arbitrage relationship to express r_{23} in terms of r_{12} and r_{13}: $r_{23} = r_{13}/r_{12}$

$$(7) \quad B_1 = C_0 + C_1 r_{12} + C_2 r_{13} + C_3 r_{23}/r_{12} + C_4 \frac{1}{r_{12}}$$

$$+ C_5 r_{12}/r_{23} + C_6 \frac{1}{r_{13}} + C_7 r_{12}^2 + C_8 r_{12} r_{13}$$

$$+ C_9 r_{13}^2$$

Equation (7) is cubic in r_{13}; this can be seen by multiplying both sides by $r_{12}r_{13}$.

Balance of payments of country 2.

Substitute structural equations into (4). Equations for X_{21} and X_{12} are already specified on pp. 20 and 21 above. The equation for K_{12} is on p. 20. Recall the relationship $K_{ij} = -r_{ij}K_{ji}$. The remaining structural equations for (4) are developed here.

$$X_{23} = P_2 q_{23} = \phi_0 P_2 + \phi_1 Y_3 + \phi_2 \frac{P_2}{r_{23}} + \phi_3 P_3 + \phi_4 \frac{P_1}{r_{13}}$$

$$X_{32} = P_3 q_{32} = \psi_0 P_3 + \psi_1 Y_2 + \psi_2 r_{23} P_3 + \psi_3 P_2 + \psi_4 \frac{P_1}{r_{12}}$$

$$K_{21} = - r_{21} K_{12} = - \frac{K_{12}}{r_{12}}$$

$$K_{23} = c_0 + c_1 (r_{12} - r^*_{12}) + c_2 (r_{13} - r^*_{13}) + c_3 (r_{23} - r^*_{23})$$

Substitute these equations into (4):

$$\begin{aligned}(8)\quad B_2 = {} & \gamma_0 P_2 + \gamma_1 Y_1 + \gamma_3 P_1 + \gamma_2 r_{12} P_2 + \gamma_4 r_{13} P_3 \\ & + \phi_0 P_2 + \phi_1 Y_3 + \phi_3 P_3 + \phi_2 \frac{P_2}{r_{23}} + \phi_4 \frac{P_1}{r_{13}} \\ & - \frac{1}{r_{12}} (\alpha_0 P_1 + \alpha_1 Y_2 + \alpha_3 P_2) - \frac{\alpha_2 P_1}{r_{12}^2} - \alpha_4 \frac{r_{23}}{r_{12}} P_3 \\ & - r_{23}(\psi_0 P_3 + \psi_1 Y_2 + \psi_3 P_2) - \psi_4 P_1 \frac{r_{23}}{r_{12}} - \psi_2 P_3 r_{23}^2 \\ & - \frac{1}{r_{12}}(a_0 - a_1 r^*_{12} - a_2 r^*_{13} - a_3 r^*_{23}) - a_1 - a_2 \frac{r_{13}}{r_{12}} - a_3 \frac{r_{23}}{r_{12}} \\ & + (c_0 - c_1 r^*_{12} - c_3 r^*_{13} - c_1 r^*_{23}) + c_1 r_{12} + c_2 r_{13} + c_3 r_{23}\end{aligned}$$

Again collecting terms, we have:

$$(9)\quad B_2 = D_0 + D_1 r_{12} + D_2 r_{13} + D_3 r_{23} + D_4 \frac{1}{r_{12}} + D_5 \frac{1}{r_{13}} + D_6 \frac{1}{r_{23}} + D_7 \frac{r_{23}}{r_{12}} + D_8 \frac{r_{13}}{r_{12}} + D_9 r_{23}^{\;2} + D_{10} \frac{1}{r_{12}^2}$$

where the D's are functions of the parameters and exogenous variables in the model.

Employ arbitrage relationship to express r_{23} in terms of r_{12} and r_{13}:

$$(10)\quad B_2 = D_0 + D_1 r_{12} + D_2 r_{13} + D_3 \frac{r_{13}}{r_{12}} + D_4 \frac{1}{r_{12}} + D_5 \frac{1}{r_{13}} + D_6 \frac{r_{12}}{r_{13}} + D_7 \frac{r_{13}}{r_{12}^{\;2}} + D_8 \frac{r_{13}}{r_{12}} + D_9 \frac{r_{13}^{\;2}}{r_{12}^{\;2}} + D_{10} \frac{1}{r_{12}^{\;2}}$$

Multiplying both sides of (10) by $r_{13}\, r_{12}^2$, we can see that this equation is cubic in r_{12}.

C. Reduced Form Equations for Exchange Rates

At this point, we have determined the form of two simultaneous equations, (7) and (10), in two unknown exchange rates, r_{12} and r_{13}. These equations are cubic in the arguments. A closed solution to this system would provide explicit functional forms for the exchange rates in terms of the exogenous variables. Two methods of linear approximations to (7) and (10) are discussed here: (1) expression of the system in first difference changes (in place of levels) and (2) first order Taylor series approximation. Neither approach needs to be fully implemented here; a good deal can be learned about the problem merely by setting up the approximations.

We show that linear approximation and the ensuing solution of the linear system involves too many terms to be practical. Specification of the reduced form equations is not facilitated by derivation from the structural system. One remaining hope for simplification lies in modification of the export functions. This reduces the extent of nonlinearity in exchange rates, but not sufficiently to change the conclusion on feasibility.

First differences.

If the changes in variables were infinitesimally small, then the first derivatives of equations (5) and (8) would give exact linear equations in the changes. Since the changes in exchange rates, incomes, and prices are finite, the first derivative is only an approximation to the first difference equation. Begin by taking the first derivative of equation (5). (Use the notation Δ for the derivative operator (in place of d) to indicate that first differences would be used in estimation.)

$$\Delta B_1 = \alpha_0 \Delta P_1 + \alpha_1 \Delta Y_2 + \alpha_3 \Delta P_2 + \alpha_2 \left(\frac{r_{12} \Delta P_1 - P_1 \Delta r_{12}}{r_{12}^2}\right)$$

$$+ \alpha_4 P_3 \Delta r_{23} + \alpha_4 r_{23} \Delta P_3$$

$$+ \beta_0 \Delta P_1 + \beta_1 \Delta Y_3 + \beta_4 \left(\frac{r_{23} \Delta P_2 - P_2 \Delta r_{23}}{r_{23}^2}\right)$$

$$+ \beta_2 \left(\frac{r_{13} \Delta P_1 - P_1 \Delta r_{13}}{r_{13}^2}\right) + \beta_3 \Delta P_3$$

$$- r_{12} \gamma_0 \Delta P_2 - P_2 \gamma_0 \Delta r_{12} - \gamma_1 Y_1 \Delta r_{12} - \gamma_1 r_{12} \Delta Y_1$$

$$- \gamma_3 r_{12} \Delta P_1 - \gamma_3 P_1 \Delta r_{12} - 2 \gamma_2 P_2 r_{12} \Delta r_{12}$$

$$- \gamma_2 r_{12} \Delta P_2 - \gamma_4 P_3 r_{13} \Delta r_{12} - \gamma_4 P_3 r_{12} \Delta r_{13}$$

$$- \delta_0 r_{13} \Delta P_3 - \delta_0 P_3 \Delta r_{13} - \delta_1 Y_1 \Delta r_{13} - \delta_1 r_{13} \Delta Y_1$$

$$- \delta_3 P_1 \Delta r_{13} - \delta_3 r_{13} \Delta P_1 - \delta_4 P_2 r_{12} \Delta r_{13}$$

$$- \delta_4 P_2 r_{13} \Delta r_{12} - \delta_4 r_{13} r_{12} \Delta P_2$$

$$- 2 \delta_2 P_3 r_{13} \Delta r_{13} - \delta_2 r_{13}^2 \Delta P_3$$

$$- a_1 \Delta r_{12}^* - a_2 \Delta r_{13}^* - a_3 \Delta r_{23}^* + a_1 \Delta r_{12}$$

$$+ a_2 \Delta r_{13} + a_3 \Delta r_{23} - b_1 \Delta r_{12}^* - b_2 \Delta r_{13}^*$$

$$- b_3 \Delta r_{23}^* + b_1 \Delta r_{12} + b_2 \Delta r_{13} + b_3 \Delta r_{23}$$

Without explicitly taking the first derivative of equation (8) we can sketch the solution to the simulatenous linear equations in changes in exchange rates. Group terms on Δr_{12} and Δr_{13} and incorporate all changes in exogenous variables in E_0 and F_0.

$$\Delta B_1 = E_0 + E_1 \, \Delta r_{12} + E_2 \, \Delta r_{13}$$

$$\Delta B_2 = F_0 + F_1 \, \Delta r_{12} + F_2 \, \Delta r_{13}$$

$$\begin{bmatrix} \Delta B \\ \\ \end{bmatrix} = \begin{bmatrix} E_0 \\ F_0 \end{bmatrix} + \begin{bmatrix} E_1 & E_2 \\ F_1 & F_2 \end{bmatrix} \begin{bmatrix} \Delta r_{12} \\ \Delta r_{13} \end{bmatrix}$$

$$\begin{bmatrix} \Delta r_{12} \\ \Delta r_{13} \end{bmatrix} = \begin{bmatrix} E_1 & E_2 \\ F_1 & F_2 \end{bmatrix}^{-1} \begin{bmatrix} \Delta B_1 - E_0 \\ \Delta B_2 - F_0 \end{bmatrix}$$

$$= \frac{1}{(E_1 \, F_2 - E_2 \, F_1)} \begin{bmatrix} F_2 & - E_2 \\ -F_1 & - E_1 \end{bmatrix} \begin{bmatrix} \Delta B_1 - E_0 \\ \Delta B_2 - F_0 \end{bmatrix}$$

For simplicity, assume intervention is zero and therefore $\Delta B_1 = \Delta B_2 = 0$.

$$(11) \quad \begin{bmatrix} \Delta r_{12} \\ \Delta r_{13} \end{bmatrix} = \frac{1}{(E_1 \, F2 - E_2 \, F_1)} \begin{bmatrix} -F_2 E_0 + E_2 F_0 \\ F_1 E_0 + E_1 F_0 \end{bmatrix}$$

From the equation for ΔB_1 we can get an idea of the number of terms involved in solving equation (11). E_0 has 25 terms, E_1 has 13 terms, and E_2 has 13 terms. Then to obtain the cross products necessary to solve for Δr_{12}

$$(12) \qquad \Delta r_{12} = \frac{E_2F_0 - F_2E_0}{E_1F_2 - E_2F_1}$$

involves expansion to approximately 1000 terms.

Several aspects of the reduced form approach become clear without proceeding further:

(1) The expressions for Δr_{12} and Δr_{13} are rational functions of combinations of exogenous variables. There is no way to obtain a linear regression specification. Although (12) could be estimated with non-linear methods, it does not attain the gains in simplicity sought by recourse to a reduced form approach. Linearization of (12) would entail yet another level of approximation.

(2) The numerator and denominator of (12) contain cross products of exogenous variables. Aside from the rational function problem, all the cross products would consume more degrees of freedom than there are observations. Estimation would require a principal components approach and entail the concomitant additional approximations.

(3) There is no hope of determining unambigous signs on the reduced form coefficients from a priori assumptions regarding the structural parameters.

Taylor Series Expansion.

For completeness, consider the characteristics of a first order Taylor series approximation to the solution of equation (5) and (8).

Taylor Series for single variable:

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots$$

First order Taylor expansion for multivariate function:

$$f(x_1, x_2, \ldots x_n) = f(a_1, a_2, \ldots a_n) + \sum_{k=1}^{n} \frac{\partial f}{\partial x_k}\Big|_{x=a}(x_k - a_k)$$

First order Taylor series expansion of $f(r_{12}, r_{13})$ and $g(r_{12}, r_{13})$ about a point (r_{12}^o, r_{13}^o):

$$f(r_{12}, r_{13}) \approx f(r_{12}^o, r_{13}^o) + \frac{\partial f}{\partial r_{12}}\Big|_{r_{12}^o, r_{13}^o}(r_{12}-r_{12}^o) + \frac{\partial f}{\partial r_{13}}\Big|_{r_{12}^o, r_{13}^o}(r_{13}-r_{13}^o)$$

$$g(r_{12}, r_{13}) \approx g(r_{12}^o, r_{13}^o) + \ldots .$$

Here, there are no derivatives of the exogenous variables. It is easier to begin with equation (7) expressing B_1 in terms of compound coefficients C_i. Let $f(r_{12}, r_{13})$ represent (7) in implicit form. After taking first derivatives, expanding, and grouping terms, the approximation to (7) is expressed as

$$f(r_{12}, r_{13}) = \{C_0 - B_1 + C_3 \frac{r_{13}^o}{r_{12}^o} + 2C_4 \frac{1}{r_{12}^o} + C_5 \frac{r_{12}^o}{r_{13}^o} + 2C_6 \frac{1}{r_{13}^o} - C_7 r_{12}^o - C_8 r_{12}^o r_{13}^o - C_9 r_{13}^o\}$$

$$+ r_{12}(C_1 - C_3 \frac{r_{13}^o}{r_{12}^o} - C_4 \frac{1}{r_{12}^{o2}} + C_5 \frac{1}{r_{13}^{o2}} + 2C_7 r_{12}^o + C_8 r_{13}^o)$$

$$+ r_{13}(C_2 + C_3 \frac{1}{r_{12}^o} - C_5 \frac{r_{12}^o}{r_{13}^{o2}} - C_6 \frac{1}{r_{13}^{o2}} + C_8 r_{12}^o + 2C_9 r_{13}^o).$$

The approximation to (10) is similar in form.

Solution of these two linear equations in r_{12} and r_{13} takes the same form as that for the difference equations (11). C_i and D_i contain somewhat fewer terms than the components of E_i and F_i. The Taylor series approximation offers the slight benefit of involving one third fewer coefficients than the first difference approach but suffers the same fundemental limitations.

Simplification of Export Functions.

It is possible that reduction of the extent of nonlinearity in r_{12} and r_{13} in equations (5) and (8) could simplify the reduced form system to a point of manageability. There is some scope for simplification in the form of the export functions.

Modify functional form.--On the face of it, linear export functions might seem the simplest case. Yet when prices are multiplied by the inverse of an exchange rate, this form increases the extent of nonlinearity in exchange rates. Consider exports from country 1 to country 2:

$$X_{12} = \alpha_0 P_1 + \alpha_1 Y_2 + \alpha_2 \frac{P_1}{r_{12}} + \alpha_3 P_2 + \alpha_4 r_{23} P_3$$

P_1 is the exports own--price. When viewed as import demand, P_1 should have a negative price effect. This can be accomplished with a negative sign on α_2. But a downward sloping demand curve can also be described with a positive coefficient on the inverse of price.

$$\begin{aligned} X_{12} &= \alpha_0 P_1 + \alpha_1 Y_1 + \alpha_2^* \frac{1}{r_{21} P_1} + \alpha_3 P_2 + \alpha_4 r_{23} P_3 \\ &= \alpha_0 P_1 + \alpha_1 Y_2 + \alpha_2^* \frac{r_{12}}{P_1} + \alpha_3 P_2 + \alpha_4 r_{23} P_3 \end{aligned}$$

The new specification incorporates the exchange rate r_{12} linearly rather than as an inverse. This helps in B_1 where X_{12} enters by itself. It also helps in B_2 where X_{12} is multiplied by r_{21}.

$$r_{21}\, X_{12} = \frac{X_{12}}{r_{12}} = \frac{1}{r_{12}} (\alpha_0\, P_1 + \alpha_1\, Y_2 + \alpha_3\, P_2) + \alpha_2^* \frac{1}{P_1} + \alpha_4 \frac{r_{23}}{r_{12}} P_3$$

With the linear functional form, the entry for X_{12} and B_2 was

$$r_{21}\, X_{12} \qquad = \frac{1}{r_{12}} (\alpha_0\, P_1 + \alpha_1\, Y_2 + \alpha_3\, P_2) + \frac{\alpha_2 P_1}{r_{12}^2} + \alpha_4 \frac{r_{23}}{r_{12}} P_3$$

This simplification removes terms in r_{12}^2, r_{13}^2, $1/r_{12}$, and $1/r_{13}$ from B_1 and B_2. It has little effect on the first difference approximation (because exogenous variables are differentiated along with exchange rates) and only reduces the number of terms from 1000 to 900. Removing these terms has greater effect on the Taylor series approximation (since only derivatives with respect to r_{12} and r_{13} are taken in the first place). Here, the number of terms is reduced to 200 from 600. This is still too many terms to be practical and the basic difficulties of the reduced form approach remain.

Predetermined trade flows.--The ultimate in simplification of the trade flows with respect to exchange rates is to make the flows functions of the previous periods' prices and incomes. This would not be an unreasonable assumption in a quarterly model where a good case can be made for lags in response to price and income changes. Postulate $q_{12,t} = f(Y_2,\, P_2,\, \frac{P_1}{r_{12}},\, r_{23}\, P_3)_{t-1}$ for simplicity, normalize the arguments of f ( ) on P_{1t} so that when q_{12} valued at current prices the numerator still drops out. Write out B_1 where $\bar{X}_{ij}$ indicates a trade flow as a function of predetermined variables and use the same capital flow functions.

$$B_1 = \bar{X}_{12} + \bar{X}_{13} - r_{12}\,\bar{X}_{21} - r_{13}\,\bar{X}_{31} + K_{12} + K_{13}$$

Substitute capital flow functions and group terms on exchange rates. Then

$$B_1 = X_{12} + X_{13} + a(r^*) + b(r^*) + r_{12}(a_1 + b_1 - X_{21}) + r_{13}(a_2 + b_2 - X_{31}) + r_{23}(a_3 + b_3)$$

where $a(r^*) = a_1 r_{12} + a_2 r^*_{13} + a_3 r^*_{23}$ and $b(r^*)$ is similarly defined.

$$B_1 = X_{12} + X_{13} + a(r^*) + b(r^*) + r_{12}(a_1 + b_1 - X_{21}) + r_{13}(a_2 + b_2 - X_{31}) + \frac{r_{13}}{r_{12}}(a_3 + b_3)$$

B_1 remains nonlinear in exchange rates as a result of the appearance of r_{23} in the capital flow equations. Thus, the degree of nonlinearity is diminished but linear approximations are still required to solve B_1 and B_2 for reduced form equations in r_{12} and r_{13}.

D. Conclusion on the Practicality of Reduced Form Equations for Exchange Rates

Reduced form equations for exchange rates as linear regressions on the exogenous variables of the BOP system would offer a simple means of endogenizing exchange rates. However, were we merely to write down a linear specification, we would have no idea of the underlying structure it represented. Consequently, we have sought to characterize the reduced form implied by the simplest multi-country balance-of-payments system. The extent of nonlinearity in exchange rates in the balance-of-payments structure can be reduced by altering the functional form of the trade flow equations to the most favorable case or by specifying the trade flows as functions of predetermined (previous period) variables. In either case, however, the balance-of-payments equations retain some nonlinearity in exchange rates due to the currency conversion of the capital and trade flows and the arbitrage condition which determines the cross-rate.

Even if the two balance-of-payments equations were linear in exchange rates, the reduced form would be nonlinear in exogenous variables as a result of solving for exchange rates. The resulting reduced forms are rational functions of exogenous variables that could, in principle, be estimated by nonlinear procedures. The rational functions could, also in principle, be approximated linearly. By this stage, however, we have two successive levels of approximations. The representativity argument against an a priori linear reduced form would apply to the final product of this procedure.

The approximation and solution to obtain the reduced form would result in a large number of cross-products of exogenous variables--too many for the number of observations available. A principal components procedure could be applied to these independent variables but it would constitute an additional approximation in itself. It is unlikely that all the variables affecting exchange rates would yield significant coefficients.

Finally, the standard criticisms of reduced form modeling apply. Reduced form coefficients are generally regarded as less stable over time than structural parameters. Moreover, the model should be capable of exploring alternative kinds of decision rules on intervention policy. These are structural changes and each would require a separate reestimation of a reduced form system. Therefore the need to take explicit account of alternative forms of intervention behavior means that the use of reduced forms is not practical.

IV. Research Strategy

In this section we describe some of the key topics in developing a U.S. balance-of-payments model in which exchange rates are determined endogenously by resorting to structural models for the balance of payments of non-U.S. countries. Little attention is given to specific components of the balance of payments of the U.S. The concern here is in putting together a multi-country model.

One topic is to specify a simple three-country pilot model, such as that described in Part III, with arbitrary parameters, and solve it for the two exchange rates. The purpose in doing this is to investigate the computational difficulties involved in solving highly nonlinear models. It is possible that the usual linear approximation techniques will break down in this case. The model might be extended to four or more countries to see if this raises further complications.

Another is to decide which major countries ought to be included in this truncated world model. In Table 1 are listed the fifteen countries with the largest share of world imports in 1972. These countries (together with others in the top fifteen for other categories) are also ranked in Table 1 by shares in U.S. imports and exports, in U.S. direct investment, and in U.S. short-term liabilities to foreigners. Positions 2-6 in the world imports rankings are highly correlated with the other

Table 1

TRADE AND INVESTMENT BY MAJOR COUNTRIES

(Billions of U.S. Dollars)

	World Imports (1972)	U.S. Trade (1972)		U.S. Direct Investment (1973)[5]		U.S. Short-Term Liabilities To Foreigners (1973)[6]	
		Imports[3]	Exports[4]	Stock	Flow (1973-1971)	Stock	Flow (1973-1971)
UNITED STATES	54.566(1)						
W. GERMANY	46.850(2)	4.249(3)	2.814(3)	7.954(3)	2.740	13.227(1)	6.631
JAPAN	28.380(3)	9.065(2)	4.941(2)	2.733(7)	0.915	6.417(2)	-7.877
FRANCE	26.573(4)	1.369(7)	1.611(7)	4.259(5)	1.246	3.483(5)	0.333
UNITED KINGDOM	25.113(5)	2.986(4)	2.659(4)	11.115(2)	2.174	6.148(3)	-1.226
CANADA	23.055(6)	14.909(1)	12.416(1)	28.055(1)	4.025	3.862(4)	0.421
ITALY	18.964(7)	1.756(5)	1.425(8)	2.301(11)	0.441	1.404(12)	-0.484
NETHERLANDS	17.125(8)	0.639(15)	1.831(6)	2.255(12)	0.583	2.886(8)	2.615
BELGIUM	15.958(9)	0.968(11)	1.138(10)	2.514(10)[1]	0.699	1.483(10)	0.782
SWEDEN	9.124(10)	0.601(18)	0.472(22)	0.846(20)	0.157	1.885(9)	1.094
SWITZERLAND	7.574(11)	0.619(16)	0.652(15)	2.593(8)	0.709	3.377(6)	0.129
AUSTRALIA	7.293(12)	0.807(13)	0.848(13)	4.526(4)	0.822	3.131(7)	2.215
SAUDI ARABIA	5.540(13)	0.194(38)	0.314(29)	N.A.	N.A.	N.A.	N.A.
DENMARK	4.455(14)	0.367(22)	0.258(31)	0.847(19)[2]	0.490	0.659(19)	0.491
U.S.S.R.	4.404(15)	0.096(52)	0.550(20)	0.0	0.0	0.022(43)	0.008
SUBTOTAL	295.424	38.625	31.929	70.009	15.001	47.984	5.132
GRAND TOTAL	390.601	55.557	49.703	107.268	21.267	65.426	11.794
% OF GRAND TOTAL	75.6%	69.5%	64.2%	65.3%	70.5%	73.3%	43.5%

Source: World Imports, Direction of Trade, Annual 1968-1972, World Totals, p. 001.
U.S. Trade, Direction of Trade, Annual 1968-1972, United States, p. 111.
U.S. Capital Stock, SCB, "U.S. Direct Investment Abroad in 1973," August 1974, Table 10A.
Short-Term Liabilities to Foreigners, FRB Bulletin, September 1973 and February 1974, Table 9, p. A79.

[1] Includes Luxembourg.
[2] Includes Ireland.
[3] Remaining top fifteen countries and ranks: Mexico 1.632(6), Venezuela 1.298(8), China 1.294(9), Hong Kong 1.250(10), Brazil 0.942(12), and Korea 0.707(14).
[4] Remaining top fifteen countries and ranks: Mexico 1.982(5), Brazil 1.244(9), Spain 0.972(11), Venezuela 0.925(12), and Korea 0.736(14).
[5] Remaining top fifteen countries and ranks: Brazil 3.199(6), Venezuela 2.591(9), Mexico 2.249(.3), Panama 1.665(14), and Argentina 1.407(15).
[6] Remaining top fifteen countries and ranks: Venezuela 1.468(11), Mexico 1.284(13), Norway 0.965(14), and Argentine 0.914(15).

categories, so that West Germany, Japan, Canada and the United Kingdom will definitely be included. Thus four countries -- and therefore four exchange rates -- plus the United States, plus the rest of the world, will constitute the basic model. These four countries accounted for 56% of U.S. imports in 1972 and substantial fractions of the other components.

Of course, we may decide to include more countries. Inspection of the top ten rankings in Table 1 reveals that France, Italy and perhaps Australia are possible additions to the model. Also, Switzerland is sixth in U.S. short-term liabilities, and some of its variables could be included in the model, along with Eurodollar market variables. The point is that if a bilateral exchange rate is to be made endogenous, a model of the country corresponding to this rate is needed. Variables for a country for which the exchange rate will not be endogenous may be endogenized with reduced forms. Such reduced forms as those used by Marston and Herring in their interest rate equation are among those that we have in mind.

A third topic involves specifying the minimum model that can be used for each of the structural country models. At this point, it must be mentioned that such specification interacts with matters described below, namely, deciding on periodicity for the country models and collection of data and firm contacts abroad to supply us with a continuing, timely flow of those data. We avoid discussing the detail of the specification at the present time, since it will in large degree depend on the quality of the empirical results. Our principle in specification will be to hold the number of equations to a minimum, consistent with reasonable and significant parameter estimates.

Prior to this discussion, however, the question might be raised as to how this model differs from Project LINK or other multi-country models. First, our model will include the endogenization of exchange rates, something that is completely absent from LINK.
Second, these sectors will be linked together in a way not implemented in other models. Third, the model will be smaller; fewer countries are involved. Finally, the focus is asymmetric: the U.S. is of primary importance. It follows that other countries are modeled in a summary way.

A Minimum Country Model

Each model must determine GNP and bilateral trade flows with all the other countries. The price level, the domestic supply of and demand for money, and the interest rate will similarly be endogenous. It may also be necessary to make the wage rate and the unemployment rate endogenous. About five or six variables in each country model would be exogenous, and we would make an effort to use variables that are regularly forecast elsewhere in the Division of International Finance.

Notable features of the model might include an allocation demand system for import demands to explain all elements of the trade matrix, import prices linked bilaterally to other countries' export prices, interest rate linkages, and the liberal use of endogenous foreign variables in each model.

Solving for Exchange Rates

The way exchange rates are determined is crucial to the specification of the model. Two approaches to solving for exchange rates may be distinguished. First, if all the components of a country's balance of payments are determined, including intervention, they sum to zero. This constraint for all countries involves a simultaneous system that clears the spot foreign exchange markets, yielding a set of equilibrium bilateral exchange rates, with cross rates determined by arbitrage.

For example, a general model of this sort for five countries--the U.S. and four others--would look like the following:

$$BB_1(\overline{X}_1, \overline{r}) + STK_1(\overline{r}, \overline{r}*, \overline{i}) + I_1(r_1) = 0$$

$$BB_2(\overline{X}_2, \overline{r}) + STK_2(\overline{r}, \overline{r}*, \overline{i}) + I_2(r_2) = 0$$

$$BB_3(\overline{X}_3, \overline{r}) + STK_3(\overline{r}, \overline{r}*, \overline{i}) + I_3(r_3) = 0$$

$$BB_4(\overline{X}_4, \overline{r}) + STK_4(\overline{r}, \overline{r}*, \overline{i}) + I_4(r_4) = 0$$

where:

BB_i = basic balance of the ith country

$\overline{X}_i$ = vector of exogenous variables in the ith country

$\overline{r}$ = vector of the four endogenous exchange rates

STK_i = short-term capital account of the ith country

$\overline{r}*$ = vector of expected spot exchange rates

$\overline{i}$ = vector of interest rates

I_i = foreign exchange market intervention of the ith country

The U.S. balance of payments would be included as one of the four equations. The coefficients for the BB_i and the STK_i accounts would be estimated, and the coefficients for the intervention functions, I_i, would either be estimated or would be set at plausible values. The four endogenous exchange rates, $\bar{r}$, would then be solved using these parameters and the values of the exogenous variables.

To compute these market-clearing exchange rates, iterative methods for the solution of nonlinear equations, such as Gauss-Seidel or Newton-Raphson, can be employed. An alternative procedure, if the system is differentiable, is to solve the system for first order changes using its Jacobian.[8] As in most nonlinear problems, one is not sure beforehand of a unique solution or whether the method will converge at all. However, our problem is not different in kind from other nonlinear numerical procedures.

A second technique would rely on the monetarist approach to exchange-rate determination. This involves re-specifying the model and deriving an explicit equation determining each exchange rate as a function

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[8] Given the conditions $B_i = 0$, $i = 1,\ldots,n$,

$$\frac{\partial B_i^o}{\partial e_j}\frac{\partial e_j}{\partial x_k} + \frac{\partial B_i^o}{\partial x_k} = 0 \quad , \text{ where the}$$

x_k are exogenous variables, e_j are exchange rates, and o denotes equilibrium values. Similar calculations can be performed in such a system for other exogenous variables. The total change is derived by summing the $\partial e_j/\partial x_k$. Note that this is the same principle applied above in section III for the first difference approximation. There, the idea was to obtain a functional form for the right-hand side of an exchange rate equation; here, the parameter estimates are known and numerical values for changes in exchange rates are sought.

of the excess demand for money. This follows from the monetarist view that it is more convenient to regard exchange rates as determined by the excess demand for money, rather than as the excess demand for foreign exchange generated by the two other markets, i.e., the goods and bond market.

Since the three markets (goods, bonds and money) are tied to each other by Walras' Law, it follows that the excess demand generated by the goods and bond markets is equal to the excess supply in the money market. Consequently, one can look at the demand for foreign exchange as emanating either from two of the markets, as is done above, or from the money market alone.

A complication arises in applying the latter, i.e., monetarist, approach to exchange rate determination because we shall in any case be specifying a disaggregated model of the trade and capital accounts of the United States. Because of the relationships among the three markets described above, explicit equations for exchange rates derived from monetarist assumptions would have to be made consistent with the trade and capital flow equations. Since our experience is limited in how to impose such consistency in an econometric model, we have decided to proceed at this point with the first alternative, namely, solving the trade and capital flow equations for market-clearing exchange rates.

Periodicity and data

To a large extent, periodicity chosen for each model will be determined by data availability. Table 2 details the availability of quarterly data for the countries considered.

Table 2

Quarterly Data Availability - Sources

Country	NIA	Monetary	BOP
U.S.	SCB	FRB	SCB, TB
Germany	Series 4 DB	MRDB	MRDB
Japan	Estat MBJ	Estat MBJ	BOP Monthly
U.K.	Blue Book	BOE-QB	BOE-QB
France	INSEE-U	BMBF	BMBF
Canada	NIE-SC	BOCR	QEBOP-SC
Italy	ISCO-U	Bolletino BI	Supplemento-BI
Australia	NIE-AU	MSS	MSS

Abbreviations

SCB - Survey of Current Business
FRB - Federal Reserve Bulletin
TB - Treasury Bulletin
Series 4 DB - Series 4 of the Deutsche Bundesbank
Estat MBJ - Economic Statistics Monthly of the Bank of Japan
BOP monthly - Balance of Payments monthly
BOE-QB - Bank of England-Quarterly Bulletin
INSEE-U - INSEE, unpublished
BMBF - Bulletin Mensuel de la Banque de France
NIE-SC - National Income and Expenditure - Statistics Canada
BOCR - Bank of Canada Review
QEBOP-SC - Quarterly Estimates of the BOP
ISCO-U - Istituto per la Congiuntura, unpublished
Bolletino BI - Banca d'Italia Bolletino e Supplemento
MRDB - Monthly Report of the Deutsche Bundesbank
NIE-AU - National Income and Expenditure Account, Reserve Bank of Australia
MSS - Monthly Statistical Summary, Reserve Bank of Australia

Certainly the data are available, and the choice of periodicity is one we can make with impunity. We favor quarterly data for linkage with a quarterly U.S. model.

The two sources of unpublished data, for France and Italy, will be accessible through our French and Italian connections. The LINK telex network might be helpful in this regard.