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CENTRAL BANK OPERATIONS IN FOREIGN AND DOMESTIC ASSETS
UNDER FIXED AND FLEXIBLE EXCHANGE RATES

by

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Under the Bretton Woods system, national authorities agreed to intervene in the exchange markets to keep exchange rates within narrow limits around pre-established and rarely altered par values. Foreign balances obtained in exchange market intervention were normally used to purchase interest earning assets, mainly U.S. Treasury securities, although sizable amounts of reserves were held as gold and SDR's. These exchange market and reserve management operations were carried out along side conventional open market operations. While exchange market operations were narrowly directed at preventing the exchange rate from moving outside its band, domestic open market operations were, depending on time and circumstances directed at operating targets, such as monetary aggregates or interest rates, in order to achieve desired values for primary targets such as price levels, incomes, or balances of payments. Under the recent, post Bretton Woods system of mixed fixed and floating exchange rates, many central banks have continued to intervene in exchange markets, even when not obliged to do so by any international agreement.

In this paper, an analytical framework is developed that focuses on the markets for the various assets used by the authorities in their monetary operations. Exchange market interventions and reserve management operations, like conventional domestic open market operations, normally involve an exchange of assets. International operations, unlike domestic open market operations, may involve the exchange of assets denominated in different currencies.

Placing exchange market, reserve management, and domestic open market operations of central banks on a more equal footing by viewing them all as asset market operations, though in different assets, facilitates analysis of a number of important policy problems. Three such problems are addressed in this paper. First, it is shown that a change in the exchange rate from one fixed value to another usually alters desired asset holdings in such a way that the devaluing country gains reserves. The amount
of the reserve gain and the effects of the devaluation on other financial variables depend crucially on the policy responses of the monetary authorities. Second, it is demonstrated that under flexible rates open market operations and exchange market operations, while having different impacts on the exchange rate and interest rates in some cases, are essentially quite similar and are identical in a limiting case. Third, it is argued that under some special, but relevant, assumptions the presence of a forward market does not alter the opportunities available to asset holders and that, therefore, the effects of forward market intervention on the exchange rate and interest rates are identical to the effects of other policies already available to the authorities.

The Model

The model developed in this paper is designed to analyze financial asset markets in a short-run, partial equilibrium context. Disturbances to financial asset equilibrium cause instantaneous adjustments in interest rates, the financial asset holdings of both wealth holders and central banks, and, under flexible exchange rates, the exchange rate. Given the short-run nature of the model, the effects of savings on wealth and investment on the capital stock are ignored. Income, the employment level, and the price level measured in domestic currency are assumed to remain fixed in both countries.

The model contains two countries, the United States (U.S.) and the United Kingdom (U.K.). The U.S. currency is the dollar; the U.K. currency is the pound. The behavior of four groups is analyzed: ultimate wealth holders in the U.S., ultimate wealth holders in the U.K., the U.S. central bank (the Federal Reserve, or FR), and the U.K. central bank (the Bank of England, or BOE).
It is assumed that there are only four types of assets: dollar deposits (U.S. money), pound deposits (U.K. money), dollar denominated securities (U.S. securities), and pound denominated securities (U.K. securities). All deposits are liabilities of one of the two central banks, and each central bank accepts only deposits denominated in its country's currency. The nominal interest rate on deposits is fixed at zero. Securities are fixed in nominal value in the currency of the country in which the issuer resides and have variable nominal interest rates.

The exchange rate, $S$, defined as the dollar price of pounds, may be fixed by the central banks or may be flexible. The expected percentage change in the exchange rate, $s$, is the same for all wealth holders. $s$ is assumed to be a given number, usually zero, under fixed rates and is taken to be a decreasing function of the gap between the current exchange rate and its "normal" level, $\bar{S}$, under flexible rates:

$$s = s(S - \bar{S}), \quad s_{S - \bar{S}} < 0.$$ 

This formulation implies that expectations are stabilizing, and it is retained through most of the analysis though occasionally reference is made to the case of destabilizing expectations ($s_{S - \bar{S}} > 0$).

Wealth holders in each country hold domestic money and both types of securities. They regard the three assets they hold as imperfect substitutes.

Wealth holders in the U.S. base their nominal demands in terms of dollars for U.S. securities, $B^d$, for U.K. securities, $SF^d$, and for U.S. money, $M^d$, on their existing dollar denominated nominal wealth, $W$. It is assumed that the fraction of their nominal wealth which they wish to hold in each of these three assets depends on the interest rate on U.S. securities, $r$, and the interest rate on U.K. securities adjusted for the expected rate of change in the exchange rate, $r' + s$, but not on their current savings so that $\frac{\partial}{\partial S}$
\[ B^d = b(r, r' + s)W, \]  
(1)  
\[ S^d = f(r, r' + s)W, \]  
(2)  
\[ M^d = m(r, r' + s)W. \]  
(3)

Similarly, U.K. wealth holders base their pound denominated nominal demands for U.S. securities, \( \frac{1}{S}B^d \), for U.K. securities, \( F^d \), and for U.K. money, \( N^d \), on their existing pound denominated nominal wealth, \( W' \), so that

\[ \frac{1}{S}B^d' = b'(r-s, r')W', \]  
(4)  
\[ F^d' = f'(r-s, r')W', \]  
(5)  
\[ N^d' = n'(r-s, r')W'. \]  
(6)

\( W \) and \( W' \) are given by

\[ W = M^A + B^A + SF^A, \]  
(7)  
\[ W' = N^A' + \frac{1}{S}B^A' + F^A'. \]  
(8)

where the superscript \( ^A \) denotes actual holdings of a given asset.

The balance sheet constraint for U.S. wealth holders requires that the sum of their nominal demands for all assets must be equal to their wealth which is defined to be the sum of the nominal values of the securities and money they currently hold. This constraint implies that b, f, and m must sum to one and that the sum of the partial effects on the three asset demands of a change in either of the two interest rates must be zero. U.K. wealth holders face a similar balance sheet constraint. Thus only two of the three asset demand functions are independent in each country. The two securities and home country money are assumed to be strict gross substitutes in the portfolios of wealth holders in each of the two countries. This assumption
means that if the interest rate on a given security rises, desired holdings of that security increase while the desired holdings of the other security and money decline.

The liabilities of the FR are the stock of U.S. money held by the U.S. public, \( M^S \), and the stock of U.S. money held by the BOE, \( M^C \); its assets are its holdings of U.S. securities, \( B^C \), and the dollar value of its outside reserve assets, \( cR \).\(^2\) \( R \) is the FR holding of outside reserve assets such as gold or SDR's, and \( c \) is the dollar price of outside reserve assets. The FR balance sheet is

\[
M^S = B^C + cR - M^C + A
\]

where \( A \) is a balancing item which changes to offset changes in \( c \).

For the BOE we have

\[
N^S' = F^C + \sigma'R^S + \frac{1}{S}(B^C + M^C') + A'
\]

where \( N^S' \) is the stock of U.K. money held by the U.K. public, \( F^C \) is the BOE holding of U.K. securities, \( \sigma' \) is the dollar price of outside reserve assets, \( R' \) is the BOE holding of outside reserve assets, \( S \) is the dollar price of pounds, \( B^C' \) is the dollar value of the BOE holding of U.S. securities, \( M^C' \) is the dollar value of the BOE holding of U.S. money, and \( A' \) is a balancing item which changes to offset changes in \( \sigma' \) and \( S \). The total world supply of outside reserve assets \( \bar{R} \) is fixed, and

\[
\bar{R} = R + R'.
\]

Equilibrium in the world market for U.S. securities requires that the supply of U.S. securities available to the public, \( B^S \), given by the total supply of U.S. securities, \( \bar{B} \), minus the holdings of the FR and the BOE, equal the demand for these securities by wealth holders in the two countries:

\[
B^S = \bar{B} - B^C - B^C' = b(r, r'+s)W + Sb'(r-s, r')W'.
\]
Similarly, the supply of U.K. securities available to the public in dollar terms, $S_t^S$, given by the total supply of foreign securities minus the holdings of the BOE, both in dollar terms, must equal the dollar denominated demand for these securities by wealth holders:

$$S_t^S = S(R - R') = f(t_1, r, r'+s) w + Sf(r-s, r') w'. \quad (11)$$

Using the central bank balance sheet identities and the fact that the total supply of outside reserve assets is fixed, the money market equilibrium conditions for the two countries can be written as follows:

$$M_t^S = B_t^C + cR = M_t^C + A = m_t(r, r'+s) w, \quad (12)$$

$$S_t^{n1} = S[R^C + c'(R-R)] + B_t^C + M_t^C + S_A = S_n(r-s, r') w'. \quad (13)$$

Three of the four market equilibrium conditions are independent, so three endogenous variables can be determined. Different sets of three variables are taken to be endogenous at different stages in the study depending upon the situation which is being analyzed. In one important case under fixed rates, for example, the two interest rates, $r$ and $r'$, and the stock of reserves held by the FR, $R$, are regarded as being endogenously determined, and all the other variables are considered to be given exogenously.

In Figure 1, four schedules are plotted which show the pairs of $r$ and $r'$ which are compatible with equilibrium in each of the four financial asset markets. The curve labeled $B_{00}$ gives the combinations of $r$ and $r'$ for which the private demand for securities issued in the U.S. is equal to the fixed supply of securities issued in the U.S. minus the holdings of U.S. securities by the FR and the BOE. The curve slopes upward; an increase in $r$ causes an excess demand for U.S. securities so that $r'$ must rise in order to cut demand back until it matches the fixed supply of U.S. securities available to wealth holders. Minus (plus) signs in a region near the schedule for a given market
indicate that the interest rate pairs in that region imply excess supply (demand) in that market; vertical (horizontal) arrows represent the direction of pressure on \( r \) (\( r' \)) in each of the regions. The curve labeled \( F_0^F_0 \) represents the combinations of \( r \) and \( r' \) which insure that wealth holders' demands for securities issued in the U.K. are equal to the available supply. The \( F_0^F_0 \) curve is positively sloped; an increase in \( r \) is required to offset the excess demand caused by an increase in \( r' \) if private wealth holders are to continue to be satisfied holding the available supply of U.K. securities. Combinations of \( r \) and \( r' \) that equate the demand for and supply of U.S. money, given a fixed U.S. money supply available for the public to hold, are plotted as the \( M_0^M_0 \) curve. The \( M_0^M_0 \) curve is negatively sloped since an increase in either \( r \) or \( r' \) reduces the demand for U.S. money. Combinations of \( r \) and \( r' \) that equate the demand for the fixed supply of U.K. money are plotted as the \( N_0^N_0 \) curve. The \( N_0^N_0 \) curve is negatively sloped since an increase in either \( r \) or \( r' \) reduces the demand for U.K. money.

The assumption that the three assets held by wealth holders in each of the two countries are strict gross substitutes, insures that the slope of the \( F_0^F_0 \) curve must be greater than the slope of the \( B_0^B_0 \) curve. Suppose that all four markets are in equilibrium as at \( a_0 \). At every point in the quadrant northeast of \( a_0 \), there are excess supplies of the two kinds of money. The balance sheet constraints for wealth holders imply that the sum of the excess supplies for all assets must be zero, so at no point in the quadrant can there be excess supplies of both of the two securities. This possibility is ruled out only if \( F_0^F_0 \) is steeper than \( B_0^B_0 \).

It is assumed that wealth holders in each country regard their currency as a better substitute for the security denominated in that currency than for the security denominated in the other currency. This plausible assumption is sufficient to insure that the \( N_0^N_0 \) curve has a steeper (more negative) slope than the \( M_0^M_0 \) curve.
Interest rate changes alone are not sufficient to insure that three independent schedules will have a common intersection point. A third variable not shown explicitly on the graph, for example, the stock of reserves held by the FR, must be free to vary in order to insure that a common intersection exists. If three of the four schedules intersect at a common point, the fourth schedule must also pass through that point. It is useful to retain all four schedules even though in a given situation three schedules are sufficient to determine the equilibrium values of the endogenous variables.

The Effects of a Devaluation on Financial Asset Holding

It has often been argued that since an exchange rate change has no effect on the relative attractiveness of domestic and foreign financial assets, there is no incentive for wealth holders to rearrange their portfolios of financial assets following a devaluation. Those who make this argument usually assume that wealth holders do not expect the exchange rate to change and that once the rate changes they do not expect it to change again.

Under these conditions, a U.S. investor compares the proceeds from one dollar placed in U.S. securities, \( e^0 \) dollars, with the proceeds from one dollar placed abroad, \( \left[ \frac{1}{S_0} e^{r_1} S_0 \right] \) dollars, at time zero before the devaluation. If nothing else has changed and the exchange rate changes from \( S_0 \) to \( S_1 \), the same investor must compare \( e^0 \) dollars with \( \left[ \frac{1}{S_1} e^{r_0} S_1 \right] \) dollars. Since the exchange rate cancels out in both comparisons, U.S. and U.K. securities have the same relative attractiveness after the exchange rate change as before it. Thus, the argument proceeds, there should be no incentive for wealth holders to alter their portfolios.
This argument is correct as far as it goes. What it ignores is the fact that some wealth holders experience capital gains and others experience capital losses as a result of the devaluation and that these changes in wealth are realized completely in the first instance as changes in the home currency value of foreign assets. Both the losers and the gainers from the devaluation find themselves with the wrong mix of assets at the prevailing interest rates given their changed wealth and the form in which the change has occurred. Thus, when the wealth effects of the devaluation are taken into account, there is an incentive for wealth holders to alter their portfolios.  

Consider a devaluation of the dollar in the framework of our model. U.S. citizens experience a capital gain of \( f_A dS \) dollars while U.K. residents suffer a capital loss of \( -e_A' dS \) pounds where \( S \) is the dollar price of pounds, taken to be unity initially, and \( f_A \) and \( e_A' \) are the initial holdings of U.K. securities by U.S. residents and the initial holdings of U.S. securities by U.K. residents, respectively. The increase in wealth in the U.S. comes in the first instance as an increase in the dollar value of U.K. holdings of U.S. securities, and the decrease in wealth in the U.K. comes initially in the form of a decrease in the pound value of U.K. holdings of U.S. securities. Following the devaluation, U.S. residents have an excess demand for U.S. securities and U.S. money and an excess supply of U.K. securities, and U.K. residents have an excess demand for U.S. securities and an excess supply of U.K. securities and U.K. money at the original interest rate pair. Thus, there is a world excess demand for U.S. securities and an excess supply of U.K. securities. In Figure 2, the pre-devaluation equilibrium is at the intersection of the schedules with the \( (0) \) subscript. The impact effect of the devaluation is shown by the schedules labeled with the \( (1) \) subscript. \( B_1 B_1 \)
must lie below \( B_0 B_0' \) since, for any value of \( r' \), a lower value of \( r \) will clear the market for U.S. securities following the devaluation. Similar arguments can be employed to establish the positions of the other schedules. \( B_1 B_1' \) and \( F_1 F_1' \) must intersect in the region of excess demand for U.S. money and excess supply of U.K. money. When security markets both clear, excess demands in the two money markets must be equal and opposite in sign, and, given our assumptions regarding the relative slopes of the \( MM \) and \( NN \) schedules and the shifts in all the schedules implied by the devaluation, there must be an excess demand for U.S. money.

The main point of this section is that under most plausible policy responses by the two central banks, the FR gains foreign assets. However, the ultimate configuration of interest rates and the magnitude of the increase in FR reserve holdings depend crucially upon the policy responses of the FR and the BOE.

It is instructive to begin with the case in which both central banks stabilize their interest rates and the BOE holds its changes in reserves as outside reserve assets or U.S. money. If interest rates are held constant by central bank action, the proportion of wealth held in each asset by residents of both countries remains constant. U.S. residents want to divest themselves of \( (1-f)F^A ds \) in U.K. securities and hold \( m^F^A ds \) of the proceeds in U.S. money and \( b^F^A ds \) in U.S. securities. U.K. residents want to sell \( (f' + n')B^A' ds \) pounds for dollars in order to buy \( (1-b')B^A' ds \) U.S. securities. If interest rates are to remain unchanged, the FR must meet the demand for U.S. securities and the BOE must absorb the unwanted U.K. securities. U.K. reserve losses are equal to the sales of U.K. securities by U.S. citizens plus the purchases of U.S. securities by U.K. residents or \( [(1-f)F^A + (1-b')B^A'] ds \). Of course, the
effect of the actions by central banks described above is to shift BB and FF so that they intersect at \( a_0 \) again. MM and NN return to their initial positions as the U.S. money stock increases and the U.K. money stock declines.

If the BOE uses U.S. securities instead of outside reserve assets or U.S. money to finance imbalances, it will increase the supply of U.S. securities by the full amount of the sales of U.K. securities by U.S. citizens, plus the amount of the purchase of U.S. securities by U.K. residents. This increase in the supply of U.S. securities exceeds the initial excess demand for U.S. securities at the original interest rate pair because U.S. citizens are selling off U.K. securities partly in order to increase their money balances. In this case, the FR must buy U.S. securities in an amount equal to the initial excess demand for money in the U.S. at the original interest rate pair in order to keep U.S. interest rates from rising.

Returning again to the assumption that outside reserve assets are used to finance payments imbalances, we find that if both central banks make no purchases or sales of domestic securities, the new equilibrium is at the intersection of \( B_{11} \) and \( F_{11} \) at \( a_1 \). \( r \) must fall, and \( r' \) must rise in order to remove the initial excess demand for U.S. securities and excess supply of U.K. securities. MM and NN must shift down until they pass through \( a_1 \). The U.S. money supply increases and the U.K. money supply declines by an equal amount. The shift of reserves from the U.K. to the U.S. can be represented by either the shift in MM or the shift in NN.

Complete monetary sterilization by the two central banks implies that equilibrium must be at point \( a_2 \) where \( M_{11} \) and \( N_{11} \) intersect. The FR sells enough U.S. securities and the BOE buys enough U.K. securities to cause the U.S. interest rate to rise and the U.K. interest rate to decline by the amount necessary to remove the initial excess demand for money in the U.S. and the
excess supply of money in the U.K. The loss of reserves by the BOE is equal to the purchases of securities by the BOE which are equal in amount to the sales of securities by the FR and can thus be represented by the shift in either FF or BB from $F_{1.1}$ or $E_{1.1}$ to the position they would have to have if they were to pass through $a_2$.

Point $a_3$ is the equilibrium point if the U.S. money supply and the supply of U.K. securities remain unchanged. This point would be reached if payments imbalances are financed with outside reserve assets and the BOE is passive while the FR sterilizes the U.S. money supply. Asset equilibrium would also be at point $a_3$ if the BOE sells U.S. securities to finance its deficit and both central banks are passive. In this case, the U.K. interest rate must rise in order to induce residents of both countries to hold the unchanged supply of U.K. securities. The U.S. rate may rise or fall (as in Figure 2) since either the FR or the BOE sells off enough U.S. securities to keep the U.S. money stock constant, and these sales may exceed or fall short of the excess demand for U.S. securities at an unchanged U.S. interest rate given the U.K. interest rate on the $F_{1.1}$ schedule corresponding to the unchanged U.S. interest rate.

It is important to note that the BOE can avoid reserve losses due to the devaluation. Suppose the FR is passive. If the BOE pursues a large enough contractionary open market operation, it can assure that the new equilibrium is at point $a_4$ where $E_{1.1}$ and $M_{1.1}$ intersect. Since neither the U.S. money stock nor FR holdings of U.S. securities is changed at $a_4$, U.S. reserves must be unchanged. Of course, this policy response by the BOE implies an increase in the U.K. interest rate and a decline in the U.K. money supply.
The graphic analysis can be used to determine the relative magnitude of BOE reserve losses implied by some of the equilibrium positions we have considered. The technique for ranking each pair of points is to find an item on the balance sheet of one central bank or the other which remains unchanged between the two points and then compare the changes in another item between the two points; this comparison allows us to determine how the third item must have changed. We showed that point $a_4$ was an equilibrium which involved no reserve loss for the BOE by looking at the balance sheet of the FR. Since neither the U.S. money stock nor the FR holdings of U.S. securities has changed, the U.S. must not have gained any reserves. Point $a_1$ involves some reserve loss by the BOE since the U.K. money supply declines while the BOE holdings of U.K. securities remain fixed. An even greater U.K. reserve loss is implied by point $a_3$ since the U.K. money supply declines further while BOE holdings of U.K. securities are still constant. The U.K. reserve loss (U.S. reserve gain) implied by point $a_3$ can also be represented by the shift in BB from $B_1$ to the position it would have if it passed through $a_3$ since the U.S. money stock is the same at $a_4$ and at $a_3$. The same line of argument leads to the conclusion that $a_2$ involves a larger U.K. reserve loss than $a_3$ since the BB curve must shift even further to reach $a_2$. Thus, ranking points from least to greatest U.K. reserve loss, we obtain the following: $a_4 < a_1 < a_3 < a_2$. 19/ 

The magnitude of U.S. reserve gains from the portfolio adjustments due to the wealth effects of a devaluation depends upon the policy responses of the two central banks, which in turn depend upon what policies the central banks
deem appropriate from a domestic point of view and how convinced the BOE is that
the U.S. should be allowed to accumulate reserves.

If a devaluation by the U.S. had been expected, asset demands would have been
shifted away from U.S. securities and U.S. money and in favor of U.K. securities
and U.K. money at some point before the devaluation. After the devaluation, asset
demands might well shift back to their original configuration; that is, the demand
for U.S. securities and U.S. money would rise at the expense of the demand for U.K.
securities and U.K. money. If these shifts in asset demands occurred, all the
schedules would shift in the same direction, but farther, at the time of the devalu-
ation. These increased shifts mean that the size of the adjustments in the quantity
variables in the model required in order for equilibrium to be reattained are larger.
For any given combination of policy responses by the FR and BOE, changes in stocks
of money and securities held by the U.S. and U.K. publics, and reserves held by the
central banks will be at least as large, in absolute value, as they would have been
without the additional shift in asset demands. 20/

Open Market Operations and Exchange Market Operations Under Flexible Rates

The purpose of this section is to explore the similarities and differences between
open market operations and exchange market operations under flexible exchange rates.
If the effects of these operations are different, each type of operation is an independ-
ent policy tool. Knowing exactly how many independent policy tools they have availab
is very important to the authorities no matter what the exchange rate regime, and this
knowledge may affect their willingness to commit one of their tools to an exchange rate
target as they must under a fixed exchange rate regime. The crucial question is, can
central banks achieve something different by trading home currency assets for foreign
currency assets than they can achieve by open market operations?
Suppose the BOE undertakes an expansionary open market operation and that
the FR remains passive. The original equilibrium is given by the intersection
of the schedules with the (r) subscript at a\textsubscript{0} in Figure 3, and the BOE open
market purchase shifts FF and NN to F\textsubscript{1} F\textsubscript{1} and N\textsubscript{1} N\textsubscript{1}, respectively. At unchanged
values of r, r', and S, there is an excess supply of U.K. money and an excess
demand for U.K. securities. The excess demand for U.K. securities puts
downward pressure on r' which tends to create excess demand for U.S. securities
with resulting downward pressure on r; the excess supply of U.K. money puts
downward pressure on S, the dollar price of pounds. The depreciation of the
pound stimulates demand for pound assets shifting N\textsubscript{1} N\textsubscript{1} and F\textsubscript{1} F\textsubscript{1} upward for two
reasons. First, according to the arguments given in the analysis of devaluation,
the dollar value of U.S. wealth falls and the pound value of U.K. wealth rises.
But these changes accrue completely in the form of reduced values of pound
holdings in the U.S. and increased pound values of dollar holdings in the U.K.,
so that both groups want more pound assets. Both groups also want fewer
dollar assets, so M\textsubscript{0} M\textsubscript{0} shifts down and B\textsubscript{0} B\textsubscript{0} shifts up. Second, given our
assumption of stabilizing expectations, a decline in S causes s to rise, a
movement which stimulates demand for pound assets and reduces demand for dollar
assets. Even if expectations are destabilizing, the schedules will shift in
the directions described above if the wealth effects are strong enough. The
new equilibrium pair of r and r' must lie within the kite-shaped area bounded
by the M\textsubscript{0} M\textsubscript{0}, N\textsubscript{1} N\textsubscript{1}, B\textsubscript{0} B\textsubscript{0}, and F\textsubscript{1} F\textsubscript{1} schedules. Thus, the expansionary open
market purchase by the BOE with the FR passive lowers r' and S, but the net
effect on r is indeterminate. The U.S. interest rate is more likely to fall
the greater the responsiveness of the demand for U.S. money to exchange rate
changes relative to the responsiveness of the demand for U.S. securities to
exchange rate changes. An interesting special case arises when money
demands do not depend on wealth and when the expected change in the exchange rate does not depend on its level so that money demands in both countries are independent of $S$. In this case, the equilibrium is at $a_1$, and $r$ definitely rises.

Now, consider the effects of an exchange-market operation by the BOE designed to depreciate the pound. Such an action when undertaken primarily in order to stimulate the domestic economy has been referred to pejoratively as a competitive devaluation. Suppose that the exchange-market operation takes the form of a BOE purchase of dollar deposits with pound deposits. We call this an exchange market intervention of Type I. The BOE purchase of dollars with pounds shifts the money market equilibrium schedules to $N_1 N_1$ and $N_1 N_1$. Assume that the FR is passive. The excess supply of pounds and excess demand for dollars lead to a depreciation of the pound. This depreciation increases world demand for pound assets, and reduces world demand for dollar assets, and the $F_0 F_0$, $N_1 N_1$, and $B_0 B_0$ schedules shift upward while the $M_1 N_1$ schedule shifts downward. Although one would expect that $r$ would rise and $r'$ would fall, this is not necessarily the case. The larger (smaller) the responsiveness of the demand for U.S. securities to exchange rate changes relative to the responsiveness of the demand for U.K. securities to exchange rate changes, the more likely it is that both interest rates will rise (fall). In the special case in which money demands are not responsive to exchange rate changes so that the responses of U.S. and U.K. security demands to exchange rate changes are equal but opposite in sign, equilibrium is at $a_2$ where $r'$ is lower and $r$ is higher.

The BOE would probably not hold dollar deposits but would convert the proceeds of its intervention activities into U.S. securities. The net result of all these operations is an exchange of home money for foreign securities which we call an exchange market operation of Type II. This type of exchange market operation is particularly interesting because European monetary authorities, especially the British, have, at times, performed their "open market operations" using U.S. securities; that is, they have conducted "monetary policy" over the exchanges. In the graph, an
exchange market operation of Type II can be represented by shifts of \( N_0 N' \) to \( N_1 N' \)
and \( B_0 B' \) to \( B_1 B' \). If the FR remains passive, the equilibrium point lies somewhere
in the kite-shaped region bounded by \( N_1 N', F_0 F', B_1 B' \) and \( M_0 M' \). Private portfolios
must be adjusted to absorb more U.K. money at the expense of U.S. securities, and
this adjustment will involve declines in \( r' \) and \( S \). The effect on \( r \) is indeterminate;
\( r \) is more likely to fall the greater the responsiveness of the demand for U.S. money
to exchange rate changes compared to the responsiveness of the demand for U.K. secu-
rities to exchange rate changes. If changes in exchange rates do not alter money
demands, \( a_1 \) is the new equilibrium point. Thus, in this special case, open market
operations and exchange market operations have identical effects on interest rates
though the effects on the exchange rate are different. In fact, in this polar case,
the two actions have identical interest rate effects for any given FR policy response.

If the BOE wanted to depreciate its exchange rate without affecting its money
supply and wanted to hold its reserves in interest bearing form, it would sell U.K.
securities and use the proceeds to purchase U.S. securities, a series of actions
designated exchange market operation Type III. Some analysts regard Type III inter-
vention as potentially very important because they believe it might come to be used
quite widely by cooperating central banks to affect exchange rates without alter-
ing money supplies. This operation shifts the BB and FF schedules from
\( B_0 B' \) and \( F_0 F' \) to \( B_1 B' \) and \( F_2 F' \) in Figure 3. With the FR passive, an operation of
Type III results in an increase in \( r' \), a decline in \( r \), and a fall in \( S \).

While open market operations and exchange market operations have similar effects,
there are clearly some differences between them in the general case. The comparative
statics effects of each of the four BOE policy actions when the FR is passive are
shown in Table I. All of the operations compared are equal in terms of the value of
the assets exchanged. The relative magnitudes are ranked ordinally with the number
one indicating the policy action which results in the largest decline or the smallest
rise in the variable under consideration. It is interesting to compare the effects...
<table>
<thead>
<tr>
<th>Policy Action</th>
<th>Effect on</th>
<th>r</th>
<th>r'</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open Market Operation</td>
<td>Sign</td>
<td>2</td>
<td>1</td>
<td>3 or 4</td>
</tr>
<tr>
<td>$(\Delta F' C')$</td>
<td>Rank</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exchange Market Operation Type I</td>
<td>Sign</td>
<td>3</td>
<td>cannot be ranked</td>
<td>1</td>
</tr>
<tr>
<td>(Sale of Pound Deposits for Dollar Deposits)</td>
<td>Rank</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(\Delta M' C')$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exchange Market Operation Type II</td>
<td>Sign</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>(Sale of Pound Deposits for Dollar Securities)</td>
<td>Rank</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(\Delta B' C')$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exchange Market Operation Type III</td>
<td>Sign</td>
<td>cannot be ranked</td>
<td>3</td>
<td>3 or 4</td>
</tr>
<tr>
<td>(Sale of Pound Securities for Dollar Securities)</td>
<td>Rank</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(\Delta B' C' = - \Delta F' C')$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
of an open market operation and an exchange market operation of Type II, especially since some Europeans use the latter to conduct "monetary policy". An open market operation lowers the U.K. interest rate more and depreciates the exchange rate less than an exchange market operation of Type II. Both operations stimulate both the domestic and trade account components of aggregate demand, but open market operations have a greater effect on the domestic component while exchange-market operations of Type II have a greater effect on the trade account. The ranking of exchange market operations of Types I and III with respect to the depreciation of the pound (decline in S) arises because interest rate movements are relatively more effective in removing disequilibria in the securities markets than disequilibria in the money markets, so less movement in the exchange rate is required in the case of an exchange market operation of Type III. Given the attention devoted to exchange market intervention of Type III as a likely method for cooperative intervention, it is noteworthy that its effectiveness in terms of bang per buck for changing the exchange rate is relatively low. Although the ranking in Table I depends on the assumption that the FR is passive, the table can be easily altered to reflect alternative assumptions about FR behavior.

It is important to note that only three of the four policies in Table I are independent. The effect of an exchange market operation of Type II is the same as the effect of undertaking simultaneously an open market operation and an exchange market operation of Type III. Even if all the policy options open to the FR were considered, it would be true that the two central banks taken together have only three independent policies which, of course, can be used to attain desired values for three target variables.

This situation changes markedly if the two securities are perfect substitutes. In this case, open market operations and Type II exchange market operations have identical effects, and Type III exchange market operations have no effect. A sufficient condition for the two securities to be perfect substitutes under flexible rates is that wealth holders be risk neutral. Risk neutrality implies that the returns on the two
securities differ only by the expected rate of change in the exchange rate, no matter what the composition of world supply of securities. Since the composition of the world security supply is inconsequential, purchases of securities denominated in either currency with U.K. deposits have the same effect and exchanges of securities for securities have no effect. The effects of open market operations are, however, different from the effects of a purchase of U.S. deposits with U.K. deposits. Thus, the degree of substitutability between the two securities is a crucial factor in determining how different the impact of the different types of central bank operations are.

The Redundancy of the Forward Exchange Market in an Important Special Case

In addition to making portfolio allocation decisions regarding money and securities, participants in international financial markets may enter into forward exchange contracts. The authorities may also buy or sell forward exchange. Contrary to what one might expect, if U.S. securities (dollar securities) are regarded as perfect substitutes for U.K. securities (pound securities) except for exchange risk and if capital markets are perfect, the explicit introduction of a forward exchange market does not add to the alternatives available to wealth holders in the model of the preceding section in any material way and, therefore, does not affect the conclusions derived from that model. In this important special case forward market intervention does not add to the ability of the authorities to achieve desired combinations of money stocks, interest rates, and the exchange rate since the effects of forward intervention are identical to the effects of policies already available.

When constructing a model which includes forward contracts, it is important to be explicit about the time period to which the model refers and the types of assets available to wealth holders. In order to keep the analysis simple, it is assumed that asset markets meet at time \( t \) and do not meet again until time \( T \), some time in the future. The assets traded are dollar securities, pound securities, and forward contracts, all of which mature at time \( T \), as well as dollar deposits and pound deposits. Wealth holders may either borrow (issue securities) or lend (hold securities), and borrowing and lending
rates are identical. Buying or holding a dollar (pound) security involves paying $1 (II) at time t in return for receiving $e^{r(T-t)}[1 + e^{r'(T-t)}] at time t. Selling or issuing a security involves a receipt at t and a payment at T. Buying dollars forward (selling pounds forward) involves agreeing at t to receive $1 at T in return for paying an amount of pounds equal to \( \frac{1}{V} \) at T where V is the forward exchange rate, the price of a pound at T in terms of dollars at T. Selling dollars forward (buying pounds forward) involves receiving pounds and paying dollars at T.

Consider the situation of a U.S. resident at time t who has holdings of dollar deposits, dollar securities, pound securities, and net forward commitments. He can shift the composition of his portfolio in several ways. There are two ways in which he can increase his claims to dollars payable at time T, his open dollar position, at the expense of his money holding. By reducing his holdings of dollar deposits by one dollar and increasing his holdings of dollar securities by one dollar he can obtain a net dollar return of

\[ $[e^{r(T-t)} - 1]$.\]

By reducing his holdings of dollar deposits by one dollar and placing the dollar in covered pound securities he can earn

\[ $[e^{(r' + \varphi)(T-t)} - 1]$,

where \( \varphi \) represents the forward premium on pounds and is defined by

\[ \frac{V}{S} = e^{\varphi(T-t)} \].

There are also two ways in which he can increase his claims to pounds payable at time T, his open pound position, at the expense of his money holdings. If he replaces a dollar deposit by one dollar's worth of uncovered pound securities, he can expect a dollar return of

\[ $[e^{(r' + s)(T-t)} - 1]$,
where $s$ represents the expected rate of change in the spot rate and is defined by

$$\delta^s = e^{s(T-t)}$$

and $\delta$ is the spot exchange rate expected to prevail at $T$ by all wealth holders in the U.S. and U.K. If he replaces a dollar deposit by a dollar security accompanied by a sale of the proceeds forward, he can expect to receive

$$\$\left[e^{(r+s-\varphi)(T-t)-1}\right].$$

Finally, he can increase his open pound position by reducing his open dollar position, leaving his money holdings unchanged, in two ways. He can reduce his holdings of dollar securities by one dollar (or borrow a dollar) and buy a pound security (or reduce his pound borrowings) and expect to earn

$$\$\left[e^{(r+s)(T-t)}e^{r(T-t)}\right] = \$\left[e^{(r+s-r)(T-t)-1}\right] e^{r(T-t)}.$$

Alternatively, he can sell $e^{r(T-t)}$ dollars (one dollar in present value terms) forward and expect to receive

$$\$\left[e^{(s-\varphi)(T-t)-1}\right] e^{r(T-t)}.$$

The pound returns from portfolio rearrangements open to a U.K. resident who has holdings of pound deposits, dollar securities, pound securities, and net forward commitments can be expressed in similar fashion:

<table>
<thead>
<tr>
<th>Portfolio Rearrangement</th>
<th>Expected Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>pound securities (+), pound deposits (-)</td>
<td>$$\left[e^{r'}(T-t)-1\right]$</td>
</tr>
<tr>
<td>covered dollar securities (+), pound deposits (-)</td>
<td>$$\left[e^{(r-\varphi)(T-t)-1}\right]$</td>
</tr>
<tr>
<td>dollar securities (+), pound deposits (-)</td>
<td>$$\left[e^{(r-s)(T-t)-1}\right]$</td>
</tr>
<tr>
<td>pound securities (+, forward dollars (+)</td>
<td>$$\left[e^{(r'-s+\varphi)(T-t)-1}\right]$</td>
</tr>
<tr>
<td>forward pounds (-), pound deposits (-)</td>
<td>$$\left[e^{(r-s-\varphi)(T-t)-1}\right] e^{r'(T-t)}$</td>
</tr>
<tr>
<td>dollar securities (+), pound securities (-)</td>
<td>$$\left[e^{(s-\varphi)(T-t)-1}\right] e^{r'(T-t)}$</td>
</tr>
<tr>
<td>forward dollars (+), forward pounds (-)</td>
<td>$$\left[e^{(s-\varphi)(T-t)-1}\right] e^{r'(T-t)}$</td>
</tr>
</tbody>
</table>
Given the assumptions that dollar securities and pound securities are perfect substitutes except for exchange risk and that capital markets are perfect, interest arbitrage insures that

$$r = r' + \varphi.$$  

To see that the familiar interest parity condition must hold, suppose that $r > r' + \varphi$. If this were the case, neither U.S. nor U.K. residents would want to hold pound securities for any purpose. By inspection of the returns described above, it can be seen that dollar securities would dominate covered pound securities and dollar securities combined with forward sales of dollars would dominate pound securities respectively as substitutes for money for residents of both countries. Similarly, selling dollars forward would dominate reducing dollars security holdings and buying pound securities as a means of taking an open position in pounds without running down money holdings. Reducing pound security holdings and buying dollar securities would dominate buying dollars forward as a means of taking an open position in pounds without running down money holdings. Likewise, if $r < r' + \varphi$, neither U.S. nor U.K. residents would want to hold dollar securities for any purpose. The assumptions of this section are the weakest set of assumptions sufficient to insure that interest parity holds. They are not the only set, however. Interest parity would also hold if wealth holders were risk neutral.

The wealth allocation behavior of U.S. residents can now be specified in more detail. U.S. residents have demands for money, $M^d$, for open dollar positions, $D^d$, and for open pound positions measured in dollars, $S^p^d$. The demand for open dollar positions can be satisfied by either U.S. securities or purchases of dollars forward so

$$D^d = [B + Ce^{-r(T-t)}]^d$$

where $C$ represents net forward purchases of dollars by U.S. residents and $Ce^{-r(T-t)}$ is the present value of these purchases. The demand for open pound positions can be satisfied either by U.K. securities or by purchases of pounds (sales of dollars) forward, so
$S R^d = S[F + H e^{-r'(T-t)}]^d$

where $H$ represents net forward purchases of pounds by U.S. residents and $H e^{-r'(T-t)}$ is the present value of these purchases. Given that $r = r' + \varphi$, the returns on all the alternatives facing U.S. residents can be written as functions of $r$ and $r' + s$. We continue to assume that asset demands are homogeneous in nominal wealth so that

$$M^d = m(r, r' + s)W$$  \hspace{1cm} (14)$$

$$[B + C e^{-r(T-t)}]^d = b(r, r' + s)W$$ \hspace{1cm} (15)$$

$$S[F + H e^{-r'(T-t)}]^d = f(r, r' + s)W$$ \hspace{1cm} (16)$$

where

$$W = M^A + B^A + S R^A + C^A e^{-r(T-t)} + H^A e^{-r'(T-t)}$$ \hspace{1cm} (17)$$

and where the superscript $(^A)$ denotes actual holdings of a given asset. Both $b(\cdot)$ and $f(\cdot)$ are the same functions that were used above to describe security demands. The presence of forward contracts makes available more ways of achieving open positions in dollars and pounds, but it does not affect the amount of open positions demanded. A rise in $r$ with $r'$ constant and $\varphi$ varying to maintain interest parity, for example, prompts U.S. residents to switch out of U.S. money into either dollar securities or covered pound securities and to either switch out of pound securities into dollar securities or buy dollars forward.

The demands by U.K. wealth holders for pound deposits, $N^d$, pound denominated open positions in dollars, $\frac{1}{S}[B' + C' e^{-r(T-t)}]^d$, and open positions in pounds, $[F' + H' e^{-r'(T-t)}]^d$ are given by

$$N^d = n'(r - s, r')W'$$ \hspace{1cm} (18)$$

$$\frac{1}{S}[B' + C' e^{-r(T-t)}]^d = b'(r - s, r')W'$$ \hspace{1cm} (19)$$

$$[F' + H' e^{-r'(T-t)}]^d = f'(r - s, r')W'$$ \hspace{1cm} (20)$$

where

$$W' = N^A' + B^A' + F^A' + C^A' e^{-r(T-t)} + H^A' e^{-r'(T-t)}$$ \hspace{1cm} (21)$$
It is assumed that only the U.K. central bank intervenes in the forward market. Its net purchases of forward dollars and forward pounds are $C^{c'}$ and $H^{c'}$, respectively. Net sales of forward dollars (pounds) by the public must be matched by net purchases of forward dollars (pounds) by the public so,

\[
C^{c'} = -C^{A} - C^{A'} \quad \text{(22)}
\]

\[
H^{c'} = -H^{A} - H^{A'} \quad \text{(23)}
\]

Increases in dollar purchases must be equal in value to increases in pound sales, so

\[
dC^{c'} = -VdH^{c'} \quad \text{(24)}
\]

The equilibrium conditions for the model with forward contracts are

\[
\bar{B} - B^{c} - B^{c'} - C^{c'} e^{r(T-t)} = b(r,r'+s)W + Sb'(r-s,r')W' \quad \text{(25)}
\]

\[
S[F - F^{c'} - H^{c'} e^{r'(T-t)} = f(r,r'+s)W + Sf'(r-s,r')W' \quad \text{(26)}
\]

\[
B^{c} + C^{c'} + M^{c'} + A = m(r,r'+s)W \quad \text{(27)}
\]

\[
S[F^{c'} + \sigma'(\bar{R} - R)] + B^{c'} + H^{c'} + S' = Sn'(r-s,r')W' \quad \text{(28)}
\]

\[
r = r' + \sigma \quad \text{(29)}
\]

Only three of the four equations (25)-(28) are independent as can be seen by using (17) and (21) to eliminate $W$ and $W'$, and (22) and (23) to eliminate $C^{c'}$ and $H^{c'}$ and then adding together the four equations.

The model of equations (25)-(29) can be analyzed using the graphical framework described above. The BB and FF schedules are now interpreted as showing pairs of $r$ and $r'$ at which the public is satisfied with open positions in dollars and pounds, respectively. The forward premium is determined graphically by drawing a line with a slope of positive one through the intersection of the BB, FF, MM, and NN curves and extending it until it hits the $r$ axis. If this line hits the $r$ axis above zero, as in Figure 4, the pound is at a premium.

It should be clear that none of the results derived in preceding sections is affected by adding a market in forward exchange. All we need to know to determine the
two interest rates and the spot exchange rate are the supplies of the two moneys and the supplies of net open positions in dollars and pounds available to the public. The composition of the supplies of net open positions is immaterial. The forward premium is determined recursively given the two interest rates which, together with the spot rate, clear the asset markets. Even in the absence of a forward market, \( \omega \) can be determined as the difference between the two interest rates. In this case it can be interpreted as the "shadow cost" of reducing an open position in pounds and obtaining an open position in dollars, a set of actions which has an expected return of \(-s\).

Under the assumptions of this section, the purchase of forward dollars (\( dC' \)) and sale of an amount of forward pounds equal in value (\( dH' = -\frac{1}{V} dC' \)) by the BOE has the same effect as BOL purchases of an amount of dollar securities equal in present value to the forward dollar purchases (\( dB' = e^{-r(T-t)} dC' \)) financed by sales of pound securities (\( dF' = -\frac{1}{S} dB' = e^{-r'(T-L)} dH' \)). Both actions shift \( BB \) and \( FF \) from \( B_0B_0 \) and \( F_0F_0 \) to \( B_1B_1 \) and \( F_1F_1 \) respectively in Figure 4. The final equilibrium is at the same point, say \( a_1 \), in both cases, and the forward premium is reduced from \( \omega_0 \) to \( \omega_1 \).

This result demonstrates our contention that when securities denominated in different currencies are perfect substitutes except for exchange risk, forward market intervention does not provide the authorities with an additional policy tool.

Even when official intervention in the forward market is analytically equivalent to simultaneous intervention in both securities markets, the authorities may find forward market intervention useful for operational or political reasons. A monetary authority might not possess the securities it needs to sell in order to conduct a desired intervention in the securities markets. It was probably for this reason that the German authorities used the forward market to offset the effects of shifts in private portfolio preferences away from dollar securities and toward mark securities under the Bretton Woods System. It may appear impolitic at times
from either a domestic or foreign relations point of view to enter a foreign securities market, so forward intervention may be used instead. These practical considerations, while important, should not deflect attention from the fact that money stocks, interest rates and the exchange rate are affected in the same way by both types of intervention.
This paper was presented at the United States Department of the Treasury Conference on Effects of Exchange Rate Adjustments held April 4 and 5, 1974, in Washington, D.C. It will appear in Clark, Logue, and Sweeney (1976). The part of the paper in which the effects of a devaluation on financial asset holding are discussed is drawn from Girton and Henderson (1973).

The authors would like to express their appreciation to their colleagues in the International Finance Division for useful comments and suggestions. Especially helpful were Don Roper, Robert Townsend, Jeffrey Shafer, and Michael Dooley, whose own work (Dooley (1974)) on the problem considered in the final section of the paper has influenced us significantly. No one but the authors is responsible for the paper's remaining shortcomings. The analysis and conclusions of this paper should not be interpreted as representing the views of the Board of Governors of the Federal Reserve System or anyone else on its staff.

1/ The model used in the first three sections of the paper contains no forward market. A forward market is introduced in the final section of the paper.

2/ The model can be modified to allow both central banks to accept deposits denominated in both currencies so long as some assumptions are made which determine the desired liability composition for each central bank. Some of the implications of introducing a fractional reserve commercial banking system into the model are explored in Girton and Henderson (1976b). There it is shown that, under appropriate assumptions, the central bank and the commercial banking system in each country can be consolidated and high-powered money can be cancelled out as an intrasector item without losing sight of the essential features of the problem under consideration.

3/ The assumption that all wealth holders expect the same percentage change in the exchange rate makes possible considerable simplification in the exposition of the model, but this assumption can be dropped without affecting any of the conclusions of the model.

4/ Wealth holders in each country might regard the two securities as imperfect substitutes for two sets of reasons. First, the two securities are issued in different countries and wealth holders might believe that the returns to the two securities are uncertain and that these returns are not perfectly correlated either because they perceive that fluctuations in economic activity are not perfectly correlated across countries, or because they view governments as having different degrees of "responsibility"; that is, business and political risks might be different in the two countries. Second, the two securities are denominated in different currencies so actual or potential exchange rate movements or exchange controls add exchange risk to the other risks associated with holding "foreign" assets. Either set of reasons is sufficient to insure that wealth holders in both countries would, in general, want to hold well defined amounts of both securities.

5/ The demand functions used in the text can be arrived at from more general demand functions by ignoring interest payments, assuming first degree homogeneity in wealth, and subsuming variables held constant into the functional form.
- 2 -

6/ See Tobin (1969). The U.S. balance sheet constraint is

$$W = B^A + SF^A + M^A = B^d + SF^d + M^d,$$

where $B^A$, $F^A$, and $M^A$ are predetermined actual holdings. This identity and equations (1) through (3) imply

$$M^d = m(r, r' + s)W = [1 - b(r, r' + s) - f(r, r' + s)]W,$$

so

$$b_r + f_r + m_r = b_{r + s} + f_{r + s} + m_{r + s} = 0,$$

where $b_r$ is the partial derivative of $b(r, r' + s)$ with respect to $r$, etc. Similar relationships hold for the U.K.

7/ We assume that the FR does not hold U.K. securities or U.K. money.

8/ The total supply of U.S. securities $\overline{B}$ is equal to the sum of cumulated U.S. government deficits and the value of net claims to the income of the capital stock of the U.S. non-financial corporate sector. $\overline{F}$ might also include consumer debt, but we assume that obligations of ultimate wealth holders to one another are netted out. $\overline{F}$ has a similar interpretation.

9/ Instead of including in $\overline{B}$ and $\overline{F}$ all securities issued in the two countries, it may be useful to restrict attention to some subset of securities. A significant part of short-run financial capital movements seems to be made up of changes in holdings of short-term (liquid) assets. If people quickly balance money and short-term security holdings according to rates of return independently of their holdings of long-term (illiquid) securities, then, for some purposes, defining $W$ and $W'$ to include only short-term securities and money may be useful. The treatment here is theoretical, and no attempt is made to prejudge the question of what is the best empirical definition of allocatable wealth in any particular application.

10/ Adding the left hand sides of (10), (11), (12) and (13) we obtain the total dollar value of all assets available to wealth holders, which can be expressed as $\overline{B} + SF + cR + A + SA'$ since $S = c'/\sigma'$. Adding the right hand sides of (10), (11), (12), and (13) and taking account of the fact that $b + f + m = b' + f' + n' = 1$, we obtain world wealth in dollar terms of $W + SW'$. $\overline{B} + SF + cR + A + SA'$ is identically equal to $W + SW'$, so only three of the four market equilibrium conditions are independent.

11/ Of course, if a devaluation is anticipated, asset demands shift away from the money and securities of the country which is expected to devalue. After the devaluation has occurred, these asset demand shifts probably are reversed if no further devaluation is expected. We consider here the less realistic case of an unanticipated devaluation in order to highlight what we believe to be a necessary modification of conventional arguments. Later we consider the case in which a shift in asset demands occurs at the same time as a devaluation.
There may be other avenues in addition to those we consider below in the text through which an exchange rate change can affect the relative attractiveness of U.S. and U.K. securities. For example, if a country devalues in order to free itself from an underemployment-deficit dilemma situation, investors might expect that increased economic activity resulting from the direct impact of the devaluation and a relaxation of constraints on expansionary policies would lead to a higher return on the securities of the devaluing country. Our model is not well suited for an investigation of these effects, so we abstract from them here.

The devaluation may be reflected in a change in \( \sigma \), the dollar price of outside reserve assets; in \( \sigma' \), the pound price of outside reserve assets; or in both. Of course, at least one currency price of reserve assets must change. The relationship between \( dS \), \( d\sigma \), and \( d\sigma' \) is given by

\[
\frac{dS}{\sigma^2} = \frac{d\sigma}{\sigma^2} - \frac{d\sigma'}{\sigma^2}.
\]

Changes in \( S \), \( \sigma \), and \( \sigma' \) alter the domestic currency value of the initial reserve holdings of both central banks. The size of these effects, the impact effects of a devaluation on the value of reserve holdings, depends upon how much of the devaluation is reflected in \( \sigma \) and how much in \( \sigma' \). We want to assume that these impact effects themselves cause no changes in the money supply of either country, so we assume that the two central banks simply change \( A \) and \( A' \) so as to offset the impact effects of changes in \( S \), \( \sigma \), and \( \sigma' \) on the domestic currency value of their assets,

\[
dA = -R \frac{d\sigma}{\sigma^2},
\]

\[
dA' = -(R - R)\frac{d\sigma'}{\sigma^2} + (B' \sigma' + M' \sigma')dS.
\]

We continue to assume that asset demands are homogeneous of degree one in nominal wealth and that output prices are constant in home currency.

Throughout this section and the next we assume that \( F^A \) and \( B^A \) are positive; that is, we assume that residents of both the U.S. and the U.K. have positive net holdings of foreign securities. If either \( F^A \) or \( B^A \) or both are negative, the description of how the schedules shift when the exchange rate changes must be modified somewhat, and some of the results may be changed.

In graphical terms, the question is whether or not the distance between \( M \) and \( M_0 \) measured along a line through \( (r_0, r^*_0) \) and parallel to the \( r^*_0 \) axis is greater than the distance between \( F^0 \) and \( F^1 \) measured in the same way. An algebraic analysis reveals that the answer to this question is indeterminate. The shift in \( HH \) is given by

\[
\left. \frac{d\frac{F^A}{M}}{dS} \right|_{r=r_0} = -\frac{mF^A}{m^M},
\]

and the shift in \( FF \) is given by

\[
\left. \frac{d\frac{F^A}{M}}{dS} \right|_{r=r_0} = -\frac{mF^A + (bF^A + f'F^A')}{m^M + f'F^A' + f^2'M'},
\]

The expressions for both shifts are positive, and the difference between them may be positive or negative.

The two central banks cannot, of course, attain inconsistent reserve targets.
18/ It was explained in footnote 13 that a devaluation has impact effects on the values of initial reserve holdings. In describing the reserve changes consequent upon a devaluation, we ignore the impact effects and concentrate instead on reserve changes which reflect changes in the two money supplies and changes in central bank holdings of securities.

19/ We have not ranked $a_0$ because the graphical technique is insufficient in this case. However, it can be proved algebraically that the reserve loss for point $a_0$ is ranked between the losses for points $a_1$ and $a_2$. It is shown in Girton and Henderson (1976b) that for any given shock to asset equilibrium, of which devaluation is one example, it is more efficient in terms of the reserve shift required for the attainment of equilibrium for central banks to keep their holdings of securities constant and allow money supplies to change, that is, to rely completely upon what we call pure money adjustment, than for central banks to keep money supplies constant and allow their holdings of securities to adjust, that is, to rely completely upon what we call pure security adjustment. Pure money adjustment alone leads to point $a_1$ in Figure 2, while pure security adjustment alone leads to point $a_2$. It can also be shown that pure money adjustment alone is more efficient and pure security adjustment alone is less efficient than what we call mixed money-security adjustment which leads to points which lie in the four-sided figure in Figure 2, the corners of which are $a_1$, $a_2$, $a_3$, and $a_5$.

20/ Examples can be constructed to show that the same statement cannot be made for the required changes in interest rates.

21/ For a discussion of the effects of an open market operation by one central bank under fixed exchange rates given various possible policy responses by the foreign monetary authorities, see Girton and Henderson (1973) and (1976b). Further analysis of the effectiveness of monetary policy under flexible rates is contained in Girton and Henderson (1976a).

22/ In this case the U.S. asset demands, for example, are given by

\[
M^d = M(r, r' + s),
\]

\[
B^d = B(r, r' + s, W),
\]

\[
SF^d = F(r, r' + s, W),
\]

where $s$ is a parameter; the signs of the interest rates are the same as before, and

\[
\tilde{B}_W + \tilde{F}_W = 1.
\]

23/ Kindleberger (1969) contains a clear statement of this proposition. Dooley (1974) constructs a theoretical model of short term capital movements based on the assumptions used in this section and reaches many of the same conclusions. He tests hypotheses derived from the model on U.S. capital account data.
24/ In the language of footnote 4, it is assumed that business and political risks are perfectly correlated across the two countries. It is also assumed that forward contracts are not subject to default risk.

Introducing a forward market when business and political risks are not perfectly correlated across countries makes the model somewhat more difficult to analyze. However, the major conclusions of the model of the preceding section remain essentially unchanged under plausible assumptions. For a model which contains a forward market and is consistent with the assumption that business and political risks are not perfectly correlated across countries, see Black (1973).

25/ We continue to assume that residents in each country do not hold deposits denominated in the currency of the other country.

26/ The net return can be expressed as

\[ \frac{V}{S} e^{r(T-t)} - 1. \]

27/ The net return can be expressed as

\[ \frac{\bar{S}}{S} e^{r(T-t)} - 1. \]

28/ The net return can be expressed as

\[ \frac{\bar{V}}{V} e^{r(T-t)} - 1 = \left( \frac{\bar{S}}{S} \right) \left( \frac{\bar{V}}{V} \right) e^{r(T-t)} - 1. \]

29/ The net return can be expressed as

\[ \frac{\bar{S}}{S} e^{r(T-t)} - e^{r(T-t)}. \]

30/ The net return can be expressed as

\[ \frac{\bar{V}}{V} - 1 \left( e^{r(T-t)} \right) = \left( \frac{\bar{S}}{S} \right) \left( \frac{\bar{V}}{V} - 1 \right) e^{r(T-t)}. \]

31/ Marston (1973) tests to see whether the interest parity condition is met by Euro-currency interest rates and using monthly data on the Euro-dollar, Euro-pound, Euro-mark, and Euro-Swiss franc rates from the years 1966 and 1968-71 finds that "Euro-currency rates hold quite strictly to the interest parity relationship." Interest parity does not appear to hold between short term money market rates from various countries. Marston has suggested that deviations of interest differentials from forward premia are explained primarily by capital controls. Dooley (1974) has argued forcefully not only that these deviations are due to capital controls, but also that they are independent of the relative supplies of the assets involved. According to this view, there is a set of short term money market instruments traded in various national money markets which are perfect substitutes except for exchange risk when their interest rates are adjusted for the explicit penalties of capital controls or the implicit costs of circumventing them.
32/ Actually, a slightly weaker set of assumptions is sufficient. The securities need not be perfect substitutes except for exchange risk and borrowing and lending rates need not be equal for all market participants. The assumptions must be met for a group of market participants sizable enough that interest parity is maintained.

33/ Forward purchases of dollars represent dollars at time T and must, therefore, be discounted back to time t before they can be added to B which represents the value of bond holdings at time t.

34/ If $C_A$ is positive, $H_A$ is negative, but even when the interest parity condition is satisfied, the expression

$$C_A e^{-r(T-t)} + SH_A e^{-r'(T-t)}$$

is not in general equal to zero. This is because $H_A$ represents promises to pay or receive pounds already made, so it is not, in general, equal to $\frac{1}{V}$ $C_A$ where $V$ is the current forward rate.

35/ Exchange rate changes affect the schedules in the manner described in previous sections if $[F_A + H_A e^{-r'(T-t)}]$ and $[B_A + C_A e^{-r(T-t)}]$ are positive, that is, if residents in both the U.S. and the U.K. have positive net open positions in foreign currency. If either $[F_A + H_A e^{-r'(T-t)}]$ or $[B_A + C_A e^{-r(T-t)}]$, or both, are negative, the description of how the schedules shift when the exchange rate changes must be modified somewhat, and some of the results may be changed. This possible complication does not depend in any way on the presence of a forward market. See footnote 15.
REFERENCES


