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THE GOODS MARKET AND THE LABOR MARKET OF THE MULTI-COUNTRY MODEL

by

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In "Modeling the International Influences on the U.S. Economy: A Multi-Country Framework," a project involving the construction of a multi-country linked model with endogenous exchange rates is described. The structure of the goods and labor markets in a typical or prototype country sub-model, already sketched in that paper, is treated in this paper in detail. The paper is organized as follows: a discussion of accounting, assumptions and methodology is followed by the behavioral equations for various agents within each of the two markets. References to other companion papers in this series will be made as needed.

Specification of a macro model that is compact and that also captures the essential features of domestic and international economic activity is the primary goal of this companion paper. Compactness is a desideratum because a smaller model is easier to estimate, simulate and maintain. Disaggregation may be undertaken at a later date. It is extremely important, however, that the model builder be in full control of his model: he must assimilate it and "have it in his head". Only in this way is it possible to build in and check on desirable model properties. Compactness is also a goal since the purpose of the present model is primarily the analysis of macro policies on macro aggregates. More specifically, monetary policy and

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its influence on economy activity, as it is transmitted in the U.S. and fed back via other countries, is the primary focus. So, for example, detailed treatment of the income side and tax functions in the model is foregone. Similarly, labor markets are treated in a summary way. This does not imply that a casual approach to specification is taken here. On the contrary, every effort has been made to ensure a rigorous, theoretically justifiable and consistent model.

I. Accounting, Assumptions, and Methodology

The multicountry model described in the summary paper is based in part on the assumptions that there is one good produced in each country, and that it is purchased by three agents in every country: households, firms, and non-central bank government. Thus, each of these three agents is supposed to have a demand (possibly zero) for each of the n goods, in being the number of countries in the model. For example, firms produce the domestic good and purchase it and imported goods for investment and inputs. Firms also hold stocks of the domestic good (finished goods, work in progress, and materials) and of imported goods (materials).

The "good" that each country produces in this model is aggregate value added, or GNP (GDP). Since GNP or GDP is actually a bundle of goods and services and since it is output net of inputs, there are some problems involved in jumping from the concept of "production" of a "good" to GDP.

First, exports of services are receipts for travel, transport and the like, and investment income. Thus, exports in the model (like imports) are disaggregated into goods and services; the "output" of investment income is not an operational concept. Domestically, goods and services are aggregated. Export output is of goods only. Thus, while the exported goods and domestically produced goods are identical, the services are not identical. Consequently, it is assumed that there are two "outputs", and therefore two production functions in the model -- one for domestic sales, and one for exports of goods output. The firms producing these two outputs are monopolists in both markets (although their behavior reflects imperfect competition, as discussed below).

Second, intermediate goods play a role as inputs, as imports, and as stocks held by firms. Furthermore, "goods" are really goods and services, so that while changes in stocks (inventories) are solely goods, GNP includes services, and market clearing for goods is a somewhat blurred concept. In this model, a goods concept important for the discussion below is that of domestic absorption: GNP minus net exports, or C + IF + II + G, where the usual symbols denote GNP components (II is change in inventories).

A third issue concerns the origin of "goods" that are purchased as final demand. National accounts data on expenditures are for "apparent" final use of n goods (and services) per final demand category. That is, consumption in the national accounts is the sum of both the domestic and imported components. It is thus clear that a "consumption function" is the summation of household demands for n goods, or is the demand for the "bundle" called consumption. It should be equally clear that an aggregate "import" demand function is the sum of several agents' demands for final and intermediate goods.

The price deflators that correspond to such expenditure components are, consequently, weighted averages of the price of domestic-origin goods and services and those that are imported. Details on the explicit assumptions involved in the deflators used in this prototype submodel will be found in the paper by Howard Howe.[1] In the present discussion, the fact that there is but a single deflator for domestic absorption is important. It is consistent with the assumption that all output is sold domestically at the same price. This means that while the GNP identity still holds in current and constant prices, the components of domestic absorption in constant prices will differ from those found in the national accounts, since each of the components has its own deflator. Their sum will be equal to GNP net of the trade balance in constant prices as found in the national accounts.[2] Deflators for exports and imports that are distinct from that for domestic absorption are used in this model. This is implied by the assumption of a distinction between domestic

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[1]"Price Determination in the Multi-Country Model."

[2]It can easily be shown that an implication of a single domestic absorption deflator is that for every component of domestic absorption, imports make up the same fraction in the total.

output and export output. The imports deflator is for goods and services, and is employed to eliminate the "artifact" prices that are used below, notably the price of domestic sales.

Thus, the standard GNP identities hold:

(1) $GNP \equiv C + IF + II + G + XGS - MGS$

and

(2) $GNPV \equiv P(C + IF + II + G) + XGSV - MGSV$,

where the usual notational conventions are used, except

IF is fixed investment,

II is inventory change,

XGS, MGS denote goods and services exports and imports,

P is the domestic absorption deflator, $\equiv \frac{CV+IFV+IIV+GV}{C+IF+II+G}$,

and V denotes current prices.

II. Behavior of Agents and Markets

While the demands detailed in what follows are supposed to be the summation of a particular agent's demands by country of origin, it will not always be possible to derive them as such. Other considerations may be more important. For example, in deriving the consumption function, intertemporal aspects of the consumption decision are of primary importance. As noted above, however, derivation of the demand for imports will follow as the addition of several schedules.

A. Consumers

It is assumed that there is a particular sequence to the transactions that characterize consumer behavior. Purchasers of goods and services are also sellers of services in the labor market. In order to make the consumption-saving choice, disposable income must be determined. Thus, it is assumed that consumers are all sellers of labor services, and that the transaction "period" begins with a known portfolio of assets (wealth) inherited from the previous period.

The first transaction in the sequence occurs when the would-be consumer offers his labor services in the labor market, and is either employed to the extent he wishes at the given wage rate (he is on his _notional_ supply-of-labor schedule) or not (he is on his _effective_ supply schedule). In general, the supplier of labor will not be able to sell all the labor services he desires. Therefore, as in, e.g., Barro and Grossman (1971), he is contrained off his notional demand schedule for goods and services by the actual labor income received - the notional demand being _inter alia_ a function of expected income based on the sale of his labor services from his notional supply schedule and income from holding his stocks of assets.

This consumer still makes an optimal choice between saving and consuming, but he cannot make it until he knows his gross income and taxes. At that moment, he is at time zero for his planning horizon, and can solve the problem by maximizing an intertemporal utility function subject to a wealth constraint.

An aggregate consumption function is specified, thus ignoring the well known distinction between purchases of durables and the services they yield. Durables purchases are more properly treated as a form of investment. Such purchases _qua_ investment are notoriously difficult to specify at the macro level, however, due to lack of macro data on stocks, cost of capital, and other crucial variables.[1]

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[1]Cf. Houthakker and Taylor (1970), Garcia dos Santos (1972).

In order to capture the effects of monetary policy on consumption expenditures, the life-cycle hypothesis of saving is employed.[1] Following Heien (1972), a specific functional form is chosen for an intertemporal utility function,

$$(A.1) \quad U = U(C_0, C_1, \ldots, C_{N-1}),$$

a function of consumption expenditures in real terms over the next N periods. Heien uses a modified CES form; a modified Cobb-Douglas form is used here to reduce the degree of nonlinearity in estimation. Therefore

$$(A.1') \quad U = \sum_{i=0}^{N-1} \delta^i (C_i - \gamma_i),$$

where δ is a subjective discount factor,

γ_i is "subsistence" or minimum acceptable consumption in period i,

and N is the number of periods remaining in the life of the representative consumer.[2]

The existence of the γ_i in the utility function will yield a consumption function that bears a family resemblance to the persistence or ratchet models of Brown (1952) and Duesenberry (1949), respectively. The optimal plan for a consumer will never involve starvation in any period.

The utility function (A.1') is maximized subject to the following intertemporal budget constraint:

$$(A.2) \quad \sum_{i=0}^{N-1} C_i(1+r_0)^{-i} = \frac{NW_{-1}}{P} + \sum_{i=0}^{N-1} Y_i^e (1+r_0)^{-i} = V_0$$

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[1] See Modigliani and Brumberg (1954) and Ando and Modigliani (1963).

[2] N may be chosen as one half the average adult lifespan of a typical resident.

where V is lifetime (<u>not</u> permanent) real income,

Y_i^e is expected real income in period i,

NW is net worth at the end of the period,

r is the rate of interest (long term).

The notation V_0 implies that at time zero (in planning time) the consumer will choose a consumption path consistent with (A.2) as he now perceives it. Thus, stationary interest rate expectations are assumed. Since expectations are in real terms, however, he incorporates his expectations of inflation automatically in Y_i^e.

It is assumed that

$$(A.3) \quad Y_i^e = Y_0 \, (1+g_0)^i,$$

where g_0 is a growth rate for real income, chosen to be the mean over his past history. Thus, the righthand side of (A.2) is particularized to

$$(A.4) \quad V_0 = \frac{NW_{-1}}{P} + Y_0 \sum_{i=0}^{N-1} (1+r_0)^{-i} \, (1+g_0)^i,$$

where DYP ≡ (GNPV − TV − TRANV − CCAV)/P is substituted for Y_0. This proxy for disposable personal income includes corporate retained earnings, an unfortunate consequence of aggregation of the income side of the model. Maximization of (A.1') subject to (A.2) as modified by (A.4) yields N planning time consumption functions, all of the form

$$(A.5) \quad C_i = \gamma_i + (\sum_{t=0}^{N-1} \delta^t)^{-1} [V_0 - \sum_{t=0}^{N-1} (1+r_0)^{-t} \gamma_t], \; i = 0 \ldots, N-1.$$

Now the consumer is assumed to replan each period, so attention can be focused on (A.5) for $i = 0$.[1] Noting that $(\sum_{t=0}^{N-1} \delta^t)^{-1}$ is a power series in a parameter, δ, it can be represented by a constant, say a_2. A habit

[1]The implication is that i=0 for each observation in the sample period, so that the r_i and g_i that are used are contemporaneous interest and growth rates.

formation hypothesis is employed to explain γ_t:

(A.6) $\gamma_t = a_0 + a_1 C_{t-1}$.

Constancy of tastes and therefore of minimum consumption can be tested via the null hypothesis $a_1 = 0$. Substituting (A.6) and a_2 into (A.5) and using RL for the long-term interest rate yields

(3) $C = a_0 + a_1 C_{-1} + a_2 [V - RRL (a_0 + a_1 C_{-1})] + u$

where $RRL \equiv \sum_{i=0}^{N-1} (1+RL)^{-i}$,

and u is an N.I.D. $(0, \sigma^2)$ error term.

Lifetime income in this model incorporates financial variables in two ways: first, lagged real net worth enters the budget constraint, and second, the long term interest rate is used to discount planned future consumption and expected future real income. Monetary policy may therefore have a powerful influence on consumption.

The Cobb-Douglas form of the intertemporal utility function means that the interest rate alters consumption only through its effect on lifetime income (wealth). Nonetheless, these effects can be significant. The effect on consumption of the interest rate is always negative, since

(A.7) $$\frac{\partial C_t}{\partial RL_t} = a_2 \, DYP \sum_{i=0}^{N-1} - i(1 + RL_t)^{-i-1}(1 + g_t)^i$$

$$- a_2(a_0 + a_1 C_{t-1}) \sum_{i=0}^{N-1} - i(1 + RL_t)^{-i-1}$$

Notice that a_2, DYP, $(a_0 + a_1 C_{t-1})$, RL_t and g_t are all positive, so that a negative influence of the interest rate on consumption depends on a_2 and a_1 being less than one ("required" lifetime income is reduced by an increase in the interest rate, as is lifetime income, but by not as much).

The short-run marginal propensity to consume (out of disposable income) in this model is a decreasing function of the interest rate and positively related to the growth rate of income:

$$\text{(A.8)} \quad \frac{\partial C_t}{\partial DYP_t} = MPC^{SR} = a_2 \sum_{i=0}^{N-1} (1 + RL_t)^{-i}(1 + g_t)^i$$

The long-run consumption function (where $C = C_{-1}$) is given by

$$\text{(A.9)} \quad C_t^{LR} = (1 - a_1 + a_2\, a_1\, RRL)^{-1}(a_0 + a_2[V - a_0\, RRL]),$$

and the long-run MPC is thus

$$\text{(A.10)} \quad MPC^{LR} = MPC^{SR}\,(1 - a_1 + a_2\, a_1\, RRL)^{-1}$$

Assuming that the expression in parentheses is less than one, the long-run MPC will be larger than the short-run MPC, and both will be functions of the interest rate.

For NW_{-1}, the short-run MPC is simply a_2, and the long-run MPC is a_2 deflated by the expression in parentheses in (A.10). Hence, the short-run and long-run propensities to consume out of DYP and NW_{-1} are not identical. The difference arises from the fact that DYP is used in the formation of expectations about future income, whereas NW_{-1} is predetermined. Expectations therefore depend on the contemporaneous interest rate.

Though this section deals with households, both firms and households pay taxes. The aggregate tax function is included next under this heading to follow the rule in exposition that variables that appear on the right-hand of a behavioral equation shall be explained directly below. This approximates the causal flow in the model.

In a complex econometric model, one might want to replicate the tax tables to obtain tax revenues from the tax base. Aside from the fact that this procedure is undesirable from the point of view of maintaining compactness, it requires that all tax rates are present in the model, and that all tax bases and revenues be present as well. Clearly this is not possible in the context of the present model. The question of whether or not it is appropriate in any model is left open, although an attempt for the U.K. has been made by Dorrington and Renton (1974).

Hence, what is called by Klein (1974) an "institutional" relationship is needed; i.e., one that relates tax revenue to the tax base. Since aggregate revenues are used here, the base is net national product. Depreciation is deductable from corporate taxes, and is therefore not included

A simple linear function, therefore, is

$$\text{(A.11)} \qquad TV = b_0 + b_1(GNPV - CCAV) + u,$$

where again, u is a random error term. Such a tax function can be shifted in its intercept term (b_0) with the usual sort of constant adjustment, but it is also desirable to shift its slope for policy simulations. Further, addition of a slope correction for each period will eliminate the error u in simulation, so that the error in this equation in simulation will be entirely due to errors in predicting the base. In other words, it puts the model a little closer to the actual simulation path.

Define

$$\text{(A.12)} \qquad TSL \equiv (TV - \hat{b}_0 - \hat{b}_1[GNPV - CCAV])/(GNPV - CCAV)$$

so that TSL is a slope adjustment, the data for which are derived so that the error in (A.11) is set to zero; the hats denote estimated coefficients from (A.11). The tax equation used in the model for simulation and forecasting is thus

(4) $$TV = b_0 + (b_1 + TSL)(GNPV - CCAV),$$

where TSL is considered to be an exogenous variable.

Transfers are paid from government to firms as well as to households but are included here. They include subsidies, other business transfers, social security and unemployment compensation. Hence, transfers are made dependent on the level of activity and on the number of unemployed, UE. UE is defined as

(A.13) $$UE \equiv (UN/100) \cdot (CU/100) \cdot LFP,$$

where UN is the unemployment rate, CU is the rate of capacity utilization, and LFP is potential labor force.[1] This is necessary because neither employment nor labor force appear explicitly in the model. The transfers function is

(5) $$TRANV = c_0 + c_1 GNPV + c_2 UE + u.$$

[1]The reason UE is defined this way is that LFP serves as the labor force variable in the model. Neither employment nor actual labor force appear in the model; they are subsumed in the reduced form for the unemployment rate; see Section D below. Potential labor force is defined by "blowing up" actual labor force data (not a variable in the model) by the inverse of the capacity utilization rate, implying that labor and capital utilization are at the same rates.

B. Firms

Firms in this model produce the single homogeneous good, GNP, that is used for all purposes. Firms invest, pay taxes, and hold inventories. This activity is summarized in what follows.

The Concept of Output in the Model

Prior to discussing behavioral relationships for firms, the concept of output in the model must be clarified. Gross national product in nominal terms is

(B.1) $GNPV \equiv P \cdot A + PX \cdot X - PM \cdot M,$

where

(B.2) $A \equiv C + IF + II + G.$

GNP is also the value of gross output net of intermediate demands:

(B.3) $GNPV \equiv QDS \cdot PDS + X \cdot PX - DIV - MIV \equiv QD \cdot PD - DIV - MIV + XSV$

where

DIV and MIV are nominal domestic and imported origin intermediate demands, QD is domestic output, distinct from domestic sales, and XSV is services exports.

As discussed in the companion paper on price determination, we have a concept of domestic output (QD) that is net of domestic intermediates but includes imported intermediate goods. This serves two purposes. First, it ensures that the influence of import prices on domestic prices via cost (input) channels is explicit. Second, it makes it possible to specify a production function in quasi-value-added terms.

The phrase "quasi-value-added" means that domestic intermediates are eliminated with a separability assumption, and imported intermediates appear in the dual formulation of the problem that underlies the price equations. The gross output production function is

(B.4) $Q = Q(K, L, MI, DI),$

where DI and MI are quantities of domestic and imported intermediates. Separability for QD means that B.4 can be written as

(B.5) $Q = Q(QD(K, L, MI), DI).$

The discussion above indicates that, in an accounting sense, QD is defined by

(B.6) $PD \cdot QD \equiv QV - DIV.$

In turn, the QD function is assumed to be separable:

(B.7) $QD = QD(GNP(K, L), MI).$

We have chosen a functional form (Cobb-Douglas) that is additively separable, and it therefore satisfies all these assumptions. Thus, the functional form for QD in B.7 is

(B.8) $$QD = \phi \, e^{\rho T} K^{\gamma_1} L^{\gamma_2} MI^{\gamma_3}, \quad \sum_i \gamma_i = 1 .$$

It is not appropriate to include as a produced service net factor payments since these are net income from overseas investments, and are counted in the national income accounts to make the transition from gross national to gross domestic product:

(B.9) $GNP \equiv GDP + XYS \text{ net}.$

GDP is not the basic output concept used in the model, however, in spite of the fact that it is really the only output concept for which data are readily available. Three output components are identified in the model: QXG, output of goods exports, QDS, output of domestic sales (goods and services), and XOS net, output of net exports of non factor payments services. Corresponding to these three components are three prices: PXG, PDS and PXOS. QD is the "bundle" or total of these two components that is produced according to the technology described in (B.8); the corresponding deflator is PD, a weighted average of PXG, PDS, and PXOS (see B.3). While value added is one of the "bundles" that is an <u>input</u> to QD (see B.7), the split on the output side is not subdivided into intermediates and final demand (although input-output accounting and common sense indicate that it could be so split).

Thus, QD is split via a transformation frontier into three components: XOS net, XG, and DS (domestic sales). The functional form of this frontier is a bit different from those used elsewhere in the model; it is a two-level CES function, as used by Sato (1967) for production functions:

$$\text{(B.10)}\quad QD = \gamma\,[\alpha_{xos} XOS_{net}^{\ -\rho} + \alpha_z \{ \beta_{xg}\, QXG^{\ -\rho_z} + \beta_{ds}\, QDS^{\ -\rho_z}\}^{\rho/\rho_z}]^{-1/\rho},$$

where $\rho = (1-\sigma)/\sigma$, $\rho_z = (1-\sigma_z)/\sigma_z$,

Z is the bundle consisting of XG and DS,

σ and σ_z are elasticities of transformation, assumed constant and the α's and β's are allocational parameters.

This two-level function describes a constant elasticity of transformation frontier, on which every point corresponds to a constant level of QD, say $\overline{QD}$:

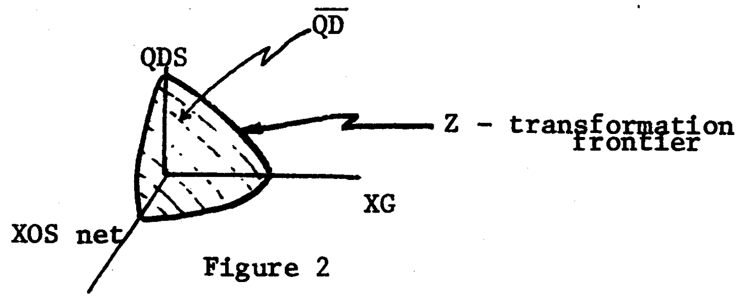

Figure 2

Powell and Gruen (1968) describe such a frontier for two products, which is derived to be:

(B.11) $$X_1^{1-k} + X_2^{1-k} = B(1-k),$$

where $k = 1 + \rho$ in the notation of (B.10),

and B(1-k) is the size of the transformation frontier.

Here a particular scale of output is chosen using QD, so that $B\ (1-k) \equiv QD^{-\rho}$. It is evident from (B.10) that the function for Z is CES:

(B.12) $$Z = \{ \beta_{xg} QXG^{-\rho_z} + \beta_{ds} QDS^{-\rho_z} \}^{-1/\rho_z}$$

A normalization rule for the α's and β's is needed; the most frequently used one is

(B.13) $$\alpha_{xos} + \alpha_z = 1,$$

$$\beta_{xg} + \beta_{ds} = 1.$$

It should be clear that this CET frontier and its projection on the Z -plane is not a production function. It is an aggregator that enables us to go from goods that sell for different prices to an aggregate bundle, QD. Its use will influence the functional forms of the behavioral relations derived below.[1] The derivation of this frontier is a little less restrictive than that for production functions below in that it is allowed for $\sigma \neq \sigma_z \neq 1$. If $\sigma = \sigma_z \neq 1$, (B.10) combined with (B.13) becomes:

$$(B.10') \quad QD = \gamma\ [\alpha_{xos} XOS_{net}^{-\rho} + \alpha_{xg} QXG^{-\rho} + (1-\alpha_{xos}-\alpha_{xg})\ QDS^{-\rho}]^{-1/\rho}.$$

And, of course as σ and σ_z both approach unity, in the limit (B.10) and (B.13) imply the Cobb-Douglas form:

$$(B.10'') \quad QD = \gamma\ XOS_{net}^{\beta_{xos}}\ QXG^{\beta_{xg}}\ QDS^{(1-\beta_{xos}-\beta_{xg})}.$$

Use of (B.10") would simplify our estimation problem. The major equations in which this function plays a role are for fixed investment and prices. In both of these, we need a proxy for resultant expression when QDS is involved, since that variable is unobserved. If the proxy does not depend on the functional form of(B.10), then we may as well use the Cobb-Douglas form. The CET permits more generality in the future, however.

Depreciation

Depreciation, or capital consumption allowances, is the last of the variables mentioned in Section A. It is both an accounting and a physical concept, since the value of depreciation reported depends

[1]Further elaboration of alternatives for the CET frontier is found in the companion paper by Howard Howe, op. cit.

on the method used, the tax lives of the assets being depreciated, and other incentives to minimize corporate tax liability. The physical aspect of depreciation (including capital losses) is proxied by the inclusion of lagged capital stock, K, and the price of capital goods, P.[1] The accounting aspect is introduced with a retained earnings proxy,

(B.14) $RE \equiv GNPV - TV - CCAV - CV,$

where CCAV is depreciation and CV is private consumption. Since CCAV appears on the left hand side of equation (6), it may be substituted out of (B.1) in the equation for CCAV,

(6) $CCAV = d_0 + d_1 K_{-1} + d_2 P_{-1} + d_3 (GNPV - TV - CV) + u.$

Capital stock data will be generated using a benchmark figure, gross fixed investment (IF) and a depreciation rate consistent with that in the investment equation, δ. Beginning with the identity

(B.15) $K \equiv \Delta K + K_{-1},$

where ΔK is net investment, note that

(B.16) $\Delta K \equiv IF - \delta K_{-1}.$

(B.16) may be substituted in (B.15) to obtain

(7) $K \equiv (1-\delta)K_{-1} + IF.$

Fixed investment in this model is aggregated investment in plant and equipment (IPE) and housing investment (IH). In some of the country submodels, it will no doubt be desirable to disaggregate these two components. However, the equation specified immediately below is for IF,

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[1] Again, notice that P is used to deflate all absorption (expenditure components).

although its functional form, strictly speaking, is most appropriate for IPE. Following this equation, an optional housing investment equation is specified.

Investment in Plant and Equipment

Capital stock demand by firms is treated as a factor demand, following the Jorgensonian (1963) neoclassical theory. Firms are monopolists, selling their product in the home market, D and the export market, X. The firm's profits are therefore

(B.17) $\pi = PDS \cdot QDS + PXG \cdot QXG + PXOS \cdot XOS_{net} - W \cdot L - UC \cdot K - PMI \cdot MI,$

where PDS is the price of domestic sales,

QDS is domestic sales (output of the domestic good),

PXG is the price of exports of goods

QXG is export sales of goods (output of the export good),

PXOS is the price of net exports of other services (XOS_{net}),

W is the wage rate,

L is manhours employed

UC is the user cost of capital (rental price),

K is capital stock,

PMI is the price of imported intermediates,

and MI is imported intermediates.

Domestic intermediates are netted out of the production function for QD (see B.5 and B.7 above), which is technologically related to inputs by a Cobb-Douglas production function:

(B.18) $QD = a_o K^{a_1} L^{a_2} MI^{a_3} e^{gt}, \quad \sum_{i=1}^{3} a_i = 1$

It is assumed that producers seek to maximize profits (B.17) subject to the production constraint (B.18), with the CET frontier (B.10") substituted on the output side for QD. The Lagrangian is:

$$\text{(B.19)} \quad \Lambda = \pi - \lambda \, [a_o \, K^{a_1} L^{a_2} \, MI^{a_3} e^{gT} - \gamma \, XOS_{net}^{\beta_{xos}} \, QXG^{\beta_{xg}} \, QDS^{(1-\beta_{xos}-\beta_{xg})}]$$

First order conditions for a maximum are:[1]

$$\text{(B.20)} \quad \Lambda_{QDS} = PDS \; (1 - 1/ \; \eta_D) + \lambda \; (1-\beta xos-\beta xg) \; QD/QDS = 0$$

$$\text{(B.21)} \quad \Lambda_{QXG} = PXG \; (1 - 1/ \; \eta_X) + \lambda \; \beta xg \; QD/QXG = 0$$

$$\text{(B.22)} \quad \Lambda_K = - \; UC - \lambda \; a_1 \; QD/K = 0$$

$$\text{(B.23)} \quad \Lambda_L = -W - \lambda a_2 \; QD/L = 0$$

$$\text{(B.24)} \quad \Lambda_{MI} = - \; PMI - \lambda a_3 QD/MI = 0$$

The goal is to use B.22, the first order condition for efficient capital stock usage, to derive a desired capital stock demand function. B.22 says that the marginal product of capital equals its rental. The marginal product in this case involves output in either of two sectors: DS and XG (see footnote below).

The marginal revenue from sales in either market (as in B.20 and B.21) must be equal, and producers set output to equate marginal revenue with marginal cost. The expression for marginal cost under oligopolistic

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[1] The first order condition for $\Lambda_{XOS\ net} = 0$ is analogous to B.20 and B.21; since the slope of the CET frontier on a particular $\bar{z}$ - line (i.e., for any XOS net) is independent of XOS net, this condition is ignored here. See Howe, op. cit., p. 21 for more detail.

pricing involves demand elasticities, η_D and η_X; see the companion paper by Howard Howe, op. cit. Either B.20 or B.21 can be used in B.22 to obtain the desired capital stock expression. Making the substitution from B.21 for λ yields:

$$\text{(B.25)} \quad PXG\ (1 - 1/\eta_X)\ \frac{QXG}{QD} \cdot \frac{a_1}{\beta_{xg}}\ \frac{QD}{K} = UC.$$

Now (B.25) is solved for K as a desired capital stock demand, denoted K^*:

$$\text{(B.26)} \quad K^* = a_1 \frac{QD}{UC}\ PXG\ (1 - 1/\eta_X)\ \frac{QXG}{QD}$$

$$= a_1\ \frac{QD}{UC} \left(\frac{PXG}{PGDP}\right)^{\varepsilon}_{-1} / \left(1 - \sum_{j \neq i} z_{ij} CU_j/100\right),$$

where, by proxy,

$$\text{(B.27)} \quad \frac{QXG}{\beta_{xg} QD} = \left(\frac{PXG}{PGDP}\right)^{\varepsilon}_{-1},$$

and

$$\text{(B.28)} \quad PXG\ \ (1-1/\eta_X) \doteq 1/\left(1 - \sum_{j \neq i} z_{ij}\ CU_j/100\right).$$

It is assumed that firms are monopolists, as mentioned, so that they will never operate in that region of the demand curve where $|\eta_D|$ or $|\eta_X|$ are equal to or less than unity. The capacity utilization rate, then, is a pressure of demand variable that allows η_D and η_X to vary. For (B.28), the markup times price, which equals marginal cost for the export price, a weighted average of foreign (countries j) capacity utilization rates proxy for pressure of demand. The weights z_{ij} represent the share of exports to country j by country i as a fraction of total exports of country i.

Firms' decisions on the fraction of output to allocate to the export sector are based on lagged relative prices in B.27. An alternative formulation for desired capital stock is obtained by substituting B.20 (the domestic marginal cost condition) in B.22:

$$\text{(B.29)} \quad PDS \frac{(1-1/\eta_d)}{(1-\beta xos-\beta xg)} \frac{QDS}{QD} \alpha_1 \frac{QD}{K} = UC.$$

Substitution in the analogous expression for the fraction of domestic output

$$\text{(B.30)} \quad \frac{QDS}{(1-\beta xos-\beta xg)QD} = \left(\frac{PDS}{PGDP}\right)^{\zeta}_{-1} = 1 - \left(\frac{PXG}{PGDP}\right)^{\varepsilon}_{-1},$$

and for the domestic markup, analogous to B.28,

$$\text{(B.31)} \quad PDS \quad (1-1/\eta_D) \cong 1/(1 - \frac{CU}{100}),$$

in B.25 yields desired capital stock:

$$\text{(B.32)} \quad K^* = \alpha_1 \frac{QD}{UC} \left[1 - \left(\frac{PXG}{PGDP}\right)^{\varepsilon}_{-1}\right] /(1 - CU/100).$$

The expression involving PDS in B.30 might be used, but then the following identity has to be substituted for PDS:[1]

$$\text{(B.33)} \quad PDS \equiv P^{1/\beta} \, PM^{(\beta-1)/\beta},$$

which results from

$$\text{(B.34)} \quad \ln P = \beta \ \ln PDS + (1-\beta) \ \ln PMF,$$

and

$$\text{(B.35)} \quad \ln PMF = \mu \ \ln PMG + (1-\mu) \ \ln PMS = \ln PM.$$

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[1]As detailed in "Price Determination in the Multi-Country Model," PDS is an "artifact" price, the price of domestic sales, related to the absorption and final imports deflators as in B.34. PMF, it turns out, is equivalent to PM, the deflator for all imported goods and services, since it is a weighted average of PMG and PMS. (PMS and PMG are discussed in the companion paper, "Price Determination in the Multi-Country Model.")

Given a prior estimate of β, this is fine; otherwise the nonlinearity in B.32 becomes severe.[1]

It is appropriate to use GDP as a proxy for QD. It is still a proxy and not exact, because GDP does not include imported intermediates, whereas that is clearly one of the inputs in QD from B.7.

To implement empirically (B.26) or (B.32), theories of adjustment, of gross investment vs. net, and of the determination of UC are needed.

Adjustment from desired to actual capital stock

From the desired stock, first differences yield desired change, or desired net investment. The adjustment to actual from desired net investment takes place over time; this adjustment is represented by a distributed lag:

(B.36) $\Delta K = e_0 + E(L)\ \Delta K^*,$

where E(L) is a lag operator.

Replacement investment

Replacement investment is done to offset depreciated capital stock at a rate δ consistent with that in the capital stock identity (7); where IF is gross fixed investment:

(B.37) $IF = e_0 + E(L)\ \Delta K^* + \delta K_{-1}.$

[1] This also explains the use of PGDP (or PGNP) for PD; PXOS is unobserved, and B.33 is needed for PDS.

Rental price of capital (user cost)

The rental price of capital is derived from Jorgenson's neoclassical theory in which the price of a new capital good is equated to the present discounted value of the service flow from the stock:

(B.38) $$q(t) = \int_t^\infty e^{-r(s-t)} UC(s)\, e^{-\delta(s-t)} ds,$$

where q is the price of a new capital good, and

r is the interest rate.

Jorgenson shows that maximization of the present discounted value of the firm implies

(B.39) $UC = q(r+\delta) - \dot{q}$.

Jorgenson assumes, and he is followed here, that $\dot{q} = 0$. Thus, the simplest form of UC in the present notation is

(B.39') $UC \equiv P(RL + \delta)$.

Extensions to this can be made for allowance for investment tax credits and the tax write-off on depreciation, see Hall and Jorgenson (1967), (1972). The modified formula is

(B.40) $UC = q(r+\delta)(1-k)(1-uz)/(1-u)$

where u is the tax rate (corporate)

k is the tax credit rate

z is the present discounted value of the depreciation allowance for tax purposes:

(B.41) $z = \int_0^\infty e^{-rs} D(s)\, ds,$

where D(S) is the depreciation function. A straight-line depreciation function is

(B.42) $D(s) = \frac{1}{r\tau}(1-e^{-r\tau})$

where τ is the life of the equipment. This yields

(B.43) $UC = [(1-k)/(1-u)]\ P(RL+\delta)[1- \frac{u}{RL\tau}(1-e^{-RL\tau})].$

(B.43) may be used in those cases where it figures importantly, as for the U.S.

<u>The Complete Investment Function</u>

Putting these pieces together, and substituting GNP for QD yields

(8) $IF = e_0 + E_1(L)\Delta\{[\hat{a}_1(GNP) \cdot (1 \ -(\frac{PXG}{PGNP})^{\varepsilon}_{-1}) \ / \ (1-CU/100)]/UC\}$

$+ \delta K_{-1} + u,$

or

(8') $IF = e'_o + E'_1\ (L)\ \Delta\{[\hat{a}_1(GNP) \cdot (\frac{PXG}{PGNP})^{\varepsilon}_{-1} /(1-\sum_{j\neq i} z_{ij}CU_j/100)]/UC\}$

$+ \delta' K_{-1} + u',$

where

(9) $CU \equiv (GNP/GNPP) \cdot 100,$

(from B.39')

(10) $UC = P(RL+\delta),$

GNPP is capacity output, and the coefficient $\hat{a}_1$ is the aggregate output elasticity of capital. This average output elasticity $\hat{a}_1$ can be measured from factor shares data as in Klein and Preston (1967), renormalized so that the subfunction

(B.44) $\quad GNP = \psi \, e^{\zeta T} K^{\alpha} L^{(1-\alpha)}$

exhibits constant returns to scale, where $\alpha = a_1/(a_1 + a_2)$.[1]

Housing Investment

As mentioned above, housing investment may be separated from plant and equipment investment. The specification is detailed in what follows. It will be up to the constructor of a country submodel whether or not this disaggregation is made.

Suppose that a production function similar to that used above (B.35) relates output of the housing sector to the inputs it uses:

(B.45) $\quad QH = Ae^{gT} KH^{\alpha} L^{(1-\alpha)}$,

where QH is output of housing services (=CH, consumption of these services),

KH is the stock of housing,

and L is labor inputs into the housing industry.

Profits of the housing sector are

(B.46) $\quad \pi_H = P \cdot QH - UCH \cdot KH - W \cdot L$,

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[1] $\hat{a}_1$ = mean (non-labor income/GNPV).

where UCH is the rental price of housing, and the same wage rate (on average) prevails in this sector as in the rest of the economy.

Using arguments analogous to those developed for IF, desired housing stock is derived as

$$(B.47) \quad KH^{*} = \alpha \frac{P \cdot QH}{UCH} .$$

The same pieces as were employed for IF are needed to complete the picture here: an adjustment theory, a replacement investment theory, and a UCH theory. In addition, a theory for QH is needed. The first two are also analogous to those used for IF, yielding

$$(B.48) \quad IH = f_0 + F_1(L)\ \Delta KH^{*} + \delta_H\ KH_{-1}.$$

The theory for UCH is analogous to that for fixed investment, except that here, the earlier Jorgenson formula (1963) involving the interest payments tax deduction is used:

$$(B.49) \quad UCH \equiv P \cdot (\frac{1-uV}{1-u}\ \delta_H + \frac{1-uX}{1-u}\ RL),$$

where again P is the price of new housing (a new capital good),

V is the percentage of depreciation possible to write off against tax liabilities

X is the percentage of interest payments allowable as write offs,

u is the personal income tax rate (proxied by $[b_1 + TSL]$ from equation (4) here).

CH in the national income accounts includes domestic and imported services for owner-occupied housing (imports being fuels), so it reasonably accurately represents the services yielded by the stock. The demand for CH is derived from a homogeneous indirect translog utility function with arguments CH, CM (consumption of imported goods) and CO (other consumption).[1] This tripartite division reflects the need for a CH and for a CM proxy (the latter is used for derivation of imports demand). Given the consumption function (3), the components of consumption are allocated exhaustively by the translog demand system, of which the following equation is a part:

$$\frac{CHV}{CV} = \alpha_H + \beta_{HH} \ln(\frac{UCH}{CV}) + \beta_{HM} \ln(\frac{PMF}{CV}) + \beta_{HO} \ln(\frac{PO}{CV}). \quad \text{(B.50)}$$

This equation is not estimated. Rather, proxies for the unobserved variables (PMF, PO) are determined, and the entire function is inserted in (B.47) to substitute for QH. As in (B.35), ln PMF = ln PM, and it is assumed that

$$\ln PO = \gamma \ln P. \quad \text{(B.51)}$$

Defining

$$CH \equiv CHV/UCH, \quad \text{(B.52)}$$

(B.34), (B.51) and (B.52) can be substituted in (B.50) to obtain

$$\hat{CH} = \frac{CV}{UCH} [\alpha_H + \beta_{HH} \ln(\frac{UCH}{CV}) + \beta_{HM} \ln(\frac{PM}{CV}) + \beta_{HO} \gamma \ln(\frac{P}{CV})] \quad \text{(B.53)}$$

[1] See Christensen, Jorgenson and Lau (1975) for details on the translog family.

Now (B.53) and (B.44) can be substituted in (B.47), which, substituted in turn in (B.48) yields the estimating equation

$$(B.54)\quad IH = f_o + F_1(L)\ \Delta[\alpha \frac{P\cdot \hat{CH}}{UCH}] + \delta_H\ KH_{-1} + u.$$

Notice that the parameter α remains here - in the IPE (IF) equation α_1 and β_1 were estimated a priori. In this case, it is a constant that will scale the lag distribution coefficients in $F_1(L)$.

Housing stock data are derived from a benchmark and gross investment, analogous to the stock of plant and equipment:

$$(B.55)\quad KH \equiv (1-\delta_H)\ KH_{-1} + IH.$$

Capacity Utilization

Capacity output in the typical country submodel is specified as the "true" production frontier, that is, at full employment. The data for capacity output (GNPP) are generated by

$$(B.56)\quad GNPP \equiv GNP \cdot 100/CU,$$

where CU is the Wharton capacity utilization index for the country in question. This is a peak-to-peak index, with 100 representing a peak.[1] The production function is Cobb-Douglas with constant returns to scale, consistent with that used in the rest of the model where T is a time trend, and EP is potential employment:

$$(B.57)\quad \ln GNPP = \ln A + gT + \hat{\alpha}\ \ln K + (1-\hat{\alpha})\ \ln EP + u.$$

[1] See Klein and Summers (1966), Klein and Preston (1967).

A prior estimate of $\hat{\alpha}$ can be had à la Klein and Preston (1967) from factor shares data:

(B.58) $\hat{\alpha}$ = mean (non-labor income/GNPV).

The estimating equation is thus

(11) $\ln GNPP - \hat{\alpha} \ln K - (1-\hat{\alpha}) \ln EP = \ln A + gT + u.$

EP is derived in the section on the labor market below (Section D).

Producer Behavior in Disequilibrium

The structure of the prototype country submodel described thus far has been, with the exception of the equation for capacity output above, exclusively demand oriented. Little has been said about the disequilibrium behavior of producers. Producers' short-run output decisions and supply-demand disequilibria are represented by the specification of the next equation - that for changes in stocks (inventories). As mentioned in the summary paper, the goods market in the submodels are assumed to clear - but "clear" means that effective and not necessarily notional excess demand are zero, see Clower (1965), Tucker (1969), Barro and Grossman (1971). As described above, a Marshallian mechanism is at work, in which households are effectively constrained off their notional goods demand schedules by the level of actual labor income received in a disequilibrium labor market. In turn, this level of labor income corresponds to effective, not notional, labor supply schedules. This is where the sequence begins. The system is dynamically recursive, in that no recontracting in the period takes place in the labor market as a result of, say, inflation. That adjustment takes place in the following period, when the sequence of transactions begins again.

In this model, as implicitly in many macro models, what makes the goods market "clear" is not price adjustment - although prices do eventually adjust to partially choke off excess demand. In the short-run, however, it is the adjustment of quantities that clears the goods market. Producers cut back on production in the semi-short-run when inventories of finished goods build up faster than they would like, due to realized demand being lower than what was previously expected. And in the very short run, then, it follows that inventories take up all the slack. With these two adjustments, it is possible to have a good market that clears.

Specifically, rather than have a separate output decision rule function, as is done in some macro models that are somewhat more disaggregated than is this one,[1] output (GNP) is equal to the sum of the components of national expenditure, including change in stocks. Thus, the inventory change equation in this model embodies two decisions: that to hold inventories for speculative reasons, and that to adjust the rate of production.

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[1] See McCarthy (1972). See also most of the theoretical literature on inventories; e.g. Bryant (1975), Holt, et. al. (1960), Childs (1969), and Hay (1970).

A central problem in a one-good macro model is that the dynamics underlying the components of output are not uniform. Specifically, changes in output are not synchronous with changes in orders for plant and equipment investment. An order for such output will be reflected in a sustained increase in output until completion. Work-in-progress inventories will build up as the plant or equipment nears completion and then decline - inventory changes are first positive, then negative. An even sharper rise and decline cycle is induced in raw materials inventories. In fact, these inventories may first dip, as producers use materials on hand, then rise, and finally decline again.

Suppose that producers follow the simple output decision rule (for <u>intended</u> supply) given in B.59:

$$\text{(B.59)}\quad PR = \alpha + \beta C_{-1} + \gamma(L)\, NO + \delta \left[\frac{1}{4} \sum_{i=1}^{4} (S/SL)_{-1} - (S/SL)_{-1}\right] \cdot SL,$$

where NO are manufacturers' real new orders, SL are sales of goods,

$\gamma(L)$ is a polynominal lag operator, PR is production (QD),

and S are inventory stocks.

Output is thus assumed to respond to lagged consumer demand (C)[1] and over time for orders by manufacturers (plant and equipment). The average inventory (stock) sales ratio over the preceding year represents the desired stock-sales ratio; producers adjust with a lag to the gap between desired and actual stocks/sales, with δ being the adjustment coefficient.[2]

[1] Producers may adjust quickly, but there are lags in the transmission of information about demand.

[2] Of course, alternative proxies for the desired stock/sales could be used, for example, involving the interest rate and price expectations.

NO, new orders, are determined by consumption demand (again, a quick adjustment), and the determinants of anticipated plant and equipment investment:

(B.60) $$NO = \sigma + kC + \Theta(L)\ \Delta(GNPV/UC)_{-1} + \lambda K_{-1}$$

where "the determinants" of investment are crudely represented by the change in GNP deflated by user cost and lagged capital stock. Obviously, one cannot use "future" investment in this equation without making it computationally difficult to solve the model (it becomes a dynamic programming problem).

Substituting B.60 into B.59, and letting $D^* = C+IF+XG = SL$[1] gives

(B.61) $$PR = \alpha' + \kappa(L)C_{-1} + \vartheta(L)\ \Delta(GNPV/UC)_{-1} + \lambda(L)K_{-1} + \delta\left[\frac{1}{4}\sum_{i=1}^{4}(S/D^*)_{-1} - (S/D^*)_{-1}\right] \cdot D^*$$

Inventory change *ex post* is the difference between production and sales. In an open economy, imports must be added as a source of supply. Thus, following McCarthy (1975), in real terms:

(B.62) $$\Delta S \equiv PR + MG - SL,$$

where ΔS is the change in stocks,

PR is production of goods,

MG is merchandise imports,

SL is sales of goods.

Now the change in stocks has two well-known components: planned (SP) and unplanned (SU) changes. In turn, following Caton and Higgins (1974), the unplanned changes have two components: that due to buffering

[1] Government purchases of goods and services, G, are not included in D* because the bulk are wages, salaries and other services. If goods purchases could be identified (as in the U.S. government defense orders data), they could also be included.

the gap between actual domestic supply and actual demand, ex post, and that due to unexpected imports. Thus,

(B.63) $\Delta S = \Delta SB + \Delta SM + \Delta SP = \Delta SU + \Delta SP.$

It becomes clear that B.62 is a behavioral relationship ex ante if ΔSP is substituted for ΔS:

(B.62') $\Delta SP = PR^e + MG^e - SL^e,$

where the right hand side variables are now expected values. Substituting B.62' into B.63 yields

(B.64) $\Delta S = \Delta SB + \Delta SM + PR^e + MG^e - SL^e.$

Obviously ΔSB and ΔSM are both unobserved. Analogously to Caton and Higgins, ΔSM is represented by

(B.65) $\Delta SM = \phi MGU,$

where MGU is the vector of residuals from the imports of goods equation estimated below in Section C.[1]

From (B.63) and (B.64) it is apparent that

(B.66) $\Delta SU = \Delta SB + \Delta SM = \Delta S - \Delta SP$

$$= (PR - PR^e) + (MG - MG^e) - (SL - SL^e).$$

Naturally, the gap between actual and expected goods imports (the second term of the second line of B.66) is represented by ΔSM, for which (B.65) provides a proxy representation. Then ΔSB, the domestic buffering component of unexpected ΔS, either takes up the slack between actual and expected sales (complete buffering) or producers adjust production

[1]Deflated by PMG.

somewhat in the current period. That is, since

$$(B.68) \qquad \Delta SB = (PR - PR^{e}) - (SL - SL^{e}),$$

if $PR = PR^{e}$, obviously $\Delta SB = -(SL - SL^{e})$. To generate a buffering rule (incomplete buffering) for producers, suppose that

$$(B.69) \qquad \Delta SB = \Theta\,(PR - PR^{e}), \; -\infty < \Theta < -1$$

which implies incomplete buffering since substitution of (B.68) in (B.69) gives

$$(B.70) \qquad (PR - PR^{e}) = -\frac{1}{\Theta-1}(SL - SL^{e}),$$

which, for the range of Θ given in (B.69) implies incomplete adjustment of production to the sales gap.

To obtain a representation of ΔSB, under the incomplete buffering hypothesis, begin with actual production from the definitions of ΔS and ΔSP:

$$(B.71) \qquad PR = SL + \Delta SP + \Delta SU - MG$$

Subtracting (B.71) from PR^{e} gives

$$(B.72) \qquad PR^{e} - PR = (PR^{e} - MG) - (SL + \Delta SP) - \Delta SM - \Delta SB,$$

and substitution of (B.69) into (B.72) yields, where

$\mu = (1 - \Theta)/\Theta$, or $\Theta = -1/(1-\mu)$:

$$(B.73) \qquad \Delta SB = \mu\,[(PR^{e}+MG)-(SL+\Delta SP) - \Delta SM]+u, \; 0 \leq \mu < 1,$$

where u is a random error term.

Rearranging (B.71) gives

(B.71') $SL + \Delta SP = PR + MG - SU,$

and thus, from (B.65), (B.66) and (B.73),

$$\text{(B.74)} \qquad \begin{aligned} \Delta SU &= \Delta SB + \Delta SM \\ &= \mu[PR^e + MG - (PR + MG - \Delta SU) - \Delta SM] + \Delta SM \\ &= \mu[PR^e - (PR - [\Delta SU - \phi MGU])] + \phi MGU + u. \end{aligned}$$

Also, by definition, $\Delta SU = \Delta S - \Delta SP$, so that

$$\text{(B.75)} \qquad PR - [\Delta SU - \phi MGU] = PR - [\Delta S - \Delta SP - \phi MGU] = DSAL + \Delta SP$$

where $DSAL = PR - \Delta S + \phi MGU$. Thus, from (B.63), (B.74),

$$\text{(B.76)} \qquad \begin{aligned} \Delta S &= \Delta SP + \mu[PR^e - (DSAL + \Delta SP)] + \phi MGU + u \\ &= (1-\mu)\Delta SP + \mu[PR^e - DSAL] + \phi MGU + u \\ &= (1-\mu)(PR^e + MG^e - SL^e) + \mu[PR^e - DSAL] + \phi MGU + u \\ &= PR^e + (1-\mu)[MG^e - SL^e] - \mu DSAL + \phi MGU + u. \end{aligned}$$

Now substituting (B.61) for PR^e, $\eta \frac{1}{4} \sum_{i=1}^{4} MG_{-i}$ for MG^e, $D^{**} = \frac{1}{4} \sum_{i=1}^{4} (C + IF + XG)_{-i}$ for SL^e, and $GNP - G$ for PR in

$$\text{(B.77)} \qquad DSAL \equiv GNP - II + \phi MGU - G$$

for DSAL into (B.70) gives[1]

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[1]The two equations (MG,II) are recursive in that order, in that MGU appears in the equation for II(as does MG), but neither II n or components thereof currently appear in the MG equation. Caton and Higgins (1974) include ΔSP as an explanatory variable in their imports equation. In the present case, (B.61) could be substituted in (B.62') and solved for a $P \cdot \Delta SP$ that would be added to DSV in the imports equation below. Note that (B.62') would involve actual values for SL, ηMG for MG^e, and (B.61) for PR^e. Note also that DSAL is close to $D^* = C + IF + XG$.

(12) $$II = \alpha' + \kappa\ (L)C_{-1} + \Theta'(L)\ \Delta\ (GNPV/UC)_{-1}$$

$$+\ \lambda(L)K_{-1}\ +\ \delta[\ \frac{1}{4}\sum_{i=1}^{4}(S/D^*)_{-1} - (S/D^*)_{-1}]\ \cdot D^*$$

$$+\ (1-\mu)\ \eta\ \frac{1}{4}\sum_{i=1}^{4}MG_{-i}\ -\ (1-\mu)\ D^{**} - \mu DSAL + \phi MGU + u,$$

where D^* is C + IF + XG, as before,

II is ΔS, and

(13 $$S \equiv S_{-1} + II$$

is constructed using a benchmark for S_0. of course,

(B.78) $$IIV \equiv II \cdot P.$$

C. International Transactions

Exports

Exports of goods and services in the national income accounts do not exactly match the sum of exports of goods, of factor income receipts and other service income receipts. The discrepancy is due to the fact that the data are collected on a somewhat different basis. For example, for the U.S., unilateral military arms shipments are counted as exports in the BOP data, but not in the NIA data.

As a result, a "bridge" equation is used to reconcile the two totals which would otherwise be an identity:

(14) $XGS = g_0 + g_1[XG + (XSYV + XSOV) / PXS] + u$,

where XGS is exports of goods and services, (NIA),

XG is exports of goods (BOP),

XSYV is factor income receipts,

XSOV is other services exports (travel, transport, insurance),

and PXS is (XGSV - XGV) / (XGS - XG).

In a complete model of world trade, the "centerpiece" is the matrix of trade shares, denoted A.[1] If the share matrix (exports per unit of imports from country i to country j) is explained, and we are given from elsewhere in the model an n-vector of goods import demands, assuming the trade matrix is adjusted for f.o.b./c.i.f. differentials, the n-vector of the exports follows from the matrix identity

(C.1) $X \equiv AM.$

Although the A matrix is nxn, since

(C.2) $\iota' A = \iota',$

only (n-1)xn of its elements need be explained.[2] Bilateral trade flows modeling, even if simplified via a Resnick-Truman (1975) tree approach, in which prior separability assumptions about trade groupings are used, is beyond the scope of this model at the present time, however.

Instead, a technique closer to that of Project LINK is used.[3] This requires n-1 separate export demand equations.[4] In its original form, it also requires all n import demand equations. This is fine for LINK, a world model, but in the present study, the rest of the world and thus, the determinants of its import demands, are not subjects of interest per se. The following modification to the LINK method will handle the ROW imports difficulty, given that is is undesirable to build a complete

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[1] See Hickman (1973), Rhomberg (1970), Taplin (1973).

[2] ι is the unit vector.

[3] See Ball (1973).

[4] n equations are required if the data yield $\Sigma X - \Sigma M \neq 0$.

ROW model, and given that even the determinants of ROW's imports are not included in our model, and given that these determinants are likely to be impossible to specify with any accuracy for the majority of ROW countries.

Partition equation (C.1) into a "model" sector and an ROW sector, denoted F and R, respectively (R may, in this case, consist of one or more countries, depending on whether or not it is judged desirable to break out developed countries from LDC's). Thus,

(C.1')

$$\left[\begin{array}{c} X_F \\ \hline X_R \end{array}\right] = \left[\begin{array}{c|c} A_{FF} & A_{FR} \\ \hline A_{RF} & A_{RR} \end{array}\right] \left[\begin{array}{c} M_F \\ \hline M_R \end{array}\right],$$

or,

(C.3) $X_F = A_{FF}\ M_{FF} + A_{FR}\ M_R,$

and

(C.4) $X_R = A_{RF}\ M_F + A_{RR}\ M_R.$

Solve (C.4) for M_R:

(C.5) $M_R = A_{RR}^{-1}\ (X_R - A_{RF}\ M_F)$

substitute (C.5) in (C.3):

(C.6) $X_F = A_{FF}\ M_F + A_{FR}\ A_{RR}^{-1}\ (X_R - A_{RF}\ M_F)$

$$= (A_{FF} - A_{FR}\ A_{RR}^{-1}\ A_{RF})M_F + A_{FR}\ A_{RR}^{-1}\ X_R.$$

Given the (n-k) vector M_F and the k vector X_R, a foundation for the LINK approach can be built without considering M_R, which has been substituted out.

In the LINK approach, contemporaneous shares cannot be used; lagged shares are employed. This is because the n(n-1) elements of A are not identified. In LINK, the <u>difference</u> between the right and left-hand side of (C.1) is explained by relative prices. This is a sort of linear expenditure system with no intercepts. However, the point of estimating this quasi-identity is not only that explaining the shares matrix is undesirable, but that the data do not necessarily add up. A trade shares matrix adjusted for f.o.b./c.i.f. differentials covers a multitude of other sins, such as changes in coverage of the data. Equation (C.6) is used to generate data (converted to local currency units) for the following simple constant price export demand equation for goods:

(15) $$XG = h_0 + h_1\ XGVD/PXG + H_2(L)(PXG/PC) + u,$$

where $PC_i \equiv \sum_k R_{ik} \cdot PXF_k\ a_{ki}$,

$R_{ik} \equiv R_i/R_k$,

(local currency/dollar exchange rates)

and XGVD is X_F from (C.6).

This equation has the same rationale as the LINK export equations. However, instead of imposing a unit coefficient on XGVD/PCG by making the dependent variable XG - XGVD/PXG, h_1 is estimated here to further allow for data discrepancies. It may well be that the shipping lags from exports to imports will raise a problem with this equation: the imports are recorded one quarter after exports. In that case, XGCD must be led one quarter, and the model must be iterated back and forth between the two periods. This problem does not arise in LINK, which is an annual model. Hopefully, it will not arise in the present model.

The equation might be specified in logs, so that exchange rates can be broken out from other terms. The relative price term includes competitiors' prices, a weighted average of all other countries' export prices.

In estimating the model, trade matrices for each observation may be used; in simulation, the previous period's trade matrix must be used since A is not endogenous. The A matrix will be updated recursively using the RAS method, so that its row and column sums equal the export and import vectors, respectively. By recursive is meant that after the model is solved for period t, the A matrix is updated. This matrix, denoted A_t, will then be used in the solution for period t+1.

A second problem in this model is that M_R is really determined residually, although the constraints on the system will not permit it to act as a "sink" by assuming unreasonable values. A constraint that must be imposed to assume this, however, is that the long-run elasticities in (15) should both be unity in absolute value. Specification of the equation in logs and imposing unity on h_1 is easily done; then the sum of the lag coefficients in $H_2(L)$ should be minus one.

Exports of goods in nominal terms are obtained from the identity

(16) $XGV \equiv XG \cdot PXG.$

Exports from the rest of the world are required to close this system. Having rejected using a ROW imports function that would require inclusion of ROW "domestic" variables, the obvious move is to make the export equation a function of variables in the model; i.e., those belonging to the group of countries included in the model. Such an ROW export demand equation is the complement of that of a single country which depends on weighted averages of (exogenous) foreign variables. Here, the weighted averages are of variables endogenous to the present model, and are "foreign" to the ROW.

Denoting by F a foreign weighted (with trade weights) average variable, we have (PP is the primary product price, and PC are competitive prices):

$$(17)\quad XGV_R = m_0 + m_1\, FGNPV + m_2\, FP + m_3\, FR + m_4\, PP/PC + u,$$

R representing exchange rates, for exports of goods. Now, exports from the ROW obviously include intratrade; that is, trade among the ROW countries. Possibly included in the F variables might be some from the important ROW countries or country groups, such as France, Italy, Switzerland among the developed countries, or OPEC among the LDC's. Reserves of the latter group might prove particularly convenient in this equation. Admittedly, this equation is somewhat ad hoc, but its flexibility allows inclusion of variables that a more formal functional form might preclude, such as reserves.

In a manner analogous to exports, the NIA and BOP imports data do not match precisely; hence the following bridge equation relating NIA imports to the BOP sum:

(18) $$MGS = n_0 + n_1[MG + (MSYV + MSOV)/FP] + u.$$

FP is a weighted average of foreign (to the country for which the equation is specified) prices, similar to that employed in (17).

Identically,

(19) $$MG \equiv MGV/PMG,$$

where PMG is the price index for imports of goods. The implication from this identity is that import demands are specified in nominal terms. The reasons for such specification will become clear in the development that follows.

As argued in detail in Berner (1976), the demand for imports is the sum of several demands across at least two important agents: producers and consumers. Data by end-use category have made it possible to distinguish those two demands by agents, and further, by type of usage; e.g., investment and intermediate demands by producers. Unfortunately, this project cannot presently afford the increased size of the model that would be necessitated by such disaggregation, since corresponding prices and exports (and trade matrices) would also require disaggregation. The case for aggregation is based on compactness. In specifying the equation, however, attention is given to the components of demand.

Imports demand is an amalgam of at least three demands: that of consumers and that of producers for intermediates and for plant and equipment investment.[1] Each of these three demands is derived below as if it were a single demand equation in a complete system of demand equations, where it is supposed that each demand system allocates a total explained elsewhere in the model among its (inter alia) domestic and imported components. The demand equations in the consumer demand system were sketched in the discussion of housing investment above; see equation (B.32). These are income-compensated, while the factor demands of producers are not, since the latter are derived from a cost function, to be minimized subject to target output which is in volume terms.

That consumers' demands are income-compensated while producers' demands are not poses problems for aggregation. On the basis of letting the data tell us which type of demand is more important (the general case is that both are), minor variations on the functional form are proposed in the

[1]Producers who order imports to stock and hold them might be treated as having an inventory demand for imports. In fact, consumers do not really demand imports, they demand imported goods sold to them by importers. In what follows, the retailer veil is stripped away, although inventory behavior is taken into account explicitly. Notice that imports and inventories are thus highly simultaneous, see Section B on inventories.

following discussion to allow for the importance of both types of demands.

1. Consumption demand is similar to the housing services demand in equation (B.32); it is one in a system of equations derived from a homogeneous indirect translog utility function. The desired demand equation is:

$$(C.7) \quad \frac{CMV^*}{CV} = \alpha_{MC} + \beta_{MH} \ln(\frac{UCH}{CV}) + \beta_{MM} \ln(\frac{PMF}{CV}) + \beta_{MO} \ln(\frac{P}{CV}),$$

where CMV is the value of consumers' imports,

UCH is the user cost of housing,

and PMF is the price of final goods imports, an artifact.

Including UCH, the user cost of housing, is predicated on the specification of a housing investment demand equation that requires a proxy for housing services demand. Notice that the translog form makes the dependent variable a share, while prices used to explain the shares are in logs.

2. Investment imports demand is specified as a factor demand, derived minimizing a homogeneous translog cost function. The "inputs" are domestic and imported capital stock, and the "output" is total capital stock. The two equation system thus exhaustively allocates the total between the two components.[1]

The desired stock demand function for imports is thus

$$(C.8) \quad \frac{KMV^*}{KV} = \alpha_{MK} + \beta_{MDK} \ln P + \beta_{MMK} \ln PMF,$$

where it is assumed that the user cost of capital as between the two types

[1] If housing investment is disaggregated in a country model, the relevant capital stock variable here is for plant and equipment; otherwise, the stock corresponding to IF (gross fixed investment) is used.

of capital increments differs only by the purchase price. Notice that this factor demand is not income compensated.

Taking first differences and adding a replacement investment term and distributed lags for partial adjustment of desired to actual capital stock yields

$$\text{(C.9)} \quad \frac{IMV}{IFV} = \alpha_{MK} + \beta_{MDK}(L)\ D(P) + \beta_{MMK}(L)\ D(PMF) + \delta_M\ KV_{-1},$$

where the $\beta(L)$ are lag operators and the $D(\cdot)$ operator means $\Delta \ln$.

3. Intermediates are factor demands par excellence, and the functional form used is similar to that for investment demand, allocating total intermediate inputs (IN) among the domestic and imported components.[1] This demand is also a stock demand, since producers have inventories of intermediates that have some desired level. Hence, the stock adjustment (to total inventory stock) in this first differenced equation:

$$\text{(C.10)} \quad \frac{INMV}{INV} = \alpha_{MI} + \beta_{MDI}(L)\ D(PDS) + \beta_{MMI}(L)\ D(PMI) + \lambda SV_{-1},$$

where PDS is supposed to be the price of domestic intermediate inputs. The variable INV must be proxied as some multiple of GNP, since it is not observed. Intermediates are both raw materials and semi-finished goods, so that these three demands exhasut total imports of goods.

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[1] Actually, the first order condition in B.12 above would be used if a genuine demand for MI were to be distinguished. This would yield MI as a factor demand consistent with the underlying production function. The functional form would be analogous to B.14 for K^*, with α_3 and β_1 instead of α_1 and β_1, and PMI instead of UC. The function in C.10 is not inconsistent with this formualtion, however, and its functional form is more conducive to aggregation with the other components of import demand.

These three equations cannot be exactly aggregated, since the consumption demand is specified as being income compensated, while the factor demands are not. First order linear approximations derived either by Taylor's series expansion, or an approximation to the Taylor's series using cross product terms, involve a rather large number of terms on the right hand side (over twenty). An additional difficulty results from using artifact prices. The following relationships hold:

(C.11a) $\ln PMF = \mu_1 \ln PMG + (1 - \mu_1) \ln PMS = \ln PM$

(C.11b) $PMI = PMG$

(C.11c) $\ln P = \beta \ln PDS + (1 - \beta) \ln PMF$

(For details, see the companion paper by Howard Howe.)

It is apparent that aggregation of C.7, C.9, and C.10 would involve PMG and P on the right hand side. Notice that D(PMG) can be decomposed into D(PMGF) and D(R), the goods import price in foreign currency and the exchange rate, respectively.

An approximate aggregate import demand function having a similar functional form as the above three equations seems to be the only alternative since aggregation is a must. The choice of a scale variable for the "share" is not immediately apparent. Two appealing candidates are domestic sales,

(C.12) $DSV \equiv GNPV - XV$,

and import - content weighted GNP components.[1]

If the vector of import content weights is μ, then FDV, the nominal import content weighted activity variable is

$$\text{(C.13)} \quad FDV \equiv \mu' \, GV,$$

where GV is the vector of final demand components. Using AV as either FDV or DSV, the aggregate import demand equation will be

$$\text{(20)} \quad \frac{MGV}{AV} = \alpha_M + \beta_M(L)\ D(PMGF) + \beta_r(L)\ D(R) + \beta_D(L)\ D(P) + \delta_M\ KV_{-1}$$

$$+ \lambda SV_{-1} + a\ STR + b\ DSTR + u,$$

where STR is a domestic strike dummy (country-specific)

DSTR is a dock strike dummy (country-specific)

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[1]See Barker (1970). The COMET and DESMOS models both use import content weights for activity variables in import demand, see Barten, et. al (1976) and Waelbroeck and Dramais (1975), respectively.

Import content weights may be derived as follows. Consider the input - output balance equations

$$(*)\ Y = (I-A)X,$$

where Y are final demands, X is gross output, and A is the I-0 coefficient matrix. Suppose that X=BM and Y=HG, where B is a diagonal matrix of import-output coefficients, M is a vector of imports, H is a bridge table between industrial final demands Y and their counterparts in the national accounts, such that $\iota' H = \iota'$, and G is the vector of NIA final demands (C, IF, II, G, EX,[exports]). Substitution of these assumptions into (*) gives (**) $M = B^{-1}(I-A)^{-1} HG=CG$. To get total imports, sum (**): (***) $\iota' M = \iota' CG = \mu' G$, where μ is a vector of "import content of final demand" weights. Barten's weights for the final demand categories for Germany are, to give an example:

C	IF	II	G	EX
.172	.151	.213	.134	.156

These average import content weights need have no normalization constant such as unity, for they express the fraction of final demand in each category that is satisfied by imports.

The imports for inventories buildup,[1] is captured in two ways: First, in the import content variable, and second, in the lagged stock term (SV). This effect can be extremely important, as importers actually order to stock, and arrival dates can be quite uncertain.[2] Hence, inventories of imports may fluctuate wildly, a fact that ought to increase their optimal level. This aggregate form may not perform well if consumer imports dominate, since the demand in (20) is not income compensated. Additional price terms in levels that are so compensated may be added or used to replace the specification in (20).

Analogously to exports (equation 14) and imports (equation 18) in constant prices, nominal bridge equations are required for NIA exports and imports:

$$(21)\quad XGSV = s_0 + s_1\ [XGV + XSYV + XSOV] + u,$$

$$(22)\quad MGSV = t_0 + t_1\ [MGV + MSYV + MSOV] + u.$$

International Service Transactions and Transfers[3]

International services transactions are divided into two groups: factor income transactions and non-factor income services. Factor incomes from abroad are remitted dividends, branch profits and interest receipts. Non-factor income services cover travel spending, transportation fares and miscellaneous services such as telephone calls.

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[1]See Rees and Layard (1972), Caton and Higgins (1974).

[2]See Hooper (1976) for the use of inventories in explaining shifts in the import demand function for the U.S.

[3]This section is essentially due to Sung Kwack.

Factor income (payments) on foreign assets owned (liabilities issued) is a flow obtained as the product of the relevant rate of return and the relevant measure of the existing stock of assets (liabilities). Assets and liabilities may be denominated in local or foreign currencies, irrespective of the currency of the issuing country. Assets and liabilities are in reality found in a wide variety of maturities. These two factors are what complicate the equations for factor receipts and payments, since it is undesirable to model the currency denomination or maturity structure of the stocks giving rise to these flows.

In the case in which foreign assets are denominated in foreign currency and have but a single maturity, the income flow is $SYV = E(R \cdot A_{-1})$, where E is the exchange rate, R the rate of return, and A the asset value in foreign currency. To convert asset value to local currency, use is made of $E = E_{-1} + \Delta E$, so that $SYV = R(1 + \frac{\Delta E}{E}) \cdot B_{-1}$ where $B = E \cdot A$. It is assumed that capital flows during the period do not affect the income stream, so that lagged stocks may be used.

In the present model, claims and liabilities are both aggregated across countries and currencies. Currency denomination will be accounted for in the equations that follow by including separate terms for the local currency interest rate and for a weighted average of foreign currency rates.

The maturity of financial assets is aggregated in this model into total financial claims (liabilities), which equals short plus long term portfolio claims (liabilities). However, direct claims and liabilities are separated, and a separate term is included for these stocks. Separate terms on appropriate weighted averages for both short and long foreign and domestic rates will partially account for differing maturities of stocks.

Further, assets with maturity longer than one quarter are not assumed to pay interest at the current, but rather the issue, rate. To explicitly capture this phenomenon would require series on issues and redemptions of debt that are not available and modeling them is not a goal of this project. Instead, a distributed lag on the product of stock times rate is included as a proxy for the maturity composition of long-term claims and liabilities. The exchange rate at which these income streams must be valued is the contemporaneous one, however, so the exchange rate term is carried outside the lag distribution (which is normalized to sum to unity). For local currency-denominated assets, this exchange rate term is unnecessary. The exchange rate is not merely the dollar rate, but rather a weighted average of other countries' rates vis-à-vis the local currency (computed from the arbitrage condition with the dollar). The weights are the same as those used in the weighted average of interest rates.

Assets and liabilities of the central bank or exchange authority bear interest, and thus generate payments and receipts, respectively. A separate term is included for these. Finally, seasonality in such equations is likely to be proportional to the stocks of claims or liabilities; it is unlikely to be additive. This fact is treated with multiplicative seasonal dummies.

Putting all these components together results in the following equations:

$$\text{(23)}\quad XSYV = c_o + c_1\, RS\cdot FCP_{-1} + C_2(L)\, RL\cdot FCP_{-1}$$

$$+ c_3(1 + \Delta ZE)\, FRS\cdot FCP_{-1} + \alpha_1(1 + \Delta ZE)C_4(L)\, FRL\cdot FCP_{-1}$$

$$+ (1 + \Delta ZE)\, C_5(L)\, FRL\cdot LTDC_{-1}$$

$$+ (1 + \Delta ZE)\, C_6(L)\, FRS\cdot NFAEOQ_{-1}$$

$$+ [c_7 S1 + c_8\, S2 + c_9 S3]\cdot FC_{-1} + u$$

where $c_2 + \iota' C_2(L) + c_3 + \alpha_1 = .01, \iota' C_4(L) = 1.0,$

$\iota' C_5(L) = \iota' C_6(L) = 0.01,$

$FC \equiv FCP + LTDC + NFAEOQ,$

XSYV is nominal factor income receipts,

RS is the local currency short-term interest rate,

RL is the local currency long-term interest rate,

FRS is a geometrically weighted average of foreign short-term interest rates, with the fixed weights representing the currency composition of claims (BIS data),

FCP is the stock of outstanding private financial claims on foreigners,

LTDC is the stock of outstanding direct claims on foreigners

FRL is a weighted average of foreign long-term interest rates, similar to FRL,

NFAEOQ is the stock of net foreign assets (end of quarter) of the central bank,

E is a weighted average of spot exchange rates vis-à-vis local currency.

S1, S2, S3 are seasonal dummies,

$C_3 + \alpha_1$ is the foreign currency share of FCP,

$C_1 + \iota' C_2(L)$ is the local currency share of FCP,

and u is an error term.

(24) $$MSYV = d_o + d_1 RS \cdot FLP_{-1} + D_2(L)\ RL \cdot FLP_{-1}$$

$$+ d_3(1 + \Delta\%E)\ FRS \cdot FLP_{-1} + \beta_1(1+\Delta\%E)\ D_4(L) FRL \cdot FLP_{-1}$$

$$+ D_5(L)\ RL \cdot LTDL_{-1} + D_6(L)\ RS \cdot LO_{-1}$$

$$+ [d_7 S1 + d_8 S2 + d_9\ S3] \cdot FL_{-1}$$

$$+ u$$

where $d_1 + \iota' D_2(L) + d_3 + \beta_1 = 0.01$, $\iota' D_4(L) = 1.0$,

$\iota' D_5(L) = \iota' D_6(L) = 0.01$,

$FL \equiv FLP + LTDL + LO$,

MSYV is nominal factor income payments,

FLP is the stock of outstanding private financial liabilities to foreigners,

LTDL is the stock of outstanding direct liabilities to foreigners,

LO is the stock of outstanding official liabilities to foreigners,

and the other symbols are as above.

The terms on direct and official claims and liabilities, respectively, might be aggregated if the lag distributions are sufficiently similar. Separate terms are included here for currency composition, but these might be aggregated using prior weights in a geometric mean.

The supply of non-factor services is assumed to be perfectly elastic at the own-currency prices in the supplying country, adjusted for exchange rate variations. Thus, the actual volume of the services is determined by the demand for the services. It is hypothesized that the demand for the services provided abroad depends on real income, relative prices, and import volume. Since import volume is influenced by real income, import volume can be disregarded. Consequently, we have two equations:

(25) $\ln(XSOV/P) = b_0 + B_1(L)\ \ln FGNP + B_2(L)\ \ln(FP/P) + u$

(26) $\ln(MSOV/FP) = a_0 + A_1(L)\ \ln GNP + A_2(L)\ \ln(P/FP) + u$

Both FGNP and FP are weighted averages (trade weights might be used), and the domestic absorption deflator is used here because it has a large service component.

Transfer receipts are assumed to depend on a weighted average of foreign disposable incomes, FDYPV:

(27) $XTRANV = a_0 + a_1\ FDYPV + u,$

while transfer payments depend on domestic disposable income, DYPV:

(28) $MTRANV = b_0 + b_1\ DYPV + u.$

For countries that have a high proportion of foreign workers, e.g. Germany, the income variable can be supplemented by another variable: the wage rate time the number of foreign workers.

D. The Labor Market

Labor markets differ markedly among the countries that have been selected for this model. The United States market is heterogeneous with a high degree of mobility and segmentation among age, racial and sex groups. Canada and Germany are similar in that both experience a lot of labor migration. Canadian migration is in both directions, while German migration is mostly inward (outward migration if foreign workers returning home). Japan's paternalistic, low mobility employment system is still pervasive, while unions in the U.K. still manage to rule the roost in the face of many declining industries.

In the face of such diversity, it still is advantageous to maintain commonality in specification, which is here rather compact, yet realistic. Ironically, the two key equations are reduced forms. Disequilibrium in the labor market makes it advantageous to specify the unemployment rate as a reduced form, although one might think that supply not being equal to demand would make a structural treatment ideal. However, the unemployment rate is a key variable, especially since it proxies for excess supply in the wage rate equation. One can obtain much more accurate predictions of the unemployment rate as a behavioral equation rather than as an identity (where E is employment and LF is labor force) such as

(D.1) $UN \equiv (1 - E/LF)\ 100.$

This is because the unemployment rate is a residual, and small errors in LF or E mean large errors in UN.

Labor supply is derived as a deviation from the trend in labor force (potential labor force); this captures the short run, cyclical variations in supply as well as the long-run trends.

Potential labor force is generated (data) by

(D.2) $LFP \equiv LF \cdot 100/CU,$

where LF is a data series not actually used in the model.[1] LFP is a function of population and time (making the peak participation rate depend on a trend):

(D.3) $\ln LFP = a_0 + a_1 \ln POP + a_2 T + u.$

(This is equation 33 in the summary paper)

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[1]This assumes that the hidden labor force is proportional to unused capacity. As a recession starts, workers drop out of the pool of those "seeking jobs."

Potential employment is an identity:

(D.4) $EP \equiv LFP - UEF$, where

(D.5) $UEF \equiv LFP \cdot (CU/100) \cdot (UNMIN/100)$.

Thus, potential employment equals potential labor force minus frictional unemployment, and UNMIN is a constant. UNMIN is the minimum or frictional unemployment rate. Notice that LFP·(CU/100) gives actual LF (see D.2), which, multiplied by UNMIN/100 gives the number of frictionally unemployed. Potential employment is used in the capacity output production function, equation (11). (D.4 is equation 34 in the summary paper)

Labor supply or labor force, as noted above, is not actually used in the model but a function for LF is specified as logarithmic deviations from trend. Short-run labor supply is derived from consumers making a choice (limited as it may be) between labor and leisure. If it is assumed that consumption and leisure are substitute "goods",[1] and that there is an inverse, linear relationship between labor and leisure (by definition), the labor-leisure choice can be represented by including consumption in the labor supply equation.

The traditional labor supply function involves the real wage; Ashenfelter and others are followed by allowing for the inclusion of the

[1]They may be complementary goods, if Becker's theory of time is used as a basis for consumption/time allocation. However, it may be assumed that, up to a point, more income is required to consume more, and that to earn more, one must consume less leisure. This ambiguity results in an ambigous sign on consumption in the labor supply equation; see the discussion in the text. See also Barnett (1975).

real wage squared to represented a backward bending (quadratic) supply function.

As noted above, labor migration may significantly influence labor supply. If labor force includes foreign workers, the stock of foreign workers will have a positive influence on LFP, since it is a component thereof. If labor force is a "domestic" concept, the influence will be negative, because foreign workers "discourage" domestic workers by working at lower wage rates.[1]

These components yield the following labor force equation:

$$\text{(D.6)} \quad \ln(LF/LFP) = b_0 + b_1 \ln C + b_2 \ln(W/P) + b_3 \ln[(W/P)^2] + b_4 \ln MIG + u.$$

Labor demand is derived by using the first order condition on efficient factor usage from the Cobb-Douglas production function that is used consistently throughout the goods market. This function is the sub-function in value added:

$$\text{(D.7)} \quad GNP = Ae^{gT} K^{\alpha} L^{(1-\alpha)},$$

noting that

$$\text{(D.8)} \quad QD = Ae^{gT} K^{\alpha_1} L^{\alpha_2} MI^{\alpha_3}$$

so that

$$\text{(D.9)} \quad (1-\alpha) = \alpha_2/(\alpha_1 + \alpha_2).$$

The first order condition is

$$\text{(D.10)} \quad GNP_L \equiv (1-\alpha) \frac{GNP}{L} = \frac{W}{PV},$$

where

$$\text{(D.11)} \quad PV = GNPV/GNP.$$

[1]See Berner (1973), (1976).

Desired labor demand is obtained by solving (D.10):

(D.12) $L^* = (1-\hat{\alpha})\ GNP/(W/PV) = (1-\hat{\alpha})\ GNPV/W$,

and $\hat{\alpha}$ is estimated from the mean of factor shares, as for equation (11). Assuming that average hours worked are constant, labor inputs may be expressed in terms of people rather than person-hours.[1] Hence, E^* can be substituted for L^*.

Assume that actual labor demand adjusts slowly to desired demand:

(D.13) $E = g_0 + G_1(L)\ E^*$,

where $G_1(L)$ is a polynominal lag operator.

The unemployment rate is related to the employment rate by the identity

(D.14) $E/LF \equiv 1 - UN/100$.

Taking the log of D.14 and substituting for E from D.13 and for LF from D.6 yields

$$(32)\quad \ln(1-\frac{UN}{100}) + \ln LFP = -z_0 - z_1 \ln C - z_2 \ln(W/P) - z_3 \ln[(W/P)^2]$$
$$Z_1(L) \ln MIG + Z_2(L) \ln[(1-\hat{\alpha})\ GNP/(W/PV)]$$

where a distributed lag has been added to lnMIG to accomodate the stock-build-up. Notice that the coefficient of lnLFP is -1 by assumption.[2]

The wage rate equation is derived from two basic hypotheses

Hypothesis 1 (Lipsey, 1960): The change in wage rates is an increasing

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[1] Known in the trade previously as manhours.

[2] Notice that E/LF is derived in estimation from D.14.

function of relative excess demand in the labor market:

(D.15) $\dot{W} = f([d-s]/s)$.

Lipsey approximates the argument of $f(\cdot)$ by 1/UN, noting that there is a positive UN at which d-s = 0. Hence, $\dot{W} = f(1/UN)$. Aggregation over labor markets result in "Lipsey loops", with the change in the unemployment rate displacing the macro relationship upward from the micro (individual market) relation (like Gordon's dispersion argument). Representing the change in UN in discrete form yields

(D.16) $\dot{W} = f(1/UN, 1/\Delta\%UN)$.

<u>Hypothesis 2</u> (Friedman - Parkin): There is no money illusion in wage-setting, and the UN at which d-s = 0 (no influence on wage rates) is the "natural rate" of unemployment. Hence the wage equation should be specified in real terms. This can be tested by including $\Delta\%P$; a test of the natural rate would involve the null hypothesis $\beta_P - 1 = 0$.

In addition to these hypotheses, the construction of the wage rate, defined as wage bill/person-hours, as well as other factors, must be taken into account. These are:

1. The wage bill includes employer contributions to social insurance. Since this variable is not included in the country submodels, it is proxied by TV, tax revenues.
2. Minimum wage rates force up the whole structure of wage rates when changed (WMIN).
3. Strikes increase negotiated wage rates (STR).

4. Foreign workers are willing to work at lower wages and may discourage domestic workers (and the unemployment rate may be unchanged); hence MIG is needed.

5. The short run Phillips curve may be considerably flatter than the long-run curve (which may be vertical). A distributed lag on 1/UN can be used to capture this phenonemnon.

6. Incomes policies, according to Parkin and Laidler, flatten the Phillips curve; dummies can be added for the periods when the policy is "on".

7. The percentage change may be specified in a variety of ways:

$$\Delta\%W = (W - W_{-4})/W_{-2} \tag{D.17}$$

$$\Delta\%W = (W - W_{-4})/W_{-4} \tag{D.18}$$

$$\Delta\%W = (W - W_{-1})/W_{-1} \tag{D.19}$$

$$\Delta\%W = (W/\tfrac{1}{4} \sum_{i=1}^{4} W_{-i}). \tag{D.20}$$

The frequency of wage rounds may determine the form to be used.

The estimating equation is

$$\Delta\%W = b_0 + B_1(L)/UN + b_2/\ \Delta\%UN + b_3\ \Delta\%P + b_4\ \Delta\%TV + b_5\ \Delta\%WMIN + b_6\ \Delta\%MIG + b_7\ STR + u. \tag{35}$$

This equation completes the specification of the goods and labor markets of the typical country submodel.

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