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PRICE DETERMINATION IN THE MULTI-COUNTRY MODEL

by

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The goal of this paper is to obtain consistent specifications for domestic, export, and import price equations for use in the multi-country model project being undertaken by the Quantitative Studies Section. The multi-country model, described in [3], consists of small-scale macroeconomic sub-models for the United States, Canada, Japan, Germany, and the United Kingdom linked together to capture international economic and financial interactions. The domestic and export price equations are derived from the structure of the prototype country sub-model. The import price equation is developed as a weighted average of the appropriate exporting countries' export prices and an exogenous price of primary products.

In the first section, the supply and demand relationships of a simplified macroeconomic model are reviewed to illustrate the departures from perfect competition and perfect adjustment that are permitted in the country sub-model. The second section presents the price indexes that will be used in the actual country sub-model. Section three explains how domestic and export prices are allowed

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to diverge. It is assumed that domestic output can be transformed into domestic goods and services, export goods, and export services according to a constant elasticity of transformation (CET) frontier. Output is then divided between the domestic and export markets under the assumption of discriminating monopoly. Desired prices are set to maximize profits, subject to the CET constraint, in the face of different price elasticities of demand in these segregated markets. Marginal cost is derived from a production function for domestic output using imported intermediate inputs. A Cobb-Douglas production function is assumed, but an alternative marginal cost expression from a vintage capital production function is also developed. Finally, the marginal-cost and price-markup terms are combined with the price indexes to obtain behavioral equations for the domestic absorption deflator and the export price index.

Throughout the paper, the major objective is to obtain the simplest form of price equations consistent with the structure of the model. The assumptions necessary to attain this goal are addressed explicitly and areas of refinement for possible future elaboration of the model are pointed out.

1. Interpretation of the Aggregate Price Equation

The form of a price equation depends on the structure of the rest of the model. An illustrative model of a closed economy is reviewed here to show the "reduced-form" nature of the price equations and the departures from perfect competition and perfect adjustment that will be embodied in them. This illustrative model has a standard Keynesian framework and is devoid of enhancements such as wealth effects on consumption, depreciation, and anticipated inflation. Consumption depends only on income. Investment is a function of the difference between the marginal product of capital and the interest rate. The capital stock is so large relative to investment that no change is perceived during the period of analysis. All other variables are permitted to change instantaneously. There are three markets: goods, labor, and money. The system is expressed in six equations.

(1.1)	production function: output is a function of labor and capital; diminishing marginal product of labor ($Y_{NN} < 0$).	$Y^S = Y(K, N)$
(1.2)	demand for labor: real wage equals marginal product of labor. Nominal wage, $\bar{w}$, is institutionally determined.	$\frac{\bar{w}}{p} = Y_N$
(1.3)	consumption function.	$C = C(Y)$
(1.4)	investment function.	$I = I(Y_K - r)$
(1.5)	goods market equilibrium; $Y^S = Y^D$.	$Y = C + I + \bar{G}$
(1.6)	money market equilibrium; $M^S = M^D$.	$\frac{\bar{M}}{p} = m(r, Y)$

For theoretical applications, this kind of model is generally manipulated in an IS-LM framework. Equations (1.1), (1.3), (1.4) and (1.5) are collapsed into the IS curve which gives the combinations of output and interest rate which clear the goods market. Equations (1.1), (1.2) and (1.6) are collapsed into the LM curve which gives the combinations of output and interest rate which clear the money market. Price is jointly determined with the other endogenous variables (N, Y, C, I, r) of the system.

The same model can be analyzed in terms of aggregate supply and demand. Equations (1.1) and (1.2) together can be interpreted as an aggregate supply curve relating output to the price level. Since capital is fixed, the aggregate supply curve amounts to a simple transformation of the demand for labor curve. The diminishing marginal product of labor leads to an inverse relationship between employment and the real wage. Thus, an increase in the price level lowers the real wage, increases the demand for labor and, thereby, raises output. The aggregate supply curve is upward sloping in (p,Y) space.

Equations (1.3) to (1.6) provide an aggregate demand curve. The aggregate demand curve shows the combinations of price level and output at which the goods and money markets are in equilibrium. As long as the marginal propensity to save exceeds the marginal propensity to invest out of income, the aggregate demand curve is downward sloping.

In empirical models, the demand side is generally applied by estimating structurally the parameters of the consumption, investment, and money demand functions. The parameters are then substituted in the market clearing conditions, $Y^S = Y^D$ and $M^S = M^D$, to obtain an aggregate demand equation. On the supply side, however, the parameters of the production function are not usually estimated in this structural fashion. Rather, the aggregate supply equation implicit in the production function (1.1) and the marginal productivity condition (1.2) can be inverted to provide a price equation for direct estimation. Suppose, for example, that equation (1.1) is a Cobb-Douglas production function; $Y = \phi K^{\alpha_1} N^{\alpha_2}$, $\alpha_1 + \alpha_2 = 1$. Then the marginal product of labor equals $\alpha_2 Y/N$. With capital fixed, the marginal product of labor and the production function can be substituted into (1.2) to provide a price equation.

$$(1.7) \qquad p = \frac{\overline{w}}{\alpha_2} \phi^{-1/\alpha_2} K^{-\alpha_1/\alpha_2} Y^{\alpha_1/\alpha_2}$$

The way the marginal productivity condition (1.2) is stated assumes that price is given and the firm adjusts employment to equate the marginal product of labor to the real wage. But now the price equation is to be estimated as an aggregate market relationship (as opposed to a cross-section micro relationship). So in (1.7) price is viewed as a response to the institutional wage, the capital stock and aggregate demand.[1]

Imperfect competition.--The goods market in this illustrative case has been assumed perfectly competitive. In the actual country sub-models, output is assumed to be supplied monopolistically. There is no aggregate supply curve, but, rather, supply is characterized as a point determined by the marginal revenue and long-run marginal cost of the producer. With monopoly pricing permitted, marginal revenue is not constant but depends on the price elasticity of aggregate demand. In this case, the price equation is no longer a strictly "structural" representation of the supply side of the model. More correctly, it is a kind of "reduced-form"[2] equation because it incorportates elements of the demand side of the model.

Strict consistency would require that the price elasticity of demand employed in the price equation be the same as that embodied in the aggregate demand equation. But the elasticity expression for even a simple model is

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[1]Even though constant returns to scale are specified in the production function, price is an increasing function of output here. This is because capital is assumed fixed. If capital were allowed to vary, price would be constant and the aggregate supply curve perfectly elastic.

In the case of a fixed coefficient production function, $Y = \min\,[K/\beta_1, N/\beta_2]$, the aggregate supply curve would be either horizontal or vertical. When capital is not fully employed, the marginal product of labor is constant and the aggregate supply curve is infinitely elastic. Once capital is fully utilized, the aggregate supply curve becomes inelastic at $Y = Y_{max}$.

[2]It is not a reduced form equation in a strict econometric sense because other endogenous variables appear on the right-hand side.

too involved to be substituted directly into the price equation. Written in differential form, the aggregate demand function implicit in equations (1.3) - (1.6) is

$$(1.8)\quad dY\left(1 - C' - I' \frac{Y_{KN}}{Y_N} - \frac{m_Y}{m_r} I'\right) = - \frac{I'}{m_r}\left(\frac{dM}{M} - \frac{M}{p}\frac{dp}{p}\right) + dG.$$

One thing is clear despite the complexity -- the price elasticity is not constant.[1] For constant elasticity, the aggregate demand curve would have to be a hyperbola, $Y^D = \phi p^{-\eta_D}$. There is no reason a priori to impose this functional form. Moreover, it is desirable to permit policy-induced expansionary (contractionary) shifts in the aggregate demand schedule, $\left.\frac{dY}{dM}\right|_{dp=0}$ and $\left.\frac{dY}{dG}\right|_{dp=0} > 0$, (<0) to reduce (increase) the absolute value of the price elasticity of aggregate demand. As a result of the complexity of the price elasticity expression and the desirability of permitting a variable elasticity, the monopoly markup of the price equations (a function of the price elasticity of demand) will be approximated as a function of excess demand in the relevant (domestic or foreign) economy.

Lagged adjustment.--In this simple model, with price assumed perfectly flexible, the aggregate supply and demand schedules together determine output. If price follows some lagged adjustment pattern, however, price

[1]The slope of the aggregate demand curve (1.8), $\left.\frac{dY}{dp}\right|_{dM = dG = 0}$, can vary with respect to the level of aggregate demand only through the term Y_{KN}/Y_N. The effect on the slope of a change in Y_{KN}/Y_N will not exactly offset the movement in Y/p along the demand curve.

movement alone does not normally clear the market. Although the price level does eventually adjust to eliminate partially excess demand, it is the adjustment of quantity that clears the goods market in the short run. In the actual country sub-models, it is assumed that the (representative) firm makes a decision at the beginning of the period on output, price, and inventory simultaneously based on expected demand for the period. During the period, the producer attempts to adjust output by adjusting labor to meet demand in the period. Even so, complete adjustment to unexpected demand changes may be too costly and, thus, inventory change takes up the slack between supply and demand in the short run.[1] In the following sections, equations for desired prices are developed under neoclassical assumptions of perfect flexibility. Then the actual price equations are postulated to be functions of distributed lags on the determinants of the desired prices.

Open economy.--The illustrative model reviewed in this section represented a closed economy. The actual country sub-models will represent open economies where domestic and export prices are permitted to diverge.

2. Important Prices

Six price indices are required to capture the international interdependence sought with the linked country models:

(1) domestic-absorption deflator.--This index is defined as the ratio of the current value of domestic absorption to real domestic absorption;

$$(2.1) \quad P \equiv (CV + IFV + IIV + GV) \;/\; (C + IF + II + G)$$

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[1] See Berner [2, pp. 30-37] for further discussion of the treatment of inventories.

where C is consumption, IF is fixed investment, II is inventory investment, G is goverment expenditure, and the suffix V denotes current value. The absorption deflator is used to inflate real absorption into current-value terms; it is also used as a determinant in the wage equation. Since domestic absorption is composed of domestic sales of domestic output (QDS) as well as final imports (MF), the domestic absorption deflator can also be approximated as a geometric mean of the prices of these components.[1]

$$(2.2) \quad P \equiv PDS^{\beta} PMF^{(1-\beta)}$$

Domestic sales of domestic output consists of goods and services produced from three factors: capital services, labor, and imported intermediate goods. For the moment, assume that the equation for the domestic sales price resulting from profit maximization takes the form[2]

$$(2.3) \quad PDS = m_D \kappa_D W^{a_1} UC^{a_2} PMI^{a_3}$$

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[1]The aggregation P·A=PDS·QDS+PMF·MF implies that an arithmetic mean of the price of domestic sales and the price of final imports should be used. (see Appendix).

$$P = \left(\frac{QDS}{A}\right) PDS + \left(\frac{MF}{A}\right) PMF$$

A geometric mean approximation is proposed, however, for ease in later substitution of an expression for the price of domestic sales (PDS) which will include factor prices raised to exponential powers.

Note that the share weights QDS/A and MF/A are not to be substituted for the weights β and 1-β. The weight β will appear as a component of the coefficients to be estimated in the final price equations. The estimated coefficients will therefore incorporate an implicit estimate of β (different from QDS/A) such that (2.2) would hold.

[2]Equations (2.3) and (2.4) represent a markup (m_i) over marginal cost where the marginal cost expression proceeds from a Cobb-Douglas production function; see section 3 below.

where m_D is the markup on domestic sales, W is the wage rate, UC is the user cost of capital, and PMI is the price of imported intermediate goods. Specific production functions and the price equations emerging from them will be discussed in sections 3 and 4 below.

(2) price of primary products.--The price of world-traded primary products (PP) is assumed to be exogenous and is expressed in U.S. dollars.

(3) price of exported goods.--Exported goods (XG) comprise all goods except those included in the category "primary products." Since exports are sold in a different market, the markup is not the same as for domestic sales. The export price equation depends on marginal cost in the same way as (2.3) but has a different transformation factor (κ_X) and a different markup (m_X).

$$(2.4) \quad PXG = m_X \kappa_X W^{a_1} UC^{a_2} PMI^{a_3}$$

(4) price of imported goods.--To facilitate the exposition, equations (2.2)-(2.4) were written as if import prices could be disaggregated by final and intermediate goods. This disaggregation is not possible in the data. Another way to view PMF and PMI is to view them as (different) weighted averages of the price of imported goods (PMG) and the deflator for services imports (PMS).

The price of final imports (PMF) can be written as the geometric mean

$$(2.5) \quad PMF = PMG^{\mu_1} PMS^{(1-\mu_1)}$$

Imported intermediates are assumed to consist entirely of goods; so the price of imported intermediates (PMI) equals the price of imported goods.

$$(2.6) \quad PMI \equiv PMG$$

The price of imported goods for a given country is a weighted average of the price of primary products and the prices of exported goods of the relevant exporting countries, all converted into local currency units by the appropriate exchange rate (R_{ij}, R_{i1} is the U.S. dollar exchange rate of currency i.) Since the weighting scheme is complex and because the average will be inexact if every single supplier is not included, PMG_i can be estimated as a quasi-identical function of its determinants.

(2.7) $PMG_i = PMG_i(R_{i1}PP,\ R_{i1}PXG_1, \ldots R_{iN}PXG_N)$

Since prices $R_{i1}PXG_1$ through $R_{iN}PXG_N$ are likely to be highly collinear, the function $PMG_i(\ )$ probably cannot be estimated satisfactorily. It is likely that the export prices would have to be combined first into a weighted average where the weights w_{ij} are shares of manufactured imports from country j to country i as a fraction of total manufactured imports of country i.

(2.8) $\ln PMG_i = \gamma_{1i} \ln(R_{i1}PP) + \gamma_{2i} \sum_{j \neq i}^{N} w_{ij} \ln(R_{ij}PXG_j)$

In empirical studies of import price behavior, Wilson [14] and Ahluwalia and Hernández-Catá [1] have found that the dynamic effects of exporter-currency price changes and exchange rate changes can differ appreciably in the short run. In any model that allows for adjustment effects, this argues for separate distributed lags on the export price and exchange rate terms. By allowing for a proportion of export contracts to be denominated in local currency and for lags in delivery of goods, Ahluwalia and Hernández-Catá obtained an equation for the import price as a linear combination of current and lagged values of the foreign export price and the exchange rate.[1]

$$(2.9) \quad \ln PMG = \sum_{\tau=0}^{T} \Theta_{\tau} \ln PXG_{t-\tau} + \sum_{\tau=0}^{T} \Theta'_{\tau} \ln R_{t-\tau}$$

Applying the same decomposition of price and exchange rate terms as in (2.9) to the price index for imported goods (2.8), results in the distributed lag equation

$$\begin{aligned}(2.10) \quad \ln PMG_i = & \sum_{\tau=0}^{T} \gamma_{1\tau} \ln PP_{t-\tau} + \sum_{\tau=0}^{T} \gamma'_{1\tau} \ln R_{i1,t-\tau} \\ & + \sum_{\tau=0}^{T} \gamma_{2\tau} \left(\sum_{j\neq i}^{N} w_{ij} \ln PXG_j\right)_{t-\tau} \\ & + \sum_{\tau=0}^{T} \gamma'_{2\tau} \left(\sum_{j\neq i}^{N} w_{ij} \ln R_{ij}\right)_{t-\tau}\end{aligned}$$

The import price index then, employs a weighted average exchange rate. However, the weighted average is applied only to the merchandise trade

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[1] Strictly speaking, the Ahluwalia and Hernández-Catá import price equation holds only for a bilateral trade realtionship. An aggregate import price equation requires superscripting (2.9) by country of origin and then aggregating with trade share weights w_{ij}, shares of total manufactured imports from country j to country i as a fraction of total imports of country i.

$$\ln PMG_{it} = \sum_{j\neq i}^{N} w_{ij} \ln PMG^j_{it}$$

$$\ln PMG_{it} = \sum_{j\neq i}^{N} \sum_{\tau=o}^{Tj} w_{ij} \Theta^j_{\tau} \ln PXG^j_{t-\tau} + \sum_{j\neq i}^{N} \sum_{\tau=0}^{Tj} w_{ij} \Theta'^j_{\tau} \ln R_{ij,t-\tau}$$

To make the double summation requires the assumption of equal delivery lag weights (Θ^j_τ), delivery periods (T_j), and contract-currency proportions; these assumptions are made here. Otherwise, separate distributed lags would be required for each exporter's local price and exchange rate.

account with trade weights. It is not an overall effective exchange rate applied to all international transactions accounts.[1]

In the country sub-models, equation (2.10) is estimated as a quasi-identity. Using lag operator notation,[2] the final estimating equation for the price of imported goods is written

$$(2.11)\quad \ln\ PMG = a_0 + A_1(L)\ln\ PP + A_2(L)\ln\ R_{i1} + A_3(L)\left(\sum_{j\neq i}^{N} w_{ij}\ln\ PXG_j\right) + A_4(L)\left(\sum_{j\neq i}^{N} w_{ij}\ln R_{ij}\right).$$

(5) deflator for imported services.--Import services are explained separately from imported goods. Although agregation of all imports would be desirable from the standpoint of simplicity, imported services are separated for the following reasons:

(a) a matrix of trade shares is to be used to generate export equations from the import demand equation.[3] The baseline trade share matrix is based on merchandise trade only. It would be inaccurate to assume that the merchandise weights were identical to total-trade weights;

(b) factor income payments (MSYV) are not trade in the usual sense. They are the income realized by foreigners from holding domestic liabilities; and

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[1] However, such an effective exchange rate could readily be calculated after the fact since the multi-country model will determine all the bilateral rates independently.

[2] $A(L)\ x \equiv \sum_{\tau=o}^{T} a_\tau x_{t-\tau}$

[3] See Berner [2, pp. 38-44].

(c) this disaggregation is already made in the data.

An import-services deflator (PMS) can be calculated from total imports of goods (MG). The deflator is the ratio of services in value terms to real services.

(2.12) $PMS \equiv (MGSV - MGV) / (MGS - MG)$

Since all the constituents of PMS are determined elsewhere in the model,[1] PMS could appear as an explanatory variable of the domestic absorption deflator (P) through equations (2.5) and (2.2). In fact, the final equation for P will contain a large number of explanatory variables and collinearity problems may dictate that PMS be dropped. Such a concession to tractability would amount to assuming that the weight μ_1 in equation (2.5) be close to one. This seems not to be a very serious restriction.

(6) GNP deflator.--Since the domestic absorption deflator is the key behavioral equation for the domestic price level, the GNP deflator is not actually required for the functioning of the country sub-model. The GNP deflator is of interest, however, for comparing simulation and forecast results of this multi-country model with those of other national models and sources. The components of GNP are determined in the model in real and value terms, so the GNP deflator is simply determined implicitly from

(2.13) $PGNP \equiv GNPV / GNP.$

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[1] See equations (18), (19), (20), (22), (24) and (26) in part iv of [2].

Of the nine prices introduced here, only six (P, PP, PXG, PMG, PGNP, and, possibly, PMS) will be applied empirically in the model; PDS, PMF, and PMI are artifacts introduced to obtain consistency between P and PXG. For actual estimation, equation (2.3) for PDS will be substituted into (2.2) to obtain the specification for the domestic absorption deflator (P). Expressions (2.5) and (2.6) will also be substituted for PMF and PMI. The other behavioral price equation to be estimated is (2.4), the price of exported goods (PXG).

3. Production, Transformation, and Market Structure

It is desirable to permit prices in the domestic and export markets to diverge and at the same time link the export price to cost conditions in the producing country. One means of capturing this kind of relationship is to view domestic producers as discriminating monopolists who sell to residents and foreigners. Transportation costs and barriers to trade permit segmentation of the market into two areas.

From the definitions of Section 2, it is seen that the two behavioral relationships to be estimated, P and PXG, are not alike in coverage. The domestic absorption deflator, P, covers both goods and services. For exports, the behavioral equation explains the price of goods only, PXG. These different coverages can still be handled within a framework of discriminating monopoly by viewing domestic output (QD) as a formless amalgam. QD is produced with capital, labor, and imported intermediates. It can be transformed into domestic goods and services, export goods, and exports of services (other than investment income) according to a constant elasticity of transformation (CET) frontier. Prices are then set to maximize profits in the face of different elasticities of demand in the separate markets.

Production of domestic output. -- Domestic output is defined as output net of domestic intermediates but inclusive of imported intermediates. In this way, the price of imported goods enters naturally into the marginal cost expression. The Appendix presents the justification for writing domestic output in that way. For simplicity, a Cobb-Douglas functional form is chosen.

$$(3.1) \qquad QD = a_o \, e^{gT} K^{a_1} L^{a_2} MI^{a_3}, \quad a_1 + a_2 + a_3 = 1.$$

Disembodied technical change occurs at the rate g. Constant returns to scale is posited for simplicity of exposition although this restriction will not be imposed during estimation of the price equations.

Transformation of domestic output.--Since we are not interested in the separate production functions for domestic goods and services and for exported goods, it is convenient to assume that domestic output can be transformed into these physically distinct components according to a CET frontier.[1] Powell and Gruen[11] derived the family of CET production possibility schedules which are analogous to constant elasticity of substitution (CES) isoquants. They worked with a two-product frontier described by

$$(3.2) \qquad x_1^{1-k} + Ax_2^{1-k} = B(1-k)$$

where x_1 and x_2 are the two products, k is the reciprocal of the elasticity of substitution (which is negative to insure convexity of the transformation frontier), A is a "bias" parameter affecting the marginal rate of transformation, and B is a "scale" parameter which results in product-neutral shifts of the production frontier. Powell and Gruen illustrated how arbitrarily high and low elasticities of

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[1] Lawrence Lau suggested the CET approach as an alternative to separate production functions.

transformation can make the CET frontier approximate a straight line or a technologically-fixed output mix, respectively.

In the present case, output is transformed into three products: a domestic goods and services mix (QDS), export goods (QXG), and (net) export services (excluding investment income) (XOS_N). We are interested only in the prices of the domestic sales mix and the price of exported goods. In the country sub-model, the price of export services is determined residually. We can legitimately limit our attention to these two behavioral relationships because the CET function is separable; the marginal rate of transformation between the domestic sales mix (QDS) and export goods (QXG) is unaffected by the level of export services being provided. We therefore concentrate on the two-dimensional projection of the CET surface onto the QDS-QXG plane. We will write the CET function for our three-product case in the standard CES form (rather than with the normalizations implicit in the Powell-Gruen form, (3.2)) and will make use of two special cases for simplification.

The CET function for the three product bundles (with equal pairwise elasticities of transformation) can be written:

$$QD = \gamma[\alpha_1 QDS^{-\rho} + \alpha_2 QXG^{-\rho} + (1-\alpha_1-\alpha_2)\ XOS_N^{-\rho}\ ]^{-1/\rho} \tag{3.3}$$

where the allocation parameters, α_i, are normalized to sum to 1, $\rho = (1-\sigma)/\sigma$, and σ is the elasticity of transformation ($\sigma < 0$). Restrictive assumptions on the size of $|\sigma|$ can limit the transformation frontier to the two special cases of interest: a Cobb-Douglas surface and a flat plane.

Cobb-Douglas.--If $|\sigma| = 1$, the CET frontier of equation (3.2) becomes a Cobb-Douglas function.

$$(3.4) \qquad QD = \gamma^* \, QDS^{\beta_1} \, QXG^{\beta_2} \, XOS_N^{1-\beta_1-\beta_2}$$

If in addition $\beta_1 = \beta_2$, the projection of the frontier on the domestic sales and export goods axes would be a circle.

Flat plane.--If the elasticity of transformation were set arbitrarily high, the CET frontier in the limit $|\sigma| \to \infty$ would approach a flat plane.

$$(3.5) \qquad QD = \gamma^{**}[\beta_1 QDS + \beta_2^* \, QXG + (1-\beta_1^*-\beta_2^*)XOS_N]$$

Although this case is very restrictive, it is useful for simplification of the price equations.

Discriminating monopoly.--The component parts of the system can now be put together in the framework of discriminating monopoly. The representative firm seeks to maximize its profit from the sales of the three bundles of goods in the segregated home and export markets subject to the Cobb-Douglas production function (3.1) and the Cobb-Douglas transformation frontier (3.4). This problem can be written as

$$(3.6) \; \max \Pi = PDS\,(1-LTR)\,QDS + PXG \cdot QXG + PXOS \cdot XOS_N$$

$$-W \cdot L - UC \cdot K - PMI \cdot MI$$

subject to $QD = a_o e^{gT} \, K^{a_1} \, L^{a_2} \, MI^{a_3}$

and

$$QD = \gamma^* \, QDS^{\beta_1} QXG^{\beta_2} \, XOS_N^{(1-\beta_1-\beta_2)}$$

where ITR is the indirect tax rate on domestic sales. The constrained maximization is

$$(3.7)\quad \max\ \Lambda = \pi - \lambda\ [a_o e^{gT} L^{a_1} L^{a_2} MI^{a_3} \\ -\gamma^*\ QDS^{\beta_1}\ QXG^{\beta_2}\ XOS_N^{(1-\beta_1-\beta_2)}\ ]$$

and the first order conditions for a maximum are:[1]

$$(3.8)\qquad \Lambda_{QDS} = (PDS + QDS\cdot\partial PDS/\partial QDS)\ (1-ITR) + \lambda\beta_1 QD/QDS = 0$$

$$(3.9)\qquad \Lambda_{QXG} = PXG + QXG\cdot\partial PXG/\partial QXG + \lambda\ \beta_2\ QD/QXG = 0$$

$$(3.10)\qquad \Lambda_K = -UC - \lambda a_1\ QD/K = 0$$

$$(3.11)\qquad \Lambda_L = -W - \lambda\ a_2\ QD/L = 0$$

$$(3.12)\qquad \Lambda_{MI} = -PMI - \lambda a_3 QD/MI = 0$$

Equations for PDS and PXG proceed directly from (3.8) and (3.9). The negative of the Lagrangian multiplier (λ) is the marginal cost of output QD. It can be expressed in terms of factor prices by substituting the factor demand equations from (3.10) to (3.12) back into the production function (3.1) and rearranging.

$$(3.13)\qquad -\lambda = MC = \frac{1}{a_o}\ e^{-gT}\ \left(\frac{UC}{a_1}\right)^{a_1} \left(\frac{W}{a_2}\right)^{a_2} \left(\frac{PMI}{a_3}\right)^{a_3}$$

The derivatives of price with respect to quantity appearing in (3.8) and (3.9) can be expressed in terms of the respective price

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[1]No first order condition for PXOS is obtained because we do not need a behavioral equation for it. Since the CET function (3.2) is separable, the conditions for setting PDS and PXG are independent of XOS_N.

elasticities. $QDS \cdot \partial PDS/\partial QDS = PDS/\eta_D$, where η_D is the price elasticity of domestic sales. The similar substitution is made for PXG. Equations for the desired (indicated by*) price levels are then written as

(3.14) $\quad PDS^* \ (1-ITR) = (1+1/\eta_D)^{-1} \ MC \ \beta_1 QD/QDS$

(3.15) $\quad PXG^* = (1+1/\eta_X)^{-1} \ MC \ \beta_2 QD/QXG$

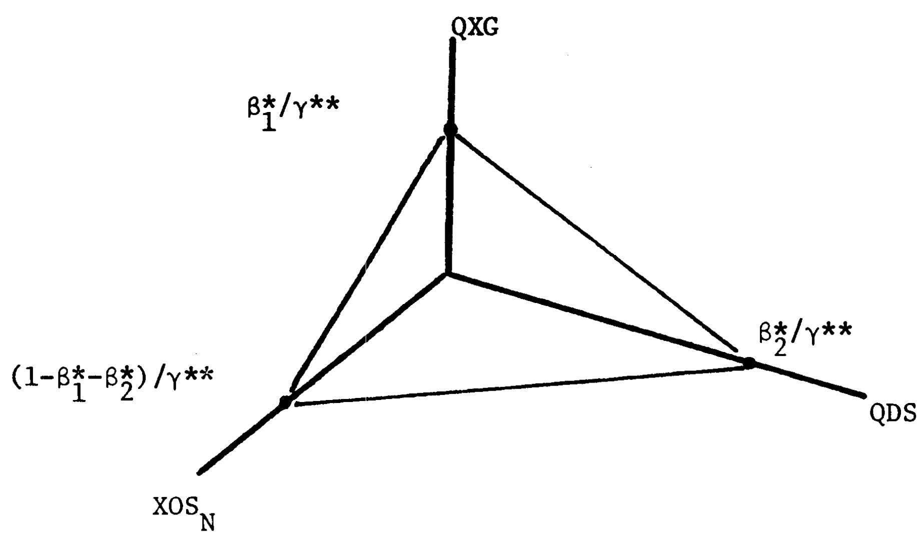
These equations are similar to the familiar markup-over-marginal-cost price equations except for being multiplied by the inverse share of domestic output assigned to the respective market. Because QDS and QXG depend, in turn on their prices, (3.14) and (3.15) are subject to simultaneous equation bias in estimation.

The difficulty in obtaining closed-form price equations from a discriminating monopoly framework in any but the simplest case was illustrated by Clark[4]. He applied a discriminated markets approach in an attempt to specify export price equations for (homogeneous) manufactures. He used a Cobb-Douglas production function and Cobb-Douglas demand functions in a one good - two market profit maximization problem. Combining the factor demand equations (following Nordhaus [10] with the marginal revenue conditions, Clark obtained two simultaneous equations in domestic price and export price. Only under the assumption of constant returns to scale in production was it possible to obtain closed-form expressions for domestic and export prices.

In the present case, even though the production function is specified to have constant returns to scale, the nonlinearity of the transformation frontier results in the presence of quantity terms in the price equations. One way to overcome the simultaneity problems presented in (3.14) and (3.15) would be to substitute the lagged terms $(GDP/QDS)_{-1}$ and $(GDP/QXG)_{-1}$ as proxies for the respective quantity ratios.[1] Because the price equations will contain a large number of terms, it is desirable as a first stage to seek the further simplifying assumptions that will eliminate the quantity terms from (3.14) and (3.15).

In the case of an arbitrarily high elasticity of substitution, the CET frontier approaches the plane shown in Figure 1. Replacing the

Figure 1. CET Frontier
with Arbitrarily High Elasticity of Transformation



[1]Another possibility employed by Berner [2] would be to approximate the quantity ratios by a function of the lagged price ratios $(PGDP/PDS)^{\varepsilon_1}$ and $(PGDP/PXG)^{\varepsilon_2}$, where PDS could be obtained as a weighted average of P and PMF from (2.2).

transformation frontier (3.5) in the maximization problem (3.7) results in the Lagrangian expression

$$(3.16)\quad \max\ \Lambda = \pi - \lambda\ [a_o e^{gT}\ K^{a_1} L^{a_2} MI^{a_3}$$

$$-\gamma^{**}(\beta_1^* \ QDS + \beta_2^* \ QXG + (1-\beta_1^* - \beta_2^*) XOS_N)]$$

where first order conditions (3.8) and (3.9) are simplified and (3.10) - (3.12) remain the same. Then the price equations (3.14) and (3.15) are replaced by

$$(3.17)\quad PDS^*(1-ITR) = (1 + 1/\eta_D)^{-1} MC\ \gamma^{**}\ \beta_1^*$$

$$(3.18)\quad PXG^* = (1 + 1/\eta_x)^{-1}\ MC\ \gamma^{**}\beta_2^*$$

With constant elasticities, these become the familiar constant-markup price equations. With variable elasticities as suggested in Section 1, they are variable markup-over-marginal-cost equations.

<u>Determinants of price elasticity</u>. --Since an exact expression for the price elasticity of aggregate demand is complex beyond justification, the elasticity expressions will be replaced by the determinants of the respective elasticities. In the case of the domestic markup, several "demand pressure" variables can be used in place of the actual policy variables as the determinants of demand elasticity.

Before substituting the determinants of elasticity, it is helpful to review the arithmetic of the markup term. As defined for use in (3.14) and (3.15), the elasticity is a negative number. The markup term can be rewritten for compactness as

$$(3.19)\quad (1+1/\eta)^{-1} = \eta/(1+\eta).$$

When demand is infinitely elastic ($\eta = -\infty$), perfect competition prevails, the markup equals 1 and price equals marginal cost. When the elasticity is finite and greater than 1 in absolute value, the markup is greater than 1. Unit elasticity represents the lower bound on the applicability of a markup framework; there marginal revenue equals zero and the implied markup would be infinite.

In his survey of price equations developed through 1970, Nordhaus [10] listed three types of demand variables that have been used in price equations (although not necessarily in the explicit monopoly markup formulation presented here): capacity utilization, inventory investment, and unfilled orders (or new orders) as a fraction of sales. Unfilled orders or new orders are not particularly helpful proxies for excess demand in the country sub-model because either would require an additional explanatory equation. To conserve on the number of endogenous variables in the model, capacity utilization is chosen as the proxy for excess demand in the markup expression. Nordhaus (p. 41) reported that Eckstein and Fromm, Solow, and Klein have had success in using the Wharton index of capacity utilization as a measure of demand.

The manner in which the markup depends on excess demand has important implications for the dynamic behavior of the model. If the level of the markup term depends on the level of excess demand (ED),

$$m = ED^{\sigma_1} \text{ or}$$

$$\ln m = \sigma_1 \ln ED; \quad \dot{m}/m = \sigma_1 \dot{ED}/ED \tag{3.20}$$

where m indicates the markup term, $\eta/(1+\eta)$. Then, as long as excess demand is stable, the price level (or more precisely, the ratio of price to cost) will be stable. If, however, it is assumed that price adjusts to eliminate excess demand as in Eckstein and Fromm [6], then the rate of change of price relative to cost depends on the level of excess demand,

$$(3.21) \qquad \dot{m}/m = \sigma_2 \, ED.$$

In this case, as long as excess demand persists, prices continue to rise. Rather than make an a priori choice between these two assumptions, Gordon [7] combined (3.20) and (3.21) to permit the data to assist in making the choice.

Domestic price markup.--De Menil [5] approximated the logarithm of the markup as a linear function of the level of excess demand (unfilled orders as a fraction of capacity output). Following de Menil, we will approximate the log of the markup as a function of the level of capacity utilization

$$(3.22) \qquad \ln \, m_D \simeq \sigma_0 + \sigma_1 \, CU.$$

This results in a rate of change of the markup proportional to the change in capacity utilization -- $\dot{m}/m = \sigma_1 \, \dot{CU}$. This is close to assumption (3.20) where stable excess demand implies stable prices.

Equation (3.22) is convenient to manipulate because the final form of the price equations will be obtained by substituting expressions for the component parts of the log form of equations (3.17) and (3.18). If alternative price equations embodying the assumption that price adjusts to eliminate excess demand are desired, the log forms of (3.17) and (3.18) should first be differentiated to obtain rates of change and then equation (3.21) should be substituted for the rate of change of the markup.

Export price markup.--Excess demand in foreign economies is one determinant of the export, markup, but it is not the only one. The distribution of a given country's imports among potential origins is determined in part by their relative prices. Exporters to that given country are not likely to have the degree of monopoly power in their export markets that they are able to exercise in the domestic market. Domestic markets can be protected by tariff and non-tariff barriers. In its export market, the firm is subject to competitive pressure from suppliers in other countries. Thus, the export prices of other supplier countries are likely to have an effect on the markup of the export price.

Starting with the same form of markup expression as (3.22), the excess demand measure is first made to cover all customer countries; a trade-weighted index is specified. It has been argued that domestic excess demand should also be a determinant of the export markup. Excess demand in the home market provides alternative outlets for domestic output and thereby puts upward pressure on the export price, as well as the domestic price. To capture this effect, CU_i should be included in the markup expression. Finally, the export price indexes of all other countries, in terms of country i's currency, should be included to account for the effect of competitors' prices on the markup term.

$$\ln m_X \simeq \sigma_0^* + \sigma_1^* CU_i + \sigma_2^* \sum_{j \neq i}^{N} z_{ij} CU_j + \sigma_3^* \sum_{j \neq i}^{N} w_j^* (\ln R_{ij} PXG_j) \tag{3.23}$$

Here, the weights z_{ij} represent the share of exports to country j as a fraction of total exports of country i. The weights w_j^* could be based on the share of country j's exports of goods in total world trade of goods. The world-trade basis is better than the share of country j in country i's exports because it allows for competition in third markets.

4. Consolidation of Price Indexes

In section 2, it was indicated that the price of domestic sales (PDS) was only an artifact to use in deriving the specification for the domestic absorption deflator (P). Now, all the component parts are ready for this consolidation.

Domestic absorption deflator.--Substitution of the marginal cost from (3.13) into (3.17) yields an expression for the desired domestic sales price (PDS*). Substitution of this expression into (2.2) yields an equation for the domestic absorption deflator in terms of the wage rate, user cost of capital, and imported final and intermediate prices. The domestic markup can be replaced by its determinants as specified in equation (3.22). The resulting expression for P* (where P* is the level of P corresponding to the desired domestic sales price, PDS*) is written here in terms of logarithms and all the constants are consolidated in c_0.

$$(4.1) \qquad \ln P^* = c_o + \beta\sigma_1 CU - \beta \ln(1-ITR) - \beta gT + \beta a_1 \ln UC + \beta a_2 \ln W + \beta a_3 \ln PMI + (1-\beta) \ln PMF$$

The prices of imported intermediates (PMI) and final imports (PMF) can then be substituted out. As specified above in (2.6), imported intermediates are assumed to consist exclusively of goods, so PMI=PMG. But final imports are comprised of goods and services, so the price index (2.5) must be used to replace PMF.

$$(4.2) \qquad \ln P^* = c_o + \beta\sigma_1 CU - \beta \ln(1-ITR) - \beta gT + \beta a_1 \ln UC + \beta a_2 \ln W + [\beta a_3 + \mu_1(1-\beta)] \ln PMG + (1-\beta)(1-\mu_1) \ln PMS$$

Finally, the price of imported goods (PMG) as a function of the price of primary products (PP) and the prices of other countries' exported manufactures (PXG) can be substituted from equation (2.10).

Before substituting (2.10), it is useful to put (4.2) in the form of the actual level of P corresponding to PDS*. Then, actual price could be regarded as a partial adjustment to the desired price of (4.2) as in de Menil [5], for example. However, rather than specifying geometrically declining lag weights, polynomial distributed lags on (4.2) are preferred because they:

(1) allow different lag shapes and lengths on each independent variable, and

(2) avoid the econometric problems associated with the use of a lagged dependent variable on the right-hand side.

Wilson [14] has demonstrated that even if the true lag structure is not geometrically declining, results obtained by imposing a Koyck-type structure may appear to support the Koyck hypothesis. Conversely, if the true lag structure is geometrically declining, general distributed lag estimates can identify such a pattern.

In the initial stages of estimating the price equations, the restrictions among structural parameters (i.e. $\Sigma a_k = 1$) are not employed. Thus, there is no need to preserve further their identities in the price equations, and only reduced form coefficients will be expressed. Substituting (2.10) into (4.2) for PMG, specifying distributed lags on the remaining independent variables,[1] and submerging all structural parameters into single coefficients, the

[1] To reduce the number of distributed lag terms, the indirect tax rate term is specified contemporaneously.

final estimating equation for the domestic absorption deflator becomes[1]

$$(4.3) \qquad \ln P = c_o - \beta \ln(1-ITR) + c_1 T + C_2(L)CU + C_3(L) \ln UC$$
$$+ C_4(L) \ln W + C_5(L) \ln PP + C_6(L) \ln R_{i1}$$
$$+ C_7(L) \sum_{j \neq i}^{N} w_{ij} \ln PXG_j + C_8(L) \sum_{j \neq i}^{N} w_{ij} \ln R_{ij} + C_9(L) \ln PMS.$$

Export price index.--The desired price of export goods is obtained directly by substituting (3.13) into (3.18). Then the export markup is replaced by its determinants as approximated in equation (3.23). The resulting price equation is of the same form as (4.1) with differences of the markup term and the absence of PMF.

$$(4.4) \qquad \ln PXG^* = b_o - gT + \sigma_1^* CU_i + \sigma_2^* \sum_{j \neq i}^{N} z_{ij} CU_j$$
$$+ \sigma_3^* \sum_{j \neq i}^{N} w_j^* \ln(R_{ij} PXG_j) + a_1 \ln UC + a_2 \ln W + a_3 \ln PMI.$$

The competitive price term (whose coefficient is σ_3^*) can be decomposed in the same way as equation (2.10). PMG and, thereby, equation (2.10) can be substituted for PMI (as per equation (2.6)). Finally, distributed lags are specified on all remaining variables to obtain an

[1]Equation (4.3) differs in two details from equation 29 presented in [3]: (1) Equation 29 included ln CU along with CU, ln CU is dropped here; and (2) ln PMS was eliminated from equation 29 by assuming PMF ≡ PMG; it is included explicitly here.

equation for the actual level of the export price index from the expression for PXG*.[1]

$$(4.5)\quad \ln PXG = b_o - gT + B_1(L)\, CU_i + B_2(L) \sum_{j\neq i}^{N} z_{ij} CU_j$$
$$+ B_3(L) \ln UC + B_4(L) \ln W + B_5(L) \ln PP + B_6(L) \ln R_{i1}$$
$$+ B_7(L) \left(\sum_{j\neq i}^{N} w_j^* \ln PXG_j + \sum_{j\neq i}^{N} w_{ij} \ln PXG_j\right)$$
$$+ B_8(L) \left(\sum_{j\neq i}^{N} w_j^* \ln R_{ij} + \sum_{j\neq i}^{N} w_{ij} \ln R_{ij}\right)$$

Note how the decompositions of the competitive price effect ($w_j^* \ln R_{ij}\ PXG_j$) and the import price effect ($w_{ij} \ln R_{ij}\ PXG_j$) are combined in the terms with B_7 and B_8. The only way to avoid duplicate terms in PXG_j and R_{ij} is to impose the same lag length on the competitive price effect and the intermediate-input price effect.[2] Two separate weighted averages are used to capture the two price effects. The competitive price effect uses trade weights, w_j^*, that represent the share of country j's goods exports in world trade; the input price effect uses weights, w_{ij}, that represent the share of manufactured imports from country j to country i as a fraction of total manufactured imports of country i.

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[1] Equation (4.5) differs in two respects from equation 30 presented in [3]: (1) equation 30 included the lag of the weighted average of customer-countries' CU, it is dropped here; and (2) equation 30 did not include the effect of domestic CU on the markup for PXG.

[2] If third degree polynomials (allowing an inflection point) were used, the single lag distribution could be regarded as an approximation to the sum of two second degree distributions of different length. If second degree polynomials are used, the same lag length must be assumed for both price effects.

Equations (4.3) and (4.5) are likely to contain too many terms to be estimated satisfactorily. PMS could be eliminated from (4.3) by the approximation PMF $\simeq$ PMG. Depending on the significance of coefficients, distributed lags on some terms could be eliminated and some terms dropped entirely. Equations (4.3) and (4.5) were developed to generate a plausible list of the most important determinants of P and PXG, consistent with the structure of the country sub-model. They are intended as starting points for estimation.

5. Alternative Production Assumptions

Equations (3.17) and (3.18) for optimal prices have the same form as that which Nordhaus [10] obtained for the single market case with a Cobb-Douglas production function. Nordhaus made three points about the optimal price derived from this function:

(1) the elasticity of optimal price with respect to the wage rate is less than 1,

(2) user cost of capital (UC) is an important component of optimal price, and

(3) productivity enters(smoothly) through the time trend rather than explicitly.

Nordhaus also pointed out three shortcomings of the neoclassical approach itself, regardless of the particular production function assumptions:

(1) The demand model is unrealistic but applicable where the firm is a monopolist. For oligopolistic markets with few firms, the demand model is clearly misspecified;

(2) Growth and risk-aversion are excluded from the firm's objective function. Firms will generally minimize costs even if they do not maximize profits; the cost component (3.13) would remain unchanged but the markup term from (3.22) would differ; and

(3) The treatment of capital is unsatisfactory; it assumes that capital is rented rather than owned.

A vintage-capital model overcomes the third shortcoming.

Putty-clay production function.--As an alternative to the smooth neoclassical substitutability of the Cobb-Douglas production function, the vintage capital model of de Menil [5] is reviewed here. Capital put in place is non-malleable; each vintage has a fixed-coefficient production function

$$(5.1) \qquad QD_\nu = \min \left[\frac{L_\nu^{\alpha_1} \; MI_\nu^{\alpha_2}}{\beta e^{-\rho_1 \nu - \rho_2 t}} \; , \; \frac{K_\nu}{\gamma} \right] .$$

QD_ν is the output from machines of vintage ν, K_ν is the amount of capital of vintage ν still in existence, and L_ν and MI_ν are the man hours and imported materials being used with these machines. Labor (L) and imported intermediates (MI) are combined by a Cobb-Douglas production function into a composite variable input. Constant returns to scale are assumed for this intermediate process; $\alpha_1 + \alpha_2 = 1$. Embodied technical change increases the efficiency of new machines at rate ρ_1 and disembodied technical change raises the efficiency of all machines at rate ρ_2. Every machine in operation runs at full capacity. Output is varied by putting individual machines into or taking them out of production. Total output (QD) is obtained by integrating QD_ν over vintages from the age of the oldest machine in operation to the present. Total capacity (QD*) is the integral over vintage from the age of the oldest machine in existence to the present.

The short-run production function is the implicit relation between QD and variable inputs. It is here that a useful property of putty-clay models comes into play. The marginal cost for the (representative) firm is equal to the marginal cost on the oldest machine in operation.

Marginal cost is obtained in virtually the same way as with the Cobb-Douglas function. Because variable inputs are always the constraining factor in the short run, short-run output on the oldest machine in operation, ν', is simply the Cobb-Douglas function

$$(5.2) \qquad QD_{\nu'} = \frac{1}{\beta} e^{\rho_1 \nu' + \rho_2 t} L_{\nu'}^{\alpha_1} MI_{\nu'}^{\alpha_2}$$

The marginal product of labor is then

$$(5.3) \qquad \frac{\partial QD_{\nu'}}{\partial L_{\nu'}} = \frac{\alpha_1}{\beta} \left(\frac{MI_{\nu'}}{L_{\nu'}}\right)^{\alpha_2} e^{\rho_1 \nu' + \rho_2 t}$$

and marginal cost is obtained as the ratio of the wage rate to the marginal product of labor. Making the appropriate substitutions from the expansion path of (5.2) yields the marginal cost expression

$$(5.4) \qquad MC = \frac{1}{\beta} e^{-(\rho_1 + \rho_2) t + \rho_1 U} \left(\frac{W}{\alpha_1}\right)^{\alpha_1} \left(\frac{PMI}{\alpha_2}\right)^{\alpha_2}$$

where U is the age of the oldest machine in operation ($U = t - \nu'$).

Equation (5.4) can be compared directly to (3.13) the marginal cost under a Cobb-Douglas function. Aside from the different normalizations used in the coefficients α_i, the vintage capital model replaces the user cost of capital term $(UC/a_1)^{a_1}$ with the age of the marginal machine term, $e^{\rho_1 U}$. The vintage capital model, therefore, requires explicitly that marginal cost increases with output (as older plant is drawn into production) while the Cobb-Douglas formulation would permit constant costs if the user cost of capital remained constant.

Two adjustments to (5.4) are necessary to obtain an estimable aggregate price equation. Data on the age of the oldest machine in operation are not available and β varies with factor prices in period ν'. On the first problem, de Menil employed a relationship between U and capacity utilization (CU) and average age of machinery (AA) that can be approximated by

$$U \approx s_0 + s_1 CU + s_2 AA. \tag{5.5}$$

In our case, we would need to endogenize AA. Replacing AA in (5.4) by its determinants would be preferable to estimating a separate equation for it. Making some simplifying assumptions regarding equal deterioration of vintages, the average age could be approximated by the ratio of lagged investment to lagged capital stock,

$$AA \approx k_0 + k_1 \frac{I_{-1}}{K_{-1}} . \tag{5.6}$$

Then (5.6) and (5.5) together could be substituted into (5.4) for estimation.[1]

Unit variable input requirements, β, can vary with expected factor prices in period ν'. De Menil asserted that movements in β are partly reflected in movements in the average product of labor, QD/L. He replaced $e^{-\rho_1 \nu - \rho_2 t}$ with a weighted average of that expression and the average product of labor in an attempt to capture any major continuous movements in the marginal product of labor.

<u>Comparison of neoclassical and putty-clay production models</u>.--There are other combinations of functional form and capital malleability

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[1]Peter Isard suggested that the assumption of "one-hoss-shay" depreciation and that obsolescence occurs before physical decay could provide a simplified relationship between U, capacity utilization, and lagged investment.

assumptions in addition to the two presented here. In principle, a vintage capital assumption could be employed without imposing the putty-clay assumption of a fixed-coefficient ex post production function. A putty-putty Cobb-Douglas production function could be substituted for (3.1). Then the integration over vintages could be carried out and the implicit relationship between QD and L and MI obtained. However, this relation is complex and empirical implementation would require data on the capital stock and technical progress by vintage.[1] Alternatively, capital could be regarded as homogeneous and fixed in quantity as in the simple Cobb-Douglas and fixed-coefficient cases reviewed above in section 1. However, these alternatives do not resolve the central questions in achieving consistency with the other parts of the country sub-model. Three considerations affect the choice between the neoclassical and the putty-clay production theories:

(1) the presence of the user cost of capital as a short-run determinant of price,

(2) the consistency of the rental concept of capital with the possibility of unemployed capital, and

(3) the consistency of the short-run production function with the long-run production function for potential output.

User cost of capital.--Opinion is divided on the appropriateness of the user cost of capital as a short-run determinant of price. Tobin [13]

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[1]See, for example, Solow [12].

objected to the presence of capital costs in a short-run price equation. He contrasted Nordhaus' [10] neoclassical contention that the elasticity of price with respect to labor cost should be less than one with empirical results that showed prices moving in proportion to standard unit labor cost. Only in the case of a balanced growth path with a constant capital-to-output ratio and where the interest and depreciation rates are constant could the two results be consistent. Tobin preferred a Marshallian short-run view wherein prices are related to marginal variable costs. If the quasi-rents earned in the short run diverge from long-run capital costs, there will be long-run output adjustments by investment and disinvestment. Eventually, Nordhaus' equilibrium conditions would hold, but Tobin maintained that it is not plausible to include them in price equations.

Although the central estimate of the elasticity of price with respect to the wage rate is one in the empirical studies summarized by Tobin, some studies had estimates less than one. Eckstein and Wyss [7] and Heien and Popkin [9] found that interest rates affected prices directly in certain concentrated and regulated industries. More recently, Gordon [8] applied Nordhaus' neoclassical price formulation and found significant cost-of-capital effects.

Clearly, the putty-clay production model more elegantly approximates the reality of fixed and heterogeneous plant than does the assumption that all capital is rented. However, the simplicity of the variable- and homogeneous-capital model constitutes an advantage[1] and, if it is

[1] The Cobb-Douglas neoclassical model also facilitates the explicit derivation of an investment function. See Berner [2].

applied, it seems reasonable to test empirically for the significance of a cost-of-capital effect.

Capacity utilization.--With homogeneous capital, a rate of capacity utilization less than 100% implies a marginal product of capital equal to zero. A zero marginal product of capital is inconsistent with an investment function. Positive investment can be reconciled with less than full capacity only with rather extensive explanation. Variations in demand across regions and products cause some operators to be operating below capacity and others at full capacity. Those at full capacity are investing (even though economy-wide capacity is less than full) because of the high transportation or transactions costs involved in renting the underutilized plant. The putty-clay production function, on the other hand, has the compatibility of underutilization with investment built right into the model through the greater efficiency of new plant.

Long-run production function.--Potential output (against which current output is compared to obtain capacity utilization) is measured on a GDP basis. Long-run potential output should be a function of primary factors (capital and labor) only. There is no way to implement potential supply constraints on intermediate inputs. Both production functions for short-run output considered here include imported intermediates and do not proceed directly from the long-run production function for potential output. At this stage, it is necessary to assume compatibility of the long-run and short-run production functions.

Conclusion.--The putty-clay production model is theoretically superior to the Cobb-Douglas neoclassical model in terms of handling fixed plant and variable capacity utilization. But, because of its simpler form, greater familiarity, and consistency with the investment function, the Cobb-Douglas is proposed as the production function underlying the initial price equations. The putty-clay formulation (5.4) will be held in reserve as a backup equation in case the Cobb-Douglas model performs badly. Once estimation begins, pilot tests on both price equations can be made to determine if one demonstrates any empirical superiority over the other.

APPENDIX

Domestic Output, Separability, and Domestic Absorption

Gross output (Q) is a function of capital, labor, imported intermediate goods (MI), and domestic Intermediates (DI)

$$Q = Q(K, L, MI, DI).$$

We wish to be able to use a concept of domestic output which is net of <u>domestic</u> intermediate goods but includes <u>imported</u> intermediate goods.[1] We therefore define domestic output (QD) by

$$PD \cdot QD \equiv QV - DIV. \tag{A.1}$$

To do this we must assume that the input variables K, L, and imported intermediates are separable from domestic intermediates. The gross output production function must look like

$$Q = Q(QD(K, L, MI), DI). \tag{A.2}$$

In this way, the price of imported intermediate goods will enter naturally the cost function of domestic output. Writing the production function in this nested form amounts to requiring that the marginal rates of substitution between K, L, and MI are independent of the level of domestic intermediates.

Although this form of the production function is a consequence of our desired concept of domestic output, the assumptions implicit in (A.2) are no more restrictive than those underlying commonly applied value-added production functions. The domestic output function, QD( ), can

[1] Domestic and foreign goods are taken to be different; see the discussion of the goods market in Berner [2].

itself be separable in K, L, and/or MI. In fact, the two functional forms for QD considered here -- Cobb-Douglas and fixed coefficients -- are separable functions. The gross output function could therefore be expressed for these specific cases in the familiar value added form,

$$Q = Q(GNP(K, L), MI, DI)$$

$$Q = Q(GNP(K, L), INT). \tag{A.3}$$

The concept of domestic output can be worked through the accounting framework of the typical country sub-model to obtain an expression for domestic absorption. The GNP definition in current value terms is

$$GNPV = CV + IFV + IIV + GV + PX \cdot X - PM \cdot M$$

and can be rewritten in terms of domestic absorption as

$$GNPV = P \cdot A + PX \cdot X - PM \cdot M, \tag{A.4}$$

where A represents real domestic absorption (C + IF + II + G). The value of GNP equals the value of gross output net of the value of intermediates

$$GNPV = QV - DIV - MIV$$

and the value of domestic output is defined in (A.1) as QV - DIV; so

$$GNPV = PD \cdot QD - MIV. \tag{A.5}$$

Setting the RHS of (A.4) and (A.5) equal yields

$$PD \cdot QD - PMI \cdot MI = P \cdot A + PX \cdot X - PM \cdot M$$

$$PD \cdot QD - PX \cdot X = P \cdot A - (PM \cdot M - PMI \cdot MI). \tag{A.6}$$

The value of domestic output minus the value of exports equals the value of domestic sales (DS). The difference between the value of imports and the value of intermediate imports is the value of final imports (MF). Then,

$$PDS \cdot DS = P \cdot A - PMF \cdot MF.$$

Rearranging terms yields the accounting equivalence for domestic absorption

$$P \cdot A = PDS \cdot DS + PMF \cdot MF \tag{A.7}$$

that underlies the expression for the domestic absorption deflator (2.2) introduced above.[1]

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[1] See footnote 1, p. 9.

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