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A MULTI-COUNTRY MODEL OF THE INTERNATIONAL INFLUENCES ON THE U.S. ECONOMY: PRELIMINARY RESULTS

by

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A Multi-Country Model of the International Influences on the U.S. Economy: Preliminary Results

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INTRODUCTION

The major responsibility of the Quantitative Studies Section has been the construction of a model of U.S. international transactions that will provide answers to a broader range of questions than has hitherto been possible using existing models at the Board. The most important of these questions are the following: what effect does U.S. monetary and fiscal policy have on U.S. trade and capital flows, and how do these flows affect important domestic variables such as GNP, interest rates, the price level and unemployment; how do exchange rates respond to changes in macroeconomic policies and what is the effect of exchange rate movements on income and prices; what is the repercussion of economic developments in the United States on other countries, and what are the feedbacks to the United States from those countries; and finally, what is the direct impact on this country of changes in foreign economic conditions and policies.

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The views expressed herein are those of the authors and do not necessarily represent the views of the Federal Reserve System.

In fulfilling this assignment we have constructed and linked quarterly econometric models of the United States and four other countries: Canada, Germany, Japan and the United Kingdom. These models, which are of roughly similar size, explain the main domestic variables and international transactions of each country. The international linkages among these countries -- through trade and capital flows, changes in international reserves, exchange rates and prices -- are specified in considerable detail. The most important instruments of monetary policy (reserve requirements, the discount rate, and net central bank holdings of domestic assets and foreign assets) and fiscal policy (government expenditures and tax rates) are integrated into each country model. Since the sample period runs from 1964 to 1975, the country models have been constructed so that they are operational both under a regime of pegged rates (up to 1973 for most of our countries) and under a regime of managed floating (from 1973).

All five models have been simulated in isolation, and they have also been linked with each other and with a small set of equations explaining the merchandise trade of an aggregated rest-of-the-world sector and short- and long-term Eurodollar interest rates. The entire multi-country model (MCM) was simulated successfully over the period of 1964-1975. The overall tracking ability of the individual country models, both when simulated separately and when linked together, compares favorably with the performance of existing econometric models of the countries in question. For certain countries (most notably Germany) the performance of the country models is improved in the MCM, i.e., when the interaction of all countries and the rest-of-the-world sector is taken into account.

One particularly noteworthy result is the ability of our models to track the movements in four bilateral dollar exchange rates (vis-a-vis the Canadian dollar, the DM, the pound and the yen) during the period of managed floating. Furthermore, the models perform as well, if not better in many respects, during the period from 1973 to 1975 than during the previous fixed-rate period. These results are especially gratifying, since our project constitutes the first attempt to explain several exchange rates simultaneously in a multi-country setting.

The period 1973-1975 was characterized not only by managed floating but by considerable variability in both activity and price variables. This variability, due mainly to the unprecedented rise in prices of primary products, especially petroleum, is captured in the simulation results of our multi-country model: the extremely rapid rise in both import and domestic prices, the high interest rates and decline in both real GNP and capacity utilization are all fairly well tracked. These results provide an indication that our set of linked country models should perform well in describing both the transmission of external shocks among countries and the domestic effects of these shocks on the income, prices, interest rates and levels of employment of individual countries.

This report consists of three parts. Part I provides a summary discussion of the basic structure underlying the econometric models of our five countries. Part II describes the direct and indirect channels through which the country models are linked together, as well as the abbreviated rest-of-the-world sector that closes our multi-country model. Finally, Part III presents an evaluation of the model's performance, namely, the in-sample tracking ability of the individual country models (both by themselves and when linked together) in fixed as well as floating exchange rate periods.

It needs to be stressed that the findings presented here are based on the initial empirical results of the multi-country project. Our simulations are limited to the sample period, and we have neither calculated the multipliers of the system nor performed out-of-sample forecasting tests. Furthermore, the regression results are preliminary; further work in specification and estimation remains to be done. The multi-country project is therefore by no means complete. We nevertheless thought it useful to present the preliminary empirical results at this time to demonstrate that our set of linked country models can be solved successfully for exchange rates and other important variables. In our concluding remarks we outline the program for future research and refinement of the individual country models and the system as a whole.

I. THE PROTOTYPE COUNTRY MODEL

There are numerous differences among the five country models, largely reflecting differences in institutional detail. There is, however, substantial similarity in their basic structure, and this structure can be usefully described in terms of a highly stylized prototype model. The main features of this model are discussed in Part I, and a condensed list of equations is given in the Appendix.

In each model, prices and quantities are determined by the behavior of four classes of economic agents: the monetary authorities (including the central bank and other holders of official foreign assets), the government, commercial banks, and the private nonbank sector (firms and consumers). Each country is assumed to produce a different composite consumption-investment commodity. The domestic and the foreign demands for this commodity, as well as the domestic and export prices, are determined in the expenditure and pricing sector of the model. The labor market contains equations explaining the wage rate and the unemployment rate. The short-term interest rate is determined in the monetary sector, which is based on the identity between the sources and uses of the monetary base. A term structure equation is employed to explain the long-term interest rate. Finally, the balance of payments equations are used to determine the four bilateral dollar exchange rates in the multi-country model.

Part I of this report provides a brief description of the salient features of each sector in the prototype model. It needs to be emphasized that only the most important elements of the structure underlying this model are presented here. For a more detailed description the reader should consult a longer summary paper and the much more extensive papers devoted to individual sectors.[1/]

A. Domestic Output and Price Determination

The market for domestic output is described in terms of three sectors: (1) aggregate demand and expenditure, (2) potential output and capacity utilization, and (3) price determination.

1. Domestic expenditure sector

Aggregate demand (GNP) is broken down into five major components: personal consumption, fixed investment, inventory investment, exports and imports. Consumption depends upon private disposable income[2/] and net worth, while gross fixed investment (following the neoclassical approach) is positively related to current and lagged changes in GNP and negatively related to current and lagged changes in the user cost of capital. Since the long-term interest rate is an important determinant of the user cost of capital, the investment function provides a key link between the monetary and real sectors of the model. Inventory investment is assumed to depend upon the gap between expected sales (a function of moving averages of GNP components) and current production. The change in inventories thus contributes, together with the movement in prices, to the absorption of any discrepancy between final demand and the supply of domestic output.

1/ See Richard Berner, et al., "Modeling the International Influences on the U.S. Economy: A Multi-Country Approach," (IFDP #93), Richard Berner, "The Goods Market and the Labor Market of the Multi-Country Model," (IFDP #97), Peter Clark and Sung Kwack, "Asset Markets and Interest Rate Determination in the Multi-Country Model," (IFDP #94), Howard Howe, "Price Determination in the Multi-Country Model," (IFDP #98), Guy Stevens, "Alternatives for Modeling the World Outside the United States," (IFDP #96), and "Balance of Payments Equations and Exchange Rate Determination," (IFDP #95).

2/ The disposable income variable used in the model is a proxy obtained by adding total government transfers to GNP, and subtracting taxes and capital consumption allowances.

Imports and exports of goods and services are broken down into merchandise, investment income and other services[1/]. Investment income flows are related to lagged stocks of claims and liabilities vis-à-vis foreigners and to the corresponding interest rates. Imports of goods and imports of other services are functions of domestic economic activity, domestic prices and import prices. Since import prices depend upon foreign export prices and exchange rates, the import equation plays a key role in the transmission of external influences to a country sub-model[2/]. The import equation is also important in that the exports of each country are influenced by the imports of the other four countries and those of the rest of the world (ROW). This feature of the model -- which insures that world imports are equal to world exports at every point in time -- is described in more detail in Part II.

Private net worth is another important variable determined within the real sector of the model. As mentioned earlier, this variable is included in the consumption function, and it is also used as a scale variable in all the asset demand equations (Section I.C). The change in private net worth is essentially derived as the difference between private disposable income and personal consumption. The stock of private net worth is simply the cumulated sum of private savings and therefore excludes capital gains and losses.

2. Potential output and capacity utilization

The rate of capacity utilization is defined as the ratio of

1/ In the U.S. and Japanese models, merchandise imports are further disaggregated into fuels and nonfuels.

2/ These linkages are systematically described in Part II.

actual to potential GNP. Potential GNP, in turn, is assumed to be related to the capital stock and to potential employment through a Cobb-Douglas production function. Substituting for potential GNP yields an equation in which capacity utilization is directly related to the output-labor ratio and inversely related to the capital-labor ratio.

3. Price determination

There are three main price variables in the prototype model: the deflator for domestic absorption expenditures (consumption, investment and government spending), P, the export unit value index, PXG, and the import unit value index, PMG. In each country PMG is determined by the export prices of the four other countries (expressed in the currency units of those countries), the export unit value index of the rest of the world, and the exchange rates which convert these foreign-currency export prices into domestic currency. In turn, each country's export price is determined as a mark-up over wage costs, changes in labor productivity, and the cost of imports (PMG), where the mark-up depends on domestic and foreign capacity utilization rates as well as competitors' export prices. Thus price and exchange rate developments are transmitted directly among the countries in our model through these import-export price linkages.

The deflator for domestic expenditures -- which includes domestic as well as foreign goods and services -- is also explained as a mark-up over wage rates, labor productivity and import prices, with the mark-up varying in response to the level of domestic capacity utilization. Changes in aggregate demand will have an influence on the domestic price level not only through the impact of capacity utilization, but also through demand pressures affecting wage rates. Changes in foreign prices of intermediate and final goods, together with exchange rates, have a direct effect on the domestic price level through import prices.

B. The Labor Market

The rationale behind the equations of the labor market is that, due to the existence of union contracts and minimum wage laws, wages do not adjust enough to clear the market. Hence there is typically a disequilibrium in the form of excess labor supply.

The important variables determined in this sector are the wage rate in manufacturing and the unemployment rate. The rate of change in nominal wages is a function of the unemployment rate and the expected rate of change in the deflator for aggregate expenditure. Unemployment is viewed as the difference between supply and demand for labor. Labor demand is determined by lagged adjustment to the desired labor input (the value of output divided by the wage rate), a relation which follows from the equality between the real wage and the marginal physical product of labor. Labor supply is a function of population and the real wage rate.[1/]

C. The Monetary Sector: Asset Demand and Interest Rate Determination

In spite of many differences in existing financial institutions, we have found that a common framework for the specification of the monetary sector can be applied to three of the five countries. This is the framework described below. A somewhat different treatment of certain aspects

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[1] In the model, labor demand is identified with the level of employment and labor supply with the labor force. In the Canadian and U.K. models, the equations for these two variables are combined into a single unemployment rate equation. In the other three models, employment and labor force are handled separately. The Japanese model also includes the unfilled vacancy ratio since this variable is known to be a considerably more sensitive indicator of labor market conditions than the unemployment rate.

of the monetary sector was required for Japan and the United Kingdom, as explained at the end of this section.

The basic building block in this sector is the balance sheet of the central bank. The balance sheet identity specifies the link between the main sources of the unborrowed monetary base--net foreign assets (NFA) and the net government position (NGP) --and its uses: required reserves (RR), free reserves (RF, which are excess reserves minus borrowed reserves), and currency (CUR)

$$NFA + NGP \equiv RR + RF + CUR.$$

Required reserves are calculated by multiplying reserve requirement ratios by the stock of deposits (mainly demand and time deposits) demanded by the non-bank public. The demands for deposits, free reserves and currency are a function of the domestic short-term interest rate (and of course other variables). Hence for a given stock of the unborrowed base, the short-term interest rate will adjust so as to equilibrate the given stock with the direct and indirect demands for base money: RR, RF and CUR.

For estimation purposes we re-normalized the equation for free reserves so as to make the short-term rate the left-hand-side variable. This equation, which was used in estimation, thus explains the short-term rate in terms of free reserves (RF), the discount rate and other variables. RF is determined by the balance sheet identity, and consequently reflects changes in the supply of and demand for base money other than RF. These supply and demand factors will therefore affect the short-term interest rate through their influence on

RF.[1]

Except for the model of the United Kingdom, we do not introduce explicitly the demand and supply for long-term securities. Instead, we assume that these securities are close substitutes for short-term money-market instruments, and therefore we use term structure equations which express the long-term rate as a weighted average of current and past values of the short-term rate.

The equations explaining the demand for both domestic and foreign assets follow basically the portfolio balance approach and are specified in stock form. Due to the lack of benchmark data, however, in several instances the capital flow equations have been estimated in first-difference form. They include private net worth, a transactions variable, and a vector of rates of return as the main explanatory variables. Different transactions variables are used in different equations: consumption in the currency equation, GNP in demand and time deposit equations, and exports (imports) in short-term claims (liabilities) equations.

The equations explaining financial claims and liabilities vis-à-vis foreigners contain not only domestic and foreign interest rates, but also a variable representing the expected change in the bilateral U.S. dollar exchange rate, since this is part of the rate of return on foreign assets. In several countries the forward rate is used as a proxy

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1/ Initially we estimated the free reserve equation with RF as the dependent variable. Under this specification we had basically three asset demand equations (deposits, currency and RF); these equations, together with the balance sheet identity, are sufficient to solve for the quantities of the assets demanded and the short-term interest rate. The simulation results of this procedure, however, were inferior to those obtained using the approach described in the text.

for the expected future spot rate.[1/] The forward rate or premium is itself explained as a function of the interest rate differential as well as variables designed to reflect expectations concerning future spot rates. The two variables used most often, either separately or combined together, are the ratio of imports to net foreign assets (lagged one period) and the country's export price divided by a weighted average of other countries' export prices. The former proxy variable is particularly useful during the fixed rate period, when it serves as an indicator of the ability of the central bank to continue pegging its exchange rate against the dollar. We hypothesize that an increase in these variables generates an expectation of a future depreciation of the currency, which shows up as a larger forward discount (smaller premium).

A key empirical feature of our model is that the net capital account in each country model responds in a stabilizing manner to a change in the current exchange rate. Given the expected future spot rate -- which, as described above, is determined endogenously -- a depreciation of the exchange rate in the current period will generate a net capital inflow that will attenuate the depreciation. In our model, therefore, expectations are basically regressive: a depreciation (appreciation) generates expectations of a future appreciation (depreciation). These regressive expectations prevent exchange rates from fluctuating sharply in response to shocks to the balance of payments.

1/ In the original specification of the model, the expected future spot rate was assumed to equal the actual spot rate observed one quarter ahead plus a random error. Except for Canada, this assumption did not provide good estimation results; therefore the forward rate and the other expectational variables discussed below were used for Germany, Japan, the U.K. and the U.S. In the Canadian model, the specification in terms of next period's value of the spot rate was retained. In simulation, this actual value is replaced by an estimated value obtained from a regression using past exchange rates and changes in NFA as explanatory variables.

Another key feature of the model is that changes in net foreign assets (ΔNFA) for all five countries are endogenous variables under pegged exchange rates. (See section I.D.) Hence changes in monetary policy instruments (discount rate, reserve requirements and control over the central bank's net government position) will have induced effects on the country's NFA component of the monetary base. Changes in NFA may, however, be sterilized by the central bank though changes in its net government position (NGP). We assume that, during the sample period, there was full sterilization in the United States and United Kingdom, and that partial sterilization occurred in Canada. For both Germany and Japan NGP is taken as exogenous. In future work, however, we shall also take account of the response of NGP in these two countries to variations in NFA and other variables.

Finally, we should point out a number of linkages between the monetary and real sectors. First, a rise in GNP will increase the demand for bank deposits, thereby raising the short-term interest rate which, through the term structure equation, will raise the long-term rate. The higher interest cost will reduce the level of investment, thereby moderating the initial rise in GNP. Second, a change in private net worth will have interest rate effects (assuming a given stock of the monetary base) and consequently affect investment. Third, when a country does not maintain a pegged rate, a change in monetary policy will have both interest and exchange rate effects. For example, expansionary monetary policy will reduce (at least in the short run) the domestic interest rate and depreciate the country's exchange rates vis-à-vis the four other countries. These exchange rate changes will tend to improve its trade balance (expressed in real terms) by raising the price of imports and increasing demand for its exports. Hence, under managed floating, monetary policy affects domestic income and prices not only through its familiar effect on investment expenditure, but also through changes in exchange rates.

The current version of the Japanese financial model is centered around a consolidated balance sheet of the monetary authorities (Bank of Japan and Foreign Exchange Fund). The balance sheet identity is similar to that discussed earlier for the prototype financial model except that the variable RB (reserves borrowed by city banks from the Bank of Japan) is used instead of the free reserve variable RF. (Japanese banks hold virtually no excess reserves with the Bank of Japan; and commercial banks other than the large city banks do not borrow from the central bank). As in the prototype model NGP is exogenous. The model includes demand equations for currency, demand deposits and time deposits. The domestic interest rate included in these asset demand equations is not the short rate (as in the prototype model) but the rate on bank debentures (RL), which is controlled by the monetary authorities. This is due to the fact that the short-term rate used in the Japanese model (the rate on call money) is strictly an interbank rate and is therefore not directly relevant to the portfolio decisions of the public non-bank sector. The equation for time deposits also includes an exogenous interest rate on time deposits. The treatment of required reserves is similar to that given in the prototype model, and the call money rate is determined by inverting the equation for RB (borrowed reserves). In the resulting expression, the main determinants of the call rate are the Bank of Japan's discount rate and the banks' borrowing/deposit ratio. The balance sheet identity is then used to determine RB.

The Japanese model also features a bank loan rate (RLN) which plays an important role in the private fixed investment equation. This interest rate is explained by inverting the banks' loan supply function and assuming the stock of bank loans to be exogenously determined by the monetary authorities through the so-called window-guidance mechanism.

The U.K. financial sector follows the London Business School model in that the balance sheet of the commercial banking system -- rather than that of the Bank of England -- is modeled. Reserve assets in the U.K. are not all liabilities of the Bank of England. The balance sheet of the Exchange Equalization Acount, which contains net foreign assets, is aggregated with that of the Treasury. For these and other reasons, the monetary base concept is not operational for the U.K. Attempts to determine the short rate (local authority) implicitly using the balance sheet of the banking system proved fruitless, largely because nondeposit liabilities were nearly impossible to model. Currently, these liabilities are determined residually, and the short rate is derived as a reduced form based on other short rates, mainly the minimum lending rate. By contrast, the long rate (war loan) is estimated using an inverted supply function, with the authorities assumed to control new debt issues to manipulate the rate. (It is generally agreed that monetary policy in the U.K. is done in gilts and not bills.) Eurodollar assets and liabilities to foreigners are modeled explicitly since they enter in the balance sheet of the commercial banks. In turn, these deposits influence U.K. banks' loans in foreign currency to U.K. residents, part of which enters in the balance of payments. Other links between the balance of payments and the financial sector are represented by the appearance of net short-term inflows in the equation for Eurodollar loans to overseas residents, and by the inclusion of portfolio claims in the equation for loans in foreign currency to U.K. residents for investment overseas.

D. Exchange Rates and Reserve Changes.

Between 1970 and 1973 many countries -- including the five treated separately in the multi-country model -- have undergone a transition from a system of pegged exchange rates to one of limited exchange rate flexibility, or managed floating. This transition is explicitly introduced in the MCM. Indeed, the structure of each country submodel shifts when that country evolves from one exchange regime to the other, notably by changing the way in which the spot exchange rate (E) and the change in official net foreign assets (ΔNFA) are determined.1/ Under both regimes, balance of payments equations play an important role in the determination of these variables.2/

During the pegged exchange rate period, each country's spot exchange rate vis-à-vis the U.S. dollar is exogenous3/ while the

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1/ ΔNFA is a proxy for exchange market intervention. It is defined as the change in net foreign assets held by the monetary authorities net of SDR allocations and valuation changes.

2/ In previous papers we have discussed how (and proved that) a country's balance of payments equation can be substituted for any of the other equilibrium conditions in the model (See the Summary paper, IFDP #93, pp. 46 ff. and IFDP # 95). For reasons discussed in these papers, we have taken the course of substituting the balance-of-payments equation for the market-clearing condition in the short-term securities market.

3/ These exchange rates may exhibit minor fluctuations around the par value during this period; and they may also undergo sharp discontinuous changes (like the 1969 revaluation of the DM). But these changes are exogenous to the model.

The four bilateral exchange rates considered in the model are the rates for the DM, the Canadian dollar, the U.K. pound, and the Japanese yen, respectively, in terms of the U.S. dollar. The model also includes an "effective" exchange rate for the U.S. dollar which is simply a weighted average of the four bilateral rates previously mentioned. Cross-exchange rates (e.g. between the Pound and the DM) are obtained by assuming perfect triangular arbitrage.

change in this country's stock of net foreign assets is endogenously determined by using the balance of payments equation. During the period of managed floating, the 4 bilateral exchange rates in the model become endogenous. The balance of payments equation need not be used to determine ΔNFA since the rate is not pegged; therefore this equation can be used to solve for the spot exchange rate. The treatment of exchange rate and intervention variables is summarized in the following table.

Variable	Under Pegged Exchange Rates	Under Managed Floating
Exchange Rate vis-à-vis U.S.$ (E)	exogenous	endogenous (given by balance of payments equation)
Change in NFA	endogenous (given by balance of payments equation)	exogenous (German and U.K. models) endogenous (given by reaction function -- Canadian and Japanese models)

With regard to intervention during the floating rate period, one possibility is to assume that it is exogenously determined by the monetary authorities. This assumption was initially adopted for all countries in the MCM. In the current version of the model, however, it was decided to experiment with reaction functions for ΔNFA in the Canadian and Japanese models. The reaction functions assume that the monetary authorities "lean against the wind" -- that they intervene to moderate changes in the exchange rate. Currently the form of these equations is quite simple and much work remains to be done in this area. It is encouraging, however, that the introduction of intervention functions improved the simulation results for both the Japanese yen and the Canadian dollar, presumably by moderating the effect on these exchange rate variables of errors in the various balance of payments components.

II. FOREIGN INFLUENCES ON COUNTRY MODELS AND LINKAGES AMONG MODELS

This section discusses the key influences of foreign variables on a typical country model and the important linkages among models. Directly or indirectly, every foreign variable affects every endogenous variable in a country model because all variables are determined simultaneously. In this part of the report, these linkages are explained first in terms of direct influences, and then by tracing through the impact on a country model of changes in some important foreign variables. The consistency of model linkages and the structure of the rest of the world sector are discussed in the final sections of Part II.

A. Direct influences

The accompanying table presents the signs of important foreign variables in the equations for selected domestic variables. The table illustrates the fact that current exchange rates appear in virtually every component of the balance of payments. It also highlights the different roles played by the spot rate in different equations; for example, an exchange rate appreciation has a negative influence on net liabilities to foreigners, due to the role played by the current spot rate in the formation of expectations concerning future spot rates.[1/] On the other hand, the effect of exchange rates on the investment income account is ambiguous, since both the valuation of income from foreign investment and of payments from overseas investment are negatively related to spot rates. Spot exchange rates also influence the capital account indirectly through the forward premium, due to the introduction of expectations proxies in the forward premium equation.

[1/] In addition there is a valuation effect which stems from converting the value of foreign wealth into local currency (this is simply a translation effect and does not take portfolio composition into account: each country's net worth is assumed to be denominated in local currency).

Key Direct Foreign Influences

Foreign Variable / Domestic Variable	Spot Exchange Rate 1/	Imports of Other Countries	Foreign Interest Rates	Foreign Prices 2/	Foreign GNP, Foreign Net Worth
Goods Imports 3/	+			-	
Goods Exports	-	+		+	
Import Price	-			+	
Domestic Price 3/	-			+	
Export Price	-			+	
Investment Income	-		+		
Investment Payments	-		+		
Services Imports	+			-	
Services Exports	-			+	+
Claims on Foreigners	+		+		
Liabilities to Foreigners	-		-		+
Interest Rates			+		
Forward Premium	-		+	+	

1/ Exchange rates are defined in units of foreign currency per unit of local currency.
2/ Including the price of primary products.
3/ The effect of exchange rate changes on the volume of imports and on the domestic price takes place via the price of imports.

Merchandise export equations in the country models are treated as quasi-identities. If bilateral trade data were available without errors, exports of one country would be the sum of other countries' imports from that country. Rather than attempting to reconcile data discrepancies and to specify bilateral import demand equations, weighted averages of other countries' imports are constructed using variable (but exogenous) trade share weights, and the difference between a country's merchandise exports and the constructed average of foreign imports is explained by pressure of demand variables, such as foreign capacity utilization. As indicated in the table, foreign prices and spot rates influence exports through their influence on other countries' imports. The trade shares sum to unity, and this yields a consistent treatment of world exports and imports, discussed in more detail below.

As seen in the table, foreign interest rates influence both the current and capital accounts. While the influence on net liabilities to foreigners of foreign interest rates is negative, their influence in both investment payments and receipts is positive. As a result of interest arbitrage, the direct influence on the forward premium of foreign interest rates is positive (but not necessarily with a coefficient of unity).

Foreign prices influence imports via the domestic import price variable. Similarly, the positive influence of foreign prices on exports is transmitted via the demand for imports in other countries, which respond positively to increases in these countries' domestic prices. The influence of foreign prices via the import price extends to local prices, as discussed in Part I above. Finally, the effect on the forward premium reflects the expectational considerations referred to above.

The only variables directly affected by foreign GNP (or income) are services imports and exports; but of course foreign GNP has a strong positive influence on exports via other countries' imports. Foreign net worth (or claims) is a scale variable for liabilities to foreigners, viewed as a foreign asset demand.

B. Tracing Indirect Influences of Foreign Variables on a Country Model

Three illustrative cases will be considered: an increase in foreign income, an increase in foreign interest rates, and an increase in foreign prices (equivalent to a devaluation during the fixed rate period).

The discussion is cast in terms of a single country model, with all foreign variables exogenous, and a change is made only in the foreign variable in question. Thus, these cases do not fully describe how the MCM works -- they merely trace the effects on a domestic model of an exogenous change in a variable explained elsewhere in the MCM.

1. Increase in foreign GNP

An increase in foreign GNP will increase the home country's exports and these will have effects similar to an exogenous increase in domestic demand: higher GNP, higher imports, lower unemployment, higher wages and prices, higher net worth and therefore higher demand for domestic assets. The increased demand for domestic assets will tend to increase domestic interest rates, increasing foreign demands for domestic assets. On the other hand, the increase in domestic net worth will increase domestic demands for foreign assets.

Assuming that the increase in foreign income increases the home country's exports more than its imports, and that the higher domestic interest rates generate a net capital inflow, there will be an increase in reserves under fixed exchange rates. In the absence of sterilization, the monetary base will increase, reducing the upward pressure on the domestic interest rate. In those countries that are assumed to sterilize (the United States, United Kingdom, and, to some extent, Canada), the base will be fully or partially insulated from reserve changes. Hence, the responsiveness of imports to income changes, the interest elasticity of capital flows and the extent to which the authorities sterilize reserve changes will influence the magnitude of the movement in the domestic interest rate.

Under floating exchange rates there will be an exchange rate appreciation rather than an inflow of reserves if there is no official foreign exchange intervention. The extent of the appreciation will depend on the short-run elasticity of the trade account, as well as on the degree to which the current appreciation generates expectations of a future depreciation, which will induce a desired capital outflow. In any case, the effects on domestic income, price, etc., caused by the increased foreign income will be much smaller than in the pegged rate case. In the case of managed floating, these domestic effects will approach those described above for fixed rates, the more the central bank intervenes in the foreign exchange market.

2. Increase in foreign interest rates

Two subcases must be distinguished here, depending on the exchange rate regime. Under fixed exchange rates, an increase in foreign interest rates will cet. par. generate a capital outflow and a concomitant decline in reserves. The decrease in reserves will, in the absence of sterilization, contract the unborrowed base, raise domestic interest rates, and eventually choke off the outflow. The increase in domestic interest rates will eventually reduce aggregate demand, mainly via investment. The effect on the net investment income account is ambiguous, since both receipts and payments will increase.

In a floating rate regime, an increase in foreign interest rates will result in a depreciation of the exchange rate. Expectations of a future appreciation, given the spot depreciation, will attenuate the downward movement in the rate. It is not at all clear what will be the effects of an increase of foreign interest rates on the current account and therefore on domestic demand and interest rates even given the knowledge of the interest elasticity of capital flows. To the extent that there is a depreciation, however, the domestic price level and the forward premium will tend to rise -- which would tend to offset any current account improvement.

3. Increase in foreign prices, or a devaluation

An exogenous increase in foreign prices will tend to improve the real current account of the home country -- again, possibly with a lag-- but domestic prices will also rise, attenuating the initial current account improvement. Thus, the aggregate demand effects are similar to those caused by an increase in foreign GNP. The principal difference between the two cases is that the increase in foreign GNP results in a direct effect on aggregate demand, with only an induced effect on home-country domestic and export prices, while an increase in foreign prices results in a direct domestic price effect and an induced effect on aggregate demand through the current account balance.

In both cases, the effects on aggregate demand will result in higher interest rates, generating a capital inflow. Additional effects will stem from induced changes in the variables representing exchange rate expectations: the competitive and reserve asset position of the home country. An improvement in the competitive position due to an increase in foreign prices will generate expectations of an exchange rate appreciation, leading to a further capital inflow. To the extent that domestic prices rise, the magnitude of these expectations and the inflow will be dampened.

An exogenous devaluation in a fixed rate regime will have current and capital account effects similar to those resulting from an increase in foreign prices. The primary difference between the two cases is that a devaluation, in addition to increasing import prices, decreases home-country export prices in foreign currency, yielding a direct increase in foreign demand for domestic exports that is absent in the case of a foreign price increase.

C. Consistency of Model Linkages

The trade shares mentioned above sum to unity; given n countries, this ensures that rest of the world (ROW) exports can be obtained by adding up the exports of the other n-1 countries and subtracting this result from total world imports. This is a restatement of the world trade constraint, which means that world imports equal world exports. In fact, two world trade constraints must hold -- one in current prices, and one in constant prices. These two constraints plus two behavioral equations, to be described in the section on the ROW that follows, determine ROW exports and imports and their prices.

Similar consistencies cannot be enforced either on the asset side of the model or on other current transactions because of data limitations. However, the use of foreign claims or foreign net worth as scale variables in liabilities equations does ensure that an increase in foreign claims or foreign net worth will yield an increase in liabilities to foreigners in the home country.

D. Rest of the World

The ROW is represented by equations for trade and trade prices, the Eurodollar and Eurobond interest rates; and three exogenous price variables. This strategy precludes, at this point, modeling feedbacks between the countries in our model and say, ROW income. However, it captures the important trade interactions, and endogenizes the world trade constraints and the Eurodollar rate. Detail for other countries is always a possibility for future work.

1. Trade

The volume of ROW imports is related to an exogenous ROW industrial production index and the prices of ROW imports and exports. The ROW export deflator is a function of the export prices of the five countries in the model, the price of primary products, and a weighted average of endogenous exchange rates.

2. Eurodollar rates

The three month Eurodollar rate is determined as a reduced form from the supply and demand for three month Eurodollars. Entering the reduced form are the U.S. short rate, an average of other countries' short rates, a weighted average of forward premia vis-à-vis the dollar, foreign currency borrowing by the U.K. government, the differential between the Regulation Q ceiling and the CD market interest rate (when positive), and special U.S. securities sold to foreign branches of U.S. banks. The Eurobond rate is a function of past short-term Eurodollar rates in a term structure equation.

3. Exogenous prices

Three exogenous prices enter the models: a dollar price index of primary products other than cereals (including petroleum), a dollar price index for cereals, and the Bremen index of liner freight rates. The primary products prices enter equations for both import and export unit values (see table above), while the freight rate index influences the price of imported services (for the UK, Canada, and Japan).

III. EVALUATION OF MODEL PERFORMANCE

Our evaluation of the performance of the multi-country model (MCM) is divided into four sections. First, the simulation results from individual models and from the MCM are compared. In the second section, the tracking ability of key variables in the MCM is analyzed in more detail. Third, MCM performance is compared in the two exchange rate regimes. Finally, the within-sample predictive performance of the model in the wake of the oil price shock is examined.

A. Individual Models Compared with the MCM.

To evaluate the performance of the MCM and its component country models, it is natural to compare the tracking ability of each model alone with the ability of the MCM. Thus, the individual models and the MCM were simulated dynamically over their common sample period, 1964:4 through 1975:4. The simulation results reveal acceptable performance not only for the country models alone but also for the MCM, indicating that the errors in individual models are not compounded when the models are linked together. The statistics for the variables representing the expenditure, price, labor, monetary and balance of payments sectors are given in Table 1. ME denotes the mean error and is included as an indicator of bias, and RMSE denotes the root mean square error. Both statistics are in terms of percentage errors so that units of different variables do not hinder comparability.[1/]

[1/] RMSE in percentage terms distorts evaluation of the performance of variables that have zero mean and large variance, such as the change in NFA. This distortion is also present for UN and RS since, for example, a one percentage point error could be a fifty percent error.

Table 1: Percent Errors for Key Variables in the Multicountry Model and Individual Country Models (Simulation Period: 1964:4 to 1975:4)

			GNP	P	CU	UN	RS	MG	PMG	XG	PXG	E	NFA	RED
Canada														
	MCM	ME	0.28	-1.35	1.44	-0.41	0.13	3.81	-5.11	0.79	-2.54	1.44	22.15	
		RMSE	2.4	2.1	4.2	17.2	14.3	9.9	7.7	6.5	4.7	5.3	32.5	
	Country	ME	0.02	0.18	0.77	0.85	-6.9	0.05	-0.07	0.01	0.15	0.31	38.33	
		RMSE	1.8	0.7	3.5	13.7	21.0	7.0	1.81	4.5	2.65	3.0	54.4	
Japan														
	MCM	ME	-0.79	-0.72	-0.80	-2.8	2.49	2.68	-3.61	-0.09	-0.51	-0.79	-4.63	
		RMSE	3.7	1.5	5.9	24.4	7.5	7.7	5.4	5.2	3.5	3.0	13.3	
	Country	ME	-0.4	0.6	0.1	1.0	1.0	0.0	0.9	0.0	1.5	-3.4	5.7	
		RMSE	3.5	1.4	6.4	25.7	8.9	6.6	3.7	4.2	2.4	5.2	24.2	
UK														
	MCM	ME	0.18	-0.42	0.16	7.64	2.43	-0.06	-2.87	-0.57	-1.62	1.87	*	
		RMSE	2.6	2.9	1.8	20.9	6.9	4.2	4.3	4.2	2.6	6.5	*	
	Country	ME	-0.12	0.23	0.05	5.6	0.91	-0.81	-1.0	-0.60	-0.44	1.10	*	
		RMSE	2.7	3.4	1.9	18.0	5.6	3.5	2.7	4.0	1.8	5.0	*	
US														
	MCM	ME	0.45	-0.85	-1.02	10.32	2.69	1.88	-2.84	12.46	-10.22	0.53	23.49	-0.40
		RMSE	4.1	1.9	8.6	32.1	19.8	8.7	5.4	18.0	13.6	2.3	38.5	19.7
	Country	ME	0.42	-0.64	-1.04	10.24	3.57	1.05	-1.69	12.25	-9.49	***	35.08	0.82
		RMSE	4.0	1.7	8.5	31.1	19.8	7.5	4.0	17.7	12.6	***	48.5	17.1
W.Germany														
	MCM	ME	-1.01	-0.23	-0.11	**	3.54	0.22	-1.91	-1.8	-1.41	4.68	-4.56	
		RMSE	2.4	1.5	3.4	**	13.4	7.9	4.4	4.8	2.9	9.4	11.2	
	Country	ME	0.43	1.29	1.55	**	0.92	2.68	1.15	0.30	1.21	-3.53	-0.54	
		RMSE	4.4	4.3	5.2	**	17.7	7.7	3.6	4.0	3.7	9.2	15.8	
Rest of the World														
	MCM	ME						1.17	-2.80	-1.05	-1.21			
		RMSE						3.3	3.9	3.1	2.0			

Variables: GNP = gross national product (gross domestic product for the U.K.), P = absorption deflator, CU = capacity utilization rate, UN = unemployment rate (unfilled vacancy ratio for Japan), RS = short-term interest rate, MG = imports of goods, PMG = import price, XG = exports of goods, PXG = export price, E = exchange rate index in U.S. dollars per unit of local currency (weighted average of the four bilateral rates for the U.S.), NFA = net foreign assets of the monetary authorities, RED = Eurodollar rate. [ME =mean percentage error; RMSE = root mean square percentage error].

* Statistics not relevant because the variable takes on negative as well as positive values.

** Variable not used in the German model

*** Exchange rate variables are exogenous in the U.S. model when simulated individually.

The country models perform well, judging from the statistics in the table and from the simulation results of variables unreported here. On average, the errors in GNP and prices are low, especially for a 45 period dynamic simulation over a change in exchange rate regimes. The U.S. export price is the exception to this appraisal. The export price affects export volume proportionally and therefore GNP and other variables.[1/]

Despite the high degree of simultaneity within and across models, the performance of the MCM is more than adequate. Interest rates (except for the UK) and reserves (NFA) all perform better in the MCM than in the individual country model simulations. This is due to the fact that, under fixed exchange rates, an error in income leading to an error in domestic imports will generate a larger error in the current account balance when other countries' imports are exogenous than when other countries' imports are endogenous. This follows from the dependence of domestic exports on all other countries' imports. Our treatment of the ROW is apparently satisfactory in terms of closing the model; the ROW is not the repository of large residual errors. This is due to the consistent way in which we treat world imports and exports.

West Germany is the country most affected by the improvement in NFA resulting from linkage to other countries. The reduction in the errors for NFA, and the resultant reduction in the errors in the monetary base, reduces errors in both short- and long-term interest rates. For

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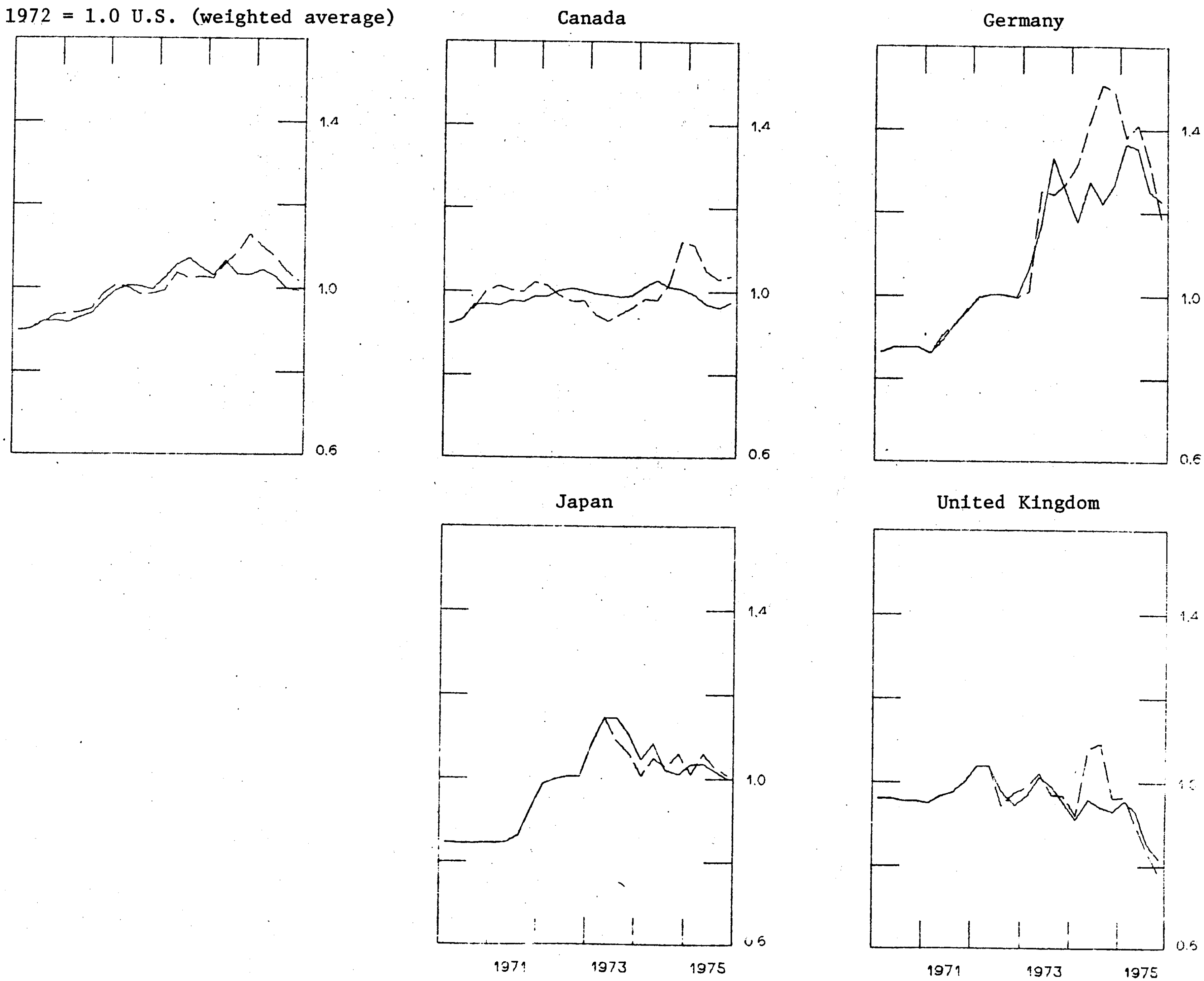
[1/] Introduction of an alternative export price equation in the U.S. model alone (neither reported in the table nor included in MCM due to time constraints) reduces the ME and RMSE for U.S. exports to -5.5 and 7.4, respectively, and the ME and RMSE for GNP are respectively decreased to -0.02 and 3.7 percent.

both Canada and Japan, interest rates also appear to perform better in the MCM as a result of an improvement in NFA. The U.S. monetary base is exogenous, due to the assumption of complete sterlization. Hence, the performance of the U.S. short-term interest rate is insignificantly affected when the United States is part of the MCM. Finally, as a result of the reduction in errors in interest rates, German investment and therefore GNP are dramatically improved when Germany is linked with all other countries.

The RMSE's of the GNP variables are, by generally accepted criteria (2 - 3%), highly satisfactory both in the MCM and in each country model alone. That GNP errors are offsetting across countries is indicated by the mixed signs on the ME's (these signs give an indication of the direction of the errors' bias.) In comparison with the other variables in Table 1, the RMSE's of UN and RS in percentage terms may appear to be quite high. However, these relatively large errors result from both the nature of the data and the inability of models in general to predict these variables with accuracy. For example, in the Federal Reserve - MPS model, the ratio of the RMSE of UN to its sample mean is 11.9%, while the corresponding statistic for the US unemployment rate in the MCM is 19.3%. The analogous figures for the short-term interest rate are MPS - 20.4% and MCM - 28.8%.

No comparable previous modeling effort has simultaneously endogenized the four exchange rates that are present in this model. The errors on these variables range from 3.0 to 9.4% in the MCM, and the four country average has a 2.3% RMSE. Figures 1.a - 1.e plot the actual and simulated values for the exchange rate indices over the period 1970:1 - 1975:4. Except for overpredictions in 1974:2 - 1974:3, the model

Figure 1. Actual (solid line) and Simulated (dashed line) Values of Exchange Rates
(US $ per unit of foreign currency, quarterly average)



appears to perform remarkably well.

The performance of the model cannot be fully described by the two statistics discussed so far. In what follows, therefore, the tracking ability of the MCM in different subperiods is analyzed in more detail.

B. In-Sample Tracking

This section examines the ability of the multi-country model to reproduce history over its own sample period. Other checks on the quality of the model (relationships among critical parameters and multiplier analysis) are yet to be made. However, acceptable tracking over the sample period is a necessary condition to qualify the model for forecasting and policy analysis applications.

Table 2 presents four tracking measures for six endogenous variables: GNP, the domestic absorption deflator (P), capacity utilization (CU), the short-term interest rate (RS), the import price index (PMG), and the exchange rate index (E). For the rest-of-world, the relevant activity and price variables are imports of goods (ROWMG) and the export price index (ROWPXG), respectively. Figures 2a - 2e present actual and simulated values for each country over the period 1964:4 - 1975:4.

The root mean square percentage errors for these 27 variables are reproduced from the MCM portion of Table 1. The next statistic presented is Theil's U-statistic or inequality coefficient.[1/] This is the square root of the mean square prediction error of changes in the variable as a fraction of the mean square of the actual changes. The inequality coefficient constitutes a measure of how much better the model predicts than a naive no-change extra-

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[1/] H. Theil, Applied Economic Forecasting, Amsterdam: North Holland 1966, pp. 26-29.

Table 2

Measures of In-Sample Tracking Performance

for the Period 1964:4 to 1975:4

Country / Variable[a]	Canada	Germany	Japan	U.K.	U.S.	R.O.W.
GNP						ROWMG
RMSE	2.4	2.4	3.7	2.6	4.1	3.3
U-statistic	.39	.47	.29	.53	.52	.57
opposite direction[b]	8	13	8	18	8	20
error runs[b]	4	13	5	16	4	27
P						ROWPXG
RMSE	2.1	1.5	1.5	2.9	1.9	2.0
U-statistic	.18	.25	.43	.20	.13	.17
opposite direction	1	4	2	4	0	10
error runs	6	7	8	7	2	7
CU						
RMSE	4.2	3.4	5.9	1.8	8.7	--
U-statistic	.66	.61	.61	.32	.78	
opposite direction	19	14	18	18	28	
error runs	6	12	6	10	4	
RS						RED
RMSE	14.3	13.4	7.5	6.9	19.9	19.7
U-statistic	.33	.32	.24	.26	.27	.30
opposite direction	10	10	7	9	11	11
error runs	14	17	13	20	7	8
PMG						
RMSE	7.7	4.4	5.4	4.3	5.4	--
U-statistic	.51	.49	.24	.26	.16	
opposite direction	15	12	10	8	8	
error runs	4	12	4	5	2	
E[c]						
RMSE	5.3	9.4	3.0	6.0	3.3	--
U-statistic	.70	.61	.46	.48	.52	
opposite direction	9	7	2	2	8	
error runs	4	8	5	5	4	
total observations in floating period	22	15	11	14	22	

[a] GNP = gross national product (gross domestic product for the U.K., ROWMG = rest of world imports), P = absorption deflator (ROWPXG for rest of world export price), CU = capacity utilization rate, RS = short-term interest rate (Eurodollar rate for R.O.W.), PMG = import price, E = exchange rate index in U.S. dollars per unit of local currency (weighted average of the four bilateral rates for the U.S.).

[b] For all variables except E, the total number of observations is 45.

[c] E is only tracked over the floating rate period for each country; note the relevant number of observations presented in the last row.

Figure 2a. United States (solid line = actual; dashed line = simulated)

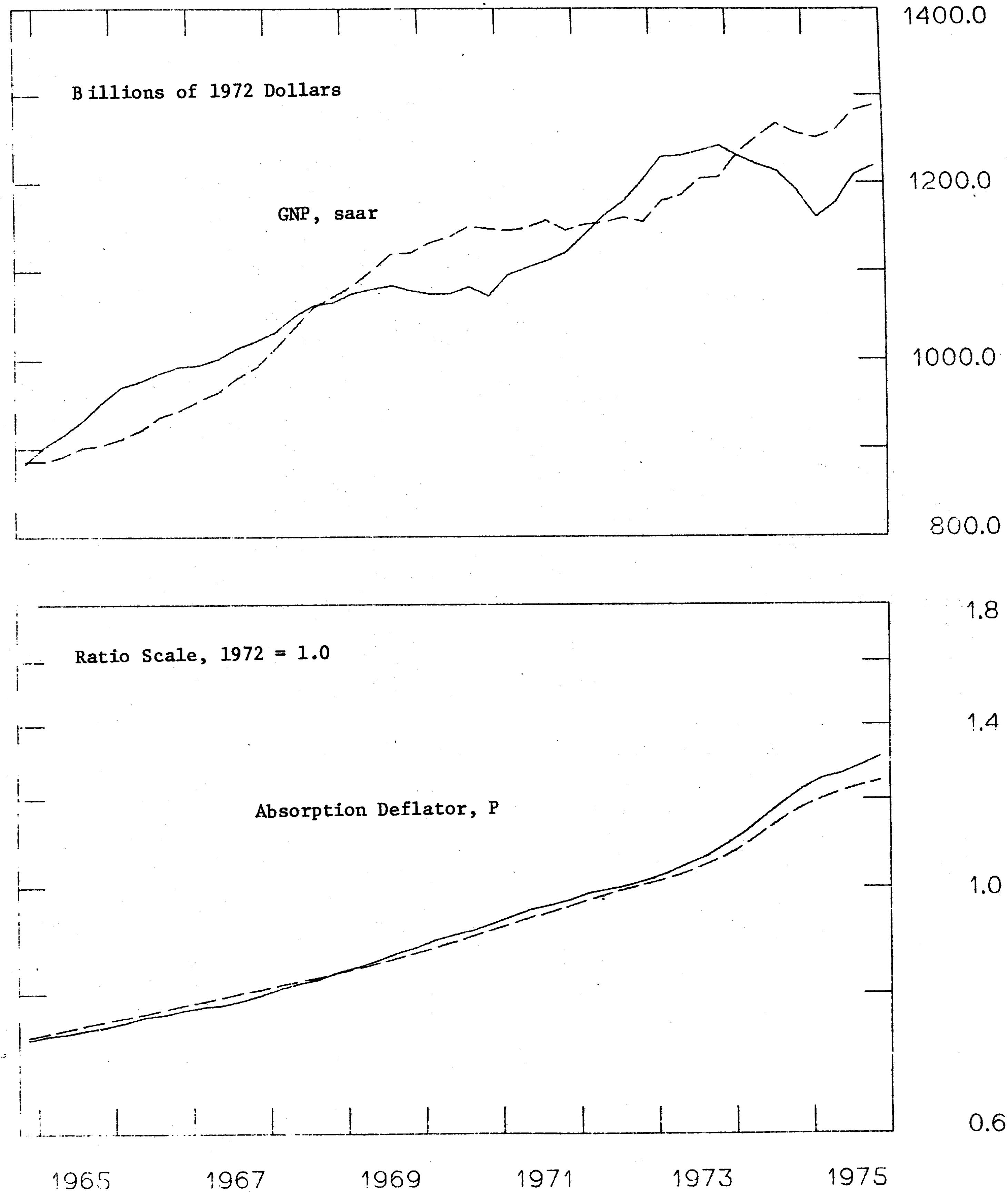


Figure 2b. Canada (solid line = actual; dashed line = simulated)

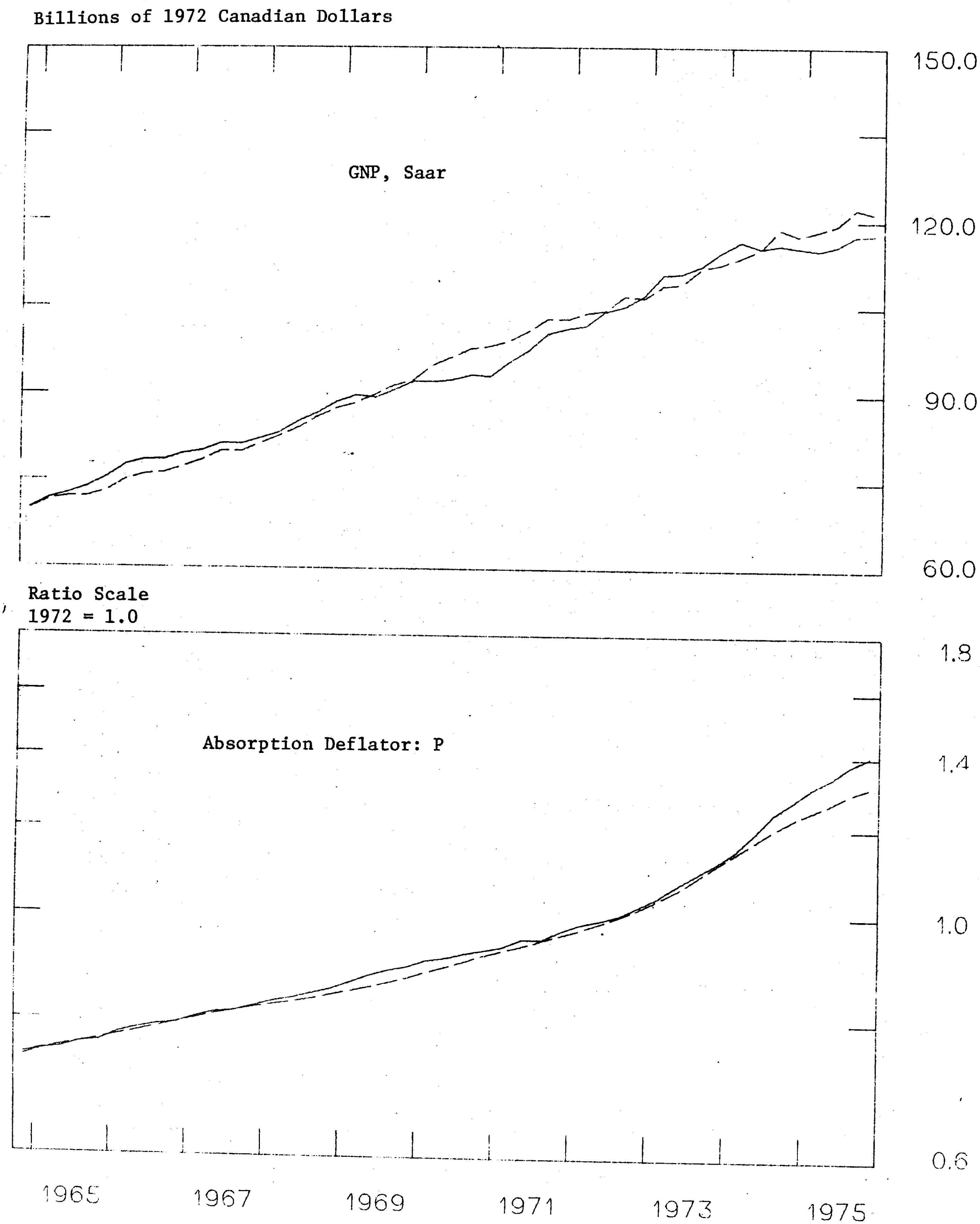


Figure 2c. Germany
(solid line = actual; dashed line = simulated)

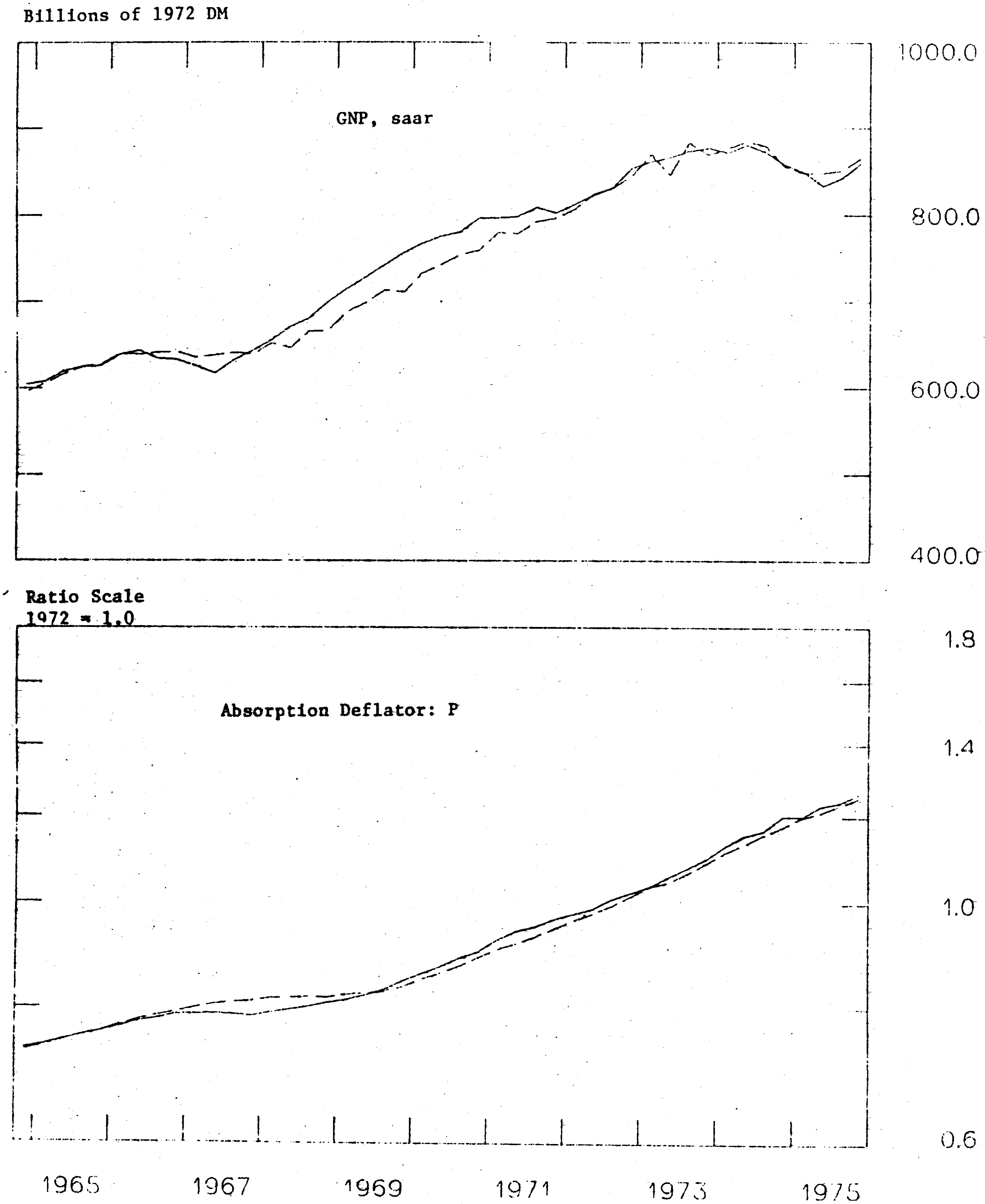


Figure 2d. Japan (solid line = actual; dashed line = simulated)

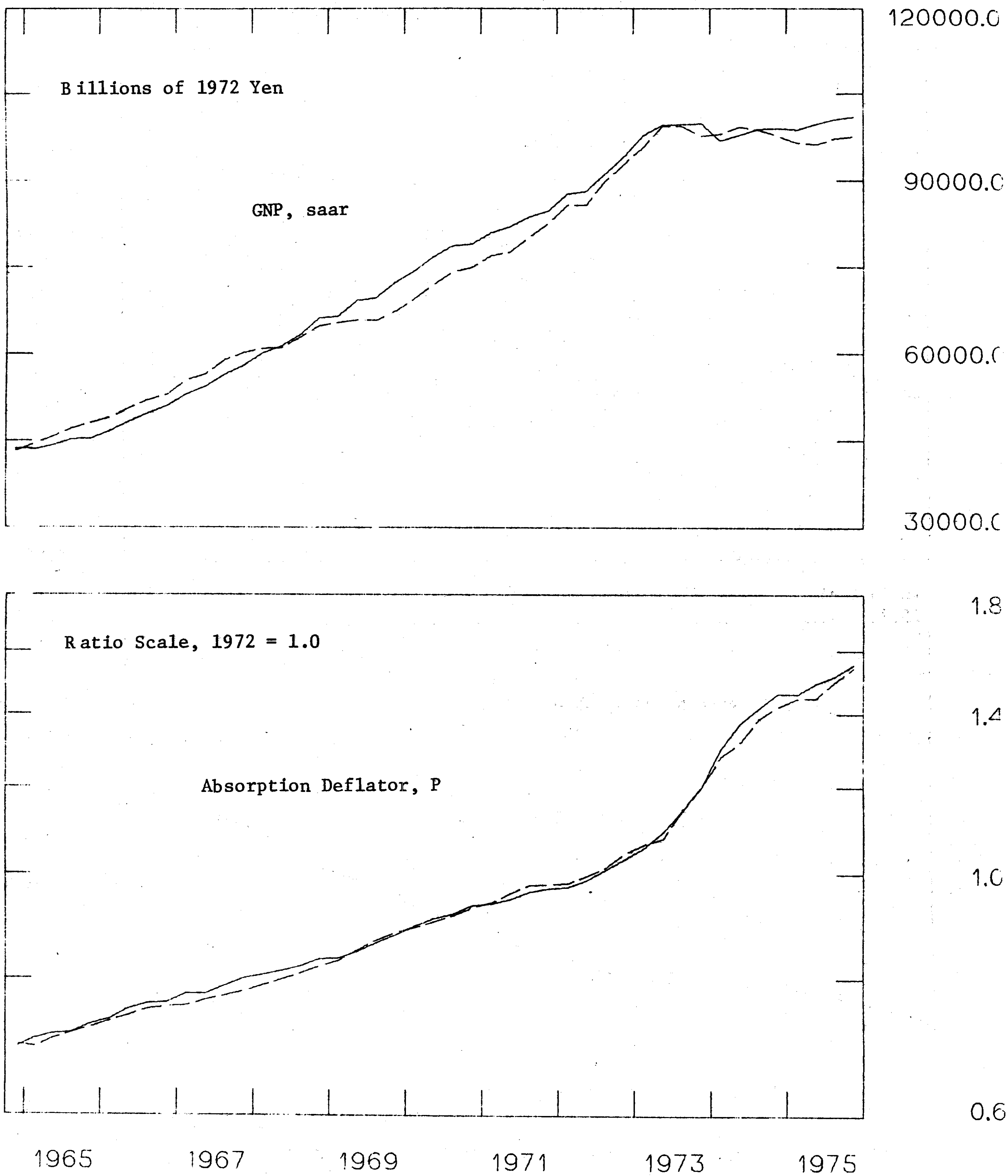
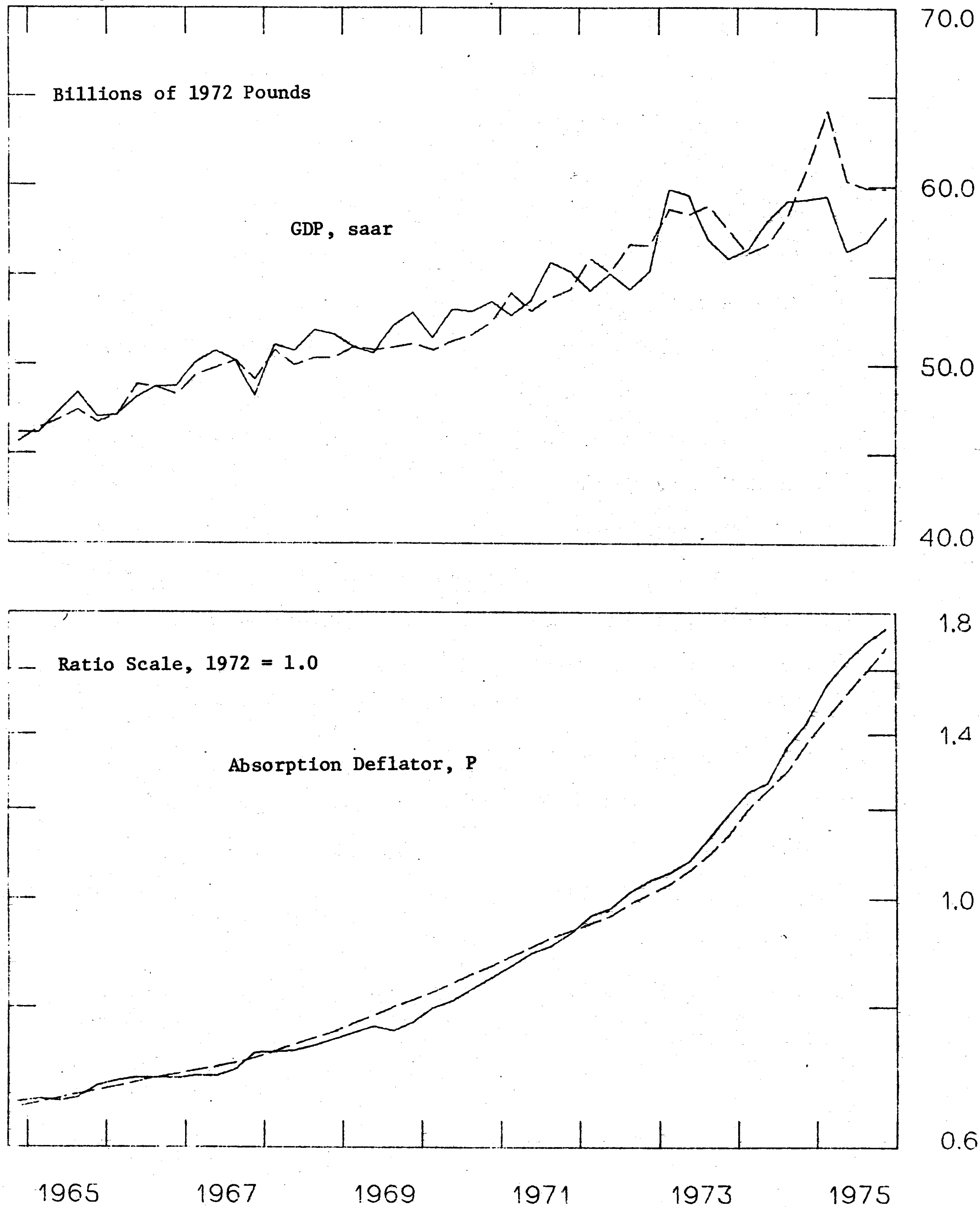


Figure 2e. United Kingdom (solid line = actual;dashed line = simulated)



polation. When all predictions are perfect, U = 0; under the no-change hypothesis, U = 1. Lower U values indicate better tracking.

To obtain a rough measure of turning-point errors, the directions of change in the actual and predicted series were compared. The third row for each variable in Table 2 presents the number of times (out of a possible 45) the predicted and actual series are moving in the opposite directions.[1/] This is a rather inexact measure on several counts. It does not take account of the magnitude of the divergence; small oscillations would penalize the prediction more than a single large spike. Nor does it take account of the necessity of some changes in direction if prediction errors are to be cancelled in subsequent periods.

The final measure is a count of the error runs above and below the actual series. This is a simple measure of the randomness of prediction errors. If the runs were counted for deviations above and below the _median_ value of the error, the null hypothesis of random errors could be tested statistically. For 45 observations, the null hypothesis of random errors is rejected at the 5% confidence level if the number of runs is less than 18.[2/] In the present case, since there is no certainty that the median error is zero, statistical testing is not appropriate. However, most of the series in Table 2 have many fewer than 18 runs and are there-

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[1/] Theil _op. cit._, pp. 21-26, refers to these differences in direction as turning point errors. Actually, this measure overstates the number of turning point errors whenever the actual and predicted series continue to move in opposite directions after the turning point has occured. Thorough analysis of turning point performance would require sophisticated criteria on the timing, magnitude and duration of the divergences of the two series.

[2/] For additional information on the runs test see Freund and Williams, _Elementary Business Statistics_, Englewood Cliffs: Prentice-Hall 1964, pp. 350-351 or P. Hoel, _Introduction to Mathematical Statistics_, New York: Wiley, 1962, pp. 335-341.

fore likely to have significantly autocorrelated prediction errors. These swings in prediction error are apparent in the plots appearing in this part of the report.

There is no simple answer to the problem of autocorrelated prediction errors. Clearly it is not a desirable property. But it is not uncommon for an econometric model covering a long sample period to have autocorrelated errors. In a heuristic sense, it is an indication that aside from the autocorrelated errors that may or may not have been accounted for in estimation, the system of equations (as opposed to each single equation) has an error mechanism of its own that is autocorrelated. In forecasting practice, the run of errors before the forecast period is often taken into account to devise a correction factor.

1. GNP

As might be expected, GNP shows less relative variation of RMSE among countries than the other variables; all fall in the range of 2.4 to 4.1%. The relevant activity variable for ROW, goods imports, has a RMSE of 3.3%. Although Japan does not have the lowest RMSE, it does have the lowest inequality coefficient (U-statistic), .29. By the same token, the UK has one of the lowest RMSE's (2.6%) but one of the highest U-values (.53).[1/] The U.S., Canada and Japan have the fewest number of opposite changes in direction between actual and predicted (8) series.

The usefulness of opposite direction changes in the case of a heavily trended variable such as GNP is questionable. In fact, the undesirability of a low number of opposite changes in the case of a trended variable becomes clear when considering the low number of error runs for the U.S., Canada and Japan. The 4 or 5 error runs in these

[1/] GDP in the UK is substantially less autocorrelated than is GNP in the other four countries. Hence, the no-change naive hypothesis is more nearly valid for the UK than it is for countries with strongly trended GNP's.

countries indicate long runs of over- and under-prediction.

Plots of the actual and simulated paths of GNP help impart a feel for the model's tracking performance beyond the statistical measures presented in Table 2. Differences in scale in the plots do not permit a good discrimination among RMSE's, although the relatively greater dispersion of the U.S. and U.K. models is evident. Note in the German case that the high number of opposite changes in direction does not hurt its cyclical tracking in the main, whereas the low number of opposite changes in direction for the U.S. does not adequately reveal its difficulty in capturing the cyclical movements.

The relatively higher coefficients of inequality for the U.S., U.K. and Germany do reflect the relatively greater dispersion of their predictions.[1/] The low numbers of runs for the U.S., Japan and Canada show up clearly in the long swings of over- and under-prediction.

2. Domestic price.

The RMSE of the domestic absorption deflator varies from a low of 1.5% (Germany and Japan) to a high of 2.9% for the U.K. But by the coefficient of inequality, Japan shows the least improvement over a no-change extrapolation (.43). The U.S. shows the best performance; that is, the RMSE for the U.S. domestic price level predicted by the MCM is 13% of what it would have been with a no-change extrapolation of past values.[2/]

Since price is so strongly trended, it is not surprising that the number of opposite changes is low and approximately equal for all countries. An exception is the ROW where the ROW export price index has

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[1/] The inequality coefficients also reflect the greater appropriateness of the no-change null hypothesis for these countries relative to Canada and Japan.

[2/] Inequality coefficients for no-change on <u>inflation rates</u> are: US: .62; Canada: .74; Japan: .65; Germany: .83; UK: .81.

10 opposite changes in direction between actual and predicted values.[1/] The number of error runs is also low for all price variables; the price predictions all have long swings of over- and under-shooting. The relatively low RMSE's, however, indicate that the magnitudes of the errors are in the main, not large.

Plots of the actual and simulated values of P corroborate the measures presented in Table 2. The relatively larger RMSE of the U.K. is evident in the wider dispersion of the actual and simulated paths. The good performance of the Japanese model in tracking the steep upswing in prices in 1974 is not evident in the statistics of Table 2 but is clear in Figure 2d. The low dispersion of the U.S. price path and its autocorrelation are clear in Figure 2a.

The predicted Canadian price path drifts away from the actual path in 1975 worse than in the other models. Canada's import price was most affected by the MCM linkage; its RMSE (shown in Table 1) increased from 1.8 to 7.7%.[2/] The underprediction of PMG for Canada in 1974-75 can be seen clearly in Fig. 3b below. The underprediction of PMG for Canada, in turn, is likely a major cause of the underprediction of the Canadian domestic price level at the end of the sample period. The same would hold true in lesser degree for the U.K.

[1/] Because of the strong trend in price, a more appropriate variable to inspect for turning points is the inflation rate. When changes in direction of ΔP were compared, Canada had the fewest number of cases (17) where the actual and predicted changes were opposite. Japan had the most cases of divergence (23). The counts for the other countries were: U.S. (18), ROWPXG (18), U.K. (21) and Germany (22). See fn. 2, p. 40.

[2/] It is likely that this is a result of the large error in the U.S. export price equation. The U.S. export price has a weight of .85 in the Canadian import price equation for 1975. Correction of the U.S. PXG equation should greatly improve Canada's PMG performance.

3. Capacity utilization

Because of the cyclical nature of capacity utilization, its RMSE is appreciably higher than that of the trended variables GNP and P. The UK has the lowest RMSE (1.8%) and the lowest inequality coefficient (.32). In the worst case, the RMSE of CU for the U.S. predicted by the MCM was 78% of what it would have been if no change was hypothesized.[1/] The number of opposite changes in direction is higher for CU than for any other variable in Table 2. Yet, the number of error runs is still low enough that significant autocorrelation of forecast errors is likely.

4. Short-term interest rate

As with many macroeconomic models, the short-term interest rate (RS) was the most troublesome of endogenous variables in the MCM. Although the RMSE's are appreciably higher than those of the other variables in Table 2, the coefficients of inequality are not. Among countries, the UK has the lowest RMSE (6.9%). Canada has the largest inequality coefficient of all. But even here, the RMSE of 14.3% was only 33% of what it would have been under a no-change extrapolation.

In all cases but the U.S. and the Eurodollar rate, the number of error runs was high enough that autocorrelation of prediction errors was less serious with interest rates than the other endogenous variables inspected. Considering the extent and complexity of the financial linkages between the foreign and domestic sectors built into the MCM, the tracking performance of RS is considered better than adequate for the present state of development.

[1/] An alternative CU equation for the U.S. is presently being tested.

5. Import prices

As indicated above in Part II, the import price index incorporates one of the most important international linkages. There is great scope for volatility from both foreign prices and exchange rates. RMSE's range from 4.3 to 7.7%. Here, Canada is somewhat of an outlier among countries. Its RMSE is 1/3 higher than the next worse case (Japan and U.S.); Canada's coefficient of inequality is also very high (.51 vs .16 for the U.S.). The likely cause of these poor tracking indicators is the inability of the Canadian import price index to follow the actual increase in prices in 1974 and 1975. This can be seen in the plots of PMG presented below in section D.[1/] In all countries except possibly Germany, the low number of error runs indicates the presence of error autocorrelation.

6. Exchange rates

The RMSE of exchange rates ranges from a low of 3% (Japan) to a high of 9.4% (Germany). It is useful to recall that the countries with lowest RMSE (Japan and Canada) use reaction functions for central bank intervention during the floating period. Again, there is divergence between the RMSE and coefficient of inequality performance. Although Canada has the second lowest RMSE, this is fully 70% of what it would have been under a no-change hypothesis. In one sense, Canada is bound to look particularly bad in terms of the U-statistic. Compared with other exchange rates, the actual Canadian rate varied very little over the floating rate period. Thus the denominator of the U-statistic is relatively small, and the no-change hypothesis is correspondingly harder to beat.

[1/] See related discussion above in section B.2, Domestic prices.

Japan and UK had only two cases of the actual and predicted values moving in opposite directions. This could be explained by the trended depreciation experienced during the float by these countries. This is not so for Germany, however, where the DM appreciated; in 7 of 15 cases the actual and predicted rates moved in opposite directions. Even with the lower number of observations during the floating period, no case had a sufficient number of runs to insure a random pattern of prediction errors.

C. Fixed vs. Floating Rate Periods

As explained in section I.D, exchange rate variables in the model are exogenous during periods of pegged rates and endogenous during periods of managed floating. Since the starting date for the floating rate period differs from one country to another,[1/] such a period cannot be defined unambiguously for the multi-country model taken as a whole. It is of interest, however, to compare simulation results for the period ranging from the second quarter of 1973 to the fourth quarter of 1975 -- during which all five countries in the model were under a regime of managed floating -- with results for the earlier part of our sample period. The comparison is given in Table 3 in terms of the percentage root mean square errors for a set of key variables.[2/]

[1/] The periods of managed floating are defined as follows: Canada 1970:3 to 1975:4; Germany 1971:1 to 1971:4 and 1973:1 to 1975:4; Japan 1973:2 to 1975:4; and U.K. 1972:3 to 1975:4. In the Canadian case there is a previous period of floating rates which extends from 61:1 to 62:1. This affects the estimation of some Canadian equations but has no direct influence on simulation results which start only in 1964:4.

[2/] The results are not calculated from different periods of the same dynamic simulation, but rather from two dynamic simulations: one beginning in 1972:2 and one beginning in 1964:4.

Table 3

Comparative Simulation Results:
Root Mean Square Errors for the Period 1964:4 to 1973:1 (Line 1)
and for the Period 1973:2 to 1975:4 (Line 2)

Variable a/ \ Country		Canada	Germany	Japan	U.K.	U.S.	R.O.W.
GNP	(1)	2.4	2.6	4.1	1.9	3.8	--
	(2)	1.5	2.4	1.5	2.4	4.0	
P	(1)	1.2	1.6	1.2	2.1	1.1	--
	(2)	1.1	2.2	1.3	2.0	0.4	
CU	(1)	4.5	3.6	5.6	1.4	8.8	--
	(2)	1.5	3.3	5.0	1.3	7.1	
UN	(1)	19.4	b/	24.0	16.8	34.0	--
	(2)	5.0		14.4	11.7	17.8	
RS	(1)	15.5	12.7	8.1	7.1	22.2	21.4
	(2)	6.3	9.3	4.0	5.0	12.4	14.9
MG	(1)	10.3	6.3	6.3	3.8	6.3	3.2
	(2)	3.4	2.6	6.7	2.8	7.4	3.2
PMG	(1)	4.2	2.4	3.4	2.4	2.1	2.5
	(2)	4.5	3.5	5.6	3.8	1.8	2.5
XG	(1)	6.2	3.1	5.0	4.4	10.2	2.9
	(2)	6.8	5.7	5.3	3.3	6.8	3.0
PXG	(1)	3.2	1.2	2.3	1.3	6.8	1.8
	(2)	3.5	2.0	3.0	3.2	4.6	1.7
E	(1)	3.1	c/	c/	6.0	1.6	--
	(2)	4.5	6.6	4.0	6.8	2.6	
NFA	(1)	36.1	11.6	15.0	d/	36.3	--
	(2)	9.5	c/	6.1	c/	c/	

a/ GNP = gross national product (gross domestic product for the U.K.), P = absorption deflator, CU = capacity utilization rate, UN = unemployment rate (unfilled vacancy ratio for Japan), RS = short-term interest rate (Eurodollar rate for R.O.W.), MG = imports of goods, PMG = import price, XG = exports of goods, PXG = export price, E = exchange rate index in U.S. dollars per unit of local currency (weighted average of the four bilateral rates for the U.S.), NFA = net foreign assets of the monetary authorities.

b/ Variable not used in the German model.

c/ Exogenous.

d/ Statistic not relevant because the variable takes on negative as well as positive values.

Compared to the earlier period, the period 1973:2 to 1975:4 is characterized by a considerably greater degree of variability, not only for exchange rates, but also for activity and price variables. Yet most entries in Table 3 suggest that the multi-country model performs better during the floating rate period.[1/] In particular, it is interesting to note that this is the case for the two cyclical variables, unemployment and capacity utilization, for all five countries in the model. It is also worth noting that the same result holds for the short-term interest rate.

There are three variables in Table 3 for which simulation errors are, in most cases, larger during the period of managed floating: export and import prices, and the volume of exports. To some extent, the endogeneity of exchange rates during the more recent period could account for this result in the case of PXG and PMG, which are strongly influenced by exchange rate movements, and therefore also by errors in the simulated exchange rates. In the case of exports, the relatively weaker performance during the floating rate period reflects to some extent the fact that export volumes are obtained by deflating nominal exports by the export price index, so that errors in the latter variable generate errors in real exports.

[1/] To some extent, this may reflect the fact that there is more scope for error accumulation over the longer period 1964:4 - 1973:1.

D. The Oil Price Hike and Recession of 1974 - 1975

The sample period over which the MCM has been estimated includes both the oil price hike in 1973 - 1974 and the recession of 1974 - 1975. It is of interest to trace out how these shocks are reflected in the MCM. The price of oil is a major component of one of the main exogenous variables in the model, viz., an index (expressed in U.S. dollars) of the prices of primary products other than cereals, PPO.[1/] This price index of primary products is the main determinant of the index of export prices of the rest-of-the-world sector in our model, ROWPXG. The tremendous rise in the price of primary products is shown in Figure 3f, which gives actual and simulated values of ROWPXG.[2/]

The linkage between the prices of oil and other primary products, and the domestic prices of our five countries, occurs through import prices. The merchandise import unit value index of each country, PMG, is a function of ROWPXG, the export price indexes of the other four countries, and the exchange rates vis-à-vis the four other countries. The domestic price index,P, is in turn determined by the price of imports, an index of wage rates, capacity utilization, and in certain cases by labor productivity.

The actual and fitted values of PMG between 1972:1 - 1975:4 for each of the five countries in the model are plotted in Figures 3a-3e.[3/] The rise in PPO, due primarily to the four-fold increase

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1/ The price of cereals is also exogenous to the model and is a determinant of both U.S. and Canadian export prices.

2/ These data are drawn from the dynamic simulation beginning in 1964:4.

3/ See Figures 2a - 2e above for actual and simulated values of P.

in the price of oil, works through RQWPXG to the import prices, which in turn affect the domestic prices of each country. As mentioned already in Parts I and II, there is a direct effect on the deflator for domestic expenditures because imported final goods and intermediate inputs form part of domestic absorption, and there is an indirect effect through the response in wages to the higher domestic price level. As can be seen from the figures, the simulated values follow the actual upward movement of the values, but for all countries there is an underprediction of the rise in imports prices in 1974 - 1975.

Figures 3a - 3f also indicate that errors in import prices are affected by errors in exchange rates, for which the actual and simulated values are depicted in Figure 1 above. Thus in Germany, for example, the overprediction (appreciation) of the DM against the U.S. dollar between 1974:3 and 1975:1 is reflected in an underprediction (a decline) in the German import unit value index (Figure 3c) over the corresponding period.

The period 1973 - 1975 was also characterized by the worst post-war recession experienced by the major industrial countries. The large increase in domestic price levels in the five countries raised interest rates, which led to a precipitous decline in real investment. As a result all five countries experienced a decline in real GNP mirrored in a decline in the rate of capacity utilization, CU. Figure 4a - 4e plot actual and fitted values for CU for the five countries over the period 1972:1 - 1975:4. A comparison of actual and fitted values for this series indicates that with the exception of the United States, our model captures the main course of their cyclical development.

Figure 3. Import Prices (PMG), 1972-1975

(Solid line = actual value; dashed line = simulated value)
(ratio scale, 1972 = 1.0)

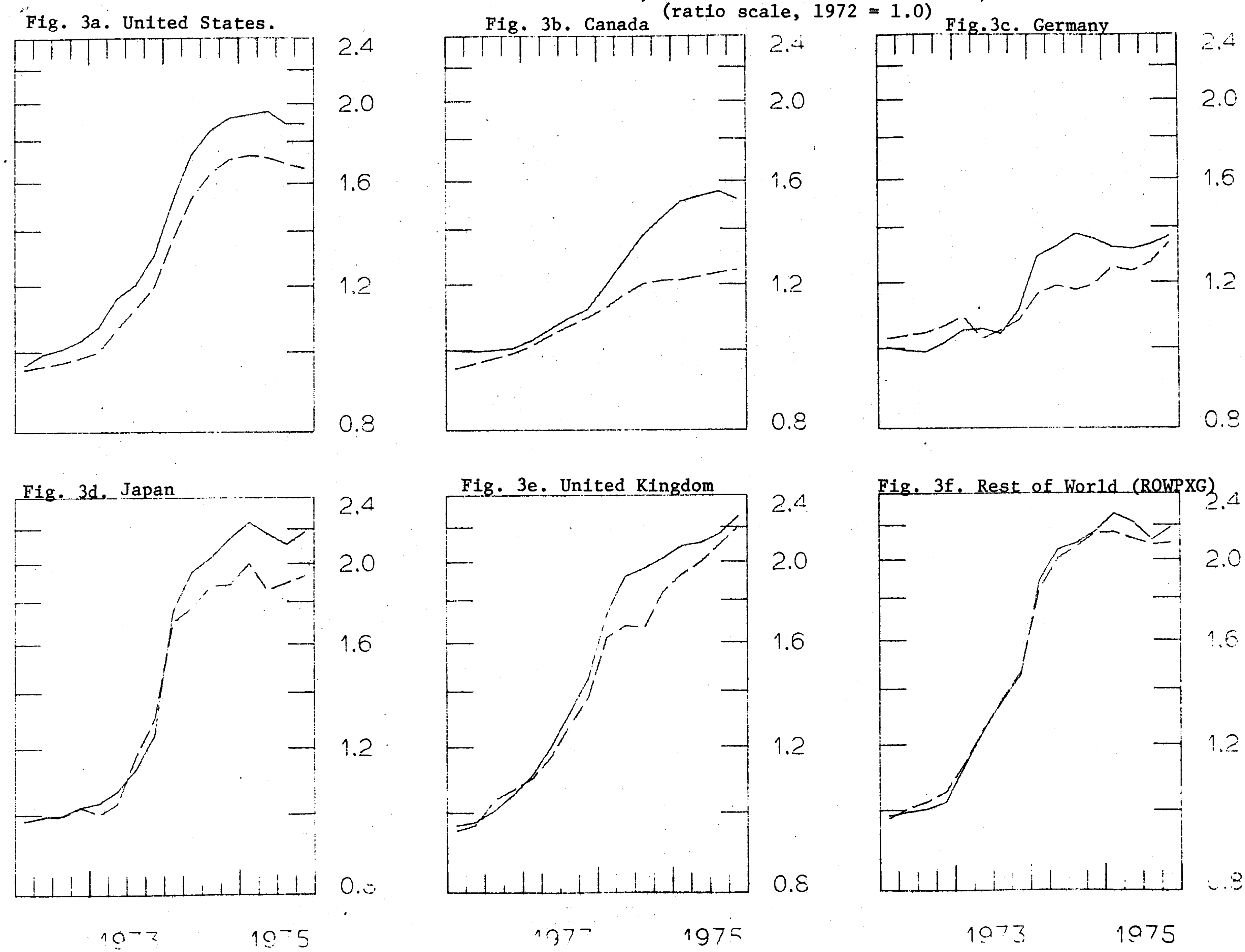
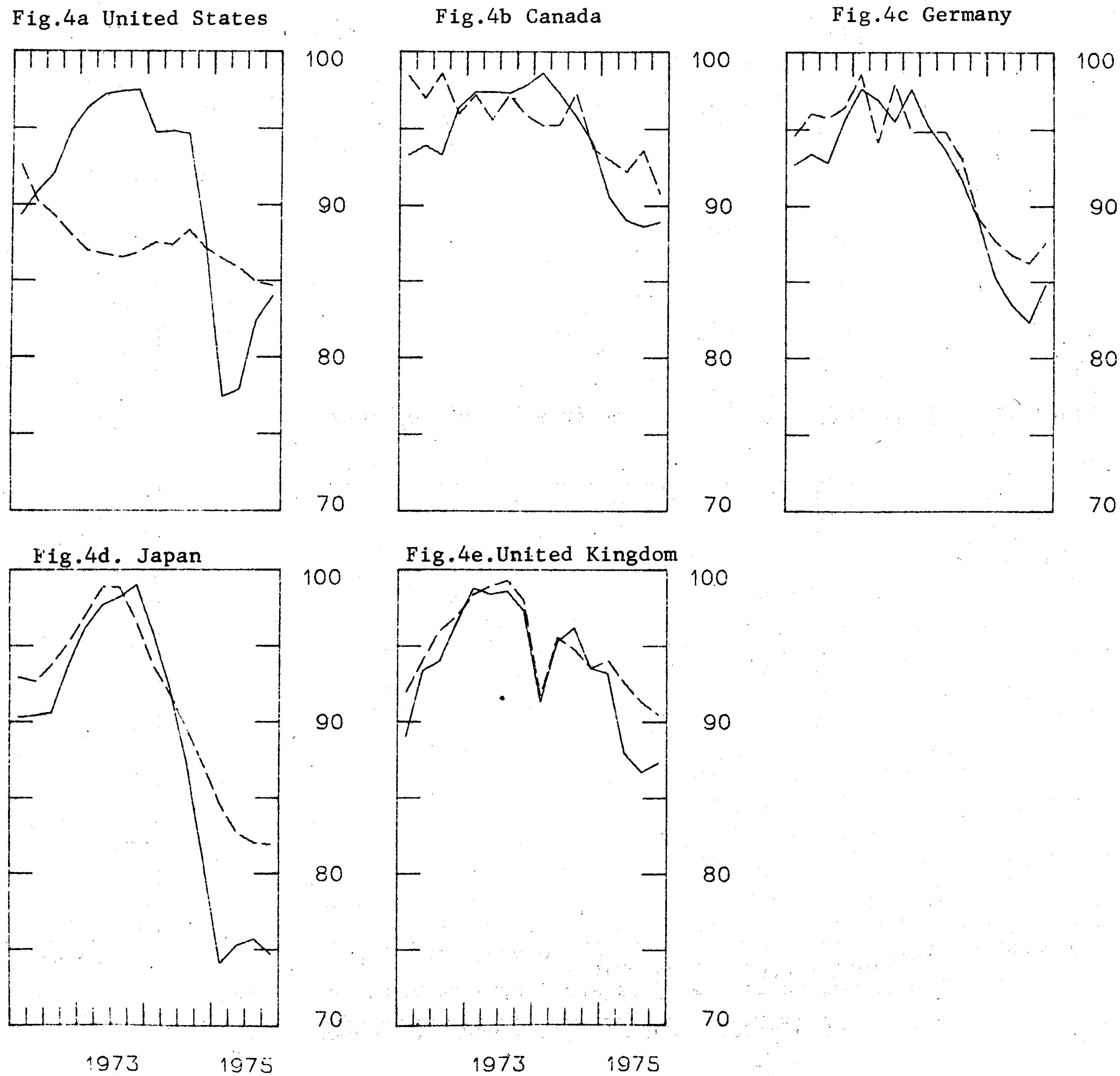


Figure 4. Capacity Utilization (CU), 1972-1975
(Solid line = actual value; dashed line = simulated value)



IV. Future Work

The goal of the first stage of this project was to determine the feasibility of developing a model of the U.S. economy that endogenizes exchange rates and other key foreign variables and that models explicitly the major linkages with important foreign economies. With the results presented above, we feel we have achieved that goal and answered the basic question positively.

There remains, however, much to be done before the multi-country model can become an operational tool for policy analysis or forecasting.

First, the model's performance within the sample period must be analyzed further. Multipliers for the system must be calculated, e.g. the effects on important endogenous variables of changes in key exogenous variables or parameters (such as the monetary base, government expenditures, the price of oil, and, depending on the exchange regime, the exchange rate or net foreign assets) must be computed. Such simulations will be carried out both for the country models taken in isolation and for the fully linked system. Further, the models will be put through a set of simulation experiments more complicated than simple multiplier calculations. These would include simulating the effects of (1) fundamental structural changes in the system -- such as shifting from fixed to floating exchange rates at various dates, or vice versa -- and (2) multiple policy changes (in this non-linear model, the effects of multiple changes will not usually be the sum of the effects of changes taken individually). Finally, it may also be useful to undertake a more thorough analysis of the multi-country model's ability to predict turning points and to out-perform naive models.

As the result of the above, parts of the model will undoubtedly be restructured or re-estimated. Major data revisions in Germany and Japan will necessitate the complete re-estimation of these two country models. In addition, an effort will be made to assess the simultaneous equations bias introduced by our use of ordinary least squares.

With regard to making improvements in the structure of the country models, the preliminary simulations have already suggested a number of areas of concentration. A major line of research will be to endogenize the trade-share matrix which translates imports into exports; in the preliminary simulations this matrix was exogenously determined. Making government intervention in the exchange market endogenous led to significant reductions in the root mean square errors for the exchange rate in Canada and Japan; we, therefore, wish to explore the use of intervention functions for the remaining countries. The possibilities for endogenizing certain other government policy instruments will also be investigated. A third area of work will be an effort to improve a number of the monetary sectors--such a need being evidenced in part by the large errors shown in Table 1 for certain short term interest rates (RS).

The final major task will be to explore the out-of-sample forecasting ability of the multi-country model. An initial step will be to update the model's data banks through mid-1977 and to run ex-post forecasts for up to six quarters beyond the end of the sample period (ex-post in the sense that the actual values of the exogenous variables will be known). Subsequently, we will move into ex-ante forecasting to assess the model's ability to contribute to the forecasting effort within the Board's Division of International Finance.

Capital Consumption Allowance

9. $CCAV = CCAV\ [K_{-1}, P_{-1}, (GNPV - TV - CV)]$

Private Net Worth Proxy

10. $\Delta NWV = (XGSV - MGSV + XTRANV - MTRANV) + [(G \cdot P) + TRANV - TV]$
$+ [(IF + II) \cdot P - CCAV]$

10a. $NWV = NWV_{-1} + \Delta NWV/4$

B. Government Sector

Government Transfers

11. $TRANV = TRANV\ (GNPV, LU, RL \cdot GD_{-1})$

Government Debt

11a. $GD = GD_{-1} + (G \cdot P + TRANV - TV)/4$

Tax Function

12. $TV = a_o + a_1\ (GNPV - CCAV)$

C. Current Account

Imports of Goods[1/]

13. $MG = MG\ [(GNP - II), II, CU, PMG^*, P^*]$

1/ For Japan and the U.S., the dependent variable is the volume of nonfuel imports (the volume of fuel imports being treated as exogenous); and the import price variable on the right hand side is the unit value for non-fuel imports. The U.K. equation has the ratio of imports to domestic sales as dependent variable, and includes the lagged stocks of capital and inventories as explanatory variables.

14. $MGV = MG \cdot PMG$

Exports of Goods

15. $XGV = \sum_{\substack{j=1 \\ i \neq j}}^{5} \delta_{ij} \, MGV_j \cdot E_j/E_i + d_o + d_1 t$

where the δ_{ij}s are elements of a 6 X 6 trade share matrix. E_i and E_j are the exchange rates for the exporting country and the importing countries, respectively, each defined in dollars per unit of national currency.

Investment Income

(i) Payments

16. $MSYV = MSYV \; [\; \emptyset \; (R \cdot FLT_{-1})^* + (1 - \emptyset)(FR \cdot X \cdot FLT_{-1})^*, (RL \quad LTDL_{-1})^*]$

(ii) Credits

17. $XSYV = XSYV \; [\psi \; (R \cdot FCT_{-1})^* + (1 - \psi)(FR \cdot X \quad FCT_{-1})^*, (FRL \cdot X \cdot LTDC_{-1})^*]$

The investment income equations are based on the assumption that liabilities (claims) denominated in local currency are a constant franction $\emptyset$ (ψ) of total external financial liabilities (claims). Direct investment liabilities are assumed to be denominated in local currency, direct investment claims in foreign currency. The term $X = [1 + \Delta FE/FE_{-1}]$ is used to convert into local currency the interest payments on foreign-currency denominated claims or liabilities. (FE is a weighted average of foreign exchange rates, and is here expressed in units of foreign currency per unit of local currency).

Japan and the U.S. have separate equations for income on direct and nondirect investment. Further, the U.S. distinguishes between private and government investment income.

Imports of Other Services

18. $MSO = MSO\ (GNP,\ PMS^*,\ P^*,\ MG^*)$

19. $MSOV \equiv MSO \cdot PMS$

Exports of Other Services

20. $XSO = XSO\ (FGNP,\ PXS^*,\ FP^*,\ XG^*)$

21 $XSOV \equiv XSO \cdot PXS$

External Transfers

22. $MTRANV = d_o + d_1\ YD$

23. $XTRANV \equiv e_o + e_1\ FYD$

Link from Balance of Payments to National Income Accounts[1/]

24. $MGSNIV = f_o + f_1 MGSV$ 24a. $MGSNI = MGS$

25. $XGSNIV = g_o + g_1\ XGSV$ 25a. $XGSNI = XGS$

26. $MGSV = MGV + MSYV + MSOV$

27. $XGSV = XGV + XSYV + XSOV$

1/ In the U.S. model, MGSNIV and MGSNI exclude interest payments on U.S. government liabilities to foreigners.

D. Price Determination and Capacity Utilization

Domestic Absorption Deflator

28. P = P [W*, PMG*, CU, t, { GNP/(LE · H) }*]

W is the hourly wage rate in manufacturing (labor compensation in the non-farm business sector for the U.S.). Japan and the U.K. have WPI equations which are similar in structure to (28).

Import Unit Value

29. PMG = PMG [FPXG*, FE*, t]

Note that the average foreign export price FPXG includes the R.O.W. export price. Both FPXG and FE are geometric means, so that the logarithms of these two variables can be multiplied to obtain a foreign export price converted into local currency.

In the U.S. and Japanese models, the dependent variable is the unit value of nonfuel imports.

Export Unit Value

30. PXG = PXG [W*, PMG, CU, FCU, t, FPXG, FE, {GNP/(LE · H}]

The U.S. adds the dollar price index for cereals (PPC) to the list of explanatory variables.

Import and Export Prices for Services

31. PMS = PMS (FP, FE, FRTL)

32. PXS = PXS (P, FRTL)

Capacity Utilization

The equation for capacity utilization is derived by combining the Cobb-Douglas production function $GNPP = AK^{\alpha}(EP)^{(1-\alpha)}e^{gt}$ with the following definitions of potential GNP and potential employment:

$$GNPP = GNP/CU \quad \text{and} \quad EP = (1-u)\ LF/CU^{\gamma}$$

(Frictional unemployment is assumed to be a constant fraction u of the potential labor force). Taking logarithms and re-arranging yields the equation used in estimation:

33. $\log CU = h_o + h_1 \log (GNP/LF) - h_2 \log (K/LF) - h_3\, t$

E. The Labor Market

Wage Rate in Manufacturing

34. $\Delta\%W = W\ [(1/UN)^*,\ (\Delta\%P)^*]$

where Δ% indicates percentage change. The U.S. equation also includes a minimum wage variable.

Employment

35a. $\log LE = LE\ [\log (GNP \cdot P/W)^*,\ \log CU]$

Labor Force[1/]

35b. $\log LF = LF[\log(POP),\ \log (W/P)]$

1/ Exogenous in the German and Japanese models.

Unemployment Rate

35c. $UN = (1 - LE/LF) \cdot 100$

In the Canadian and U.K. models equations 35a, 35b and 35c are condensed into a single unemployment rate equation.

F. Domestic Asset Demand and Interest Rate Determination

The "prototype" financial model described in this section applies with relatively minor variations to the Canadian, German and U.S. models. The structure of the Japanese and U.K. sectors, -- which differs from that of the "prototype" model -- is summarized in the main text (Part I).

The Central Banks' Balance Sheet

36. $NFA + NGP + RB + OTH \equiv RT + (CUR + CURB)$

Total reserves (RT) are the sum of required reserves (RR) and excess reserves (RX), and free reserves are defined as $RF \equiv RX - RB$ (where RB denotes borrowed reserves). Using these definitions, the balance sheet identity can be rearranged as follows:

36a. $BU \equiv NFA + NGP + OTH \equiv RR + RF + (CUR + CURB)$

Stock of Net Foreign Assets

37. $NFA = NFA_{-1} + \Delta NFA/4$

Required Reserves

38. $RR^{c} \equiv a\ DD + b\ TD$

"Computed" reserve requirements are obtained by adding up demand deposits and time deposits, each multiplied by the corresponding (exogenous) reserve ratio. Given the complexity of reserve requirement regulations, "computed" reserves may differ from actual reserves (for example because reserve requirements differ according to the size of banks). Actual required reserves are then derived from the following "bridge" equation:

39. $RR = C_{o} + C_{1}\ RR^{c} + e$

In simulation the error term e is added to the right hand side of the equation, so that any error in simulating RR must come from errors in simulating demand and time deposits.

Banks' Demand for Free Reserves

40. $RF/NDD = RF\ [RD, RS, \Delta(BU - CUR), \Delta RR]$

where $NDD = (1-a)\ DD$ is net demand deposits and $\Delta RR = \Delta a(DD_{-1}) + \Delta b\,(TD_{-1})$. The German equation also includes a stock-adjustment formulation where the speed of adjustment depends on the level of the rediscount quotas set by the Bundesbank. In the current version of the model, this equation is estimated by normalizing for the short-term rate RS.

Banks' Demand for Currency

41. CURB = g (DD + TD)

Asset Demand Equations

In each model, demand equations are estimated for at least 3 domestic assets: currency, demand deposits and time deposits.[1/] The general form of these equations is:

42. $A_i/NWV = f$ [GNPV/NWV, RS*, FRS*, (EE -E)/E]

where A_i is the demand for the $i^{\underline{th}}$ domestic asset by the nonbank public. (EE-E)/E denotes the expected change in the spot exchange rate. The determinants of this unobservable variable (see equation 51) are substituted into the asset demand equations. In the currency equation the scale variable is CV rather than GNPV. The foreign interest rate and expected exchange rate terms are omitted from the currency equation and -- except in the Japanese case -- play no role in the equation for demand deposits.

Term Structure Equation for the Long-term Rate

42. RL = RL (RS*)

[1/] The U.S. time deposit equation includes a time deposit rate RTD (the own rate). It is an average of the (exogenous) rate on consumer-type time and savings deposits and of the rate paid on large negotiable certificates of deposit. The CD rate is itself related to the commercial paper rate RS (through arbitrage) and to a variable designed to capture the impact of Regulation Q. The Japanese equations also include an exogenous rate on time deposits.

G. Capital Movements, the Balance of Payments and the Forward Rate

Portfolio Capital

The demand for foreign assets and liabilities follows the portfolio balance approach already used for domestic assets. The equations are specified in stock form, although they are often estimated in first difference form owing to the lack of benchmark data. The typical equations for foreign portfolio claims and liabilities are:[1/]

43a. FCP = FCP [NW, FNW, RS*, FRS*, RL*, FRL*, XGV*, (EE-E)/E]

43b. FLP = FLP [NW, FNW, RS*, FRS*, RL*, FRL*, MGV*, (EE-E)/E]

It may be noted that the U.S. equation also includes a variable (equal to zero before the third quarter of 1973 and to the price of imported fuel thereafter) which serves as a proxy for the change in OPEC financial wealth.

Direct Investment Capital

44. DLTDC = f (ΔFGNPV*)

45. DLTDL = g (ΔGNPV*)

[1/] The Japanese model has a separate equation for short-term bank-reported claims, short-term bank-reported liabilities and other short-term liabilities. The U.K. and Canada distinguish between long and short-term financial claims while these two items are aggregated in the U.S. and German models. Each country has a separate equation for "errors and omissions" which is similar in structure to equation (43).

H. Balance of Payments and Exchange Rates

46. $\Delta NFA = (XGV + XSYV + XSOV + XTRANV) - (MGV + MSYV + MSOV + MTRANV + (\Delta FLP + \Delta LTDL) - (\Delta FCP + \Delta LTDC) + NGKA + EO$

46a. $\Delta NFA^{T} = \Delta NFA + SDR + VAL$

As explained in Part II of this paper, the balance of payments equation is used to solve for the change in net foreign assets during the fixed rate period, and for the spot exchange rate during the floating rate period.[1/] This result is achieved by using the following set of equations.

47. $\Delta NFA = \Delta NFA_{fx} + \Delta NFA_{fl}$

where $\Delta NFA_{fl} \equiv \lambda DNFA$, $\Delta NFA_{fx} \equiv (1-\lambda)\ \Delta NFA$, and λ is a switch equal to one during the floating rate period and zero otherwise.

Similarly we have:

48. $E = E_{fx} + E_{fl} \qquad (E_{fl} \equiv \lambda E, \quad E_{fx} \equiv (1-\lambda)\ E)$

Finally, the equation

49. $(1-\lambda)\ E_{fl} + \lambda \Delta NFA_{fx} = 0$

insures that the two variables will appropriately switch from one regime to another.

[1/] In the U.K. and U.S. models, the left-hand side of (46) is $\Delta NFA - DLO$, where DLO denotes the change in liabilities to foreign official agencies (other than liabilities reported by the monetary authorities, which are netted out of ΔNFA).

Forward Premium

The forward premium equation combines covered arbitrage (the interest rate differential) with a speculative element (the expected change in the spot rate).

50. $(EF - E)/E = h\ [(RS - FRS)^*,\ (EE - E)/E, \lambda\ ]$

Expected Change in the Spot Exchange Rate

31a. $(EE-E)/E = F[(FPXG/PXG - E),\ (NFA^T/MGSV)_{-1},\ \lambda]$

51b. $(EE-E)/E = (E_{+1}-E)/E + \eta$

Equation 51b is used in the Canadian model, equation 51a in the other four models. (E_{+1} is the one-period ahead value of the spot rate, and η is a random error term).

I. Rest of the World Sector

Imports and Import Price

52. $ROWMG = \sum_{i=o}^{5} XG_i - \sum_{i=o}^{5} MG_i + ROWXG$

53. $ROWMGV = ROWPMG \cdot M[ROWIP, (ROWPXG/ROWPMG)^*]$

54. $ROWPMG = ROWMGV/ROWMG$

ROWIP is an average of industrial production indexes for 9 rest-of-the-world countries.[1/]

[1/] Belgium, France, Italy, Korea, Mexico, the Netherlands, Norway, Switzerland and Taiwan.

Exports and Export Price

55. $ROWXGV = \sum_{i=o}^{5} MGV_i \cdot E_i - \sum_{i=o}^{5} XGV_i \cdot E_i + ROWMGV$

56. $ROWXG = ROWXGV/ROWPXG$

57. $ROWPXG = PX\,[(FPXG \cdot FE)^*,\ PPQ^*]$

Euro-Dollar Rate

$RED = F(RS_{US},\ FRS,\ (FEF - FE)/FE,\ D]$

where FRS, FEF and FE are weighted averages of short-term interest rates, forward rates, and spot rates, respectively, for Canada, Germany, Japan and the U.K. D is a vector of control variables including dummies for U.S. regulations Q and M, foreign currency borrowing by the U.K. government and special U.S. securities sold to foreign branches of U.S. banks.

Euro-Bond Rate

$RMEB = G(RED^*)$

Definition of Variables

All balance of payments and national income accounts variables are at annual rate. Interest rates are in per cent per annum, and price variables are indexes based 1 in 1972. Exchange rates are expressed in U.S. dollars per local currency.

The symbol "x" indicates an exogenous variable.

x a = reserve requirement against demand deposits

x b = reserve requirement against time deposits

BU = unborrowed monetary base, i.e., the total base minus reserves borrowed by commercial banks.

C = Private consumption

CCAV = capital consumption allowances

CU = capacity utilization

CUR = currency in the hands of the non-bank public

CURB = currency in the hands of banks

DD = demand deposit component of M1.

DLO = change in liabilities to foreign official institutions (U.K. and U.S. only)

E = exchange rate

EE = exchange rate expected next quarter.

EF = forward exchange rate

EO = errors and omissions item

FE = weighted average exchange rate (trade weights)

FCP = private short- and long-term portfolio claims on foreigners

FR = weighted average of foreign interest rates (weights based on foreign claims or liabilities).

FCT = total financial claims on foreigners, including international reserves

FLP = private short- and long-term portfolio liabilities to foreigners

FLT = total financial liabilities to foreigners

FP = weighted average of foreign absorption deflators

FPXG = weighted average of foreign export prices

FNWV = domestic currency equivalent of foreign net worth

x FRTL = liner freight rate (Bremen index)

x G = government expenditures

GD = proxy for the stock of government debt

IF = fixed investment

II = inventory investment

K = stock of capital

LTDC = long-term direct claims on foreigners

LTDL = long-term direct liabilities to foreigners

MG = merchandise imports

MGS = imports of goods and services, balance of payments basis

MGSNI = imports of goods and services, national income accounts basis

MSOV = other service account payments

MSYV = investment income payments

MTRANV = BOP transfers (debits)

NFA = net foreign assets of the monetary authorities, excluding the cumulated value of SDR allocations and valuation adjustments

NFAT = NFA + VAL + SDR

x NGKA = net government capital account

x NGP = net government position of the central bank

NWV = private net worth (cumulated sum of private savings)

OTH = all other assets of the central bank, net

P = deflator for aggregate domestic expenditures

PMG = Price of merchandise imports

PMS = price of imported services

x PPC = dollar price index of cereals

x PPO = dollar price index of primary products other than cereals

PXG = price of merchandise exports

PXS = price of exported services

R = weighted average of RS and RL

RD = discount rate

RF = free reserves = excess reserves minus borrowed reserves

RL = long-term interest rate[1/]

RLN = average interest rate on Japanese bank loans

RMEB = Euro-bond rate

RS = domestic short-term interest rate[2/]

ROWIP = rest of the world (ROW) average industrial production index

ROWMG = ROW imports

ROWPMG = ROW import price index

ROWPXG = ROW export price index

[1/] Canada = Government bonds (10 years or over); U.K. = yield on war loans; Japan = yield on bank debentures; Germany = market yield on private bonds; U.S. issue rate on AAA utility bonds.

[2/] Canada: 90 day finance company paper rate; Germany: 3 months inter-bank rate; Japan: call money rate; U.K.: Treasury bill rate; U.S.: Commercial paper rate; Euro-dollar: (RED) 3 months deposits in London.

ROWXG = ROW exports

$\Delta\overline{RR}$ = autonomous change in reserve requirements due to alteration in statutory reserve ratios

x SDR = allocation of special drawing rights

SII = stock of inventories

x t = linear time trend

TD = time deposit component of M2

TRANV = total government transfers

TV = total government tax receipts

UC = user cost of capital

x VAL = change in net foreign assets caused by valuation adjustments

XG = merchandise exports

XGS = exports of goods and services, balance of payments basis

XGSNI = exports of goods and services, national income account basis

XSOV = other service account receipts

XSYV = investment income receipts

XTRANV = BOP transfers (credits)

YD = proxy for disposable income

x δ = depreciation rate

x σ = rate at which capital stock is discarded