THE EFFECTS OF FOREIGN AID ON OPTIMAL SAVINGS AND DEBT

by

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Since the hypothesis that foreign aid may retard domestic savings in a developing country was first raised by Trygve Haavelmo,\textsuperscript{1} this issue has been at the center of an ongoing debate among development economists. For example, the findings by Weisskopf\textsuperscript{2} and others of evidence that an increased aid flow tended to reduce the level of domestic savings in LDC's has given support to the view that foreign aid may bring about a temporary or permanent increase in the degree of dependence on outside resources among recipient countries.

In recent papers by Bhagwati and Grinols and by Wasow\textsuperscript{3} empirical estimates of these aid-related shifts have been incorporated into multi-period models to explore the relationship between aid, savings, and growth. While these studies are of considerable interest inasmuch as they trace out the dynamic implications of induced savings shifts, they are bound by their dependence on relatively rigid Harrod-Domar-type models and by the fact that the aid-related savings shifts are taken as given data.

This paper examines these and related questions from a slightly different point of view. Here we investigate the effect

\textsuperscript{1} T. Haavelmo (1965).
\textsuperscript{2} T. Weisskopf (1972); see also K.A. Areskoug (1969) and M.A. Rahman (1968). For a general discussion of this and related issues, see G. Papanek (1972).
\textsuperscript{3} J. Bhagwati and E. Grinols (1975) and B. Wasow (1978).

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I have benefitted greatly in the preparation of this paper from the comments of many members of the Federal Reserve's International Finance Division, especially Val Koromzay and Steve Salant.
of foreign aid on the optimum savings program in the context of a more flexible neo-classical growth model. This approach has the virtue of allowing the savings rate to vary over time and, by endogenizing the savings decision, of providing an interpretation of the observed aid-related shift. It is demonstrated that not only is the level of savings likely to fall under the introduction of additional foreign assistance, but that the marginal propensity to absorb resources out of new foreign aid can exceed unity in the early part of the optimum program. Furthermore, under a wide range of plausible assumptions about planning objectives, the level of savings will be less with aid than without over the entire planning horizon.

When the model is extended to allow for borrowing in international capital markets as well as grants, it turns out that aid and borrowing bear a complementary relationship to one another. On the optimal program an increase in foreign aid brings forth an increase in borrowing, rather than the reverse. The model also confirms the point made by Wasow in a slightly different context that the effect of aid on savings is strongly influenced by the time profile of the aid program.

The Model

To explore these issues, we first consider a simple planning model in which the planner seeks to maximize the functional

\begin{equation}
V = \int_0^T u(c(t))e^{-\gamma t} dt,
\end{equation}

where
where \( u(c) \) is the community's social welfare function (utility function),
c is per capita consumption, and \( \gamma \) is an appropriate time rate of discount.
This maximization is subject to initial and terminal conditions on the
per capita stock of capital, \( k \), namely:
(2a) \( k(0) \leq k_0 \),
(2b) \( k(T) \geq k_T \),
and to the income accounting relationship \(^4\),
(2c) \( \dot{k} = f(k) - c - \delta k + a \),
in which \( \delta \) is the fixed growth rate of the labor force, \( f(k) \) is a neo-
classical production function with the usual properties, and \( a \) is the
level of per capita foreign aid received in each period. For the moment
we shall assume that this aid flow is constant. \(^5\)

To maximize (1) subject to the given constraints we form the
Hamiltonian function
(3) \( H = [u(c(t)) + q(t) (f(k) - c - \delta k + a)] e^{-\gamma t} \),

\(^4\) We omit the time argument when the interpretation is clear. Imbedded
in (2c) is an assumption that aid inflow is exactly offset by net
imports -- that is, that the real transfer of resources associated
with aid is accomplished by continuous adjustment of the exchange rate
or some other equivalent mechanism.

\(^5\) A somewhat similar optimal program was considered in Rahman (1967).
where \( q(t) \) is the adjoint variable associated with the state equation, (2c). Maximization of (3) with respect to \( c \) and \( q \) produces the following two well-known necessary conditions that characterize the optimal path:

\[
(4) \quad u' = q
\]

and

\[
(5) \quad \dot{q} = - (f' - (\delta + \gamma))q
\]

Since \( q = u' \), equation (5) is easily transformed to

\[
(6) \quad \dot{c} = - [(f' - (\delta + \gamma))c] / (1 - \beta)
\]

where \( \beta \), the elasticity of utility with respect to consumption, is defined by

\[
\beta = \frac{du}{dc} \cdot \frac{c}{u} \text{.}
\]

If we assume that the target level of the capital stock is below the level corresponding to the modified golden-rule level for this system, that there is a positive gap between the target level and initial stock, and that the level of foreign aid is small relative to this gap, it is well established that under a relatively wide range of specifications for \( f(k) \) and \( u(c) \) both \( c \) and \( k \) will rise monotonically on their optimal paths.

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6. We also adopt the conventional assumption that \( 0 < \beta < 1.0 \) and that \( \beta \) is constant.
Sensitivity of the optimal path to changes in aid

To investigate the effect on these trajectories of small changes in the level of aid, we differentiate the equations of motion of the system, 7 (2c) and (5), with respect to a and obtain

(7a) \( \dot{k}_a = (f' - \delta) k_a - c' q_a + 1 \)

(7b) \( \dot{q}_a = -f'' q_k - (f' - (\delta + \gamma)) q_a \)

where \( k_a = dk/da \); \( q_a = dq/da \); \( \dot{k}_a = d^2k/dadt \); \( \dot{q}_a = dq^2/dadt \).

It should be apparent that this pair of equations constitutes a system of differential equations in \( q_a \) and \( k_a \). In order to interpret the motion of this derived system, in Figure 1 we have set the left hand sides of both (7a) and (7b) to zero and have shown sample locations for these two zero-growth frontiers. Evaluation of the slopes and intercepts of each equation reveals that the two frontiers must divide the \( (q_a, k_a) \) space into four quadrants within which the corresponding directions of motion are indicated.

If the change in foreign aid does not affect the value of

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7. This technique for performing sensitivity analyses on an optimal program is considered more fully in Oniki (1972).
Figure 1
either initial resources \((k_0)\) or the target terminal value \((k_T)\), then
the system must start and end on the \(q_a\)-axis. Accordingly, only one
type of path for \((q_a,k_a)\) is consistent with the directions of motion
indicated in Figure 1 and thereby qualifies as a solution. Such a path
must start on the \(q_a\) - axis below the intercept of the \(k_a\) - frontier,
move into the region in which both \(k_a\) and \(q_a\) are negative, and terminate on
the \(q_a\) - axis above the \(k_a\) - intercept but below the origin. Since \(q = u'\) and
\(u'\) is inversely related to \(c\), this path for \(q_a\) suggests that additional
aid must raise the level of optimum consumption at every point. In
addition, with the exception of the end points, the movement of \(k_a\)
indicates that the capital stock and, hence, output are reduced throughout.
Evidently, in this version of the model at least, per capita domestic
savings (defined by \(s = f(k) - c\)) will be reduced at every moment by
the introduction of additional aid.

It is, of course, not unexpected that on the optimal program
some part of the additional resources from foreign aid should go to
current consumption, and this possible outcome has been discussed
elsewhere. Figure 1, however, implies a somewhat stronger conclusion.

Taking note of the fact that

\[
(8) \quad \frac{dc_a}{da} = c_a = c' q_a
\]

the location of the path in Figure 1 indicates that \(c_a(0) > 1\) and
8
\(c_a(T) < 1\).

8. This point is discussed in greater detail in a brief appendix.
It does not depend, however, on the presence of a positive discount
rate.
Apparently, the aid-related shift in consumption is biased toward the early part of the path — that is, during some portion of the optimal path in the early stages of the program, the marginal propensity to consume out of additional foreign aid exceeds unity. In this segment not only is domestic savings diminished, but total investment is less as well. Foreign aid allows the domestic capital stock to be accumulated more slowly with the difference being made up in later periods as the marginal propensity to consume out of aid falls below unity.\(^9\)

Not surprisingly, the results depend closely on the specification of initial and terminal conditions. To illustrate this point we need only consider a case in which the prospect of additional aid brings about an upward revision of the terminal target. In terms of the phase diagram for \(q_a\) and \(k_a\) this means that the path of \((q_a, k_a)\) must terminate on a vertical line located at \(k_a = \frac{dk(T)}{da}\) rather than on the \(q_a\)-axis. This change allows for a variety of paths, three of which are illustrated in Figure 2. When the change in target is small, the main conclusions of the previous example still apply with the exception that the level of \(k\) is increased near the end of the program to meet the new higher target. If, however, the target adjustment is considerably larger, the path of \((q_a, k_a)\)

\(^9\) The implications of additional aid for the infinite horizon version of the problem can also be inferred from Figure 1. In the example, discussed above, the system would start from a point on the \(q_a\)-axis below the intercept of the \(k_a\)-frontier and move through the southwest quadrant to the intersection of the two zero-growth frontiers. Consumption would be increased and domestic savings and the level of domestic capital would be reduced throughout. As time proceeds, the system would approach a steady state in which the extra aid is divided between consumption and investment. For more details on sensitivity analysis of an infinite horizon problem, see Oniki (1972)
may move well in to the southeast quadrant and even into the northeast quadrant. Since the impact of aid on savings is given by

\[ s_a = f'k_a - c'q_a ; f' > 0, c' < 0, \]

movement into the southeast quadrant raises the possibility that \( s_a \) may be positive. *A fortiori*, passage into the northeast quadrant assures that outcome. In the latter case consumption near the end of the program must actually be reduced to meet the greatly increased terminal goal.

Conclusions about the effect of aid on the optimal path are also very sensitive to the assumed time profile of aid flows. To illustrate we modify (2c) to

\[ (2c') \quad k = f(k) - \delta k - c + a_0 e^{\theta t} , \]

where \( a_0 \) is the initial level of per capita aid and \( \theta \) an instantaneous rate of growth. The advantage of (2c') is that it now allows us to consider the effects of non-uniform shifts in aid.\(^{10}\) For example, a heavily front-loaded aid profile and shift are obtained with a large negative value for \( \theta \)(e.g., \( \delta + \gamma < -\theta < 1.0 \)). Differentiating (2c') with respect to \( a_0 \) we obtain

\[ (7a) \quad k_{a_0} = (f' - \delta) k_{a_0} - c'q_{a_0} + \theta e^{\theta t} . \]

\(^{10}\) The case of a constant absolute level of aid -- the case considered by Bhagwati and Grinols and criticized by Wasow -- is obtained with with \( \theta = -\delta \).
The presence of the time dependent term, $e^{qt}$, in (7a') greatly expands the range of possible outcomes. In terms of the phase diagram for $(q, k)$, the k-frontier may move upward at a sufficiently rapid rate that it can "overtake" $(q, k)$ in mid-path. If this is the case, it will permit trajectories of the type shown in Figure 3, in which the entire path is located in the southeast quadrant. Depending on the system's parameters, the marginal effect on domestic output and income may be strong enough relative to the effect on consumption for savings to be increased during the middle portion of the path. The underlying reason for such a pattern should be clear. When additional aid is extremely heavy in early periods, not all of the resource flow should be absorbed by increased consumption; at some point it becomes optimal for part of the additional resources to be invested in capital to provide against later periods in which the aid flow is greatly reduced.

*Foreign aid and international debt*

The model used so far makes no allowance for international capital-flows other than aid. We can easily extend the analysis, however, to include foreign borrowing by making several simple modifications. To do this we first need to distinguish between net owned capital and capital located within national borders. Net investment will impact on the former, whereas it is the latter.

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11. The most extreme case of a front-loaded aid program -- i.e., the case in which aid is given as a lump-sum transfer -- is easily analyzed by differentiating equations (2c) and (7a) with respect to $k_0$. As might be expected, in the resulting phase diagram for $q_0$ and $k_0$ the solution path will lie entirely in the southeast quadrant, implying that both the level of capital and consumption will be increased at all points by aid. If the resulting increase in output and income are large enough, in the early part of the program domestic savings may also be increased.
that corresponds to the capital input in the national production function. Accordingly, if we use the symbol \( b \) for the per capita version of the former concept and retain \( k \) for the latter, the level of per capita net international debt, \( d \), is given by

\[
d = k - b
\]

(10)

In order to be specific about the role of international borrowing, we shall also assume that the debtor country faces a loan supply schedule on which the borrowing rate, \( r_F \), depends on the level of per capita borrowing as follows:

\[
r_F = L(d) = \rho_0 + \rho_1 d^q ; \; \Theta > 1 ; \; \rho_0 > (\delta + \gamma) ; \; \rho_1 > 0 ,
\]

(11)

where \( \Theta \) is the elasticity of the interest rate with respect to the level of debt.

With these changes the optimization problem for the planner in the aid-receiving country becomes

\[
\text{Max} \quad \int_0^T u(c(t)) e^{-\gamma t} \, dt,
\]

subject to

\[
(13a) \quad b(0) \leq b_0 ,
\]

\[
(13b) \quad b(T) \geq b_T ,
\]

and

\[
(13c) \quad b = f (b+d) - \delta b - c - Ld + a .
\]
Notice that initial and terminal conditions are now expressed in terms of owned resources, and the income constraint includes terms for the level of current interest payments on international debt. The necessary conditions for optimization of this system are now

\[ (14a) \quad q = u' \]

\[ (14b) \quad \dot{q} = -(f' - (\delta + \gamma))q \]

\[ (14c) \quad f' = L + L'd \]

The new element is the solution is equation (14c), which requires that along the optimum path the marginal return to investment must be set equal to the marginal cost of borrowing. In most respects movement of this system on its optimal trajectory will be similar to the no-debt case --i.e., both c and b will increase monotonically to their terminal values. Since the level of debt is inversely related to b, this implies that net debt and interest payments will decrease monotonically on the optimal path.

When (14b) and (13c) are differentiated with respect to a, the resulting relationships are

\[ (15a) \quad \dot{b}_a = (f' - \delta) b_a + (-c') q_a + 1 \]

and

\[ (15b) \quad \dot{q}_a = -(f'' q)(1 + \frac{dd}{db}) b_a + (-f' - (\delta + \gamma)) q_a \]
which should be compared directly to (7a) and (7b). Although the two systems differ in some details, it is easily confirmed that the corresponding coefficients have the same signs. Accordingly, the phase diagram of Figure 1 and solution path, with appropriate reinterpretation of variables, may serve for both. In the system with borrowing, however, it should be evident that in cases where \( b_a < 0 \), it will also be true that \( d_a > 0 \). Thus, the lower level of domestically owned capital induced by extra aid will encourage additional capital inflows.

12. These conclusion are, of course, subject to the same important qualifications raised for earlier cases -- e.g., this "complementary" between aid and borrowing could be reversed by a heavily front-loaded aid program or an aid-related upward revision of the terminal target. Notice, however, that this complementarity does hold for the infinite horizon version of this problem.
Concluding remarks

Several points that have a bearing on the debate over the effect of aid on savings and growth emerge from the analysis above. First, when the aggregate level of consumption is a choice variable, in general savings propensities will not be constant over time or invariant to changes in the flow of aid. Under fairly general and plausible specifications of production and preferences, an additional inflow of foreign assistance will typically be divided between extra savings and extra consumption -- with the result that savings out of domestic income is likely to be reduced over a significant portion of the planning horizon. As the discussion in the text makes clear, the extent to which this is an accurate characterization of the outcome depends on assumptions both about the effect of aid on planning targets and the time profile of the aid flow. If aid results in a sufficiently large upward shift in the terminal target or if the aid program is heavily front-loaded (e.g., in the extreme case, an immediate lump-sum transfer), these features may offset the effects described above to the extent that domestic savings is unaffected or even increased on at least part of the optimal program.

A point that has not been fully appreciated in the formal literature on these issues is the strong bias toward early period consumption that a fully anticipated aid flow can produce. Our findings suggest that under a fairly wide range of circumstances the marginal propensity to absorb extra resources out of aid may exceed unity in the early part of the optimal program. Accordingly, the associated large reduction in
domestic savings means that the level of owned capital will be reduced over the full program. Furthermore, in an economy that is open to capital inflow as well as aid, the higher domestic return implied by this shift will tend to attract additional lending -- a result that may be unsettling to those who regard aid or concessional loans as a means of inducing less commercial borrowing.

It must be emphasized, however, that these conclusions derive from a highly abstract economic model and depend in large measure on its assumptions. Hence, the exercises we have carried out above can only suggest the full range of outcomes that might be obtained with a more flexible and more realistic framework. In an extended analysis for example, consideration ought to be given to such specific features as foreign exchange constraints and uncertainty. Despite these obvious limitations, however, the overall conclusions are broadly consistent with the limited empirical evidence, and provide a plausible interpretation of the observed consumption-inducing effects of aid.
Appendix

This appendix is intended to provide additional support for the assertion in the text that on the optimal path \( q_a(0) < 1/c'(q(0)) \). The validity of this relationship is made somewhat less than apparent by the fact that on the optimal program the position of the \( k \)-frontier is not fixed, but in general will change over time. In fact, the \( k \)-frontier will typically move upward through time, raising the possibility that the optimal path for \( (q_a, k_a) \) may start above the \( k \)-frontier but be overtaken by it in mid-path, implying a quite different trajectory from that shown in Figure 1.

To demonstrate that this cannot be the outcome for the case of constant per capita aid, we first recognize that the terminal level of \( k \) may be written as a function of \( q(0) \) and other parameters, i.e.,

\[
(A.1) \quad k(T) = F(q(0), a, k_0) .
\]

Differentiation of (A.1) implies that

\[
(A.2) \quad q_a(0) = -\frac{\partial F}{\partial a} = \frac{-k_T a}{k_T q_0}
\]

Hence, to evaluate \( q_a(0) \) we need to compare the level of \( k_T a \) (on a path on which \( (q_a(0), k_a(0)) = (0,0) \) with \( k_T q_0 \) (on a path on which \( (q_0(0), k_0(0)) = (1,0) \)).

The equations of motion for \( (q_a, k_a) \), as derived in the text, are

\[
(A.3) \quad \dot{q}_a = (-f''q)k_a + (-f' - \delta + \gamma)q_a .
\]
(A.4) \[ \dot{k}_a = (f' - \delta)k_a + (-c')q_a + 1. \]

The equations of motion for \((q_{q_0}, k_{q_0})\) are

\[
(A.5) \quad \dot{q}_{q_0} = (-f''q)k_{q_0} + (-f' - (\delta + \nu))q_{q_0},
\]

\[
(A.6) \quad \dot{k}_{q_0} = (f' - \delta)k_{q_0} + (-c')q_{q_0}.
\]

Direct comparison of these two systems is made easier by a change in variables.

Let \(\dot{q}_a = q_a + \frac{D S_a}{S_q - \delta_k}\), \(\dot{k}_a = k_a + \frac{D}{S_q - S_k}\);

and \(\dot{q}_{q_0} = Dq_{q_0}\), \(\dot{k}_{q_0} = Dk_{q_0}\),

where \(D = -\frac{1}{c'((q(0))}\), \(S_q = \frac{-f''(k_0)q(0)}{f'(k_0) - (\delta + \nu)}\), \(S_k = \frac{f'(k_0) - \delta}{c'((q(0))}\).

With these changes the new equations of motion can be written as

\[
(A.3)' \quad \dot{q}_a = (-f''q)\dot{k}_a + (-f' - (\delta + \nu))\dot{q}_a,
\]

\[
(A.4)' \quad \dot{k}_a = (f' - \delta)\dot{k}_a + (-c')\dot{q}_a,
\]

and

\[
(A.5)' \quad \dot{q}_{q_0} = (-f''q)\dot{k}_{q_0} + (-f' - (\delta + \nu))\dot{q}_{q_0},
\]

\[
(A.6)' \quad \dot{k}_{q_0} = (f' - \delta)\dot{k}_{q_0} + (-c')\dot{q}_{q_0}.
\]
The equations of motion for the two modified systems are analytically identical; the systems differ only in their initial conditions. Furthermore, it is easy to show by substituting the initial conditions into (A.4)' and (A.6)' that
\[ \dot{k}_a(0) = \dot{k}_q(0) \quad \text{and} \quad 0 = \dot{q}_a(0) > \dot{q}_q(0). \]
Accordingly, by applying the lemma established below it follows that
\[ \dot{k}_a > \dot{k}_q \quad \text{for all} \quad t, \quad 0 < t \leq T, \]
and
\[ \int_{0}^{T} \dot{k}_a \, dt > \int_{0}^{T} \dot{k}_q \, dt. \]
Since
\[ \int_{0}^{T} \dot{k}_i \, dt = \dot{k}_T - \dot{k}_0, \quad i = a, q, \]
\[ \dot{k}_Ta - \dot{k}_0a > \dot{k}_Tq - \dot{k}_0q. \]
Thus,
\[ k_Ta > \frac{k_Tq_0}{c'(q(0))}, \]
and referring back to (A.2),
\[ q_a(0) = -\frac{k_Ta}{k_Tq_0} < \frac{1}{c'(q(0))}, \]
which justifies the placement of the solution path shown in Figure 1 of the text. By a similar argument, it can also be demonstrated that
\[ q_a(T) > \frac{1}{c'(q(T))}. \]
A.4

Lemma: if \( \dot{k}_a, \dot{q}_a, \dot{q}_o \) and \( \dot{k}_o \) are given by (A.3)', (A.4)', (A.5)' and (A.6)', respectively, and \( \ddot{k}_a(0) = \ddot{q}_o(0) \) and \( \ddot{k}_a(0) > \ddot{k}_o(0) \), then \( \ddot{k}_a > \ddot{k}_o \) for all \( t, 0 < t \leq T \).

Proof: Let \( y = \dot{k}_a - \dot{q}_o ; z = \dot{q}_a - \dot{q}_o \).

\[
y' = \frac{\ddot{k}_a}{dt} - \frac{\ddot{q}_o}{dt} ; \quad z' = \frac{\ddot{q}_a}{dt} - \frac{\ddot{q}_o}{dt}
\]

\[
y'' = \frac{2 \dddot{k}_a}{dt^2} - \frac{2 \dddot{q}_o}{dt^2} .
\]

Substitution in (A.3)' - (A.6)' yields

(A.7) \( y' = (f' - \delta) y - c' z \),

(A.8) \( z' = (-f''q) y - (f' - (\delta + \gamma)) z \).

Differentiation of (A.7) produces

(A.9) \( y'' = (f' - \delta) y' + (f''q) y + (-c') z' + (-c''q) z \).

Consequently, substitution of (A.7) and (A.8) in (A.9) results in

(A.10) \( y'' = by' + cy \)

where \( b = b(t) = \left( \frac{1}{1-\beta} \right) \left( 1 - \frac{f'\dot{k}}{\dot{f}'} \right) \left( \frac{c}{k} - (1-\beta) \frac{\dddot{k}}{\dot{k}} \right) - (f' - \delta) \left( f' - (\delta + \gamma) \right) \)

\( c = c(t) = \frac{(2-\beta)}{(1-\beta)} \left( f' - (\delta + \gamma) \right) + \gamma > 0. \)
Equation (A.10) is a homogeneous second-order differential equation with variable coefficients. Although a particular solution to (A.10) is extremely difficult to obtain, it is easy to establish that, when $c(t) > 0$ for all $t \geq 0$ and $y'(0) = 0$ and $y(0) > 0$, then $y'(0)$ for all $t > 0$. Since these conditions are met on the optimal path (by virtue of the fact that $y'(0) = \dot{k}_a(0) - \dot{k}_q(0)$ and $y(0) = \dot{k}_a(0) - \dot{k}_q(0)$, then $\dot{k}_a - \dot{k}_q$ for all $t > 0$. 
References


