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by

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1. Introduction and Overview

This paper presents empirical evidence that month-to-month fluctuations in the dollar-Deutschemark exchange rate conform suitably to the predictions of a portfolio-balance model with rational expectations. Unlike monetarist models of exchange-rate behavior (e.g., Bilson, 1978; Dornbusch, 1976; Frenkel 1976; and Girton and Roper, 1977), our approach emphasizes that the exchange rate between two currencies depends on the relative supplies of a wide range of financial assets denominated in those currencies, not only on the relative supplies of moneys. In this context we can distinguish between the impacts on asset supplies -- and hence on exchange rates -- of three different policy instruments: changes in the supply of base money, fiscal budget deficits, and official exchange-market interventions.

Exchange rates depend as well on the relative demands for financial assets denominated in different currencies. In formulating the demand side of our model we follow Kouri (1976) and Branson (1976) by emphasizing the effects of the accumulation and shifting residence of wealth, and we pay particular attention to the dramatic growth of OPEC wealth since 1973. Current-account imbalances are viewed to affect exchange rates by shifting the residence of wealth between asset holders with

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*/ The analysis and conclusions of this paper do not necessarily represent the views of the Federal Reserve System or anyone else on its staff. We would like to acknowledge enlightening discussions with Dale Henderson and participants in the International Trade Workshop at the University of Chicago.

different sets of portfolio preferences. Unlike the portfolio-balance models of Porter (1977) and Branson, Halttunen and Masson (1977), we retain wealth variables in our estimating equation.

The demand side of our model also allows for risk-averse behavior by portfolio managers. During our sample period, the absolute values of month-to-month changes in the spot rate exceeded 2 cents per mark (an annual rate of 60 per cent) for 6 out of 50 months, and exceeded 1 cent per mark during 20 months. Interest differentials were small relative to these exchange-rate movements. Thus, to the extent that such large exchange-rate movements were expected, market participants must have been strongly risk averse; otherwise they would have taken positions to prevent exchange rates from ever getting so far out of line with their expected future values. Obversely, to the extent that the large exchange-rate movements were unexpected, market participants were caught in a world in which they had good reason to be risk averse. Accordingly, we explicitly incorporate a risk-aversion parameter into our estimating equation, which predicts exchange-rate changes significantly better than the current one-month forward rate. This emphasizes the importance of filtering information on spot rates and interest differentials through a model that allows for risk aversion, rather than viewing the world as a risk-neutral environment in which forward rates can be interpreted as expected future spot rates.

One of our main objectives is to move away from a model in which future (or current) spot rates are predicted from current (or lagged) spot or forward rates toward a model in which current and expected future spot rates are explained simultaneously and consistently. We do this by adopting an iterative estimation procedure. Our model provides an equation for estimating the current spot rate as a function of the expected future spot rate and other variables that we either treat as exogenous or replace with predetermined instruments. In our first iteration this equation is estimated under an arbitrary specification of the time path of the expected future spot rate. In subsequent iterations, however, the time path of the expected future spot rate is generated by applying the parameter estimates from the previous iteration to assumed expected future time paths of the other exogenous variables and predetermined instruments. (These latter expected future time paths are generated without sophistication from the in-sample time-series behavior of the exogenous variables.) The details of this procedure are described below. We continue to iterate until the predicted time path of the spot rate is equal (within a tolerance limit) to the time path of the expected future spot rate lagged one period. This essentially forces the current spot rate and the expected future spot rate to fit the same model, and in this sense our expectations are "rational."

In order to be as consistent as possible with the portfolio-balance framework, we choose to observe the exchange rate at a single point in time at the end of each month, rather than averaging daily observations within each month. We know that our observed exchange rates are exchange rates that clear asset markets in a tautological sense, but we reject the assumption that asset holdings reflect continuous portfolio-balance equilibrium, and we do not attempt to model asset holdings as moving almost continuously along paths that adjust smoothly toward desired portfolio holdings. Instead, we assume that asset holdings fluctuate randomly about portfolio-equilibrium levels[1/] such that the difference between observed and equilibrium exchange rates is a serially-uncorrelated variable with zero mean (and constant variance). This assumption is implicit in the use of our exchange-rate observations as the dependent variable in a model of the exchange-rate path that would be consistent with continuous portfolio-balance equilibrium.

We use a nonlinear specification to estimate the equilibrium path of the exchange rate, and our iterative procedure for imposing rational expectations only converges to a sensible solution when we give strong weight to our prior notions about the parameters that determine portfolio shares.[2/] Although we cannot provide theoretical justification for the modified-Bayesian approach and iterative procedure that we adopt, our model correctly predicts the direction

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1/ This view is also expressed by Christopher Sims in the general discussion of Kouri and de Macedo (1978).

2/ There is some precedent for imposing priors in exchange-rate estimation. Bilson (1978) gives strong weights to his priors, and Armington (1978) imposes priors on his portfolio-share matrix.

of 35 out of 50 month-to-month changes in the observed exchange-rate, as compared with 27 out of 50 changes (little better than a coin toss) that are correctly predicted by the forward rate. The coefficient of correlation between the changes predicted by our model and observed changes is .41,whereas the changes predicted by the forward rate have a small negative correlation with observed changes. In addition, our predictions of exchange-rate changes are more accurate -- albeit only slightly -- than predictions based on several measures of purchasing-power parity. Such results are nothing to crow too loudly about, but the portfolio-balance rational-expectations framework is much richer than either the forward-exchange or purchasing-power-parity theories, and there is scope for improving our empirical results by refining the model in several directions.

2. The Basic Framework

Our portfolio model resembles Girton and Henderson (1976) in a number of its basic features. We assume a world with two currencies, called the dollar and the mark, and we divide the world into four "countries" or wealth-holding regions, called the United States (US), Germany (G), the oil-exporting countries (OPEC) and the rest of the world (ROW). We distinguish two types of outside assets (or claims on official agencies) in each currency: the non-interest-bearing monetary base, and interest-bearing "bonds" or government debt. Our exchange-rate equation is derived by focussing on the market-clearing condition for dollar-denominated assets. In theory we could just as well work with the market-clearing condition for mark-denominated assets.

The supplies of U.S. base money and outside dollar-denominated bonds are assumed to be exogenously determined by the interaction of the Federal Reserve's monetary policy, the level of the U.S. government debt, and official U.S. and foreign exchange-market intervention involving dollar-denominated assets. This assumption of exogeneity is strong, though conventional.[3/] It implies that policy makers do not react <u>systematically</u> to the exchange rate or to variables that influence the exchange rate. We know, of course, that intervention policy can be viewed predominantly as a reaction to exchange rates, and that monetary policy and government-debt policy react to variables that influence exchange rates. However, it is difficult to model policy-reaction functions as systematic, and we have not attempted to do so.

We let MB denote the U.S. monetary base and B denote the supply of outside dollar-denominated bonds, by which we mean the <u>net</u> stock of dollar-denominated bonds supplied by official institutions. B is viewed to equal the cumulative U.S. budget deficit (DEF) minus

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3/ Two exceptions that incorporate policy reaction functions are Artus (1976), who embeds the dollar-mark exchange rate in a model of Germany's monetary sector, and Branson, Halttunen and Masson (1977).

the stock of bonds removed from private circulation through the open-market operations of the Federal Reserve (which we take to equal MB), minus cumulative official intervention purchases of dollar-denominated bonds (INT).

$$(1) \quad B = \int DEF - MB - \int INT$$

We assume that the U.S. monetary base is held entirely by U.S. residents, but that the supply of dollar bonds is allocated among (a) private U.S. wealth holders, (b) private German wealth holders, (c) private and official OPEC residents, and (d) private and official residents of ROW. The isolation of U.S. and German wealth holders is dictated by our focus on the dollar-mark exchange rate. The isolation of OPEC is based on the rapid increase in OPEC wealth during our sample period, combined with indications that OPEC has different portfolio preferences than other wealth holders. The inclusion of ROW is necessary to make the different components of demand add up to supply.4/

Before turning to our behavioral assumptions about demands for the two types of dollar assets, we can write the market-clearing conditions

4/ See Armington (1978) for an n-country model that is capable of estimating n-1 exchange rates simultaneously; and see Berner et al. (1977) for a 6-region model with 5 simultaneously-determined exchange rates.

for these assets as

$$(2) \quad MB = MB^d_{US}$$

$$(3) \quad B = B^d_{US} + B^d_G + B^d_{OPEC} + B^d_{ROW}$$

where a superscript "d" connotes demand and subscripts refer to the source of demand. Let F^d_{US} denote private U.S. demand for interest-bearing assets denominated in marks (or for bonds denominated in currencies other than the dollar), let x denote the exchange rate in dollars per mark, and let W denote private U.S. wealth. Then by definition, the balance sheet of private U.S. wealth holders must satisfy the condition.[5/]

$$(4) \quad W = MB^d_{US} + B^d_{US} + xF^d_{US}$$

In order to derive a market-clearing condition for the two types of dollar-denominated assets combined, we add (2) and (3) and substitute from (4) to get:

$$(5) \quad MB + B = (W - xF^d_{US}) + B^d_G + B^d_{OPEC} + B^d_{ROW}$$

Alternatively, by combining (5) and (1):

$$(6) \quad \int(DEF-INT) = (W - xF^d_{US}) + B^d_G + B^d_{OPEC} + B^d_{ROW}$$

We will substitute behavioral assumptions for the demand variables, and then manipulate condition (5) to yield an equation for the exchange rate, which is one of the arguments of our demand functions.

No information is gained by working with the market-clearing condition for money and bonds combined, rather than

5/ We make the standard assumption that private U.S. residents do not hold foreign non-interest-bearing money, and that the U.S. monetary base is entirely held by U.S. residents.

working with separate market clearing conditions, provided we introduce behavioral assumptions appropriately drawn from a portfolio-balance framework. The combined market-clearing condition is appealing, however, partly because of its broader scope and partly because of the symmetry between F^d_{US} and $B^d_{FOREIGN}$ (FOREIGN = G, OPEC, ROW).

3. Behavioral Assumptions

Let i and i_G denote one-month own-currency rates of interest on dollar-denominated and mark-denominated bonds, let x^e be the spot exchange rate (in dollars per mark) currently expected to prevail one month in the future, and let Y denote the nominal income of private U.S. wealth holders (measured in dollars). Then a complete model of portfolio behavior for the U.S. private sector (relevant to the menu of assets that we are considering) can be specified as:

$$(7)\quad MB^d_{US} = m(i, i_G + (x^e - x)/x, Y/W, \ldots).W$$

$$(8)\quad B^d_{US} = b(\qquad\qquad\qquad\qquad).W$$

$$(9)\quad xF^d_{US} = f(\qquad\qquad\qquad\qquad).W$$

such that m+b+f=1. We view (7)-(9) as a description of equilibrium levels about which actual portfolio holdings are assumed to fluctuate randomly. The first two arguments of the functions m, b, and f respectively represent the expected nominal dollar-equivalent yields on domestic bonds and foreign bonds; the nominal yield on base money is zero. The third argument, Y/W, is a transactions demand variable that allows the demand for money to increase (relative to wealth) as income increases (relative to wealth). Ideally, wealth, defined to satisfy condition (4), would also be modeled as a portfolio choice variable rather than treated as predetermined; but lack of appropriate data makes it difficult to treat wealth as endogenous, as will be discussed below.

The specification of (7)-(9) in terms of nominal rather than real expected yields, which gives no explicit consideration to expected rates of inflation, implicitly assumes that portfolio choices between money and bonds are independent of the expected yields on stocks of goods or other assets. We let r^e denote the expected differential yield on mark-denominated bonds relative to dollar-denominated bonds:

$$(10)\quad r^e = i_G + (x^e - x)/x - i$$

Since there is one-to-one correspondence between $(i, i_G + (x^e - x)/x)$ and (i, r^e), conditions (7)-(9) can be transformed into

$$(7a)\quad MB^d_{US} = m'(i, r^e, Y/W, \ldots).W$$

$$(8a)\quad B^d_{US} = b'(\qquad\qquad).W$$

$$(9a)\quad xF^d_{US} = f'(\qquad\qquad).W$$

Ideally we would like to estimate the complete U.S. portfolio model (7a)-(9a) together with analogous models for our three other regions, and to then solve our estimated structural equations for the relationship between the exchange rate and those variables that we assume to be exogenous. Unfortunately, data are not available on B^d_{US} and F^d_{US}, since we do not know exactly how much of the U.S. government debt is held by domestic residents, and neither do we know the currency composition of U.S. claims on foreigners. This lack of data motivates our approach of substituting a behavioral assumption for xF^d_{US} into condition (5), along with behavioral assumptions for B^d_G, B^d_{OPEC} and B^d_{ROW}, to arrive at a reduced-form relationship that can be manipulated to solve for the exchange rate as a function of other variables in the model.

We do have data, however, on MB^d_{US}; and the efficiency of our estimates may be increased by incorporating this information into our assumption about the behavior of xF^d_{US}. Because we may also want to make use of estimated information about the relationship of MB^d_{US} to i, Y and W, we assume that (7a) - (9a) have the specific simplified form

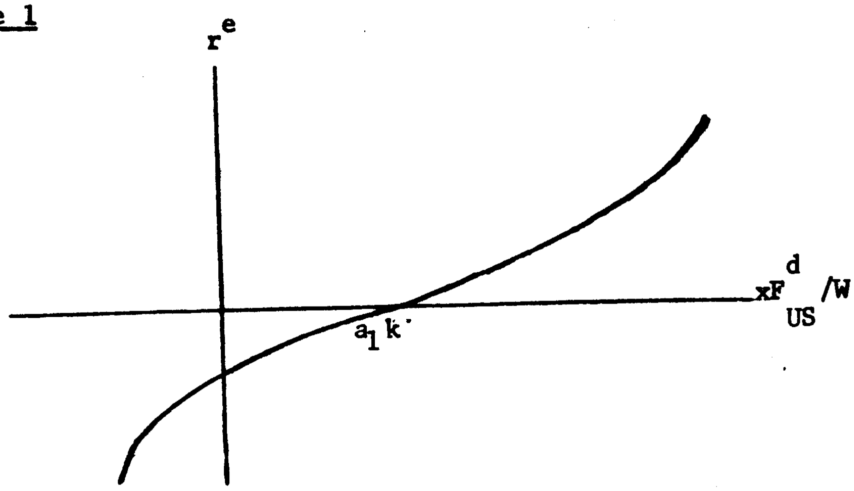
$$\text{(7b)}\quad MB^d_{US}/W = 1 - k(i, Y/W, \ldots)$$

$$\text{(8b)}\quad B^d_{US}/W = (1 - a_1 - b_1\sqrt[3]{r^e}).k(\ ) \text{ with } b_1 > o$$

$$\text{(9b)}\quad xF^d_{US}/W = (a_1 + b_1\sqrt[3]{r^e}).k(\ )$$

Condition (7b) simplifies condition (7a) by assuming that the allocation of wealth between money and bonds is independent of r^e and, in particular, does not require knowledge of the expected rate of exchange-rate change. Given MB^d_{US}/W and hence k , conditions (8b) and (9b) assume that the allocation between domestic and foreign bonds depends only on r^e, the expected differential yield between these two types of bonds, according to a functional form that is illustrated by Figure 1. With $r^e = 0$, U.S. private portfolio holders allocate the fraction a_1k of their wealth to foreign bonds, with k being a function of i and Y/W as specified

Figure 1



in condition (7b), and with the parameter a_1 presumably valued between 0 and 1. In the range $r^e > 0$, successive unit increments in r^e, ceteris paribus, lead to positive but successively-smaller increments in the share of U.S. private wealth allocated to foreign bonds, reflecting aversion toward risk in the home-currency valuation of portfolio holdings. A symmetric assumption is made for the range $r^e < 0$. The cubic-root equation is adopted as the simplest specification that exhibits the important properties of monotonicity and risk aversion. Note that the ability to issue debt denominated in either foreign or domestic currency makes it feasible to have either $xF_{US}^{d}/W < 0$ or $xF_{US}^{d}/W > k$. Note also that the greater is the parameter b_1, the less is the degree of risk aversion; for $b_1 = \infty$, respresenting the case of risk neutrality, xF_{US}^{d} would equal infinity whenever $r^e > 0$ and minus infinity whenever $r^e < 0$.

Condition (9b) can be substituted into (5) to replace xF_{US}^{d}. In addition we require behavioral assumptions for B_G^d, B_{OPEC}^d and B_{ROW}^d. For B_G^d we assume the symmetric analog of (9b):

$$(11) \quad B_G^d/xW_G = (a_2 + b_2\sqrt[3]{-r^e}).k_G(\ ) \quad \text{with } b_2 > 0$$

or

$$(11a) \quad B_G^d = (a_2 + b_2\sqrt[3]{-r^e}).k_G(\ ).W_G^{\$}$$

where W_G is German wealth measured in marks, $W_G^{\$} = xW_G$ is German wealth measured in dollars and $-r^e$ is the expected differential yield in favor of dollar bonds. For OPEC and ROW we do not have data on supplies of base money. Consequently, we assume

$$(12) \quad B_{OPEC}^d = (a_3 + b_3\sqrt[3]{-r^e}).W_{OPEC}^{\$}$$

$$(13) \quad B_{ROW}^d = (a_4 + b_4\sqrt[3]{-r^e}).W_{ROW}^{\$}$$

where the k functions are treated as constants and absorbed into the a and b parameters, and where $W_{OPEC}^{\$}$ and $W_{ROW}^{\$}$ denote wealths denominated in dollars.

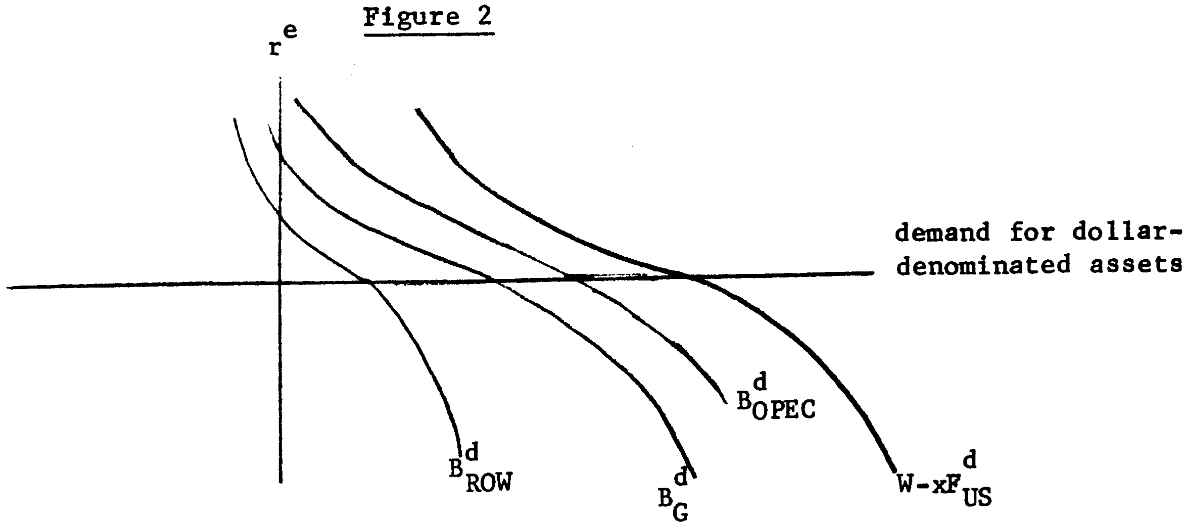
4. Graphical Illustration

Substitution of (9b), (11a), (12) and (13) into (5) yields

$$(14) \quad MB + B = (1 - (a_1 + b_1\sqrt[3]{r^e})k)W + (a_2 - b_2\sqrt[3]{r^e})k_G W_G^{\$} + (a_3 - b_3\sqrt[3]{r^e})W_{OPEC}^{\$} + (a_4 - b_4\sqrt[3]{r^e})W_{ROW}^{\$}$$

The four components of demand on the right-hand-side of (14) are illustrated in Figure 2 for given values of the k and W variables. (In reality the relative positions and slopes of the four curves may be different than this particular illustration.)

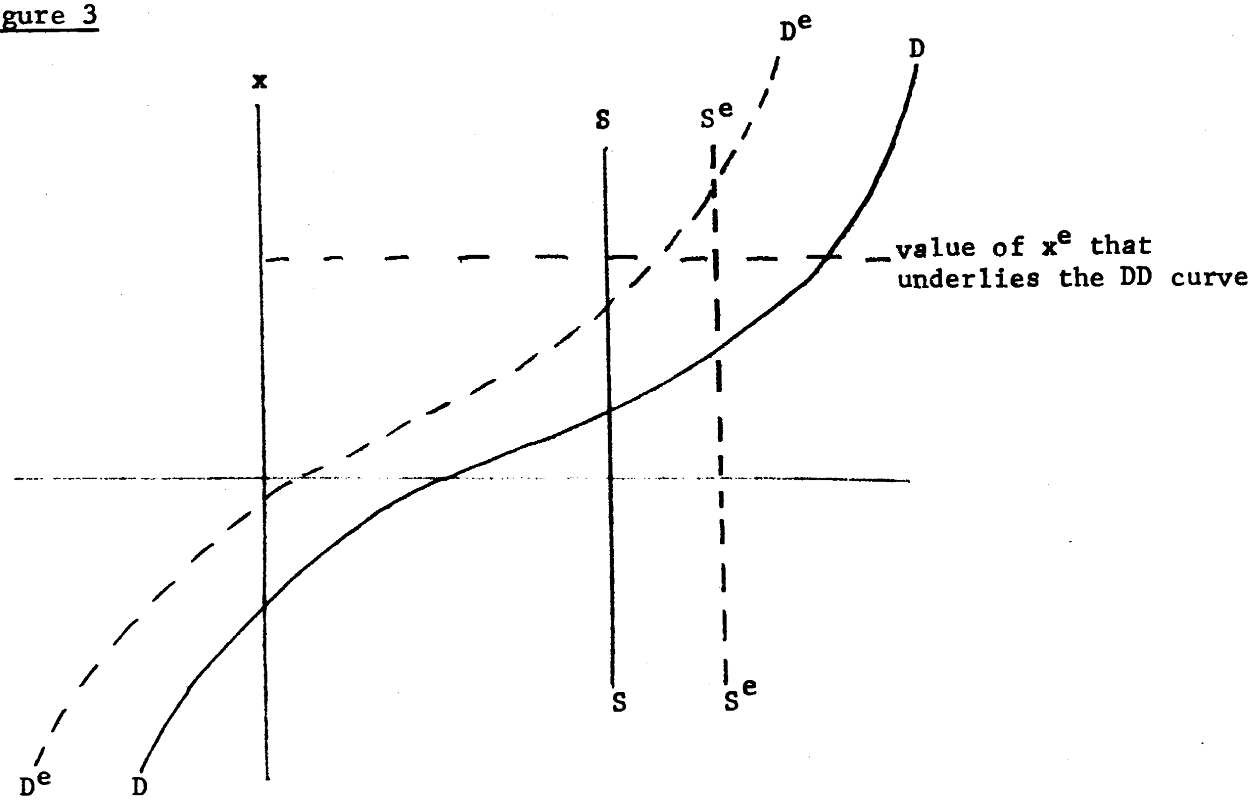
Figure 2



To illustrate the procedure we use to estimate the determination of observed and expected exchange rates, Figure 3 shows aggregate supply (the SS curve) and demand (the DD curve) -- corresponding to the left and right-hand sides of condition (14) -- as functions of the observed exchange rate (x), for a given value of the exchange rate currently expected to prevail next month (x^e). The transformation from Figure 2 to Figure 3 uses condition (10). The SS curve is vertical since MB + B is predetermined, independently of x.

We observe or introduce proxy time series on aggregate supply (MB + B), all variables other than x^e that underlie the position of the DD curve in each time period, and the market-clearing exchange rate x. Accordingly, for any specified time series of x^e we can estimate the aggregate demand function, and (given the values of predetermined variables) we can plot the DD curve for each time period. Using our estimated values of the parameters of the aggregate demand function, together with assumptions about current expectations of next period's values of all predetermined variables, we can also plot the aggregate supply and demand curves that currently are expected to prevail next period -- represented in Figure 3 by S^eS^e and D^eD^e. Unless the intersection of S^eS^e and D^eD^e coincides with our specification of x^e, however, the estimated parameters of our aggregate demand function are not consistent with rational expectations. Accordingly, we estimate the aggregate demand function using an iterative procedure that begins by adopting an arbitrary initial specification of x^e and then iterates through successive respecfications of x^e until we converge to estimated aggregate demand parameters that are consistent with rational expectations. The result is a model that simultaneously and consistently explains the determination of observed and expected exchange rates, given observed and expected values of the predetermined variables that enter the aggregate supply and demand functions.

Figure 3



5. Data Inadequacies and Our Choice of Wealth Variables

Our model is limited in a number of important ways by lack of data on the currency composition of international debts, and hence by lack of data on the currency composition of U.S. and foreign portfolio holdings.[6/] Without such data we cannot estimate a complete portfolio-balance model, and it is difficult to gear the model to a world with more than two currencies.

Without such data we are also forced to adopt measures of wealth that cannot be revalued appropriately when exchange rates change. In other words, without data on B^d_{US} and F^d_{US} we cannot construct W to satisfy condition (4) identically. Instead we have chosen to construct W from the national-income accounting identities. We know that private savings in any time period equals private investment plus the government budget deficit plus the current-account surplus on international transactions. Thus, abstracting from capital gains and losses, we generate our wealth variable (which excludes the value of equities) by estimating an initial value from the best information available and then adding in each period the cumulative value, since the beginning of the sample period, of private savings minus private investment--or the cumulative sum of the government budget deficit plus the current-account

6/ Branson, Halttunan and Masson (1977) sidestep the problem by assuming that international debt is entirely denominated in a single currency and can only be accumulated or reduced through current-account imbalances.

surplus. Government budget deficits add to private wealth, and current-account imbalances shift the residence of wealth between countries.[7/]

We construct data on W_G, the mark value of private German wealth, by the same method that we construct W. In estimating our model we adopt the strong but convenient assumption that the dollar value of private German wealth, constructed as $W_G^{\$} = xW_G$, is predetermined and not influenced by the contemporaneous exchange rate.

We construct $W_{OPEC}^{\$}$ by assuming an initial value of zero at the beginning of the floating-rate period in March 1973--i.e., by assuming an initial balance between financial claims on and debt to non-OPEC countries. Thereafter we increase $W_{OPEC}^{\$}$ each month by an estimate of OPEC's current-account surplus measured in dollars. To the extent that OPEC invests part of its current-account surpluses in real assets or equities, $W_{OPEC}^{\$}$ will overstate OPEC's combined holdings of dollar-denominated and foreign-currency-denominated bonds.

7/ Flow-of-funds data on the net financial worth of households and nonfinancial businesses (excluding their holdings of corporate equity) provide an alternative to constructing a U.S. wealth variable from data on budget deficits and current-account surpluses. Our suspicion, however, is that the flow-of-funds wealth variable contains more measurement error than our constructed wealth variable, so we have rejected this approach.

We construct $W^{\$}_{ROW}$ as an initial value (to be described below) plus the cumulative current-account balance of ROW since March 1973. The current-account of ROW is measured as the balance that equates the global current account to zero, given our estimates of the current-account balances for the United States, Germany and OPEC. To the extent that ROW "finances" part of its current-account deficits by selling real assets or equities, $W^{\$}_{ROW}$ will understate ROW's combined holdings of dollar-denominated and foreign-currency-denominated bonds.

6. The First-Stage Estimation Procedure

In the empirical work reported in this paper we treat k and k_G as endogenous, because in (7b) we view the shares of portfolios that are allotted to base-money holdings to be functions of the usual arguments of money demand functions, including interest rates. We are not willing to assume that interest rates are predetermined at the same time that we assume the stock of official dollar debt (MB+B) to be predetermined. We do assume, however, that policy variables, income levels and current-account balances are predetermined in the sense of responding to changes in exchange rates or interest rates with lags of at least one month.

Because we view interest rates as endogenous we have adopted a modified two-stage least-squares procedure. We thus sidestep the specification and estimation of a full model of interest rates by regressing i, i_G, k and k_G on the list of our predetermined variables, and by substituting fitted values for i, i_G, k and k_G in the second-stage estimation of our exchange-rate equation. In hopes of increasing the efficiency of our estimates of i, i_G, k and k_G, we modify the conventional first-stage procedure by adding to the conventional list of regressors some predetermined variables that do not appear in our second-stage exchange-rate equation, but that would appear in a full model of interest rates. Specifically, we regress i, i_G, k and k_G on the four wealth variables ($W, W_G^{\$}, W_{OPEC}^{\$}, DW_{ROW}^{\$}$), four asset stock variables (MB, MB_G, B, B_G), two scale-of-transactions variables (Y/W, Y_G/W_G) and constant terms, where MB_G and B_G are the German analogs of MB and B, and where $DW_{ROW}^{\$}$ represents the cumulative change in $W_{ROW}^{\$}$ from its base-period value in March 1973.[8/]

7. The Second-Stage Estimation Procedure

Condition (14) can be manipulated to yield

$$(15) \quad r^e = \left(\frac{(1-a_1 k)W + a_2 k_G W_G^{\$} + a_3 W_{OPEC}^{\$} + a_4 W_{ROW}^{\$} - MB - B}{b_1 k W + b_2 k_G W_G^{\$} + b_3 W_{OPEC}^{\$} + b_4 W_{ROW}^{\$}} \right)^3$$

8/ We do not include expected rates of inflation among this list of regressors since we consider expected inflation rates to be themselves determined by the list of regressors, simultaneously with the determination of interest rates and exchange rates. We may nevertheless imagine that the regressors affect nominal interest rates in part by affecting expectations of inflation. Consider two equilibrium time paths of the world that differ only by the fact that on path 2 the nominal stocks of U.S. base money and outside dollar bonds grow g per cent per month faster than on path 1. There should be no real differences between these states of the world; and it may be instructive to note that none would arise in our model provided that the U.S. nominal interest rate was a uniform g per cent per month higher under state 2. For then a g per cent per month faster depreciation of the dollar in state 2 would be consistent with the same time paths of real variables as would emerge under state 1.

and condition (10) can be manipulated to yield

$$(16) \quad x = x^e/\ (1+r^e+i-i_G)$$

We can substitute (15) into (16) to obtain an expression for x in terms of x^e, a list of predetermined variables (Z) and the set of parameters to be estimated (p). Z includes the variables i, i_G, k and k_G, which henceforth denote the fitted (and thus predetermined) values of these variables based on the first stage regressions.

With time subscripts added the model takes the general form

$$(17) \quad x_t = g(E_t x_{t+1},\ Z_t,\ p) \quad \text{for } t=1,\ \ldots,T$$

where x_t and Z_t are observed at the end of each month t and where $E_t x_{t+1}$, corresponding to x^e, is the unobserved expectation, held at the end of month t, of the value of x_{t+1}. Rather than estimate (17) under an ad hoc specification of the time series $\{E_t x_{t+1}\}$ we adopt an iterative procedure intended to converge to a time series $\{E_t x_{t+1}\}$ that is consistent with model (17) in the sense of satisfying

$$(18) \quad E_t x_{t+1} = g(E_t x_{t+2},\ E_t Z_{t+1}, \hat{p}) \quad \text{for } t=1,\ldots,\ T.$$ [9]

Here $\hat{p}$ denotes the list of estimated parameter values that best fit model (17), $E_t Z_{t+1}$ represents an assumption about period-t expectations of Z_{t+1}, and $E_t x_{t+2}$ is an assumption about period-t expectations of x_{t+2}. Our particular choices for $E_t Z_{t+1}$ are based on the time-series history of Z. In particular, the predetermined asset-supply, wealth and income variables are each regressed on six lagged values of themselves (representing one to six-month

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9/ Note that $E_t x_{t+2}$ is a simplification of $E_t E_{t+1} x_{t+2}$.

lags), and fitted values of Z_{t+1} from those equations are adopted for E_tZ_{t+1} 10/. In addition, period-t expectations of the period-t+1 values of i, i_G, k and k_G are generated from the first-stage estimates by replacing period-t+1 values of predetermined asset-supply, wealth and income variables with their period-t expectations.

Our iterative solution procedure is as follows. We start with observations on the time series $\{x_t\}$ and $\{Z_t\}$, and with assumptions about the time series $\{E_tZ_{t+1}\}$. For the first iteration we begin with an arbitrary specification of the time series $\{E_tx_{t+1}\}$-- call it $\{E^o_tx_{t+1}\}$-- and we fit model (17) to obtain a vector of parameter estimates $\hat{p}^1$. We then generate a new time series $\{E^1_tx_{t+1}\}$ using (18) subject to $\hat{p} = \hat{p}^1$ and $E_tx_{t+2} = E^o_{t+1}x_{t+2}$. And we continue in this fashion, using $\{E^m_tx_{t+1}\}$ to generate $\{E^{m+1}_tx_{t+1}\}$. If the procedure converges in the sense of reaching an m for which $\{E^{m+1}_tx_{t+1}\}$ and $\{E^m_tx_{t+1}\}$ differ by less than a small tolerance limit, 11/ then the solution has the property

10/ There are two exceptions here. $E_tWG^{\$}_{t+1}$ is based on the six-month history of WG, rather than $WG^{\$}$, and $E_tWROW^{\$}_{t+1}$ is constructed as a residual, consistent with the manner in which $WROW^{\$}_t$ is constructed to satisfy the global current-account adding-up condition.

11/ We define convergence as an average absolute percentage difference of less than one-tenth of one per cent, which is less than .05 cents per mark.

that

and

$$\hat{x}_t = g(E_t^m x_{t+1}, Z_t, \hat{p}^m) \quad (17a)$$

$$E_t^m x_{t+1} = g(E_{t+1}^m x_{t+2}, E_t Z_{t+1}, \hat{p}^m) \quad (18a)$$

Thus, the fitted path of the exchange rate is based on exchange-rate expectations that are themselves generated consistently from the same exchange-rate model and the same parameter values.12/

Several points should be made about this iterative procedure. First, convergence is not guaranteed; and if convergence is achieved, it is important to check the sensitivity of the final parameter estimates to the initial specification of $\{E_t^o x_{t+1}\}$.

Second, if the model does converge this procedure forces equality between expectations of future exchange rates and the exchange rates that the model predicts under a specific set of assumptions about the expected future values of the other variables in the model. Exchange-rate expectations can be wrong, but our procedure constrains them to be consistent with the model's estimates of the exchange rates that are consistent with portfolio equilibrium.

12/ It may be noted that the consistency here is based on using $E_{t+1}^m x_{t+2}$ in place of $E_t^m x_{t+2}$ as a right-hand-side argument in condition (18a). We lose generality but avoid an infinitely-recursive model by making this substitution. Our procedure thus ensures that $E_t x_{t+1}$ and x_t are estimated consistently, but stops short of attempting to guarantee consistency between $E_t x_{t+1}$ and $E_t x_{t+2}$, between $E_t x_{t+2}$ and $E_t x_{t+3}$ and so forth *ad infinitum*. See Bilson (1978) for a different empirical conceptualization of rational expectations.

8. The First-Stage Estimation Results

The first-stage estimation results are shown in Table 1. Since our model imposes a loose correspondence between changes in the sum of the four wealth variables ($W+W_G^{\$}+W_{OPEC}^{\$}+DW_{ROW}^{\$}$) and changes in asset supplies ($MB+B$, or MB_G+B_G) -- the correspondence would be tight if MB_G+B_G were measured in dollars -- we have excluded B_G from the i and k regressions and B from the i_G and k_G regressions. Thus, in the i and k regressions we view a change in B_G to be the counterpart to ceteris paribus changes in one of the wealth or asset-supply regressors, while changes in B are behind the scenes in the i_G and k_G regressions.

Our regression equations are not developed from a structural model, but we nevertheless have prior expectations about the signs of some of the parameters. In the k regression we expect an increase in the transactions variable Y/W to lower the ratio of bond holdings to wealth in U.S. portfolios (i.e., to lower k), ceteris paribus; and we also expect k to rise in response to increases in W or reductions in MB, ceteris paribus. Our results are consistent with these expectations and with similar expectations about the relationship of k_G to Y_G/W_G, $W_G^{\$}$, and MB_G.

The interest-rate regressions, however, are not as consistent with our prior expectations. While they confirm our expectation that increases in Y/W or Y_G/W_G, ceteris paribus, should raise both i and i_G, they do not entirely confirm our expectation that an increase in any of the four wealth variables, ceteris paribus, should reduce both i and i_G. And it is especially disconcerting: (a) that

Table 1: First-Stage Regressions

Dependent Variable	Constant	W	$W^{\$}_G$	$W^{\$}_{OPEC}$	$DW^{\$}_{ROW}$	MB	MB_G	$MB+B$	MB_G+B_G	Y/W	Y_G/W_G	$\bar{R}^2$	D.W.	RHO
$i(x100)$	-10.9	.0134	.00369	-.0234	-.0275	-.000470	-.0295	-.00977		2.13	6.08	.918	1.75	.552
	(-3.87)	(2.12)	(1.11)	(-2.05)	(-2.51)	(-.0106)	(-1.02)	(-1.36)		(2.60)	(2.97)			(4.73)
$i_G(x100)$	-14.0	-.000227	.000429	-.0331	-.0161	.0909	.0235		-.00656	.720	2.55	.779	1.87	.177
	(-5.81)	(-.0216)	(-.0881)	(-2.78)	(-1.28)	(1.37)	(.506)		(-.369)	(.536)	(.833)			(1.28)
$k(x100)$	82.6	.0528	-.00235	-.00178	-.00173	-.184	-.00158	-.0133		-.701	-.766	.999	1.11	.928
	(48.1)	(17.2)	(-2.02)	(-.302)	(-.312)	(-12.5)	(-.165)	(-4.17)		(-2.02)	(-.988)			(17.8)
$k_G(x100)$	72.0	.0127	.00602	-.0764	-.0910	.0728	-.297		.0437	4.59	-19.8	.999	1.73	.439
	(37.3)	(1.97)	(2.00)	(-8.78)	(-10.3)	(1.88)	(-10.9)		(4.01)	(5.54)	(-9.11)			(3.49)

Estimated for April 1973 - June 1977 with Cochrane-Orcutt correction for first-order serial correlations. All stock variables are measured in billions of dollars except MB_G, B_G and W_G, which are billions of marks. Y is measured in billions of dollars at an annual rate. Y_G is measured in billions of marks at an annual rate. $i(x100)$ and $i_G(x100)$ are measured as per cents per month. $k(x100)$ and $k_G(x100)$ are per cents. $DW^{\$}_{ROW}$ represents the cumulative change in $W^{\$}_{ROW}$ from its base-period value for March 1973. Numbers in parentheses are t-values.

i_G is pushed up by an open market operation that increases MB_G (holding constant MB_G+B_G); and (b) that exchange-market intervention to increase B (sell dollar bonds) by reducing B_G (buying mark bonds) has the effect of pushing i down and i_G up.

Although none of the estimated parameters with wrong signs is very significant, except perhaps for the coefficient of W in the i regression, our failure to estimate significant parameters with correct signs precludes the use of our model for estimating policy impacts. This underscores the importance of following Artus (1976) in embedding the exchange rate in a more adequate model of the monetary sector.

It should be emphasized that the shortcomings of our first-stage regressions do not affect the rationale for using first-stage fitted values as second-stage instruments for i, i_G, k and k_G. Instruments of some sort are desirable to avoid inconsistent second-stage estimates, and we have extended our list of first-stage regressors in hopes of increasing the efficiency of our instruments relative to what would emerge from an unmodified two-stage least squares procedure. The use of actual values instead of instruments for i, i_G, k and k_G could further increase the efficiency of the second-stage estimates, but in view of the high $\bar{R}^2$ measures associated with our first-stage results the gain in efficiency would not be sufficient to warrant (under our subjective preferences) the inconsistency that would be implied by using the actual values of i, i_G, k and k_G. All this is not to deny, however, that a more complete model of the monetary sector might result in more efficient instruments than those provided by the first-stage results reported here.

9. The Second-Stage Results

The second-stage problem is to choose the a_j and b_j parameters (for j=1,...,4) to minimize a sum of squared errors defined as

$$(19) \quad \sum_t \text{error}_t^2 = \sum_t (x_t - \text{RHS}_t(16))^2$$

where x_t is the exchange rate observed at the end of month t and $\text{RHS}_t(16)$ is the value at the end of month t of the right-hand side of equation (16), after substituting (15) for r^e and first-stage fitted values (instrumental variables) for i, i_G, k and k_G.

We have the following priors about the a_j parameters -- i.e., about the portfolio shares that market participants would choose if they perceived a zero expected differential yield between dollar-denominated and foreign-currency denominated bonds (i.e., when r^e=0):

(20a) a_1 = 0 to .3	i.e., private U.S. residents would denominate 70 to 100 per cent of their interest-bearing portfolios in dollars
(20b) a_2 = 0 to .3	i.e., private German residents would denominate 0 to 30 per cent of their interest-bearing portfolios in dollars
(20c) a_3 = .5 to .8	i.e., residents of OPEC would denominate 50 to 80 per cent of their portfolios in dollars
(20d) a_4 = .15 to .45	i.e., residents of ROW would denominate 15 to 45 per cent of their portfolios in dollars

The b_j parameters describe how these shares change as r^e moves away from zero. Our priors are that all four groups of wealth holders change their portfolio shares in the same proportion as r^e changes

$$(20e)\ b_j = va_j \qquad \text{for } j=1,\ldots,4$$

where

$$(20f)\ v = 0 \text{ to } 1$$ [13]

In the results reported below we impose (20e) as a constraint and estimate the five parameters a_1, a_2, a_3, a_4 and v.[14]

13/ Conditions (9b), (11), (12) and (13) assume that portfolio shares vary in proportion to $\sqrt[3]{r^e}$ rather than r^e. Ex post differential yields during our sample period frequently turned out to be .02 to .04 per month (in absolute value), or to have cube roots in the range .27 to .34. To the extent that differential yields were expected to be in this range ex ante, our priors are that wealth holders would not have chosen dollar holdings that differed by more than 27 to 34 per cent from the levels they would desire at an expected differential yield of zero. Hence we expect $v \leq 1$.

14/ The initial value of $W^{\$}_{ROW}$ is also estimated (recall section 5) to equal that value for which the numerator of the right-hand side of condition (15) has a zero mean during the sample period. (The estimate of this initial value is adjusted after each iteration to be consistent with changing estimates of the a_j parameters.) This procedure is almost equivalent to assuming that r^e has a zero mean during the sample period, although it ignores an upward trend in the denominator of the right-hand side of condition (15).

The minimand in our least-squares problem is a nonlinear function of the parameters to be estimated, and when we give no weight to our priors our estimates are nonsense.[15/] Accordingly we adopt a modified Bayesian approach of asking the computer to minimize each of two alternative loss functions:

$$(21)\quad \sum_t (\text{error}_t)^2 + \begin{cases} 0 & \text{if } \bar{p}_j - m_j \leq p_j \leq \bar{p}_j + m_j \text{ for all } j \\ \infty & \text{otherwise} \end{cases}$$

and

$$(22)\quad \sum_{t=1}^{T} (\text{error}_t)^2 + gT/5 \sum_{j=1}^{5} ((p_j - \bar{p}_j)/m_j)^2$$

Here the p_j represent the five parameters a_1, a_2, a_3, a_4, and v; the $\bar{p}_j$ represent our "point priors" for the p_j, which we set equal to the midpoints of the ranges (20a) - (20d) and (20f); the m_j in the "flat priors" case with minimand (21) define ranges for the p_j within which we attach no loss to whatever parameter estimates emerge, but outside of which we attach an infinite loss; and g is a prespecified positive weight which, under minimand (22), imposes a loss proportional to the sum of the squared percentage nomalized deviations between the estimated p_j and our point priors.[16/]

15/ Specifically, each of the five parameter estimates exceeds one million in absolute value for this case.

16/ Under minimand (22) the m_j are set equal to the half widths of the ranges (20a)-(20d) and (20f), thereby providing normalized measures of the deviations between the p_j and $\bar{p}_j$. Given the scale factor T/5 (the number of months divided by the number of parameters), an average absolute error of one cent per mark conveys the same loss as an average absolute normalized difference of $100/\sqrt{g}$ per cent between the estimated parameter values and our point priors.

For the case of flat priors, using minimand (21) and several alternative specifications of the m_j, our iterative procedure only converged to solutions consistent with rational expectations when we constrained the parameters within relatively narrow bounds. In these convergent cases the estimated parameters all took boundary values and reflected local minima that were inferior to the interior solutions generated for the case of pointed priors.[17/]

For the case of pointed priors, using minimand (22), the procedure converged for values of $g \geq 1$. Results for g=1, 36 and 10,000 are shown in Tables 2-4. (Our choice of these particular values, and their translation, will be discussed below.) The first column of Table 2 shows the observed path of the exchange rate between May 1973 and June 1977. Columns 2-4 show the corresponding one-month-ahead predictions that the model generates (with g=1,36 and 10,000 respectively) between April 1973 and May 1977 -- that is, the fitted values of the future spot rates that the model expected (one month ahead) to prevail between May 1973 and June 1977. It is noteworthy that fluctuations in the exchange rates that the model expects have lower amplitudes than fluctuations in observed

17/ We tried four alternative specifications of flat priors, in each case constraining v to lie between 0 and 1 and alternatively allowing each of the a_j to deviate from our point priors by an absolute value no greater than m=.05, .1, .15 or .3. The model converged to a boundary solution, but not a global interior minimum, for m=.05 and .1. The model failed to converge for m=.15 and .3.

Table 2: Observed and Predicted Exchange Rate Levels
(cents per mark)

	Observed Spot Rate	Model Predictions g=1	g=36	g=1000	Lagged Forward Rate	XWPPP	XCPPP	EXWPPP	EXCPPP
May	36.20	37.81	40.19	40.25	35.38	36.62	37.61	37.47	37.52
	41.15	39.23	40.92	40.34	36.34	37.18	37.61	38.03	37.57
	43.47	40.37	40.92	41.02	41.28	37.73	37.62	36.74	37.60
	40.63	41.20	40.90	40.99	43.54	36.93	37.61	40.22	38.86
1973	41.41	41.96	40.98	40.91	40.73	38.97	38.29	37.73	38.25
	41.19	42.64	41.09	41.00	41.63	38.25	38.37	37.11	38.19
	38.12	42.96	41.09	41.01	41.30	37.75	38.39	36.38	37.89
	37.01	43.00	40.97	40.90	37.98	37.46	38.19	39.41	38.13
	35.21	42.91	40.66	40.66	36.95	37.84	38.10	37.29	38.14
	37.62	42.76	40.18	40.28	35.15	38.00	38.18	38.07	38.49
	39.48	42.83	40.03	40.20	37.56	37.73	38.33	37.88	38.93
	40.46	42.87	39.89	40.11	39.47	37.72	38.64	37.87	38.46
	39.34	42.85	39.79	40.05	40.57	37.69	38.64	38.09	38.88
1974	39.25	42.66	39.63	39.92	39.44	37.90	38.83	37.75	39.16
	39.03	42.60	39.66	39.94	39.36	38.01	39.04	39.78	39.24
	37.56	42.44	39.76	39.98	39.20	39.03	39.24	41.38	39.87
	37.63	41.89	39.83	39.99	37.69	40.25	39.69	39.44	40.16
	38.86	41.44	40.02	40.08	37.71	40.08	40.04	40.31	39.99
	40.44	40.89	40.18	40.19	38.89	40.43	40.16	41.15	40.03
	41.36	40.17	40.32	40.31	40.52	40.80	40.22	40.26	40.40
	42.74	39.48	40.38	40.35	41.43	40.71	40.38	39.22	39.78
	43.91	39.06	40.39	40.36	42.73	40.29	40.19	40.31	40.29
	42.65	39.18	40.47	40.46	43.95	40.15	40.26	39.63	40.16
	41.95	39.00	40.53	40.57	42.72	39.96	40.23	40.39	39.98
1975	42.67	38.81	40.57	40.65	42.00	40.15	40.14	40.64	40.04
	42.52	38.70	40.61	40.73	42.71	40.38	40.07	40.57	40.18
	39.60	38.77	40.64	40.79	42.58	40.50	40.10	41.32	40.96
	38.74	38.82	40.68	40.86	39.66	40.93	40.53	41.34	40.72
	37.52	38.85	40.75	40.96	38.85	41.14	40.71	41.17	40.45
	39.10	38.79	40.79	41.01	37.61	41.25	40.70	41.58	40.91
	38.09	38.87	40.86	41.11	39.19	41.50	40.83	41.19	40.98
	38.30	38.87	40.89	41.15	38.18	41.36	40.96	41.24	40.87
	38.63	38.89	40.89	41.20	38.36	41.39	41.01	40.94	40.46
	38.93	38.97	40.97	41.28	38.68	41.19	40.78	40.86	40.45
	39.18	39.14	41.08	41.41	38.99	40.92	40.58	40.69	40.56
	39.43	39.24	41.15	41.48	39.25	40.82	40.54	41.13	40.37
1976	38.57	39.30	41.23	41.55	39.50	40.85	40.45	40.94	40.73
	38.80	39.35	41.34	41.62	38.65	40.82	40.54	41.10	40.77
	39.33	39.40	41.41	41.68	38.85	40.99	40.64	41.20	41.43
	39.61	39.44	41.47	41.71	39.38	41.06	41.06	40.68	40.92
	40.40	39.52	41.54	41.78	39.66	40.84	41.10	40.98	41.35
	41.67	39.53	41.57	41.81	40.44	40.97	41.27	41.22	41.36
	41.56	39.55	41.61	41.85	41.71	41.06	41.41	41.34	41.30
	42.37	39.51	41.63	41.84	41.56	41.23	41.44	41.66	41.15
	41.14	39.78	41.73	41.93	42.37	41.57	41.35	41.32	41.07
	41.77	40.09	41.81	41.98	41.15	41.42	41.21	42.10	41.62
1977	41.82	40.16	41.89	42.05	41.78	41.74	41.38	42.22	41.58
	42.48	40.45	41.98	42.11	41.82	42.04	41.49	42.84	41.64
	42.41	40.68	42.07	42.18	42.49	42.39	41.62	42.42	41.67
June	42.45	41.16	42.22	42.29	42.45	42.56	41.68	42.12	41.76

exchange rates. This relatively smooth nature of our particular description of expected exchange rates may be related to the time-series processes that we use to generate expected future values of our exogenous variables. To the extent that more realistic processes would generate more volatility in expected time paths of asset supplies and other exogenous variables, the expected time path of the exchange rate might also be more volatile.

Column 1 of Table 3 shows the month-to-month changes in the observed exchange rate, starting with the change between April and May 1973 and ending with the change between May and June 1977. Columns 2-4 show the corresponding changes expected by the model (under g=1,36 and 10,000 respectively) -- namely, the differences between columns 2-4 of Table 2 and the exchange-rate path observed between April 1973 and May 1977.

Table 3: Observed and Predicted Exchange Rate Changes

(cents per mark)

	Observed Change	Model Predictions g=1	g=36	g=1000	Forward Rate Change	XWPPP	XCPPP	EXWPPP	EXCPPP
May	.96	2.57	4.95	5.02	.15	1.38	2.37	2.23	2.28
	4.95	3.03	4.72	4.14	.14	.98	1.41	1.83	1.37
	2.32	-.78	-.23	-.13	.13	-3.42	-3.53	-4.41	-3.55
	-2.84	-2.27	-2.57	-2.48	.07	-6.54	-5.86	-3.25	-4.61
	.78	1.33	.35	.28	.10	-1.66	-2.34	-2.90	-2.38
1973	-.22	1.23	-.32	-.41	.22	-3.16	-3.04	-4.30	-3.22
	-3.07	1.77	-.10	-.18	.11	-3.44	-2.80	-4.81	-3.30
	-1.11	4.88	2.85	2.78	-.14	-.66	.07	1.29	.01
	-1.80	5.90	3.65	3.65	-.06	.83	1.09	.28	1.13
	2.41	7.55	4.97	5.07	-.06	2.79	2.97	2.86	3.28
	1.86	5.21	2.41	2.58	-.06	.11	.71	.26	1.31
	.98	3.39	.41	.63	-.01	-1.76	-.84	-1.61	-1.02
	-1.12	2.39	-.67	-.41	.11	-2.77	-1.82	-2.37	-1.58
1974	-.09	3.32	.29	.58	.10	-1.44	-.51	-1.59	-.18
	-.22	3.35	.41	.69	.11	-1.24	-.21	.53	-.01
	-1.47	3.41	.73	.95	.17	-.00	.21	2.35	.84
	.07	4.33	2.27	2.43	.13	2.69	2.13	1.88	2.60
	1.23	3.81	2.39	2.45	.08	2.45	2.41	2.68	2.36
	1.58	2.03	1.32	1.33	.03	1.57	1.30	2.29	1.17
	.92	-.27	-.12	-.13	.08	.36	-.22	-.18	-.04
	1.38	-1.88	-.98	-1.01	.07	-.65	-.98	-2.14	-1.58
	1.17	-3.68	-2.35	-2.38	-.01	-2.45	-2.55	-2.43	-2.45
	-1.26	-4.73	-3.44	-3.45	.04	-3.76	-3.65	-4.28	-3.75
	-.70	-3.65	-2.12	-2.08	.07	-2.69	-2.42	-2.26	-2.67
	.72	-3.14	-1.38	-1.30	.05	-1.80	-1.81	-1.31	-1.91
	-.15	-3.97	-2.06	-1.94	.04	-2.29	-2.60	-2.10	-2.49
1975	-2.92	-3.75	-1.88	-1.73	.06	-2.02	-2.42	-1.20	-1.56
	-.86	-.78	1.08	1.26	.06	1.33	.93	1.74	1.12
	-1.22	.11	2.01	2.22	.11	2.40	1.97	2.43	1.71
	1.58	1.27	3.27	3.49	.09	3.73	3.18	4.06	3.39
	-1.01	-.23	1.76	2.01	.09	2.40	1.73	2.09	1.88
	.21	.78	2.80	3.06	.09	3.27	2.87	3.15	2.78
	.33	.59	2.59	2.90	.06	3.09	2.71	2.64	2.16
	.30	.34	2.34	2.65	.05	2.56	2.15	2.23	1.82
	.25	.21	2.15	2.48	.06	1.99	1.65	1.76	1.63
	.25	.06	1.97	2.30	.07	1.64	1.36	1.95	1.19
1976	-.86	-.13	1.80	2.12	.07	1.42	1.02	1.51	1.30
	.23	.78	2.77	3.05	.08	2.25	1.97	2.53	2.20
	.53	.60	2.61	2.88	.05	2.19	1.84	2.40	2.63
	.28	.11	2.14	2.38	.05	1.73	1.73	1.35	1.59
	.79	-.09	1.93	2.17	.05	1.23	1.49	1.37	1.74
	1.27	-.87	1.17	1.41	.04	.57	.87	.82	.96
	-.11	-2.12	-.06	.18	.04	-.61	-.26	-.33	-.37
	.81	-2.05	.07	.28	-.00	-.33	-.12	.10	-.41
	-1.23	-2.59	-.64	-.44	-.00	-.80	-1.02	-1.05	-1.30
	.63	-1.05	.67	.84	.01	.28	.07	.96	.48
	.05	-1.61	.12	.28	.01	-.03	-.39	.45	-.19
1977	.66	-1.37	.16	.29	-.00	.22	-.33	1.02	-.18
	-.07	-1.80	-.41	-.30	.01	-.09	-.86	-.06	-.81
June	.05	-1.25	-.19	-.12	.04	.15	-.73	-.29	-.65

Columns 1-3 of Table 4 show the estimated values of the model parameters, the coefficients of correlation between the model's predictions and the actual observations of both exchange-rate levels and changes, and the percentages of exchange-rate changes whose direction (sign) the model predicts correctly. The results are quite insensitive to whether in the first iteration we specify the expected future spot rate as the forward rate or as the observed (perfect-foresight) future spot rate. (Tables 2-4 present results that correspond to the forward-rate setting.)

Recalling the discussion of minimand (22), it should be noted that under the values g=1,36 and 10,000 respectively, an average absolute difference of one cent per mark between observed and fitted exchange rates conveys the same loss as average absolute normalized differences of 100 per cent, 16-2/3 per cent and 1 per cent between the estimated parameter values and our point priors. As g is reduced below 1 the estimated parameter values become more and more inconsistent with our priors and the iterative procedure tends to converge very slowly. The

Table 4: Parameter Estimates and Goodness-of-Fit Statistics

	point priors	Model g=1	Model g=36	Model g=10,000	Forward Rate	XWPPP	XCPPP	EXWPPP	EXCPPP
estimate of a_1	.15	-.05	.127	.14993					
" of a_2	.15	.40	.154	.15001					
" of a_3	.65	.65	.652	.65001					
" of a_4	.30	.66	.320	.30007					
" of v	.50	1.26	.518	.50006					
correlation with level of exchange rate		-.232	.353	.285	.761	.308	.309	.224	.279
correlation with change in exchange rate		.223	.414	.388	-.019	.357	.372	.287	.342
per cent of changes predicted with correct sign		58	70	68	54	74	64	68	64

value g=10,000 essentially imposes our point priors on the model. The value g=36 produced the best goodness-of-fit statistics out of a half-dozen specifications on a grid between 1 and 100. The parameter estimates, goodness-of-fit statistics and fitted exchange-rate paths varied smoothly as g was moved over this grid.

Table 4 indicates that our model does not yield high correlation coefficients by conventional standards. Nevertheless our results are slightly better than the predictions of several other popular models. One-month-ahead predictions based on 30-day forward rates are more highly correlated than our model's predictions with observed exchange rate levels. But the forward rate does miserably in predicting exchange rate changes, as indicated (a) by the goodness-of-fit statistics, (b) by infering from Table 3 that the forward rate predicts only small changes and therefore is always surprised by large changes, and (c) by discerning from Table 2 that the forward rate misses all the turning points.

The last four columns of Tables 2-4 refer to the exchange rate predictions consistent with alternative views of purchasing-power parity (PPP). XWPPP is a ratio of wholesale price indexes, scaled to be as favorable as possible to PPP, and prevailing one-month in advance of the date for which the exchange-rate is being predicted; XCPPP is a similar ratio of consumer price indexes. EXWPPP is a one-month-ahead forecast based on a simple time-series regression of XWPPP on six-lagged values of itself (representing one to six-month lags); and EXCPPP is a similar forecast based on the time-series behavior of XCPPP. Thus EXWPPP and EXCPPP are constructed by the same procedure that we use to generate expected future values of the exogenous variables in our model.

Table 4 reveals that our model predicts only slightly better than the PPP models of equilibrium exchange-rate paths. Thus our particular empirical results should not be applauded too loudly, since PPP has been discredited as a hypothesis about the short-run behavior of exchange rates. We are quite encouraged, however, by the fact that we have found a procedure that is capable of providing estimates of the exchange-rate path consistent with both portfolio equilibrium and rational expectations. And we are hopeful that several refinements of our model will lead both to better second-stage goodness-of-fit statistics and to first-stage results that allow significant estimates of the impacts that various policy changes have on the path of exchange rates.

10. Conclusions

Our empirical results have demonstrated that exchange-rate behavior conforms suitably to the predictions of a portfolio-balance model with rational expectations. The retention of wealth variables in our empirical specification, the incorporation of rational expectations, and an explicit allowance for risk aversion represent important features that distinguish our exchange-rate model from others cited in this paper.

Our results nevertheless suggest several directions in which it would be interesting to extend the model before applying it to forecasting. The shortcomings of our first-stage results suggest the desirability of extending the model to include at least a small-scale specification of the monetary sector, following the spirit of Artus (1976), since the model in its present state cannot provide sensible estimates of the impacts that policy actions have on interest rates and exchange rates. To the extent that policy-reaction functions are believed to be systematic or well-defined, they could in theory be easily incorporated into the model.

It would also be particularly interesting to explore more sophisticated specifications of the processes that are assumed to generate expectations of future asset stocks, wealths and incomes. Without a full model of the entire economy we cannot generate these expectations rationally, but by basing them on more relevant information than simple time-series behavior we may be able to attribute more of the variance in observed exchange rates to the variance in the exchange-rate levels consistent with portfolio equilibrium.

11. Data Appendix

Exchange rates and interest rates are observed on the last Friday of each month (and for holiday Fridays, on the last previous day that markets were open). Spot exchange rates represent noon buying rates in New York, from Federal Reserve data.. Interest rates are 1-month Eurodollar and Euromark bid rates in London as reported by Reuters (through September 1976) and various issues of Money Manager (beginning in October 1976). To avoid possible inconsistencies resulting from differences of several hours in the times that spot exchange rates and interest rates are observed, forward exchange rates are constructed to satisfy the interest-rate parity condition. Given that our interest rates reflect Eurocurrency yields, the legitimacy of this procedure is well established.[18]

End-of-month data on U.S. base money are from the Federal Reserve Board data bank, seasonally adjusted and also adjusted for reserve requirements. Data on German base money, seasonally adjusted, are from Bundesbank publications and are also adjusted for reserve requirements. Monthly budget deficits are measured as changes in public borrowings by the U.S. and German Federal Governments, from the Federal Reserve data bank and Bundesbank publications. We seasonally adjusted these deficits ourselves using the Census X-11 program.

18/ See Herring and Marston (1976).

Monthly data on U.S. and German current-account balances (in dollars and marks, respectively) are constructed from seasonally-adjusted quarterly current-account data by starting with seasonally-adjusted monthly data on merchandise trade balances and adding to each monthly trade balance one-third of the difference between the current-account and trade balances for the corresponding quarter. Monthly data on OPEC's current-account balances (in dollars) are based on internal Federal Reserve Board estimates (as of January 1978) of OPEC's annual current-account surpluses between 1973 and 1977; we assumed that each of these estimated annual surpluses was uniformly distributed over the months in the corresponding year. Monthly data on the current-account balances of ROW (in dollars) are constructed to be equal and opposite-in-sign to the sum of the estimated current-account balances of the United States, Germany and OPEC, after converting the German data into dollars at end-of-month exchange rates.

The construction of wealth variables is largely described in section 5 and footnote 14 of the text. The initial value of U.S. wealth (as of end-of-February 1973) is specified as $422.35 billion, which equals net Federal government debt (other than to the Federal Reserve System) plus total liabilities of the Federal Reserve System minus net U.S. liabilities to foreigners. The initial value of net Federal debt, $387.49 billion, is also assumed to represent the initial supply of outside dollar-denominated bonds. (Sources are the Federal Reserve Board's Annual Statistical Digest: 1971-1975 and the April 1973 Federal Reserve Bulletin.)

We estimated the initial value of German wealth (as of end-of-February 1973) to be DM 205.83 billion, reflecting DM 147.36 billion of net Federal government debt (based on published Bundesbank statistics) and DM 82.9 billion of interest-bearing claims on foreigners (based on data from the Bundesbank Monthly Report for November 1974). We equated the initial value of the stock of outside mark-denominated interest-bearing debt to DM 48.36 billion, the net Federal debt minus the initial German monetary base.

Monthly data on U.S. and German income levels (in dollars and marks respectively) are based on quarterly seasonally-adjusted GNP data. The middle month of each quarter was assumed to have the same GNP (at an annual rate) as the quarter as a whole, and GNP levels for other months were based on linear interpolations between the mid-quarter months.

Our purchasing-power parity indexes are based on the U.S. wholesale price index for all commodities, the German producers price index for industrial production for the home market (excluding the tax on value added), the U.S. consumer price index, and the German CPI cost of living index. These price data are not seasonally adjusted.

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