International Finance Discussion Paper

Number 153

January 1980

DYNAMIC FACTOR DEMAND SCHEDULES FOR LABOR AND CAPITAL UNDER RATIONAL EXPECTATIONS

by

Richard Meese

Dynamic Factor Demand Schedules for Labor and Capital Under Rational Expectations

By Richard Meese*

Dynamic factor demand schedules for labor and capital are derived assuming a representative firm maximizes the present value of expected profit when factor inputs are subject to increasing marginal adjustment costs. The analysis is an extension of Sargent's (1978) one factor model of labor demand. The derived factor demand equations are jointly estimated with the autoregressive processes for the real wage and the rental price of capital, and the cross equation restrictions implied by the rational expectations hypothesis are imposed. Both the order of the factor price autoregressions and the structural parameters of the model are estimated. A hypothesis test of the overidentifying restrictions results in marginal rejection of the theoretical restrictions implied by the rational expectations hypothesis.

I. Introduction

The derivation and estimation of dynamic factor demand schedules for aggregate U.S. manufacturing has a rich history.[1/] This literature is based on the supposition that a representative firm maximizes the present value of its profit stream when factor inputs are subject to increasing marginal adjustment costs.[2/] Adjustment costs are non-linear costs which arise when firms change endogenous variables over time. Many authors (Brechling (1975), Gould (1968), Kennan (1979), and Nerlove (1972) have recognized that without adjustment costs expectations of future exogenous variables are generally irrelevant. If a firm can adjust instantaneously and without cost to changes in market conditions, it has no need to predict future market conditions. It is the interdependence of adjustment costs and expectation formulation which dichotomizes the existing work on factor demand schedules derived from a multiperiod theory of the firm with adjustment costs. To derive factor demand equations which are linear in the variables of the model, and are characterized by time invariant parameters, researchers have pursued two different strategies. The first and more conventional strategy is to assume static expectations on the course of future exogenous variables and derive the firm's factor demand schedules using general production and cost of adjustment functions. Berndt, Waverman, and Fuss (1977) provide the most sophisticated example of this approach. The second strategy is to model the firm's expectations of future exogenous variables while assuming simplistic (linear-quadratic) production and cost of adjustment functions. The second strategy has been employed in recent work by Kennan (1979) and Sargent (1978). These authors combine the rational expectations hypothesis with the adjustment cost literature. The advantage of this approach is that by formulating and estimating models which discriminate between effects of structural parameters of the objective functional and the constraints, and the effects of parameters describing the evolution of exogenous variables, these authors are not subject to Lucas' (1975) critique of ad hoc estimation.

In this paper we pursue the second strategy and contribute to the literature by expanding Sargent's (1978) one factor model of labor demand to include capital. We allow for interaction terms between labor and capital in quadratic production and cost of adjustment functions. Unlike Sargent's (1978) original one factor analysis, the two factor model is marginally rejected by the data. Several aspects of this result are worth noting. Factor demand equations with time invariant parameters derived from a stochastic maximum problem are necessarily the product of an overly simplistic model of firm behavior. Hence it is not surprising that such a model finds little support from aggregate U.S. manufacturing data. It has been a frustration of empirical macro-economists that aggregate U.S. time series do not contain sufficient information to distinguish between competing hypothesis of economic behavior (Sims (1977), p. 26). The theoretical restrictions of the rational expectations model derived in this paper are not strongly supported by the data; the marginal confidence level for a test of the theoretical restrictions is .98005.[3/]

Although we can formulate and estimate models which are immune to Lucas' critique of ad hoc estimation and which imply empirically refutable behavioral hypotheses on the part of economic agents, such models are difficult to decisively reject (or accept) given the information content of aggregate U.S. time series data.

The structure of this paper is as follows. The two factor model is derived in section two. This model requires that the stocks of capital, k(t), and production workers, n(t), and the rental price of capital, c(t), do not Granger cause real wages, w(t), and that (n(t),k(t),w(t)) do not Granger cause c(t).[4] A formal test of this hypothesis is carried out in appendix two using a multivariate Granger test (1969) proposed by Geweke (1978).[5] In section three we report estimates of the multiple equation system derived in section two and test certain theoretical restrictions. For the unrestricted version of the model we estimate the order as well as the coefficients of the vector autoregression in the real wage and rental price of capital. Concluding remarks are found in section four.

2. A derivation of dynamic factor demand equations for labor and capital.

In this section we derive estimable factor demand equations for labor and capital assuming a representative firm maximizes its real present value, faces increasing marginal costs of adjusting factor inputs, and has knowledge of the stochastic processes generating the exogenous variables. Following Sargent (1978) we assume that a representative firm faces the quadratic production function[6/], [7/]

$$\begin{aligned} f(n(t), k(t)) = (f_o + a_1(t))n(t) - \frac{f_1}{2} n(t)^2 + \\ (f_2 + a_2(t))k(t) - \frac{f_3}{2} k(t)^2 + f_4 n(t)k(t), \end{aligned} \tag{1}$$

where n(t) and k(t) represent the stocks of production workers and capital respectively, $a_1(t)$ and $a_2(t)$ are exogenous stochastic processes affecting the productivity of each factor input, and f_o, f_1, f_2, f_3, and f_4 are firm-specific parameters. The firm is also assumed to face costs of adjusting employment and capital,

$$\begin{aligned} \frac{d}{2} (n(t) - n(t-1))^2 + \frac{e}{2} (k(t) - k(t-1))^2 \\ +h(n(t) - n(t-1))(k(t) - k(t-1)), \end{aligned} \tag{2}$$

where d, e, and h are firm specific parameters. We place restrictions on the parameter values f_o, f_1, f_2, f_3, f_4, d, e, and h below.

Last, the firm is assumed to face a stochastic process for the real wage $w(t)$ which is not Granger caused by $(c(t), k(t), n(t))$, and a stochastic process for the rental price of capital $c(t)$ which is not Granger caused by $(w(t), k(t), n(t))$, an assumption tested and accepted in appendix two. The firm's wage bill is $w(t) \cdot n(t)$ and its costs of capital services $c(t) \cdot k(t)$.

The firm chooses contingency plans for $n(t)$ and $k(t)$ to maximize real present value,[8/]

$$pv(t) = E_t \sum_{j=0}^{\infty} b^j [(f_0 + a_1(t+j) - w(t+j))n(t+j) - \frac{f_1}{2} n(t+j)^2 - \frac{d}{2} (n(t+j) - n(t+j-1))^2 \tag{3}$$

$$+ (f_2 + a_2(t+j) - c(t+j))k(t+j) - \frac{f_3}{2} k(t+j)^2$$

$$- \frac{e}{2} (k(t+j) - k(t+j-1))^2 + f_4 n(t+j)k(t+j) -$$

$$h(n(t+j) - n(t+j-1))(k(t+j) - k(t+j-1)],$$

where n(t-1), k(t-1) and the stochastic processes for w, c, a_1, and a_2 are available to the firm at time t. The parameter b is a real discount factor, $0 < b < 1$. The expectation operator E_t is defined by $E_t(x) = E(x \mid Q(t))$ where x is a random variable and Q(t) is the information set available to the firm at time t. We assume that Q(t) = $\{n(t-1), k(t-1), a_1(t), a_1(t-1), \ldots, a_2(t), a_2(t-1), \ldots, w(t), w(t-1), \ldots, c(t), c(t-1), \ldots\}$. The firm maximizes (3) by choosing contingency plans for n and k that are functions of the information set. To insure that there exists a bounded solution to the dynamic optimization problem we require that the stochastic process $\underline{x}(t) = (a_1(t), a_2(t), w(t), c(t))'$ satisfy

$$\lim_{j\to\infty} b^j E_t(\underline{x}(t+j)\ \underline{x}(t+j)') = 0. \tag{4}$$

In particular we assume that the stochastic processes for w(t), c(t), $a_1(t)$ and $a_2(t)$ have the following form;

$$\begin{aligned} &(a)\quad a_1(t) = \epsilon_1(t) \\ &(b)\quad a_2(t) = \epsilon_2(t) \end{aligned} \tag{5}$$

$$\text{(c)}\quad w(t) = \gamma_0 + \sum_{j=1}^{\ell} \gamma_j w(t-j) + \epsilon_3(t)$$

$$\text{(d)}\quad c(t) = \theta_0 + \sum_{j=1}^{\ell} \theta_j c(t-j) + \epsilon_4(t),$$

where $\underline{\epsilon}(t) = (\epsilon_1(t), \epsilon_2(t), \epsilon_3(t), \epsilon_4(t))'$ is a 4-variate normal vector with $E_t(\underline{\epsilon}(t+j)) = \underline{0}$ and $E_t(\underline{\epsilon}(t+j)\ \underline{\epsilon}(t+j)') = \Sigma$ for all $j \geq 0$. The zeros of $1-\sum_{j=1}^{\ell} \gamma_j Q^j = 0$ (Q complex) and $1-\sum_{j=1}^{\ell} \theta_j Q^j = 0$ lie outside the unit circle.

The maximization problem (3) falls into a general class of optimization problems that have been studied by engineers.[9/] This general optimization problem for a finite time horizon T can be stated as

$$\max_{\underline{v}(t),\ldots,\underline{v}(t+T-1)} E_t \sum_{j=0}^{T} b^j[\underline{z}(t+j)'\ \Gamma\ \underline{z}(t+j) + \underline{v}(t+j)'\ \pi \underline{v}(t+j)], \tag{6}$$

subject to $\underline{z}(t+j+1) = \psi \underline{z}(t+j) + \Delta\underline{v}(t+j) + \underline{u}(t+j+1)$, where (7)

the $\underline{u}(t+j) \sim N(\underline{0}, \Omega)$ and are independent for all $0 \leq j \leq T$. (8)

Equation (6) is the firm's present value functional, equation (7) is the state equation which describes the evolution of the system in terms of the previous value of the (nxl) state vector $\underline{z}$, the (mxl) vector of control variables $\underline{v}$, and the normally distributed random process $\underline{u}$. The maximization problem (3) can be fit into the general framework (6-8) by defining $\underline{z}$, $\underline{v}$, $\underline{u}$, Γ, π, ψ, and Δ in the following manner:

(a) $\underset{(1\times(7+2\ell))}{\underline{z}(t+1)'} = (n(t),\ k(t),\ a_1(t+1),\ a_1(t),\ a_2(t+1),\ a_2(t),\ 1,\ w(t+1),\ldots,$

$w(t-\ell+2),\ c(t+1),\ldots,\ c(t-\ell+2)),$

(b) $\underset{(1\times 2)}{\underline{v}(t)'} = (n(t)-n(t-1),\ k(t) - k(t-1)),$ (9)

(c) $\underset{(1\times(7+2\ell))}{\underline{u}(t+1)'} = (0,\ 0,\ \epsilon_1(t+1),\ 0,\ \epsilon_2(t+1),\ 0,\ 0,\ \epsilon_3(t+1),\ 0,\ldots,0,\ \epsilon_4(t+1),\ 0,$

$\ldots,0),$

(d) $$\underset{(7+2\ell)\times(7+2\ell)}{\psi} = \left[\begin{array}{ccccccc|cccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & & \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & & & 0 & & & \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & & \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & & & & & & \\
\hline
 & & & & & & & \gamma_0 & \gamma_1 & \cdot & \cdot & \cdot & \gamma_\ell \\
 & & & & & & & 0 & 1 & 0 & \cdot & \cdot & 0 \\
 & & & 0 & & & & 0 & 0 & \cdot & \cdot & \cdot 1 & 0 \\
 & & & & & & & 0_\theta & \theta_1 & \cdot & \cdot & \cdot & \theta_\ell \\
 & & & & & & & 0 & 1 & 0 & \cdot & \cdot & 0 \\
 & & & & & & & 0 & 0 & 1 & \cdot & \cdot & 0 \\
 & & & & & & & 0 & 0 & \cdot & \cdot & \cdot 1 & 0
\end{array}\right],$$

(e) $\underset{((7+2\ell)\times 2)}{\Delta} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \multicolumn{2}{c}{0} \end{bmatrix}$, (f) $\underset{(2\times 2)}{\pi} = -\frac{1}{2}\begin{bmatrix} d & h \\ h & e \end{bmatrix}$, and

(g) $$\underset{(7+2\ell)\times(7+2\ell)}{\Gamma} = \frac{1}{b}\begin{bmatrix}
-f_{1/2} & & & & & & & \\
f_{4/2} & -f_{3/2} & & & & & & \\
0 & 0 & 0 & & & & & \\
1/2 & 0 & 0 & 0 & & & & \\
0 & 0 & 0 & 0 & 0 & & & \\
0 & 1/2 & 0 & 0 & 0 & 0 & & \\
f_{0/2} & f_{2/2} & 0 & \cdots & & 0 & & \\
0 & 0 & 0 & \cdots & & & \ddots & \\
-1/2 & 0 & 0 & \cdots & & & & \\
0 & 0 & 0 & \cdots & & & & \\
\vdots & \vdots & \vdots & & & & & \\
0 & 0 & 0 & \cdots & & & & \\
0 & -1/2 & 0 & \cdots & & & & \\
0 & 0 & 0 & \cdots & & & & \\
\vdots & \vdots & \vdots & & & & & \\
0 & 0 & 0 & \cdots & & & & 0
\end{bmatrix},$$

where $\Gamma' = \Gamma$. We shall require π to be negative definite, a condition equivalent to the restrictions $d > 0$ and $de > h^2$.

Define the maximum expected present value for a T period problem as

$$V_T(\underline{z}(t)) \equiv \max_{(\underline{v}(t),\ldots,\underline{v}(t+T-1))} (E_t \sum_{j=0}^{T} b^j[\underline{z}(t+j)' \Gamma \underline{z}(t+j) + \underline{v}(t+j)' \pi \underline{v}(t+j)]). \tag{10}$$

By the principle of optimality[10] $V_{T+1}(\underline{z}(t))$ satisfies

$$V_{T+1}(\underline{z}(t)) = \max_{\underline{v}(t)} E_t[bV_T(\underline{z}(t+1)) + \underline{z}(t)' \Gamma \underline{z}(t) + \underline{v}(t)' \pi \underline{v}(t)]. \tag{11}$$

and by using an inductive procedure[11] $V_T(\underline{z}(t))$ can be shown to take the form

$$V_T(\underline{z}(t)) = \underline{z}'(t)\Phi_T\underline{z}(t) + \zeta_T, \text{ where} \tag{12}$$

$\Phi_t = 0$, $\zeta_t = 0$, and the Φ_j and ζ_j, $t < j \leq T + t$, are recursively determined by

$$\text{(a)} \quad \Phi_{j+1} = b\psi'\Phi_j\psi + \Gamma - b^2\psi'\Phi_j\Delta(b\Delta'\Phi_j\Delta + \pi)^{-1} \Delta'\Phi_j\psi, \text{ and} \tag{13}$$

$$\text{(b)} \quad \zeta_{j+1} = b\zeta_j + b \text{ trace } (\Omega\Phi_j).$$

The optimal control in period t is the $\underline{v}(t)$ maximizing (11),

$$\underline{v}(t) = - b(b\Delta'\Phi_T\Delta + \pi)^{-1}\Delta\Phi_T\psi\underline{z}(t). \tag{14}$$

Equation (13a) is known as the matrix Ricatti equation.

Given that the stochastic processes for $a_1(t)$, $a_2(t)$, $c(t)$, and $w(t)$ satisfy assumption (4), and given appropriate restrictions on ψ and Δ it can be shown that iterations on the matrix Ricatti equation (13a) converge, and the solution to the infinite time horizon problem is given by

$$\begin{aligned}
&\text{(a)} \quad \Phi_\infty = b\psi' \Phi_\infty \psi + \Gamma - b^2 \psi' \Phi_\infty \Delta (b\Delta' \Phi_\infty \Delta + \pi)^{-1} \Delta' \Phi_\infty \psi, \\
&\text{(b)} \quad \zeta_\infty = b \text{ trace } (\Omega \Phi_\infty)/(1-b), \text{ and} \\
&\text{(c)} \quad \underline{v}(t) = - b(b\Delta' \Phi_\infty \Delta + \pi)^{-1} \Delta' \Phi_\infty \psi \underline{z}(t).
\end{aligned} \tag{15}$$

To simplify notation, rewrite the firm's decision rule (15c) as

$$\underline{v}(t) = P\underline{z}(t), \text{ where } P = \{\rho_{i,j}\} = - b(b\Delta' \Phi_\infty \Delta + \pi)^{-1} \Delta' \Phi_\infty \psi \tag{16}$$

or

$$\begin{aligned}
&\text{(a)} \quad n(t) = (1 + \rho_{1,1})n(t-1) + \rho_{1,2}k(t-1) + \rho_{1,7} + \rho_{1,8}w(t) + \ldots \\
&\qquad + \rho_{1,7+\ell}w(t-\ell+1) + \rho_{1,8+\ell}c(t) + \ldots + \rho_{1,7+2\ell}c(t-\ell+1) + \\
&\qquad \rho_{1,3}\,\epsilon_1(t) + \rho_{1,5}\epsilon_2(t), \text{ and} \\
&\text{(b)} \quad k(t) = \rho_{2,1}n(t-1) + (1 + \rho_{2,2})k(t-1) + \rho_{2,7} + \rho_{2,8}w(t) + \ldots \\
&\qquad + \rho_{2,7+\ell}w(t-\ell+1) + \rho_{1,8+\ell}c(t) + \ldots + \rho_{1,7+2\ell}c(t-\ell+1) \\
&\qquad + \rho_{2,3}\,\epsilon_1(t) + \rho_{2,5}\,\epsilon_2(t).
\end{aligned} \tag{17}$$

Four characteristics of the decision rules (17a and b) are worth noting. First, n(t) and k(t) are linear functions of n(t-1), k(t-1), w(t-i), and c(t-i), $i=0,\ldots \ell-1$, but are highly nonlinear functions of the structural parameters in ψ, Δ, Γ, and π. Second, the order of the distributed lags in w(t) and c(t) is given by the specification of the autoregressive (AR) processes (5c and d). Since the assumption of ℓ-th order AR processes for the real wage and the rental price of capital is arbitrary, we estimate the order of these AR processes (along with the other parameters of the model) in section three below. Third, the one period lagged labor and capital terms in equations (17a and b) is a consequence of the specification of the costs of adjustment function (2). Last, the rational expectations approach to deriving dynamic decision rules for the firm results in estimating equations which closely resemble the more traditional distributed lag formulations. The difference between the traditional and rational expectations approaches is the explicit modeling of the processes generating w(t) and c(t), and the imposition of the cross equation restrictions implied by the rational expectations hypothesis to achieve parameter identification.

We must perform two more operations on the two equation system (17) before we arrive at estimable decision rules for n(t) and k(t). First, since the model will be estimated from data that are deviations from means and trends,[12] we drop the constant terms from both equations. Second, we substitute expressions (5c and d) for the current period real wage and rental price of capital in equations (17). In addition to equations (17a and b) we will estimate the autoregressions for $w(t)$ and $c(t)$. The complete four equation system to be estimated has the following form:

$$
\begin{aligned}
&\text{(a)}\ n(t) = (1+\rho_{1,1})n(t-1) + \rho_{1,2}k(t-1) + (\rho_{1,8}\gamma_1 + \rho_{1,9})w(t-1) \\
&\qquad +\ldots+ \gamma_1\rho_{1,8}w(t-\ell) + (\rho_{1,8+\ell}\theta_1 + \rho_{1,9+\ell})c(t-1) \\
&\qquad +\ldots+\theta_1\rho_{1,8+\ell}c(t-\ell) + u_1(t), \\
&\text{(b)}\ k(t) = \rho_{2,1}n(t-1) + (1+\rho_{2,2})k(t-1) + (\rho_{2,8}\gamma_1 + \rho_{1,9})w(t-1) \\
&\qquad +\ldots+ \gamma_1\rho_{2,8}w(t-\ell) + (\rho_{2,8+\ell}\theta_1 + \rho_{1,9+\ell})c(t-1) \\
&\qquad +\ldots+ \theta_1\rho_{2,8+\ell}c(t-\ell) + u_2(t), \\
&\text{(c)}\ w(t) = \gamma_1 w(t-1) +\ldots+ \gamma_\ell w(t-\ell) + u_3(t), \qquad\qquad (18) \\
&\text{(d)}\ c(t) = \theta_1 c(t-1)+\ldots+ \theta_\ell c(t-\ell) + u_4(t), \text{ where}
\end{aligned}
$$

$$\{\rho_{i,j}\} = P = -b(b\Delta'\Phi_\infty\Delta + \pi)^{-1}\Delta'\Phi_\infty\psi \text{ and}$$

$$\underline{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \end{bmatrix} = \begin{bmatrix} \rho_{1,3}\epsilon_1(t) + \rho_{1,5}\epsilon_2(t) + \rho_{1,8}\epsilon_3(t) + \rho_{1,8+\ell}\epsilon_4(t) \\ \rho_{2,3}\epsilon_1(t) + \rho_{2,5}\epsilon_2(t) + \rho_{2,8}\epsilon_3(t) + \rho_{2,8+\ell}\epsilon_4(t) \\ \epsilon_3(t) \\ \epsilon_4(t) \end{bmatrix}. \tag{19}$$

Since $\underline{\epsilon}(t)$ is distributed as a 4-variate normal (page 7). $\underline{u}(t)$ has a 4-variate normal distribution with zero mean vector and a covariance matrix V that depends on Σ and P. There are $(4+6\ell)$ regressors in the system (18a, b, c, and d). The free parameters of the model consist of the set $\mu = \{f_1, f_3, f_4, d, e, h, \gamma_1, \ldots \gamma_\ell, \theta_1, \ldots, \theta_\ell\}$, so there are $(6+2\ell)$ parameters to be estimated. The model is over identified for all positive intergral ℓ.

3. Estimation of the four equation system (18) using quarterly U.S. manufacturing data 1947I - 1974IV.

We first obtain maximum likelihood (ML) estimates for the unconstrained version of the four equation system (18). Let $\underline{\tau}$ be the $(4 + 6\ell) \times 1$ vector of regression coefficients of the "stacked" multivariate system (18). The log likelihood function of a sample of observations $t = 1, \ldots, T$ is

$$\ell n\, L(\underline{\tau}) = -\frac{1}{2}(4T)\, \ell n\, 2\pi - \frac{T}{2}\, \ell n \mid V \mid - \frac{1}{2} \sum_{t=1}^{T} \underline{u}(t)' V^{-1} \underline{u}(t). \tag{20}$$

In Table 1 we report the ML estimate of $\underline{\tau}$, the ML estimate of the covariance matrix, $\hat{V}$, and the log of the likelihood function (20). Since ℓ, the length of the autoregression in w(t) and c(t) is unknown, we use the multivariate CAT criterion (Parzen (1975)) to estimate its value.[13/]

For a 4 equation system, the multivariate CAT criterion selects an order ℓ^* which minimizes

$$\mathrm{CAT}(\ell) = \mathrm{trace}\left(\frac{4}{T} \sum_{j=1}^{\ell} \hat{\hat{V}}_j^{-1} - \hat{\hat{V}}_\ell^{-1}\right) , \quad \ell=1,\ldots, 5. \qquad (21)$$

The estimated covariance matrices $\hat{\hat{V}}_\ell^{-1}$, $\ell=1,\ldots,5$ are equal to the maximum likelihood estimates $\hat{V}_\ell^{-1}$ with the appropriate corrections for degrees of freedom. To limit computational costs, the maximum value of ℓ was chosen be 5. The minimum value of CAT occurred for $\ell^* = 2$, and since the values of CAT(ℓ), ℓ = 3, 4, and 5 increase monotonically, there appears to have been no harm in restricting the analysis to autoregressions of order 1-5. A second order autoregression also corresponds to the maximum value of the log likelihood functions associated with the five different versions of the unconstrained model (18), $\ell=1,\ldots, 5$. We report CAT(ℓ), $\ell=1,\ldots, 5$ and the log likelihood function associated with each model in Table 1 below.

The estimation of the constrained version of the model (18) is a difficult task. Few software routines are capable of estimating a model of such complexity, as the theoretical restrictions summarized in (16) are nonlinear implicit functions of the set μ of free parameters of the model. Usually, maximum likelihood software routines require nonlinearities to be explicitly representable as functions of the free parameters of the model.[14/] In our case when $\ell=2$, the 16 regression coefficients cannot be expressed as explicit functions of the set μ of free parameters. In order to estimate the model while imposing the theoretical restrictions, it was necessary to modify the likelihood function (20). It is well known that maximizing (20) is equivalent to minimizing the determinant of $\hat{V}$ with respect to $\underline{\Gamma}$, where

$$\hat{V} = \frac{1}{T} \sum_{t=1}^{T} \hat{\underline{u}}(t)\hat{\underline{u}}(t)', \tag{22}$$

and $\hat{\underline{u}}(t)$ is the sample (4x1) residual vector, (Bard (1974)). In order to incorporate the nonlinear constraints we append the concentrated likelihood function $|V|$ with a penalty function. The new function to be minimized has the form

$$\ln |V| + \sum_{i=1}^{21} \alpha_i^{-1/2} \omega_i^2, \tag{23}$$

where the ω_i are the 21 independent equality constraints implied by (15a),[15/] and the α_i are weights to be used during the iterative process. The α_i are set to an arbitrary starting value, $\alpha_i = .010$ for $i = 1, \ldots, 21$, and the function (23) is minimized using a numerical derivative version of a Davidon-Fletcher-Powell (DFP) algorithm. Each α_i is then divided by two and the function (23) is minimized again using the first round parameter estimates as starting values. The process continues until the appended log likelihood function (23) changes by less than .10. This convergence criterion is arbitrary and is a consequence of the fact that our efforts to reduce α_i below .00125 and minimize (23) resulted in a failure to the DFP algorithm to converge. We specified convergence of the DFP algorithm to be no change in the first 4 significant figures of each parameter estimate for two successive iterations. For the parameter estimates associated with the minimum value of (23) ($\alpha = .00125$) reported in table 2 below, the sum of the squared constraints, $\sum_{i=1}^{21} \omega_i^2$, was equal to .1366. Ideally, this value should be zero. Since the constraints do not hold exactly, our likelihood ratio test can be interpreted as a lower bound for a test statistic of the theoretical restrictions.

The likelihood ratio statistic for the test of the validity of the theoretical restrictions is equal to 15.04. Under the null hypothesis that the theoretical restrictions are valid, the likelihood ratio statistic is asymptotically distributed as a $\chi^2(6)$. The number of degress of freedom is equal to the number of parameters in the unconstrained model, 16, minus the number of free parameters in the constrained model, 10. The calculated value of the likelihood ratio statistic indicates rejection of the theoretical restrictions at any significance level greater than .01995. Since the theoretical restrictions are neither decisively rejected or accepted, it is useful to analyze the plausibility of the estimates of the free parameters of the model. Our second order condition for the optimization problem (3) is satisfied as π is negative definite. Also, the autoregressive processes for the real wage and rental price of capital are stable. Neither the second order conditions nor the stability conditions were imposed during the estimation prodedure.

The regression coefficients of the unconstrained model (Table 1, p. 20) and the implied regression coefficients of the constrained model (Table 2, p. 25) are similar. Total own price and cross price effects for both lags of the real wage and rental price of capital in the employment and capital equations are negative. We have not reported "z-ratios" of coefficient estimates to their asymptotic standard errors for Table 2, p. 23 as the standard errors are impkicit nonlinear functions of the estimated covariance matrix of the parameters of μ. The results from the unconstrained estimation of the vector autoregression (18) indicate that the real wage and employment exhibit more explanatory power (they have higher z ratios) in the equation describing capital, than the rental price of capital and the capital stock in the equation describing labor. The rental price of capital and the capital time series were constructed using the formulas described in appendix one, and these formulas are based upon restrictive assumptions. One of the least tenable assumptions is that the depreciation rate of capital for aggregate U.S. manufacturing was constant over the period 1947I - 1974IV. There are few quarterly capital and rental price of capital time series in existence; we followed the procedure used by Nadiri and Rosen (1973) to generate these series. If more sophistication were used in the generation of this data, the payoff might be more precise parameter estimates.

Last, using the estimates of the free parameters in μ we derive approximate estimates of the elasticity of substitution between capital and labor for the sample period 1947III - 1974IV.[16/] These values range from a low of .364 to a high of 3.87. The average value of the capital-labor elasticity of substitution for our sample period is 1.85, a value which is slightly larger than the Allen partial elasticities of substitution between labor and structures (range of 1.37 - 1.79) and between labor and equipment (range of 1.23 - 1.43) estimated by Berndt and Christensen (1973, p.98) for aggregate U.S. manufacturing 1929-68 using a three-input translog function, annual observations, and static theory. Although the explicit modeling of the firm's expectations of present and future factor prices necessitates our use of less flexible functional forms, we have not imposed a unitary capital-labor elasticity of substitution _a priori_. Some of the earlier attempts to estimate dynamic multivariate factor demand systems (Nadiri and Rosen (1973) and Coen and Hickman (1970)) relied on a Cobb-Douglas production technology, where the elasticity of substitution must be constrained to be unity.

Table 1

Results from the ML estimation of the autoregressive order ℓ of the unconstrained four equation system (18)

ℓ	CAT(ℓ)	Value of the log likelihood
1	-263,020	1550.55
2	-275,143	1572.94
3	-271,586	1560.76
4	-265,420	1549.93
5	-257,319	1538.79

Estimate of the covariance matrix of disturbances for ℓ = 2.

$$\hat{V} = \begin{bmatrix} .506\text{-}03 & .655\text{-}05 & .688\text{-}05 & .241\text{-}04 \\ & .107\text{-}04 & .222\text{-}05 & -.124.05 \\ & & .161\text{-}03 & -.116\text{-}05 \\ & & & .656\text{-}05 \end{bmatrix}$$

Table 1 continued

ML coefficient estimates for the unconstrained four equation system (18), $\ell = 2$.
(statistics in parentheses are ratios of coefficient estimates to their asymptotic standard errors[17/])

Equation	Lags	Real wage coefficient	Rental price of capital coefficient	Employment coefficient	Capital stock coefficient
(a)	1	1.32(1.03)	.276(.435)	.952(17.1)	-.043(-.639)
	2	-1.41(-1.16)	-.800(-1.13)		
(b)	1	-.899(-1.37)	-.098(-.276)	.106(2.98)	.952(22.0)
	2	.664(1.23)	-1.39(-.351)		
(d)	1		1.55(11.9)		
	2		- .587(-4.40)		
(c)	1	.824(6.32)			
	2	-.118(-1.12)			

Log of the likelihood function = 1572.94

Table 2

ML Coefficient Estimates of the Free Parameters of the Constrained Four Equation System (18). (Statistics in parentheses are ratios of coefficient estimates to asymptotic standard errors)

parameter	estimate	
d	6.585	(1.382)
e	.3103	(.4260)
h	.2217	(.9456)
f_1	.0164	(.1151)
f_3	.0011	(.0065)
f_4	.0029	(.0191)
γ_1	.7637	(5.537)
γ_2	-.0693	(-.5668)
θ_1	1.498	(11.54)
θ_2	-.4892	(-3.979)

Estimate of the covariance matrix of disturbances for the constrained system (18).

$$\hat{V} = \begin{bmatrix} .5364\text{-}03 & .8412\text{-}05 & -.9176\text{-}06 & .2365\text{-}04 \\ & .1112\text{-}04 & .2314\text{-}05 & -.1203\text{-}05 \\ & & .1640\text{-}03 & -.1410\text{-}05 \\ & & & .6583\text{-}05 \end{bmatrix}$$

The sum of the squares of the components of the gradient vector evaluated at the solution = 5.736-11.

Table 2 continued

Implied estimates of the regression coefficients of the constrained four equation system (18). [18/]

Equation	Lags	Real wage coefficient	Rental price of capital coefficient	Employment coefficient	Capital stock coefficient
(a)	1	-.2860	-.8221	.9665	.0049
	2	.0284	.3977		
(b)	1	-.3811	.1276	.0975	.9563
	2	.0388	-.3983		
(c)	1	.7637			
	2	-.0693			
(d)	1		1.498		
	2		- .4892		

Log of the likelihood function = 1565.42.

4. Conclusions

The derived decision rules for n(t) and k(t) (equations 18a and b) are complex and difficult to estimate. This is true despite the simple functional forms employed in the theoretical development of section two, and the restriction of the analysis to those firms which produce a single output. In section two above we assumed that a representative firm maximized real present value. This simplification would not be appropriate for firms producing multiple outputs unless a price index for those outputs was available. In a recent paper Berndt, Fuss, and Waverman (1977) examine more flexible functional forms for the production technology and input adjustment costs. These authors derive their estimating equations from a dynamic maximum problem so that lagged adjustment mechanisms are not tacked on to their model in an ad hoc fashion. Many authors, (Berndt, Fuss, and Waverman (1977), Nerlove (1972), Woodland (1977), and others), have criticized the extant body of empirical work on factor demand equations for its tenuous link with economic theory. Although the recent innovative work by Berndt, Fuss, and Waverman results in estimating equations which are both econometrically tractable and accurately characterize theoretical models, they have largely ignored multiple outputs and avoided the difficult problem of modeling expectations and the stochastic processes generating the exogenous variables.

In their recent theoretical paper "Linear Rational Expectations Models for Dynamically Interrelated Variables", written in part as a response to Lucas' critique of ad hoc estimation, Hansen and Sargent (1979b) have analyzed general models of which ours is a special case. They show how to obtain the decision rules of the firm as explicit functions of the structural parameters of the model. Given the estimation problems endemic to the methodology employed in this paper, the new Hansen Sargent methodology may prove to be the most useful approach to employ in future work.

*International Finance Division, Board of Governors of the Federal Reserve System. The views expressed herein are solely those of the author and do not necessarily reflect those of the Federal Reserve System. An abbreviated version of this paper is to appear in the 1980 issue of the Annals of Applied Econometrics, dedicated to ongoing research work in the Federal Reserve System. Computational assistance by Elizabeth Christensen and helpful comments from John Geweke, Dale Henderson and some anonymous referees are gratefully acknowledged.

1/ Theoretical and empirical work on dynamic factor demand schedules derived from a multiperiod theory of the firm with adjustment costs begins with Eisner and Strotz (1963), and is extended in various directions by Berndt, Waverman, and Fuss (1977), Brechling (1975), Coen and Hickman (1970), Crane (1971, 1975), Gould (1968), Kennan (1978), Lucas (1967), Nadiri and Rosen (1973), Sargent (1978a, 1979), Schramm (1970), Taylor (1970), Tinsley (1970, 1971), Treadway (1969, 1970, 1971), and Woodland (1977).

2/ The assumption of increasing (convex) marginal adjustment costs is not universally accepted, although it is required in the derivation of the flexible accelerator model of Eisner and Strotz (1963). If marginal adjustment costs are decreasing (concave) then "bang-bang" solutions may ensue, see Rothschild (1971) or Nerlove (1972) for a discussion of this point. Chenery (1952) was one of the first to note that economies of scale could lead to lumpy investment. Steve Peck (1974) explores this alternative investment model of the firm using micro data and contrasts it to a distributed lag or flexible accelerator model.

3/ Let T_o denote the value of the calculated test statistic and let $\phi(x)$ denote the cumulative density function of the test statistic assuming the null hypothesis is true. The marginal confidence level of To is equal to the Prob $(x \leq To) = \phi(T_O)$.

4/ A time series $\{x(t)\}$ Granger causes a time series $\{y(t)\}$ if present y can be predicted better by using past values of x than by not doing so, other relevant information (including the past of y) being used in either case.

5/ The reader of this paper may find the results of the Granger causality tests reported in appendix B to be theoretically unpalatable as they do not accord with accepted factor supply behavior. It is my own view that Granger causality tests are useful as early tests of specification error. The model in the text requires certain Granger causality relationships to hold between variables; these relationships are consistent with the data, but difficult to explain on economic grounds. As the model is partial equilibrium in nature, the results of our Granger causality tests can also be interpreted by appeal to partial equilibrium analysis. The rental price of capital and the real wage are clearly endogenous when attention is focused on the aggregate economy, but their failure to be Granger caused by the stocks of capital and production workers in aggregate U.S. manufacturing suggests the processes by which real factor prices are determined may depend upon the past values of other economic variables and/or other sectors of the economy.

6/ We restrict attention to quadratic production and cost of adjustment functions so that maximization of a firm's present value remains a tractable problem. Given a quadratic objective functional, a firm's optimal decision rules are linear in the variables of the model (but nonlinear in the structural parameters), and display the certainty equivalence property, Malinvaud (1969).

7/ The assumption of a representative firm is only a convenience as the solutions to the dynamic optimization problem for each firm can be aggregated to obtain industry wide factor demand equations for k(t) and n(t).

8/ Expression (3) in the text is the real present value of the firm. The choice of an objective functional for the firm is an unresolved issue. There is no general consensus on whether the firm maximizes profits, expected utility, or another criterion. Our choice of the firm's objective functional merits further comment. We shall asume that the firm buys capital, or secures a rental contract for capital that extends over the entire lifespan of the equipment. If this were not the case then the renter of capital this period, due to adjustment costs, could exercise monopoly power over the firm in the next period once the capital was in place.

9/ See Kushner (1971) Chapter 9.

10/ The principle of optimality states that "an optimal policy has the property that, whatever the initial state and decision (control) are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision," (Bellman (1957)).

11/ When T = t, there is no control as the system starts and stops at time t. Given $\Phi = 0$ and $\zeta_t = 0$, (12) holds for time t. Assume (12) holds for time T + t and substitute (12) and (7) into (11). Differentiating the result with respect to v(t) yields the optimal control (14). After substituting (14) back into the expression for $V_{T+1}(\underline{Z}(t))$, (12) and (13) are seen to hold for T+t+1.

12/ The analysis summarized in Tables 1 and 2 of the text was performed using the residuals from the regressions of n(t), k(t), w(t), and c(t) on a constant, trend and trend squared.

13/ Many model selection criteria can be used to select the finite unknown order of the autoregressive processes for w(t) and c(t). Geweke and Meese (1979) have shown that the asymptotic distribution of the model order selected is the same for the criteria of Akaike (1974), Amemiya (1976), Mallows (1974), Parzen (1974), and Sawa (1978). Furthermore, these criteria do not estimate autoregressive order consistently. Geweke and Meese (1979) also describe a class of model selection criteria which do estimate autoregressive order consistently. The Schwarz (1978) criterion is a member of this class. When a consistent estimator of lag length is used to select the order of the w(t) and c(t) processes, the results of Table 1 are unchanged.

14/ In a recent unpublished working paper Hansen and Sargent (1979b) discuss alternative estimation strategies for multiple factor models of which ours is a special case. In an appendix they show that it is possible to represent the regression coefficients as explicit functions of the free parameters of the model using frequency domain methods. We have not yet experimented with this new methodology.

15/ For example, let $\Phi_\infty = \{\varphi_{ij}\}$ where the indices i and j run from 1 to 6 when $\ell = 2$. Then from (15a) ω_1 has the following form:

$$\omega_1 = (\varphi_{11} - b\varphi_{11} - f_4/2b + b(b_1\varphi_{11}^2 + 2b_2\varphi_{11}\varphi_{12} + b_3\varphi_{12}^2), \text{ where}$$

$$b_1 = (-b/s)(b\varphi_{22} - e/2)$$

$$b_2 = (-b/s)(h/2 - b\varphi_{12})$$

$$b_3 = (-b/s)(b\varphi_{11} - d/2)$$

$$s = (b\varphi_{11} - d/2)(b\varphi_{22} - e/2) - (h/2 - b\varphi_{12})^2.$$

There are 21 constraints of this form as Φ_∞ is symmetric.

16/ The estimates of the capital-labor elasticity of substitution, $\sigma_{k,n}$, reported in the text are approximate since we did not estimate the production function parameters f_o and f_2. These parameters are part of the constant terms in equation (17) that were suppressed to simplify estimation. As f_o and f_2 are components of the expressions for the marginal products of n(t) and k(t) respectively, we approximated these components by the sample means of each variable when calculating $\sigma_{k,n}$.

17/ The asymptotic standard errors are calculated from the inverse of the information (Hessian) matrix evaluated at the final parameter estimates. The DFP minimization routine produces an estimate of this matrix which improves with the number of iterations required for convergence. We have not performed sufficient experiments with this algorithm to have confidence in the estimates of the final Hessian matrix. We did estimate the unconstrained four equation system (18) using the iterative Zellner technique to achieve ML estimates of the regression coefficients, and found that the standard errors of these estimates were on the same order of magnitude as those of the DFP algorithm.

18/ For example, the regression coefficient on n(t-1) in equation (a) of (18) is equal to

$$1 + b_1 \Phi_{11} + b_2 \Phi_{12},$$

where b_1, b_2, Φ_{11}, and Φ_{12} are defined in footnote 15.

Data Appendix

Eight quarterly time series were collected in order to generate the four series needed for the analysis of section three and appendix two. These series are:

1.) The consumer price index (CPI), converted to the base 1972 = 1.00.
2.) Investment (I) in new plant and equipment for all manufacturing industries in billions of dollars.
3.) Production (or nonsupervisory) workers (n) on private nonagricultural payrolls in millions of people.
4.) Average hourly gross earnings (W) per production (nonsupervisory) worker on private nonfarm payrolls, all manufacturing in dollars.
5.) Bond yields (r) from Moody's investors service, corporate averages over four different ratings (Aaa, Aa, A, Baa) for railroads, public utilities and industrials in percentage terms.
6.) The fixed investment deflator (FID), converted to the base 1972 = 1.00.
7.) The effective corporate income tax rate (CIT).
8.) The investment tax credit (ITC).

Series 1-5 are seasonally adjusted and taken from various issues of Business Statistics, 1947-1974. The series 1, 3, 4, and 5 are quarterly averages of monthly data. Series 6 is seasonally adjusted and obtained from the National Income and Product Accounts, 1974. Last, series 7 and 8 were extracted from the FRB-MIT-Penn data tape at the University of Wisconsin, Madison. The real wage (w) is equal to W/CPI. The employment series n was used without modification. The capital stock (k) was generated using the recursive formula

$$k(t) = (1-\delta)k(t-1) + (I(t)/FID(t)),$$

where δ = .0273 is the constant rate of depreciation of plant and equipment in U.S. manufacturing taken from Jorgenson and Stevenson (1967). The benchmark capital stock was taken to be 94.64 billion 1972 dollars. The rental price of capital (c) was constructed by the formula stated in Hall and Jorgenson (1967):

$$c(t) = \left[\frac{(1-ITC(t))(1-CIT(t)z)}{(1-CIT(t))}\right] FID(t)(r(t) + \delta),$$

where z = .365 = $(1/\tau d)(1-\exp(\tau d))$ assuming straight line depreciation. The values of τ and d, the lifetime of capital for tax purposes and the discount rate respectively, were taken to be 72 quarters and .035 from Nadiri and Rosen (1973).

Although the data used in this study are seasonally adjusted, not all series were adjusted by the same method. As is well known, the non-conformity of seasonal adjustment procedures can distort parameter estimates. It is also true that two-sided filtering techniques (Census X-11) can alter lead-lag relationships thus affecting the results of Granger causality tests reported in the following appendix, and can weaken the case for covariance stationary as recent data are filtered differently than earlier data. It would have been preferable to use seasonally unadjusted data, but some of the series were not available in the unadjusted form.

Appendix 2

This appendix extends the recent findings of Neftci (1978) and Sargent (1978) who both report stronger evidence for Granger causality flowing from real wages to employment in aggregate U.S. manufacturing than for Granger causality in the reverse direction. We extend these results by performing a multi-variate Granger test of the null hypothesis that in a four variable system, the rental price of capital and the stocks of capital and production workers do not Granger cause real wages, and that the real wage and the stocks of capital and production workers do not Granger cause the rental price of capital. When performing a test of Granger causality, the researcher must choose a parameterization that offers a compromise between the criteria of unbiasedness which requires a generous parameterization, and power which necessarily diminishes as the parameter space expands, Geweke (1978, p. 178). Under the null hypothesis that (k(t), n(t), c(t)) do not Granger cause w(t) and (k(t), n(t), w(t)) do not Granger cause c(t), the model for the Granger test of Granger causality has the following form:

$$\begin{aligned}
&\text{(a)}\quad w(t) = \sum_{j=1}^{M} w(t-j)\cdot\alpha_1(j) + \sum_{j=1}^{N} c(t-j)\cdot\beta_1(j) \\
&\qquad + \sum_{j=1}^{N} n(t-j)\cdot\gamma_1(j) + \sum_{j=1}^{N} k(t-j)\delta_1(j) + \varepsilon_1(t) \\
&\text{(b)}\quad c(t) = \sum_{j=1}^{N} w(t-j)\cdot\alpha_2(j) + \sum_{j=1}^{M} c(t-j)\cdot\beta_2(j) \\
&\qquad + \sum_{j=1}^{N} n(t-j)\cdot\gamma_2(j) + \sum_{j=1}^{N} k(t-j)\,\delta_2(j) + \varepsilon_2(t).
\end{aligned} \tag{24}$$

Let $\underline{\varepsilon}(t)' = (\varepsilon_1(t), \varepsilon_2(t))$. We assume $\underline{E(\varepsilon(t))} = \underline{0}$, $E(\underline{\varepsilon}(t)\underline{\varepsilon}(t)') = \Sigma$ for all t, and all non-contemporaneous covariances are zero.

Following Geweke (1978, p. 178) we choose a generous parameterization for the real wage and rental price of capital, M=12, since omitting past values of these variables with large coefficients can lead to spuriously significant coefficients on lagged values of employment and the capital stock, and we choose N=2 since if the null hypothesis is false, it seems reasonable that the first few lagged values of employment and the capital stock are likely to have nonzero coefficients. A constant term and trend were also included in the regression equations (24). All variables are in levels, and the period of observation on the dependent variable is 1950I - 1974IV.

We report the coefficient estimates for equations (24) in Table 3. Table 4 contains the estimate of the unconstrained covariance matrix of disturbances, and the Wald F statistic (Silvey (1959)) for a test of the hypothesis that all past values of capital and employment have zero coefficients in both equations, all past values of the rental price of capital have zero coefficients in equation (a) and all past values of the real wage have zero coefficients in equation (b). The results of this test indicate the null hypothesis that (k(t), c(t), n(t)) do not Granger cause w(t) and (k(t), n(t), w(t)) do not Granger cause c(t) is a tenable specification.

Table 3

Coefficient estimates for the Granger test that (k(t), n(t), c(t)) do not Granger cause w(t).

Equation (a)*

Lags	real wage coefficient	rental price of capital coefficient	employment coefficient	capital stock coefficient
1	.834 (6.797)	-.019 (-.023)	-.0129 (-.863)	-.0034 (-.438)
2	-.088 (-.621)	-.612 (-.689)	.0065 (0.440)	.0041 (0.529)
3	.242 (1.782)			
4	.031 (0.228)			
5	-.136 (-.969)			
6	-.194 (-1.386)			
7	-.016 (-.113)			
8	.141 (1.039)			
9	-.084 (-.617)			
10	-.027 (-.200)			
11	.230 (1.676)			
12	.014 (0.121)			

* t statistics are shown in parentheses.

Table 3

Coefficient estimates for the Granger test that (k(t), n(t), w(t)) do not Granger cause w(t).

Equation (b)*

Lag	real wage coefficient		rental price of capital coefficient		employment coefficient		capital stock coefficient	
1	-.0045	(-.288)	1.350	(12.03)	.0037	(1.915)	-.0005	(-.459)
2	.0005	(0.029)	-.455	(-2.391)	-.0036	(-1.757)	0.0006	(.568)
3			.217	(1.095)				
4			.155	(0.790)				
5			-.356	(-1.750)				
6			.267	(1.289)				
7			-.304	(-1.470)				
8			-.039	(- .191)				
9			.086	(0.427)				
10			-.214	(-1.050)				
11			.020	(0.098)				
12			.241	(1.867)				

* t statistics are shown in parentheses.

Table 4

Covariance matrix of unconstrained residuals for the null hypothesis that (k(t), n(t), c(t)) do not Granger cause w(t) and (k(t), n(t), w(t)) do not Granger cause c(t).

$$\hat{\Sigma} = \begin{pmatrix} .4512\text{-}01 & 1.484\text{-}04 \\ 1.484\text{-}04 & 8.415\text{-}04 \end{pmatrix}$$

(F statistic for the null hypothesis.)

F(12,160) = 1.608 Marginal confidence level = .906

Bibliography

Akaike, H., "A New Look at the Statistical Model Identification," IEEE Transactions on Automatic Control, AC-19, (1974), 716-723.

Amemiya, T., "Selection of Regressions," Technical report No. 225, Economic Series, Institute for Mathematical Studies in the Social Sciences, Stanford University, 1976.

Bard, Y., Nonlinear Parameter Estimation, New York, Academic Press, 1974.

Bellman, R., Dynamic Programming, Princeton, New Jersy, Princeton University Press, 1957.

Berndt, E.R., and L. Christensen, "The Translog Function and the Substitution of Equipment Structures, and Labor in U.S. Manufacturing 1929-68," Journal of Econometrics 1 (1973), 81-114.

Berndt, E.R., M.A. Fuss, and L. Waverman: "Dynamic Models of the Industrial Demand for Energy." Electric Power Research Institute, November 1977.

Brechling, F., Investment and Employment Decisions. Manchester University Press, 1975.

Business Statistics, the biennial supplement to the Survey of Current Business, U.S. Department of Commerce, Bureau of Economic Analysis.

Chenery, H.B.: "Overcapacity and the Acceleration Principle," Econometrica, 20 (1952), 1-28.

Coen, R.M., and B.G. Hickman: "Constrained joint estimation of factor demand and production functions,: Review of Econ. and Stat., 1970, 257-300.

Crane, R.: "Optimal Distributed Lag Responses and Expectations," American Economic Review, December, 1971.

__________: "Investment, Adjustment Costs and Uncertainty, International Economic Review, October 1975.

Eisner, R., and R. Strotz: "Determinants of Business Investment," pp. 59-236 in Impacts of Monetary Policy, a series on research studies prepard for the Commission on Money and Credit. Englewood Cliffs: Prentice-Hall, 1963.

Geweke, J.F., "Testing the exogeneity specification in the complete dynamic simultaneous equation model," Journal of Econometrics, 7, (1978), 163-185.

Geweke, J.F., and R. Meese, "Estimating Distributed Lags of Unknown Order," unpublished paper presented at the North American Econometric Society Meetings, Montreal, June 27-30, 1979.

Gould, J.P., Adjustment Costs in the Theory of Investment of the Firm," Review of Econ. Studies., 35 (1968), 47-56.

Hall, R., and D. Jorgenson, "Tax Policy and Investment Behavior," American Economic Review 57, 1967, 391-414.

Hansen, L.P. and T. Sargent, "Formulating and Estimating Dynamic Linear Rational Expectations Models," Federal Reserve Bank of Minneapolis, March 1979a.

_________, "Linear Rational Expectations Models for Dynamically Interelated Variables," Federal Reserve Bank of Minneapolis working paper #135, 1979b.

Jorgenson, D., and J.A. Stephenson, "The Time Structure of Investment Behavior in U.S. Manufacturing, 1947-60," Review of Economics and Statistics 49, Feb. 1967, 16-27.

Kennan, J.: "The Estimation of Partial Adjustment Models with Rational Expectations," Econometrica 47 (1979), 1441-1456.

Kushner, M., Introduction to Stochastic Control, New York, Holt, Rinehart and Winston, Inc., 1971.

Lucas, R.E.: "Optimal Investment Policy and the Flexible Accelerator," International Economic Review, 8 (1967), 78-85.

Lucas, R.E.: Econometric Policy Evaluation: A Critique," in The Phillips Curve and Labor Markets, K. Brunner, ed., Supplement to the Journal of Monetary Economics, April, 1976.

Mallows, C.L., "Some Comments on Cp," Technometrics, 15, 661-675.

Meese, R., "Distributed Lag Order Determination with an Application to the Multiperiod Theory of the Firm," Unpublished Ph.D. thesis, University of Wisconsin-Madison, 1978.

Nadiri, M.I., and S. Rosen, A Disequilibrium Model of Demand for Factors of Production, National Bureau of Economic Research, Monograph No. 99, New York: Columbia University Press, 1973.

The National Income and Product Accounts of the U.S. 1929-1974, Statistical Tables, a supplement to the Survey of Current Business, 1975, U.S. Department of Commerce, Bureau of Economic Analysis.

Neftci, S.N., "A Time Series Analysis of the Real Wage-Employment Relationship," Journal of Political Economy, Vol. 86 No. 2, Part 1, April 1978, pp. 281-291.

Nerlove, M., "Lags in Economic Behavior," Econometrica, March 1972, 40, 221-251.

Parzen, E., Multiple Time Series: Determing the Order of Approximating Autoregressive Schemes," Technical Report 23, State University of New York at Buffalo, 1975.

Peck, S.C.: "Alternative investment models for firms in the electric utilities industry," Bell Journal of Economics, Autumn 1974, 421-435.

Rothschiled, M.: "On the cost of adjustment," Quarterly Journal of Economics, November 1971.

Sargent, T., Macroeconomic Theory, New York, Academic Press, 1979.

__________, "Estimation of Dynamic Labor Demand Schedules Under Rational Expectations," Journal of Political Economy, October 1978.

Sawa, T., "Information Criteria for Discriminating Among Alternative Regression Models," Econometrica, 46, 1973-1282.

Schramm, R.: "The influence of relative prices, production conditions and adjustment costs on investment behavior," Review of Economic Studies, July 1970.

Schwarz, G., "Estimating the Dimension of a Model," Annals of Statistics, 6, 461-464.

Silvey, S.D., "The Lagrange Multiplier Test," Annals of Mathematical Statistics, 30 (1959), 389-407.

Sims, C.: "Macro-economics and Reality," Discussion Papers No. 77-91, Center for Economic Research, Department of Economics, University of Minnesota, Dec. 1977.

Taylor, L.: "The Existence of Optimal Distributed Lags," Review of Econ. and Stud., 37 (1970), 95-106.

Theil, H. Principles of Economics, New York, John Wiley and Sons, 1971.

Tinsley, P.A.: "On Optimal Dynamic Adjustment of Quasi-fixed Factors," Federal Reserve Board Special Studies Paper No. 9, March 1970.

__________: "A Variable Adjustment Model of Labor Demand," International Economic Review, October 1971, pp. 482-510.

Treadway, A.B.: "On rational entrepreneurial behavior and the demand for investment," Review of Econ. Stud., April 1969.

__________: "Adjustment costs and variable inputs in the theory of the competitive firm," Journal of Economic Theory, December 1970.

__________: "The rational multivariate flexible accelerator," Econometrica 39, 1971, 845-55.

Woodland, A.D.: "Estimation of a variable profit and of planning price functions for Canadian manufacturing, 1947-1970," Canadian Journal of Economics, August, 1977, 355-377.