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by

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## Introduction

The U.S. interpretation of the current international monetary regime has been explained by Anthony Solomon (1978), Undersecretary of the Treasury for Monetary Affairs, as follows:

> The basic philosophy of the new monetary system... is that international monetary stability cannot be imposed from without, but must be developed by countries from within, through the application of sound underlying economic and financial policies.
>
> In line with that concept, our program for assuring a strong and healthy dollar relies on fundamental economic performance, not on market operations to hold or attain a particular exchange rate or maintain a particular exchange rate zone. We do recognize, of course, that markets can become disorderly, subject to great uncertainty, dominated by psychological factors and speculation. We have made clear that we are fully prepared to intervene in the markets to counter such disorders.

Controlling the fluctuations caused by non-fundamental or psychological factors has become a major goal of U.S. intervention policy.

All sorts of things are thought to be capable of generating these fluctuations. News of political or institutional change is often blamed, as is the reporting of new figures on inflation rates, trade balances, oil imports, etc. Of course, these pronouncements

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may well herald the working of fundamental economic forces that would, unless offset by Solomon's sound underlying policies, eventually affect exchange rates; however, these fundamental economic forces can not be the cause of the "knee jerk" market reaction that often follows such announcements. Instead, these fluctuations are usually attributed to psychological or speculative factors. The recent instability in gold markets has also been identified as a possible source of instability in exchange markets, even in quarters that had previously admitted little or no casuality in that direction.[1/]

Most of the existing models of destabilizing speculation postulate speculators that make some sort of systematic prediction error.[2/] By contrast, exchange market participants are generally thought to be rather efficient in their use of information. The purpose of this paper is to present a characterization of destabilizing speculation that does not depend upon systematic prediction errors, and to examine its implications for a stabilizing intervention policy.

The notion that rational speculators might build "psychological" factors into stock market prices, and be right in doing so, is not new. Keynes (1936) makes the colorful analogy to a beauty contest in which the judges are allowed to bet upon the outcome. For each speculator-judge, guessing what the other judges will take as a sign of beauty becomes as important as discerning the innate pulcritude of the individual contestants. In fact, it becomes all important unless the winning judge is awarded some claim on the actual beauty of the winning contestant. In the same way, exchange market speculators must consider both the fundamental soundness of individual currencies and the capital gains and losses that will be caused by the actions of other speculators. If one speculator expects others to react to changes in, say, the price of gold, then he too will react to changes in the price of gold. And if enough speculators behave in this manner, exchange rate movements will indeed reflect movements in the price of gold, fulfilling the speculators' expectations and needlessly disrupting exchange markets.

This kind of phenomenon can be captured in rational expectations models. Shiller (1978), for example, has noted that extraneous variables can become a part of the solution to such models, causing more uniqueness problems of the type discussed by Taylor (1977), and indirectly by Sargent (1973) before him. However, the rational expectations literature has tended to view these uniqueness problems as a nuisance; they are universally assumed away. Here the extraneous variable problem will be focused upon as a characterization of rational, but destabilizing, speculative behavior.

It turns out that there is a simple intervention policy which, if imposed, will minimize the disorder created by this kind of speculation; unfortunately, there may be some reason to believe that it is not the kind of policy that is usually envisioned. In particular, it will be shown in a model incorporating the "new" monetary approach to exchange rate determination that a policy of leaning against the wind only makes matters worse. The correct policy, according to this view, is one of accomodation: a depreciation should be met with an increase in the money supply. This unlikely policy conclusion is not a general implication of the characterization of destabilizing speculation that is being proposed here; rather, it is an implication of the (admittedly strong) version of the "new" monetary approach that has been adopted for illustrative purposes. A counterexample will be provided (in an appendix) in which the correct policy is to lean against the wind. On the other hand, the present characterization of destabilizing speculation is somewhat unconventional in that certain uniqueness problems can not be simply assumed away in a manner that has become standard in the rational expectations literature, and this fact does seem to affect the way that models incorporating the "new" monetary approach work. So a more conventional stabilization problem will be presented for comparison; it will be seen that when the conventional assumptions are imposed to achieve uniqueness, a policy that leans against the wind can stabilize the exchange rate in models incorporating the "new" monetary approach to exchange rate determination.

## I. A Model of Rational Destabilizing Speculation

A simple small country framework can be used to illustrate this characterization of destabilizing speculation. This may not be considered the appropriate setting for a discussion of U.S. intervention policy, but it does yield a clear and unemcumbered view of the destabilizing speculation and its implications for monetary policy. The basic conclusions will be true in more complicated settings.

Consider a small open economy whose output is a perfect substitute for foreign goods. Purchasing power parity implies

(1) $p_t = \bar{p}^* + e_t$

where p and p* are the domestic and foreign prices of the home product (in logs), and e is the home currency price of foreign exchange (again in logs); the foreign price is fixed at p* which, for simplicity, will be set equal to zero. Domestic producers can sell all they want at the going world price; the (log of) output is fixed at its full employment level, $\bar{y}$.

(2) $y_t = \bar{y} = s_o + s_1(1/\sigma_e^2)$

Full employment output is inversely related to the asymptotic variance of the exchange rate, $\sigma_e^2$; exchange rate volatility is assumed to impede trade flows.[3/]

The existence of $\sigma_e^2$ in (2) plays no role in the present characterization of destabilizing speculation. It turns out that the monetary authority can limit the size of $\sigma_e^2$, and the "fixed versus flexible rate" literature suggests that there is some reason for its doing so. The inclusion of $\sigma_e^2$ in (2) is the simplest way of modeling this benefit in the present setting.

So where does the destabilizing speculation come from? In this model it originates in financial markets. There are only two assets, home money and foreign exchange. Home residents must divide their wealth between these two assets, and they are assumed to hold the entire home money stock. They hold home money for transactions purposes and for speculative gain. The real return on foreign money is zero (since p* is fixed), while the real return on home money is a random variable, $-(e_{t+1} - e_t)$. From the portfolio manager's point of view, the mean of this return is

$$(3) \qquad E_t[-(e_{t+1} - e_t)] \equiv -(e_{t+1|t} - e_t)$$

and its variance is

$$(4) \qquad E_t[-(e_{t+1} - e_t) + (e_{t+1|t} - e_t)]^2 = E_t[e_{t+1} - e_{t+1|t}]^2 \equiv \hat{e}^2_{t+1|t}$$

$E_t[\cdot]$ is the conditional expectation operator; $e_{t+1|t}$ and $\hat{e}^2_{t+1|t}$ are the "rational" mean and variance of $e_{t+1}$ based upon information available at the end of period t.[4/] It will turn out that $e_{t+1} - e_t$ has a normal conditional distribution; so a risk averse portfolio manager's demand for home money will be of the form

$$(5) \qquad m_t - p_t = -\alpha(e_{t+1|t} - e_t) - \gamma\hat{e}^2_{t+1|t} + \eta y_t$$

where $\alpha$, $\gamma$ and $\eta$ are positive constants.[5/]

Notice that conditional means and variances appear in the money demand function (5) while asymptotic variances appear in the supply curve (2). It is often asserted that financial markets respond more quickly to new developments than do goods markets, or that labor markets are encumbered by long-term contracts. Differential conditioning of moments is one way of modeling this asymmetry. Each period, portfolio managers watch prices as they form and chose the portfolio that maximizes their expected utility of return. The output supply structure, on the other hand, does not respond to day by day occurances; it responds only to changes in asymptotic moments.

Equations (1), (2) and (5) determine $e_t$, $p_t$ and $y_t$. Using (1) and (2) to eliminate $p_t$ and $y_t$ in (5), the money market equilibrium condition can be solved for an equation that determines $e_t$:

$$(6)\quad e_t = c + \alpha(e_{t+1\,|\,t} - e_t) + m_t \qquad c \equiv \gamma\hat{e}^2_{t+1\,|\,t} - \pi\bar{y}$$

It will turn out that $\hat{e}^2_{t+1\,|\,t}$ is not time dependent, so c may be regarded as a constant.

If the money supply is fixed at $\bar{m}$, then

$$(7)\quad e_t = \bar{e} \qquad \bar{e} \equiv c + \bar{m} = \bar{m} - \pi\bar{y}$$

is an obvious solution of (6). Here there are no random elements in the model, so (6) becomes an ordinary (but unstable) difference equation with a stationary solution at $\bar{e}$. Others have discussed stability and uniqueness problems that arise in the deterministic case;[6/] the present paper focuses upon the "extraneous variable" problem.

The extraneous variables in this case might be a series of proclamations, forecasts, or statistical reportings by governments or by private agents, or they might be changes in prices of irrelevant goods like (in the present model) gold or oil. In short, they can be anything that one speculator might think other speculators will take as a leading indicator of exchange rate movements. These variables will be modeled as sequence $\{u_t\}$ of serially uncorrelated, normally distributed random variables with zero mean and variance $\sigma_u^2$. (Adding serial correlation merely complicates the mathematical expressions without changing the conclusions in any important way; the case of serially correlated extraneous variables will be discussed in an appendix. It may also be interesting to note that the entraneous variable need not be stochastic; that is, it may be perfectly predictable. Things do work out somewhat differently in this case; it too is treated in an appendix.)

Can this extraneous variable become a true, self-fulling leading indicator? If so,

$$(8)\quad e_t = \bar{e} + \Sigma_{i=0}^{t} w_i u_{t-i}$$

should also be a solution of equation (6) (for some appropriately chosen values of the coefficients $\bar{e}$, $w_0$, $w_1$, $w_2$,...). Equation (8) says that the extraneous variable actually becomes a part of the solution at some time $t = 0$. If equation (8) is indeed a valid solution, then

$$(9)\quad e_{t+1\,|\,t} = \bar{e} + \Sigma_{i=1}^{t+1} w_i u_{t+1-i} = \bar{e} + \Sigma_{i=0}^{t} w_{i+1} u_{t-i}$$

and

$$(10)\quad e_{t+1\,|\,t} - e_t = \Sigma_{i=0}^{t} (w_{i+1} - w_i) u_{t-i}$$

Substituting (10) into (6) (with $m_t$ set equal to $\bar{m}$), that expression becomes

(11) $e_t = \bar{m} + c + \alpha\Sigma_{i=0}^{t}(w_{i+1} - w_i)u_{t-i}$

and (8) is consistent with (11) if

(12) $\bar{e} = \bar{m} + c$ $\qquad$ or $\qquad$ $\bar{e} = \bar{m} + c$

$w_i = \alpha(w_{i+1} - w_i)$ $\qquad\qquad$ $w_i = [(\alpha + 1)/\alpha]^i w_0$

So if the coefficients $\bar{e}$, $w_0$, $w_1$,... are chosen to satisfy the constraints in (12), then (8) will be a valid solution of the model. Speculators' use of the extraneous information embodied in $u_t$ can indeed cause the exchange rate to fluctuate needlessly. And this case of destabilizing speculation can not be attributed to systematic prediction errors since speculators' expectation formation is "rational." The extraneous information can become a true leading indicator.

Before going on, there is one technical matter that should be disposed of. Earlier it was asserted that the conditional variance $\hat{e}^2_{t+1|t}$ would turn out to be time independent, so that c could be treated as a constant. It is time to verify that assertion, and this is easily done. From equations (8) and (9).

(13) $e_{t+1} - e_{t+1|t} = w_0u_t$ and $\hat{e}^2_{t+1|t} = w_0^2\sigma_u^2$

There is a uniqueness problem associated with this characterization of destabilizing speculation. Note that (8) is a valid solution of equation (6) for any set of coefficients that satisfy (12), but (12) places no restriction upon $w_0$. There is a difference equation generating the $w_i$, but no initial condition is given. Put another way, with restrictions (12), equation (8) becomes

$$(14) \quad e_t = \bar{e} + w_0 \Sigma_{i=0}^{t} \beta^i u_{t-i}$$

$$\bar{e} = \bar{m} + c = \bar{m} + \gamma w_0^2 \sigma_u^2 - \eta \bar{y} \qquad \beta \equiv (\alpha + 1)/\alpha$$

and (14) is a valid solution of the model for any choice of $w_0$.

There are several ways of comparing the solutions corresponding to different choices of $w_0$. One approach is to compute the unconditional variance of $e_t$:

$$(15) \quad \sigma_{e_t}^2 = w_0^2 \sigma_u^2 \Sigma_{i=0}^{t} \beta^{2i}$$

For any $w_0 \neq 0$, this variance increases over time, and it increases without bound since $\beta > 1$. As long as speculators continue to view $u_t$ as a leading indicator, the bubble will continue to grow.[7/] (And there is no obvious reason for them to quit taking $u_t$ as a leading indicator, for expectation formation is rational and $u_t$ has actually become a part of the solution.)

Another approach is to compute expected future exchange rates. Forwarding equation (14) to period T and taking the conditional expectation

$$(16) \quad e_{T|t} = \bar{e} + w_0 \Sigma_{i=0}^{T} \beta^i u_{T-i|t} = \bar{e} + w_0 \beta^{T-t} x_t$$

$$x_t \equiv \beta^t u_0 + \beta^{t-1} u_1 + \ldots + u_t$$

$e_{T|t}$ is the exchange rate that portfolio managers espect to obtain T - t periods from now. Unless a highly improbably canceling has occured (so that $x_t = 0$), $e_{T|t} \to \pm\infty$ as $T \to \infty$ if $w_0 \neq 0$. That is, for all choices of $w_0$ other than zero, the exchange rate is expected to explode or implode.

This uniqueness problem always arises in rational expectations models that include expectations of future prices in the equations that determine the current price.[8/] In the past, it has been handled in various ways. Sargent's (1973) "no speculative bubbles" condition ruled out any solution for which the price is expected to explode or implode, while Taylor (1977) chose the solution that minimizes the asymptotic variance. Here, either criterion implies a choice of $w_0$ equal to zero, but setting $w_0$ equal to zero in (14) results in equation (7). The Sargent-Taylor solution excludes the extraneous variable altogether; they simply assume that speculators collectively choose to ignore the leading indicator.[9/] In the present context this amounts to throwing the baby out with the bath water. In what follows it is assumed that $w_0$ is some number other than zero, but the indeterminacy remains.

It should also be noted that $\bar{e}$ depends upon $w_0$. In addition, this long-run on average exchange rate depends upon the degree of risk aversion ($\gamma$) and the volatility of the leading indicator ($\sigma_u$).

## II. Policy Implications

Does there exist a practical intervention policy that will stabilize exchange rates against this kind of speculative disturbance? The answer depends upon what the monetary authority is assumed to know. It also depends upon what is meant by the word stabilize. There would seem to be two stabilization goals worth considering. The monetary authority could try to minimize the asymptotic variance of the exchange rate, the variance that curbs production and trade flows. (See equation (2).) Or it could try to limit the conditional variance $\hat{e}^2_{t+1|t}$ that lowers portfolio managers' (ex ante) welfare.

If the policy maker thinks he has identified the leading indicator that is causing the disturbances, then he may be tempted to try to stabilize the indicator itself instead of intervening in exchange markets. (If $\sigma^2_u$ can be set equal to zero, then both $\sigma^2_e$ and $\hat{e}^2_{t+1|t}$ will vanish.) Recent flirtation with the idea of trying once again to stabilize the price of gold would appear to stem from this kind of thinking.[10/] The present author has serious doubts about this approach. First, an unsuccessful stabilization effort would only draw attention to the indicator and probably make matters worse. Second, there are many extraneous variables that could concievably generate speculative disturbances; there may be several operative at the same time. Discerning the popular fancy may be difficult, and once again, mistakes only draw attention to potential sources of instability. Finally, stabilizing all of the potential sources of instability would, even if possible, be prohibitively expensive. The present paper considers instead policies that intervene in exchange markets directly.

In particular, consider simple policy rules of the form

(17) $m_t = \overline{m} + g(e_t - \overline{e})$

where g is an as yet undetermined policy parameter.11/ 12/ Substituting (17) and (10) into (6) results in

(18) $e_t = \overline{m} + c + \Sigma_{i=0}^{t} [\alpha w_{i+1} + (g - \alpha)w_i]u_{t-i}$

and (18) is consistent with (8) if $\overline{e}$ and the $w_i$ are chosen to satisfy

(19) $\overline{e} = \overline{m} + c$ or $\overline{e} = \overline{m} + \gamma w_0^2\sigma^2 - \pi\overline{y}$

$w_i = \alpha w_{i+1} + (g - \alpha)w_i$ $w_i = [\beta - (g/\alpha)]^i w_0$

$\beta \equiv (\alpha + 1)/\alpha$

With these restrictions, solution (8) becomes

(20) $e_t = \overline{e} + w_0\Sigma_{i=0}^{t}[\beta - (g/\alpha)]^i u_{t-i}$

$\overline{e} = \overline{m} + \gamma w_0^2\sigma_u^2 - \pi\overline{y}$

where $w_0$ is once again undetermined, and the variances in question become

(21) $\hat{e}^2_{t+1\,|\,t} = w_0^2\sigma_u^2$

$\sigma^2_{e_t} = w_0^2\sigma_u^2\Sigma_{i=0}^{t}[\beta - (g/\alpha)]^{2i}$

$\sigma^2_e = \dfrac{w_0^2\sigma_u^2}{1-(\beta - g/\alpha)^2}$ (if $|\beta - (g/\alpha)| < 1$)

The choice of the policy parameter g has no effect upon the conditional variance $\hat{e}^2_{t+1|t}$; portfolio managers can not be helped in an ex ante sense.[13/] However, the unconditional variance $\sigma^2_{e_t}$ can radically modified. If g is chosen to make $|\beta - (g/\alpha)| < 1$, then $\sigma^2_{e_t}$ no longer grows without bound, and the asymptotic variance is finite. Speculative disturbances will continue to grow, but not without bound. The optimal intervention policy and the solutions it implies are:

$$(22)\quad \left.\begin{aligned} &g = g^* \text{ where} \\ &g^* = \alpha\beta = \alpha + 1 \end{aligned}\right\} \Longrightarrow \begin{aligned} &e_t = \bar{e} + w_0 u_t \\ &\sigma^2_e = \sigma^2_{e_t} = \hat{e}^2_{t+1|t} = w_0^2\sigma_u^2 \end{aligned}$$

This policy rule reduces all of the variances to the conditional variance.

It may be interesting to compare these solutions to the Sargent-Taylor solution discussed in the last section. If the optimal policy is imposed, all of the solutions satisfy Sargent's "no speculative bubbles condition": that is, for all $w_0$

$$e_{T|t} = \bar{e} + w_0 u_{T|t} = \bar{e} \quad \text{for all } T > t$$

and the exchange rate is not expected to blow up. Technically, all of the solutions in (22) qualify as Sargent solutions; as Taylor pointed out, Sargent's condition does not guarantee uniqueness. Taylor's procedure of chosing the solution that minimizes $\sigma^2_e$ still requires setting $w_0$ equal to zero; his condition always achieves uniqueness, and it rules out destabilizing speculation.

It is important to note that the stabilization policy described above is not the conventional lean against the wind policy that is usually envisioned. In fact, it is clear from (21) that leaning against the wind (or setting $g < 0$) actually increases variances. Policy must be accommodating ($g > 1$) to stabilize exchange rates; if the exchange rate depreciates, the money supply should be increased and more then proportionately.

This policy prescription may sound strange at first, but it is not difficult to see why it works in the present model: If the exchange rate rises above $\bar{e}$, it must be forced back down; an appreciation must be engineered. "Rational" portfolio managers will foresee this appreciation and raise their demand for money; the (real) supply of money must be increased to accommodate this new demand. Put another way, the monetary authority must accommodate the demand for money that is consistent with the expected appreciation or depreciation that moves the exchange rate in the desired direction.

Finally, it should be noted that portfolio manager's risk aversion makes the average exchange rate $\bar{e}$ depend upon $w_0$, no matter what policy is instituted.[14/] It will fluctuate in a manner that the policy maker can not predict as bubbles come and go. This fact may or may not in itself be a problem, but it will certainly make the success or failure of any stabilization effort hard to document.

## III. A More Conventional Stabilization Problem

It should not be thought that the accommodating policy prescription of the last section is the general policy implication of the new monetary approach to exchange rate determination when expectations are assumed to be rational. In fact, the results of the last section run counter to the implications of more conventional rational expectations models as well. More conventional models often imply that leaning against the wind can be a beneficial policy. An example is provided below.

How is the present model unconventional? The standard practice with rational expectations models is to rule out solutions for which the exchange rate is expected to blow up. The speculative bubbles described in the first section would be ruled out by assumption,[15] and the stabilization problem discussed in the second would never come up. It is in this sense that the model outlined in the first section is unconventional.

It should be emphasized that the present characterization of destabilizing speculation does not depend upon this explosive nature of expectations. A model is provided in Appendix 1 in which solutions incorporating extraneous information satisfy Sargent's "no speculative bubbles" condition. The model outlined above does exhibit explosive behavior, and it has affected the qualitative nature of the appropriate stabilization policy. To see why this is so, it is helpful to consider a more conventional stabilization problem.

Suppose the money demand function is respecified as follows:

$$(23) \quad m_t - e_t = -\alpha(e_{t+1|t} - e_t) - c + u_t$$

Here the random variable $u_t$ is a part of money demand. It represents a fundamental economic force, and not an extraneous variable. In fact, extraneous variables will play no role in this example; in keeping with tradition, the explosive solutions will be ignored. The money supply rule is

(24) $m_t = h + ge_t$

and the purpose of this example is to show that negative values of g will lower $\sigma_e^2$. Leaning against the wind stabilizes the exchange rate.

Substituting (24) into (23) and rearranging

(25) $e_t = \phi\alpha e_{t+1 \mid t} + \phi(h + c) - \phi u_t$

$\phi \equiv (1 + \alpha - g)^{-1}$

Forwarding (25) j periods and taking the (conditional) expectation,

(26) $e_{t+j \mid t} = \phi\alpha e_{t+j+1 \mid t} + \phi(h + c)$

since $u_t$ is not serially correlated. If (26) is successively substituted into (25) (with j set equal to 1, then 2 and so on), the result is[16/]

(27) $e_t = \lim_{T\to\infty} (\phi\alpha)^T e_{t+T \mid t} + \phi(h + c)\Sigma_{i=0}^{\infty} (\phi\alpha)^i - \phi u_t$

Note that $\phi\alpha < 1$ for all $g \leq 0$, so the only solution for which $e_{t+T \mid t}$

does not blow up is

$$(28) \quad e_t = \phi(h + c)(1 - \phi\alpha)^{-1} - \phi u_t$$

$$\sigma_e^2 = \phi^2\sigma_u^2 = \sigma_u^2/(1 + \alpha - g)^2$$

Negative values of g clearly stabilize the exchange rate.

Why does eliminating the explosive solutions change the qualitative nature of the stabilization rule? The present author is not entirely sure, but some intuition may be gained by considering two non-stochastic versions of the model. In particular, the money demand function can be rewritten as

$$(23)' \quad m_t - e_t = - \alpha(e_{t+1} - e_t) - c$$

or

$$(23)'' \quad m_t - e_t = - \alpha(\bar{e} - e_t) - c$$

Equation (23)' is the perfect foresight version of the model. Rearranged, (23)' is a difference equation generating next period's exchange rate:

$$e_{t+1} = [(\alpha + 1)/\alpha]e_t - (1/\alpha)(c + m_t)$$

As is well known, this difference equation is unstable; however, the policy rule

$$m_t = h + ge_t$$

changes the dynamics to

$$e_{t+1} = [1 + (1 - g)/\alpha]e_t - (1/\alpha)(c + h)$$

and it takes values of g greater than unity to stabilize the equation. This is reminicent of the results in section II; in fact, setting $g = g^* = \alpha + 1$ reduces the equation to

$$e_{t+1} = - (1/\alpha)(c + h)$$

which is comparable to (22).

In (23)", the expected future exchange rate is fixed at $\bar{e}$, a constant. A decrease in the money supply produces an appreciation of the home currency; this is the only way to create the expectation of a depreciation that is consistent with a lower demand for money. In this model, leaning against the wind works. Shocks that would tend to depreciate the currency can be offset with a tight money policy. This is reminicent of the more conventional stabilization problem just considered.

With (23)', the expected future exchange rate does all of the adjusting to equate supply and demand for money in the current period. With (23)", the forward rate is fixed, and the spot rate must do the adjusting. Clearly, the rational expectations models (with either (17) or (23)) fall somewhere in between; both $e_t$ and $e_{t+1 \mid t}$ adjust to equilibrate the market at time t. However, it appears that the conventional rational expectations model, with its long-run expectations pinned down, behaves more like the deterministic model with (23)".

## IV. Conclusion

The extraneous variable problem associated with rational expectations models has been suggested as a characterization of destabilizing speculation that does not depend upon systematic or perverse prediction errors. Speculators build extraneous variables into their forecasts of future exchange rates simply because they expect other speculators to do the same; their collective behavior then ratifies the forecasts and exchange rates fluctuate for psychological or non-fundamental reasons.

There is a simple intervention policy that will stabilize the exchange rate, but it is an accommodating policy that seems to run counter to conventional wisdom. In response to a depreciation, the monetary authority should increase the <u>real</u> supply of money, thereby creating a level of demand that is consistent with an appreciation of the currency. This policy prescription appears to stem from the fact that without an effective stabilization policy the exchange rate would be expected to blow up. The present characterization of destabilizing speculation does not depend upon this explosive nature of expectations; it could have been explicated in a more stable setting. However, the most straightforward modeling of the new monetary approach to exchange rate determination does exhibit this explosive behavior unless speculative bubbles are simply ruled out by assumption.

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## Footnotes

1/ The New York Times (October 3, 1979) quotes Anthony Solomon as saying the gold fever "not only reflects concerns about inflation, but becomes an engine of inflationary expectations." The Times goes on to point out that this was the first time an American official had formally acknowledged a spillover from gold market activity to exchange markets.

2/ See, for example, Stern's (1973) discussion of stabilizing and destabilizing speculation.

3/ Criticisms of flexible rate regimes are often based upon this assertion. Stern (1973) discusses some of the issues involved in this long literature; the present model is not sufficiently rich to explore them. One way of justifying the inclusion of $\sigma_e^2$ in the present model is to allude to the work of Sandmo (1971) and Coes (1977). They show that risk averse firms will contract in response to greater price uncertainty. Friedman (1977) has also argued that the natural rate of employment depends upon the variance of prices.

4/ The rational expectations hypothesis asserts that speculators' subjective views about the moments of future random variables are identical to the model's own mathematical expectations conditioned upon information available when the predictions are made. $e_{t+1 | t}$ and $\hat{e}^2_{t+1 | t}$ are conditioned upon all variables dated t or earlier. Shiller's (1978) survey of the rational expectations literature is excellent background reading for the present discussion.

5/ $\alpha$ and $\gamma$ need not be positive if income effects are large; see, for example, Tobin (1958). The postulated signs are generally considered to be the relevant ones.

6/ See Sargent and Wallace (1973) or Kouri (1976). There are similar problems in models that add random disturbances to the structural equations; see Sargent (1973).

7/ Why is it that risk averse portfolio managers continue to hold home money while the variance of the exchange rate is increasing without bound? This apparent discrepancy is resolved when it it recalled that portfolio managers re-evaluate their portfolios each period and base their decisions upon the conditional variance of the exchange rate which is stationary; see equation (13).

8/ See Shiller (1978) or Aoki and Canzoneri (1979).

9/ There have been a number of attempts, generally in a deterministic framework, to rule out "unstable" solutions on microeconomic grounds (see Brock (1975) or Minford (1978), but it is hard to see how their arguments pertain to finite lived utility maximizers like the portfolio managers postulated in the present model. It should be noted that the explosive nature of the present model is not a necessary feature of this characterization of destabilizing speculation. Appendix 1 provides a model in which solutions incorporating extraneous information satisfy Sargent's "no speculative bubbles" condition. It will also be seen that when the optimal stabilization rule is imposed in the present model, solutions reflecting extraneous information can still occur, but they will satisfy Sargent's condition.

10/ Various rumors of such a policy were reported by the press about the time of the IMF meeting in Belgrade (October, 1979). George Willis of the U.S. Treasury told the American Mining Congress (Los Angeles, Sept. 26, 1979) that official sales of U.S. gold stocks "responded to conditions in the gold markets last year, which had contributed to the adverse psychological atmosphere in the foreign exchange market."

11/ Here again, the information available to the policy maker has been limited to make the problem interesting. The optimal full information policy rule is obviously

$$m_t = \bar{m} - \alpha(e_{t+1\,|\,t} - e_t)$$

If it were implemented, equation (6) would reduce to (7), and once again both variances would vanish. However, to calculate the expected rate of depreciation, the policy maker must have identified the extraneous variable, and he must know the value of $w_0$. See equation (10). This would not appear to be the revelant case. On the other hand, private agents are assumed to be able to calculate $e_{t+1\,|\,t}$, and it is not entirely clear why the policy maker should be at an information disadvantage.

12/ Some would prefer a feedback rule of the form

$$m_t = \bar{m} + g(e_{t-1} - \bar{e})$$

where the policy maker is not assumed to know the current value of the exchange rate. However, the dating structure on expectations in the money demand function implies that portfolio managers are able to watch the exchange rate as it settles upon its equilibrium value and modify their demands accordingly. Here, it is difficult to see why the policy maker should have an information disadvantage; nevertheless, this case is treated in an appendix. The mathematics becomes more complicated, but it turns out that the results are basically unchanged, provided that $\alpha > 1$.

13/ This is essentially the familiar Sargent-Wallace (1975) result. Policy rules can not affect price prediction errors unless they incorporate some information advantage. In the case of unconditional variances, they do precisely that; the feedback term used data that was not used in the (unconditional) prediction.

14/ At first glance, this fact would seem to make the policy rule (17) untenable; by assumption, the monetary authority does not know $w_0$. However, (17) can be expressed in a form that is usable.

$$m_t = \bar{m} + g(e_t - \bar{e})$$

$$= h + ge_t \qquad h \equiv \bar{m} - g\bar{e}$$

The monetary authority sets g and h; setting g equal to g* will again result in solution (22). The $w_0$ that obtains will determine $\bar{e}$ and the average money supply $\bar{m}$ (= h + $g\bar{e}$).

15/ It will be recalled that $\lim_{T\to\infty} e_{T|t}$ was not finite unless $w_0$ was set equal to zero.

16/ This solution technique originated with Sargent (1973).

## Appendix 1: An Example of Non-Explosive Destabilizing Speculation

There have been a number of attempts to justify Sargent's "no speculative bubbles" assumption on microeconomic grounds; see Brock (1975) or Minford (1978). It is difficult to see how their arguments pertain to finite lived utility maximizers like the portfolio managers postulated above. However, the jury is not yet in on this issue, so it is worth noting that the explosive behavior described above is not an essential feature of the present characterization of destabilizing speculation. A model is presented below in which solutions incorporating extraneous information satisfy Sargent's "no speculative bubble" condition; that is, the exchange rate is not expected to explode or implode. The non-uniqueness will of course remain.

This counterexample is essentially Taylor's (1977). Suppose real money balances are a factor of production, so the supply curve (2) is replaced by

$$(2)' \quad y_t = \bar{y} + s_2(m_t - p_t)$$

With (2)' in place of (2), (6) becomes

$$(6)' \quad e_t = \tilde{c} + \tilde{\alpha}(e_{t+1|t} - e_t) + m_t$$

$$\tilde{c} \equiv c/(1-\eta s_2) \qquad \tilde{\alpha} \equiv \alpha/(1-\eta s_2)$$

and a solution to (6)' can be found by replacing $c$ and $\alpha$ with $\tilde{c}$ and $\tilde{\alpha}$ in (14):

$$(14)' \quad e_t = \bar{e} + w_o \Sigma_{i=o}^{t} \tilde{\beta}^{i} u_{t-i}$$

$$\tilde{\beta} \equiv (\tilde{\alpha}+1)/\tilde{\alpha} = 1 - (\eta s_2 - 1)/\alpha$$

The exchange rate expected to obtain T-t periods hence is

$$(16)' \quad e_{T|t} = \bar{e} + w_o \; \tilde{\beta}^{T-t} x_t \qquad x_t \equiv \tilde{\beta} \; u_0 + \tilde{\beta}^{t-1} u_1 + \ldots + u_t$$

and the unconditional variance of $e_t$ is

$$(15)' \quad \sigma_{e_t}^2 = w_0^2 \sigma_u^2 \Sigma_{i=o}^{t} \; \tilde{\beta}^{2i}$$

Now suppose $1 < \eta s_2 < 1 + 2\,\alpha$, so that $|\tilde{\beta}| < 1$. In this case,

$$\lim_{T \to \infty} e_{T|t} = \bar{e} \qquad \text{and} \qquad \sigma_e^2 = w_0^2 \sigma_u^2 / (1 - \tilde{\beta}^2)$$

for all values of $w_o$. All of the solutions satisfy Sargent's "no speculative bubbles" condition, and the unconditional variance approaches a finite bound, the asymptotic variance $\sigma_e^2$.

So here is an example of non-explosive destabilizing speculation. Once again, extraneous information can become a part of the solution, but the explosive, snow-balling effect has been eliminated. In the main body of the paper, it was shown that a properly designed monetary policy can play the same role that real balances in the production functions plays here; either renders the model "background stable" and creates uniqueness problems even when Sargent's condition is imposed. Taylor (1977) demonstrated this result for real balances in the production function; F. Black (1974) demonstrated it it for accomodative policy rules.

It may be interesting to note that leaning against the wind is an appropriate policy in the present model if $1 < \eta s_2 < 1 + \alpha$. Demonstration of this fact is left as an exercise for the interested reader.

## Appendix 2: Serially Correlated Extraneous Variables

Serial correlation does not seriously alter any of the results in the main text. It does cause the optimally controlled solutions (the solutions corresponding to (22)) to cycle.

To see this, consider three equations

$$\text{(I)} \quad e_t = c + \alpha(e_{t+1|t} - e_t) + m_t$$

$$\text{(II)} \quad u_t = \rho u_{t-1} + \varepsilon_t$$

$$\text{(III)} \quad m_t = \overline{m} + g(e_t - \overline{e})$$

(I) is equation (6) in the main text. (II) explains the correlation structure of the extraneous variable; $\varepsilon_t$ is a serially uncorrelated normal random variable with zero mean and variance $\sigma^2$. (III) is the monetary policy rule.

Once again, a solution of the form

$$\text{(IV)} \quad e_t = \overline{e} + \Sigma_{i=o}^{t} w_i u_{t-i}$$

will satisfy (I). If (IV) is a valid solution then

$$e_{t+1|t} = \overline{e} + \Sigma_{i=o}^{t+1} w_i u_{t+1-i|t}$$

$$= \overline{e} + w_o \rho u_t + \Sigma_{i=1}^{t+1} w_i u_{t+1-i}$$

$$= \overline{e} + (w_o \rho + w_1) u_t + \Sigma_{i=1}^{t} w_{i+1} u_{t-i}$$

and

$$e_{t+1|t} - e_t = (w_o\rho + w_1 - w_o)u_t + \Sigma_{i=1}^{t} (w_{i+1} - w_i)u_{t-i}$$

Substituting this expression and (III) into (I) and gathering like terms,

(V) $$e_t = c + \bar{m} + [\alpha w_1 - (\alpha w_1 - (\alpha - \alpha\rho - g)w_o]u_t + \Sigma_{i=1}^{t}[\alpha w_{i+1} - (\alpha - g)w_i]u_{t-i}$$

so the restrictions on $\bar{e}$ and the $w_i$ become

(VI) $$\bar{e} = c + \bar{m}$$

$$w_1 = [\beta - \rho - (g/\alpha)]w_o$$

$$w_i = [\beta - (g/\alpha)]^i w_1 \text{ for all } i \geq 2$$

$$\beta \equiv (\alpha+1)/\alpha$$

Once again, there is no restriction on $w_o$.

And as before, if $g=0$, the speculative disturbances will continue to grow as time passes. The same feedback rule can be used to limit this instability; setting $g$ equal to $g^*(=\alpha\beta)$, the solution becomes

(VII) $$e_t = \bar{e} + w_o u_t - (\rho/\alpha)w_o u_{t-1}$$

and $e_t$ has a finite asymptotic variance (assuming $|\rho| < 1$). The basic difference between (VII) and (22) is that serial correlation in the leading indicator induces persistence or cyclical effects in the controlled solution.

## Appendix 3: Lagged Feedback Rules

Some may object to the instantaneous feedback in the policies discussed in the main text. However, it turns out that policy rules of the form

$$\text{(A)} \qquad m_t = \bar{m} + g(e_{t-1} - \bar{e})$$

yield very similar results provided that $\alpha$ is greater than one.[1][2] To see this, consider once again the equations

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1. This restriction on $\alpha$ is perhaps believable. $\alpha$ is the partial derivative of the log of money demand with respect to changes in the "interest rate," $e_{t+1} - e_t$. The interest elasticity of demand $\eta$ is equal to $\alpha \cdot \bar{\dot{e}}$ where $\bar{\dot{e}}$ is the "average" rate of change of the exchange rate; so $\alpha > 1$ is equivalent to $\eta > \bar{\dot{e}}$. The interest elasticity of money demand must be greater than the average rate of change in the exchange rate.

2. It may be interesting to note that F. Black (1974) gets the same restriction.

(B) $e_t = c + \alpha(e_{t+1|t} - e_t) + m_t$

(C) $e_t = \bar{e} + \Sigma_{i=o}^{t} w_i u_{t-i}$

(B) is the reduced form for the exchange rate (equation (6) in the main text), and once again it has a solution of the form (C).

The now familiar procedure produces the following restrictions on $\bar{e}$ and the $w_i$:

(D) $\bar{e} = c + \bar{m}$

$w_1 = \beta w_o$ $\qquad\qquad$ $\beta \equiv (\alpha + 1)/\alpha$

$w_{i+2} - \beta w_{i+1} + hw_i = 0$ $\qquad\qquad$ $h \equiv g/\alpha$

Again, there is no restriction on $w_o$. The solution to the difference equation takes the form

(E) $w_i = c_1\lambda_1^i + c_2\lambda_2^i$

where $c_1$ and $c_2$ are constants that depend upon the initial conditions $w_o$ and $w_1$ $(= \beta w_o)$, and

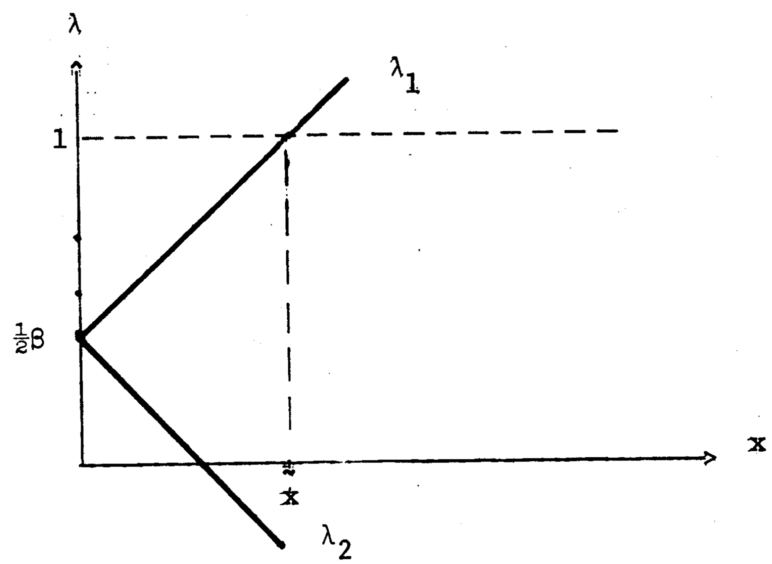
$$\lambda_1 = \tfrac{1}{2}[\beta+(\beta^2-4h)^{\frac{1}{2}}] \text{ and } \lambda_2 = \tfrac{1}{2}[\beta-(\beta^2-4h)^{\frac{1}{2}}]$$

The asymptotic variance of the exchange rate is

(F) $\sigma_e^2 = \sigma_u^2 \Sigma_{i=o}^{\infty} w_i^2 = \sigma_u^2[c_1^2\Sigma_o^{\infty}\lambda_1^{2i} + c_2^2 \Sigma_o^{\infty}\lambda^{2i} + 2c_1c_2\Sigma_o^{\infty}(\lambda_1\lambda_2)^i]$

so the policy problem is to find an h that will make the difference equation stable and the $\lambda_i$ real. Then the asymptotic variance will be finite.

Suppose first that $h \leq \beta^2/4$; then the $\lambda_i$ are real. Let $x = (\beta^2 - 4h)^{1/2}$; then if $\alpha > 1$ (so that $\frac{1}{2}\beta < 1$), $\lambda_1$ and $\lambda_2$ are functions of $x$ as shown below;




Any choice of $x$ in $[o, \tilde{x})$ will produce a stable difference equation for the $w_i$. Since $w_o$ and thus $c_1$ and $c_2$ are not known, it is not possible to tell which point in $[o, \tilde{x})$ is preferable. Now $x$ is in $[0, \tilde{x})$ if $g = \alpha h$ is in $(1, (1+\alpha)^2/4\alpha]$; so once again the optimal $g$ is greater than one.

## Appendix 4: Non-stochastic Extraneous Variables

If the extraneous variable is exactly predictable then future values of the extraneous variable will also be included in the solution. With this adjustment the results are much the same as before.

To see this consider the equations

(I) $e_t = c + \alpha(e_{t+1|t} - e_t) + m_t$

(II) $m_t = \bar{m}$, a constant

(III) $u_t \varepsilon \{u_o, u_1, \ldots\}$, a non-stochastic sequence.

and consider solutions of the form

(IV) $e_t = \bar{e} + \Sigma_{i=o}^{t} w_i u_{t-i} + \Sigma_{i=1}^{\infty} v_i u_{t+i}$

where the $w_i$ and the $v_i$ are undetermined coefficients. Here future values of the extraneous variable have been added to the solution.

Using the now familiar procedure (and noting that $e_{t+1|t} = e_{t+1}$ since all of the $u_{t+i}$ are known to speculators at time t), the interested reader will be able to show that (IV) is a valid solution if the coefficients are chosen to satisfy the restrictions.

(V) $w_i = [(1+\alpha)/\alpha]^i \, w_o$

$v_i = [\alpha/(1+\alpha)]^i v_o$

$w_o = v_o$

Once again, $w_o$ can be any number.

It should be noted that the future values of the extraneous variable must be included in the solution. The interested reader will also be able to show that if

$$\text{(VI)} \quad e_t = \bar{e} + \Sigma_{i=o}^{t} w_i u_{t-i}$$

is proposed instead of (VI) as a solution form, then the restrictions on the coefficients become $w_i = 0$. The extraneous variable can not be built into the solution unless future values are considered.