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I. Introduction

The Multi-Country Model (MCM), under development in the International Finance Division at the Federal Reserve Board, is a system of linked national macro-economic models, at the center of which is a medium-sized model of the U.S. economy. Linked to it, and to each other, are models for Canada, West Germany, Japan, the United Kingdom, and an abbreviated model representing the rest of the world. The country models are connected by trade and capital flows, interest rates and prices.

The general goal motivating the development of the MCM was to improve our ability to analyze the international influences on the U.S. economy and those on the world economy emanating from the United States. This led in turn to three more specific objectives.

The first was to specify in greater detail than heretofore the interrelationships between the United States and other economies. We have made a special effort in this area, as mentioned above, by a careful linkage of bilateral trade flows, prices, and financial sectors. Domestic prices, for example, are affected by import prices, and import prices, in turn, are linked to other countries' export prices and

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exchange rates. The financial sectors of the various countries are connected in a number of ways. Foreign interest rates affect the domestic asset demands. Further, capital flows affect the international reserves and money supplies of each country.

The second goal of the MCM project was to provide detail on the economies of the individual foreign countries included in the system. Each country model taken alone is large enough to explain that country's main domestic and international transactions: i.e., real GNP and its major components; prices; interest rates and monetary aggregates; exchange rates and the major trade, service, and capital account flows in the balance of payments. Further, the major instruments of monetary and fiscal policy in each country are integrated into the model, thus facilitating the analysis of the impact of government policy changes on both the home and foreign countries.

The third purpose, and the subject to which this paper is devoted, was to improve our ability to analyze developments in the U.S. economy, and in the world economy, under a regime of flexible exchange rates. When the project was started all econometric models of the U.S. economy could handle only a fixed exchange-rate regime. The country breakdown and linkages in the MCM now permit us to work directly with bilateral exchange rates, four of which are determined endogenously in the system.[1] More generally, the MCM can be used to study the effects of economic policies under a variety of exchange-rate systems --from fixed rates, to the current managed floating, to a pure float with no exchange market intervention.

This paper will concentrate on the process of exchange rate determination and the analysis of exchange-rate effects in the Multi-Country Model. The next section and the Appendix cover in some detail the mechanism by which exchange rates are determined in the MCM. Section III provides statistics on how well the model has performed inside and outside the sample period, with particular attention paid to exchange rates. The last two sections of the paper discuss another aspect of exchange rate determination: the multiplier or simulation effects of changes in exogenous policy variables on exchange rates. To this end, some illustrative monetary policy multipliers are presented and later analyzed in considerable detail.

II. The Determination of Exchange Rates in the Multi-Country Model.

Each of the four bilateral exchange rates in the Multi-Country Model are proximately determined by use of an ex ante balance of payments equation. We say "proximately" because of the natural qualification that in a general equilibrium model everything is interdependent; with that caveat in mind it can be stated that in the MCM the balance of payments equation for a given country is used to solve for the bilateral exchange rate between that country's currency and the dollar.[2]

Despite a rather long history of the use of balance of payments conditions, there is some controversy in the profession concerning their use as equilibrium conditions, and, in fact, a considerable theoretical effort must be mounted to justify them. Since this is a key decision in modeling flexible exchange rates, in the text and appendix to this paper we will discuss in some detail both the justification for and the merits of the use of balance of payments equations as equilibrium conditions. It is also appropriate to dwell on this issue, because much recent work in the monetary theory of the balance of payments and exchange rate has emphasized that the exchange rate, as the price of foreign currency, should be determined in the <u>money market</u>.[3] We shall show below that the apparent inconsistency between that statement and our approach is only superficial.

Recent advances in general equilibrium theory teach the would-be model builder that great care should be exercised in describing the underlying market behavior which constitutes the basic building blocks of the model. A general equilibrium model of any kind is essentially a collection of markets and the determination of every price, interest rate and exchange rate should be consciously related to behavior -- demand and supply -- in one or more markets. In focusing on the behavior in a given market, the analyst should consciously decide whether the market is best modeled as an equilibrium process, where price moves to clear supply and demand every period, or whether, alternatively, some disequilibrium process is more appropriate.

Each country model in the MCM contains five basic markets: a market of a domestically produced good (which is assumed differentiated from the goods produced by other countries), a labor market, a market for domestic money, a short-term bond market, and a long-term bond market. Given this description, it is natural to ask where exchange rates are determined and what role balance of payments equations play.

As we discuss in considerable detail in the Appendix, the balance of payments condition is substituted for the equilibrium condition in the short-term bond market in each country model. Moreover, the balance of payments equation for a given country, an ex ante concept, can be shown to be equal to a linear sum of the market clearing conditions in each market of that country model.[4] Thus, a given country model as it appears in the MCM contains market equilibrium or, in some cases, disequilibrium conditions for a domestic goods market, the labor market, domestic money, long-term bonds, and a balance of payments condition.

We assert here, and prove in the appendix and in Stevens (3), that this representation is theoretically equivalent to the original model which contains no balance of payments condition. This is true because the BOP condition contains the market-clearing condition for the short-term bond market -- as well as all other markets; thus it can be shown that when the balance of payments condition clears, along with the other markets in the model, that the short-term bond market also clears, and vice versa.

Why did we take such an indirect approach? There were a number of reasons for our decision, partly related to tradition and partly to questions concerning available data.

The first reason was our desire to maintain the consistency between the method of modeling the monetary sectors in the MCM and that of other econometric models. In no case that we knew of did any econometric model actually model the short-term bond market; in all models of the United States, for example, the short-term interest rate was and is determined not in the bond market, but in the money market. The short-term bond market was the market that was dropped in accord with Walras' Law. But Walras' Law allows one to drop only one market, bond or otherwise, in any given model; thus in a linked model of "n" countries such as the MCM, "n-1" short-term bond markets must be modeled. By using our balance of payments equation we were able to do that indirectly, rather than directly, and by using a set of equations -- capital flow and trade equations -- with which we had some experience. On the other hand, we felt much less experienced estimating directly demand and supply equations for short-term bonds. Little work had been done in this field. Moreover, given the large number of short-term instruments in existence, one either had a difficult aggregation problem or the necessity of making an arbitrary choice of one short-term bond and of ignoring all the rest.

By using a balance of payments equation, the observed demand by foreigners for all domestic short-term assets are aggregated in the data for short-term capital inflows; a similar statement holds for domestic demand for all foreign financial assets. For better or for worse, by using balance-of-payments capital flow data, the aggregation over different assets is already made for the researcher. With the possible exception of foreign demands for domestic money, which might appear separately in the money market, one therefore is not forced to identify and explain separately the foreign demands for specific domestic assets. This might appear to be a rather questionable and messy solution to a difficult issue -- and it may be! -- but it should be recalled that researchers have at least some experience estimating capital flow equations, more experience we would say than for estimating supply and demand equations for the short-term bond market.

A second reason for preferring our approach, given that it was an initial step into what promised to be a very difficult field, was that it would allow the country models in the MCM to continue solving for the short-term interest rate in the money market, as is customary. If bond markets were modeled explicitely, it would be natural to expect to solve for interest rates in the bond market and exchange rates in the only market left -- the money market. The use of balance of payments conditions allowed us to maintain this consistency between the models in the MCM and the leading econometric models for the countries represented.

A. Comparison with the Asset and Monetarist Approach to Exchange Rates

What we have said above should suggest that there is no inconsistency between the approach taken here and any theory maintaining that exchange rates are determined in asset markets. By the asset approach, we mean any theory that postulates that the exchange rate, in the shortest run, is determined in financial asset markets and, along with interest rates, equilibrates the stock demand and supply for assets.

The MCM encompasses all the above strands of argument -- and a few more. The asset demand equations in our model are all stock demand equations. Moreover, here we have asserted (and in another paper, (3), proved) the equivalence between the system we have used and one where exchange rates help clear markets for asset stocks.

On the other hand, the MCM does not adopt the more extreme additional assumptions that characterize what has often been called the monetarist approach: in addition to the assumptions behind the monetary approach, those of purchasing power parity, perfect substitutability among financial assets in different countries and, sometimes, the so-called small country assumption that postulates fixed foreign interest rates and prices. These assumptions could easily be incorporated into the model, but is our view that for most of the countries in the MCM all of them would be empirically false. We are, after all, modeling mostly large countries. Moreover, there seems to be a consensus developing that purchasing power parity does not hold in any form in the short run, if at all. More debatably, perhaps, is our rejection of the perfect substitutability assumption, by our adoption of a portfolio balance approach to capital flows; the alternative approach could be incorporated into the MCM if the preponderance of the empirical evidence supported it.

Returning to a comparison of the MCM with models that take a monetary (as opposed to monetarist) approach to the exchange rate, some discussion should be given to the flow nature of our balance of payments equilibrium condition. The BOP condition, in combining the market equilibrium conditions in all markets, must combine conditions from both flow and stock markets. The terms from the goods markets, the (domestic) demand for imports and the (foreign) demand for exports, are usually modeled as flow conditions. Asset terms in the BOP equation are, as we have explained, based on equilibrium conditions for markets where the demand and supply for stocks of assets are important.

In a period model such as the MCM, the inconsistency between stock and flow conditions is eliminated by taking the change (first-difference) in the stock equilibrium condition, thus changing it into a flow condition. To put it in other words, if markets for stocks are in equilibrium at every point in time, one way to derive the equilibrium exchange rate (and interest rates) at time t, is to find those rates that make the change in asset demands equal to the change in asset supplies. This can be shown as follows; assume that changes are equilibrated:

$$D_t(e) - D_{t-1}(e_{t-1}) = S_t(e) - S_{t-1}(e_{t-1})$$

Since it is assumed that stock equilibrium held at time t-1, we have $D_{t-1}(e_{t-1}) = S_{t-1}(e_{t-1})$; therefore, the flow equilibrium condition that exchange rates move to equilibrate changes in demand and supply implies that stock demand and supply are also equal: $D_t(e_t) = S_t(e_t)$. In more complicated cases, where asset markets need not be in equilibrium, one can derive similar correspondances between the stock and flow equilibrium conditions.[5]

Besides the first-difference form of the asset market equilibrium condition, our BOP condition contains flow terms for exports and imports of goods and services, interest payments, and so on. This emphasizes something that tends to be forgotten by believers in the monetary approach: that the exchange rate affects much more that asset markets, and that in all models except those relating to the shortest of runs, these flow effects on the exchange rate cannot be ignored. Our results, which still must be considered preliminary, show significant effects of exchange rate changes on trade and service flows within a quarter, so we feel there is some empirical evidence supporting this position.

A final point of comparison between the MCM and the asset approach is to indicate what happens to the BOP equilibrium condition when the period of the model is shortened; i.e., as one contracts the period of observation from a quarter to shorter and shorter time spans, moving in the limit toward a continuous-time model. As one would hope and expect, the flow market terms, trade and service flows, have less and less impact on exchange rate determination as the time period contracts; in fact as one approaches the limit of a continuous model, the effects of trade and service flows disappears, and the BOP conditions "collapses" to the stock market equilibrium conditions emphasized in the monetary approach.[6] Thus in the shortest run, the MCM, also, would tell the story that exchange rates are determined only in asset markets.

III. The Performance of the MCM in Explaining Exchange Rates and Other Key Variables

To evaluate the performance of the MCM and its component country models, it is natural to compare the tracking ability of each model alone with the ability of the MCM as a whole. To this end, the individual models and the MCM were simulated dynamically both over their common sample period, 1964:4 through 1975:4, and outside the sample period, from 1976:1 to 1977:1 These results reveal, in our view, acceptable performance not only for the country models alone but also for the MCM, indicating that the errors in individual models are not compounded when the models are linked together. The statistics for exchange rates and a few other key variables are given in Tables 1 and 2. The mean error (ME) is included as an indicator of bias, along with the root mean square error (RMSE). Both statistics are in terms of percentage errors so that the units of different varibles do not hinder comparability.

The country models perform well, judging from the statistics in the table and from the simulation results of variables unreported here. For the variables most commonly compared across models, GNP and prices, the errors averages are low, expecially for a 45 period dynamic simulation over a change in exchange rate regimes. It is harder to provide a basis of comparison for in-sample errors in exchange rates, since so few models have endogenous exchange rates.

The out-of-sample forecasts seem quite promising, when their root mean square errors are compared to those for the sample period.

Table 1

Within-Sample Error Statistics*
(1964:4 to 1975:4)

	US	Canada	Japan	W. Germany	U.K.
GNP (1972 prices)					
unlinked	2.4	2.4	3.0	4.5	3.4
	(-0.7)	(0.09)	(1.2)	(0.3)	(-1.1)
MCM	2.3	2.0	4.0	3.6	3.9
	(-0.9)	(-0.4)	(-2.5)	(-0.3)	(-1.6)
Absorption deflator					
unlinked	0.94	2.4	1.3	3.6	5.4
	(-0.6)	(0.1)	(0.4)	(0.3)	(2.9)
MCM	1.0	0.7	2.4	3.5	5.1
	(-0.7)	(0.3)	(-0.8)	(0.04)	(3.1)
Short-term interest rate					
unlinked	19.3	26.1	9.0	18.5	29.7
	(-15.8)	(8.7)	(2.7)	(6.1)	(8.6)
MCM	20.4	27.7	11.2	10.7	23.1
	(-17.7)	(-1.5)	(1.3)	(-2.9)	(2.2)
Exchange Rate (vis-a-vis $)**					
unlinked	a/	2.5	5.6	9.2	6.2
		(1.0)	(4.7)	(-3.7)	(-5.2)
MCM	a/	4.4	12.6	11.5	9.7
		(-3.9)	(9.5)	(-8.7)	(-6.8)
Exports of goods (1972 prices)					
unlinked	b/	1.2	6.2	4.8	3.4
		(-0.2)	(-1.8)	(1.7)	(-0.1)
MCM	5.9	5.2	9.5	2.8	6.2
	(-0.6)	(-1.2)	(-5.5)	(-0.6)	(-1.7)

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* The first figure reported is the root mean square percentage error; in parenthesis is reported the mean percentage error or bias.

** The statistics are calculated only for the period in which the exchange rate was not fixed, since inclusion of the fixed rate period would bias the results downward.

a/ No exchange rate is solved for when the U.S. model is run alone; see, above, p. 4 and footnote 2. In the MCM, the "U.S." rate is a weighted average of the four bilateral rates solved for.

b/ When the U.S. model was run alone, because neither the exchange rate nor foreign variables were endogenous, exports were held exogenous.

Table 2

Out-of-Sample Error Statistics*
(1976:1 to 1977:1)

	US	Canada	Japan	W. Germany	U.K.
GNP, 1972 prices					
unlinked	5.5	1.4	0.7	0.9	4.0
	(-5.4)	(0.4)	(0.1)	(-0.6)	(-3.7)
MCM	7.7	0.8	0.9	1.7	6.5
	(-7.5)	(-0.4)	(0.2)	(-1.7)	(-6.2)
Absorption deflator					
unlinked	0.5	1.4	2.5	1.0	3.4
	(-0.3)	(0.4)	(-2.3)	(-0.4)	(3.0)
MCM	0.5	0.6	2.1	1.0	3.9
	(-0.3)	(-0.1)	(-1.0)	(-0.4)	(3.4)
Short-term interest rate					
unlinked	9.9	17.4	3.8	11.3	6.9
	(-6.7)	(16.2)	(-0.7)	(7.8)	(2.3)
MCM	25.4	20.4	4.8	11.6	7.9
	(-21.3)	(19.1)	(-1.8)	(7.3)	(-0.7)
Exchange Rate (vis-a-vis $)**					
unlinked	a/	10.8	5.8	11.7	24.3
		(-10.1)	(5.5)	(4.1)	(23.2)
MCM	a/	13.3	11.2	12.3	20.7
		(-12.4)	(10.2)	(3.0)	(19.6)
Exports of goods, (1972 prices)					
unlinked	b/	7.3	4.1	3.6	2.9
		(5.0)	(-2.6)	(0.6)	(0.4)
MCM	3.3	3.7	5.1	2.6	9.6
	(0.9)	(-0.9)	(1.1)	(-2.3)	(-9.5)

* The first figure reported is the root mean square percentage error; in parenthesis is reported the mean percentage error or bias.

** The statistics are calculated only for the period in which the exchange rate was not fixed. Since inclusion of the fixed rate period would bias the results downward.

a/ No exchange rate is solved for when the U.S. model is run alone; see, above, p. 4 and footnote 2. In the MCM, the "U.S." rate is a weighted average of the four bilateral rates solved for.

b/ When the U.S. model was run alone, because neither the exchange rate nor foreign variables were endogenous, exports were held exogenous.

In fact, for most variables, we were surprised to find the size of the RMSE's comparable to those for dynamic simulations within the sample period.

Despite the high degree of simultaneity within and across models, the performance of the linked MCM compares favorably with that of the unlinked country models. The performance of some variables is improved dramatically, notably for a number of German variables within the sample period; however, on the other hand, there is some deterioration in the ability to predict certain Japanese variables and the U.S. interest rate. Overall, the linkage of the models had an essentially neutral effect on in-and out-of-sample prediction errors.

The RMSE's of the GNP variables are, by generally accepted criteria (2 to 4 percent, in sample), highly satisfactory both in the MCM and in each country model alone. In comparison with the other variables in Table 1, the in-sample RMSE's of the short-term interest rate (RS) in percentage terms may appear to be quite high. However, these relatively large errors result from both the nature of the data and the inability of models in general to predict these variables with accuracy. For example, in the Federal Reserve's MPS model, the ratio of the RMSE of RS to its sample mean is 20.4% while the corresponding statistic for the rate in the MCM is 28.8%.

No comparable previous modeling effort has simultaneously endogenized the four exchange rates that are present in this model. The in-sample errors on these variables range from 4.4 to 12.6% in the MCM, and slightly less in the country models alone. Outside the sample period the results, except for the U.K. and to a smaller extent, Canada, are comparable to and even superior to those for the sample period.[7]

IV. Endogenous Exchange Rates and Multiplier Analysis

The intent of the previous section was to demonstrate the MCM's ability to track with reasonable accuracy exchange rate movements within and beyond the sample period. In this section we look at some key analytical questions regarding the endogenization of exchange rates in a model such as the MCM: namely, what the model says regarding (1) the impact of changes in government policy and other exogenous variables on exchange rates (multiplier analysis); and (2) the effects of the exchange rate regime on the size of the important multipliers.

To summarize what will be shown below, as presently constructed the MCM frequently exhibits what many would say are quite large exchange rate effects -- both in the size of exchange rate movements induced by policy changes and in the importance of the effects of flexible exchange rates on other key variables. Moreover, these exchange rate effects contribute importantly to the overall size of most multipliers with respect to such key goal variables as GNP and prices. To illustrate these conclusions we will present selected aspects of typical monetary policy simulations, focusing most on multipliers for Japan and the United States. After describing the results of two typical simulations, we will discuss in the next section the reasons for these results -- their essential causes as far as we can now determine. We hope that this final analytic section will provide some guidance for econometric practitioners who may be trying to detect the causes of unusual results in simulations dealing with the exchange market.

A. Monetary Policy Multipliers in Japan and the United States

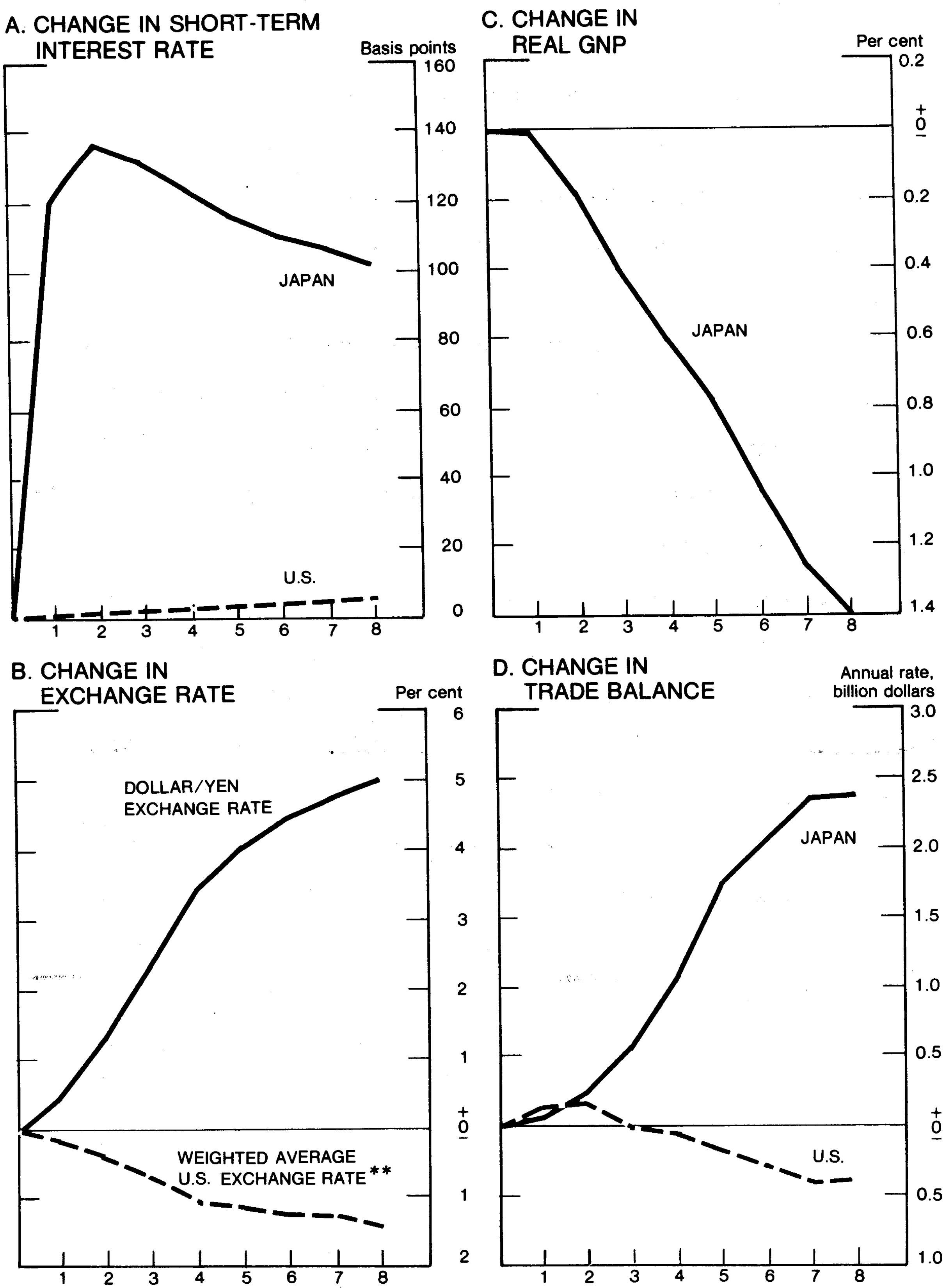
The "largeness" of a number of the exchange rate effects in the MCM first was observed in the context of calculating monetary policy multipliers. The two following cases provide good illustrations: (1) an increase in the Bank of Japan's official discount rate, and (2) a contractionary open market operation by the Federal Reserve Board in the United States.

The effects of an increase in the Bank of Japan's discount rate by one percentage point are shown in Chart 1. In panel A, the Japanese short-term interest rate is seen to increase sharply in the first two quarters and to decline gradually thereafter. Although the U.S. short-term rate rises moderately, there is, initially, a substantial increase in the interest-rate differential in favor of Japan. This increase reduces the relative attractiveness of borrowing from the U.S. and Eurodollar markets, thus leading to a large appreciation of the yen against the dollar, reaching 5% after two years. The yen appreciates vis-a-vis all other currencies as well.

The rise in domestic interest rate also has an adverse impact on fixed investment in Japan, resulting in a prolonged contraction of aggregate demand. This leads to an improvement in the Japanese trade balance (as seen in panel D) and to additional upward pressure on the Yen. Finally, as indicated in Chart 2, Japanese prices decline under the combined effects of reduced capacity utilization, increased unemployment and exchange rate revaluation; and there is also some upward

Chart 1

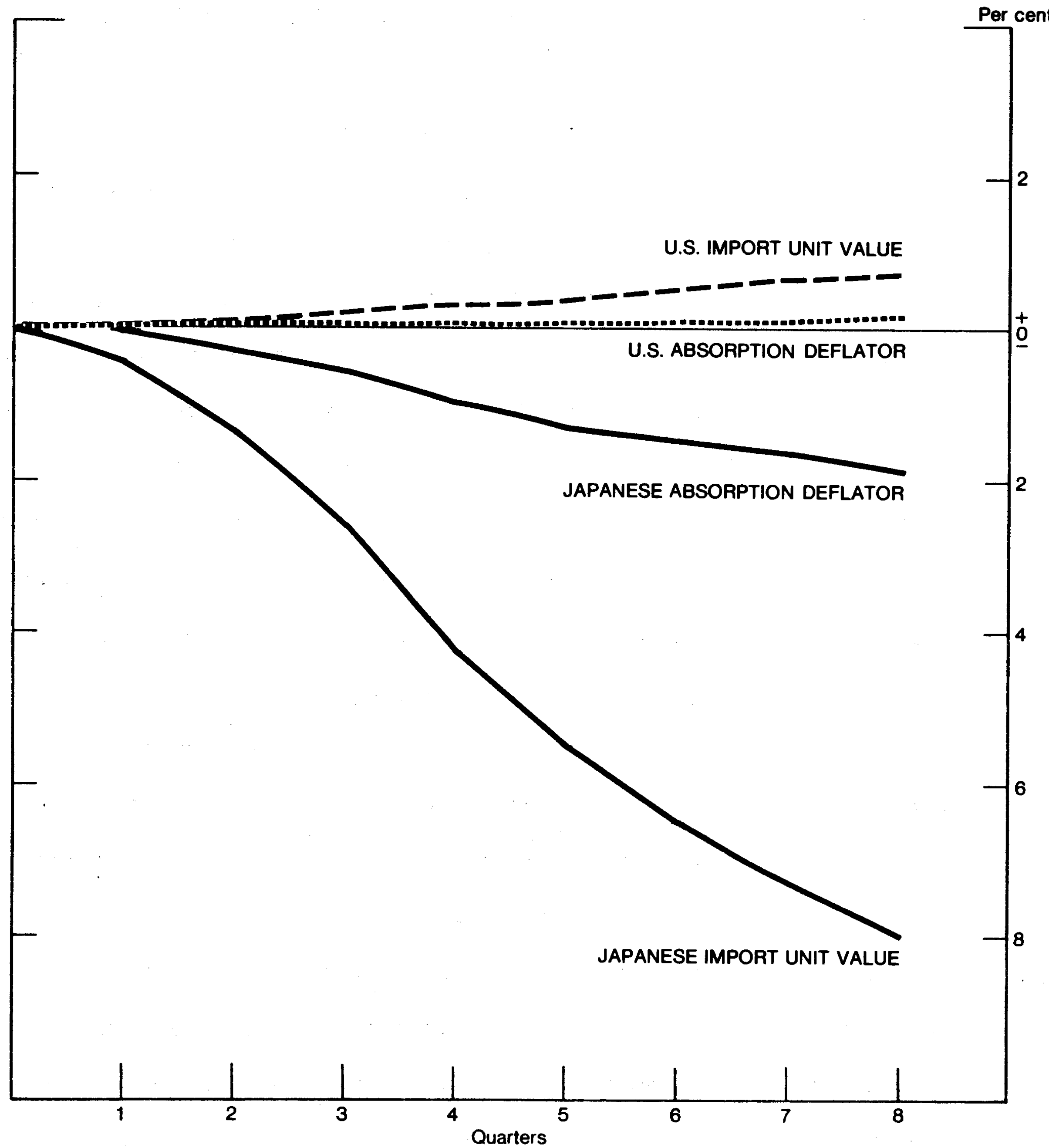
Effects of a One Percentage Point Increase in Japan's Discount Rate*



* All changes are measured relative to conditions that would prevail in the absence of policy actions.
** Units of foreign currency per dollar

Chart 2

Effects on Prices of a One Percentage Point Increase in Japan's Discount Rate*



*All changes are measured relative to conditions that would prevail in the absence of policy actions.

pressure on U.S. prices stemming from the devaluation of the dollar.

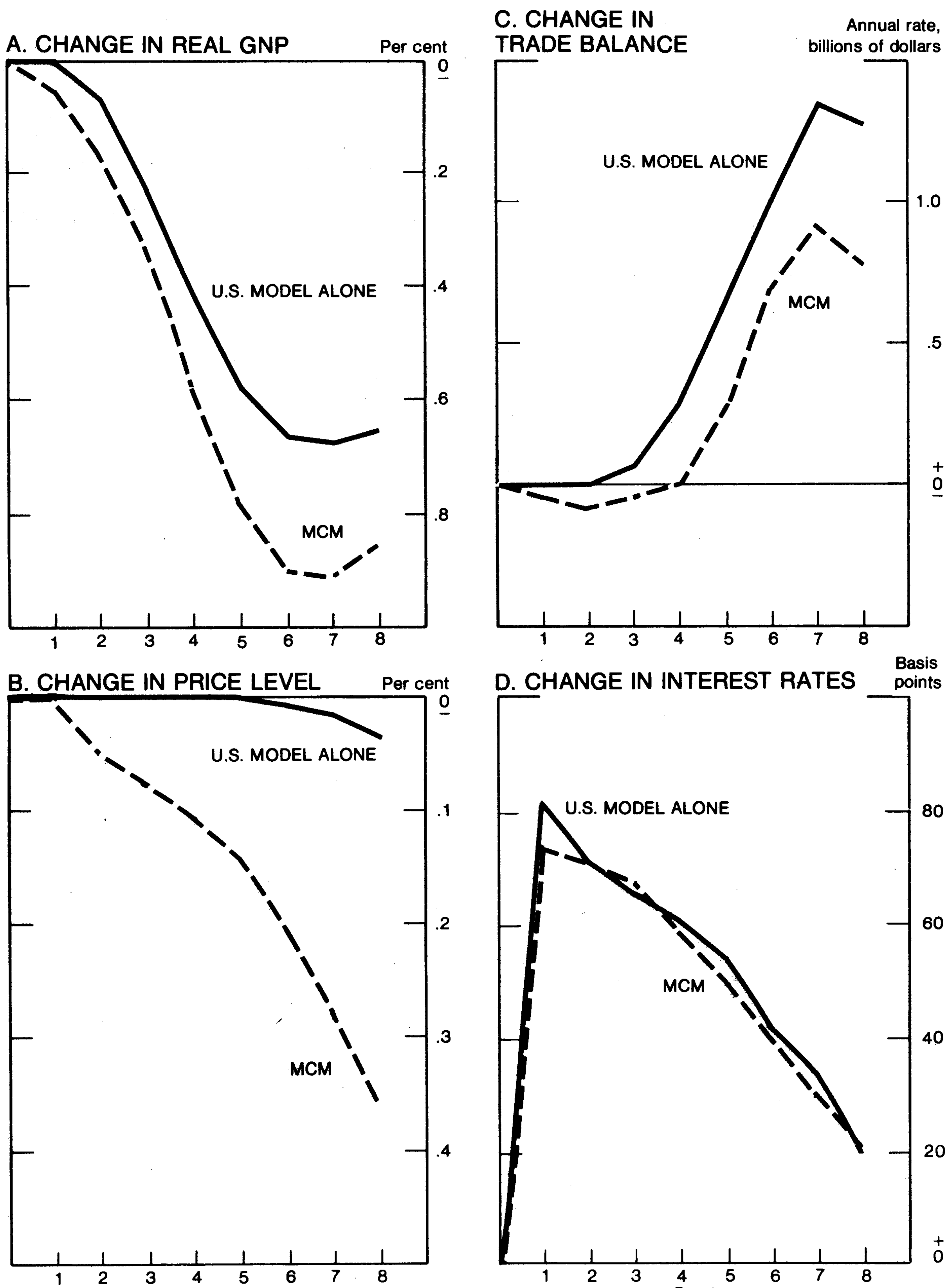
Two striking properties of this simulation that have been questioned, inside and outside the Federal Reserve Board, are (1) the relative size of the exchange rate and GNP effects and (2) the length of time over which they cumulate.

This pattern does not occur in monetary simulations for every country. In Canada, for example, a discount rate change which raises the domestic short-term rate by approximately one percentage point, causes an exchange rate appreciation of only 1 percent. However, the major characteristics of the Japanese simulations are **repeated** for typical monetary contractions in both West Germany and the United States. An investigation of the causes of such a pattern is clearly in order.

Before moving to such an analysis let us present some evidence that the large observed changes in exchange rates indeed have significant effects on other key monetary multipliers. That this type of exchange rate effect is frequently dramatically large is illustrated by the following results for U.S. monetary simulations. In Chart 3 results are presented for a given monetary contraction, a $1 billion sales of government securities, under different circumstances: (1) in the Multi-Country model, where exchange rates are endogenous, and (2) in the U.S. model alone, with the exchange rate held fixed. The major difference between the two cases is in the fixity of the exchange rate. Chart 3 shows that the results can differ significantly. With the exchange rates endogenous, U.S. GNP falls considerably more (over .25%) than when it is fixed. Much of this effect is caused by the impact of the appreciation on exports

Chart 3

Effects of a Change in U.S. Monetary Policy*



* All changes are measured relative to conditions that would prevail in the absence of policy actions.

(negative) and the trade balance (positive). Another important impact of the endogenization of the exchange rate is on the change in the domestic price level. Panel B of Chart 3 shows that with the exchange rates endogenous, the fall in the price index (absorption deflator) is, after two years, some seven times more than what it would be under fixed rates. A major element in this dramatic difference is the fall in import prices due to the three per cent appreciation of the dollar.

V. Analysis of the Exchange Rate Effects in the MCM

Despite the fact that the MCM and the individual country models therein track over the sample period and forecast quite well, there has been considerable questioning of the realism of the size and shape of the exchange rate responses observed in simulations such as those discussed in the previous section. People, both inside and outside the Federal Reserve Board, have questioned the reasonableness of exchange rate responses that in percentage terms are many times larger than the accompanying interest rate changes. Moreover, for these monetary policy simulations, they have questioned the length and progressive build-up of the response, continuing as it does more than three years after the precipitating change in monetary policy.

This section is an exercise in detective work. Its purpose is to isolate the causes of the unusual exchange rate patterns, particularly as they are manifested in the simulation of the effects of the change in the Japanese discount rate. We hope that such an exercise will prove useful, both by increasing our knowledge of the internal workings of the MCM and, perhaps more important, by providing a case history of an effort to illuminate a set of puzzling results thrown-up in the course of simulating a large econometric model.

Large models make for interesting and difficult cases for detectives, because there are always many potential "suspects," most of whom are related to and interacting with each other. However, in the case of the Japanese simulation discussed above, there are three basic channels through which any suspect cause exerts its effect. One is the capital account of the balance of payments: higher domestic interest rates attract a flow of foreign capital into Japan. In order to maintain balance of payments (BOP) equilibrium the exchange rate must appreciate: this increases the trade deficit and also reduces the capital inflow, because, in the MCM, appreciation creates expectations of a subsequent depreciation. This in turn requires adjustment on the domestic side: at the appreciated exchange rate export demand is lower, so there is an excess supply of goods, and real income must fall. The quantitative significance of capital account equations and coefficients in causing the observed exchange rate patterns will be examined in detail in sub-section B below.

The second major channel is the domestic goods market: higher domestic interest rates reduce investment and thus lower aggregate demand. Real income falls, and this tends to bring about a trade surplus, requiring a further exchange appreciation to maintain BOP equilibrium. Thus the new equilibrium which results from the increase in the discount rate must have an appreciated exchange rate and a lower real income. The trade balance is pulled in two opposite directions --toward surplus by the fall in real income and toward deficit by the appreciation --so that in general we cannot say what the final result will be. The same is true of the actual capital inflow.

The size of the final changes in income, the trade balance, and the exchange rate depends on the size of the initial shock and on the various elasticities in the system. For example, the greater is the response of either the trade balance or the capital account to an exchange rate change, the smaller is the exchange appreciation needed to maintain equilibrium. The greater is the interest elasticity of either investment or capital flows, the more the exchange rate must appreciate. This is because any factor which tends to bring about a larger capital inflow or a larger trade surplus requires a larger appreciation to restore equilibrium; similarly the more these variables respond to the exchange rate the less the exchange rate needs to change.

The final outcome is also affected by the money market: as real income falls the demand for real balances also falls, tending to lower interest rates and to move the economy back toward the original equilibrium. The larger the income elasticity of the demand for money, the more the interest rate must adjust to maintain equilibrium, and the less the exchange rate need appreciate.

Finally, interactions with the rest of the world will affect the outcome in Japan, partly at least through the channels already described. When Japan's interest rates rise, the resulting capital inflow will raise foreign interest rates as well. This tends to shut off the capital flow, and the exchange rate needs to appreciate less in order to maintain equilibrium than it would if all foreign variables remained constant. Further, a fall in Japanese income will lower foreign income as well, because Japanese demand for foreign goods is reduced. This reduces foreign demand for Japanese goods, and thus lowers the Japanese trade surplus. As a result the exchange rate must again depreciate relative to what it would otherwise have done. On the other hand, the exchange appreciation that does occur in Japan tends to stimulate foreign output, which raises the Japanese trade surplus, requiring a further appreciation in order to maintain BOP equilibrium. With the higher demand for its exports, Japan's income will rise, relatively.

Thus the magnitude of the final changes in income and the exchange rate is determined by the interaction of variables throughout the model. The key factors seem to be the responses of investment, trade, and capital flows, to changes in interest rates and the exchange rate, and it is to these that we should look to interpret the empirical results obtained in the Japanese case. (The behavior of prices and labor markets is neglected in this analysis --while in the MCM there is interaction both ways between prices and income, and between the exchange rate and prices, we assume these effects do not alter those described above.)

Unfortunately, due to the number of sectors and the level of disaggregation in the MCM, the various elasticities referred to above are not readily computed. (Because most of the equations are estimated in linear form the price and income elasticities are not generally constant, there is no ready basis for comparison with other models.)

However, some idea of the roles played by these various channels in determining the response of the model to the discount rate shock can be obtained by running the model under different assumptions about the exogenous variables or about the coefficients in key equations. Here we report some simulations which help to determine the quantitative significance of the three major channels discussed above.

A. Foreign Variables, Intervention and Fixed Investment Held Exogenous

First, in order to measure the effects of foreign variables, the same discount rate simulation was repeated with all foreign variables held exogenous. (Japanese exports and imports, and the dollar-yen exchange rate, however, remained endogenous.) Chart 4 shows the results. The exchange rate paths are very similar for the two simulations. For the first 7 quarters the exchange rate appreciates fractionally more in the unlinked model (with foreign variables exogenous); after that the appreciation is greater in the linked simulation. While the differences are measurable, they do not seem large enough to have any great economic significance, particularly in the first two years after the initial shock.

Thus in order to isolate the primary factors driving the exchange rate in the wake of the discount rate shock it was necessary to repeat the experiment with a number of other variables held constant. In order to save computer time the simulations were performed with the unlinked Japanese model: that is, foreign variables were held exogenous. Since the endogenization of foreign variables affected the exchange rate in only a minor way, the qualitative differences between simulations obtained here should also be found when these simulations are run in the fully linked model.

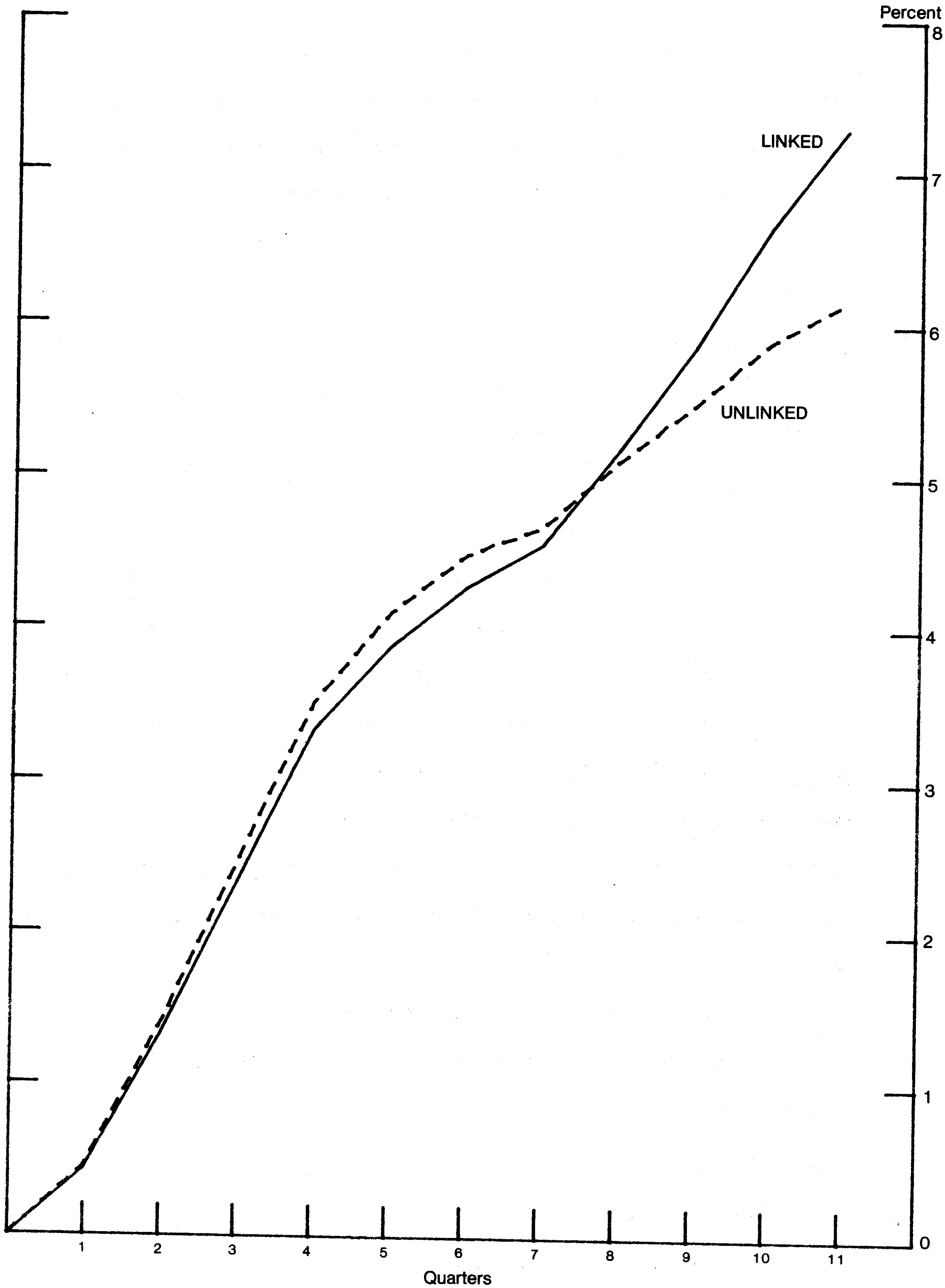
The accompanying figures show the paths of two key variables, real GNP and the exchange rate in yen per dollar, for the following simulations:

(1) the discount rate shock of 1 point alone

(2) discount rate shock with government exchange market intervention exogenous

Chart 4

One Percentage Point Increase in Japan's Discount Rate: Effect on Exchange Rate of the Exogenization of Foreign Variables



(3) discount rate shock with investment exogenous

The first simulation shows the response of the unlinked model to a discount rate shock, and is shown for purposes of comparison. Simulation (2) eliminates government intervention in the foreign exchange market. This removes a stabilizing factor from the system, since in the basic model the government is assumed to act to dampen any exchange rate movements. However, Charts 5 and 6 show that in this case government intervention is relatively unimportant.

Finally, by holding real investment constant in the final simulation we eliminate one of the major channels by which a discount rate change affects the economy. Chart 5 shows that the fall in investment is responsible for almost one half of the total change in exchange rates and is particularly important in the second half of the simulation period.

The previous simulation indicates that the interest elasticity of investment is a critical parameter in determining the magnitude of the change in the exchange rate. To investigate this factor further the discount rate exercise was run with the interest sensitivity of nonresidential investment set to alternative values and the following results were obtained:

Interest sensitivity of investment as a percentage of basic model	Exchange rate appreciation after 11 quarters (per cent)	Change in real GNP after 11 quarters (per cent)
100%	4.91	-1.92
50%	3.98	-1.12
0%	2.83	-0.32

Chart 5

One Percentage Point Increase in Japan's Discount Rate: Effect on Exchange Rate of Alternative Assumptions

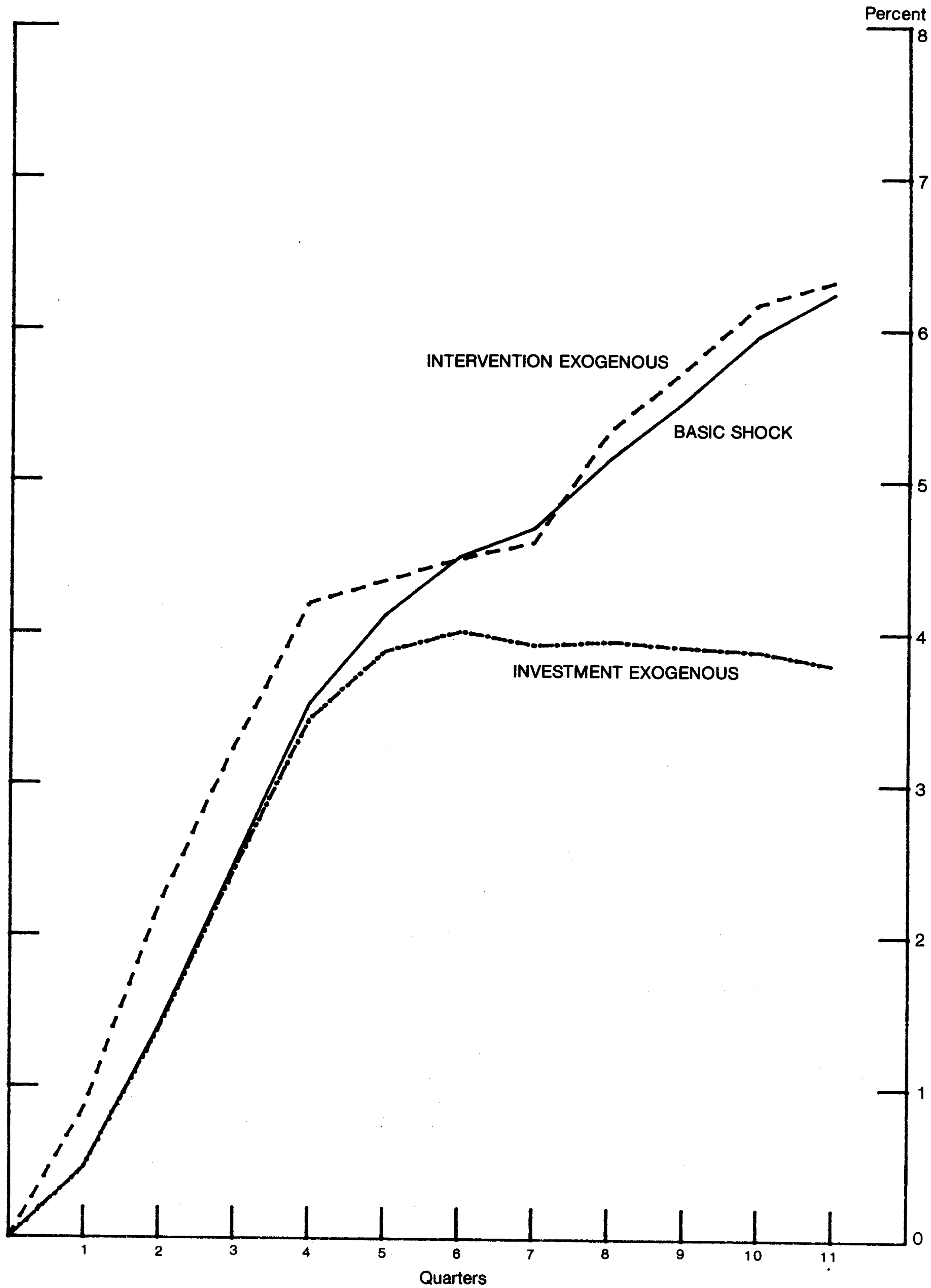
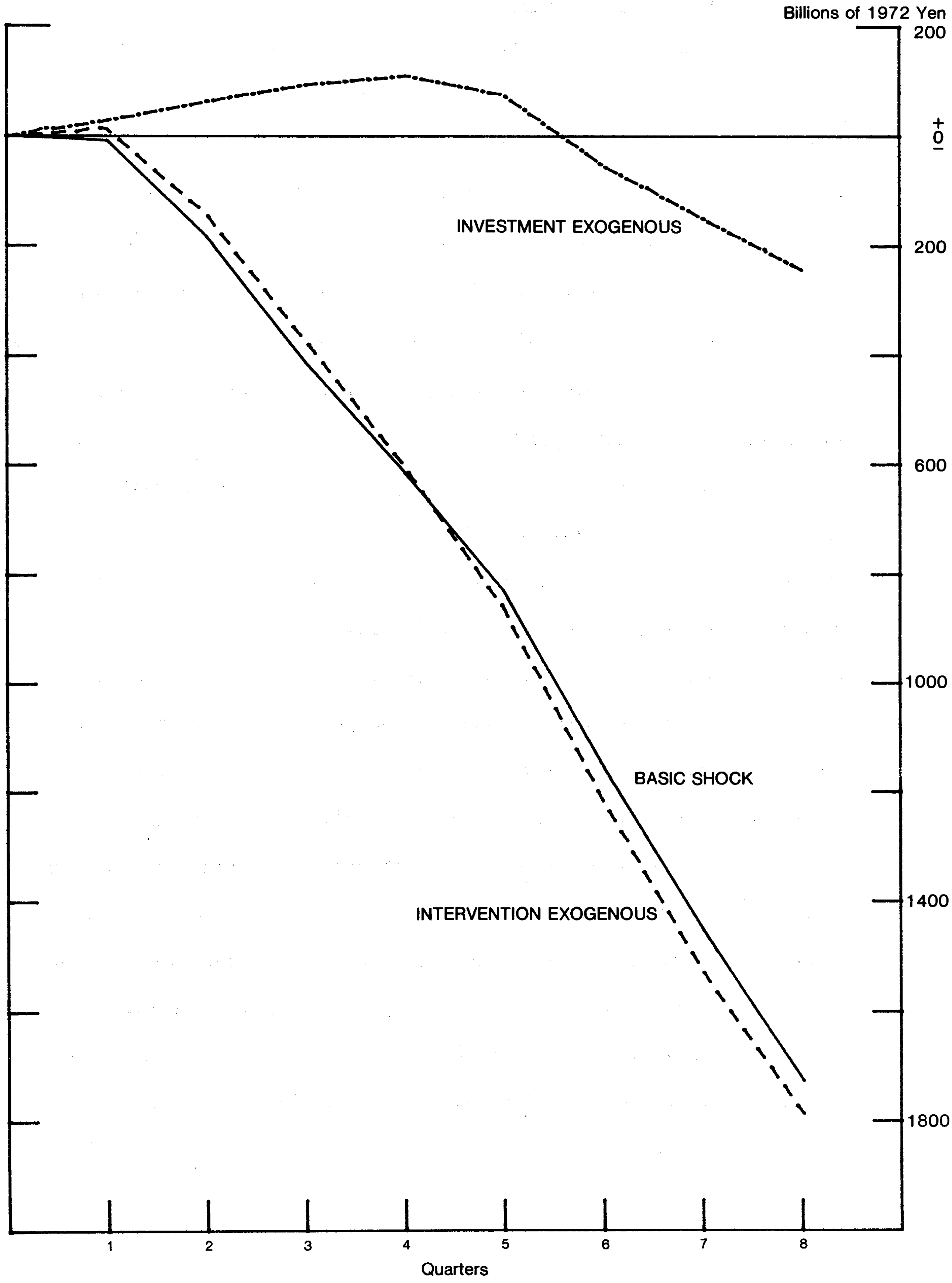


Chart 6

One Percentage Point Increase in Japan's Discount Rate: Effects on Real GNP of Alternative Assumptions



B. Capital Account Effects

Although the interest rate-investment-GNP nexus explains about half of the total exchange rate effect in the Japanese discount rate simulation, not only is another half still unaccounted for, but also that half is concentrated in the first five quarters of the simulation period. A further series of simulations indict the capital account, in general, and the estimated coefficients of the exchange rate terms, in particular, as the second major culprit.

The difference in the sizes of the estimated interest-rate and exchange-rate coefficients in many of our capital flow equations seems to be a primary cause of the large exchange rate multipliers in the MCM. A rise in domestic interest rates tends to induce a relatively large capital inflow. However, because of the smaller exchange-rate coefficients, in order to equilibrate the balance of payments, the exchange rate must appreciate (overshoot) by a much greater percentage than the change in the interest differential. To fully understand this point it would help to review briefly certain aspects of the specification of capital flows in the MCM.

Capital flows (changes in private short-term and long-term claims on and liabilities to foreigners) generally are derived from stock demand equations that include as determinants the expected returns on domestic and foreign assets, wealth proxies and various transactions demand and dummy variables. (In some cases capital flow equations are estimated directly as functions of changes in the stock demand determinants.) The expected return on domestic assets is the domestic interest rate, and that on foreign assets is the foreign interest rate plus the expected annual rate of change in the exchange rate.[8] In some cases the expected annual rate of change in the exchange rate is approximated explicitly by the fitted value of an equation relating the annualized forward premium or actual exchange rate change to deviations from purchasing power parity (as measured by relative export prices minus the current exchange rate) and the ratio of domestic imports to net foreign assets. More often, the expected exchange rate change is included only implicitly in the form of a deviation-from-PPP variable or import-to-reserve-ratio variable.

In empirical estimation, the coefficients on the interest rate and expected exchange rate change variables usually were unconstrained, although in some cases interest rates were expressed as differentials, resulting in coefficients constrained to be the same with opposite signs. Distributed lags often were included (up to 9 quarters long); the lags on interest rates generally are longer than these on expected exchange rates. The estimated long-run coefficients on domestic and foreign interest rates are generally similar in magnitude, but in cases where expected annual rates of change in the exchange rate are included explicitly (the Japanese, Canadian and UK models), the interest rate coefficients are ten times (or more) as large as the expected exchange rate coefficients.[9] These results suggest that a 1 per cent decline in foreign interest rates has about the same partial equilibrium effect on capital flows as a 10 per cent increase in the expected rate of appreciation of the domestic currency.

In order to measure the comparative sensitivity of capital flows to changes in exchange rates and interest rates, a number of partial-equilibrium simulations were run with the capital account sectors of the German, Japanese and Canadian models. The German and Japanese models were selected, as discussed above, because the mark and yen (bilateral rates against the dollar) have shown by far the greatest sensitivity to discount rate shocks. The Canadian model was also selected because, contrary to the mark and yen, the Canadian dollar is relatively stable in monetary shock simulations -- rising by about the same percentage as domestic interest rates in the long run.

These simulations involved several different shocks: (1) a 1 per cent (100 basis point) increase in all domestic interest rates; (2) a 1 per cent decline in all foreign interest rates; and (3), a 1 per cent appreciation of the domestic currency. In each case the expected exchange rate change was treated endogenously, but all other determinants of capital flows, including the relative export price and import/reserve ratio proxies for expected exchange rates were held exogenous. Thus, in equations that include the deviation-from-PPP proxy, a 1 per cent appreciation of the domestic currency, with the PPP rate (relative export prices) held constant, implies a 1 per cent increase in the expected depreciation (or decrease in the expected appreciation) of the domestic currency. This latter is not necessarily equivalent to either a 1 percentage point increase in the expected depreciation at an <u>annual</u> rate or a 1 percentage point change in the interest rate. The implied change at an annual rate depends upon the unspecified speed at which the shocked exchange rate is expected to return to equilibrium. A low responsiveness of capital flows to actual exchange rate changes relative to the responsiveness to interest rate changes could imply a very slow exchange rate adjustment speed.

The simulation results are shown in Table 3. The effects of each of the three different shocks on the private and official capital flows in the three models examined are shown in billions of local currency units for periods from 0 to 10 quarters after the shock and, cumulatively, over the whole period. Capital flows in the German and Japanese models are clearly much more sensitive to changes in interest rates than they are to changes in exchange rates, both immediately following the shock and cumulatively over an 11-quarter period following the shock. In the German case, the cumulative private capital inflow caused by the shock to foreign interest rates (DM 52.8 billion) is more than 30 times as great as the outflow caused by the exchange rate change (-DM 1.6 billion). In the Japanese case it is nearly 20 times as great.

The exchange rate changes required to equilibrate the German and Japanese payments balances after 1 per cent interest rate shocks are substantially less than 30 and 20 per cent, respectively, because the exchange rate change also induces equilibrating official capital flows and current account flows.[10] As shown in Table 2, the cumulative official capital flows resulting from exchange rate shocks (reflecting central bank intervention reaction functions) are about the same as the net private capital flows in the German case and somewhat greater

Table 3

Partial Equilibrium Impacts of 1% Changes in Interest Rates and Expected Exchange Rates on German, Japanese and Canadian Capital Accounts in the MCM

	Capital Inflows (+) and Outflows (-) in Billions of Units of Local Currency, AR. (Number of quarters following shock) 0	1	2	4	8	10	Cumulative Flow Σ 0 - 10
I. German Capital Account							
A. 1% Increase in German Interest Rates							
-- Private Capital, Net	9.6	8.5	7.0	3.3	.8	-.7	39.2
-- Official Capital	0	0	0	0	0	0	0
B. 1% Decline in Foreign Interest Rates							
--Private Capital, Net	3.5	5.5	4.7	5.2	5.8	.9	52.8
--Official Capital	0	0	0	0	0	0	0
C. 1% Appreciation of the Mark.							
-- Private Capital, Net	-1.8	-.3	.1	-.2	.1	.1	-1.6
-- Official Capital	-1.5	0	0	0	0	0	-1.5
-- Total	-3.3	-.3	.1	-.2	.1	.1	-3.1
II. Japanese Capital Account							
A. 1% Increase in Japanese Interest Rates							
-- Private Capital, Net	1283	1192	1048	467	0	0	4898
-- Official Capital	0	0	0	0	0	0	0
B. 1% Decline in Foreign Interest Rates							
-- Private Capital, Net	1227	1128	997	508	0	0	4871
-- Official Capital	0	0	0	0	0	0	0
C. 1% Appreciation of the Yen							
-- Private Capital, Net	-203	0	22	-17	-16	8	-276
-- Official Capital	-313	-4	-28	11	5	-6	-338
-- Total	-516	-4	-6	-6	-11	2	-614

(Table 2 continued)

	Capital Inflows (+) and Outflows (-) in Billions of Units of Local Currency, AR. (Number of quarters following shock) 0	1	2	4	8	10	Cumulative Flow Σ 0 - 10
III. Canadian Capital Account							
A. 1% Increase in Canadian Interest rates							
-- Private Capital	1.2	0	0	0	0	0	1.2
-- Official Capital	0	0	0	0	0	0	0
B. 1% Decline in Foreign Interest Rates							
-- Private Capital	1.4	0	0	0	0	0	1.4
-- Official Capital	0	0	0	0	0	0	0
C. 1% Appreciation of the Canadian Dollar							
-- Private Capital	-.2	0	-.2	-.1	-.1	-.1	-.6
-- Official Capital	-.5	0	0	0	0	0	-.5
-- Total	-.7	0	-.2	-.1	-.1	-.1	-1.1

than the net private flows in the Japanese case. The current account inflows are likely to be smaller than the capital flows in the near term, and would be offset by the income effects discussed above in the longer term.

In contrast to the German and Japanese results, Canadian private capital flows induced by the interest rate shocks are only about twice as large as the flows induced by the exchange rate shock. When official capital flows are combined with private flows, the exchange-rate-induced flow is almost as large as the interest-rate-induced flow. This result is consistent with the observed much lower sensitivity of the Canadian dollar to monetary policy shocks -- in the case of a discount rate increase, the percentage appreciation of the Canadian dollar was about the same as that of domestic interest rates.

The Canadian results suggests that the speed at which the Canadian dollar is expected to return to equilibrium following a shock is fairly rapid. This is confirmed by behavior of the endogenous expected exchange rate change variable in the Canadian model. In response to a 1 per cent appreciation of the Canadian dollar; *ceteris paribus*, the expected change variable fell by 2-1/2 per cent. That is, the Canadian dollar was expected to depreciate by 2-1/2 per cent at an annual rate, which means it was expected to take less than a half year for the 1 per cent appreciation to be reversed and the exchange rate to return to equilibrium.

VI. Summary and Conclusion

The goals of this paper have been to describe the theory underlying the modeling the exchange rates in the Multi-Country Model, and the empirical implications stemming from the application of this theory.

Theoretically, the MCM takes an approach which, as is customary today, emphasizes the importance of asset markets for the determination of the exchange rate; however, we have also emphasized that the exchange rate enters into many other equations and markets in a quarterly model, trade and services in particular, and that the evidence indicates that these interactions should not be ignored. The many influences on the determination of the exchange rate are all present in our use of an ex-ante balance-of-payments equation, the theoretical justification of which is treated in detail.

Empirically, the MCM at this point seems to explain and predict exchange rate movements reasonably accurately, inside and outside the sample period. However, it was noted that forecasting work, in particular, has just begun. Moreover, the multiplier analysis of the model uncovered a potential flaw in the model's ability to predict exchange rate changes: apparent overly-large exchange rate effects accompanying a number of government policy changes. As reported in the preceding section, an extensive effort has traced causes of these effects for the Japanese model to (1) the elasticity of investment with respect to the interest rate and (2) the much larger elasticity of capital flows with respect to interest rate changes than to changes in exchange rates.

This knowledge, along with the results of more extensive forecasting tests, should provide the basis for deciding whether to modify key equations which are important for the determination of MCM's four exchange rates.

OTH = other assets

CUR = currency

RT = total reserves of commercial banks at the central bank; broken down into:

a) RR = required reserves

b) RX = excess reserves

NW = net worth

The assets of the central bank constitute the "sources" of the monetary base; by increasing these claims the central bank generates an increase in its liabilities, which represent the "uses" of the monetary base. In particular, by increasing the total-reserves use of the monetary base, i.e., RT, the central bank can induce commercial banks to expand their holdings of earning assets, thereby depressing interest rates and encouraging additional expenditures.

The link between a sector's balance sheet and income and expenditure flows is its budget constraint, which states that the difference between income and expenditures, i.e., savings, is necessarily equal to the change in claims minus the change in liabilities, i.e., the change in net worth, where valuation effects, namely, capital gains and losses, are excluded from liabilities and assets. Taking the first difference of the items in the central bank's balance sheet, the budget constraint of this agent is given by:

(1) $R_C - E_C \equiv \Delta NFA + \Delta NGP + \Delta RB + \Delta OTH - \Delta CUR - \Delta RR - \Delta RX$

where:

Appendix

Deriving Ex Ante Balance of Payments Conditions

The construction of an aggregate budget constraint for a country --at this point an identity --is the first step in deriving a valid *ex ante* balance of payments equation.

To understand how this is done in the Multi-Country Model it is necessary to describe the balance sheets and budget constraints of the various actors or economic agents appearing in the model.

A. Agents and Flow of Funds

Four types of domestic decision-making agents are differentiated in a typical country model: the central bank, the commercial banking sector, the government sector and the private non-bank sector. The salient features of each sector are described in turn. We then aggregate the four sectors to obtain the transactions of the entire economy with the rest of the world.

1. The Central Bank

A typical central bank has the follwoing balance sheet:[11/]

Central Bank Balance Sheet

A	L	
NFA	CUR	
NGP	RR	} RT
RB	RX	
OTH	NW	

where:

NFA = net foreign assets (international reserves)

NGP = net government position, i.e., claims on the government minus deposits held by the government at the central bank

RB = reserves borrowed by commercial banks from the central bank

R = receipts

E = expenditures

C = our mnemonic for central bank

2. <u>The Commercial Banks</u>

The balance sheet of the commercial banks is given by:

Commercial Bank Balance Sheet

A	L
RR	RB
RX	DD
STS	TD
LTS	
FA	NW

where:

STS = short-term securities

LTS = long-term securities

FA = foreign assets

DD = demand deposits

TD = time deposits

To simplify notation we have avoided the use of superscripts and subscripts, and we hope that our explanation of the balance sheets and budget constraints will make clear which are the assets and which are the liabilities of each sector. Thus the assets of commercial banks include required reserves (RB) and excess reserves (RX), which together equal total reserves (RT).[12] As we have seen above, these appear on the liability side of the central bank's balance sheet. We assume that there are two kinds of non-deposit financial assets: short-term securities (STS) and long-term securities (LTS), and that commercial banks are net holders of both types of assets. Finally, banks also hold claims on foreigners (FA). For our purpose at this point, it is not necessary to disaggregate these claims. The liabilities of banks include reserves borrowed from the central bank (RB), as well as demand deposits (DD) and time deposits (TD).

The budget constraint of the commercial banks, like that of the central bank, expresses the fact that the difference between receipts and expenditures is identically equal to the change in claims minus the change in liabilities (again, excluding capital gains):

(2) $$R_B - E_B \equiv \Delta RR + \Delta RX + \Delta STS + \Delta LTS + \Delta FA - \Delta RB - \Delta DD - \Delta TD,$$

where the letter "B" is our mnemonic for commercial banks. We have ignored real assets in both the balance sheets and the budget constraints of central and commercial banks because relative to their financial claims, their real assets are very small.

3. The Private Non-Bank Sector

This sector contains two separate agents: firms and households. We distinguish between the behavior of these two agents when it comes to determining expenditures and prices. The theory of the firm is used to derive equations explaining investment and pricing decisions, whereas household behavior determines consumption expenditures. With regard to the supply and demand for assets, however, we do not separate firms from households, and therefore we deal here with the aggregate private non-bank sector (which, for short, we shall denote as the "private" sector).

The balance sheet of the private sector is given below.

Private Sector Balance Sheet

A	L
CUR	STS
DD	LTS
TD	
FA	
KV	NW

The private sector holds claims on the central bank in the form of currency (CUR), claims on commercial banks in the form of demand and time deposits (DD and TD), and claims on foreigners (FA).[13] A large fraction of the net worth of the private sector is in the form of real assets, which appear in the asset column of the balance sheet as the nominal value of the net capital stock (KV). The private sector issues both short- and long-term securities to other domestic sectors and to foreigners, and these appear (STS and LTS) in the liability column. Both STS and LTS are measured net of private-sector holdings of securities issued by the government, which would otherwise appear on the asset side of the balance sheet.

In the budget constraint of the private sector the change in the value of the net capital stock (ΔKV), i.e., net investment spending, is included along with all other expenditures. Therefore the change in the private sector's net claims on other domestic sectors and on foreigners is equal to total receipts (income net of taxes plus transfers) minus total (i.e., current and capital) expenditures:

(3) $$R_P - E_P \equiv \Delta CUR + \Delta DD + \Delta TD + \Delta FA - \Delta STS - \Delta LTS$$

4. The Government Sector

Because of the inherent difficulties in constructing a balance sheet for the government, we restrict outselves here to a description of its budget constraint. Our treatment of this sector differs from the others in that we shall determine the government budget surplus or deficit as an endogenous variable. Our reason for doing this is that --as described in more detail below --the government deficit or surplus, when combined with the current account surplus or deficit, will yield an estimate of the change in the net worth (exclusive of capital gains or losses) of the combined non-government domestic sectors. There are three components of the government budget: government spending on goods and services (GV), which is assumed to be exogenous, and total government receipts (TV) and total government transfer payments (TRANV), both of which are explained within the model.

The surplus or deficit of the government is reflected in changes in its claims and liabilities. These are changes in government deposits at the central bank (ΔBU), which are part of the net government position of the central bank (ΔNGP), changes in demand and time deposits (ΔDD and ΔTD), issues of short- and long-term securities (-ΔSTS and -ΔLTS), and changes in foreign assets held by the government (ΔFA). The government budget constraint is therefore given by:

(4) $$TV - TRANV - GV \equiv \Delta BU + \Delta DD + \Delta TD + \Delta FA - \Delta STS - \Delta LTS$$

B. Flow of Funds and the Balance of Payments in the Country Sub-Model

By combining the budget constraints of all the domestic agents in a country sub-model we obtain a picture of the flow of funds among the domestic sectors, and by aggregating all these budget constraints together we can derive the flow of funds between a country and the rest of the world. To facilitate the exposition we make a few simplifications in certain components of individual budget constraints. First, we assume that changes in the net government position of the central bank (ΔNGP) reflect only changes in its holdings of short- and long-term securities, so that changes in government deposits at the central bank are assumed to be zero, i.e., $\Delta BU = 0$. Second, we ignore changes in other assets of the central bank, so $\Delta OTH = 0$. Finally, since free reserves (RF) are equal to excess reserves (RX) minus borrowed reserves (RB), and since in each sub-model we shall explain RF rather than its components, we substitute ΔRF for ΔRX-ΔRB in the central bank's and commercial banks' budget constraints. After incorporating these simplifications and substitutions, and after rearranging the terms in the budget constraints, we end up with Table 3.

Table 3. Flow of Funds, Budget Constraints and the Balance of Payments of the Country Sub-Model

Real Side: Receipts - Expenditures			Financial Side: Change in Assets and Liabilities								
1. Central Bank: C	$R_C - E_C$	≡	$-\Delta CUR$	$-\Delta RR$	$-\Delta RF$			$+\Delta STS$	$+\Delta LTS$	$+\Delta NFA$
2. Commercial Banks: B	$R_B - E_B$	≡		$+\Delta RR$	$+\Delta RF$	$-\Delta DD$	$-\Delta TD$	$+\Delta STS$	$+\Delta LTS$	$+\Delta FA$
3. Private Non-bank: P	$R_P - E_P$	≡	$+\Delta CUR$			$+\Delta DD$	$+\Delta TD$	$-\Delta STS$	$-\Delta LTS$	$+\Delta FA$
4. Government: G	TV-TRANV-GV	≡				$+\Delta DD$	$+\Delta TD$	$-\Delta STS$	$-\Delta LTS$	$+\Delta FA$
5. Total: T	$R_T - E_T$	≡	0	0	0	$-\Delta DD_T$	$-\Delta TD_T$	$-\Delta STS_T$	$-\Delta LTS_T$	$+\Delta FA_T$
6. $XGSV^S$ + XTRANV - MGSV - MTRANV		≡				$-\Delta DD^S$	$-\Delta TD^S$	$-\Delta STS^S$	$-\Delta LTS^S$	$+\Delta FA$
7. $XGSV^D$ + XTRANV - MGSV - MTRANV		=				$-\Delta DD^D$	$-\Delta TD^D$	$-\Delta STS^D$	$-\Delta LTS^D$	$+\Delta FA$

New variables:
XGSV = receipts from exports of goods and services
XTRANV = receipts of transfers from abroad
MGSV = expenditures on foreign goods and services
MTRANV = expenditures in the form of transfer payments to foreigners

By aggregating the budget constraints of the four sectors in the sub-model we obtain the overall budget constraint for the country as a whole. This is shown in row 5, which is the sum of the first four rows. We assume that there is no foreign demand for currency, required reserves and free reserves, so that transactions in these assets net out to zero among the four sectors of the economy. The aggregate budget constraint for the country then states that the difference between <u>total</u> receipts and <u>total</u> expenditures (R_T-E_T) is necessarily equal to the change in aggregate liabilities to foreigners ($-\Delta DD_T-\Delta TD_T-\Delta STS_T-\Delta LTS_T$) plus the change in aggregate claims on foreigners (ΔFA_T). Since row 5 is obtained by aggregating ex post identities, it is itself an ex post identity.

When we sum all the receipts and expenditures of the four domestic sectors, transactions between sectors net out in the same way that claims and liabilities between domestic residents cancel out. On the real side of the budget constraint we end up, as shown in row 6, with total receipts from foreigners minus total payments to foreigners, which is the current account of the balance of payments, namely, receipts from exports of goods, services and transfers minus payments for imports of goods, services and transfers. Similarly, the capital account of the balance of payments, namely, the change in claims on foreigners minus the change in liabilities to foreigners, is given by the financial side of row 6. Thus we have just demonstrated that the balance of payments, when regarded as the ex post equality of the current account and the capital account, can be derived from the overall budget constraint of the country. Since one can go just as easily in the opposite direction, we can assert that ex post a country's balance of payments and its overall budget constraint are equivalent or identical.

In somewhat similar fashion one can work on the ex ante rather than the ex post level.[14] Ex ante as well as ex post each agent has a budget or wealth constraint; this states the simple fact that at any given set of prices, income, etc., the total value of an agent's demands for goods and assets must equal the total value of what he supplies to the market (including labor and the services of his assets). Another way to say it is that ex ante his sources and uses of funds must be equal.

This ex ante budget constraint is a relationship among an agent's demand and supply functions; it is a constraint on the functions and since it must hold at all sets of prices, it is an identity. By adding these ex ante identities over all domestic agents we obtain, as we did for Table 3 above, an ex ante budget identity for the country as a whole. Again, as we showed above, this overall budget identity can be expressed equivalently as a balance-of-payments identity.

One can, furthermore, transform this ex ante identity into an ex ante balance-of-payments equation or equilibrium condition. Why this is desirable is explained in section II, above.

There are many ways to convert the above identity into an equilibrium condition. The method we shall use is as follows.[15] Consider, for example, the entry for goods produced at home in the country's overall budget constraint; ex post this entry can be expressed either as the country's exports or the domestic excess supply of home goods; this is the first term on line 6 of Table 3. Ex ante the entry would be the ex ante excess supply of the home good (i.e. the difference at the given set of prices between home demand and home supply), which is represented by $XGSV^S$, where "S" denotes domestic excess supply. At the assumed set of prices, this is the country's potential exports; but we do not know ex ante whether foreigners will purchase this supply at the assumed set of prices.

In equilibrium, however, we know that foreigner's demand for the home good will be equal to the above domestic excess supply. Similarly, in equilibrium foreign demand for domestic assets equals the domestic excess supply of these assets. Thus, for conditions of equilibrium, if we substitute foreign demand functions for the home excess supply functions in the balance-of-payments identity, the resulting expression will still equal zero. However, out of equilibrium, this new expression need not equal zero, since foreign demand will not generally equal home excess supply for all sets of prices.

This is the convention we have adopted for building ex ante balance-of-payments equations: take a given country's ex ante budget constraint and substitute aggregate foreign demand for domestic excess supply wherever the latter appears. In row 7 of Table 3 we have made the appropriate substitutions, replacing the excess supply of home goods and services with foreign demand ($XGSV^D$, where "D" denotes foreign demand), and similarly, replacing equations describing the change in the excess supply of domestic assets with the change in foreign demand.

This substitution converts the balance of payments from an identity into an ex ante equation or equilibrium condition. The resulting expression equals zero in equilibrium, but can be non-zero for sets of prices (and other endogenous variables) that are not equilibrium values. It is proved by Stevens that such a balance-of-payments equation can be used as an equilibrium condition in our model and that it can be substituted for any of the other market clearing conditions.[16]

Footnotes

**/ The authors are, or at one time were, associated with the Quantitative Studies Section, Division of International Finance, Board of Governors of the Federal Reserve System, Washington, D. C., U.S.A. Messrs. Berner, Clark, Hernández-Catá, Howe, Kwack and Stevens jointly produced the Multi-Country Model and ran the dynamic simulations and multipliers reported in sections III and IV.A. Ralph Tryon planned and carried out the investigative simulations on the Japanese model reported in section V.A. Peter Hooper was responsible for the analysis of the capital flow equations and their properties reported in section V.B. This paper was written by Guy Stevens.

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1/ The bilateral exchange rates are those between the U.S. dollar and the yen, mark, U.K. pound and the Canadian dollar.

2/ The U.S. balance of payments equation would naturally be used to solve for the rate between the U.S. dollar and the "rest of the world" (ROW). Since we felt that the small ROW sector now in the MCM is too limited in scope and aggregated to make solving for an endogenous ROW exchange rate meaningful, we have not used the U.S. balance of payments equation for that purpose. At present the ROW exchange rate is assumed exogenous and the U.S. equation is used to calculate the reserve changes necessary to support the ROW exchange rate.

3/ This includes whatever official intervention occurs.

4/ See for example the papers by Frankel and Rodriguez (2).

5/ For a rigorous proof of this assertion see Stevens (3). The statement holds strictly only for markets that are in equilibrium; for a system in disequilibrium the above paper shows how an altered balance-of-payments condition may be substituted for the bond market.

6/ See Stevens (3).

7/ These points are proved in an as yet unpublished paper by Stevens which is available on request.

8/ A case can be made for the position, we think, that it would be virtually impossible to forecast the U.K. exchange rate in 1976-77 as accurately as in the preceding period. The 1976-77 period was one of great speculative activity, producing exchange rate movements far greater than those observed in the sample period.

9/In the U.S. model the foreign interest rate is a weighted average of the interest rate of the four other countries and the exchange rate is a weighted average of the four currencies. In other country models, the foreign interest rate at times is a dollar interest rate and at times a weighted average; the exchange rate is always the bilateral dollar rate (never a weighted average).

10/In one Japanese capital account equation and several U.K. equations, the expected exchange rate coefficients are not far out of line with the interest rate coefficients.

11/In MCM simulations, the mark appreciated by 12 per cent in response to a 1 per cent German discount rate increase, and the yen by 7 per cent in response to a 1 per cent Japanese discount rate increase.

12/Since institutions differ considerably, the details of central bank balance sheets and, of course, methods of monetary control differ significantly over countries.

13/Vault cash held by commercial banks is subsumed under RT, so that currency held by banks does not appear explicitly in their balance sheet.

14/As explained below, we break these claims down into two components: short-term and long-term portfolio claims, and long-term direct claims. At this point in our exposition, this disaggregation is not necessary.

15/The points made in the following paragraphs are developed in detail in Stevens (3).

16/See, for explicit proofs, Stevens (3), Section III.A.

17/See Stevens (3), Section IV.B.

References

(1) Berner, Richard, Peter Clark, Ernesto Hernández-Catá, Howard Howe, Sung Y. Kwack and Guy V.G. Stevens, "A Multi-Country Model of the International Influences on the U.S. Economy: Preliminary Results," International Finance Discussion Paper No. 115, Board of Governors of the Federal Reserve System, December 1977.

(2) Frenkel, J. and C. Rodriguez, "Portfolio Equilibrium and the Balance of Payments: A Monetary Approach," American Economic Review, Sept. 1975.

(3) Stevens, Guy V.G., "Balance of Payments Equations and Exchange Rate Determination," International Finance Discussion Paper No. 95, Board of Governors of the Federal Reserve System, December 1976.