International Finance Discussion Papers

Number 165

July 1980

RATIONAL EXPECTATIONS, RISK PREMIA, AND THE MARKET FOR SPOT AND FORWARD EXCHANGE

by

Richard A. Meese and Kenneth J. Singleton*

Rational Expectations, Risk Premia, and the Market For Spot and Forward Exchange

by

Richard A. Meese and Kenneth J. Singleton

1. Introduction

Implicit in models of exchange rate determination that assume markets for spot and forward rates are "efficient" are the assumptions of rational expectations, costless transactions, risk neutral market participants, and the absence of legal restrictions on transactions. Since these assumptions do not literally describe the foreign exchange market, the efficiency hypothesis also cannot be literally true. While this observation alone does not imply that the efficient markets model is a poor approximation to the true model, the recent empirical studies of Geweke and Feige (1979), Hakkio (1979), and Hansen and Hodrick (1980a, b) suggest that there have been significant departures from efficiency during the 1970's.

The primary purpose of this paper is to investigate the extent to which risk aversion, as manifested in a time varying risk premium, is capable of explaining these departures from efficiency. In an efficient market, the expected n-period holding period return equals the proportional n-period forward premium. We begin our empirical analysis by testing this version of the efficiency hypothesis and find that it is rejected by the data for the countries considered. To investigate the possibility that the failure of the efficiency hypothesis is a consequence of a time varying, ex ante risk premium, a testable lower bound on the variance of the ex ante risk premium is derived. For the countries considered, the test results support the hypothesis of a time varying, ex ante risk premium.

In the process of testing these hypothesis, we also discuss two practical issues that heretofore have received little formal attention in the literature. First, we attempt to overcome the potentially important problem of data alignment that is unique to foreign exchange markets. Three month forward contracts, for example, are not usually contracts for the delivery of foreign exchange ninety days hence. As a means of avoiding biases due to misalignment of forward rates with their respective future spot rates, we have constructed a new data set from daily data in which the rates are properly aligned.

Second, we formally address the problem of non-stationarity of spot and forward exchange rates. Our econometric tests of exchange market efficiency require the assumption of covariance stationarity[1/] of the variables in the model. Before studying the behavior of holding period returns, we test the null hypothesis that the natural logarithm of the exchange rate is well approximated by a random walk using the recently developed testing procedures of Fuller (1976), Dickey and Fuller (1979), and Hasza and Fuller (1979). The results of this analysis suggest that the n-period holding period return, which is approximately the n-th difference in the logarithm of the exchange rate, is covariance stationary and, hence, that the asymptotic distribution theory we employ to study risk premia is appropriate.

The paper is organized as follows. In section 2 we derive a relationship between the holding period return, the proportional forward premium, and the ex ante risk premium under the null hpothesis of efficiency. Then in section 3 we discuss the proper alignment of present forward and future spot rates, review the estimation problems inherent in exchange rate models where the sampling interval differs from the forward contract length, and present evidence that the aligned holding period return is covariance stationary. In section 4 we test and reject the holding period version of the efficiency hypothesis and present evidence of a time varying risk premium. Concluding remarks comprise section 5.

2. Rational Expectations and the Behavior of Risk Premia.

If the rational expectations hypothesis applies to the market for forward exchange, if this market is perfectly competitive, and if market participants are risk neutral, then

$$f^n(t) = E(s(t + n)|\phi(t)), \tag{1}$$

where $f^n(t)$ is the n-period forward price of foreign exchange observed in period t, $s(t + n)$ is the spot price at period $(t + n)$, and $E(s(t + n)|\phi(t))$ denotes the conditional expectation of the spot price in period $(t + n)$ based on the current information $\phi(t)$. The set $\phi(t)$ includes at least current and past $s(t)$ and $f^n(t)$. Equation (1) implies

$$s(t + n) = f^n(t) + \eta^n(t + n), \tag{2}$$

where $\eta^n(t + n)$ is the error from forecasting $s(t + n)$ using information dated t and earlier, and is therefore uncorrelated with $f^n(t)$. Since equation (2) can be estimated using standard regression techniques, it is frequently employed to conduct statistical tests of (1); see for example Frenkel (1976, 1977, 1979, 1980).

In this paper we focus on a different implication of (1), and utilize an approach to test the null hypothesis that has not been employed previously. To start with, the spot rate $s(t)$ is contained in the information set $\phi(t)$, so equation (1) implies,

$$\frac{f^n(t) - s(t)}{s(t)} = E\left(\frac{s(t + n) - s(t)}{s(t)} \,\middle|\, \phi(t)\right). \tag{3}$$

Equation (3) says that in a risk neutral world with rational expectations, the proportional n-period forward premium, $H^n(t) = (f^n(t) - s(t))/s(t)$, is an unbiased predictor of the holding period return, $H(t + n) = (s(t + n) - s(t))/s(t)$.[2/] This version of the efficiency hypothesis implies that the ex ante risk premium, which can be defined (Frankel (1979), p.6) as

$$E(H(t + n)|\phi(t)) - H^n(t) = (E(s(t + n)|\phi(t)) - f^n(t))/s(t), \tag{4}$$

is identically zero.

Statistical tests of the null hypothesis that the ex ante risk premium is identically zero can be formulated in several ways. First observe that equation (3) implies

$$H(t + n) = H^n(t) + \varepsilon^n(t + n), \tag{5}$$

where $\varepsilon^n(t + n)$ is the error from forecasting $H(t + n)$ using information in $\phi(t)$, and is thus uncorrelated with $H^n(t)$. From (5) it is clear that a regression of the forecast error or ex post risk premium, $\varepsilon^n(t + n) = (H(t + n) - H^n(t))$, on $H^n(t)$ and a constant should yield zero coefficients on the regressors. Formulated in this way, the regression test of (5) corresponds to a test of the variance inequality,

$$\mathrm{var}(H(t + n)) \geq \mathrm{var}(H^n(t)), \tag{6}$$

which is also implied by (5) when both series have finite variance.[3/] Both tests are based on the orthogonality of $H^n(t)$ and the forecast error $\varepsilon^n(t + n)$. Since (6) would still hold if $\mathrm{cov}(H^n(t), \varepsilon^n(t + n))$ were positive, the variance inequality (6) is a necessary but not sufficient condition for the null hypothesis to obtain.[4/]

Second, consider the variable $\hat{H}^n(t)$ defined as the linear least squares projection of $H(t + n)$ on $P(t)$, where $P(t)$ contains present and a finite number of past $H(t)$. Again, we can represent $H(t + n)$ as the sum of a predictor and forecast error,

$$H(t + n) = \hat{H}^n(t) + v^n(t + n), \tag{7}$$

where $\text{cov}(\hat{H}^n(t), v^n(t + n)) = 0$ by construction. Equating the right hand sides of (5) and (7) we can infer

$$H^n(t) = \hat{H}^n(t) + v^n(t + n) - \varepsilon^n(t + n), \tag{8}$$

where $\text{cov}(v^n(t + n) - \varepsilon^n(t + n), \hat{H}^n(t)) = 0$ because $P(t)$ is contained in the information set $\phi(t)$. In order to couch the regression test of (7) in terms of directly observable variables, note that

$$H(t + n) - \hat{H}^n(t) = (H(t + n) - H^n(t)) + (H^n(t) - \hat{H}^n(t)). \tag{9}$$

The forecast error $(H(t + n) - \hat{H}^n(t))$ is uncorrelated with the elements of $P(t)$. Since $(H^n(t) - \hat{H}^n(t)) = (v^n(t + n) - \varepsilon^n(t + n))$ from (8), it too is uncorrelated with the constitutents of $P(t)$. This implies that $\varepsilon^n(t + n) = (H(t + n) - H^n(t))$ is uncorrelated with $P(t)$, so a regression of the observable $(H(t + n) - H^n(t))$ on any of the constituents of $P(t)$ can be used to test (7). Again, we can associate this regression test with a test of a variance inequality, since under the null hypothesis equation (8) implies

$$\text{var}(H^n(t)) \geq \text{var}(\hat{H}^n(t)). \tag{10}$$

All of the implications of the zero ex ante risk premium hypothesis that we have discussed thus far can be tested jointly by performing a test of the null hypothesis $\underline{\alpha}' = (\alpha_1, \alpha_2, \alpha_3) = \underline{0}'$ in the regression equation,

$$(H(t + n) - H^n(t)) = \alpha_1 + \alpha_2 H^n(t) + \alpha_3 H(t) + \delta^n(t + n). \tag{11}$$

In section four we report the results of the regression test (11) based on weekly data for three currencies vis à vis the U.S. dollar.

If the foreign exchange market is not efficient, i.e., if the ex ante risk premium is not identically zero, then the ex ante risk premium has non-zero variance which is a testable implication. To be more precise, let $E(\varepsilon^n(t + n)|\phi(t))$ denote the ex ante risk premium (4), and define $Q(t)$ as the set of past and present ex post risk premia or forecast errors, $Q(t) = [\varepsilon^n(t), \varepsilon^n(t - 1), \ldots]$. Consider the linear least squares projection of $\varepsilon^n(t + n)$ on $Q(t)$, denoted $\hat{\varepsilon}^n(t)$. As before we can represent $\varepsilon^n(t + n)$ as the sum of two components,

$$\varepsilon^n(t + n) = \hat{\varepsilon}^n(t) + \Theta^n(t + n), \tag{12}$$

where the forecast error $\Theta^n(t + n)$ is uncorrelated with the predictor $\hat{\varepsilon}^n(t)$. Clearly, (12) implies $\text{var}(\varepsilon^n(t + n)) \geq \text{var}(\hat{\varepsilon}^n(t))$, and we can also deduce

$$\text{var}(E(\varepsilon^n(t + n)|\phi(t))) \geq \text{var}(\hat{\varepsilon}^n(t)) \geq 0, \tag{13}$$

as the projection set $Q(t)$ is contained in $\phi(t)$. Equation (13) provides a least lower bound on the variance of the ex ante risk premium, since the inclusion of more variables in the projection set $Q(t)$ cannot decrease the variance of the predictor $\hat{\varepsilon}^n(t)$. A test of the hypothesis that the ex ante risk premium is time varying, a sufficient but not a necessary condition for (1) not to obtain, can be conducted by performing a test that $\text{var}(\hat{\varepsilon}^n(t)) > 0$ using the asymptotic distribution theory derived by Singleton (1980). The results of this test are reported in section four.

Both the regression equation (11) and the variance inequality (13) can be interpreted as preliminary tests of the "portfolio-balance" approach to exchange rate determination, an approach that relies on the assumption of imperfect substitutability of domestic and foreign assets. In a recent paper Frankel (1979) does not reject the hypothesis of perfect asset substitutability and a zero ex ante risk premium. To conduct his analysis, Frankel specifies asset demand functions of the portfolio-balance type, and then inverts these functions to obtain an expression for the ex ante risk premium as a function of asset stocks. Under the null hypothesis of a zero risk premium and rational expectations, a regression of the ex post risk premium $\varepsilon^n(t + n)$ on the determinants of the ex ante risk premium should yield zero coefficients on the regressors. Since the coefficients on the various regressors in Frankel's estimating equations are insignificantly different from zero, he accepts the hypothesis of a zero risk premium and perfect substitutability.

One of the problems associated with empirical tests of the portfolio-balance models of exchange rate determination is that regression equations involve relative asset stocks and wealth variables. Since data for these variables is not readily available, it is advantageous to work with an estimating equation unencumbered by possible measurement error. To circumvent this problem note that past values of the forecast error are contained in the information set, so we can test Frankel's (1979) hypothesis using a regression equation of the form

$$\varepsilon^n(t + n) = \gamma_0 + \gamma_1 t + \sum_{i=2}^{K} \gamma_i \varepsilon^n(t - i + 2) + \rho^n(t + n).^{5/} \qquad (14)$$

Under the null hypothesis of efficiency, a zero ex ante risk premia, and perfect substitutability, all regression coefficients are zero. In section four we also report the results of this test.

3. The Problems of Data Alignment, Serial Correlation and Covariance Stationarity.

Existing empirical studies of the relationship between forward rates and future spot rates have largely ignored the institutional features of the exchange market which induce irregularities in the alignment of the data.[6/] For example, when two parties agree to a spot transaction of Swiss francs for dollars, the transaction is for a value date two business days following the day the transaction was closed.[7/] To be an eligible spot value date, it must be a business day in both the United States and Switzerland as both currencies are involved in the transaction. Similiarly, today's U.S. dollar-Swiss franc three month forward transaction involves settlement of a contract three months from the corresponding spot value date, provided it is an eligible day. The future spot rate that corresponds to this forward value date was contracted two business days prior to the forward value date. The use of spot and forward rate quotations to test the hypothesis that the forward rate is an unbiased predictor of the future spot rate requires knowledge of all banking holidays in all countries whose currencies were transacted over the entire sample period.

All results reported in section four are for Wednesday twelve o'clock Swiss franc-U.S. dollar, German mark-U.S. dollar, and Canadian dollar-U.S. dollar spot bid rates, and the mid-point of the twelve o'clock bid-offer spread on forward rates. These series are collected by the Federal Reserve System as part of their daily monitoring of the New York foreign exchange market. The spot and forward rates were aligned using back issues of Morgan Guarranty Trust Company's World Calendar of Holidays. Our sample period is 1976 through 1979 inclusive. The length of this sample was dictated by our inability to secure additional back issues of the World Calendar of Holidays.

The institutional features of the foreign exchange market that necessitate separate alignment of each forward rate and its corresponding future spot rate introduces an additional estimation problem. The time period between $s(t + n)$ and $f^n(t)$ cannot be considered constant, and some of the econometric procedures we employ assume a constant forward contract length n. When necessary, we use the largest value of n for each currency over the sample period. This value is $n = 14$ for weekly data and a three month foward contract. In addition, we report empirical results based on weekly data although daily observations are readily available. Weekly data are employed to circumvent the weekend or holiday unobservability problem, as economic and political events that occur when the exchange market is closed can affect the market when it re-opens. Last, the slight incomparability of spot (bid rates) and forward (mid-point of the bid-offer spread) is a potential source of distortion in our empirical results.

Estimation of equations (11) and (14) is further complicated by the fact that the regressors are predetermined, not exogenous, and the disturbances may be serially correlated. Under the null hypothesis of efficiency, the disturbances can be serially correlated whenever the interval of observation t is finer than the contract interval n. In equation (11) for example, the disturbance $\delta^n(t + n)$ may be correlated with $\delta^n(t + i)$, $0 < i < 2n$. Since the regressors in this equation are not strictly exogenous, conventional two-step generalized least squares (GLS) estimation techniques cannot be used to test the zero ex ante risk premium hypothesis. This point is ably made in a recent paper by Hansen (1980), where he develops the asymptotic distribution theory for the estimation strategy employed in this paper. This strategy involves ordinary least squares (OLS) estimation of (11) with a correction for the asymptotic bias in the OLS estimator of the covariance matrix of regression coefficients.[8/]

The asymptotic distribution theory we employ to test the ex ante risk premium hypothesis requires that the regressors in equations (11) and (14) be covariance stationary.[9/] Since there is considerable debate whether the levels (or logarithmic levels) of exchange rates have finite variance, an analysis of the stationarity assumption is clearly warranted.

Mussa (1978) has conjectured that the natural logarithm of the exchange rate is well approximated by a random walk. To support this conjecture, we report in table 1 the results of a test of the null hypothesis that the univariate autoregressive (AR) representations of $\ln(s(t))$ and $\ln(f^n(t))$ have unit roots.[10/] In all cases we reject the null hypothesis that these univariate autoregressions have two unit roots. We cannot, however, reject the null hypothesis that these autoregressions have one unit root, except for the German Mark-U.S. dollar spot exchange rate. Since fitting autoregressive processes of different orders had no effect on the hypothesis tests reported in table 1, these tests suggest that $\ln(s(t))$ and $\ln(f^n(t))$ do not have stable univariate autoregressive representations, even after mean and linear trend have been removed. Using a consistent estimator of lag length,[11/] the order selected for the univariate autoregressions of $\ln(s(t))$ and $\ln(f^n(t))$ was always one. This result coupled with our failure to reject the hypothesis that these autoregressions contains a unit root lends credence to Mussa's conjecture.

The foregoing analysis suggests that whenever empirical tests of (1) are based on information sets with only past and present $s(t)$ and $f^n(t)$, it is important to work with transformations of $s(t)$ and $f^n(t)$ for which covariance stationarity is a reasonable assumption. If the results of table 1 are taken to imply $s(t)$ and $f^n(t)$ are nonstationary processes, then we can rule out conventional tests of the efficiency hypothesis like equation (2), as the predetermined regressor $\ln(f^n(t))$ will not satisfy the requisite assumption of stationarity. If $\ln(s(t))$ follows a random walk, the holding period return $H(t)$ is approximately a finite moving average process and hence is convariance stationarity.[12/] Thus the asymptotic distribution theory required to test the efficiency hypothesis is appropriate when $H(t)$ is used as a regressor. The three tests of efficiency we report in the next section, equations (11), (14), and the variance inequality (13), require $H^n(t)$ and $H(t)$ to be jointly covariance stationary.[13/] Tests for the existence of unit roots in the joint autoregressive representation of $s(t)$ and $f^n(t)$ are not yet available, so we offer no empirical evidence on the joint stationarity of $H^n(t)$ and $H(t)$.[14/] The regression tests (11) and (14) can be conducted using only present and past values of $H(t)$ as regressors. This formulation results in test statistics qualitatively the same as those reported below.

Table 1

Tests for unit roots

All results refer to the equation:

$$y(t) = \beta_0 + \beta_1 t + \beta_2 y(t-1) + \beta_3 (y(t-2) - y(t-1)) + \text{error}.$$

Switzerland

$y(t) = \ln(s(t))$

	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$
coefficient	.1093	.1016-3	.9699	-.5593-3
OLS standard error	.6104-1	.5612-4	.1706-1	.7335-1

T = 191
R^2 = .9924
$\Phi_3(2)$ = 5343.7**, reject $H_0^{(1)}$ } see table notes below.
$\hat{t}_T$ = -1.764 , accept $H_0^{(2)}$

$y(t) = \ln(f^n(t))$

	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$
coefficient	.1041	.1013-3	.9714	.4130-2
OLS standard error	.5906-1	.5627-4	.1649-1	.7337-1

T = 191
R^2 = .9928
$\Phi_3(2)$ = 4787.5**, reject $H_0^{(1)}$
$\hat{t}_T$ = -1.734 , accept $H_0^{(2)}$

Table 1 continued

Canada

$y(t) = \ln(s(t))$

	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$
coefficient	.2388	-.6263-4	.9484	.1789
OLS standard error	.9710-1	.2630-4	.2091-1	.7161-1

T = 190
R^2 = .9933
$\Phi_3(2)$ = 156953.**, reject $H_0^{(1)}$
$\hat{t}_T$ = -2.468 , accept $H_0^{(2)}$

$y(t) = \ln(f^n(t))$

	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$
coefficient	.2807	-.6950-4	.9393	.1790
OLS standard error	.1050	.2687-4	.2267-1	.7157-1

T = 190
R^2 = .9921
$\Phi_3(2)$ = 207874.**, reject $H_0^{(1)}$
$\hat{t}_T$ = -2.678 , accept $H_0^{(2)}$

Germany

$y(t) = \ln(s(t))$

	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$
coefficient	.3407	.2049-3	.9062	.1525
OLS standard error	.1040	.6244-4	.2874-1	.7278-1

T = 191
R^2 = .9934
$\Phi_3(2)$ = 126907.**, reject $H_0^{(1)}$
$\hat{t}_T$ = -3.264* , reject $H_0^{(2)}$ at the 10% significance level

Table 1 continued

$$y(t) = \ln(f^n(t))$$

	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$
coefficient	.3038	.1898-3	.9164	.1336
OLS standard error	.9877-1	.6161-4	.2728-1	.7279-1

$T = 191$
$R^2 = .9936$
$\Phi_3(2) = 96537.$**, reject $H_0^{(1)}$
$\hat{t}_T = -3.065$, accept $H_0^{(2)}$

Table 1 notes

$H_0^{(1)}$ denotes the null hypothesis $\beta_2 = \beta_3 = 1$. The derivation of the test statistic and a tabulation of its distribution is given in Hasza and Fuller (1979). In all cases the test statistics exceed 12.31, the critical value for a 1% significance level, $T = 100$, (see page 1116 of Hasza and Fuller), so ** denotes rejection at well below the 1% significance level.

$H_0^{(2)}$ denotes the null hypothesis $\beta_2 = 1, \beta_3 < 1$. The derivation of the test statistic and a tablulation of its distribution is given in Dickey and Fuller (1979) and Fuller (1976). When $T = 100$ the critical value of $\hat{t}_T = -3.15$ for a 10% significance level, (see Fuller (1976) p.373), so * indicates rejection at the 10% significance level. $\hat{t}_T$ is the conventional t-test for $\beta_2 = 1$.

4. Some Empirical Evidence of Time Varying Risk Premia.

The results of the regression test (11) for bilateral U.S. exchange rates with Germany, Switzerland and Canada are reported in table 2. In each case the covariance matrix of the estimated parameters was corrected for serial correlation using a frequency domain procedure suggested by Hansen (1980).[15/] Our results are not supportive of the null hypothesis of a zero ex ante risk premium. The marginal confidence levels (MCL)[16/] for the tests $\underline{\alpha} = \underline{0}$ in (11) are for Germany, Switzerland, and Canada, 1.000, .8936 and .9670, respectively.

In table 3 we report the results of the regression test (14) using the same estimation procedure. The empirical work summarized in table 3 offers no support for the null hypothesis for Germany and Canada, as the marginal confidence levels of the chi-square statistics are both 1.000. The marginal rejection of the null hypothesis for Switzerland, table 2, is not reinforced by the results of table 3 where the MCL of the chi-square statistic is .5982.

Since the coefficients in our regression equations need not remain constant after a change in policy regime, we examined the subsample from November 1, 1978 to the end of 1979 to see whether the ex ante risk premium has been affected by President Carter's November 1, 1978 announcement of a dollar support package. While the marginal confidence levels of the German and Canadian regression tests (11) and (14) have not changed for this subsample period, the MCL for the Swiss regression tests are now both 1.000. Since the post November 1, 1978 subsample tests are based on approximately 33 observations, the asymptotic distribution theory may not provide accurate confidence levels for the tests we employ over this recent sample period.

Last, in table 4 we report the results of the test of the hypothesis that the lower bound on the variance of the ex ante risk premium (13) is zero. These test statistics, distributed as standard normals under the null hypothesis, indicate rejection of the null hypothesis at high marginal confidence levels for all three countries.

Table 2

Regression tests of (10)

Switzerland

	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_2$
OLS coefficient	.6537-1	-2.835	-.1175
corrected t-statistic*	2.410	-2.259	-.7169
T = 179	R^2 = .1114		
$X^2(3)$ = 6.219	Marginal confidence level = .8986		

Germany

	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_2$
OLS coefficient	.3763-1	-1.103	-.3906
corrected t-statistic	4.297	-.9657	-2.854
T = 179	R^2 = .2040		
$X^2(3)$ = 28.62	Marginal confidence level = 1.000		

Canada

	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_2$
OLS coefficient	-.1018-1	.4352-2	.1436-1
corrected t-statistic	-1.714	.4127-2	.1033
T = 178	R^2 = .0002		
$X^2(3)$ = 8.739	Marginal confidence level = .9670		

Table 2 notes

*The heading corrected t-statistic refers to the ratio of the OLS estimate of α_i to the square root of the i^{th} diagonal element of $(X'X)^{-1}\hat{S}_Z(0)(X'X)^{-1}$ the corrected covariance matrix of $\underline{\hat{\alpha}}$, see footnote 15.

Table 3

Regression tests of (13)

Switzerland

	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5
OLS coefficient	.2946-1	-.1028-3	.2070	-.9407-1	-.1407	-.7986-1
corrected t-statistic*	1.308	-.5188	.7819	-1.190	-1.835	-.4393

T = 177 R^2 = .0276
$X^2(6)$ = 6.194 Marginal confidence level = .5982

Germany

	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5
OLS coefficient	.4025-1	-.1405-3	.1092	-.2285	-.8789-1	-.2239
corrected t-statistic*	3.703	-1.288	.6273	-2.680	-1.352	-2.531

T = 177 R^2 = .2151
$X^2(6)$ = 160.8 Marginal confidence level = 1.000

Canada

	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5
OLS coefficient	-.1774-1	.6461-4	-.3245	.1383-1	.4455-1	.2521
corrected t-statistic	-1.776	.7878	-2.293	.2202	.6608	2.170

T = 176 R^2 = .0799
$X^2(6)$ = 94.61 Marginal confidence level = 1.000

*see table 2 notes.

Table 4

Variance bound tests of (12), $\mathrm{var}(\hat{\varepsilon}^n(t)) = 0$.

Switzerland

Point estimate of $\mathrm{var}(\hat{\varepsilon}^n(t))$* = 3.471 - 7	
test statistic = 4.391	Marginal confidence level = 1.000

T = 193

Germany

Point estimate of $\mathrm{var}(\hat{\varepsilon}^n(t))$ = 6.356 - 7	
test statistic = 4.667	Marginal confidence level = 1.000

T = 193

Canada

Point estimate of $\mathrm{var}(\hat{\varepsilon}^n(t))$ = 2.586 - 8	
test statistic = 4.879	Marginal Confidence level = 1.000

T = 192

*$\mathrm{Var}(\hat{\varepsilon}^n(t))$ is the variance of the n = 14 period ahead predictor of $\varepsilon^n(t)$ based on its own past. This variance was estimated and the null hypothesis $\mathrm{var}(\hat{\varepsilon}^n(t)) = 0$ tested using the procedures derived in Singleton (1980). The spectral density of $\hat{\varepsilon}^n(t)$ was estimated using an inverted V smoothing window with 27 periodogram ordinates.

5. Concluding Remarks.

In this paper we have examined the extent to which departures from exchange market efficiency over the period 1976-1979 can be attributed to a time varying, ex ante risk premium. In the process of testing this hypothesis special attention was paid to the important practical problems of data alignment unique to the foreign exchange market, and the assumptions underlying the asymptotic distribution theory required to conduct our statistical tests. The empirical results for Germany and Canada presented in tables 2-4 provide no support for the null hypothesis of a zero ex ante risk premium. The results do not, of course, imply a lack of rationality in the market for foreign exchange, since the null is a composite hypothesis of rational expectations, risk neutrality, and competitive markets without transactions costs. The empirical results for Switzerland are not nearly as clear cut. While the results of table 2 are unsupportive of the null hypothesis as the R^2 and individual t-statistics are quite large relative to the overall chi-square test, the Swiss regression results of table 3 do not reinforce the results of table 2. We can however, reject the hypothesis that the lower bound on the variance of the Swiss ex ante risk premium is zero at a very large confidence level using Singleton's (1980) variance bound procedure.

The mixed results for Switzerland and the decisive results for Canada and Germany suggest that exchange rate models need to explicitly account for factors that generate risk premia. This is hardly a new idea, as other authors have suggested the need to model exchange rates in a dynamic competitive equilibrium framework where factors which drive a wedge between forward rates and expected future spot rates can be explicitly modeled. Our split sample, post November 1, 1978 tests indicate that these factors appear to be growing, not diminishing in importance, as the estimate of the lower bound on the variance of the ex ante risk premium increased for each currency in the more recent sample period.

Footnotes

* International Finance Division and Carnegie-Mellon University respectively.

1/ A time series $[x(t)]$ is said to be covariance stationary if $\mathrm{cov}(x(t),x(\ell))$ depends only on $(t - \ell)$. Two time series $[x(t)]$ and $[y(t)]$ are said to be covariance stationary if each is covariance stationary and if $\mathrm{cov}(x(t),y(\ell))$ depends only on $t - \ell$.

2/ Since $H(t + n)$ and $H^n(t)$ are good approximations for $\ln(s(t + n)/s(t))$ and $\ln(f^n(t)/s(t))$ respectively, implication (3) of the null hypothesis avoids convexity problems associated with "Siegel's Paradox" (1972). Rogoff (1979) shows that the empirical relevance of Seigel's paradox cannot be dismissed a priori, without making specific assumptions on the joint distribution of $s(t)$ and $f^n(t)$. Most authors assume (1) holds for the natural logarithm of the spot and forward exchange rates, so the convexity problem does not arise.

3/ From (5), $\mathrm{var}(H(t + n)) = \mathrm{var}(H^n(t)) + \mathrm{var}(\epsilon^n(t + n))$, as $\mathrm{cov}(H^n(t),\epsilon^n(t + n)) = 0$ by construction. Since $\mathrm{var}(\epsilon^n(t + n))$ is non-negative, (6) follows immediately. The variance inequalities (6) and (10) are analogous to those derived by Leroy and Porter (1980), Shiller (1979), and Singleton (1980) for present value relations implied by rational expectations models of stock prices and the term structure of interest rates.

4/ This point has been noted independently by Geweke (1979), and it suggests that regression tests are robust to potentially interesting alternative hypotheses that variance bound tests lack power to reject. While regression tests may dominate variance bound tests on power considerations, the latter may be more useful when data alignment problems exist. For example, if a regression relation requires contemporaneous observations on the regressand and regressors, while actual data on these variables is published on different dates, a bias in estimation may ensue. This problem would not affect a variance bound test as all series are assumed to be covariance stationary. Variance bound tests are also unaffected by the serial correlation of the regression disturbance term.

5/ The choice of a finite K is arbitrary, and we use three lags of $\epsilon^n(t)$ in table 3 below.

6/ The work of Dooley and Shafer (1976) is a notable exception.

7/ See Reihl and Rodriguez (1977) for a discussion of foreign exchange markets.

8/ When the fraction $(n - 1)/n$ of the total number of observations are omitted by sampling every n^{th} observation, equation (11) can be estimated and the null hypothesis tested using the output from an ordinary least squares regression package. In this case the sequence of every n^{th} error $\delta^n(t)$ is serially uncorrelated so OLS is appropriate. Hansen and Hodrick (1980a) discuss the relative power of regression tests of (1) with and without all observations. Under a reasonable class of alternative hypotheses, they show that the modified OLS procedure described in the text dominates an OLS procedure using sampled data.

9/ Sufficient conditions on the regressors in (11) and (14) to obtain Hansen's (1980) results are stationarity and ergodicity.

10/ See Fuller (1976), Dickey and Fuller (1979) or Hasza and Fuller (1979) for a discussion of these tests. These authors supply asymptotic distribution theory for testing the null hypothesis that autoregressions (with and without trend and constant) have unit roots.

11/ There is a growing literature on the determination of the order of an autoregression. Since the estimators of autoregressive order are derived in the context of a stationary and ergodic time series with linear innovations, their use is less justifiable in our context. Using OLS point estimates of the autoregressive parameters, to calculate the roots of the lag polynominals we fit, it was always the case that these roots had squared moduli greater than one. While point estimates of the AR processes indicate borderline stationarity, we noted above that in virtually all cases we could not reject the hypothesis that one root is unity.

12/ Suppose $\ln(s(t))$ follows the process

$$\ln(s(t)) = \ln(s(t-1)) + \pi(t),$$

where $\pi(t)$ is the error from predicting $\ln(s(t))$ using past values of $\ln(s(t))$. Then

$$H(t+n) \doteq \ln(s(t+n) - \ln(s(t)) = \sum_{i=1}^{n} \pi(t+i),$$

a finite, noninvertible moving average process of order $(n-1)$.

In related work, Blattberg and Gonedes (1974) show that stock price holding period returns are better characterized by a Student t distribution than fat tailed symmetric-stable (Paretian) distributions for which second and higher order moments typically fail to exist. In a recent study of exchange rate changes, Westerfield (1977) concludes that exchange rate changes are better characterized by stable Paretian distributions, without finite second and higher order moments, than by the normal distribution. She does not, however, employ distributions with fatter tails than the normal whose second and higher order moments are finite.

13/ In addition, Singleton's (1980) variance bound test procedures require that the process $\epsilon^n(t) = (H(t+n) - H^n(t))$ have autoregressive representation. The foregoing analysis suggests that $H(t+n)$ itself, does not have an AR representation.

14/ Throughout the analysis, we have restricted attention to an information set $\phi(t)$ with only current and past exchange rates. Since interest rates via covered interest parity, current account imbalances, relative inflation rates, and other variables are known to influence spot and forward exchange rates, future empirical work in this area might best be conducted using vector autoregressive techniques. Our finding of unit roots in the univariate AR representations of $s(t)$ and $f^n(t)$ may be an artifact of our choice of the constituents of $\phi(t)$.

15/ Let $\underline{x}(t) = (1, H^n(t), H(t))$ be the t^{th} now of the observation matrix X. Define $\underline{Z}(t+n) = \delta^n(t+n)\underline{x}(t)$, and let T denote sample size. Hansen (1980) shows that the asymptotic covariance matrix of the OLS estimator $\hat{\underline{\alpha}}$ appropriate for hypothesis tests of (11) is $(X'X)^{-1}\hat{S}_Z(0)(X'X)^{-1}$. The middle term, $\hat{S}_Z(0)$, is the estimated spectral density matrix of $\underline{Z}(t)$ evaluated at the zero frequency. We estimate $S_Z(0)$ using T (see table 2) harmonic periodigram ordinates and a lag window (Daniell) of width 14 ordinates.

16/ The marginal confidence level is the probability of observing a random variable less than or equal to the calculated test statistic, given the random variable has the distribution of the test statistic under the null hypothesis. The marginal significance level is one minus the marginal confidence level.

Bibliography

Blattberg, Robert C., and Nicholas J. Gonedes; "A Comparison of the Stable and Student Distributions as Statistical Models for Stock Prices," Journal of Business 47 (1974), 244-280.

Cornell, Bradford; "Spot Rates, Forward Rates, and Exchange Market Efficiency," Journal of Financial Economics 5 (1977), 55-65.

Dickey, David A., and Wayne A. Fuller; "Distribution of the Estimators for Autoregressive Time Series With a Unit Root," Journal of the American Statistical Association 74, (1979), 427-431.

Dooley, Michael P., and Jeffrey R. Shafer; "Analysis of Short-Run Exchange Rate Behavior: March, 1973 to September, 1975," International Finance Discussion Paper No. 76, Washington, Federal Reserve Board, (1976).

Frankel, Jeffrey A.; "A Test of the Existence of the Risk Premium in the Foreign Exchange Market vs. The Hypothesis of Perfect Substitutability," International Finance Discussion Paper No. 149, Washington, Federal Reserve Board, (1979).

Frenkel, Jacob A.; "A Monetary Approach to the Exchange Rate: Doctrinal Aspects and Empirical Evidence," Scandinavian Journal of Economics, 78 (1976), 200-224.

______________; "The Forward Exchange Rate, Expectations and the Demand for Money: The German Hyper-inflation," American Economic Review, 64 (1977), 653-670.

______________; "Further Evidence on Expectations and the Demand for Money during the German Hyper-inflation," Journal of Monetary Economics, 5 (1979), 81-96.

______________; "Flexible Exchange Rates in the 1970's," in Stabilization Policy: Lessons from the 1970's and Implications for the 1980's, Center for the Study of American Business at Washington University, St. Louis, Missouri (1980).

Fuller, Wayne A; "Introduction to Statistical Time Series, New York: John Wiley and Sons (1976).

Geweke, John F.; "A Note on the Testable Implications of Expectations Models," University of Wisconsin mimeo (1979).

______________, and Edgar Feige; "Some Joint Tests of the Efficiency of Markets for Forward Foreign Exchange," Review of Economics and Statistics, 61 (1979), 334-341.

______________, and Richard A. Meese; "Estimating Regression Models of Finite but Unknown Order," forthcoming in the International Economic Review (1980).

Grenander, Ulf; "On the Estimation of Regression Coefficients in the Case of an Autocorrelated Disturbance, "Annals of Mathematical Statistics, 25 (1954), 252-272.

Hakkio, Craig S.; "Expectations and the Foreign Exchange Market," unpublished Ph.D. dissertation, University of Chicago (1979).

Hannan, E.J.; Multiple Time Series, New York: John Wiley and Sons (1970).

Hansen, Lars Peter; "Large Sample Properties of Generalized Method of Moment Estimators," Carnegie-Mellon University working paper (1980).

______________, and Robert J. Hodrick; "Forward Exchange Rates as Optimal Predictors of Future Spot Rates: An Econometric Analysis," forthcoming in the Journal of Political Economy (1980a).

______________; "Speculation and Forward Foreign Exchange Rates: Some Tests of Cross-Currency Restrictions," Carnegie-Mellon University Working Paper (1980).

Hasza, David P., and Wayne A. Fuller; "Estimation for Autoregressive Processes With Unit Roots," Annals of Statistics (1979), 1106-1120.

Isard, Peter; "Exchange-Rate Determination: A Survey of Popular Views and Recent Models," Princeton Studies in International Finance No. 42, (1978).

Leroy, Stephen F., and Richard D. Porter; "The Present Value Relation: Tests Based on Implied Variance Bounds," forthcoming in Econometrica (1980).

Levich, Richard; "Further Results on the Efficiency of Markets for Foreign Exchange," paper presented at the Federal Reserve Bank of Boston, Bald Peak Conference (1978).

Morgan Guaranty Trust Co. World Calendar of Holidays (1976, 1977, 1978, 1979).

Mussa, Michael; "Empirical Regularities in the Behavior of Exchange Rates and Theories of the Foreign Exchange Market," University of Chicago manuscript (1978).

______________; "A Model of Exchange Rate Dynamics," University of Chicago manuscript (1979).

Riehl, Heinz and Rita M. Rodriguez; Foreign Exchange Markets, New York: McGraw Hill (1977).

Rogoff, Kenneth S.; "Essays on Expectations and Exchange Rate Volatility," unpublished Ph.D. dissertation, Massachusetts Institute of Technology (1979).

Samuelson, Paul A.; "Proof that Properly Anticipated Prices Move Randomly," Industrial Management Review, 6 (1965), 41-49.

______________; "Is Real-World Price a Tale Told by the Idiot of Chance?," Review of Economics and Statistics, 58 (1976), 120-123.

Siegel, J.; "Risk, Interest Rates, and the Forward Exchange," Quarterly Journal of Economics, 86 (1972), 303-309.

Shiller, Robert; "The Volatility of Long-Term Interest Rates and Expectations Models of the Term Structure," Journal of Political Economy 87 (1979, 1190-1219.

Singleton, Kenneth J.; "Expectations Models of the Term Structure and Implied Variance Bounds," forthcoming in Journal of Political Economy (1980).

Stockman, Alan, "Risk, Information, and Forward Exchange Rates," in The Economics of Exchange Rates: Selected Studies, edited by Jacob A. Frenkel and Harry G. Johnson, Reading, Mass.: Addison-Wesley (1978).

Theil, Henri; Principles of Econometrics, New York: John Wiley and Sons (1971).

Tryon, Ralph; Testing for Rational Expectations in Foreign Exchange Markets, International Finance Discussion Paper No. 139, Washington, Federal Reserve Board (1979).

Westerfield, Janice Moulton; "An Examination of Foreign Exchange Risk Under Fixed and Floating Rate Regimes," Journal of International Economics 7 (1977), 181-200.