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THREE ROLES OF THE FORWARD FOREIGN EXCHANGE MARKET

by

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I. INTRODUCTION

This paper studies the behavior of a foreign exchange trader in the forward foreign exchange market. The foreign exchange trader, who may be an export/import trading merchant, an international investor, or a commercial bank portfolio manager, is assumed to receive or pay fixed amounts of domestic and foreign currencies at a specified future date. Facing an uncertain spot exchange rate that will prevail on the future maturity date, he attempts to fix his foreign currency claims or obligations in terms of the domestic currency by entering into a forward contract. Section II discusses the expected utility maximizing behavior of a risk averse foreign exchange trader and obtains the first order condition. It is shown in Section III that, if the foreign exchange trader's terminal wealth derived from all sources other than foreign exchange transactions (spot and forward) is subject to uncertainty, then his demand for forward contracting can be decomposed into three distinctive parts--pure hedging, speculative hedging and pure speculation. Importance of pure hedging and pure speculation has been emphasized in the literature (for example, Spraos [13], Tsiang [14] and Grubel [8]), but the concept of speculative hedging has not been fully investigated. Effects upon pure speculation of increases in absolute risk aversion, wealth, the expected spot rate and exchange rate uncertainty will be scrutinized in Section IV.

The paper generalizes and extends the single forward foreign exchange market results obtained by such authors as Feldstein [4], Leland [10] and

Folks [5] in that the trader maximizes the expected utility of his terminal wealth without arbitrarily specifying the form of the utility function or the joint probability distribution of random variables. When the terminal wealth depends on two random variables which are not perfectly correlated, the Arrow-Pratt measures of risk premium and risk aversion (Pratt [11] and Arrow [1]) are not sufficient to ensure the intuitively plausible proposition that the more risk averse the foreign exchange trader is, the less he speculates in forward foreign exchange. New and stronger measures of risk premium and risk aversion recently developed by Ross [12] are quite useful in analyzing this problem.

In section V, some of the conditions for stabilizing and destabilizing speculation will be discussed in the sense of Walras as has been mentioned by Feldstein [4], Williamson [15] and Driskill and McCafferty [3]. Implications for government intervention will also be developed under stabilizing and destabilizing speculation.

Although the paper concentrates on forward foreign exchange contract and speculation, identical results should obtain for the spot markets of foreign exchange and assets denominated in different currencies under the interest rate parity condition (see Tsiang [14]).

II. THE MODEL

Consider a foreign exchange trader with initial wealth W_0, who is to receive or pay Y_d and Y_f units of domestic and foreign currencies, respectively, at a future specified date, say ninety days hence. Let a positive Y_d or Y_f indicate a receipt and a negative Y_d or Y_f a payment. Assuming that the domestic currency is the trader's "preferred monetary habitat", he computes his terminal wealth in terms of the domestic currency. Facing an uncertain spot exchange rate in 90 days hence, the risk averse trader enters into a contract of purchasing H units of foreign exchange forward with a 90-day maturity. A positive H indicates a long position (purchase contract) and a negative H a short position (sales contract). His terminal wealth in domestic currency units is expressed as:

$$\tilde{W} = \tilde{W}_0 + Y_d + \tilde{S}Y_f + H(\tilde{S} - F)$$

where

$\tilde{W}$ = terminal wealth (in 90 days)

$\tilde{W}_0$ = terminal wealth from all sources of economic activities other than currency transactions

Y_d = receipt (if positive) or payment (if negative) of the domestic currency

Y_f = receipt (if positive) or payment (if negative) of foreign exchange

H = amount of forward purchase (if positive) or sale (if negative) in foreign exchange

$\tilde{S}$ = spot foreign exchange rate (in 90 days), defined as the domestic currency price of foreign exchange

F = forward foreign exchange rate (for delivery in 90 days hence)

The random variables are denoted with tildes on top of the variables. Note that the initial wealth W_0 changes randomly to $\tilde{W}_0$ in the course of 90 days due to the trader's other economic activities previously committed.

The foreign exchange trader is assumed to maximize the expected utility obtained from his terminal wealth by optimally choosing H:

$$\underset{\{H\}}{\mathrm{Max}} EU(\tilde{W})$$

where E is the mathematical expectation operator conditional on all information available at the time of decision making (W_0, Y_d, Y_f, F and the subjective joint probability distribution of $\tilde{S}$ and $\tilde{W}_0$), and U(.) is a strictly concave, thrice differentiable von Neuman-Morgenstern utility function with $U'(.) > 0$ and $U''(.) < 0$.

The first-order condition for a local optimum is

$$\frac{dEU(\tilde{W})}{dH} = EU'(\tilde{W})(\tilde{S} - F) = 0 \qquad (1)$$

The sufficient condition for a maximum is satisfied due to the strict concavity of U(.):

$$\frac{d^2EU(\tilde{W})}{dH^2} = EU''(\tilde{W})(\tilde{S} - F)^2 < 0 \qquad (2)$$

provided the marginal probability distribution of the spot exchange rate, $\tilde{S}$, is such that $\mathrm{Prob}\{\tilde{S} = F\} \neq 1$ for all $\tilde{S}$. (2) indicates that any optimum solution of H satisfying (1) is in fact a globally unique solution.

The first-order condition (1) can be rewritten as:

$$\bar{S} - F = - \frac{\mathrm{Cov}[U'(\tilde{W}^*), \tilde{S}]}{EU'(\tilde{W}^*)} \qquad (3)$$

where $\bar{S}$ is the expected value of $\tilde{S}$, and $\tilde{W}^*$ is defined by the optimum level of H*:

$$\tilde{W}^* = \tilde{W}_0 + Y_d + \tilde{S}Y_f + H^*(\tilde{S} - F)$$

III. PURE HEDGING, SPECULATIVE HEDGING AND PURE SPECULATION

In order to analyze the trader's optimum behavior in the forward foreign exchange market, the sign of $\mathrm{Cov}[U'(\tilde{W}^*), \tilde{S}]$ in (3) has to be determined. This covariance can be expressed as:

$$\mathrm{Cov}[U'(\tilde{W}^*), \tilde{S}] = \int \varphi(\tilde{S})(\tilde{S} - \bar{S})dG_s(\tilde{S})$$

where $\varphi(\tilde{S})$ is the conditional expectation of $U'(\tilde{W}^*)$ given $\tilde{S}$:

$$\varphi(\tilde{S}) = \int U'(\tilde{W}^*)dG_w(\tilde{W}_0 \mid \tilde{S}) \tag{4}$$

and $G_s(\tilde{S})$ and $G_w(\tilde{W}_0 \mid \tilde{S})$ are the marginal probability distribution of $\tilde{S}$ and the conditional distribution of $\tilde{W}_0$ given $\tilde{S}$, respectively. According to Theorem 1 stated and proven in Appendix, the sign of $\mathrm{Cov}[\varphi(\tilde{S}), \tilde{S}]$ is the same as that of the first derivative of $\varphi(\tilde{S})$ provided it is a monotone function. So the next task is to determine the sign of $\varphi'(\tilde{S})$.

We know from elementary statistics that the conditional expectation of $\tilde{W}_0$ given $\tilde{S}$ should be expressed as a function solely of $\tilde{S}$. In particular, assume that this function is linear and that $\tilde{W}_0 - E(\tilde{W}_0 \mid \tilde{S})$ is independent of $\tilde{S}$, so that $\tilde{W}_0$ may be written as:

$$\tilde{W}_0 = \bar{W}_0 + B(\tilde{S} - \bar{S}) + \tilde{V};\ E(\tilde{V} \mid \tilde{S}) = 0 \tag{5}$$

where $\bar{W}_0$ is the expected value of $\tilde{W}_0$, and $\tilde{V}$ is distributed independently of $\tilde{S}$ with mean zero.[1] In this specification, $\tilde{W}_0$ and $\tilde{S}$ are perfectly correlated if and only if $\tilde{V}$ is non-random and $B \neq 0$, and they are perfectly uncorrelated random variables if and only if $\tilde{V}$ is random and $B = 0$. If $\tilde{V}$ is non-random and $B = 0$, $\tilde{W}_0$ must be a non-random variable. Note that B is a subjective parameter (with a dimension of foreign currency unit) the foreign exchange trader may estimate through regression analysis.[2]

Using relationship (5), $\varphi(\tilde{S})$ defined in (4) can further be written as

$$\varphi(\tilde{S}) = \int U'(\tilde{W}^*)dG_v(\tilde{V})$$

where $G_v(\tilde{V})$ is the marginal distribution of $\tilde{V}$ and $\tilde{W}^*$ is now

$$\tilde{W}^* = [\overline{W}_0 + B(\tilde{S} - \overline{S}) + \tilde{V}] + Y_d + \tilde{S}Y_f + H^*(\tilde{S} - F)$$

Hence differentiation of $\varphi(\tilde{S})$ with respect to $\tilde{S}$ yields:

$$\varphi'(\tilde{S}) = (Y_f + B + H^*)\int U''(\tilde{W}^*)dG_v(\tilde{V})$$

Since $U''(.) < 0$, $\varphi'(\tilde{S})$ and hence $Cov[U'(\tilde{W}^*), \tilde{S}]$ have the opposite sign of $Y_f + B + H^*$.[3] Combining this fact with equation (3), it can be easily found that the following three conditions are all equivalent:

$$\overline{S} - F \gtreqless 0 \Leftrightarrow Cov[U'(\tilde{W}^*), \tilde{S}] \lesseqgtr 0 \Leftrightarrow Y_f + B + H^* \gtreqless 0$$

This indicates that demand for forward foreign exchange may be expressed as:

$$H^* = - Y_f - B + X^*; \quad X^* \gtreqless 0 \text{ iff } \overline{S} - F \gtreqless 0 \tag{6}$$

which introduces a new variable X, a purely speculative forward purchase of foreign exchange. Equation (6) explicitly decomposes the demand for a forward contract into three distinctive parts.[4] The first term of the right-hand side is the negative of the exact amount of foreign currency received that should be sold in the forward market if the trader were to completely hedge against exchange risk. Hence this term can be called the "pure hedging" component.

The second term of equation (6) is what I call the "speculative hedging" component. In order to clarify this concept, suppose $\tilde{S}$ and $\tilde{W}_0$ are positively correlated ($B > 0$) so that the foreign exchange trader takes a short position to this extent ($-B < 0$). If there is an unanticipated rise (decline) in that part of terminal wealth which does not involve foreign currency transactions, $\tilde{W}_0$, a positive correlation between $\tilde{S}$ and $\tilde{W}_0$ indicates that $\tilde{S}$ is likely to rise (decline). Hence the increase (decrease) in total terminal wealth, $\tilde{W}$, caused by a rise (decline) in $\tilde{W}_0$ is compensated by the probable exchange rate depreciation (appreciation); an increase (decrease) in $\tilde{S}$ creates a forward contract loss (gain) due to the trader's short position. On the other hand, suppose $\tilde{S}$ and $\tilde{W}_0$ are negatively correlated ($B < 0$) so that the trader takes a long position to this extent ($-B > 0$). Then although an unanticipated rise (decline) in $\tilde{W}_0$ would lead to an unanticipated increase (decrease) in $\tilde{W}$, an accompanied appreciation (depreciation) in the exchange rate, which will be induced by the negative correlation between $\tilde{S}$ and $\tilde{W}_0$, creates an unexpected loss (gain) in forward transactions and thus offsets the initial decrease (increase) in wealth. Thus this second term represents a smoothing-out of unanticipated fluctuations in terminal wealth, $\tilde{W}$, caused by unanticipated changes in that part of terminal wealth derived from economic activities other than foreign currency transactions, $\tilde{W}_0$. In other words, the forward foreign exchange market partially plays the role of a forward market for wealth risk, $\tilde{W}_0$, to the extent that the two random variables, $\tilde{S}$ and $\tilde{W}_0$, are correlated. Since this offsetting effect depends on the subjective parameter B it is partly "speculative", although its major role is "hedging". This effect is complete if $\tilde{S}$ and $\tilde{W}_0$ are perfectly correlated.

The last term is the "pure speculation" component, reflecting the difference between the trader's subjective expectation and the market expectation about the future spot exchange rate, $\bar{S} - F$, which is nothing but the anticipated gain per unit of foreign exchange purchased. As is expected, pure speculation is positive (long) for $\bar{S} - F > 0$, zero for $\bar{S} - F = 0$ and negative (short) for $\bar{S} - F < 0$.

IV. CHANGES IN RISK AVERSION, WEALTH AND EXCHANGE RATE DISTRIBUTION

We have seen that demand for forward foreign exchange can be decomposed into three distinctive parts. With fixed amounts of currency transaction (Y_d and Y_f) and a given degree of correlation between the exchange rate and terminal wealth acquired from other economic activites than currency transactions (B), the foreign exchange trader determines only pure speculation, X. The foreign exchange trader as an open-economy firm, an export/import merchant, an international investor or a banking portfolio manager, may decide his forward commitment together with the level of production, the amount of foreign trade or portfolio allocation. The current paper abstracts from these economic activities which the foreign exchange trader might be engaged in.[5] In other words, the trader is assumed to face given economic activities, which are carried over from the past, and then commit himself to selling or buying foreign exchange forward in a certain time interval. This assumption is not only analytically convenient but also realistic enough for many participants in the forward market. In this simplified framework, effects of changes in risk aversion, wealth, exchange rate expectation and exchange rate volatility (uncertainty) upon pure speculation can be investigated without introducing specific forms of the utility function or the probability distribution of random variables. In investigating the effects, it will be assumed that changes in exogenous parameters do not alter the subjective probability distribution of random variables.

It turns out that the approach suggested by Pratt [11] and Arrow [1] is not sufficient to draw definite conclusions regarding the impacts of exogenous changes in parameters on pure speculation. This is because the Arrow-Pratt index of risk premium and risk aversion is defined over a single random variable, whereas in our case the firm faces two random variables,

$\tilde{S}$ and $\tilde{W}_0$. Ross [12] has developed new measures of risk premium and risk aversion for a decision-maker facing two gambles. The risk premia of Arrow-Pratt and Ross, π^{AP} and π^{R} respectively, are defined as follows:

$$\left.\begin{aligned} EU(W + \tilde{\epsilon}) &= U[W + E(\tilde{\epsilon}) - \pi^{AP}] \\ EU(\tilde{W} + \tilde{\epsilon}) &= EU[\tilde{W} + E(\tilde{\epsilon} \mid \tilde{W}) - \pi^{R}] \end{aligned}\right\} \quad (7)$$

where W and $\tilde{W}$ are non-random and random wealths and $\tilde{\epsilon}$ is a random variable. These definitions of risk premia say that the decision-maker is indifferent between receiving a gamble $\tilde{\epsilon}$ and receiving an amount $E(\tilde{\epsilon}) - \pi^{AP}$ or $E(\tilde{\epsilon} \mid \tilde{W}) - \pi^{R}$. Notice in Ross's definition the decision-maker's final wealth is the sum of two random events, $\tilde{W}$ and $\tilde{\epsilon}$, and his risk premium is defined only with respect to $\tilde{\epsilon}$ rather than $\tilde{W} + \tilde{\epsilon}$. For either definition, it can be said that the higher the π, absolutely the more risk averse is the decision maker. The Arrow-Pratt premium is clearly a special case of the Ross risk premium where $\text{Prob}\{\tilde{W} = W\} = 1$ and, hence, the Ross measure of risk aversion implies the Arrow-Pratt measure, but not vice versa.

If the terminal wealth from all other economic activities than currency transactions, $\tilde{W}_0$, is a random variable which is perfectly correlated with the spot exchange rate, $\tilde{S}$, or if it is nonrandom (these two ifs correspond to the case where $\tilde{V}$ in (5) is nonrandom), then the trader faces essentially one random variable so that the Arrow-Pratt concept can be still relevant. If $\tilde{W}_0$ is random and imperfectly correlated with $\tilde{S}$ (this case corresponds to a random $\tilde{V}$ in (5)), then the Ross measure has to be used.

The results presented below apply to both cases, although the proofs can be quite different.

The first exercise is to study the effect of an increase in risk aversion on pure speculation. Our method is to compare "purely speculative" positions of two traders with identical wealth and probability distributions but with different utility functions and, hence, different Arrow-Pratt or Ross measures of risk aversion. Suppose trader 1 is absolutely more risk averse than trader 2 in the sense of Arrow-Pratt or Ross; that is, $\pi_1^{AP} > \pi_2^{AP}$ or $\pi_1^R > \pi_2^R$ where π_i^{AP} and π_i^R are the Arrow-Pratt and Ross risk premia for trader i (i = 1, 2), respectively, as defined in (7). Also suppose trader 2 attains his optimum at a nonzero level, X_2^*:

$$EU_2'(\tilde{W}_2^*)(\tilde{S} - F) = 0$$

where $U_2(.)$ is the utility function of trader 2 and

$$\tilde{W}_2^* = \tilde{W}_0 + Y_d + \tilde{S}Y_f + (- Y_f - B + X_2^*)(\tilde{S} - F)$$

Let $U_1(.)$ be the utility function of trader 1. If the sign of $EU_1'(\tilde{W}_2^*)(\tilde{S} - F)$ is determined, then one can tell whether optimum pure speculation for trader 1, X_1^*, should be greater or less than that for trader 2, X_2^*. That is, if $EU_1'(\tilde{W}_2^*)(\tilde{S} - F) > 0$ (< 0), then X has to be raised (reduced) in order to drive the expected value down (up) to zero so that $X_1^* > X_2^*$ $(X_1^* < X_2^*)$. If optimum pure speculation of trader 2 is zero $(X_2^* = 0)$, then it must be the case that $\bar{S} = F$ from (6) so that X_1^* is also equal to zero.

Theorem 2 provided in Appendix proves that

$$EU_1'(\tilde{W}_2^*)(\tilde{S} - F) \lesseqgtr 0 \text{ if } \bar{S} - F \gtreqless 0$$

for cases of nonrandom and random $\tilde{V}$, using both Arrow-Pratt's and Ross's methods. This theorem implies that if $\bar{S} - F > 0$ then $0 < X_1^* < X_2^*$, and if $\bar{S} - F < 0$ then $0 > X_1^* > X_2^*$. Needless to say, if $\bar{S} - F = 0$ then $X_1^* = X_2^* = 0$. Hence the absolute size of pure speculation, $|X|$, is reduced towards zero as the foreign exchange trader becomes absolutely more risk averse in the sense of Arrow-Pratt or Ross.

The second exercise is to investigate the effect of an increase in wealth on pure speculation.[6] Noting that $\tilde{W}$ can be expressed as

$$\tilde{W} = W^F + X(\tilde{S} - F) + \tilde{V}; \quad W^F = \bar{W}_0 + Y_d + FY_f - B(\bar{S} - F)$$

this effect can be captured by totally differentiating the first order condition with respect to W^F and X and rearranging the terms:

$$\left.\frac{dX}{dW^F}\right]_{X=X^*} = - \frac{EU''(\tilde{W}^*)(\tilde{S} - F)}{EU''(\tilde{W}^*)(\tilde{S} - F)^2} \tag{8}$$

The denominator of the right hand side is clearly negative, whereas the sign of the numerator is generally ambiguous without further restrictions. Theorem 3 in Appendix proves that

$$X^*EU''(\tilde{W}^*)(\tilde{S} - F) \gtreqless 0$$

depending upon whether the utility function of the foreign exchange trader exhibits decreasing, constant, or increasing absolute risk aversion for non zero X^*. This implies that

$$\left.\frac{dX}{dW^F}\right]_{X=X^*} \begin{cases} \gtrless 0 \text{ if } \bar{S} - F \gtrless 0 & \text{for a decreasing absolute risk aversior utility function} \\ = 0 & \text{for a constant absolute risk aversion utility function} \\ \lessgtr 0 \text{ if } \bar{S} - F \gtrless 0 & \text{for an increasing absolute risk aversion utility function} \end{cases}$$

When X^* is zero initially, it follows that $\bar{S} - F = 0$ so that a change in W^F would have no impact on X^* and it remains to be zero regardless of the trader's absolute risk aversion. In sum, an increase in wealth, W^F, raises, does not change and reduces the absolute size of pure speculation, $|X|$, for decreasing, constant and increasing absolute risk aversion utility functions. That is, pure speculation may be a "normal good" or an "inferior good" with respect to wealth depending on the nature of absolute risk aversion.

One of the interesting implications is that seemingly irrelevant variables to pure speculation, such as the domestic currency receipt or payment, Y_d, do have impacts on pure speculation and, hence, demand for forward foreign exchange through their effect on wealth, W^F. For the same reason, a change in $\bar{S}$ or F not only affects pure speculation directly, but it also influences X indirectly by inducing a valuation change in wealth.

Third, let us consider the impacts of changes in the probability distribution of the future spot exchange rate.[7] One is an upward shift in the expected value of $\tilde{S}$ with all other moments about the mean constant, and the second is an increase in exchange rate volatility (uncertainty) holding the expected spot rate constant. Such shifts of the spot rate may be represented by the new random variable $\hat{S}$:

$$\hat{S} = \bar{S} + \alpha + \beta(\tilde{S} - \bar{S})$$

where the initial position is given by $\alpha = \beta - 1 = 0$. A small increase in α from zero (with β equal to unity) raises the expected value of the spot exchange rate while holding all higher moments at the initial levels. A small increase in β from unity (with α set to be zero) raises all the moments of exchange rate, keeping the mean rate constant at $\bar{S}$.

For the analysis we replace $\tilde{S}$ and $\bar{S}$ in the first order condition by $\hat{S}$ and $E\hat{S}$, respectively, totally differentiate it with respect to α, β and X, and evaluate $\frac{dX}{d\alpha}$ and $\frac{dX}{d\beta}$ at $\alpha = 0$, $\beta = 1$ and $X = X*$. This procedure yields:

$$\left.\frac{dX}{d\alpha}\right]_{\substack{\alpha=0\\ \beta=1\\ X=X*}} = - \frac{EU'(\tilde{W}*)}{EU''(\tilde{W}*)(\tilde{S} - F)^2} + (X* - B)\, \frac{dX*}{dW^F} \tag{9}$$

$$\left.\frac{dX}{d\beta}\right]_{\substack{\alpha=0\\ \beta=1\\ X=X*}} = \left\{- X* + (\bar{S} - F)\, \frac{EU'(\tilde{W}*)}{EU''(\tilde{W}*)(\tilde{S} - F)^2}\right\} - X*(\bar{S} - F)\frac{dX*}{dW^F}$$

where $\frac{dX*}{dW^F}$ is defined in (8).

As Leland [10] realized it, $\frac{dX}{d\alpha}$ and $\frac{dX}{d\beta}$ may be divided into a type of "substitution effect", the first term of each expression which has a determinate sign, and a type of "income effect", the second term involving $\frac{dX*}{dW^F}$ whose sign depends on the trader's absolute risk aversion and can take any sign.

The "substitution effect" term of $\frac{dX}{d\alpha}$ is always positive, whereas its "income effect" term has an ambiguous sign even if the property of absolute risk aversion is known (except when it is constant). This ambiguity arises when X^* and B has the same sign. A special case is where the utility function exhibits constant absolute risk aversion or the foreign exchange trader is a pure hedger ($H^* = - Y_f$); in this case the "income effect" term vanishes. Even when the "income effect" turns out to be negative, the positive "substituion effect" may still dominate. Only when pure speculation is a "Giffen good", does the size of pure speculation decline as the expected value of the future spot rate increases.

The "substitution effect" term of $\frac{dX}{d\beta}$ is positive for $X^* < 0$ (or $\bar{S} - F < 0$) and negative for $X^* > 0$ (or $\bar{S} - F > 0$). The "income effect" term is positive for $X^* < 0$ (or $\bar{S} - F < 0$) and negative for $X^* > 0$ (or $\bar{S} - F > 0$) with a decreasing absolute risk aversion utility function, and it is negative for $X^* < 0$ (or $\bar{S} - F < 0$) and positive for $X^* > 0$ (or $\bar{S} - F > 0$) with increasing absolute risk aversion. The "income effect" again disappears if the utility function has the property of constant absolute risk aversion. When $\bar{S} = F$, the level of pure speculation is always zero so that $\frac{dX}{d\beta}$ is zero. Hence an increase in exchange-rate volatility as measured by a rise in β unambiguously reduces the absolute size of pure speculation, $|X|$, towards zero if the trader's utility function exhibits nonincreasing absolute risk aversion. If it indicates increasing absolute risk aversion, the effect of β on $|X|$ can be positive.

V. INTERVENTION POLICY AND STABILIZING AND DESTABILIZING SPECULATION

In order to consider implications for government intervention policy, we wish to further investigate the demand function for forward foreign exchange.

Since it is assumed that pure hedging and speculative hedging components are independent of the forward exchange rate,[8] only the slope of pure speculation has to be obtained. In doing so, assume that the trader's expected future spot rate can be affected by the market forward rate, so that the spot exchange rate can be expressed as

$$\tilde{S} = \bar{S}(F) + \tilde{\epsilon}; \qquad E(\tilde{\epsilon}) = 0$$

This assumption does not alter any of the arguments above, since $\bar{S}$ and F have been treated as exogenous. Then total differentiation of the first order condition (1) with respect to X and F and rearranging the terms yield:

$$\left.\frac{dX}{dF}\right]_{X=X*} = \frac{\bar{S}}{F}(\eta - \frac{F}{\bar{S}})\frac{dX*}{d\alpha} + Y_f\frac{dX*}{dW^F} \tag{10}$$

where η is the elasticity of the expected spot rate with respect to the forward rate:

$$\eta = \frac{d\bar{S}(F)}{dF}\,\frac{F}{\bar{S}}$$

and $\frac{dX*}{d\alpha}$ and $\frac{dX*}{dW^F}$ are expressions defined in (9) and (8), respectively.

The sign of $\frac{dX}{dF}$ is quite ambiguous because neither $\frac{dX*}{d\alpha}$ nor $\frac{dX*}{dW^F}$ has a determinate sign, as has been discussed in the previous section.. Even if the signs of $\frac{dX*}{d\alpha}$ and $\frac{dX*}{dW^F}$ are given, the sign of $\frac{dX}{dF}$ is still unclear because of ambiguities of the signs and magnitudes of η and Y_f. For example, when the utility function exhibits constant absolute risk aversion, $\frac{dX^*}{d\alpha}$ is unambiguously positive in sign and $\frac{dX^*}{dW^F}$ disappears; then $\frac{dX}{dF}$ is negative for $\eta < \frac{F}{\bar{S}}$, zero for $\eta = \frac{F}{\bar{S}}$ and positive for $\eta > \frac{F}{\bar{S}}$. The term $Y_f \frac{dX^*}{dW^F}$ represents a type of "income effect" in the sense that a change in F induces a valuation adjustment of Y_f and, hence, W^F and exerts its income effect upon pure speculation. Suppose pure speculation is not a "Giffen good" ($\frac{dX^*}{d\alpha} > 0$) and exchange rate expectation is inelastic ($\eta < \frac{F}{\bar{S}}$) so that the first term of (10) is always negative. In this case, $\frac{dX}{dF}$ may still be positive even with a decreasing absolute risk aversion utility function if $Y_f \frac{dX^*}{dW^F}$ is a large positive number; for example, if $\bar{S} - F < 0$ then $X^* < 0$ and, hence, $\frac{dX^*}{dW^F} < 0$ (see Theorem 3) so that $Y_f \frac{dX^*}{dW^F} < 0$ provided $Y_f > 0$ (foreign currency receipt); if $\bar{S} - F > 0$ then $X^* > 0$ and, hence, $\frac{dX^*}{dW^F} > 0$ (Theorem 3) so that $Y_f \frac{dX^*}{dW^F} < 0$ provided $Y_f < 0$ (foreign currency payment).

Hence all one can say is that a constant absolute risk aversion utility function is sufficient for the downward (upward) sloping demand schedule provided exchange rate expectation is inelastic (elastic). Under the plausible assumption of nonincreasing absolute risk aversion (Arrow [1]) and inelastic expectation (Tsiang [14]), the slope of pure speculation demand may or may not be negative. When the demand schedule is upward sloping, speculation can be defined as destabilizing (Feldstein [4], Williamson [15] and Driskill and McCafferty [3]) in the Walrasian sense.[9] That is, a Walrasian auctioneer who would call prices could not clear the market by raising (lowering) price in the presence of excess demand (supply), because an increase (decrease) in the current forward exchange rate serves to increase excess demand (supply). This notion of Walrasian destabilizing speculation is more conventional than those of dynamically destabilizing and stochastically destabilizing speculation recently discussed by such authors as Williamson [15] and Driskill and McCafferty [3], although it provides an analytical foundation for studying the latter types of destabilizing speculation.

In what follows assume that the individual trader's demand function obtained in (6) can represent the characteristics of aggregated market demand and that all the comparative static excercise results in the preceding section apply to the market demand.[10] Now consider the stabilizing case of downward sloping demand schedule under the usual assumption of nonincreasing absolute risk aversion. In Figure 1, demand for forward foreign exchange HH, is the sum of pure and speculative hedging ($- Y_f - B$), which is expressed as OA, and pure speculation (X), which would pass through $\bar{S}$ with the same slope as HH. The market equilibrium forward foreign exchange rate is determined by the intersection of the HH schedule and the vertical axis, at F*, in the absence of government intervention. Suppose foreign exchange traders become absolutely more risk averse or exchange rate uncertainty is increased; then the absolute size of pure speculation diminishes and this implies that the demand schedule HH rotates around point B clockwise to H'H'. As a result the equilibrium exchange rate declines from F* to F*'. (In this case the forward rate declines because point B is in the left-hand quadrant). To the extent that this decrease in F is not fully offset by a change in the current spot exchange rate, the forward premium narrows and the domestic interest rate tends to decline. In order to mitigate undesirable effects of declining interest rates in the period of overemployment, the government may wish to intervene in the forward market and attempt to peg the forward rate at F*. At F* there is an excess demand for a short forward position by the amount OC, which has to be eliminated by the government's offsetting long position of OD (= OC).[11] In this stabilizing case the movement of the forward foreign exchange rate can be a good indicator for the government intervention action; it can simply follow the "law of supply and demand".

Next consider the destabilizing case of upward-sloping demand. The schedule HH is depicted in Figure 2. Under the usual assumption of nonincreasing absolute risk aversion, an increase in traders' risk aversion or exchange rate uncertainty causes the schedule to tilt counter clockwise from HH to H'H'. The equilibrium forward rate rises from F* to F*', because point B is assumed to be in the left-hand quadrant in the Figure. Note that at F* there is an excess supply of forward foreign exchange by OC, which has to be matched by the government purchase of OD in order to maintain the rate at F*. In this case a rising forward rate is not an indication of excess demand. If the government reacts by selling forward foreign exchange, excess supply increases and the forward rate continues to rise. For successful government intervention, it is important to know whether the speculators are stabilizing or destabilizing. Otherwise the government can be a source of instability in the foreign exchange market, causing wider fluctuations of exchange rates.

We can also investigate the case of multiple equilibria as a theoretical possibility. Putting aside the question of whether the multiple-equilibrium case is a realistic assumption in the forward market, it produces many types of conclusions depending upon the shape of the demand schedule and the way it rotates as a result of parametric changes. One sentence summarizing all the diverse consequences would be that, with the possibility of multiple equilibria, the government intervention action finds it more and more difficult to attain market stability even with globally stabilizing speculators.

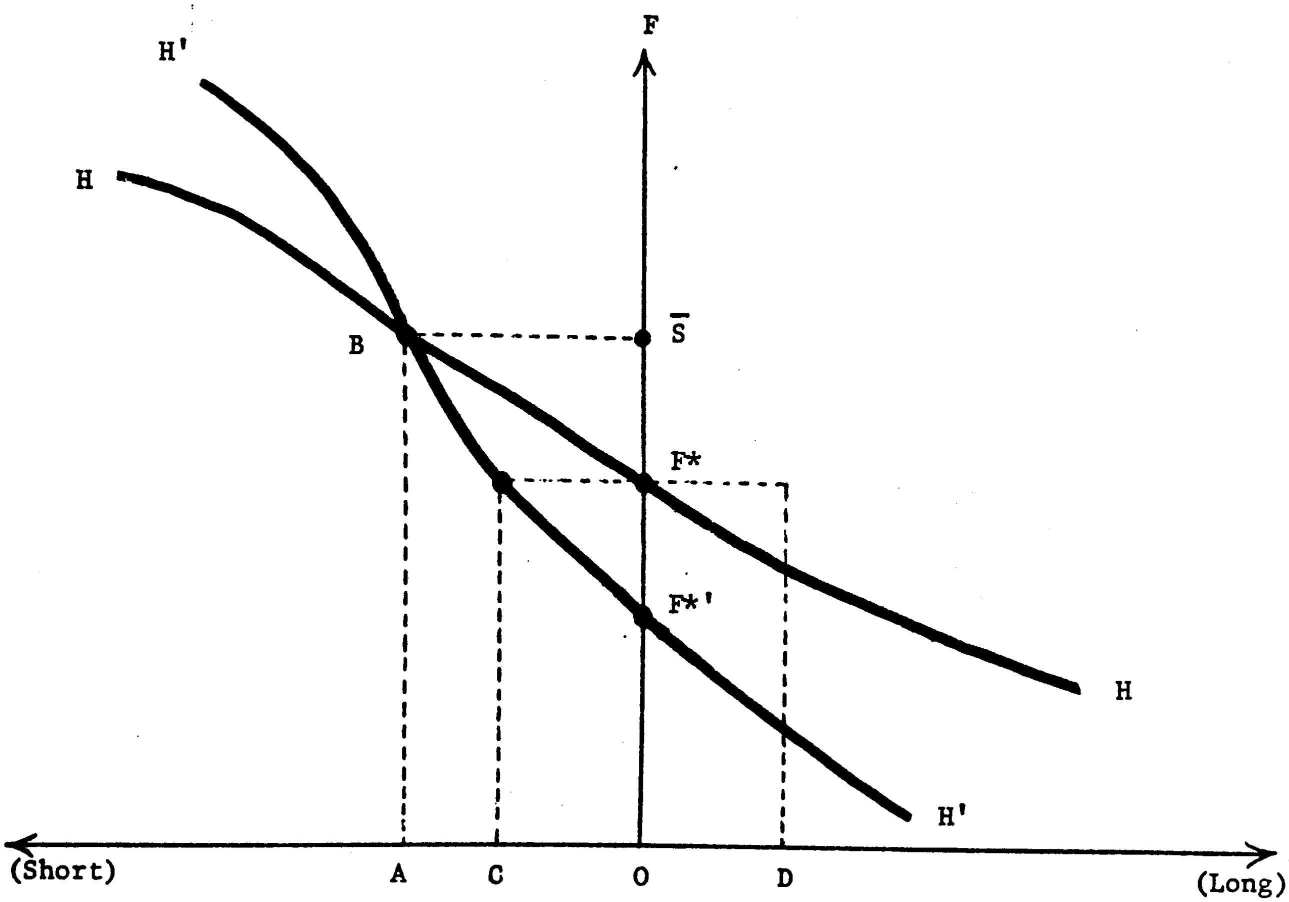

FIGURE 1

OA = demand for pure hedging and speculative hedging ($- Y_f - B$)

F* = initial equilibrium forward rate

F*' = equilibrium forward rate with increased absolute risk aversion or increased exchange rate uncertainty

OD = amount of government intervention (purchase) in the forward market

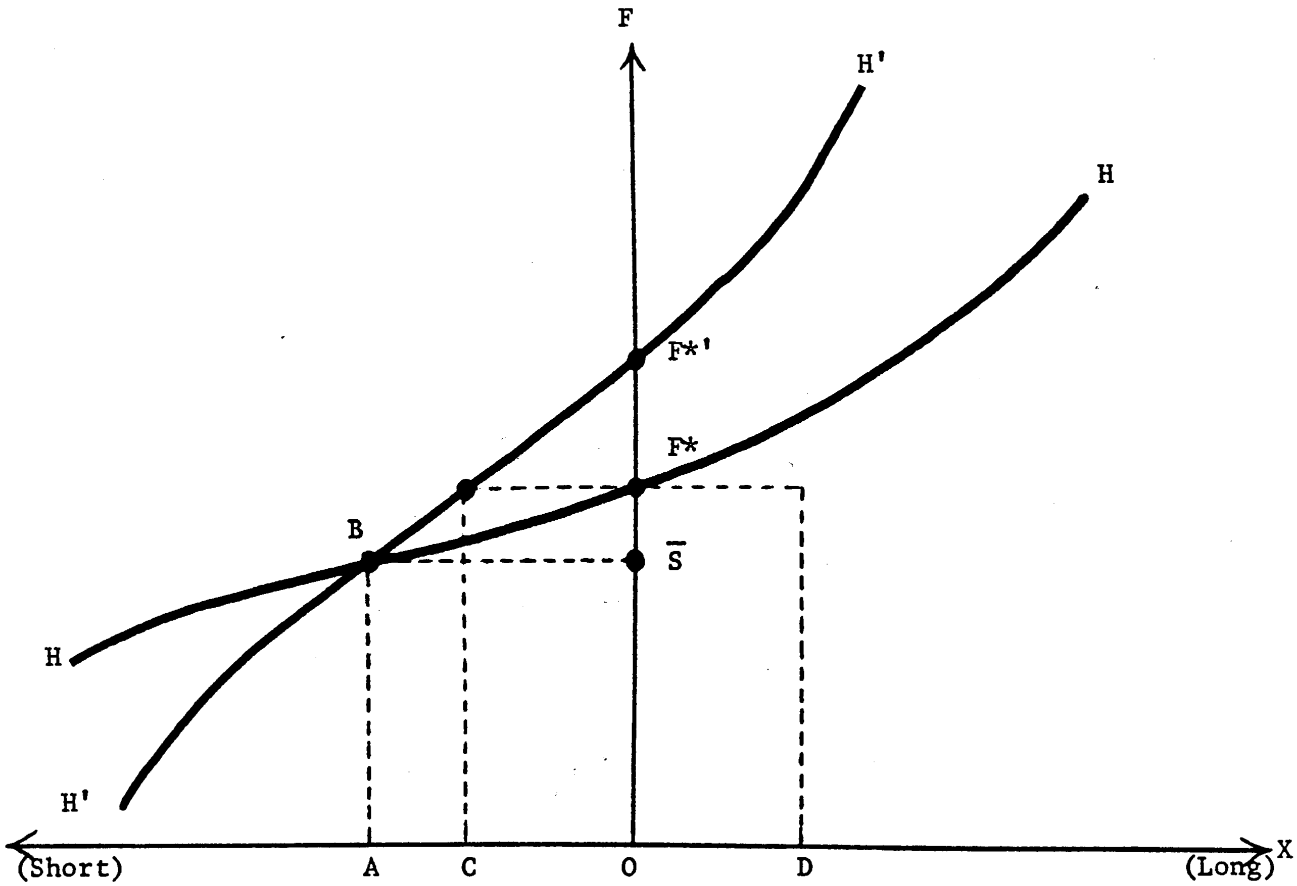

FIGURE 2

OA = demand for pure hedging and speculative hedging ($- Y_f - B$)

F* = initial equilibrium forward rate

F*' = equilibrium forward rate with increased absolute risk aversion and increased exchange rate uncertainty

OD = amount of government intervention (purchase) in the forward market

VI. CONCLUSION

The paper has shown that the foreign exchange trader, who committed himself to given economic activities in the last period and is to receive or pay fixed amounts of domestic and foreign currencies, decomposes his demand for forward foreign exchange into three distinctive parts--pure hedging, speculative hedging and pure speculation. The pure hedging and pure speculation components have received due attention in the literature. The speculative hedging component, a neglected notion in the literature, plays a role of hedging against fluctuations in terminal wealth induced by random factors other than exchange rates. This decomposition would be a testable hypothesis given a sufficiently large number of micro observations on forward contracts.

The effects upon pure speculation of increases in risk aversion, wealth, the expected spot exchange rate and exchange rate volatility have been analyzed using the conventional Arrow-Pratt measure of risk aversion as well as the stronger measure by Ross. The Arrow-Pratt measure is useful if the foreign exchange trader faces only exchange risk or if terminal wealth risk is perfectly correlated with exchange risk. When these two risks are imperfectly correlated with each other, the Ross measure has to be utilized. The Ross measure is indeed strong enough to ensure the intuitively plausible proposition that pure speculation declines as the foreign exchange trader becomes more risk averse. The effects of changes in the probability distribution of the spot rate are generally ambiguous due to the conflicting factors of "substitution" and "income" effects.

Implications for government intervention policy have been discussed in the context of Walrasian stabilizing and destabilizing speculation. A changing forward exchange rate may not suggest the presence of excess demand or supply and, hence, may fail to serve as a good indicator of how the government should intervene. Without correctly knowing the existence of excess demand or supply in the forward market, the government intervention can be a source of instability. A study of dynamically or stochastically destabilizing speculation under uncertainty and risk aversion can be developed on the basis of our microeconomic analysis and Walrasian stability or instability.

One of the important policy implications obtained from our analysis would probably be the following: if pure speculators become less risk averse and if exchange rate uncertainty is reduced, the pure speculation demand schedule becomes elastic (flat) and, therefore, with a large number of pure speculators forward exchange rates tend to fluctuate little in response to parametric changes. Hence, if the government wishes stability in the forward market, it should attempt to reduce spot exchange rate volatility, eliminate legal restrictions on speculation and promote financial policies which relax speculators' capital constraints. Being free from legal and capital constraints, the degree of speculators' risk aversion will be diminished.

Although the paper has focused on forward speculation, it would be equivalent to spot speculation as long as the interest rate parity holds. For the analysis of spot speculation the reader has only to replace F by $S_0\frac{1 + i}{1 + i*}$, where S_0 is the current spot exchange rate and i and i* are the domestic and foreign interest rates. H can be interpreted as the foreign-currency asset position plus interest. Then given the interest rates, similar implications of government intervention should apply to the spot foreign exchange market.

FOOTNOTES

[1] To use regression analysis terminology, the assumption of independence between $\tilde{S}$ and $\tilde{V}$ implies that there is no problem of simultaneity bias or errors in variables in equation (5).

[2] The most reasonable estimator of B would be σ_{ws}/σ_s^2, where σ_{ws} and σ_s^2 are the covariance between $\tilde{W}_0$ and $\tilde{S}$ and the variance of $\tilde{S}$, respectively.

[3] The linearity assumption of $E(W_0|\tilde{S})$ is important here. If $E(\tilde{W}_0|\tilde{S})$ is not linear in $\tilde{S}$, $\varphi(\tilde{S})$ will not be a monotone function because $\varphi'(\tilde{S}) = Y_f + \frac{d}{d\tilde{S}} E(\tilde{W}_0|\tilde{S}) + H^*$ can take any sign over a given range of $\tilde{S}$.

[4] These three distinctive roles of a forward foreign exchange contract are not peculiar to the forward market of currencies, and any market for contingency claims would display these attributes.

[5] The behavior of an open-economy firm which jointly determines capital accumulation, production, foreign trade, currency borrowing and forward contract is studied in Kawai [9] using a quadratic utility function.

[6] Leland [10] and Folks [5] have analyzed the impact of increases in wealth on the level of speculative transactions under the assumption that $\tilde{W}_0$ is nonrandom.

[7] Felstein [4] discusses the impacts in the mean variance framework. Leland [10] analyzes the problem in a more general framework, but he assumes that $\tilde{W}_0$ is deterministic.

[8] Even when pure hedging and speculative hedging are appropriate functions of F, our analysis in what follows remains intact. The important implicit assumption is that the forward-contracting decision is made independently of any other economic decisions.

[9] Friedman's definition of destabilizing speculation (Friedman [7]) may also apply in the sense that the trader loses money <u>on the average</u> out of speculation (a negative expected speculative profit). Needless to say, however, what concerns the trader is not the expected gain or loss of pure speculation but the expected utility of final wealth.

[10] The derivation of aggregate market demand may not be straightforward. The common procedure would be to obtain individual demand functions explicitly by using the quadratic utility function or the mean variance approach, and then to aggregate them over all market participants (see, for example, Frankel [6] and Dornbusch [Z]). The problem of the quadratic utility function is that it exhibits an undesirable property of increasing absolute risk aversion.

Assume that the utility function of individual i is of the form $U_i(\tilde{W}_i) = -\exp(-\theta_i \tilde{W}_i)$ and that $\tilde{W}_i$ is normally distributed. The parameter θ_i in this utility function is the Arrow-Pratt (but <u>not</u> the Ross) measure of absolute risk aversion, which is constant. Then the first-order condition yields the demand function of individual i:

$$H_i^* = -Y_{f,i} - \frac{\sigma_{ws,i}}{\sigma_{s,i}^2} + X_i^*; \quad X_i^* = \frac{\bar{S}_i - F}{\theta_i \sigma_{s,i}^2}$$

where the terms on the right-hand side of the first expression represent the three components of demand as discussed in Section III. By summing up over all market participants, the aggregated market demand can be obtained:

$$H^* = -Y_f - \frac{\sigma_{ws}}{\sigma_s^2} + X^*; \quad X^* = \frac{\bar{S} - F}{\theta \sigma_s^2}$$

where

$$H^* = \sum_i H_i^*$$

$$Y_f = \sum_i Y_{f,i}$$

$$\sigma_{ws} = \sum_i \frac{\sigma_{ws,i}}{\sigma_{s,i}^2} \Big/ \sum_i \frac{1}{\sigma_{s,i}^2}$$

$$\frac{1}{\sigma_s^2} = \sum_i \frac{1}{\sigma_{s,i}^2}$$

$$\bar{S} = \sum_i \frac{\bar{S}_i}{\theta_i \sigma_{s,i}^2} \Big/ \sum_i \frac{1}{\theta_i \sigma_{s,i}^2}$$

$$\frac{1}{\theta} = \sum_i \frac{1}{\theta_i \sigma_{s,i}^2} \Big/ \sum_i \frac{1}{\sigma_{s,i}^2}$$

This aggregated demand function has similar characteristics to those represented by individual functions, except that now Y_f has to be interpreted as the sum of market participants' Y_f's, σ_{ws} as the weighted average of individual σ_{ws}'s, $1/\sigma_s^2$ as the sum of $1/\sigma_{s,i}^2$, $\bar{S}$ as the weighted average of individual expected spot rates, and $\frac{1}{\theta}$ as the weighted average of individual risk aversion coefficients.

[11] It is assumed that government intervention does not alter the probability distribution of random variables or the degree of risk aversion of foreign exchange traders.

APPENDIX

Theorem 1

Let $\varphi(Z)$ be a differentiable, monotone function and Z be a random variable with finite moments. Then $\mathrm{Cov}[\varphi(Z), Z] \gtreqless 0$, if and only if $\varphi'(Z) \gtreqless 0$.

Proof

(i) $\varphi'(Z) \gtreqless 0 \rightarrow \mathrm{Cov}[\varphi(Z), Z] \gtreqless 0$

Because of the monotonicity of $\varphi(Z)$, it is always true that

$$(Z - EZ)[\varphi(Z) - \varphi(EZ)] \gtrless 0 \text{ if } \varphi'(Z) \gtrless 0$$

where EZ is the expected value of Z, and equality holds if $\varphi'(Z)$ is zero or $pr\{Z = EZ\} = 1$. The latter case is not considered here. Taking the expected value of both sides, we can obtain

$$E(Z - EZ)[\varphi(Z) - \varphi(EZ)] \gtreqless 0$$

Since

$$E(Z - EZ)[\varphi(Z) - \varphi(EZ)] = E(Z - EZ)\varphi(Z)$$

$$= E(Z - EZ)[\varphi(Z) - E\varphi(Z)]$$

$$= \mathrm{Cov}[Z, \varphi(Z)]$$

the desired result is derived.

(ii) $\mathrm{Cov}[\varphi(Z), Z] \gtreqless 0 \rightarrow \varphi'(Z) \gtreqless 0$

(iia) $\mathrm{Cov}[\varphi(Z), Z] > 0 \rightarrow \varphi'(Z) > 0$

The contraposition of this statement is $\varphi'(Z) \lesssim 0 \rightarrow \mathrm{Cov}[\varphi(Z), Z] \leq 0$, which is true because of the monotonicity of $\varphi(Z)$.

(iib) $\mathrm{Cov}[\varphi(Z), Z] = 0 \rightarrow \varphi'(Z) = 0$

The contraposition of this statement is $\varphi'(Z) \neq 0 \rightarrow \mathrm{Cov}[\varphi(Z), Z] \neq 0$ which is true because of the monotonicity of $\varphi(Z)$ and random Z.

(iic) $\mathrm{Cov}[\varphi(Z), Z] < 0 \rightarrow \varphi'(Z) < 0$

The contraposition of this statement is $\varphi'(Z) \geq 0 \rightarrow \mathrm{Cov}[\varphi(Z), Z] \geq 0$, which is true due to the monotonicity of the $\varphi(Z)$ function.

Q.E.D.

Theorem 2

Assume that trader 1 is more risk averse than trader 2 in the sense that $\pi_1 > \pi_2$, where π_i is the Arrow-Pratt or Ross risk premium for trader i ($i = 1, 2$). Let X_2^* be the nonzero optimum level of pure speculation for trader 2 and $U_i(\cdot)$ be the utility function of trader i ($i = 1, 2$). Then

$$EU_2'(\tilde{W}_2^*)(\tilde{S} - F) = 0$$

$$EU_1'(\tilde{W}_2^*)(\tilde{S} - F) \lesseqgtr 0 \text{ if } \bar{S} - F \gtreqless 0$$

where

$$\tilde{W}_2^* = \tilde{W}_0 + Y_d + \tilde{S}Y_f + (-Y_f - B + X_2^*)(\tilde{S} - F)$$

Proof (Arrow-Pratt Case)

Assume that $\tilde{W}_0$ does not depend on $\tilde{V}$, that is

$$\tilde{W}_0 = \bar{W}_0 + B(\tilde{S} - \bar{S})$$

where B may be zero or nonzero. In this case the Arrow-Pratt measure of risk premium and risk aversion can be applied.

Consider the expression

$$\int_0^F \left[\frac{U_1'(\tilde{W}_2^*)}{U_1'(W^F)} - \frac{U_2'(\tilde{W}_2^*)}{U_2'(W^F)}\right](\tilde{S} - F)dG_s(\tilde{S}) + \int_F^\infty \left[\frac{U_1'(\tilde{W}_2^*)}{U_1'(W^F)} - \frac{U_2'(\tilde{W}_2^*)}{U_2'(W^F)}\right](\tilde{S} - F)dG_s(\tilde{S})$$

where $\tilde{W}_2^* = W^F + X_2^*(\tilde{S} - F)$, $W^F = \bar{W}_0 + Y_d + FY_f - B(\bar{S} - F)$.

Then this expression has the same sign as that of $EU_1'(\tilde{W}_2^*)(\tilde{S} - F)$. We know that

$$\tilde{W}_2^* \gtrless W^F \text{ if } (\tilde{S} - F)X_2^* \gtrless 0$$

First assume that $\bar{S} - F > 0$ or $X_2^* > 0$ so that $\tilde{W}_2^* \gtrless W^F$ if $\tilde{S} - F \gtrless 0$. In this case the first integral above is negative because $\tilde{S} < F$ and

$$\frac{U_1'(\tilde{W}_2^*)}{U_1'(W^F)} > \frac{U_2'(\tilde{W}_2^*)}{U_2'(W^F)} \text{ for } \tilde{W}_2^* < W^F$$

by equation (20) in Pratt [8, p. 129]. The second integral is also negative because $\tilde{S} > F$ and

$$\frac{U_1'(\tilde{W}_2^*)}{U_1'(W^F)} < \frac{U_2'(\tilde{W}_2^*)}{U_2'(W^F)} \text{ for } \tilde{W}_2^* > W^F$$

by the same equation in Pratt [8]. Therefore if $\bar{S} - F > 0$ or $X_2^* > 0$ then $EU_1'(\tilde{W}_2^*)(\tilde{S} - F) < 0$.

Second assume that $X_2^* < 0$ or $\bar{S} - F < 0$. Then $\tilde{W}_2^* \lesseqgtr W^F$ if $\tilde{S} - F \gtrless 0$. In this case using analogous arguments one can easily show that $EU_1'(\tilde{W}_2^*)(\tilde{S} - F)$ is positive.

Q.E.D.

<u>Proof</u> (Ross Case)

It is assumed that $\tilde{W}_0$ depends on $\tilde{V}$:

$$\tilde{W}_0 = \bar{W}_0 + B(\tilde{S} - \bar{S}) + \tilde{V}.$$

In his Theorem 3, Ross [9] proves that if $\pi_1^R > \pi_2^R$ then there exist a constant λ and a function $T(.)$ such that $T'(.) < 0$ and $T''(.) < 0$ for which

$$U_1(W) = \lambda U_2(W) + T(W)$$

Hence

$$EU_1'(\tilde{W}_2^*)(\tilde{S} - F) = E[\lambda U_2'(\tilde{W}_2^*) + T'(\tilde{W}_2^*)](\tilde{S} - F)$$

$$= ET'(\tilde{W}_2^*)(\tilde{S} - F)$$

$$= ET'(\tilde{W}_2^*)(\tilde{S} - \bar{S}) + (\bar{S} - F)ET'(\tilde{W}_2^*)$$

$$= \int \psi(\tilde{S})(\tilde{S} - \bar{S})dG_s(\tilde{S}) + (\bar{S} - F)ET'(\tilde{W}_2^*)$$

$$= \text{Cov}[\psi(\tilde{S}), \tilde{S}] + (\bar{S} - F)ET'(\tilde{W}_2^*)$$

where $\psi(\tilde{S})$ is the conditional expected value defined as:

$$\psi(\tilde{S}) = \int T'(\tilde{W}_2^*)dG_s(\tilde{W}_0 \mid \tilde{S}) = \int T'(\tilde{W}_2^*)dG_v(\tilde{V})$$

Using Theorem 1, if $\psi(\tilde{S})$ is a monotone function the sign of $\text{Cov}[\psi(\tilde{S}), \tilde{S}]$ is determined by the first derivative of $\psi(\tilde{S})$:

$$\psi'(\tilde{S}) = X_2^* \int T''(\tilde{W}_2^*)dG_v(\tilde{V})$$

because $\tilde{W}_2^* = W^F + X_2^*(\tilde{S} - F) + \tilde{V}$ and $W^F = \bar{W}_0 + Y_d + FY_f - B(\bar{S} - F)$. Since $T''(.) < 0$, $\psi'(\tilde{S})$ has the opposite sign of that of X_2^*. Knowing that $X_2^* \gtrless 0$ if and only if $\bar{S} - F \lessgtr 0$, we obtain

$$\left.\begin{array}{l} \text{Cov}[\psi(\tilde{S}), \tilde{S}] \gtrless 0 \\ \\ (\bar{S} - F)ET'(\tilde{W}_2^*) \gtrless 0 \end{array}\right\} \text{ as } \bar{S} - F \lessgtr 0$$

so that

$$EU_1'(\tilde{W}_2^*)(\tilde{S} - F) \lessgtr 0 \text{ as } \bar{S} - F \gtrless 0$$

Q.E.D.

Theorem 3

Let X* be the nonzero optimum level of pure speculation and $\tilde{W}^*$ be the corresponding terminal wealth:

$$\tilde{W}* = W^F + X*(\tilde{S} - F) + \tilde{V};\ W^F = \overline{W}_0 + Y_d + FY_f - B(\overline{S} - F)$$

Depending on whether the trader's utility function exhibits decreasing, constant or increasing absolute risk aversion,

$$X*EU''(\tilde{W}*)(\tilde{S} - F) \gtreqless 0 \text{ for } X* \neq 0$$

The proof under decreasing absolute risk aversion is provided in the following. For other types of absolute risk aversion the proof is similar.

<u>Proof</u> (Arrow-Pratt Case)

Assuming that $\tilde{W}_0$ can be expressed as:

$$\tilde{W}_0 = \overline{W}_0 + B(\tilde{S} - \overline{S})$$

rearrange $\tilde{W}*$ as

$$\tilde{W}* = W^F + X*(\tilde{S} - F);\ W^F = \overline{W}_0 + Y_d + FY_f - B(\overline{S} - F)$$

then

$$\tilde{W}* \gtrless W^F \text{ if } (\tilde{S} - F)X* \gtrless 0$$

Hence if the foreign-exchange trader has a decreasing absolute risk aversion utility function in the sense of Arrow-Pratt,

$$r(\tilde{W}*) \lesseqgtr r(W^F) \text{ if } (\tilde{S} - F)X* \gtrless 0$$

where r(.) is the Arrow-Pratt measure of absolute risk aversion: $r(W) = -U''(W)/U'(W)$. This means

$$-U''(\tilde{W}*)(\tilde{S} - F)X* < r(W^F)U'(\tilde{W})(\tilde{S} - F)X*$$

Taking the expected values of both sides and using the first-order condition (1),

$$\overset{*}{X}EU''(\tilde{W}*)(\tilde{S} - F) > 0$$

Q.E.D.

Proof (Ross Case)

Since $\tilde{W}_0$ is assumed to depend on $\tilde{V}$, $\tilde{W}^*$ should be expressed as

$$\tilde{W}^* = W^F + X^*(\tilde{S} - F) + \tilde{V}; \; W^F = \bar{W}_0 + Y_d + FY_f - B(\bar{S} - F)$$

In his Theorem 4 Ross proves that, if the utility function exhibits decreasing absolute risk aversion, there exists a constant γ such that

$$\frac{U''(W + Z)}{U''(W)} < e^{\gamma Z} < \frac{U'(W + Z)}{U'(W)}$$

for any W and $Z > 0$. Applying this theorem, it is easy to see that

$$\frac{U''(\tilde{W}^*)}{U''(W^F + \tilde{V})} < e^{\gamma X^*(\tilde{S}-F)} < \frac{U'(\tilde{W}^*)}{U'(W^F + \tilde{V})} \qquad \text{if } X^*(\tilde{S} - F) > 0$$

and

$$\frac{U''(W^F + \tilde{V})}{U''(\tilde{W}^*)} < e^{-\gamma X^*(\tilde{S}-F)} < \frac{U'(W^F + \tilde{V})}{U'(\tilde{W}^*)} \qquad \text{if } X^*(\tilde{S} - F) < 0$$

From these inequality relationships, we obtain:

$$[U''(\tilde{W}^*) - e^{\gamma X^*(\tilde{S} - F)} U''(W^F + \tilde{V})] \, X^*(\tilde{S} - F) > 0$$

and

$$[U'(\tilde{W}^*) - e^{\gamma X^*(\tilde{S} - F)} \, U'(W^F + \tilde{V})] X^*(\tilde{S} - F) > 0$$

Taking the expected values of these expressions we further obtain

$$X^* E U''(\tilde{W}^*)(\tilde{S} - F) > E[e^{\gamma X^*(\tilde{S}-F)} \, X^*(\tilde{S} - F)] \, . \, EU''(W^F + \tilde{V})$$

$$0 > E[e^{\gamma X^*(\tilde{S} - F)} X^*(\tilde{S} - F)] \cdot EU'(W^F + \tilde{V})$$

because $EU'(\tilde{W}^*)(\tilde{S} - F) = 0$ from the first-order condition and $\tilde{S}$ and $\tilde{V}$ are statistically independent. It is clear from the last inequality

$$E[e^{\gamma X^*(\tilde{S} - F)} X^*(\tilde{S} - F)] < 0$$

so that the last inequality relationship but one yields

$$X^*EU''(\tilde{W}^*)(\tilde{S} - F) > 0$$

Q.E.D.

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