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THE BRITISH BANKING SYSTEM'S DEMAND FOR CASH RESERVES

by

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Summary

There has been considerable controversy in the United Kingdom about the most appropriate method of conducting monetary policy. This controversy has focused on the concept of the monetary base and its present and potential usefulness as an instrument of monetary policy. $\frac{1}{2}$ This paper is intended to help resolve this controversy by analyzing an important part of the monetary system in existence in the United Kingdom during the years 1973-1978 and referred to as the "present" system in this paper. First, a brief description of the relevant aspects of the British monetary system is presented. Next, a theoretical model of the banking system's demand for cash reserves (i.e., vault cash plus cash deposits at the Bank of England) is developed. This model is estimated and conclusions are drawn based on the empirical results. It is found that in the present British monetary system, banks' demand for cash reserves is a welldefined and well-behaved function of bank deposit liabilities and a few other observable variables. Thus a policy of achieving monetary growth targets by means of conventional open-market operations by the Bank of England -- that is, by manipulating the supply of monetary base -- appears to be feasible under present circumstances, assuming that the demand for the monetary base by nonbanks is also well defined.

I. The British Monetary System

British banks have long been subject to two required reserve ratios: a cash ratio and a liquid-asset ratio. The former stipulates a minimum proportion of eligible bank liabilities which must be matched by holdings of non-interest-bearing balances at the Bank of England and the latter a minimum proportion of liabilities which must be matched by certain defined liquid assets. There has been a longstanding disagreement about which of the two types of reserves is relevant for the determination of the U.K.'s money supply. (See, for example, the papers in part 3 of the book edited by Harry Johnson.) In 1971 a change in the British financial system (see Bank of England) reduced the size of the cash ratio, but not necessarily its importance. During the period studied in this paper -- 1973 through 1978 -- the required liquid-asset reserve ratio was 12-1/2 percent of eligible liabilities while the cash ratio was 1-1/2 percent for London clearing banks and zero for all other banks. $\frac{2}{}$ The liquid-asset requirement pertains to eligible liabilities in the current month, while the cash-balance requirement pertains to eligible liabilities in the preceding month. Eligible liabilities essentially are sterling deposit liabilities. Liquid-asset reserves include balances at the Bank of England other than special deposits and supplementary special deposits (see below), U.K. Treasury bills, call money in the London discount market, certain commercial bills, and some other assets. Cash reserves comprise the first item only, i.e., balances at the Bank of England, and do not include currency held by the banks. The cash balances at the Bank of England do not bear interest. In addition, the Bank of England can call for special deposits to be placed by the banks in the Bank. These special deposits are a certain percentage -- stipulated by the Bank -- of eligible liabilities and pay a rate of interest equal to the Treasury bill rate. Finally, the Bank can set a limit to the growth of the interest-bearing component of eligible

liabilities and require non-interest-bearing deposits to be placed with the Bank if the limits are violated. (This supplementary special deposit scheme is often referred to as the "corset.")

An important component of banks' liquid-asset reserves is call money at the discount houses. The discount houses play an important role as financial intermediaries, particularly with respect to the relationship between the Bank of England and the banks. When banks are short of cash, i.e., balances at the Bank of England, they call in money deposited with the discount houses. The discount houses, in turn, obtain the cash from the Bank either by selling it bills or by borrowing at a penal rate (the Bank's Minimum Lending Rate) at the Bank's discretion. The discount houses are particularly active in the market for U.K. Treasury bills and by agreement with the authorities are obligated to bid for the entire amount of new Treasury bills offered each week. Most, and perhaps all, of the functions of the discount houses could be performed by the banks themselves in a more integrated financial system, and in this paper the discount houses are treated as part of the banks. Although some aspects of the British financial system are obscured by this treatment, the consolidation allows one to focus on a central issue of this paper -- the reserves of the banking system. When the banks and discount houses are combined it becomes clear that call money is not a true reserve of the banking system. Indeed in the consolidated balance sheet it does not even appear; it is merely an internal accounting item. Nevertheless, because the reserve requirements refer to banks' accounts that are not consolidated with the discount houses, call money still qualifies as a liquid-asset reserve for the purpose of complying with the liquid-asset reserve requirement.

On the other hand, Treasury bills and the other bills eligible for use as liquid-asset reserves do constitute at least potential reserves of the banking system because the Bank of England often stands ready to purchase such assets at market prices. However, because in some cases nonbanks issue these

bills, the supply of these assets is not directly under the control of the monetary authorities. Therefore, because liquid-asset reserves can be either created by the banking system -- as in the case of call money -- or fairly readily produced by and obtained from nonbanks -- as in the case of bills -- the liquid-asset reserve requirement would appear not to be much of a constraint on the banking system. In addition, if such liquid-asset reserves constitute at least part of the monetary base, the ability of the authorities to control the base is called into question.

One of the methods by which the Bank of England tries to control the money supply is by altering the supply of Treasury bills (liquid assets) through funding operations. This method would seem to indicate that the Bank considers the liquid-asset ratio to be relevant to the determination of the money stock. However, in its day-to-day activities in the London money market the Bank usually operates so as to make the banking system short of balances at the Bank, thereby ensuring that the discount houses have to go to the Bank for cash. The form of assistance chosen by the Bank -- i.e., purchases of bills or lending at a penal rate -- then exerts an influence on interest rates and market conditions. This type of activity would seem to indicate that the cash ratio might be the relevant ratio because of its role in determining interest rates and hence the money stock.

In this study the conventional definition of the monetary base -- that is, cash held by banks and nonbanks including banks' cash deposits (but not special and supplementary special deposits) at the central bank -- is used. The role of liquid-asset reserves in the banking system's demand for this monetary-base variable is investigated empirically. Even though there is no mandatory requirement that banks hold vault cash, banks will hold a certain amount for prudential reasons and

for normal business transactions with their customers. Similarly, the banking system may want to hold more cash balances at the Bank of England than are needed to meet the mandatory cash requirement in order to facilitate interbank clearing. Thus it is to be expected that there will be a well-defined demand for central bank liabilities despite the lack of any mandatory requirement for the banks to hold more than a small amount of such liabilities. This study treats the demand for bank cash reserves as being derived from the needs of the banking business as well as from the reserve requirement imposed by the authorities.

II. The Model

A. The Monetary Base

The monetary base (B) is defined to be the sum of banks' cash reserves (R) and the nonbank public's holdings of cash (PC). Therefore the monetary base usually consists of the stock of central bank liabilities held by the banks and the nonbank public. In the standard monetary-base approach to monetary theory, certain definitions are presented and manipulated until the money stock (M) is shown to be related to the monetary base by a multiple involving various ratios. These ratios are then postulated to be functions of a few variables and therefore the "money multiplier", i.e., the ratio of the stock of money to the base, is also postulated to be a function of those same variables. $\frac{3}{}$ That is,

(1)
$$M \equiv \left(\frac{\frac{PC}{D} + 1}{\frac{R}{D} + \frac{PC}{D}}\right) B$$

where D represents the nonbank public's holdings of bank deposits, and MEPC+D.

In the present paper the behavior of the reserve ratio -- R/D -- is investigated.

The behavior of the currency ratio -- PC/D -- is beyond the scope of the present paper.

B. The Demand for Reserves

Before discussing the banks' demand-for-reserves function, it is necessary to define the following terms:

- R = banks' holdings of vault cash and deposits at the Bank of England exclusive of special deposits and supplementary special deposits; $\frac{4}{}$
- RR = quantity of R which the banks are required by "law" to hold; $\frac{5}{}$
- CTB = banks' holdings of commercial and U.K. Treasury bills eligible for sale to the Bank of England and qualifying as liquid-asset reserves;
 - D = sterling deposit liabilities of the banks, including certificates
 of deposit;
 - DD = sterling demand-deposit liabilities of the banks;
 - TD = sterling deposit liabilities of the banks other than demand deposits, i.e., $TD \equiv D-DD$;
 - λ = ratio of demand deposits to total deposits, i.e., λ = DD/D;
 - P = price level;
- r_{TB} = interest rate on U.K. Treasury bills, i.e., the interest rate on a close substitute for R;
- r_{LD} = the cost to the banks of raising additional funds by borrowing from the nonbank public;
- PEMLR = the expected cost to the banks of obtaining funds from the Bank of
 England -- represented by the product of the Bank's Minimum Lending
 Rate (MLR) times a proxy for the probability that the banks will
 have to borrow at MLR;
 - SSD = binary variable reflecting the operation of the supplementary
 special deposits scheme.

The demand-for-reserves function -- in general form -- is:

(2)
$$R=f(r_{TB},r_{LD},PBMLR,SSD,D,P,\lambda,RR,CTB)$$
,

where the signs indicate the predicted signs of the partial derivatives. equation (2) there is no variable representing the yield on reserve assets because in this paper a maintained hypothesis is that bank reserves consist solely of vault cash and cash balances at the Bank of England, and these assets do not bear interest. The predicted signs of the partial derivatives are readily explained. An increase (decrease) in r_{TB} increases (decreases) the opportunity cost to the bank of holding a cash-reserve asset and thus it is predicted that such increases (decreases) will decrease (increase) the demand for reserves; hence the predicted sign for r_{TB} . Similarly, an increase, for example, in r_{LD} increases the cost to the bank of obtaining funds and thus leads the bank to economize on its holdings of non-interest-bearing assets; thus the predicted negative sign. The effect of a change in PBMLR is negative because an increase, for example, in PBMLR increases the cost of obtaining reserves, which would tend to reduce the demand for $R.\frac{6}{}$ The imposition of the corset raises the cost to the banks of increasing liabilities above a specified amount; the increase in cost may tend to induce the banks to economize on reserves, as in the case of an increase in $\mathbf{r}_{l,D}$. Thus, the predicted sign of SSD is negative. An increase (decrease) in the nominal quantity of deposits, D, can be expected to increase (decrease) the demand for reserves, hence the predicted positive The price level, P, is included in equation (2) because an increase, for example, in the level of real deposits would not lead necessarily to a proportionate increase in the demand for reserves; there may well be economies of scale in the holding of reserves in that as the quantity of real deposits increases the ratio of reserves to deposits can decline without increasing risk,

hence the predicted positive sign for P. On the other hand, in the absence of money illusion on the part of both banks and depositors it would be expected that proportional increases in the price level and other nominal variables (i.e., D, RR, and CTB) would lead to a proportionate increase in the demand for (nominal) reserves. One might expect that demand deposits require more reserves than do other deposits because they are more likely to be withdrawn. If so, an increase (decrease) in the ratio of demand deposits to total deposits will increase (decrease) the demand for reserves, holding the total level of deposits constant. Therefore the predicted sign of λ is positive. An increase, for example, in required reserves can be expected to increase the demand for reserves by approximately the same amount; hence the positive sign for RR. As mentioned in the preceding section, it is possible that certain liquid assets other than cash also function as bank reserves in the United Kingdom in that legal reserve requirements are stated in terms of such assets and because the assets are fairly readily sold to the Bank of England for cash balances at the Bank. These liquid assets are denoted as CTB in this study and to the extent that they are substitutes for R, the predicted sign of the partial derivative with respect to CTB will be negative: the more CTB is held, the less cash is necessary as a reserve. $\frac{7}{}$ The CTB variable itself as well as its interest rate -- r_{TB} -- is included in equation (2) in order to represent the effect on the banks' demand for cash of the Bank of England's policy of often standing ready to exchange cash for CTB at prevailing market rates. Thus the quantity held as well as the price of CTB can be expected to affect the demand for R.

C. Functional Form

The demand-for-reserves function is specified to be:

(3)
$$R/P=\alpha+(\beta_0+\beta_1r_{TB}+\beta_2r_{LD}+\beta_3PBMLR+\beta_4SSD)(DD/P)$$

 $+(\delta_0+\delta_1r_{TB}+\delta_2r_{LD}+\delta_3PBMLR+\delta_4SSD)(TD/P)$
 $+\theta(RR/P)+\phi(CTB/P)+\eta$

where n is an error term and the absence of money illusion has been imposed by expressing the equation in real terms. The additional constraint that reserves change by the same amount as required reserves can be imposed by setting θ =1. Subtraction of (RR/P) from both sides of the equation then yields an equation that represents a demand for excess reserves and is the behavioral function that relates the banks' demand for prudential and transactions cash reserves to certain economic variables. All is this equation that is central to the question of the feasibility and desirability of a monetary policy based on the central bank's manipulation of the outstanding supply of its liabilities, both with and without mandatory reserve requirements.

Equation (3) has some useful properties: (1) the constant term, α , allows the average reserve ratio to vary as the level of real deposits varies while the ratio is unchanged with respect to proportional changes in nominal quantities and the price level;(2) the response to changes in DD and TD can differ and these differing responses are functions of a few key variables; and (3) the (RR/P) and (CTB/P) variables enter additively so that the equation is readily transformed into an excess-reserves equation (as mentioned above) and the relationship between R and CTB can be inferred directly from the coefficient of (CTB/P). If R and CTB are interchangeable as bank reserves, ϕ =-1 and equation (3) can be transformed into a demand function for cash reserves plus liquid-asset reserves by adding (CTB/P) to both sides of the equation. $\frac{9}{}$

The error term, η , can be expected to be heteroscedastic and, in particular, it can be expected to vary with the real size of the banks' deposit liabilities, i.e.,

$$\eta = \varepsilon(D/P)$$
; $\varepsilon \sim N(0, \sigma^2)$.

In order to remove the heteroscedasticity, equation (3) must be divided through by (D/P) to obtain:

(4)
$$XR/D=\alpha(D/P)^{-1}+(\beta_0-\delta_0)\lambda+(\beta_1-\delta_1)\lambda r_{TB}+(\beta_2-\delta_2)\lambda r_{LD}$$

$$+(\beta_3-\delta_3)\lambda PBMLR+(\beta_4-\delta_4)\lambda SSD$$

$$+\delta_0+\delta_1 r_{TB}+\delta_2 r_{LD}+\delta_3 PBMLR+\delta_4 SSD$$

$$+\phi(CTB/D)+\varepsilon ,$$

where XR=R-RR. Equation (4) is the equation that is estimated in this paper. In addition to the right-hand-side variables shown, the equation also includes a binary variable (DUM) equal to unity in July 1978 and zero elsewhere, a linear time trend, and seasonal binary variables. DUM removes an outlier -- in July 1978, banks' cash balances at the Bank of England were at an extraordinarily high level: £616 million compared to £399 million in June and £402 million in August. The trend term is meant to reflect such things as the effects of technological advances in information processing during the period as well as increasing familiarity with the new financial system instituted in 1971. The seasonal variables are required because the data used are not seasonally adjusted.

D. Predicted Signs

The signs of the partial derivatives of equation (4) can be predicted based on the above discussion of equation (2). They are:

$$\begin{split} & \Im(XR/D)/\Im r_{TB} = (\beta_1 - \delta_1)\lambda + \delta_1 < 0\,; \\ & \Im(XR/D)/\Im r_{LD} = (\beta_2 - \delta_2)\lambda + \delta_2 < 0\,; \\ & \Im(XR/D)/\Im PBMLR = (\beta_3 - \delta_3)\lambda + \delta_3 < 0\,; \\ & \Im(XR/D)/\Im SSD = (\beta_4 - \delta_4)\lambda + \delta_4 < 0\,; \\ & \Im(XR/D)/\Im(D/P) = -\alpha(D/P)^{-2} \le 0\,; \\ & \Im(XR/D)/\Im P = \alpha/D \ge 0\,; \\ & \Im(XR/D)/\Im P = \alpha/D \ge 0\,; \\ & \Im(XR/D)/\Im P = (\beta_0 - \delta_0) + (\beta_1 - \delta_1)r_{TB} + (\beta_2 - \delta_2)r_{LD} + (\beta_3 - \delta_3)PBMLR + (\beta_4 - \delta_4)SSD > 0\,; \\ & \Im(XR/D)/\Im(CTB/D) = \phi < 0\,. \end{split}$$

III. Estimation

Most, if not all, of the right-hand-side variables in equation (4) are likely to be correlated with the disturbance term, ε . Thus ordinary-least-squares estimation of equation (4) will not yield consistent estimates. In order to obtain consistent estimates, this paper utilizes the two-stage-least-squares (2SLS) technique. Two types of relationships are used to generate the exogenous and predetermined variables needed to compute the instruments used in the 2SLS regression: the demand-for-money function and the monetary authorities' policy-reaction functions.

The general demand-for-money function used in this paper is: $lnM* = \mu_0 + \mu_1 r + \mu_2 lny + \mu_3 lnP,$

where the asterisk indicates long-run demand, r is a vector of interest rates, y is real income, and the error term in the equation is ignored. Assuming partial adjustment one obtains: $\frac{10}{}$

$$lnM-lnM_{-1} = \gamma(lnM*-lnM_{-1}),$$

or,

$$lnM = \gamma \mu_0 + \gamma \mu_1 r + \gamma \mu_2 lny + \gamma \mu_3 lnP + (1-\gamma) lnM_{-1},$$

where γ is the adjustment parameter. In this paper the relevant demand for money is taken to be the demand for sterling M3. Monthly data are used in this study and in a monthly model it is reasonable to treat real income and the price level as predetermined. However, the r variable must be taken to be a vector of endogenous variables. Therefore the demand-for-money function supplies the following predetermined variables for use in estimating equation (4): $\ln y$, $\ln P$, and $\ln M3_{-1}$, where M3 denotes sterling M3.

During the period examined in this paper (1973-1978), the monetary authorities were mainly concerned with setting interest rates and did so primarily by manipulating the U.K. Treasury bill rate. $\frac{12}{}$ In this paper it is assumed that

the authorities had implicit or explicit targets for the rates of growth of real income, prices, the money stock, and the dollar-sterling exchange rate, and changed the Treasury bill rate according to how actual performance compared to the targets. In addition, it is assumed that the authorities took into account the level of the U.S. Treasury bill rate and the prevailing values of the authorities' other policy instruments when setting the U.K. Treasury bill rate. Thus the monetary authorities' reaction function can be represented as:

(5)
$$r_{TB}-r_{TB}_{-1} = \pi_0 + \pi_1 (lny-lny_{-1}-\tau_1) + \pi_2 (lnP-lnP_{-1}-\tau_2) + \pi_3 (lnM3-lnM3_{-1}-\tau_3) + \pi_4 (lnEX-lnEX_{-1}-\tau_4) + \pi_5 r_{USTB} + \pi_6 \rho_{SD} + \pi_7 \rho_{SSD} + \pi_8 \rho_{SD}$$

where the $\boldsymbol{\tau_i}$ represent the authorities' targets, EX is the dollar-sterling exchange rate, $r_{\mbox{USTB}}$ is the interest rate on U.S. Treasury bills, $\rho_{\mbox{SD}}$ is the rate of call for special deposits, and BOR denotes discount-house borrowing from the Bank of England. The last three variables in equation (5) are other policy instruments available to the monetary authorities -- recall that in the United Kingdom the quantity of borrowing from the central bank is determined by the monetary authorities. (The Bank's MLR is set in accordance with the $r_{\mbox{\scriptsize TB}}$ target.) The lagged variables in equation (5) are predetermined; it has been argued in the above discussion that Iny and InP can be treated as predetermined In addition $r_{\mbox{USTB}}$ can be taken safely to be exogenous to British monetary events. On the other hand, current values of M3 and EX are likely to be correlated with contemporaneous monetary disturbances and must be treated as endogenous variables. The status of the three policy variables -- $\rho_{\mbox{SD}}$, SSD, and BOR -- must now be determined. In principle each of these variables is subject to a reaction function similar to equation (5), which would make them all endogenous variables. However, only $\rho_{\mbox{SD}}$ and BOR are similar to $r_{\mbox{TB}}$ in their

flexibility and the ease with which they can be altered in order to react to approximately contemporaneous events. The corset is a much less flexible policy instrument which cannot be altered effectively from month to month. Thus it is better to consider SSD to be determined by the past history of money growth (or inflation) and not by current variables. Therefore the SSD variable can be taken to be predetermined while the other two policy variables on the right-hand-side of equation (\mathfrak{S}) -- ρ_{SD} and BOR -- are assumed to be endogenous.

In summary, the policy reaction function -- equation (5) -- provides the following exogenous and predetermined variables: $r_{TB_{-1}}$, $\ln y$, $\ln y$, $\ln y$, $\ln p$, and $\ln p$, and $\ln p$, \ln

IV. Empirical Results

Preliminary work not reported here indicates that the r_{LD} , PBMLR, and SSD variables may not be statistically significant in equation (4). A simplified version of equation (4) which excludes the r_{LD} , PBMLR, and SSD variables can be obtained by provisionally setting β_2 , β_3 , β_4 , δ_2 , δ_3 , and δ_4 equal to zero. The resulting equation is:

(6)
$$XR/D = \alpha(D/P)^{-1} + (\beta_0 - \delta_0)\lambda + (\beta_1 - \delta_1)\lambda r_{TB}$$

 $+\delta_0 + \delta_1 r_{TB} + \phi(CTB/D) + \epsilon$.

The 2SLS estimate of equation (6) is presented in Table 1. Next the hypotheses that β_2 , β_3 , β_4 , δ_2 , δ_3 , and δ_4 equal zero are investigated. The procedure for testing these hypotheses is best explained by example. If the variable r_{LD} is a significant explanatory variable in the demand-for-reserves equation, then β_2 , δ_2 , or both will be statistically significant in equation (4). Equation (6) augmented by the inclusion of r_{LD} is:

(7)
$$XR/D = \alpha(D/P)^{-1} + (\beta_0 - \delta_0)\lambda + (\beta_1 - \delta_1)\lambda r_{TB} + (\beta_2 - \delta_2)\lambda r_{LD}$$

$$+ \delta_0 + \delta_1 r_{TB} + \delta_2 r_{LD} + \phi(CTB/D) + \varepsilon .$$

The significance of δ_2 can be determined by examining its t-ratio. If it is found that the hypothesis δ_2 =0 cannot be rejected, then the following equation can be estimated:

(8)
$$XR/D = \alpha(D/P)^{-1} + (\beta_0 - \delta_0)\lambda + (\beta_1 - \delta_1)\lambda r_{TB} + \beta_2 \lambda r_{LD} + \delta_0 + \delta_1 r_{TB} + \phi(CTB/D) + \epsilon$$
,

and the significance of β_2 is determined by examining the t-ratio of β_2 . The t-ratios of δ_2 and β_2 in 2SLS regressions of equations (7) and (8), respectively, are reported in Table 2; analogous information on β_3 , β_4 , δ_3 , and δ_4 is reported in Table 2 as well. $\frac{15}{2}$

TABLE 1
2SLS Estimates of Equation (6)

<u>Variable</u>	Estimated Coefficient	t-ratio
Constant	0.0690	3.615
r _{TB}	-0.3488	2.241
λ	-0.0969	1.988
λr _{TB}	0.8796	2.210
$(D/P)^{-1}$	421.40	2.978
(CTB/D)	-0.0336	2.141
DUM	0.0065	3.808
Т	-0.00018	8.214
JAN	-0.0036	4.010
FEB	-0.0062	5.906
MAR	-0.0056	5.391
APR	-0.0037	3.764
MAY	-0.0043	4.349
JUN	-0.0032	3.217
JUL	-0.0020	1.980
AUG	-0.0037	4.203
SEP	-0.0034	3.889
OCT	-0.0039	4.538
NOV	-0.0032	3.738
R ²	0.84	
SE	0.0015	
DW	2.363	
n	72	

Notes: DUM is a binary variable equal to unity in July 1978 and zero elsewhere; T is a linear trend term; JAN, ..., NOV are seasonal binary variables; SE is the standard error of the regression; n is the sample size.

TABLE 2
Significance of Additional Explanatory Variables

<u>Variable</u>	Parameter	t-ratio
r _{LD}	β ₂ δ2	1.444 0.023
PBMLR	β ₃ δ3	0.073 0.214
SSD1	β ₄₁ δ ₄₁	0.089 0.755
SSD2	^β 42 δ42	0.658 1.163
SSD3	β ₄₃ δ ₄₃	0.066 0.258

Note: See footnote 15.

The results reported in Table 1 indicate that equation (6) performs very well as an explanatory equation for the excess-reserves-to-deposits ratio: all of the coefficients are statistically significant, the R^2 is quite high, particularly considering that the dependent variable is a ratio, the standard error of the regression is about 4 percent of the average XR/D, and there is no evidence of first-order serial correlation. As the results in Table 2 indicate, the additional explanatory variables investigated -- i.e., r_{LD} , PBLMR, and SSD-- are not statistically significant and therefore will be ignored in the rest of this paper. $\frac{16}{}$

In order to test the behavioral model presented in section II, the statistical significance of the various parameters underlying the estimates reported in Table 1 must be ascertained. Table 3 reports the t-ratios of the parameters of greatest interest. As is readily seen, only one of the parameters -- β_0 -- is not statistically significant. Setting β_0 equal to zero yields the following equation:

(9)
$$XR/D = \alpha(D/P)^{-1} + \delta_0(1-\lambda) + (\beta_1 - \delta_1)\lambda r_{TB} + \delta_1 r_{TB} + \phi(CTB/D) + \epsilon$$
.

The 2SLS estimate of equation (9) is reported in Table 4 and the estimates of the parameters of greatest interest are reported separately in Table 5. The Table 5 estimates can be used to evaluate certain partial derivatives of equation (4), which, in turn, can be compared to the predictions presented at the end of section II; Table 6 reports these partial derivatives.

TABLE 3
Estimates of Individual Parameters in Equation (6)

<u>Parameter</u>	Estimate	<u>t-ratio</u>
α	421.40	2.978
β _O	-0.0279	0.910
β ₁	0.5308	2.183
δ _o	0.0690	3.615
δ ₁	-0.3488	2.241
φ	-0.0336	2.141

TABLE 4

2SLS Estimates of Equation (9)

<u>Variable</u>	Estimated Coefficient	<u>t-ratio</u>
r _{TB}	-0.2165	3.846
$(1-\lambda)$	0.0531	6.937
λr _{TB}	0.5440	3.584
$(D/P)^{-1}$	378.70	2.800
(CTB/D)	-0.0262	1.929
DUM	0.0065	3.768
T	-0.00019	8.855
JAN	-0.0035	3.908
FEB	-0.0058	5.956
MAR	-0.0052	5.401
APR	-0.0034	3.643
MAY	-0.0040	4.266
JUN	-0.0028	3.068
JUL	-0.0017	1.741
AUG	-0.0036	4.050
SEP	-0.0033	3.769
OCT	-0.0039	4.477
NOV	-0.0032	3.742
R ²	0.83	
SE	0.0015	
DW	2.288	
n	72	

Notes: See notes to Table 1. In order to allow fully for seasonal effects, a constant term was included in a preliminary regression. However the constant term was not significant and so the regression was run again without the constant. The results of the second regression are reported in this Table.

TABLE 5

Estimates of Individual Parameters in Equation (9)

Parameter	<u>Estimate</u>	<u>t-ratio</u>
α	378.70	2.800
β ₁	0.3275	3.366
δ _o	0.0531	6.937
δ ₁	-0.2165	3.846
ф	-0.0262	1.929

Estimates of Partial Derivatives of Equation (4)

TABLE 6

<u>Derivative</u>	Expression	Estimated Value	t-ratio
a(XR/D)/ar _{TB}	$(\beta_1 - \delta_1)\lambda + \delta_1$	-0.0237	2.089
<pre>a(XR/D)/a(D/P)</pre>	$-\alpha(D/P)^{-2}$	-8×10 ⁻⁷	2.800
9(XR/D)/8P	α/D	0.0122	2.800
∂(XR/D)/∂ λ	$(\beta_1 - \delta_1)r_{TB} - \delta_0$	0.00093	0.095
<pre>a(XR/D)/a(CTB/D)</pre>	φ	-0.0262	1.929

Notes: The expressions are evaluated using the mean values of λ , (D/P), D, and r_{TB} during the sample period. The average value of (XR/D) during the sample period was 0.0357.

As can be seen by examining Table 4, equation (9) performs very well in explaining the behavior of the excess-cash-reserves ratio and all of the coefficients are statistically significant. The results indicate that there is a significant downward trend in the reserves ratio and a definite seasonal pattern in the behavior of the ratio. (In order to put the seasonal effects in perspective it should be noted that the average value of XR/D during the sample period was 0.0357.) The estimates reported in Table 5 indicate that each of the remaining individual parameters of most analytical interest in the banks' demand-for-cash-reserves equation -- equation (3) -- is statistically significant.

The theoretical predictions discussed in section II of this paper can now be tested. In Table 6 the relevant expressions are evaluated. The t-ratios indicate that the demand for cash reserves is interest-sensitive in the predicted direction [i.e., $\frac{\partial(XR/D)}{\partial r_{TB}} < 0$], that there are economies of scale in the holding of real reserves as real deposits increase [i.e., $\frac{\partial(XR/D)}{\partial(D/P)} < 0$ and $\frac{\partial(XR/D)}{\partial P} > 0$], and that U.K. Treasury bills and certain other bills are substitutes for cash reserves in bank portfolios [i.e., $\frac{\partial(XR/D)}{\partial(CTB/D)} < 0$]. However, it is also found that the demand for reserves is not significantly sensitive to changes in the ratio of demand deposits to total deposits, although the computed effect is in the predicted direction.

Nevertheless, despite the finding that $\partial(XR/D)/\partial\lambda$ is not significantly different from zero, there is a way in which the proportion of deposits consisting of demand deposits is important in the banks' demand for reserves: the ratio of demand deposits to total deposits has a significant effect on the interest-sensitivity of the demand-for-reserves function. In order to see this, note that

$$\theta^2(XR/D)/\theta r_{TB}\theta \lambda = \beta_1 - \delta_1 = 0.5440$$
,

and that the t-ratio of $(\beta_1-\delta_1)$ is 3.584. Thus an increase in λ -- i.e., an increase in the proportion of deposits consisting of demand deposits -- decreases

the interest-sensitivity of the banks' demand for excess cash reserves [that is, it makes $\partial(XR/D)/\partial r_{TR}$ less negative]. There are two possible explanations for this effect on the interest-sensitivity of demand. First, banks may be thought of as having two reserve "funds", one backing up demand deposits and the other backing up time deposits. Assuming that demand deposits are more subject to withdrawal, the banks may be less willing to alter the size of the demand-deposit reserve fund when the opportunity cost of the fund changes, hence the larger is the ratio of demand deposits to total deposits the less interest-sensitive is the demand for cash reserves. The second reason, which can be complementary to the first, is that interest-sensitivity of demand differs across banks and that the banks specializing in demand deposits -- that is, retail banking -- are more cautious in managing their reserves in that their demand for reserves is less interest-sensitive. Thus when the proportion of total deposit liabilities consisting of the liabilities of these banks increases, for example, which would probably show up as an increase in λ , the interest-sensitivity of the demand for cash reserves by the entire banking system declines.

As noted above, the sign of the interest-sensitivity of the demand for reserves and the sign of the coefficient on the (CTB/D) variable indicate that U.K. Treasury bills and certain other bills are substitutes for cash in banks' portfolios. However the relatively small size of ϕ -- the coefficient of (CTB/D) -- implies that cash reserves and bills definitely are not interchangeable and that cash reserves are not just part of a larger aggregate called liquid-asset reserves. On the contrary, the results reported in this paper indicate that cash reserves are a distinct asset and therefore the estimation of a demand function for them alone is justified and, indeed, to be preferred to aggregating them with other liquid assets.

V. Policy Implications

In order for the monetary authorities to be able to control monetary growth by controlling the growth of the monetary base, it is necessary (but not sufficient) that the banks' demand for the monetary base be related to the level of bank liabilities in a predictable and well-defined way. In terms of equation (1), the reserve ratio -- R/D -- must be a well-defined function of known and observable variables. The empirical evidence reported in this paper indicates that there is a well-defined and well-behaved banks' demand-for-cash-reserves function in the United Kingdom and that the function includes as an argument deposit liabilities, among other variables. 19/ Thus, one of the necessary conditions for the feasibility of controlling money growth by controlling the growth of the (conventionally defined) monetary base is fulfilled. 20/ Another variable in the demand-for-reserves function is the rate of interest on U.K. Treasury bills; therefore a policy of controlling interest rates by means of manipulating the supply of the monetary base also appears to be feasible. 21/

A monetary-base-oriented policy would involve the explicit recognition and use of the monetary base as an instrument of monetary policy, the formal adoption of a monetary-base growth target, and the explicit abandonment of interest-rate targets. The conclusion of this paper is that no institutional changes -- other than the obvious changes in the Bank of England's operating procedures -- appear to be necessary before a monetary-base-oriented policy can be adopted in the United Kingdom, assuming that the (PC/D) function in equation (1) is well-defined. $\frac{22}{}$

The empirical results reported in this paper indicate that there already exists a well-defined banks' demand-for-cash-reserves function that can be used for monetary policy purposes. Thus, no change in cash-reserve requirements appears to be necessary for monetary control. Furthermore, since the

reserves function behaves well even though the requirement for banks' cash deposits at the Bank of England is not really a constraint on any particular day and applies only to part of the banking system (the London clearing banks), and since there is no legal requirement on vault cash at all, one might well conclude that no mandatory reserve requirement is necessary for monetary control. As long as the banks have a well-defined demand for the liabilities of the Bank of England, the Bank can use open-market operations in its liabilities to affect the monetary base and bank deposit liabilities. Thus what is necessary for a monetary-base approach to monetary control to be feasible is that there be an incentive for the banks to use Bank of England liabilities as their reserve asset; a mandatory reserve requirement is just one way of providing such an incentive.

The discussion in this paper has not addressed the question of what the appropriate monetary policy regime is for the United Kingdom. It would appear that the present monetary system could be used to achieve a target path for monetary-base growth which is chosen as being consistent with some desired growth in the money stock or nominal income or even as being desirable in its own right for that matter. Alternatively, the system could be used in connection with feedback rules or reaction functions, where monetary developments -- money-stock growth or interest-rate changes -- elicit automatic or discretionary changes in the base. Other policy regimes -- e.g., pegging the price of Bank of England liabilities in terms of a particular commodity, group of commodities, a particular currency, or group of currencies -- are also possible. It must be noted, however, that any change in the U.K.'s monetary policy regime would very likely bring about a response in the private sector. For example, strict adherence to a money-supply-growth target by means of a monetary-base-oriented policy would represent a rather dramatic change in the way in which the monetary system is used by the

authorities, and changes in certain empirical relationships would be likely to occur -- for instance, the banks' demand for cash reserves would probably increase under such a policy regime. Thus any change in the monetary policy regime must be undertaken cautiously, perhaps gradually, and certainly only after careful consideration of the alternatives and implications $\frac{27}{1000}$, $\frac{28}{10000}$

VI. Conclusion

In this paper, a model of the British banking system's demand for cash has been developed and estimated. It was ascertained that the model performs well empirically and that the banking system's demand for cash reserves is thus a well-defined and well-behaved relationship involving a few observable variables, including bank deposit liabilities. Therefore a policy aimed at achieving a monetary growth target by manipulating the monetary base as conventionally defined appears to be feasible. Furthermore it was concluded that such a policy would not require institutional change before being implemented.

Appendix: Data

Monthly data for the period 1973-1978 are used in this study.

All data are from the Bank of England's Quarterly Bulletin and its Statistical Abstract unless otherwise indicated. The data are not seasonally adjusted. Most monetary statistics are reported for a Wednesday near the middle of the month; this date is referred to in this study as "mid-month".

Interest rates. The U.K. Treasury bill rate is the average rate of discount for three-month bills after the weekly tender, expressed as a yield, prevailing on the Friday before mid-month. The U.S. Treasury bill rate is the market selling rate in New York for 91-day bills, expressed as a yield, prevailing on the Friday before mid-month. The variable r_{LD} is defined as:

$$r_{LD} \equiv [r_{MD}/(1-\rho)] - [\rho r_{\rho}/(1-\rho)]$$
,

where r_{MD} is the rate of interest paid by the bank on a marginal deposit, ρ is the ratio of required investment in defined reserve assets to total liabilities, and r_{ρ} is the rate of interest paid on the assets fulfilling the bank's ρ requirement. $\frac{29}{}$ Because the cash requirement for the London clearing banks counts toward their liquid-asset reserve requirement, the required ratio relevant to the equation is just the 12-1/2 percent liquid-asset ratio requirement. In addition, all banks are obliged to meet any call for special deposits. In fact, special deposits can be thought of as the variable component of required liquid-asset reserves. Thus, $\rho \equiv .125 + \rho_{SD}$. $\frac{30}{}$ The rate of interest paid on special deposits is r_{TB} , and that on liquid-asset reserves other than cash balances at the Bank of England is close to r_{TB} . Thus the specification that $r_{\rho} \equiv r_{TB}$ is a reasonable simplification. Taking r_{MD} to be the rate of interest on a certificate of deposit (r_{CD}), one obtains the following specification for r_{TD} :

$$r_{LD} \equiv [r_{CD}/(.875-\rho_{SD})]-[(.125+\rho_{SD})r_{TB}/(.875-\rho_{SD})].$$

The certificate-of-deposit rate used in this study is the mean of the range of rates on the Friday before mid-month on a three-month sterling certificate of

deposit. The rate of call for special deposits is a mid-month rate. The PBMLR variable is the product of the MLR times the ratio of outstanding discount house borrowing from the Bank of England to total banking system's cash reserves. The MLR used in this study is the one announced or in effect on the Friday before mid-month. The data on discount house borrowing from the Bank, which in this study amount to data on borrowing by the banking system from the Bank, represent amounts owed to the Bank of England by the discount houses as of mid-month. The data do not measure the total amount of borrowing from the Bank by the banking system during the preceding month. The data on banks' cash reserves are discussed below.

Supplementary special deposits. There were three different episodes in which the corset was in force during the sample period: December 17, 1973 through February 28, 1975; November 18, 1976 through August 11, 1977; and June 8, 1978 through the end of the sample period. Binary variables are used to represent these episodes.

<u>Cash reserves.</u> Data on notes and coin held by U.K. banks are available for the period May 1975 through the end of the sample period. For earlier months, banks' deposits at the Bank of England must be subtracted from data on banks' total holdings of notes, coin, and balances with the Bank in order to obtain data on vault cash. Cash deposits at the Bank of England of both banks and discount houses are mid-month data, as are the other data mentioned in this paragraph. Required reserves are equal to 1-1/2 percent of the previous month's eligible liabilities of the London clearing banks.

Liquid-asset reserves. Data on banks' cash-balances at the Bank and their money at call are subtracted from data on banks' total reserve assets, i.e., liquid-asset reserves, in order to obtain a series on banks' holdings of Treasury bills and other bills eligible as reserve assets. To these data on banks' holdings of bills are added data on discount houses' holdings of U.K.

and Northern Ireland Treasury bills, local authority bills, other public sector bills, and other bills in order to obtain the amount of Treasury bills and other eligible bills (CTB) held by the banking system as defined in this study. All of the data used in the preceding calculation refer to mid-month dates. British government stocks with less than one year to maturity count as reserve assets, but data on discount houses' holdings of such assets are not available before May 1975. Therefore the variable CTB omits discount houses' holdings of these assets. (Between May 1975 and December 1978 the average amount of such assets held by the discount houses was £6 million, ranging between zero and £72 million.)

Money stock. Data on the money stock and bank deposit liabilities are for mid-month dates. The specification of DD includes all sterling sight deposits held by the U.K. private sector and thus some interest-bearing money-market bank liabilities are included in this paper's definition of demand deposits. TD includes certificates of deposit as well as time deposits.

 \underline{Prices} . The retail price index (January 1974 = 100) is used as the price level variable and is taken from the Department of Employment Gazette.

<u>Income</u>. As a proxy for real income, this study uses the index of industrial production (all industries, 1975 = 100) taken from the Central Statistical Office's <u>Monthly Digest of Statistics</u>.

Exchange rate. The exchange rate data are for the U.S. dollar/sterling spot exchange rate prevailing on the Friday before mid-month.

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Footnotes

- International Finance Division, Federal Reserve Board. This paper represents the views of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or other members of its staff. I would like to thank Richard Berner, Peter Clark, Warren Coats, Dale Henderson, Karen Johnson, Larry Promisel, Edwin Truman, and Douglas Waldo for helpful comments and suggestions on the research reported in this paper.
- See, for example, the papers by Nigel Duck and David Sheppard and by M. Foot,
 C. Goodhart, and A. Hotson, as well as the consultation paper on monetary
 control issued by the U.K. Treasury and the Bank of England.
- In December 1978, London clearing banks accounted for about half of the U.K. banking system's sterling deposits and three quarters of the system's sight deposits.
- 3/ Examples of studies in which the monetary base and/or the money multiplier figure prominently are the studies by Milton Friedman and Anna Schwartz and by Karl Brunner and Allan Meltzer.
- Special deposits are not treated as bank reserves in this study for three major reasons. First, special deposits are not clearing balances and therefore do not qualify in any way as cash. Second, even if they could be interpreted as cash reserves, the analysis would be unaffected since both total reserves and required reserves would increase by the same absolute amount, leaving the demand-for-excess-reserves function the function estimated in this paper unchanged. (All special deposits would be required reserves.) Finally, since in terms of equation (1), (R/D) and B would change by offsetting amounts if special deposits were treated as bank reserves,

movements in B would be more difficult to interpret. The exclusion of special deposits from the definition of B means that in effect the monetary base series is expressed in terms of constant reserve requirements, which facilitates interpretation of its movement over time.

- The reserve requirements in the United Kingdom are not statutory. Rather, the requirements are those agreed upon by the banks and the Bank of England in 1971 when the present system was adopted. Nevertheless they will be treated as if they were legal requirements in this paper.
- $\frac{6}{}$ Recall that the MLR is a penal rate set above the interest rate on U.K. Treasury bills.
- Liquid-asset reserves -- as defined by the authorities -- include cash balances at the Bank of England, U.K. Treasury bills and certain commercial bills, call money in the discount market, and some other assets. Cash balances are already included in R and call money is not an asset of the consolidated banking system, which includes the discount houses, used in this paper. Thus the relevant liquid-asset variable must exclude both cash balances at the Bank and call money. The variable denoted here as CTB consists primarily of the banking system's holdings of commercial and U.K. Treasury bills.
- B/ The justification for expressing the behavioral function as a demand for excess reserves is that required reserves do not represent usable reserves and are not directly a choice variable for the banks. Excess reserves are the actual banking reserves of the banking system in that they are held for prudential and transactions purposes and, as such, represent a choice variable, the demand for which, in principle, is sensitive to the variables included on the right-hand side of equation (3). At this point it should be clear why it is not very important to this paper whether special deposits are included or excluded

- from R and RR. In either case, the same behavioral function results after θ is set equal to unity and (RR/P) is subtracted from both sides of the equation.
- $\frac{9}{2}$ Note that D/P is held constant when interpreting the sign of ϕ .
- 10/ The adjustment equation used in this paper is in nominal terms, which implies that adjustment to price changes is also partial. See Stephen Goldfeld, pp. 691-92. Partial adjustment is a common assumption in empirical studies of the demand for money.
- 11/ The implicit transmission mechanism involved is that monetary developments affect interest rates contemporaneously and real income and prices with a lag.
- Foot, Goodhart, and Hotson (p. 150) have characterized British monetary policy as follows: "The Bank of England has chosen -- through its open-market operations and lender of last resort facilities -- to concentrate on influencing short-term interest rates, being prepared always to provide funds requested by the banking system but on interest-rate terms of its own choosing."
- 13/ The τ_i in equation (5) are not observable and are not used in the estimation procedure.
- 14/ In this paper a two-tailed test of statistical significance at the 10 percent level of significance is used.

 $\frac{15}{}$ A further test of the significance of δ_2 , for example, is to set β_2 =0 in order to obtain the equation:

$$XR/D = \alpha(D/P)^{-1} + (\beta_0 - \delta_0)\lambda + (\beta_1 - \delta_1)\lambda r_{TB} + \delta_2(r_{LD} - \lambda r_{LD}) + \delta_0 + \delta_1 r_{TB} + \delta_2(r_{LD} - \lambda r_{LD}) + \delta_0 + \delta_1 r_{TB}$$

and examine the t-ratio of the resulting estimate of δ_2 . Such t-ratios for δ_2 , δ_3 , δ_{41} , δ_{42} , and δ_{43} are, respectively, 1.445, 0.080, 0.075, 0.631, and 0.075, none of which is significant, where δ_{41} , δ_{42} , and δ_{43} are the coefficients of the variables representing the three separate episodes in which the supplementary-special-deposits scheme was in effect during the sample period.

- Further work on the role of MLR in the demand for cash reserves might be useful. During most of the sample period the MLR and r_{TB} were closely linked by means of a formula and so were not independent of each other. The use of PBMLR is a way of severing the link as well as capturing the effects of the Bank of England's policy concerning the form of the assistance (funds) supplied to the discount houses and the tone of any moral suasion undertaken by the Bank. There may well be better empirical specifications for the variable PBMLR than the one used in this study and further research on the specification of this variable would be useful. Similarly more research on the role of r_{LD} would also be useful.
- $\frac{17}{T}$ The sign of $\partial(XR/D)/\partial r_{TB}$ is evidence of this substitutability also.
- Note also that a change in r_{TB} will affect $\partial(XR/D)/\partial\lambda$.
- 19/ The stability of the function over time is not examined in this paper. In fact there are reasons for thinking that the 1973-78 period is bounded by two structural changes: the reform -- usually referred to as "competition and credit control" -- of the financial system implemented in late 1971, the effects of which probably lasted well into 1972, and the termination of

exchange and capital controls which took place in 1979. These two events are the main reasons for choosing the particular sample period used in this study.

- Another important necessary condition is the existence of a well-defined non-bank demand for Bank of England liabilities. The nature of this nonbank demand-for-cash function is beyond the scope of the present paper. It is assumed that the Bank of England can control the total supply of its liabilities; such control is, of course, another necessary condition for the feasibility of achieving monetary control by means of manipulation of the monetary base.
- In fact, such a policy of interest-rate control by means of open-market operations in the monetary base has been the Bank of England's policy in recent years. See footnote 12, above.
- Duck and Sheppard maintain that institutional change is necessary before a system of effective monetary control can be adopted in the United Kingdom.

 The empirical results reported in the present paper do not support Duck and Sheppard's claim.
- According to Foot, Goodhart, and Hotson (p. 150, n. 5), "there is no requirement that the [1-1/2 percent cash] ratio be maintained strictly on a day-to-day basis; daily deviations from the 1-1/2 percent ratio can be averaged over the banking month and shortfalls or excesses carried forward."
- Recall that the equation estimated in this paper is the demand for reserves in excess of those required to be held and that the equation performs very well.
- These open-market operations can be in short-term financial assets such as Treasury bills as well as long-term government bonds ("gilts"). It has long been accepted that open-market operations in gilts affect the money supply; this paper indicates that such operations in bills will also affect the money supply.

- $\frac{26}{}$ See the discussions in Foot, Goodhart, and Hotson and in the consultation paper.
- The 1971 financial reform referred to in footnote 19 included <u>inter alia</u>, a reduction in required <u>cash</u> reserves of the banking system. It appears from the data that the authorities did not offset the resulting increase in excess cash reserves through open-market operations. Thus one **perhaps unintended** product of the 1971 financial reform apparently was a sharp increase in excess monetary base, which may well have had a causal role in the subsequent acceleration in money growth and price inflation.
- It is important to note that the coefficients of the equation estimated here may very well no longer be accurate now that there are no longer exchange and capital controls in effect, although no test for this type of structural change has been done in this study. (There were insufficient data for a meaningful test for structural change when this paper was written.) Thus the coefficients cannot be trusted completely to predict the precise effects of changes in the supply of bank reserves, for example, in the post-exchange-control monetary system.
- 29/The definition of r_{LD} may require some explanation. In order to raise one pound of funds to invest freely, a bank must raise $1/(1-\rho)$ in funds. The gross cost to the bank therefore is $r_{MD}/(1-\rho)$. If the reserves bear interest, the cost is reduced to $[r_{MD}/(1-\rho)]-[\rho r_{\rho}/(1-\rho)]$.
- 30/ Actually, in the accounting system used in this paper, some of the liquid-asset reserves are bank liabilities and so more than $(1-\rho)$ of every pound raised by the banks is invested freely. Nevertheless the definition $\rho \equiv .125 + \rho_{SD}$ will be used as a simplification.