International Finance Discussion Papers

Number 196

January 1982

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Recent studies by Ederington (1979) and Frankle (1980) examined the effectiveness of Treasury bill futures contracts as instruments for hedging price risks associated with spot market transactions in Treasury bills. These studies, as well as studies of the hedging effectiveness of livestock and grain futures (Heifner, 1973) and foreign currency futures (Dale, 1981), employed a common set of procedures to estimate a measure of hedging effectiveness developed by Johnson (1960) and Stein (1961).

The present study argues that the procedures employed by the existing studies to estimate the Johnson-Stein measure of hedging effectiveness fail to distinguish reductions in price risk from reductions in price variability. The principal contribution of this study is to develop alternative procedures for estimating the Johnson-Stein measure that capture this distinction.

The new procedures are used to examine the hedging effectiveness of the 13-week Treasury bill futures contract that is traded on the International Monetary Market (IMM) of the Chicago Mercantile Exchange. An additional contribution of the study is that it considers hedges of a much larger set of spot transactions than that considered in the existing studies of Treasury bill futures. Whereas the existing studies restricted attention to hedges of spot transactions in Treasury bills for durations of 2 and 4 weeks, this study examines hedges of spot transactions in Treasury bills, commercial paper, negotiable certificates of deposit, and

Eurodollars for durations of 2 to 39 weeks. The results suggest that the Treasury bill futures contract is a highly effective instrument for hedging prices risks associated with spot transactions in all of these money market instruments.

I. The Johnson-Stein Measure

The Johnson-Stein analysis uses the concepts of mean-variance portfolio theory to formulate a precise definition of hedging and obtain a measure of hedging effectiveness. Hedging is identified with risk-minimization. Suppose that at time t a trader has an existing spot market commitment, $x_{j,t}$, that he plans to liquidate at some future date t+s, prior to or coincident with the futures contract delivery date. The trader is said to be hedging this spot commitment if his purchases or sales of futures contracts, y_t, are chosen to minimize the price risk associated with holding the portfolio of spot and futures positions $(x_{j,t}, y_t)$ from t to t+s.

Price risk is defined in terms of the trader's beliefs concerning the portfolio return. The return is given by

$$R_{t+s}(x_{j,t}, y_t) = (P_{j,t+s} - P_{j,t})x_{j,t} + (f_{t+s}^{t+T} - f_t^{t+T})y_t, \qquad (1)$$

where $P_{j,t}$ is the spot price of commodity j at time t; f_t^{t+T} is the price at time t of the futures contract maturing at time t+T; and $P_{j,t+s}$ and f_{t+s}^{t+T} are the corresponding prices at time t+s, $0 < s \leq T$. At time t the trader views $R_{t+s}(x_{j,t}, y_t)$ as a random variable. His beliefs concerning the possible realizations of $R_{t+s}(x_{j,t}, y_t)$ depends on his beliefs concerning the price changes $(P_{j,t+s} - P_{j,t})$ and $f_{t+s}^{t+T} - f_t^{t+T})$. The latter set of beliefs is represented by a subjective joint probability density function. The riskiness of the portfolio is measured by the subjective variance of $R_{t+s}(x_{j,t}, y_t)$, which can be expressed as a function of the subjective variances and covariances of the price changes:

$$\begin{aligned}\mathrm{Var}(R_{t+s}(x_{j,t}, y_t)) = {} & x_{j,t}^2 \,\mathrm{Var}(P_{j,t+s} - P_{j,t}) \\ & + y_t^2 \,\mathrm{Var}(f_{t+s}^{t+T} - f_t^{t+T}) \\ & + 2x_{j,t}y_t \,\mathrm{Cov}((P_{j,t+s} - P_{j,t}), \\ & (f_{t+s}^{t+T} - f_t^{t+T})).\end{aligned} \tag{2}$$

The risk-minimizing value of y_t is determined by differentiating (2) with respect to y_t, setting the result equal to zero, and solving for y_t. The solution is given by 1/

$$y_t^* = -x_{j,t} \frac{\mathrm{Cov}((P_{j,t+s} - P_{j,t}),(f_{t+s}^{t+T} - f_t^{t+T}))}{\mathrm{Var}(f_{t+s}^{t+T} - f_t^{t+T})} \tag{3}$$

Thus, in the Johnson-Stein framework a trader's futures position is a hedge of the spot position $x_{j,t}$ if and only if it satisfies equation (3).

The effectiveness of the futures contract for hedging the spot position $x_{j,t}$ is defined as the proportional reduction, $e(y_t^*)$, in the risk of the spot position, $x_{j,t}$, that is achieved by forming the portfolio $(x_{j,t}, y_t^*)$:

$$e(y_t^*) = \frac{\mathrm{Var}(R_{t+s}(x_{j,t},0)) - \mathrm{Var}(R_{t+s}(x_{j,t},y_t^*))}{\mathrm{Var}(R_{t+s}(x_{j,t},0))}. \tag{4}$$

Substitution for the subjective variances in (4) using (2) and (3) reveals that the effectiveness of the hedge can be measured by the square of the subjective correlation between the spot and futures price changes:

$$e(y_t^*) = \frac{Cov^2((P_{j,t+s} - P_{j,t}),(f_{t+s}^{t+T} - f_t^{t+T}))}{Var(P_{j,t+s} - P_{j,t})Var(f_{t+s}^{t+T} - f_t^{t+T})}$$

$$= \rho^2_{(P_{j,t+s} - P_{j,t}),(f_{t+s}^{t+T} - f_t^{t+T})} \quad (5)$$

II. Estimation: The Existing Studies

The trader's subjective variances and covariances, which determine the hedge position (3) and the effectiveness of the hedge (5), are, of course, unobservable. In the existing empirical studies of hedging effectiveness based on the Johnson-Stein measure it is implicitly assumed that a trader's subjective joint probability density for the price changes $(P_{j,t+s} - P_{j,t})$ and $(f_{t+s}^{t+T} - f_t^{t+T})$ is identical to the true, objective joint density function. Thus, using a "V" to denote an objective variance and a "C" to denote an objective covariance, the hedge position is

$$y_t^* = -x_{j,t} \frac{(((P_{j,t+s} - P_{j,t}),(f_{t+s}^{t+T} - f_t^{t+T}))}{V(f_{t+s}^{t+T} - f_t^{t+T})} \quad (3')$$

and the measure of hedging effectiveness is the objective squared correlation coefficient

$$e(y_t^*) = \frac{C^2((P_{j,t+s} - P_{j,t}),(f_{t+s}^{t+T} - f_t^{t+T}))}{V(P_{j,t+s} - P_{j,t})\ V(f_{t+s}^{t+T} - f_t^{t+T})}. \quad (5')$$

Given the additional assumption that the price changes are realizations of a covariance stationary time series, estimates of the hedge position and hedging effectiveness can be obtained from a time series of past realizations of the price changes. The ratio of the objective covariance of spot and futures price changes to the objective variance of the futures price change is the slope coefficient in the population linear regression of the spot change on the futures price change. A consistent estimate of that ratio can be obtained from ordinary least squares (OLS) estimation of the regression equation

$$(P_{j,t+s} - P_{j,t}) = \alpha + \gamma\ (f_{t+s}^{t+T} - f_{t}^{t+T}) + \epsilon_{j,t+s}.$$

The coefficient of determination (R^2) from the OLS estimation of this equation provides a consistent estimate of the objective squared correlation coefficient (5').

II. Estimation: An Alternative Approach

At any time t there exists a set, Φ_t, of publicity available information which a trader can use to forecast spot and futures price changes. In representing a trader's beliefs concerning spot and futures price changes by the objective unconditional distribution, the existing empirical studies ignore the availability of such information. As a result, they fail to distinguish reductions in the variability of the return on a portfolio from reductions in the riskiness of a portfolio. The riskiness of a portfolio should be measured by the variance of the portfolio return, conditional on Φ_t.

That is the approach that is taken in this study. If a trader's beliefs are identical to the objective distribution conditional on Φ_t and the spot and futures prices at time t, $P_{j,t}$ and f_t^{t+T}, are elements of Φ_t, then the hedge position (3) and the measure of hedging effectiveness (5) can be stated in terms of conditional variances and covariances of the <u>levels</u>

is a parameter in the conditional expectation of $P_{j,t+s}$ given f_{t+s}^{t+T} and φ_t. The conditional expectation is a linear function.

$$E(P_{j,t+s} | f_{t+s}^{t+T}, \varphi_t) = \alpha + \beta f_{t+s}^{t+T} + \delta' \varphi_t \tag{6}$$

where

$$\beta = Cov(P_{j,t+s}, f_{t+s}^{t+T} | \varphi_t)/Var(f_{t+s}^{t+T} | \varphi_t),$$

$$\underline{\delta} = (Var(\varphi_t | f_{t+s}^{t+T}))^{-1} Cov(P_{j,t+s}, \varphi_t | f_{t+s}^{t+T}), \text{ and}$$

$$\alpha = E(P_{j,t+s}) - \beta E(f_{t+s}^{t+T}) - \delta' E(\varphi_t).$$

Thus, β is a parameter in the linear regression equation

$$P_{j,t+s} = \alpha + \beta f_{t+s}^{t+T} + \delta' \varphi_t + \epsilon_{j,t+s} . \tag{7}$$

Given the assumption that $f_t^{t+T} \epsilon \Phi_t$, equation (7) can be rewritten as

$$P_{j,t+s} = \alpha + \beta(f_{t+s}^{t+T} - f_t^{t+T}) + \delta_1 f_t^{t+T} + \delta_2' \varphi_{2t} + \epsilon_{j,t+s} , \tag{8}$$

where $\varphi_t' = (f_t^{t+T}, \varphi_{2t}')$.

The assumption that $(f_{t+s}^{t+T} - f_t^{t+T})$ is uncorrelated with the elements of φ_t, and, in particular, φ_{2t}, implies that omission of any of the elements of φ_{2t} from equation (8) does not affect the value of the coefficient on $(f_{t+s}^{t+T} - f_t^{t+T})$. Thus, letting Φ_t^* be any subset of Φ_t such that $f_t^{t+T} \epsilon \Phi_t^*$, and letting $\varphi_t^{*'} = (f_t^{t+T}, \varphi_{2t}^{*'})$ list its elements, the slope coefficient, β, on

f_{t+s}^{t+T} in the regression equation

$$P_{j,t+s} = \alpha + \beta f_{t+s}^{t+T} + \delta_1 f_t^{t+T} + \delta' \varphi_{2t}^* + \epsilon_{j,t+s}^* \tag{9}$$

has the same value as in equation (8). Equation (9) can be estimated by ordinary least squares (OLS). Given the additional assumption that the disturbances, $\epsilon_{j,t+s}^*$, are serially uncorrelated, the OLS estimator, $\hat{\beta}$, has an asymptotic normal distribution with mean β.

Although the OLS estimator is consistent for any choice of Φ_t^* such that $\Phi_t^* \subseteq \Phi_t$ and $f_t^{t+T} \in \Phi_t^*$, its efficiency, as measured by its asymptotic variance, is not independent of the specification. More specifically, under assumptions A1 and A2 the asymptotic variance is the following function of Φ_{2t}^*:

$$\text{AsyVar}(\hat{\beta}) = \frac{\text{Var}(P_{j,t+s} \mid f_{t+s}^{t+T}, f_t^{t+T}, \varphi_{2t}^*)}{\text{Var}(f_{t+s}^{t+T} - f_t^{t+T})} .$$

In general, the more elements that are included in φ_{2t}^*, the smaller is the asymptotic variance of $\hat{\beta}$. However, if it is assumed that

(A3) there exists a variable, w_t, such that $(P_{j,t+s} - w_t)$ is uncorrelated with Φ_t,

then, if w_t is included in Φ_t^* the inclusion of additional variables will not result in a more efficient estimator. This result reflects the fact that if $(f_{t+s}^{t+T} - f_t^{t+T})$ and $(P_{j,t+s} - w_t)$ are uncorrelated with all elements of Φ_t, under the normality assumption the conditional distribution of $P_{j,t+s}$ and f_{t+s}^{t+T} given Φ_t is identical to the marginal distribution of $(P_{j,t+s} - w_t)$ and $(f_{t+s}^{t+T} - f_t^{t+T})$. An estimate of the parameter is best obtained by estimating the equation

$$(P_{j,t+s} - w_t) = \alpha + \beta(f_{t+s}^{t+T} - f_t^{t+T}) + \epsilon_{j.t+s}. \tag{10}$$

of the future spot and futures prices, $P_{j,t+s}$ and f_{t+s}^{t+T}:

$$y_t^* = -x_{j,t} \frac{Cov(P_{j,t+s}, f_{t+s}^{t+T} \mid \Phi_t)}{Var(f_{t+s}^{t+T} \mid \Phi_t)} \qquad (3'')$$

$$= -x_{j,t}\,\theta$$

$$e(y_t^*) = \frac{Cov^2(P_{j,t+s}, f_{t+s}^{t+T} \mid \Phi_t)}{Var(P_{j,t+s} \mid \Phi_t) Var(f_{t+s}^{t+T} \mid \Phi_t)} \qquad (5'')$$

$$= \rho^2_{P_{j,t+s}, f_{t+s}^{t+T} \mid \Phi_t} \,.$$

The principal contribution of this study is the development of techniques for estimating the ratio, θ, that determines the hedge position (3″) and the squared correlation coefficient that measures hedging effectiveness (5″).

In estimating those two magnitudes two basic problems must be confronted. First, the conditional distribution of the future spot and futures prices depends, in general, on the realization of the random vector, φ_t, which lists the information in Φ_t 2/. Second, as a practical matter, all of the elements of Φ_t cannot be listed. In order to make the estimation problems tractable it is assumed that

(A1) $(P_{j,t+s}, f_{t+s}^{t+T}, \varphi_t)$ is a multivariate normal, stationary time series, and

(A2) the change in the futures price, $(f_{t+s}^{t+T} - f_t^{t+T})$, is uncorrelated with φ_t.

The normality assumption is frequently given as a justification for mean-variance portfolio analysis. If a trader's subjective joint density functions for spot and futures price changes is a bivariate normal density, his preferences concerning alternative portfolios can be expressed in terms of preferences for combinations of subjective means and variances.

The change in the futures price will be uncorrelated with the elements of Φ_t if the futures market equilibrium price can be stated in terms of the expectation of the spot price on the contract delivery date, P_{t+T}, conditional on Φ_t. For example, if the futures price is an unbiased (rational) expectation of the future spot price

$$f_t^{t+T} = E(P_{t+T} \mid \Phi_t)$$

then the futures price will follow a martigale, i.e.,

$$f_t^{t+T} = E(f_{t+s}^{t+T} \mid \Phi_t).$$

The martigale property implies that changes in the futures price are uncorrelated with the elements of Φ_t 3/. Also, if the futures price is a downward-biased forecast of the future spot price

$$f_t^{t+T} = E(P_{t+T} \mid \Phi_t) + L_T,\ L_T > 0,$$

and the downward bias decreases as the delivery date approaches, then the futures price will follow a submartigale, i.e.,

$$f_t^{t+T} < E(f_{t+s}^{t+T} \mid \Phi_t).$$

The submartingale property also implies that changes in the futures price are uncorrelated with the elements of Φ_t 4/.

When the joint distribution is multivariate normal the variance of the conditional distribution of $(P_{j,t+s}, f_{t+s}^{t+T})$ is not a function of the realization of φ_t. In fact, the ratio, β, that determines the hedge position

Estimation of (10) also provides an estimate of hedging effectiveness. Equation (5″) states that hedging effectiveness, $e(y_t^*)$, is measured by $\rho^2_{P_{j,t+s}, f^{t+T}_{t+s} | \Phi_t}$. Given the equality of the conditional distribution and the bivariate distribution of $(P_{j,t+s} - w_t)$ and $(f^{t+T}_{t+s} - f^{t+T}_t)$,

$$\rho^2_{P_{j,t+s}, f^{t+T}_{t+s} | \Phi_t} = \rho^2_{(P_{j,t+s} - w_t),(f^{t+T}_{t+s} - f^{t+T}_t)}.$$

$\rho_{(P_{j,t+s} - w_t),(f^{t+T}_{t+s} - f^{t+T}_t)}$ can be estimated by the sample correlation coefficient r (the maximum likelihood estimator). As asymptotic 95% confidence interval for $\rho_{(P_{j,t} - w_t),(f^{t+T}_{t+s} - f^{t+t}_t)}$ is given by 5/

$$[\tanh(z - 1.96/\sqrt{\tau - 2}),\ \tanh(z + 1.96/\sqrt{\tau - 2})] \qquad (11)$$

where

$$z = \frac{1}{2} \log \left(\frac{1+r}{1-r}\right)$$

τ = the sample size.

Hedging effectiveness is estimated by the square of the sample correlation coefficient which is, of course, the coefficient of determination (R^2) from estimation of equation (10). A 95 percent confidence interval for hedging effectiveness is obtained by squaring the end points of the interval (11).

In cases in which it is not assumed that there exists a variable w_t which satisfies assumption A3, the best choice of Φ^*_t is not clear. One possibility is to include only the futures price, f^{t+T}_t, and the spot price, $P_{j,t}$, in Φ^*_t. An estimate of the parameter β is then obtained by estimating the equation

$$P_{j,t+s} = \alpha^{*} + \beta f_{t+s}^{t+T} + \delta_1^{*} f_t^{t+T} + \delta_2^{*} P_{j,t} + \epsilon_{j,t+s}^{*}. \qquad (12)$$

For this specification it is not clear how hedging effectiveness can be consistently estimated. Nonetheless, a lower bound on hedging effectiveness can be consistently estimated. Assumptions A1 and A2 imply that

$$\mathrm{Cov}(P_{j,t+s}, f_{t+s}^{t+T} \mid P_{j,t}, f_t^{t+T}) = \mathrm{Cov}(P_{j,t+s}, f_{t+s}^{t+T} \mid \Phi_t)$$

and

$$\mathrm{Var}(f_{t+s}^{t+T} \mid P_{j,t}, f_t^{t+T}) = \mathrm{Var}(f_{t+s}^{t+T} \mid \Phi_t).$$

However, the conditional variance of $P_{j,t+s}$ is affected by the omission of elements Φ_t from Φ_t^{*}:

$$\mathrm{Var}(P_{j,t+s} \mid P_{j,t}, f_t^{t+T}) \geqq \mathrm{Var}(P_{j,t+s} \mid \Phi_t).$$

Together, these relationships imply that the square of the partial correlation of $P_{j,t+s}$ and f_{t+s}^{t+T} given $P_{j,t}$ and f_t^{t+T} places a lower bound on the measure of hedging effectiveness:

$$\rho^2_{P_{j.t+s}, f_{t+s}^{t+T} \mid P_{j,t}, f_t^{t+T}} \leq \rho^2_{P_{j,t+s}, f_{t+s}^{t+T} \mid \Phi_t} = e(y_t^{*}).$$

$\rho_{P_{j,t+s}, f_{t+s}^{t+T} \mid P_{j,t}, f_t^{t+T}}$ can be consistently estimated by the sample partial correlation coefficient. Thus a consistent estimate of a lower bound on hedging effectiveness can be obtained by squaring the sample partial correlation coefficient. An asymptotic 95% confidence interval for this lower bound is given by:[6/]

$$[\tanh^2(z - 1.96/\sqrt{T - 4}),\ \tanh^2(z + 1.96/\sqrt{T - 4})] \qquad (13)$$

where, in this case,

$$z = \frac{1}{2} \log \left(\frac{1 + \hat{r}}{1 - \hat{r}} \right)$$

$\hat{r}$ = the sample partial correlation coefficient.

IV. The Treasury Bill Futures Contract

A. Specification

The procedures outlined in the previous section are used to examine the effectiveness of the 13-week U.S. Treasury bill (T-bill) futures contract traded on the IMM for hedging spot positions in 13-week T-bills, 3-month commercial paper (CP), 3-month Eurodollar deposits (E\$), and 3-month negotiable certificates of deposit (CDs). For each of these securities hedges with a number of different durations, s, and intervals (T-s), between the planned liquidation date and the nearest contract delivery date are investigated.

The durations chosen for investigation are 2, 4, 13, 26, and 39 weeks. The selection of 2-week and 4-week durations facilitates the comparison of the results of this study with the previous studies of the T-bill futures market. The choice of particular horizons for the longer hedges is arbitrary. For each duration selected, hedges with planned liquidation dates 3, 6, 9, and 12 weeks from the nearest contract delivery date are considered. Since the T-bill futures market features quarterly delivery dates up to two years forward, for hedges of duration of 39 weeks or less the nearest date is always within 13 weeks of the planned date of liquidation of the spot position.

Two different specifications of the subset, Φ_t^*, of the set of publicly available information are employed. For hedges of 13-week T-bills for durations of 2, 4, and 13 weeks it is assumed that the implicit forward price for delivery of 13-week T-bills s weeks in the future, i_t^{t+s}, that is embodied in the term structure of rates of return on Treasury securities satisfies assumption A3 . The implicit rate of return, r_t^{t+s}, on a forward contract for delivery of a 13-week T-bill s weeks in the future can be approximated by a linear function of the spot rate on an s-week T-bill, $r_{t,s}$, and the spot rate on an (s + 13)-week T-bill, $r_{t,s+13}$: 7/

$$r_t^{t+s} = \frac{(s+13)r_{t,s+13} - sr_{t,s}}{13} \qquad (14)$$

The implicit forward price, i_t^{t+s}, is then obtained from r_t^{t+s}.

Parkinson (1981) tested and failed to reject the hypothesis that the implicit forward price is an unbiased expectation of the spot 13-week T-bill price on the contract delivery date. As noted above, A3 is implied by the unbiased expectations hypothesis. Thus, for hedges of T-bills for durations of 2, 4, and 13 weeks the hedge position is determined by estimating equation (10) with $w_t = i_t^{t+s}$. Hedging effectiveness is measured by the R^2 from this equation. A 95 percent confidence interval for the measure of hedging effectiveness is computed on the basis of (11).

For liquidation dates more than 13 weeks in the future implicit forward prices for 13-week T-bills often cannot be computed. As equation 14 indicates, the computation of an implicit forward rate for the delivery of a 13-week T-bill s weeks in the future requires rates on an s-week T-bill and an (s+13)-week T-bill. Only 13-week and 26-week T-bills are auctioned on a weekly basis. 52-week T-bills are auctioned at 4-week intervals. If 26-week T-bills and 52-week T-bills matured on the same day of the week outstanding issues could be combined to form implicit forward contracts for 13-week T-bills for delivery on certain dates up to 26 weeks in the future. However, until November 1979 52-week T-bills matured on a Tuesday, whereas 13-week and 26-week T-bills matured on a Thursday. For the other money market instruments considered in this study, implicit forward rates are difficult to compute for any forecast horizon. Spot rates are difficult to obtain for maturities other than 1, 3, or 6 months.

Thus, for 26-week and 39-week hedges of spot T-bills and all hedges of CP, E$, and CDs, the set Φ_t^* consists of $P_{j,t}$ and f_t^{t+T}. An estimate of the optimal, risk-minimizing futures position is obtained by estimating equation (12). The square of the sample partial correlation coefficient of $P_{j,t+s}$ and f_{t+s}^{t+T} given $P_{j,t}$ and f_t^{t+T} is used to estimate the square of the population partial correlation coefficient, which is a lower bound of the measure of hedging effectiveness. A confidence interval for this lower bound is given by (13).

All estimates are based on a quarterly sampling interval. The sample period is from January 1976 to December 1979. The futures and spot T-bill prices are daily closing prices on Thursday of week t, obtained from the International Monetary Market Yearbook and the Wall Street Journal. The spot prices of the other money market instruments are weekly averages of daily prices, obtained from the Federal Reserve Bulletin.

B. Results

Point estimates and standard errors for the ratio, β, that determines the hedge position and point estimates and confidence intervals for the measure of hedging effectiveness (or lower bound thereof) are reported in Tables 1 through 9 in the appendix. Averages of the point estimates for various categories of spot positions are presented in Table 10 below.

An examination of the point estimates of β reported in the appendix reveals that they exhibit a great deal of variation, ranging from .453 to 1.696. As seen in Table 10 the estimates of β tend to be larger for hedges of spot positions in E$ and CDs than for T-bills and CP. There does not appear to be a strong systematic relationship between the estimates of β and the duration of the hedge or the proximity of the contract delivery date to the planned liquidation date.

The estimates of hedging effectiveness indicate that in general the T-bill futures contract is a highly effective instrument for reducing risk; for 58 of the 80 spot positions considered hedging eliminates at least 75 percent of the initial risk. Hedges of spot positions in CP are on average somewhat less effective than those in the other instruments considered. Both the length of the hedge and the proximity of the planned liquidation date to a contract delivery date are important determinants of hedging effectiveness. Hedges of 2 and 4 weeks are generally less effective than hedges of longer durations. As the interval between the planned liquidation date and the contract delivery date increases from 9 to 12 weeks, hedging effectiveness drops off sharply.

Examination of the standard errors of the estimates of β and confidence intervals for the measure of hedging effectiveness suggests that these results be viewed cautiously. This is particularly true for comparison of hedges of different spot positions. The standard errors are large relative to the differences between the point estimates involved in the comparison; the confidence intervals for the parameters overlap considerably. In addition, it must be remembered that in most cases the estimate of hedging effectiveness is actually an estimate of a lower bound of the true measure. To the extent that the amount by which the true value exceeds the lower bound varies across different spot positions, the comparisons can be very misleading.

Table 10

Averages of Point Estimates of $\hat{\beta}$,
Hedging Effectiveness

	$\hat{\beta}$	Hedging Effectiveness (or Lower Bound Thereof)
By Instrument		
U.S. Treasury Bills	.907	.8867
Commercial Paper	.926	.7317
Eurodollars	1.291	.8173
Certificates of Deposit	1.209	.8730
By Interval Between Delivery Date, Planned Liquidation Date		
3 Weeks	1.141	.8061
6 Weeks	1.172	.8649
9 Weeks	1.110	.8954
12 Weeks	.909	.7244
By Length of Hedge		
2 Weeks	.940	.7193
4 Weeks	1.014	.7589
13 Weeks	1.167	.8574
26 Weeks	1.176	.9116
39 Weeks	1.119	.8889

V. Discussion

The first published study of the hedging effectiveness of the T-bill futures contract (Ederington, 1979) concluded that it was a relatively poor instrument for hedging spot positions in T-bills for durations of 2 or 4 weeks. A subsequent study (Frankle, 1980) disputed this conclusion and suggested that it resulted from misspecification of spot T-bill prices.[8/] This study, based on an alternative methodology, confirms the effectiveness of this contract for hedging spot positions of T-bills for durations of 2 and 4 weeks. It also suggests that hedges of spot positions in T-bills and other money market instruments for durations of up to 39 weeks are quite effective.

Indeed, the effectiveness of hedging is found, in general, to increase with the length of the hedge. That result is really not surprising. In general, changes in prices of different financial instruments are not well-correlated over sampling intervals less than one quarter.[8/] One possible explanation for this phenomenon is that investor's portfolio choices are subject to increasing marginal adjustment costs. Given such costs, in the short run investors will find it unprofitable to make the portfolio adjustments necessary to arbitrage away discrepancies between various asset prices. However, over the long ran the adjustments will occur and movements in prices of similar financial instruments will be highly correlated.

On the other hand the effectiveness of hedges of spot transactions in commercial paper, Eurodollar deposits, and certificates of deposit contradicts widely-held views. In the existing futures market literature hedges of transactions in commodities other than those for which the futures market exists are termed crosshedges.[9/] It is often argued that crosshedges are less likely to be effective than own hedges. For example, Arak and McCurdy (1979) claim that

> When the cash asset is different from the security specified in the futures contract, the transaction ... provides much less protection than an exact hedge (p. 39).

It is true that if the spot position involves securities deliverable against the futures contract, a nearly perfect hedge is assured. For example, if the planned liquidation date of a 13-week T-bill position is a contract delivery date (s = T), then, ignoring transactions costs, $P_{t+s} = f_{t+s}^{t+T}$. By setting $y_t^* = -x_{j,t}$ the trader can form a riskless portfolio. But only rarely is a spot position in T-bills deliverable against the contract. T-bill futures contracts are available for only four delivery dates per year. There is no a priori reason to believe that hedges of spot positions in T-bills that are not deliverable are any more effective than crosshedges.[10/]

Finally, as noted in the previous empirical studies the T-bill futures market, the optimal hedge position, y_t^*, is often not equal in magnitude and opposite in sign to the spot position, i.e., $y_t^* \neq -x_{j,t}$. From equation (3) above it can be seen that $-x_{j,t}$ is the optimal position if and only if β=1. But for 24 of 80 hedges considered in this study the estimated value of β is significantly different than 1.

This is potentially important for two reasons. First, in much of the descriptive literature on hedging in trade journals and exchange publications a hedge position is defined as a futures positions that is equal in magnitude and opposite in sign to the spot position.[11/] Second, the money markets are wholesale markets where spot transactions are for multiples of a million dollars. Since the T-bill, futures market calls for delivery of bills with a par value of \$1 million, when $\beta \neq 1$ the optimal hedge position will, in general, be impossible to achieve. The position suggested in the descriptive literature may be the most attractive that is feasible.

A priori it is possible that setting $y_t = -x_{j,t}$ might result in a much smaller reduction in risk than the optimal strategy or even an increase in risk. If $y_t = -x_{j,t}$ the riskiness of the portfolio $(x_{j,t}, -x_{j,t})$ is given by

$$x_{j,t}^2[\mathrm{Var}(P_{j,t+s} \mid \Phi_t) + \mathrm{Var}(f_{t+s}^{t+T} \mid \Phi_t) - 2\mathrm{Cov}(P_{j,t+s}, f_{t+s}^{t+T} \mid \Phi_t)].$$

The proportional reduction in the risk of the spot position is

$$e(-x_{j,t}) = \frac{2\mathrm{Cov}(P_{j,t+s}, f_{t+s}^{t+T} \mid \Phi_t) - \mathrm{Var}(f_{t+s}^{t+T} \mid \Phi_t)}{\mathrm{Var}(P_{j,t+s} \mid \Phi_t)}$$

$$= \frac{\mathrm{Var}(f_{T+s}^{t+T} \mid \Phi_t)(2\beta - 1)}{\mathrm{Var}(P_{j,t+s} \mid \Phi_t)} \qquad (15)$$

Using the same specifications of Φ_t as in Section 3, (15) can be consistently estimated by

$$\tilde{e}(-x_{j,t}) = \frac{s^2(f_{t+s}^{t+T} \mid \Phi_t)(2\hat{\beta} - 1)}{s^2(P_{j,t+s} \mid \Phi_t)}, \qquad (16)$$

where $s^2(\;)$ denotes the usual unbiased estimate of the conditional variance.

Estimates of $e(-x_{j,t})$ for the spot positions considered in this study are reported in Table 11. These reveal that setting $y_t = -x_{j,t}$ in most cases does not result in a much riskier portfolio than does the optimal strategy. In only 8 of 80 cases does

Table 11

Estimates of $e(-x_{j,t})$

2-, 4-, 13-, 26-, and 39-Week Hedges

January 1976 - December 1979

		13-Week Treasury Bills	3-Month Commercial Paper	3-Month Eurodollar Deposits	3-Month Certificates of Deposit
2-Week Hedges					
Weeks from Delivery Date (T-s)	3	.9024	.4064	.6447	.7552
	6	.8863	.8372	.3017	.7714
	9	.9305	.9714	.8832	.8965
	12	.3677	-.2033	.3343	.5111
4-Week Hedges					
Weeks from Delivery Date (T-s)	3	.8440	.2605	.8176	.8728
	6	.7455	.8164	.7311	.7959
	9	.8865	.4660	.7336	.7042
	12	.7363	.3784	.3567	.6117
13-Week Hedges					
Weeks from Delivery Date (T-s)	3	.8578	.8362	.6991	.7873
	6	.7223	.9030	.8270	.8770
	9	.8058	.8544	.8092	.9074
	12	.3072	.7011	.7511	.8324

Table 11 (continued)

		13-Week Treasury Bills	3-Month Commercial Paper	3-Month Eurodollar Deposits	3-Month Certificates of Deposits
26-Week Hedges					
Weeks from	3	.9808	.7480	.7984	.8702
Delivery	6	.9811	.8444	.8561	.8845
Date(T-s)	9	.9802	.8034	.8871	.9411
	12	.9648	.6692	.8942	.9257
39-Week Hedges					
Weeks from	3	.9841	.6392	.8478	.9113
Delivery	6	.9826	.8258	.8837	.9091
Date (T-s)	9	.9738	.6202	.9173	.9536
	12	.9687	.4619	.9030	.9278

$\hat{e}(-x_{j,t})$ lie outside the confidence interval for the measure of the effectiveness of the hedged position, $e(y_t^*)$. These results reflect that the fact that, in general, estimates of $Var(f_{t+s}^{t+T} \mid \Phi_t)$ are much smaller than estimates of $Var(P_{j,t+s} \mid \Phi_t)$. The loss of effectiveness that results from pursuing the suboptimal strategy is given by

$$e(y_t^*) - e(-x_{j,t}) = \frac{Var(f_{t+s}^{t+T} \mid \Phi_t)}{Var(P_{j,t+s} \mid \Phi_t)} \quad (\beta-1)^2 \ .$$

Note that if $Var(f_{t+s}^{t+T} \mid \Phi_t) \leq Var(P_{j,t+s} \mid \Phi_t)$ the loss of effectiveness is bounded by $(\beta-1)^2$. In such a case, as long as β is in the interval [-1.3,1.3], the constraint imposed by the uniform $1 million contract size will result is a loss of hedging effectiveness of less than 10 percent.

Appendix

Table 1

Estimates of β

13-Week U.S. Treasury Bills

2-, 4-, 13-, 26-, and 39-Week Hedges

January 1976 - December 1979

		Sample Size (τ)	Coefficient Estimate	Standard Error
2-Week Hedges				
Weeks from	3	16	1.107	.093
Delivery	6	16	1.047	.099
Date (T-s)	9	15	1.071	.079
	12	15	.527	.106
4-Week Hedges				
Weeks from	3	16	.941	.107
Delivery	6	15	.900	.142
Date (T-s)	9	15	.780	.042
	12	15	.896	.145
13-Week Hedges				
Weeks from	3	15	.931	.103
Delivery	6	15	.857	.140
Date (T-s)	9	15	.766	.075
	12	14	.571	.106
26-Week Hedges				
Weeks from	3	13	.946	.040
Delivery	6	14	.979	.042
Date (T-s)	9	14	.971	.043
	12	13	.918	.052
39-Week Hedges				
Weeks from	3	13	.978	.041
Delivery	6	13	1.010	.045
Date (T-s)	9	13	.991	.054
	12	11	.944	.060

Table 2

Estimates of β

3-Month Commercial Paper

2-, 4-, 13-, 26-, and 39-Week Hedges

January 1976 - December 1979

		Sample Size (τ)		Coefficient Estimate	Standard Error
2-Week Hedges					
Weeks from		3	16	.975	.340
Delivery		6	16	.957	.121
Date (T-s)		9	16	1.023	.050
		12	15	.453	.153
4-Week Hedges					
Weeks from		3	15	.672	.281
Delivery		6	15	1.239	.158
Date (T-s)		9	15	.620	.109
		12	15	.647	.181
13-Week Hedges					
Weeks from		3	15	.996	.180
Delivery		6	15	1.201	.100
Date (T-s)		9	15	.918	.111
		12	14	.847	.165
26-Week Hedges					
Weeks from		3	13	.888	.166
Delivery		6	14	1.258	.146
Date (T-s)		9	14	1.008	.158
		12	13	.882	.201
39-Week Hedges					
Weeks from		3	13	.865	.166
Delivery		6	13	1.166	.168
Date(T-s)		9	13	.941	.244
		12	12	.968	.270

Table 3

Estimates of β

3-Month Eurodollar Deposits

2-, 4-, 13-, 26-, and 39-Week Hedges

January 1976 - December 1979

		Sample Size (τ)	Coefficient Estimate	Standard Error
2-Week Hedges				
Weeks from	3	16	1.278	.255
Delivery	6	16	.677	.245
Date (T-s)	9	16	1.194	.110
	12	15	.702	.255
4-Week Hedges				
Weeks from	3	16	1.254	.151
Delivery	6	15	1.352	.214
Date (T-s)	9	15	1.587	.201
	12	15	.718	.253
13-Week Hedges				
Weeks from	3	15	1.696	.223
Delivery	6	15	1.568	.106
Date (T-s)	9	15	1.543	.134
	12	14	1.346	.210
26-Week Hedges				
Weeks from	3	13	1.533	.162
Delivery	6	14	1.553	.069
Date (T-s)	9	14	1.394	.085
	12	13	1.265	.110
39-Week Hedges				
Weeks from	3	13	1.300	.148
Delivery	6	13	1.395	.094
Date (T-s)	9	13	1.266	.087
	12	12	1.207	.117

Table 4

Estimates of β

3-Month Certificates of Deposit

2-, 4-, 13-, 26-, and 39-Week Hedges

January 1976 - December 1979

		Sample Size (τ)	Coefficient Estimate	Standard Error
2-Week Hedges				
Weeks from	3	16	1.241	.188
Delivery	6	16	.783	.100
Date (T-s)	9	16	1.192	.101
	12	15	.810	.225
4-Week Hedges				
Weeks from	3	16	1.162	.118
Delivery	6	15	1.333	.167
Date (T-s)	9	15	1.261	.228
	12	15	.868	.202
13-Week Hedges				
Weeks from	3	15	1.500	.162
Delivery	6	15	1.424	.090
Date (T-s)	9	15	1.254	.090
	12	14	1.250	.155
26-Week Hedges				
Weeks from	3	15	1.374	.116
Delivery	6	14	1.442	.071
Date (T-s)	9	14	1.225	.063
	12	13	1.186	.092
39-Week Hedges				
Weeks from	3	13	1.182	.105
Delivery	6	13	1.307	.086
Date (T-s)	9	13	1.195	.057
	12	12	1.181	.096

Table 5
Estimates of Hedging Effectiveness
91-Day U.S. Treasury Bills
2-, 4-, 13 Week Hedges
January 1976 - December 1979

		Sample Size (τ)	Coefficient Estimate	95 Percent Confidence Interval Lower Endpoint	95 Percent Confidence Interval Upper Endpoint
2-Week Hedges					
Weeks from	3	16	.9110	.7664	.9678
Delivery	6	16	.8881	.7125	.9592
Date (T-s)	9	15	.9346	.8181	.9775
	12	15	.6540	.2729	.8670
4-Week Hedges					
Weeks from	3	16	.8473	.6223	.9436
Delivery	6	15	.7548	.4294	.9096
Date (T-s)	9	15	.9629	.8939	.9873
	12	15	.7463	.4146	.9062
13-Week Hedges					
Weeks from	3	15	.8626	.6441	.9514
Delivery	6	15	.7431	.4092	.9049
Date (T-s)	9	15	.8883	.7032	.9609
	12	14	.7075	.3339	.8946

Table 6

Estimates of a Lower Bound on Hedging Effectiveness
13-Week U.S. Treasury Bills
26-, and 39-Week Hedges
January 1976-December 1979

				95 Percent Confidence Interval	
	Sample Size (τ)		Coefficient Estimate	Lower Endpoint	Upper Endpoint
39-Week Hedges					
Weeks from	3	13	.9846	.9507	.7326
Delivery	6	13	.9827	.9447	.9403
Date (T-s)	9	13	.9739	.9174	.9901
	12	11	.9722	.9011	.7615
26-Week Hedges					
Weeks from	3	13	.9841	.9491	.9953
Delivery	6	14	.9815	.9437	.9947
Date (T-s)	9	14	.9811	.9425	.9919
	12	13	.9724	.9128	.9924

Table 7

Estimates of a Lower Bound on Hedging Effectiveness
3-Month Commercial Paper
2-, 4-, 13-, 26-, and 39-Week Hedges
January 1976 - December 1979

				95 Percent Confidence Interval	
		Sample Size (τ)	Coefficient Estimate	Lower Endpoint	Upper Endpoint
2-Week Hedges					
Weeks from Delivery Date (T-s)	3	16	.4065	.0512	.7326
	6	16	.8390	.6048	.9403
	9	16	.9719	.9219	.9901
	12	15	.4413	.0632	.7615
4-Week Hedges					
Weeks from Delivery Date (T-s)	3	15	.3655	.0242	.7167
	6	15	.8480	.6119	.9460
	9	15	.7475	.4167	.9067
	12	15	.5387	.1403	.8129
13-Week Hedges					
Weeks from Delivery Date (T-s)	3	15	.7363	.3977	.9021
	6	15	.9290	.8036	.9755
	9	15	.8613	.6412	.9509
	12	14	.7249	.3618	.9016
26-Week Hedges					
Weeks from Delivery Date (T-s)	3	13	.7602	.4034	.9195
	6	14	.8816	.6758	.9602
	9	14	.8034	.5043	.9319
	12	13	.6813	.2752	.8893

Table 7 (continued)

	Weeks from Delivery Date (T-s)	Sample Size (τ)	Coefficient Estimate	95 Percent Confidence Interval: Lower Endpoint	95 Percent Confidence Interval: Upper Endpoint
39-Week Hedges					
	3	13	.6552	.2389	.8788
	6	13	.8429	.5709	.9490
	9	13	.6226	.1977	.8653
	12	12	.6166	.1705	.8699

Table 8

Estimates of a Lower Bound on Hedging Effectiveness
3-Month Eurodollar Deposits
2-, 4-, 13-, 26, and 39-Week Hedges
January 1976 - December 1979

	Sample Size (τ)		Coefficient Estimate	95 Percent Confidence Interval Lower Endpoint	95 Percent Confidence Interval Upper Endpoint
2-Week Hedges					
Weeks from	3	16	.6769	.3202	.8724
Delivery	6	16	.3911	.0428	.7232
Date (T-s)	9	16	.9070	.7568	.9663
	12	15	.4078	.0437	.7423
4-Week Hedges					
Weeks from	3	16	.8527	.6338	.9457
Delivery	6	15	.7844	.4832	.9215
Date (T-s)	9	14	.8498	.6158	.9466
	12	15	.4221	.0516	.7506
13-Week Hedges					
Weeks from	3	15	.8406	.5960	.9432
Delivery	6	15	.9519	.8639	.9835
Date (T-s)	9	15	.9237	.7901	.9736
	12	14	.8042	.5059	.9322
26-Week Hedges					
Weeks from	3	13	.9083	.7304	.9709
Delivery	6	14	.9804	.9405	.9936
Date (T-s)	9	14	.9643	.8934	.9883
	12	13	.9354	.8042	.9797

Table 8 (continued)

		Sample Size (τ)	Coefficient Estimate	95 Percent Confidence Interval	
				Lower Endpoint	Upper Endpoint
39-Week Hedges					
Weeks from Delivery Date (T-s)	3	13	.8955	.6971	.9667
	6	13	.9608	.8777	.9878
	9	13	.9596	.8741	.9874
	12	12	.9303	.7789	.9793

Table 9

Estimates of a Lower Bound on Hedging Effectiveness
3-Month Certificates of Deposit
2-, 4-, 13-, 26-, and 39-Week Hedges
January 1976 - December 1979

				95 Percent Confidence Interval	
		Sample Size (τ)	Coefficient Estimate	Lower Endpoint	Upper Endpoint
2-Week Hedges					
Weeks from Delivery Date (T-s)	3	16	.7847	.4979	.9186
	6	15	.8352	.5970	.9388
	9	15	.9204	.7892	.9713
	12	15	.5400	.1415	.8136
4-Week Hedges					
Weeks from Delivery Date (T-s)	3	16	.8903	.7175	.9601
	6	15	.8068	.5265	.9302
	9	15	.7358	.3968	.9019
	12	15	.6261	.2365	.8545
13-Week Hedges					
Weeks from Delivery Date (T-s)	3	15	.8857	.6971	.9599
	6	15	.9581	.8807	.9857
	9	15	.9462	.8487	.9815
	12	14	.8669	.6411	.9550
26-Week Hedges					
Weeks from Delivery Date (T-s)	3	15	.9398	.8317	.9793
	6	14	.9764	.9286	.9923
	9	14	.9738	.9210	.9915
	12	13	.9490	.8430	.9841

Table 9 (continued)

		Sample Size (τ)	Coefficient Estimate	95 Percent Confidence Interval	
				Lower Endpoint	Upper Endpoint
39-Week Hedges					
Weeks from	3	13	.9334	.7986	.9791
Delivery	6	13	.9623	.8822	.9883
Date (T-s)	9	13	.9797	.9353	.9937
	12	12	.9500	.8375	.9853

FOOTNOTES

*/ Economist, Division of International Finance, Board of Governors of the Federal Reserve System. The views expressed in this paper are solely those of the author; they should not be interpreted as those of the Board or other members of its staff. The paper is from the author's doctoral dissertation ("The Usefulness of Treasury Bill Futures For Forecasting and Hedging," University of Wisconsin-Madison, 1981). The author is greatly indebted to his thesis advisor, John Geweke, for his advice and support throughout the preparation of the study. Helpful advice was also provided by Donald Hester and Dale Henderson.

1/ The sufficient second-order condition for a minimum

$$2\ \mathrm{Var}(f_{t+s}^{t+T} - f_{t}^{t+T}) > o$$

is satisfied.

2/ It is assumed that Φ_t contains a finite number of elements.

3/ See Geweke and Feige (1979).

4/ See Roll (1970), pp. 82 - 83.

5/ See Anderson (1958), p. 77.

6/ See Anderson (1958), pp. 79 and 85.

7/ For a derivation of this result see Parkinson (1981), Appendix A, or Roll (1970), p. 19.

8/ Ederington used a weekly average of daily prices for 13-week T-bills for the spot price $P_{j,t}$. Frankle argued that (1) the maturity of an inventory of T-bills changes over time so that if the maturity is 13 weeks at the liquidation date it must be (s + 13) weeks at the time is initiated and (8) weekly averages of prices should not be used. In the approach taken in this paper, the choice of a specification for $P_{j,t}$ is based on the information it provides concerning $P_{j,t+s}$, not on the basis of maturity. Frankle's contention that use of weekly averages should be avoided is correct.

9/ This contention, although part of financial economic folklore, is usually not well-documented. One source where it is well-documented is Chicago Mercantile Exchange (1980), p. 14.

10/ In fact, they should be considered crosshedges.

11/ For example, see Chicago Mercantile Exchange (1976).

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