TWO ESSAYS ON MONETARY POLICY IN AN INTERDEPENDENT WORLD:

I. MONETARY POLICY GAMES AND THE CONSEQUENCES OF NON-COOPERATIVE BEHAVIOR

II. SOME ASPECTS OF THE ADJUSTMENT PROBLEM IN AN INTERDEPENDENT WORLD

by

Matthew B. Canzoneri and Jo Anna Gray

NOTE: International Finance Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publications to International Finance Discussion Papers (other than an acknowledgment by a writer that he has had access to unpublished material) should be cleared with the author or authors.
Monetary Policy Games and the Consequences of Non-Cooperative Behavior

by

Matthew Canzoneri

and

Jo Anna Gray*

Federal Reserve Board

Washington, D.C. 20551

December, 1982
Revised January, 1982

*We would like to thank Robert Cumby, Robert Flood, Dale Henderson, Robert Hodrick, Preston Miller, Kenneth Rogoff and Jeffrey Sachs for helpful discussions. Catherine Crosby coaxed our ellipses out of the computer. Any errors are, of course, the responsibility of the authors alone.
I. Introduction and Summary

In an interdependent world, rational policy makers in one country may be expected to condition their actions on the policies pursued in other countries; policy making has unavoidable game aspects. In the absence of direct cooperation or side-payments, it is well-known that the outcomes of such games are socially inefficient; there are alternative policies that would, if implemented, make all parties better off. Unfortunately, policy makers generally have an incentive to cheat in these Pareto-improving outcomes, and politically sovereign policy makers seem to have difficulty achieving them.

In this paper we describe the games that may be played by policy makers in two structurally identical countries following a common external shock such as the oil price increase of 1973. The analysis has two objectives. The first is to investigate the nature of the inefficiencies associated with the non-cooperative Nash solutions to these games. In our framework, whether the Nash solution is too expansionary or too contractionary depends on structural features of the world economy. These structural features have changed over the post-war period and with them, it is argued, the inefficiencies associated with the Nash outcome. Thus, our analysis of the costs of non-cooperative behavior differs from previous work, particularly that of Hamada, which places its emphasis on the preferences of policy makers.

Our second objective is to explore means of achieving outcomes superior to the non-cooperative Nash solution. Because of the difficulties inherent in implementing and enforcing cooperative solutions, it is suggested that policy makers look to other non-cooperative solution concepts as alternatives to the Nash. Each of these
alternatives may be characterized by a set of assumptions about the way in which individual players' actions are constrained. The two sets of assumptions, or "rules of the game", we focus on are referred to as the Stackelberg Regime and the Fixed-Rate Regime. Each Regime is modelled as a leader-follower game with the US as the leader and the rest of the world (or ROW for short) as the follower. The Stackelberg solution concept is familiar, but our use of fixed exchange rates as a solution concept in a non-cooperative leader-follower game is new.\textsuperscript{3/} In the Fixed-Rate Regime, the follower commits itself to fixing its bilateral exchange rate with the leader. The leader then chooses an optimal policy subject to the committed response of the follower. Fixed exchange rates are, then, viewed as an alternative non-cooperative solution that the ROW may be able to impose upon the US.

The relative desirability of the Nash, Stackelberg and Fixed-Rate Regimes depends on the structural features of the world economy alluded to earlier, features that have been changing over time. Thus our analysis suggests one explanation for the way international policy regimes have adapted to an evolving world economic structure.

We begin in section II by postulating a social welfare function that policy makers in each country attempt to maximize. The welfare function includes domestic output and the long-run rate of inflation. Also described are the macroeconomic constraints that are relevant to policy makers in each country as they use the one policy tool at their disposal, domestic monetary policy, to maximize this objective function. These constraints explain how domestic and foreign monetary policy affect domestic output and long-run inflation. This section draws heavily on
the results in Canzoneri and Gray (1983). That paper analyzes a three
country model with two oil-importing countries, the US and the ROW, and
one oil producing country, OPEC. US and ROW monetary policy "matter" in
this model, as does OPEC's pricing policy, because of contractually set
wage rates in the US and the ROW. In conducting policy, monetary
policy makers face a tradeoff between short-run employment objectives and
long-run inflationary expectations. This tradeoff is due to a public
sector credibility problem that is captured, in an ad hoc way, by the
assumption that private sector expectations concerning monetary policy
are static. That is, a new policy must be run for a period before it is
fully reflected in wage setters' expectations.

The game aspects of the paper derive from the spillover effects of
monetary policy -- that is, the way one country's monetary policy affects
output in the other -- and the global shock that creates an adjustment
problem in the first place. In Canzoneri and Gray (1983), the global
shock is an oil price increase, but here it can be interpreted more
generally. The exact nature of the spillover effects depends upon
structural and institutional features of the three economies involved.
In our model, two important determinants of the sign and symmetry of the
spillover effects are the degree of wage indexation in each of the oil-
importing countries and the role played by oil.

Three of the many possible specifications of the spillover effects
of monetary policy are singled out for consideration. The first
specification produces a beggar-thy-neighbor policy world in which an
expansionary monetary policy in either country causes a contraction in
the other. Our model suggests that this specification may be relevant in
periods when indexing and oil play no major role. The second produces a
locomotive policy world in which an expansionary policy in either country causes an expansion in the other. This specification is particularly relevant for periods in which a high degree of wage indexation prevails. The third specification produces an asymmetric policy environment in which an expansion in the US induces an expansion in the ROW, while an expansion in the ROW induces a contraction in the US. Our model suggests that this specification may be relevant when OPEC is setting a dollar price of oil in the short-run. We speculate that the first, or symmetric-negative, specification is most likely to have been relevant (if ever) in the early post-war era; the symmetric-positive and asymmetric specifications refer to more recent periods.

In section III, the inefficiencies associated with the Nash solution are characterized, and a rationale is developed for considering non-cooperative, leader-follower regimes as realistic alternatives to the Nash. The nature of the inefficiencies associated with the Nash solution depend upon the macroeconomic constraints under which the game is played. In the beggar-thy-neighbor policy world, the Nash solution is too expansionary; each country would be better off if both countries could somehow manage to inflate less. The converse is true for the locomotive policy world. Welfare in both countries would be raised by more expansionary policies. In the asymmetric game, all would be better off if the US inflated more while the ROW inflated less.

The argument for considering other non-cooperative regimes as alternatives to the Nash is based upon the difficulties inherent in defining and verifying directly coordinated policies and the moral hazard that this implies. These difficulties suggest that it may be more realistic and more productive to model policy makers attempts to achieve
a better outcome in a non-cooperative setting. The non-cooperative Stackelberg and Fixed-Rate Regimes we propose appear to be historically relevant and they avoid some of the moral hazard problems associated with direct cooperation.

In section IV, it is shown that the Stackelberg and Fixed-Rate Regimes can indeed provide Pareto-superior outcomes to the Nash. With the symmetric-negative constraints, two outcomes are possible. The Fixed-Rate Regime may be the preferred set of rules for all players, in which case it is assumed that fixed rates will be implemented. However, it is also possible that the ROW will prefer to have the US lead in a Stackelberg game, while the US prefers fixed rates. In this situation there is no theoretical presumption about how the conflict over policy regime will be resolved. A similar situation arises with the symmetric-positive constraints. Fixed rates may be preferred by all. But it is also possible that the US will prefer to lead in a Stackelberg game while the ROW prefers fixed rates. Again, our analysis does not suggest a resolution to this conflict. Finally, the case in which spillover is asymmetric is the only case in which it is certain that fixed rates will not be the preferred outcome. Either both players will prefer the Stackelberg Regime, or there will be disagreement, with the US favoring the Stackelberg Regime and the ROW supporting fixed rates.

The methodology, rather than the specific results, of Section IV are what we wish to emphasize in this paper. Some of our direct comparisons depend upon the form of the welfare functions that are postulated. Further, we consider only global shocks as the driving force behind all of the games. But more importantly, no formal structure for the game of regime selection is presented. In all the games we consider,
conflict of interests may arise. Consider, for example, the game played under the symmetric-negative constraints. As already noted, the ROW may prefer the Stackelberg Regime to both the Nash and the Fixed-Rate Regimes, while the US always finds the Fixed-Rate Regime the most attractive of the three. The ROW cannot force the US to lead in a Stackelberg game, and the US cannot force the ROW to fix its exchange rate against the US dollar. Who will win out if this situation occurs? On the basis of the analysis presented in this paper, we have no way of telling. Our approach does, however, suggest which solutions or regimes a given policy maker will prefer in a given economic environment. Even in situations in which interests conflict then, it may provide some insight into the posturing and compromises of policy makers as they adapt world policy regimes to an evolving world economic structure.

II. Utility Functions and Macroeconomic Constraints

This section begins with a description of the social welfare function that policy makers in each country attempt to maximize. We then develop the macroeconomic constraints for the game played by these policy makers as they strive to achieve their objectives.

Policy makers in each oil importing country are assumed to be concerned with domestic employment and long-run expectations of the domestic rate of inflation; their utility functions take the following form:

\[ U = -(\delta x)^2 - \mu \pi^2, \quad U^* = -(\delta x^*)^2 - \mu^* \pi^*^2 \]

Here \( \delta x \) and \( \delta x^* \) represent deviations of employment or output from their
full employment values in the U.S. and the ROW, and $\pi$ and $\pi^*$ are long-run inflationary expectations. While the first argument in each of these utility functions requires no special justification, the second may. The case for including an inflation term of this particular form will be taken up later in the section.

There would, of course, be no policy game if policy makers had available as many independent instruments as they have targets. Accordingly, we assume that policy makers in each country have at their disposal only one policy tool -- the rate of growth of the domestic money supply -- to use in pursuing their dual objectives.5/

By assumption, all games are initiated by a global shock that will, in the absence of corrective policy, cause output levels in both countries to deviate from their full employment values. In Canzoneri and Gray (1983), the global shock is an unanticipated increase in the price of oil. We will continue to use that example here. The oil price shock affects output levels, and monetary policies "work", because of contractually set nominal wage rates in the two countries. Deviations of output from full employment are, however, temporary; they persist for only one contract period, which is referred to as the adjustment period. Absent further unanticipated shocks, output returns to its full employment level in the period following the adjustment period.

Monetary policy can be used to offset some or all of the short-run output effects of an oil price increase, but only at the cost of increasing the inflation rate expected to prevail subsequent to the adjustment period. This is because private agents are assumed to expect the money growth rate established during the adjustment period to be permanent.6/ In a very simple way, this captures the notion that policy
makers are subject to a credibility constraint. The important implicit assumption is that the monetary authority cannot convince the private sector of a change in policy by simply announcing it. Agents expect past policy to continue until confronted with an actual policy change. Once the new policy has been in place for a period, it, then, is believed to be permanent.

Policy makers have two choices in dealing with the higher inflationary expectations generated by their responses to the oil price increase. They can accommodate them, thereby maintaining full employment output in subsequent periods. Or they can lower them by lowering the rates of growth of their money supplies, thereby forcing their economies through additional adjustment periods. The terms \( \bar{\pi} \) and \( \bar{\pi}^* \) enter the utility functions (1) because each of these choices has undesirable consequences -- higher steady state inflation in the first case and additional periods of adjustment and unemployment in the second.\( \dagger \)

Under the assumptions discussed above, the long-run expected rates of inflation in the U.S. and the ROW are simply

\[
(2) \quad \bar{\pi} = g, \quad \bar{\pi}^* = g^*,
\]

where \( g \) and \( g^* \) are the rates of growth of the US and ROW money supplies during the adjustment period. The oil price shock has no direct effect on long-run expected rates of inflation; it affects inflation expectations only if it affects money growth rates. Letting \( \delta g \) and \( \delta g^* \) represent the changes in the growth rates of US and ROW money that occur in response to the oil price shock during the adjustment period, we have
(3) \[ \bar{\pi} = \pi_0 + \delta g, \quad \bar{\pi}^* = \pi_0^* + \delta g^*, \]

where \( \pi_0 \) and \( \pi_0^* \) are the rates of inflation in the full-information equilibrium preceding the adjustment period. These are the inherited rates of inflation that each country enters the adjustment period with. Using equations (3), the utility functions (1) may be re-written as

(4) \[ U = - (\delta x)^2 - \mu (\pi_0 + \delta g)^2, \quad U^* = - (\delta x^*)^2 - \mu^* (\pi_0^* + \delta g^*)^2. \]

For most of our analysis, we assume that \( \pi_0 \) and \( \pi_0^* \) are zero and that \( \mu \) equals \( \mu^* \).

The dependence of output levels on the price of oil and on the two monetary policy instruments is somewhat more complicated. In most macro-models concerned with the short-run determination of output, output responds negatively to unanticipated increases in the prices of imported inputs and positively to unanticipated increases in the rate of growth of the domestic money supply. However, there exists no such consensus on the spillover effects of monetary policy, which depend on the structural and institutional features of the economies involved and the role played by oil. The sign and symmetry of these spillover effects are important because they produce the game aspects of our paper.

In the case of symmetric-negative spillover, monetary policy is a "beggar-thy-neighbor" policy. An expansionary policy in either country exports unemployment to the other. In the case of symmetric-positive spillover, monetary policy has a "locomotive" effect. An expansionary
policy in either country increases output in the other. In the asymmetric case, a US expansion increases output abroad, but a ROW expansion causes output to fall in the US.

The three game situations just described are embedded in three models of output determination which are summarized by equations (5a), (5b), and (5c) below. These reduced form models describe the dependence of output in each country (δx and δx*) on the changes in the money growth rates of both countries (δg and δg*) and the oil price shock (δq). They constitute three possible sets of contrasts that may be faced by policy makers as they attempt to maximize the objective functions given by equations (4).

(5a) Symmetric-Negative Constraints:

\[
\delta x = \rho_1 \delta g - \rho_2 \delta g^* - \rho_3 \delta q, \quad \delta x^* = \rho_4 \delta g^* - \rho_5 \delta g - \rho_6 \delta q
\]

where \( \rho_i > 0 \) for \( i = 1, 2, 3 \) and \( \rho_1 > \rho_2 \).

(5b) Symmetric-Positive Constraints:

\[
\delta x = \rho_4 \delta g + \rho_5 \delta g^* - \rho_6 \delta q, \quad \delta x^* = \rho_7 \delta g^* + \rho_8 \delta g - \rho_9 \delta q
\]

where \( \rho_i > 0 \) for \( i = 4, 5, 6 \) and \( \rho_4 > \rho_5 \).

(5c) Asymmetric Constraints:

\[
\delta x = \rho_7 \delta g - \rho_8 \delta g^* - \rho_9 \delta q, \quad \delta x^* = \rho_7^* \delta g^* + \rho_8^* \delta g - \rho_9^* \delta q
\]

where \( \rho_i > 0 \) and \( \rho_i^* > 0 \) for \( i = 7, 8, 9 \).
In Canzoneri and Gray (1983), we analyze in some detail a model that can generate all three sets of constraints given above. A formal description of that model is presented in the appendix. The remainder of this section contains a summary of the model's central features and a discussion of the different structural assumptions that produce equations (5a), (5b) and (5c).

The framework developed in Canzoneri and Gray (1983) consists of three countries, one oil producing country (OPEC) and two oil importing countries (the US and the ROW). Oil is an intermediate good that is used by the US and ROW to produce two consumption goods which are consumed in all three countries. Each oil importing country is specialized in the production of one of these consumption goods. Labor is the only other variable input employed in the production of each consumption good. OPEC sets the price of oil in terms of US dollars, but this price may be partially or fully linked to a consumer price index. The nominal wage in each oil importing country is contractually fixed in terms of the local currency, but may also be partially or fully linked to an index of consumption good prices. The absence of completely flexible wage rates introduces the possibility of short-run deviations of output from its full employment level.

The model includes three exogenous policy variables: the price of oil and the rates of growth of the US and ROW money supplies. The impact of changes in these policy variables on output levels depends on structural parameters such as the degree of wage indexation in the US and ROW. The constraints given by equations (5) describe these effects under differing sets of assumptions about the sizes of such parameters. The
time horizon to which the constraints apply is the "adjustment period", a period of time determined by the length of labor contracts.

To close the model, the manner in which agents form their expectations must be specified. Expectations formation in the model is rational in a limited sense. Specifically, agents' price expectations are rational given their (static) assumptions about US and ROW monetary policies and OPEC's oil pricing policy. If their views about these policies are correct, then their price predictions will be realized.\textsuperscript{9/}

For our purposes, the most important way in which the constraints (5) differ from each other is in their specification of the spillover effects of monetary policy. These spillover effects, in turn, depend on the channels through which a country's monetary policy is transmitted abroad. A number of possible channels may be identified.\textsuperscript{10/} We will focus on four. The first two of the four, an interest rate channel and a goods demand channel, are well-known. The other two were selected because of their increasing relevance for a number of countries over the past decade. One stems from the now common practice of contractually linking nominal wage rates to an index that includes the prices of imported goods as well as domestically produced goods. The other arises from the nature of OPEC's oil pricing policy; the price of oil is denominated in dollars and it not always adjusted quickly in response to changes in the purchasing power of the dollar.

The first of our transmission channels was formally introduced by Mundell (1963). This channel, which depends critically on capital mobility, operates through the real interest rate to produce negative spillover. In the absence of fully indexed input prices, an expansionary monetary policy in one country raises that country's output and lowers
its real interest rate. The real interest rate movement is transmitted abroad, where its net impact is deflationary. This is because the lower real interest rate produces a fall in the foreign nominal interest rate and this, in turn, increases foreign money demand. The increased money demand results in downward pressure on foreign prices and output. Thus, the spillover effects that are channelled through the real interest rate are negative and symmetric across countries.

The second transmission channel we consider, which depends on the existence of two or more goods in the model, operates directly through the demand for goods. An expansion in one country raises output and expenditure in that country. Some, but not all, of the increased expenditure falls on each country's good, generating an excess supply of the expanding country's good and an excess demand for the other country's good. As a result, both the relative price and the output of the second country's good rise. Thus, the goods demand channel generates spillover effects that are positive and symmetric across countries.

Our third channel of transmission, wage indexation, depends critically on the terms of trade change generated through the goods demand channel. As discussed above, an expansion in one country raises the relative price of the good produced by the other country. If nominal wages are linked to a price index that includes the price of imports as well as the price of domestically produced output, the second country's real product wage falls and its output increases. Accordingly, the spillover effects produced through this transmission channel are also positive and symmetric.

The channel associated with the fixed dollar price of oil is, in contrast to the other three channels, an asymmetric one. An expansionary
US policy lowers the real price of oil for both US and ROW producers. In and of itself, this leads to an increase in output in both countries. An expansionary ROW policy, on the other hand, does not directly affect the real price of oil. It acts indirectly through its effect on the U.S. price level -- an effect that is ambiguous in sign. Accordingly, the spillover effects produced by this channel are positive for U.S. policy but are ambiguous in sign for ROW policy.

The nature of the game that is actually played by policy makers depends on the relative importance of the four channels of transmission just described. The relative importance of these channels, in turn, depends on the structural features of the economies involved -- features that have changed dramatically over time. On the one hand, a fully indexed price of oil, no indexation of wages, and a sufficiently high price elasticity of relative goods demand produces the negative symmetric spillover of equations (5a). By comparison, a fully indexed oil price and a sufficiently high degree of wage indexation produce the positive symmetric spillover of equations (5b). Among the conditions that produce the asymmetric spillover of equations (5c) are a fixed dollar price of oil and a large price elasticity of relative demand. The reader is referred to Canzoneri and Gray (1983) for a more extensive discussion of the economic intuition, as well as the formal analysis, underlying these results.

III. The Costs of Non-Cooperative Behavior and Alternative Rules of the Game

It is well known that, in general, the non-cooperative Nash solution to a game is inefficient; there typically exist alternative feasible outcomes that would increase the utility of all players. To
illustrate this point, consider a game characterized by the symmetric-positive constraints described in the last section. In this game policy makers are faced with a global disturbance that will, in the absence of corrective policy, produce unemployment in both the US and ROW. Policy responses are designed to balance the benefits of decreased employment losses against the costs of increased inflationary expectations. Both players opt for an expansionary monetary policy. Each is aware of the beneficial effect of the other player's expansion on home employment and appropriately internalizes that information by expanding less than he would in the absence of the other player's response. However, neither player takes into account the beneficial impact of his own actions on the other player. As a result, the Nash solution does not exploit the positive externalities associated with monetary policy enough; both policy makers would be better off if they could agree to expand more.

This result is demonstrated formally in the next section, but its logic is fairly straight-forward. In the Nash solution, the US policy maker has increased the rate of growth of the US money supply to the point where any further marginal change would have no effect on US utility. However, further increases in the growth rate of US money would increase ROW utility by lowering ROW unemployment. Similarly, a marginal increase in the rate of growth of the ROW money supply would have no effect on ROW utility, but it would increase US utility. Clearly, both policy makers would be better off if they could agree to expand more.

Analogous inefficiencies characterize the Nash solutions to the other games studied in this paper. In the symmetric-negative game, both players would be better off if they could agree to inflate less. In the
asymmetric game, both would be better off if the US inflated more while the ROW inflated less.

In each of the three games we consider, there are a variety of Pareto-improving outcomes and a Pareto-efficient set characterized by the contract curve. The basic problem with all of these "better" outcomes is that they provide incentives for one or both of the players to cheat. Consider once again the symmetric-positive game, and suppose the US and ROW policy makers have agreed to the swap described above; that is, each has promised a more expansive policy than implied by the Nash solution. In this situation, the US policy maker has an incentive to cheat on the agreement and run a less expansionary policy. This is because the more expansionary policy promised by the ROW would increase US employment, making it optimal for the US to economize to some extent on inflation. Put another way, if the US policy maker did not cheat and actually carried out the agreed upon policy, the outcome would look too inflationary to a public that did not fully understand the nature of the solution and discounted the possibility of foreign repercussions in response to a tighter US monetary policy. The political pressure to cheat could well be enormous.

The cheating problem is compounded by the difficulty of defining and verifying cooperative solutions, and by the moral hazard this implies. The various OECD countries have different procedures for implementing monetary policy and different definitions for monetary aggregates; both can be altered in subtle ways that allow policy makers to violate the spirit, without violating the letter, of their commitments. Further, the effect of a monetary policy that is well-defined in terms of implementation procedures and aggregates can always
be altered by changing the regulatory environment in financial markets. These factors provide great scope for cheating in any direct attempt at coordination of policies.

In the remainder of this section we explore ways in which a group of politically sovereign policy makers can achieve outcomes superior to the Nash solution and at the same time avoid some of the pitfalls of direct attempts at cooperation. Our approach is to ask whether policy makers can design well-defined and easily verified rules of the game that will produce a Pareto-improving outcome. To do this, we must first discuss the nature of the players in our games. We can then consider the kinds of policy regimes that they might be capable of instituting.

Up to now, we have treated the US and ROW policy makers in a completely symmetric manner. This is probably unrealistic. The US is a single country with one monetary authority; the ROW on the other hand is an aggregation of a number of smaller countries, each with its own monetary authority. This suggests that the degree of coordination ROW policy makers can hope to achieve among themselves is limited. If coordinated actions are difficult for the ROW, the role of "leader" in the global policy games we are concerned with naturally falls to the US. In fact, the US appears to have played a dominant role in the policy games of the post World War II era and may continue to do so in the 1980's. Therefore, the alternative rules of the game we have chosen to examine assign the role of leader to the US and the role of follower to the ROW. A follower, in our terminology, is a player (or group of players in the case of the ROW) that is committed to a well-defined reaction in response to any particular policy the dominant player, or leader, may choose. The leader knows the reaction function of
the follower and sets policy to maximize its own utility subject to the constraint implied by that reaction function.

Two leader-follower policy regimes are considered. The first is the Stackelberg Regime. In this case, the follower's committed response is to simply take the leader's policy as given and maximize its utility. The leader chooses its own utility maximizing policy subject to the follower's reaction function. The rules of this game are well-defined and require no verification since each player is following a utility maximizing policy. The second is the Fixed-Rate Regime. In this regime, the follower's committed response is to fix its bilateral exchange rate with the leader by matching the leaders money growth rate. Once again, the leader maximizes its own utility subject to the constraint implied by the follower's reaction function. The rules of this game are also well-defined, and since bilateral exchange rates are readily observed, adherence to the rules is also easily verified. The basic difference in the attractiveness of these two regimes lies in the difference in the reaction function the follower offers to the leader.

It should be noted that our view of the Fixed-Rate Regime is not the usual one. Here it is modelled as a non-cooperative game, with the follower enforcing the fixed rates. It is not modelled as the outcome of direct cooperation and coordination of policies; in fact, we have questioned the feasibility of such regimes because of their vulnerability to cheating. In our view, the Fixed-Rate Regime is an alternative that the follower will want to impose upon the leader if it produces an outcome more favorable than the Stackelberg or Nash solutions.
The way US and ROW policy makers rank the Nash, Stackelberg and Fixed-Rate solutions will in general depend upon the spillover effects of monetary policy. As the structural constraints implied by these spillover effects change, so will the relative merits of the Nash, Stackelberg and Fixed-Rate Regimes. In the next section, we describe how the US and ROW policy makers rank these regimes under the three sets of constraints given by equations (5) of the preceding section.

IV. Adapting Regimes to an Evolving World Economy

In this section we explore the relative merits of Nash, Stackelberg and Fixed-Rate Regimes under each of the three sets of macroeconomic constraints developed in section II. Our view is that each set of constraints may have been relevant at a different point in history. Accordingly, the discussion of this section is organized in what we regard as a plausible chronological order.

A. A Beggar-thy-neighbor Policy World:

The world economic structure characterized by the symmetric-negative constraints (5a) is one in which the interest rate channel for the transmission of monetary policy dominates. It's theoretical foundations may be found in Mundell (1963). It's policy implications are consistent with the competitive devaluations and beggar-thy-neighbor policies of an even earlier period. Our discussion in section II also suggests that it describes a period in which neither wage indexation nor oil played an important role. The time period for which this game is most likely to have been relevant, then, is the post WWI era -- a period in which policy makers found refuge in a Fixed-Rate Regime. As we will show below, these observations are consistent with our model. Fixed
exchange rates may be the preferred policy regime in a world characterized by the symmetric-negative constraints (5a) or, for that matter, the symmetric-positive constraints (5b) to be discussed later.

For symmetric-negative constraints, the situation is illustrated by Figure 1. (Figure 1B is an enlargement of the boxed area in Figure 1A.) B and B* are the US and ROW bliss points; indifference curves take the form of ellipses, with larger ellipses representing lower utility levels.\(^ {14/} \) The straight lines labeled NS and NS* are the US and ROW reaction curves for the Nash and Stackelberg games. The US curve, for example, gives the US utility maximizing response to any ROW policy.\(^ {15/} \) It and its ROW counterpart are referred to as the NS reaction functions in the discussion that follows. The Nash solution is labeled N, the Stackelberg solutions are S and S*, and the Fixed-Rate solution is F.\(^ {16/} \)

The Nash solution is located at the intersection of the two NS reaction curves. Neither the US nor the ROW has an incentive to move unilaterally from this point. However, as discussed in the last section, there exist alternative feasible outcomes that are better for one or both players. All of these Pareto-improving points are included in the shaded area of figure 1B; the Pareto-efficient outcomes lie on a hyperbola (not pictured) running from B to F to B*.\(^ {17/} \) The incentive to cheat at any of these points, also discussed in the last section, is indicated by the fact that one or both of the players must be off his NS reaction curve.

In the Stackelberg Regime, the US chooses a utility maximizing policy conditioned on the ROW's Nash reaction function. In our diagrams, the ROW restricts the US opportunity set to the curve NS*; the US picks point S, the point of tangency between NS* and a US indifference ellipse.
FIGURE 1A: THE BEGGAR–THY–NEIGHBOR POLICY WORLD
FIGURE 1B: THE BEGGAR-THY-NEIGHBOR POLICY WORL
The Stackelberg Regime has much to recommend it. Both the US and the ROW achieve higher utility at S than at N. The ROW is better off because S lies back along its reaction curve in the direction of its bliss point, B*. The US must be better off since it explicitly chooses S over N.\(^18\) The US is, however, off of its reaction curve at S. For this to be a viable regime, the US must be convinced that any attempt to move away from S toward its own reaction curve will only result in a movement along NS*. This condition will be met as long as the ROW is committed to the response function NS* and the US is aware of that commitment.

It is interesting to note that in this beggar-thy-neighbor policy world the US may be regarded as a reluctant leader. (This is not true with the symmetric-positive constraints (5b), as will be seen below.) Were it possible, the US would opt for a Stackelberg Regime in which the ROW was the leader; that is, S* is better for the US than S.\(^19\) Our assumptions preclude this possibility. However, this comparison does suggest that the US might view its leadership role in this Stackelberg Regime as a burden.

With symmetric-negative constraints, the Fixed-Rate Regime may dominate both the Nash and Stackelberg Regimes for both the US and the ROW. In the Fixed-Rate Regime, the ROW policy makers commit their monetary policies to fixing their dollar exchange rates; in our model, this amounts to the ROW setting its money growth rate to match whatever money growth rate the US adopts. In terms of our diagrams, the ROW offers the US the 45° line as its reaction curve instead of NS*; the US chooses point F, which happens to be a Pareto-efficient outcome. In this case then, the Fixed-Rate Regime is one way of achieving a cooperative solution in a non-cooperative setting.\(^20\)
The Fixed-Rate Regime is clearly better than the Nash Regime for both the US and the ROW. For the US, it is also unambiguously superior to the Stackelberg Regime.\textsuperscript{21} However, the ROW may or may not be better off at the point F than at S.\textsuperscript{22} If the ROW prefers fixed rates (as pictured in Figure 1), it is assumed that fixed rates will prevail. However, the ROW may prefer the Stackelberg Regime to fixed rates. In this situation US and ROW interests conflict, and our analysis provides no theoretical basis for determining the resolution of that conflict. In principle, of course, the ROW may commit itself to any reaction function it pleases. If it prefers S to F, it can choose to present the US with its NS reaction function instead of the 45° line. However, the US is not obligated to choose the point S in this situation. It can, instead, "punish" the ROW by moving up toward the Nash solution. This imposes costs on the US as well as the ROW. But if the US can coerce the ROW into its preferred regime through such action, the US may find it a profitable course to pursue. Unfortunately, our framework is not rich enough to allow us to evaluate and compare the costs and benefits of such strategies.

B. A Locomotive Policy World:

The world economic structure characterized by the symmetric-positive constraints (5b) is one in which the goods demand and indexing channels for the transmission of monetary policy dominate. It is most likely to be relevant in periods in which a high degree of wage indexation is prevalent and in which the price of oil is fixed in real terms (indexed to a commodity price index), rather than being fixed in terms of dollars. The 1970's meet at least the first of these requirements. However, (5b) may also describe periods such as the 1950's
and 1960's, which do not meet either of these requirements. When spillover is positive and symmetric, we find that the ROW has an unambiguous preference for a Fixed-Rate Regime, while the US may find itself in the position of opposing fixed rates.

For the symmetric-positive constraints, the situation is illustrated by Figure 2. (Figure 2B is an enlargement of the boxed area in Figure 2A.) Again, B and B* are the US and ROW bliss points. The US indifference ellipses and NS reaction curve are tilted downward, reflecting the fact that a more expansionary ROW policy decreases US unemployment and allows the US to economize on inflation. N, S, S* and F again represent Nash, Stackelberg and Fixed-Rate solutions. The locus of Pareto-efficient points is a hyperbola (not shown) running from B to F to B*. The shaded area represents the set of outcomes that are Pareto-superior to the Nash solution.

The US prefers the leadership role in a Stackelberg Regime to the Nash Regime; this is confirmed by the observation that in the Stackelberg Regime, the US chooses S over N. However, the ROW is worse off at S than at N; S is further from the ROW's bliss point (moving along NS*) than N. Were it possible, of course, the ROW would opt to lead in a Stackelberg regime; this is ruled out by assumption. As long as the ROW is unable to coordinate its actions, it runs the risk of being exploited as an unwilling follower in a Stackelberg game. One might expect the ROW to make its dissatisfaction with this arrangement evident. A likely complaint, reminiscent of the 1970's, would be that the US is shirking its critical role as a locomotive in leading the world out of recession.

With the symmetric-positive constraints, the ROW finds the Fixed-Rate Regime superior to both the Stackelberg and Nash Regimes. The ROW
FIGURE 2A: THE LOCOMOTIVE POLICY WORLD
FIGURE 2B: THE LOCOMOTIVE POLICY WORLD
prefers F to N, and therefore to S as well. The US may or may not prefer F to S, but it certainly prefers F to N.\textsuperscript{25} Once again, the Fixed-Rate Regime achieves a Pareto-efficient outcome. If the US does prefer fixed rates, it is assumed that fixed rates will prevail. However, for some parameter values, the US prefers to lead in a Stackelberg Regime. This is the case pictured in Figure 2. Here, again, US and ROW interests conflict and our analysis provides no theoretical presumption about which policy regime will be established. The ROW can be expected to seek the implementation of fixed exchange rates, while the US will oppose such efforts.

C. An Asymmetric Policy World:

The world economic structure characterized by the asymmetric constraints (5c) is one in which the spillover effects of US monetary policy are positive, but the spillover effects of ROW policy are negative. In section II, we describe a model that results in these constraints because oil is priced in terms of dollars; we speculate (but have not shown) that a similar model with a high degree of wage indexation in the ROW and a lesser degree of wage indexation in the US could also result in these constraints. Oil and wage indexation have played important roles in the more recent, flexible-rate era, and indeed we will show that the Stackelberg Regime (which assumes flexible exchange rates) is more attractive than the Fixed-Rate Regime in this case.

For the asymmetric constraints, the situation is illustrated by Figure 3. The US ellipses and NS reaction curve are tilted upward, while the ROW ellipses and NS reaction curve are tilted downward.\textsuperscript{26} The Nash solution pictured in Figure 3 lies above the 45° line. This need not be
FIGURE 3A: ASYMMETRIC POLICY WORLD
FIGURE 3B: ASYMMETRIC POLICY WORLD
the case.\textsuperscript{27} However, our conclusions are unaffected by the location of the Nash relative to the 45\degree line.

As noted earlier, the Nash solution does not exploit the comparative advantage of US monetary policy enough. Both players would be made better off by a more expansionary US monetary policy and a less expansionary ROW monetary policy. The Pareto-improving outcomes lie in the shaded area.

Both the US and the ROW prefer the Stackelberg Regime to the Nash Regime. In Figure 3, S is better for both players than N. It is better for the ROW because it lies back along its NS reaction curve in the direction of B\ast, and it is better for the US because the US chooses it explicitly over N.\textsuperscript{28}

The Fixed-Rate Regime, by contrast, comes off poorly under the asymmetric constraints. With the US leading and the ROW enforcing the fixed-rate structure, the outcome will be point F in Figure 3. In this setting, fixed rates do not lead to a Pareto-efficient outcome; in fact, F is not even a Pareto-improving outcome. It is worse than N, and thus S, for the ROW.\textsuperscript{29} The ROW has no incentive to enforce a fixed-rate structure under these circumstances. Furthermore, the US may also prefer S to F, as is the case pictured in Figure 3. In this situation, the US will lead in a Stackelberg Regime. However, it is also possible for the US to prefer the Fixed-Rate Regime to both the Stackelberg and Nash Regimes. If this occurs, US and ROW interests conflict and, as before, we venture no guess on the outcome.
Footnotes

1/ The game theoretic aspects of policy making in an interdependent world have been recognized by a number of earlier writers. See, for example, Bryant (1980), Cooper (1968 and 1969), Hamada (1974, 1976 and 1979), Henderson (1979), Johansen (1980), and Jones (1982a,b).

2/ In Hamada (1974), for example, exchange rates are assumed to be fixed and monetary policies are determined by policy makers preferences over output on the one hand and the balance of payments on the other. In Hamada (1976), policy makers objectives include inflation (rather than output) and the balance of payments. In either framework, Nash policies will be "too" contractionary if the net desire for balance of payments surpluses exceeds the creation of international reserves. Similarly, policies will be too expansionary if the net desire for balance of payments surpluses falls short of reserve creation. Which of these situations occurs depends on each country's balance of payments target and the relative importance of that target in the country's welfare function. Analogous results are obtained by Johansen (1980).

3/ Most of the literature on the game theoretic aspects of monetary policy in open economies assumes fixed exchange rates. A notable exception is Jones (1982a,b). He discusses both cooperative and Nash solutions in a multi-country setting with various exchange rate intervention strategies. His model includes more countries than ours and is tractable, in part, because he assumes commodity prices are fixed.
Some discussion of the flexible exchange rate case is contained in Hamada (1974). Under the objective function postulated by Hamada for the fixed exchange rate game, the flexible exchange game is degenerate. Accordingly, Hamada suggests alternative objective functions for policy makers in the case of flexible exchange rates -- functions that include domestic output and the domestic interest rate. The analysis of the present paper, by contrast, postulates one objective function that applies regardless of exchange rate regime or other variations in the "rules of the game". Indeed, in our analysis regime selection is an outcome of the postulated objective function and world economic structure.

4/ Many comparisons of fixed and flexible exchange rate regimes emphasize the source of disturbances; see, for example, Canzoneri (1982) and the references therein.

5/ Fiscal and commercial policy instruments are less flexible than monetary policy instruments in most countries; however, in some situations we might expect them to come into play as well. In such cases, other longer run objectives would be added to the utility functions. Or, if no one authority controlled all of its country's instruments, new players would be added.

6/ This assumption is necessary in order to make corrective monetary policy costly, given the nature of the utility functions (1). There are, of course, alternative modelling strategies, fully consistent with
rational expectations, that would produce games identical to those we study in Sections III and IV. But these alternatives have their own drawbacks. We could, for example, use current inflation rates in the objective functions (1) rather than "long-run" inflationary expectations. This formulation, however, is inconsistent with our presumption that policy makers are reluctant to run corrective policies because the resulting price increases somehow get "built into the system". Our present formulation does capture this notion. Further discussion of this point is contained in footnote 7.

7/ It has been suggested that the second argument in our utility functions should be either the short-run domestic inflation rate or the rate of depreciation of the exchange rate during the adjustment period. One reason policy makers may care about these short-run phenomena is the possibility that the resulting price increases will somehow get "built into the system." In fact, this possibility is allowed for in the macro model that underlies our paper. Further, the deleterious effects of short-run inflation and exchange rate movements are captured in our specification of policy makers' utility. These points may be demonstrated by considering the effects of a contractionary U.S. monetary policy on the ROW. This exercise is analyzed in detail in Canzoneri and Gray (1983), which provides the formal macroeconomic underpinnings for the present paper. In that framework, a contraction in the US causes an appreciation of the U.S. dollar and a depreciation of the ROW currency that reflects a deterioration in the ROW's terms of trade. Both the absolute and relative prices of the good the ROW imports from the US
rise; the price of ROW output falls. This is "inflationary" if ROW wages are indexed to a weighted average of the prices of ROW and U.S. output, not the price of ROW output alone. Under these circumstances, ROW nominal wage rates rise in absolute terms and relative to the price of ROW output. It is in this sense that the rise in import prices is built into the cost of production in the ROW. The resulting rise in the ROW real product wage will, in the absence of corrective monetary policy, produce a fall in ROW output.

Suppose now that ROW policy makers choose to prevent part of this output reduction through an expansionary monetary policy. Under the assumptions of our model, the increase in the rate of growth of the ROW money supply will be built into inflationary expectations. Thus, from the ROW's point of view, the depreciation of their exchange rate presents them with a choice between two undesirable alternatives: less output or higher expected inflation.

If ROW wages are indexed to a sufficiently high degree, a US monetary contraction produces not only a depreciation of the ROW currency but a rise in the ROW inflation rate during the adjustment period. It can be argued, as with the depreciation, that there is a sense in which this short run rise in inflation is built into the system in a way that worsens the ROW's output-inflation trade-off. Accordingly, while short-run inflation and exchange rate movements do not enter policy makers' utility functions directly, they do enter indirectly through their impact on the ultimate targets of policy -- output and inflationary expectations.
8/ A number of papers explore the effects of an oil price shock in a single country context; see, for example, Blinder (1979), Bruno and Sachs (1979), and Findlay and Rodriguez (1977). A few papers address this question in a multi-country setting. Examples include Caprio and Clark (1981), Krugman (forthcoming), Sachs (1980), and Schmid (1976). Unfortunately, none of these papers provides the analysis we require of both an oil price shock and the spillover effects of monetary policy. For further discussion of this point, see footnote 10.

9/ Of course, if policy makers' actions were appropriately constrained, our specification of the model could be made fully consistent with rational expectations. Suppose, for example, that the game described in section II is known to be a one time only game -- that the monetary policy adopted in response to the oil price shock is to be (perhaps legislatively) fixed for all time. Then the static expectations we ascribe to private agents would be fully rational.

10/ There is of course a voluminous literature devoted to the international transmission of monetary disturbances; however, explicit analyses of two (or more) country models are relatively scarce. We are unaware of any set of papers that present just the analysis we need here, although many of the individual pieces of our desired framework do appear in various other papers. Mundell (1963) for example, develops symmetric-negative constraints in a model with fixed prices and static expectations. Mussa (1979) presents an excellent discussion of macroeconomic interdependence which includes an exposition of Mundell's result. Hamada (1978) describes a transmission channel similar in spirit
to our wage indexation channel. Argy and Salop (1979) present a model very like the model developed in Canzoneri and Gray (1983), though they postulate static expectation formation. Daniel (1981) presents a rational expectations model that is also very like our model; her use of intermediate goods serves the function of our indexing. Bruno and Sach's (1979) simulation model has all of the ingredients necessary to derive the three sets of constraints we employ.

11/ Henderson and Quandt (1971) and Bryant (1980) provide useful discussions of the Nash and Stackelberg solution concepts we use in this section.

12/ Bryant (1980) and Hamada (1979) have used the Stackelberg solution concept in describing international policy games. Henderson (1979) discusses cases in which a degeneracy between targets and instruments allows Stackelberg solutions to exist when Nash solutions do not. As far as we know, we are the first to use the Fixed-Rate Regime as a solution concept in a non-cooperative game setting.

13/ As noted earlier, we can let $\delta q$ in the constraints (5) represent any global supply side disturbance; it need not represent an oil price increase.

14/ These ellipses are obtained by substituting the constraints (5a) into the utility functions (4) to eliminate $\delta x$ and $\delta x^*$. The resulting formulae are rather complicated, but discriminant analysis (see Thomas (1960), pg. 496) confirms that they are indeed ellipses. The angle $\alpha$ of
rotation of the major axis for the US ellipses (see Thomas (1960), pg. 493) is given by \( \cot 2\alpha = (\rho_1^2 - \rho_2^2 + \mu)/2\rho_1\rho_2\); so the rotation is less than 45\(^o\) for all permissible parameter values and goes to zero as \( \mu \) goes to \( \infty \). The ROW ellipses are of course the mirror images of the US ellipses if, as we assume, \( \mu = \mu^* \). The ellipses in Figure 1 reflect the parameter values \( \rho_1 = \rho_3 \delta q = 1, \rho_2 = 3/4 \) and \( \mu = 1/4 \).

15/ The US reaction function is found by maximizing \( U \) for a given \( \delta g^* \); the US function is

\[
\delta g = (\rho_1 \rho_3 \delta q + \rho_1 \rho_2 \delta g^*)/(\mu + \rho_1^2)
\]

Note that the slope of this curve is less than one since \( \rho_1 > \rho_2 \). The slope of a US indifference curve is equal to \(-U_{\delta g^*}/U_{\delta g}\), and since \( U_{\delta g} = 0 \) for any point on the US reaction function, the US reaction curve cuts the US ellipses at points where their slope is infinite. The ROW reaction function is the mirror image of the US function; it cuts the ROW ellipses at points where their slope is equal to zero.

16/ The US policies in the Nash, Stackelberg, and Fixed-Rate Regime are

\[
\delta g_N = \frac{\rho_1 \rho_3 \delta q}{(\mu + \rho_1^2 - \rho_1 \rho_2)}
\]

\[
\delta g_S = \frac{\rho_1 \rho_3 \delta q (\mu + \rho_1^2 + \rho_1 \rho_2)(\mu + \rho_1^2 - \rho_2^2)}{\mu (\mu + \rho_1^2)^2 + \rho_1^2 (\mu + \rho_1^2 - \rho_2^2)^2}
\]

\[
\delta g_F = (\rho_1 - \rho_2) \rho_3 \delta q / [\mu + (\rho_1 - \rho_2)^2],
\]
All are positive since $p_1 > p_2$. For the parameter values specified in footnote 14, $\delta g_N = 2$, $\delta g_S = 1.6$ and $\delta g_F = .8$.

17/ The Pareto-efficient set of outcomes is the locus of tangencies of the US and ROW ellipses; that is, the points ($\delta g$, $\delta g^*$) such that $-U_{\delta g^*}/U_{\delta g} = -U^*_{\delta g^*}/U^*_{\delta g}$. The formula for this locus is rather complicated but discriminant analysis confirms that it is a hyperbola and that it must run through the points $B$, $F$ and $B^*$.

18/ The point $S$ must be distinct from $N$ and in the direction indicated because the slope of the US indifference is infinite at $N$. See footnote 15.

19/ With the symmetric-negative constraints, both players would rather be followers in a Stackelberg Regime. The US for example can be seen to prefer $S^*$ to $S$ as follows: The US indifference curve at $S^*$ (not pictured) has infinite slope (see footnote 15), so $S^*$ must be preferred to $S$ if $\delta g^*_S < \delta g^*_S$ (that is, if ROW's policy as a Stackelberg leader is less expansionary than as a Stackelberg follower). Since $S$ must lie below the $45^\circ$ line (see footnote 18), $\delta g_S < \delta g^*_S$, and by symmetry, $\delta g_S = \delta g^*_S$; so finally $\delta g^*_S = \delta g_S < \delta g^*_S$. Both players are better off in a Stackelberg solution than in the Nash solution, but neither wants to lead.

20/ Since the ROW indifference curves are the mirror image of the US indifference curves, a ROW indifference curve must be tangent to the $45^\circ$ line at the same point a US indifference curve is. Clearly, the strength
of this result depends on the symmetry we have imposed on the utility functions. In particular, if \( \mu \) were not equal to \( \mu^* \), then \( F \) would not be in the Pareto-efficient set.

21/ The latter result should also be clear from Figure 1; the Fixed-Rate Regime offers the US a better "budget constraint" than the Stackelberg Regime. However, it can also be verified directly by comparing the US utility at \( S, L_S \), with US utility at \( F, U_F \). It turns out that

\[
U_F - U_S = \frac{(\rho_3^2 \delta \bar{q})^2 \mu \rho_2^2 \left[ \mu + \rho_1 (\rho_1 + 2 \rho_2) \right]}{[\mu + (\rho_1 - \rho_2)^2][\mu (\mu + \rho_1^2)^2 + \rho_1^2 (\mu + \rho_1^2 - \rho_2^2)^2]} > 0
\]

22/ The expression for \( U_F - U_S \) is rather complicated, and as it turns out, unsigned. We have found for example that if \( \rho_2 = \rho_1 = 1 \) (\( \rho_2 \) must be less than \( \rho_1 \)) and \( \mu \) is sufficiently large, then \( U_F^* > U_S^* \) (as in Figure 1). On the other hand, if \( \rho_2 = \rho_1 = 1 \) and \( \mu \) is sufficiently small, then \( U_F^* < U_S^* \).

23/ Again, discriminant analysis confirms that the indifference curves are ellipses, and the angle \( \alpha \) of rotation of the major axis for the US ellipses is given by \( \cot 2(180^\circ - \alpha) = (\rho_4^2 - \rho_5^2 + \mu) / 2 \rho_4 \rho_5 \) (see footnote 14). \( \alpha \) is greater than \( 135^\circ \) for all parameter values and goes to \( 180^\circ \) as \( \mu \) goes to \( \infty \). The US reaction curve is

\[
\delta g = (\rho_4 \rho_5 \delta \bar{q} - \rho_4^2 \delta g^* / (\mu + \rho_4^2))
\]
and cuts the US indifference curves where their slope is infinite. The ROW indifference curves and reaction curve are mirror images of their US counterparts. The US Nash, Stackelberg and Fixed-Rate policies are

\[
\delta g_N = \rho_4 \rho_6 \delta q / [(\mu + \rho_4^2) + \rho_4 \rho_5]
\]

\[
\delta g_S = \frac{\rho_4 \rho_6 \delta q (\mu + \rho_4^2 - \rho_4 \rho_5) (\mu + \rho_4^2 - \rho_5^2)}{\mu (\mu + \rho_4^2)^2 + \rho_4^2 (\mu + \rho_4 - \rho_5^2)^2}
\]

\[
\delta g_F = (\rho_4 + \rho_5) \rho_6 \delta q / [\mu + (\rho_4 + \rho_5)^2]
\]

Figure 2 reflects the same parameter values used in Figure 1; that is, \(\rho_4 = \rho_6 \delta q = 1, \rho_5 = 3/4\) and \(\mu = 1/4\). For those parameter values, \(\delta g_N = .50, \delta g_S = .40\) and \(\delta g_F = .53\).

\[24/\] S must be distinct from N and further out along the ROW reaction curve since the US indifference curve has infinite slope at N.

\[25/\] With the symmetric-positive constraints,

\[
U_F - U_S = \frac{(\rho_6 \delta q)^2 \mu^2 \rho_5^2 [\mu + \rho_4 (\rho_4 - 2 \rho_5)]}{[\mu + (\rho_4 + \rho_5)^2][\mu (\mu + \rho_4^2)^2 + \rho_4^2 (\mu + \rho_4 - \rho_5^2)^2]}
\]

and can take either sign.
The US ellipses and reaction curve have the properties described in footnotes 14 and 15; the ROW ellipses and reaction curve have the properties described in footnote 23. Again, the parameter values are $a_7 = \phi_7 = \psi_7 \delta q = \phi_{7*} \delta q = 1$, $\rho_3 = \rho_{7*} = 3/4$ and $\mu = 1/4$.

For the Nash solution, we have

$$\delta q_N / \delta q = \frac{\rho_{7*} + \mu/\rho_7}{-\rho_{7} + \phi_7 + \mu/\rho_7}$$

If $\rho_{7*}$ is close to $\rho_7$, or if the spillover coefficients $\rho_{7}$ and $\rho_{7*}$ are big, then the Nash will lie above the 45° line as shown. If $\rho_7$ is large relative to $\phi_7$, and the spillover coefficients are small, the Nash will lie below the 45° line.

Again, the reason for this result lies in the infinite slope of the US indifference curve at $N$.

ROW utility must be lower at $F$ than at $N$ if $\delta g_F$ is less than $\delta g_N$ because the ROW ellipse at $N$ has a zero slope. $\delta g_F$ must be less than $\delta g_N$ because the US indifference ellipse at $N$ has an infinite slope.
Appendix

Here we present the structural model that generates equations (5) of the text. Only the equations for the adjustment period are given. A more complete description of the model and its solution can be found in Canzoneri and Gray (1983). The model is expressed in log deviation form, which is consistent with the solutions presented in the text.

Notation

Note: With the exception of interest rates, lowercase letters denote the log value of a variable and uppercase letters denote the variable itself. An asterisk refers to the ROW, while a superscript "o" refers to OPEC.

\[
x \quad \text{real output}
\]
\[
h \quad \text{the composite input, consisting of equal numbers of units of labor and oil}
\]
\[
l \quad \text{labor employed in production of x}
\]
\[
o \quad \text{oil employed in production of x}
\]
\[
w \quad \text{nominal wage rate}
\]
\[
p \quad \text{domestic currency price of x}
\]
\[
p_i \quad \text{consumer price index}
\]
\[
q \quad \text{dollar price of oil}
\]
\[
e \quad \text{the exchange rate (units of ROW currency per dollar)}
\]
\[
\gamma \quad \text{degree of wage indexation}
\]
\[
\gamma^o \quad \text{degree of oil price indexation}
\]
\[
c \quad \text{real expenditure}
\]
\[
y \quad \text{real income}
\]
\[
t \quad \text{terms of trade (relative price of U.S. output)}
\]
\[
b \quad \text{OPEC's net holdings of real bonds issued by U.S. residents}
\]
\[
b^* \quad \text{OPEC's net holdings of real bonds issued by ROW residents}
\]
\[
r \quad \text{the real interest rate}
\]
\[
i \quad \text{the nominal interest rate}
\]
\[
m \quad \text{the nominal money stock}
\]
\[
g \quad \text{the rate of growth of the money stock}
\]
\( \delta(\cdot) \) denotes the deviation of the current value of a variable from its full equilibrium no-shock value. Thus, for example, \( \delta \) represents the deviation of the log value of output from the log value of its full employment level. For small changes, \( \delta \) approximates the percentage deviation of a variable from its full equilibrium no-shock value.

**The Model**

Notes: The model has been log-linearized around its no-shock equilibrium and written in log deviation form. For real variables, the model’s no-shock equilibrium is identical to its pre-shock equilibrium. A bar over a variable indicates its no-shock (or, equivalently, its pre-shock) equilibrium value. The two oil-importing countries are assumed to be identical in all respects in the pre-shock equilibrium. Subscripts refer to time. Unless otherwise indicated, Greek letters represent parameters.

**Production functions and derived demand:**

(A1) \[ \delta x = (1-\alpha)\delta h \]
\[ \delta x^* = (1-\alpha)\delta h^* \]

where \( (1-\alpha) = \frac{(W/P) + (O/P)}{W/P} \)

(A2) \[ \delta o = \delta q = 0 = \frac{-1}{\sigma} = \frac{B(\delta w - \delta p) + (1-\beta)(\delta q - \delta p)}{1-\beta} \]
\[ \delta h^* = \delta l^* = \delta o^* = \frac{-\sigma}{\sigma} = \frac{B(\delta w - \delta p^*) + (1-\beta)(\delta q - \delta e - \delta p^*)}{1-\beta} \]

where \( \sigma = \frac{(W/P) + (O/P)}{W/P} \)

**Factor pricing and price indices:**

(A3) \[ \delta w = \gamma \delta p \]
\[ \delta w^* = \gamma^* \delta p^* \]
\[ \delta q = \delta q + \gamma \delta p \]
(A4) \[ \delta p_i = 0.5(\delta p + \delta p^* + \delta e) \]
\[ \delta p_i^* = 0.5(\delta p^* + \delta p - \delta e) \]

Consumption functions, income definitions, and the terms of trade:

(A5) \[ \delta c = \delta y - \sigma \delta r \]
\[ \delta c^* = \delta y^* - \sigma \delta r \]

(A6) \[ \delta c^0 = 0 \]

(A7) \[ \delta y = \frac{(X/Y)(\delta x - 0.5\delta t) - (O/P)(O/Y)(\delta q - \delta p i + \delta o) - (rB/Y)\delta b - (B/Y)\delta r}{(rB/Y)\delta b} \]
\[ \delta y^* = \frac{(X/Y)(\delta x^* - 0.5\delta t) - (O/P)(O/Y)(\delta q - \delta p i + \delta o^*) - (rB/Y)\delta b^* - (B/Y)\delta r}{(B/Y)\delta r} \]

(A8) \[ \delta t = \delta p - \delta p^* - \delta p^* \]

Bond accumulation and goods market equilibrium:

(A9) \[ \delta b = \frac{(Y/B)(\delta c - \delta y)}{(Y/B)(\delta c^* - \delta y^*)} \]

(A10) \[ \delta x = 0.5(C/X)[\delta c + \delta c^* - (1+2\psi)\delta t] \]
\[ \delta x^* = 0.5(C/X)[\delta c + \delta c^* + (1+2\psi)\delta t] \]

Money markets:

(A11) \[ \delta m = \delta p + \delta x - \lambda \delta i \]
\[ \delta m^* = \delta p^* + \delta x^* - \lambda \delta i^* \]
\[(A12) \quad \delta i = \delta r + \delta p_{+1} - \delta p \]
\[
\delta i^* = \delta r + \delta p_{+1}^* - \delta p^* 
\]

\[(A13) \quad m = m_{-1} + g 
\]
\[
m^* = m^*_{-1} + g^* 
\]

Equations (A7) and (A10) may warrant clarification. To arrive at (A7), we define the level of real income in each oil-importing country to be nominal net national income deflated by the domestic consumer price index. Accordingly, \(Y\) and \(Y^*\) are given by

\[Y = (P/PI)X - (O/PI)O - rB\]
\[Y^* = (P^*/PI^*)X^* - (O/PI)O^* - rB^*\]

Because consumption patterns are identical in the two countries, and because the law of one price holds, \((O/PI) = (O^*/PI^*)\); the real price of oil in the US is equal to the real price of oil in the ROW. Log linearizing \(Y\) and \(Y^*\) around their no-shock equilibrium values yields equations (A7).

The demands for the two goods (in levels) are given by

\[X^d = (PI/P)(P*E/P)^\Psi (C+C^*) + OPEC demand\]
\[X^*d = (PI^*/P^*)(P/P^*E)^\Psi (C+C^*) + OPEC demand\]

Equations (A10) are obtained by log linearizing \(X^d\) and \(X^d^*\) around their no-shock equilibrium values, setting the change in OPEC's demand for both
goods equal to zero, and equating output demand to output supply. The \( \psi \) which enters equations (A10), then, is the negative of the own relative price elasticity of the expenditure shares of the two goods. The term \( (1+2\psi) \) which enters equations (A10) is the negative of the own relative price elasticity of the levels of expenditure on the two goods.
References


Some Aspects of the Adjustment Problem
in an Interdependent World*

by

Matthew Canzoneri

and

Jo Anna Gray

Federal Reserve Board

Washington, D.C. 20551

September, 1982
Revised October 1982
Revised December 1982

*Forthcoming in The Future of the International Monetary System, eds.
Tamir Agmon, Robert Hawkins and Richard Levich. We would like to thank
Robert Cumby, Dale Henderson, Robert Hodrick, Preston Miller and Jeffrey
Sachs for helpful discussions. Any errors are, of course, the
responsibility of the authors alone.
I. Introduction and Summary

In a world characterized by short-term wage or price rigidities, unanticipated supply-side disturbances such as an oil price increase may cause temporary deviations of output from its full-employment level. While corrective monetary or fiscal policies may be effective in neutralizing the effects of such shocks on output and employment, they may do so at the cost of a higher price level and increased inflationary expectations. These short-run macroeconomic costs constitute an important part of the adjustment burden that is shouldered by the world economy following global disturbances such as the oil price increases of the 1970's. The purpose of the present paper is to identify some of the ways in which the interdependence of the world's economies affects the adjustment to an oil price shock and the associated adjustment burden.

The focus of our analysis is on the spillover effects of monetary policy -- that is, the way in which one country's monetary policy affects output and employment in other countries. These spillover effects depend on the structural and institutional features of the individual countries involved, and may be positive or negative for any given country. They create a situation in which policy makers in one country may be expected to condition their actions on the policies pursued in other countries; policy has unavoidable game aspects.\(^1\) In studying the outcome of this global policy game, we find that the overall size of the adjustment burden is unambiguously increased by the absence of cooperation among policy makers. It is further noted that the output losses associated with an oil price shock do not, in general, provide an adequate measure of the reduction in social welfare caused by such disturbances.
We begin, in section II, by constructing a three-country model with one oil producing country (OPEC) and two oil importing countries (the U.S. and the ROW). This framework is used to develop the constraints for the game played by policy makers in the two oil importing countries. As indicated above, an important element of these constraints is the spillover effects of monetary policy. These spillover effects, in turn, depend on the channels through which a country's monetary policy is transmitted abroad. A number of possible channels may be identified. The present paper focuses on four. The first was formally introduced by Mundell in the early 1960's. This channel, which depends critically on capital mobility, operates through the real interest rate. The second channel is also well-known. This channel, which depends on the existence of two or more goods in the model, operates directly through the demand for goods. The other two channels were selected because of their increasing relevance for a number of countries over the past decade. One stems from the now common practice of contractually linking nominal wage rates to an index that includes the prices of imported goods as well as domestically produced goods. The other arises from the nature of OPEC's oil pricing policy; the price of oil is denominated in dollars and is not always adjusted quickly in response to changes in the purchasing power of the dollar.

The first channel described above produces negative spillover, a beggar-thy-neighbor effect. In the absence of fully indexed input prices, an expansionary monetary policy in one country raises that country's output and lowers its real interest rate. The real interest rate movement is transmitted abroad, where its net impact is deflationary. (The lower real interest rate produces a fall in the
foreign nominal rate and this, in turn, increases foreign money demand.) Thus, the spillover effects that are channelled through the real interest rate are negative and symmetric across countries. By comparison, the channels that operate through goods demand and wage indexation generate spillover effects that are positive and symmetric. An expansion in one country raises output and expenditure in that country. Some, but not all, of the increased expenditure falls on each country's good, generating an excess supply of the expanding country's good and an excess demand for the other country's good. As a result, both the relative price and the output of the second country's good rise. Thus, an expansion in either country increases output in the other by generating increased demand for the other country's good. Our third channel of transmission, wage indexation, depends critically on the terms of trade change generated by this increased demand. Consider again an expansion in one country and the accompanying rise in the relative price of the good produced by the other country. If nominal wages are linked to a consumer price index, the second country's real product wage falls and its output increases. The channel associated with the fixed dollar price of oil is, in contrast to the other three channels, an asymmetric one. An expansionary U.S. policy lowers the real price of oil, for both US and ROW producers and this can lead to an increase in output in the ROW. An expansionary ROW policy, on the other hand, has an effect on real oil prices that is ambiguous in sign. Accordingly, the spillover effects produced by this channel are positive for U.S. policy but are ambiguous in sign for ROW policy.

The nature of the game that is actually played by policy makers depends on the relative importance of the four channels of transmission
just described. It can be reasonably argued that as the institutional structures of the world's larger economies have changed, so have the relative importance of these channels. This perspective may be particularly useful in attempting to understand apparent inconsistencies in policy prescriptions over time and across countries.

In section III of the paper we show that regardless of the specific nature of the game played by world policy makers, the non-cooperative solution to that game is destructive in the sense that there generally exist coordinated policies that would make all players better off. In a symmetric game in which spillover effects are negative, the non-cooperative solution is too inflationary; both countries would be better off if they could both somehow manage to inflate less. The converse is true for a symmetric game in which spillover effects are positive. Welfare in both countries would be raised by more expansionary policies. In the asymmetric game in which the dollar price of oil is fixed, all would be better off if the U.S. inflated more while other countries inflated less. Clearly, then, the overall size of the adjustment burden following an oil price shock is increased by the absence of cooperative behavior on the part of policy makers.

The games discussed in section III are treated in more detail in Canzoneri and Gray (1982b). Stackelberg and fixed exchange rate solution concepts are also discussed in that paper; only a sample of the results are presented here. Section IV concludes with a brief discussion of the difficulties involved in selecting an observable measure of the adjustment burden borne by an individual country or by the world economy as a whole following an oil price shock. Output losses are shown to be a generally inadequate measure of this burden.
II. A Three-Country Model

In this section, we develop an analytical framework that is used to study the effects of both an oil price shock and the monetary policy responses to that shock. Of particular interest are the spillover effects of monetary policy. It is these effects that produce the policy games analyzed in section III of the paper.

The one period, discrete time model of this section is a two-good extension of the framework developed in Canzoneri and Gray (1982a).\(^2\) There are three countries, one oil producing country called OFEC and two oil importing countries called the U.S. and the rest of the world (or ROW). Oil is an intermediate good that is used by the U.S. and ROW to produce two consumption goods which are consumed in all three countries. Each oil importing country is specialized in the production of one of these consumption goods. Labor is the only other variable input employed in the production of each consumption good; it is used in fixed proportions with oil. OPEC sets the price of oil in terms of U.S. dollars, but this price may be partially or fully linked to a consumer price index. The nominal wage in each oil importing country is contractually fixed in terms of the local currency, but may also be partially or fully linked to an index of consumption good prices. The absence of completely flexible wage rates introduces the possibility of short run deviations of output from its full employment level. There are three assets in the system: U.S. money which is held only by U.S. residents, ROW money which is held only by ROW residents, and a real bond that is held by the residents of all three countries. The model includes three exogenous policy variables: the price of oil and the rates of
growth of the U.S. and ROW money supplies. Shocks to the model take the form of unexpected once-and-for-all changes in these policy variables.

The model can be summarized as follows:

**Notation**

Note: With the exception of interest rates, lowercase letters denote the log value of a variable and uppercase letters denote the variable itself. An asterisk refers to the ROW, while a superscript "o" refers to OPEC.

- **x**  real output
- **h**  the composite input, consisting of equal numbers of units of labor and oil
- **l**  labor employed in production of x
- **o**  oil employed in production of x
- **w**  nominal wage rate
- **p**  domestic currency price of x
- **pi**  consumer price index
- **q**  dollar price of oil
- **e**  the exchange rate (units of ROW currency per dollar)
- **γ**  degree of wage indexation
- **γ^o**  degree of oil price indexation
- **c**  real expenditure
- **y**  real income
- **t**  terms of trade (relative price of U.S. output)
- **b**  OPEC's net holdings of real bonds issued by U.S. residents
- **b^***  OPEC's net holdings of real bonds issued by ROW residents
- **r**  the real interest rate
- **i**  the nominal interest rate
- **m**  the nominal money stock
- **g**  the rate of growth of the money stock

\( δ(·) \) denotes the deviation of the current value of a variable from its full equilibrium no-shock value. Thus, for example, \( δx \) represents the deviation of the log value of output from the log value of its full employment level. For small changes, \( δ(·) \) approximates the percentage deviation of a variable from it's full equilibrium no-shock value.
The Model

Note: The model has been log-linearized around its no-shock equilibrium and written in log deviation form. For real variables, the model's no-shock equilibrium is identical to its pre-shock equilibrium. A bar over a variable indicates its no-shock (or, equivalently, its pre-shock) equilibrium value. The two oil importing countries are assumed to be identical in all respects in the pre-shock equilibrium. The superscripts d and s denote demand and supply. Subscripts refer to time. Unless otherwise indicated, Greek letters represent parameters.

1. \[ \delta x = (1-\alpha)\delta h \]
   \[ \delta x^* = (1-\alpha)\delta h^* \]
   where \( (1-\alpha) = \frac{[(W/P) + (0/P)]H/X} \)

2. \[ \delta h = \delta l = \delta o = -(1/\alpha)[\beta(\delta w - \delta p) + (1-\beta)(\delta q - \delta p)] \]
   \[ \delta h^* = \delta l^* = \delta o^* = -(1/\alpha)[\beta(\delta w^* - \delta p^*) + (1-\beta)(\delta q - \delta e - \delta p^*)] \]
   where \( \beta = \frac{(W/P)L/[(W/P)L + (0/P)O]} \)

3. \[ \delta w = \gamma \delta p_i \]
   \[ \delta w^* = \gamma \delta p_i^* \]
   \[ \delta q = \delta q + \gamma \delta p_i \]

4. \[ \delta p_i = .5(\delta p + \delta p^* + \delta e) \]
   \[ \delta p_i^* = .5(\delta p^* + \delta p - \delta e) \]

5. \[ \delta c = \delta y - \sigma \delta r \]
   \[ \delta c^* = \delta y^* - \sigma \delta r \]
(6) $\delta c^0 = 0$

(7) $\delta y = \frac{X/Y}{(O/Y)(\delta x + 0.5\delta t)} - \frac{Q/P}{(O/Y)(\delta q - \delta p + \delta o)} - \frac{rB/Y}{(B/Y)\delta r}$

$\delta y^* = \frac{X/Y}{(\delta x^* + 0.5\delta t)} - \frac{Q/P}{(O/Y)(\delta q - \delta p + \delta o^*)} - \frac{rB/Y}{(B/Y)\delta r^*}$

(8) $\delta t = \delta r - \delta p^* - \delta p^*$

(9) $\delta b = \frac{Y/B}{(\delta c - \delta y)}$

$\delta b^* = \frac{Y/B}{(\delta c^* - \delta y^*)}$

(10) $\delta x = 0.5(C/X)[\delta c + \delta c^* - (1+2\psi)\delta t]$

$\delta x^* = 0.5(C/X)[\delta c^* + (1+2\psi)\delta t]$

(11) $\delta m^S = \delta p + \delta x - \lambda \delta i$

$\delta m^* = \delta p^* + \delta x^* - \lambda \delta i^*$

(12) $\delta i = \delta r + \delta p_{+1} - \delta p$

$\delta i^* = \delta r + \delta p_{+1}^* - \delta p^*$

(13) $m^S = m^S_{-1} + g$

$m^* = m^*_{-1} + g^*$

Equation (1) gives the production technologies of the two oil importing countries. Output of each consumption good is proportional to
the amount of the composite input employed in its production. One unit of the composite input consists of one unit of labor and one unit of oil, which are used in fixed proportions.

The conditions for profit maximization yield equation (2), which gives the derived demands for the composite input, as well as the derived demands for labor and oil in each oil importing country. These demands depend on the real product price of the composite good -- that is, on its price in terms of the domestically produced consumption good. This, in turn, is equal to the sum of the real product wage and the real product price of oil. Accordingly, equation (2) shows derived demands in each country to depend negatively on the domestic currency prices of labor and oil, and positively on the price of domestic output.

Nominal wage rates and the dollar price of oil are determined by equation (3). As in Gray (1976), nominal wages are set at the beginning of each period (before any oil price or monetary policy shocks are realized) at a level that is expected to clear the labor market. In addition, each country's nominal wage is linked to its price level by an indexing parameter $\gamma$ (or $\gamma^*$) which is generally assumed to lie between zero and one, inclusive. The price of oil is set in dollars and is linked to the U.S. price level by the indexing parameter $\gamma^0$. The term $\delta q$ represents unanticipated disturbances to the dollar price of oil, or oil price shocks.$^4$

Equation (4) defines each country's price level as a weighted average of the domestic currency prices of the two consumption goods. Since the two countries are identical in all respects in the pre-shock equilibrium, the two goods are assigned equal weights of one-half.
In equation (5), total spending in each of the two oil importing countries is shown to be an increasing function of domestic net income and a decreasing function of the real rate of interest. Both spending and income are measured in terms of the same consumption basket used to define the price levels of the two countries.

A critical feature of the model is the assumption that OPEC is unable to immediately adjust its level of consumption to changes in the level of its income. Specifically, OPEC is assumed to have short-run marginal propensity to save of one. This assumption is captured by equation (6).

Real income in the two oil importing countries is given by equation (7).\textsuperscript{5/} Real income in each country is an increasing function of the level of its output and the relative price of its output. It is a decreasing function of its real oil bill, OPEC's net holdings of its bond, and the real interest rate paid on those bonds. The relative price of U.S. output is defined in equation (8) to be "the" terms of trade.

Equation (9) states that OPEC's net accumulation of each oil importing country's bond is equal to that country's excess of spending over income. Thus OPEC's total saving -- or, equivalently, its current account surplus -- is equal to the sum of U.S. and ROW dissaving.

Equation (10) is the log deviation form of the goods market equilibrium conditions.\textsuperscript{6/} The demand for each good must equal its supply. Demand for each good is an increasing function of total expenditure in the two oil importing countries and a decreasing function of its relative price.

Equation (11) gives the equilibrium conditions for the U.S. and ROW money markets. For simplicity, the income elasticity of money demand
has been set equal to unity. Nominal interest rates are defined in equation (12). And the evolution of the U.S. and ROW money stocks is described by equation (13). Monetary policy in each country takes the form of setting a constant rate of growth, \( g \) (or \( g^* \)), of the domestic money stock. Any change in monetary policies -- that is, any change in \( g \) or \( g^* \) -- is assumed to be unexpected and, once it occurs, permanent.

The model outlined in equations (1) through (13) above involves expectations of future prices and policy variables. To complete the model the manner in which agents form their expectations must be specified. It is assumed that agents' expectations are "rational" given their assumptions about U.S. and ROW monetary policies and OPEC's oil pricing policy. If their views about these policies are correct, then their price predictions will be realized.

The remainder of this section will be spent discussing the model's solutions for the levels of U.S. and ROW output. Of particular interest are the effects of the monetary policy adopted in one of the two oil importing country's on the output of the other. For simplicity, attention is limited to two special cases of the general model outlined in equations (1) through (13) above. We treat first the special case in which the degree of wage indexation in the two oil importing countries is variable, but the price of oil is fully indexed. This case produces the macroeconomic constraints necessary to yield the two symmetric games studied in the next section. We then turn to the special case in which both nominal wage rates and the dollar price of oil are fixed. This case produces the one asymmetric game studied in section III.
A Fixed Real Price of Oil

Equations (14) and (15) below give the solutions for U.S. and ROW output for the special case in which the price of oil is fully indexed ($\delta^0=1$) but the degree of wage indexation may lie anywhere between zero and one, inclusive ($0\leq \gamma \leq 1$). Each country's output is a linear function of the unanticipated changes in the model's three policy variables -- the rates of growth of U.S. and ROW money and the price of oil.

\[(14) \quad \delta x = \rho_1 \delta g + \rho_2 \delta g^* - \rho_3 \delta q \]
\[(15) \quad \delta x^* = \rho_1 \delta g^* + \rho_2 \delta g - \rho_3 \delta q \]

where

\[\nu_1 = (1/D_1 D_2)(1+\lambda)^2(1-\gamma)\theta[\xi\Delta[\theta(\lambda+\gamma)+(1+\lambda)(\phi+2\gamma)+2\theta(1-\gamma)] + \tau\lambda\theta(1-\gamma)] > 0\]

\[\rho_2 = (1/D_1 D_2)(1+\lambda)^2(1-\gamma)\theta[\xi\Delta[\theta(\lambda+\gamma)+(1+\lambda)\phi] - \tau\lambda\theta(1-\gamma)] \quad \text{(unsigned)} \]

\[\rho_3 = (1/D_1 D_2)2D_1\xi[(1+\lambda)\Delta \phi + \nu\lambda\theta(1-\gamma)] > 0\]

and

\[\tau = (\nabla/\chi)(1+4\psi) > 0\]
\[\Delta = (\nabla/\chi)[1-\tau] + (B/\gamma)] > 0\]
\[\xi = 1/(1-\gamma) > 0\]
\[\nu = (1-\alpha)(1-\beta) > 0\]
\[\theta = (1-\alpha)\beta/\alpha > 0\]
\[\phi = (1-\alpha)(1-\beta)/\alpha > 0\]
\[D_1 = \lambda(\theta+\phi+\tau) + \tau + \tau\theta(1-\gamma) + \theta\gamma + \phi > 0\]
\[D_2 = 2\xi\Delta(1+\lambda) + 2\theta(1-\gamma)(\xi\Delta+\lambda) > 0\]
As expected, each country's output responds positively to an increase in the rate of growth of its own money stock as long as wages are not fully indexed. Monetary policy "works" due to the absence of completely flexible wages in the short run. An unexpected increase in the rate of growth of U.S. money, for example, produces an incipient excess supply of money that is offset, in part, by a rise in the U.S. price level. Provided U.S. wages are not fully indexed, this "price surprise" lowers the U.S. real wage product and induces an increase in U.S. output and employment.

A rise in the real price of oil lowers output in both oil importing countries. It does so by increasing the price firms must pay for the composite input. This reduces the amount of the composite input employed in production and, therefore, the level of output.

As equations (14) and (15) show, the spillover effects of monetary policy are symmetrical but ambiguous in sign. For a sufficiently high degree of wage indexation, we see that these effects are positive. That is, an expansionary monetary policy in one country raises output in the other. If the degree of wage indexation is less than one, spillover can be negative; an expansionary monetary policy in one country may produce a contraction in the other. The larger is \( \tau \) -- that is, the higher is the responsiveness of relative goods demand to relative price -- the more likely is this outcome. Some intuition into these results can be gained by considering three competing "channels" through which one country's monetary policy may be transmitted to other countries. The first of the three channels produces negative spillover, while the other two produce
positive spillover. The actual spillover effects of monetary policy depend, then, on the relative importance of these three channels.

The first of our three transmission channels was formally introduced by Mundell in the early 1960's. This channel, which depends critically on capital mobility, operates through the real interest rate. In tracing this channel, we observe first that an increase in the rate of growth of the U.S. money supply leads to a lower real interest rate. This occurs, in our model, because an expansionary monetary policy raises U.S. output and, simultaneously, U.S. oil imports. This, in turn, leads to a rise in OPEC's income. Because OPEC's short run marginal propensity to save is assumed to be one, the increase in OPEC's income generates an equal increase in their desired saving. At an unchanged real interest rate, this produces an incipient world excess supply of goods. A fall in the real interest rate is required in order to equilibrate the goods markets.

The fall in the real interest that accompanies an expansionary U.S. monetary policy has, in and of itself, a contractionary impact on the ROW. This can be seen most easily by considering the ROW money market. At unchanged ROW prices and output, a fall in the real interest rate induces a fall in the ROW nominal rate and an excess demand for ROW money. At an unchanged terms of trade, this excess demand can be eliminated by either a fall in the ROW price level, a fall in ROW output, or both. If the terms of trade are held constant, a fall in the ROW price level means a fall in the price of domestic output. In the absence of full indexed wage rates, this produces a rise in the ROW real product wage and, accordingly, a fall in ROW output. Thus, at an unchanged
terms of trade, an excess demand for ROW money will result in a fall in both ROW prices and output. We conclude, then, that the spillover
effects of monetary policy that operate through the real interest rate
channel alone are negative.

The second of our transmission channels, which depends on the
existence of two or more goods in the model, operates directly through
the demand for goods. In and of itself (abstracting from the real
interest rate channel), this channel produces positive spillover. To see
this, consider once again an increase in the rate of growth of the U.S.
money supply. The resulting increase in U.S. output leads, at an
unchanged real interest rate, to an equal increase in U.S. expenditure.
At an unchanged terms of trade, this expenditure increase would fall
equally on the U.S. good and the ROW good, generating an incipient excess
supply of the U.S. good and an excess demand for the ROW good. This
induces, in turn, an equilibrating rise in ROW output and fall in the
terms of trade.\[9\]

Of course, increased production of the ROW good will occur only if
the ROW real product wage falls. This is possible because the increased
demand for the ROW good causes a rise in its relative price -- a fall in
the terms of trade. A rise in the relative price of ROW output can
produce a rise in its absolute (domestic currency) price even if the
overall ROW price level falls.\[10\] This, in turn, lowers the ROW
real product wage and induces increased production of ROW output. It
follows that the output response to the increased demand for the ROW good
will be larger, the larger is the terms of trade change it produces.
Further, if the induced terms of trade effect is sufficiently strong, the
positive spillover channeled through goods demand will dominate the
negative spillover channeled through the real interest rate. The change in the terms of trade will be larger, the lower the responsiveness of relative demands to relative price, or the lower $\tau$ (which is positively related to the elasticity of relative goods demand with respect to relative price). Thus we see from equations (14) and (15) that a sufficiently small $\tau$ will always produce positive spillover, or a positive $\rho_2$.

Our third transmission channel, for which the terms of trade change discussed above is a prerequisite, is wage indexation. It is a channel which has become increasingly relevant for a number of countries over the past two decades. Like the goods demand channel, it produces positive spillover. To demonstrate this effect, we consider once more an expansionary U.S. monetary policy. As already noted, such an expansion is accompanied by a rise in the relative price of ROW output. Even if the absolute (domestic currency) price of ROW output remains unchanged, this relative price change will produce an increase in ROW output as long as ROW wages are indexed to some extent. This occurs because ROW wages are indexed to a weighted average of the prices of ROW and U.S. output, not to the price of ROW output alone. If the absolute price of ROW output remains unchanged, a rise in its relative price will be associated with a fall in the overall price level. Provided wages are indexed to some extent, this will result in a fall in the ROW nominal wage rate, and accordingly, a fall in the real product wage (the nominal wage rate deflated by the price of domestic output). This, in turn, leads to a rise in ROW output. Examination of the terms entering $\rho_2$ reveals that for a sufficiently high degree of wage indexation (a sufficiently large $\gamma$) the positive spillover effects associated with this
channel will always dominate the negative spillover effects channeled through the real interest rate.

The three channels of transmission discussed above, and the spillover effects they generate, are symmetrical for the two oil importing countries. The effects of a U.S. expansion on ROW output are identical to the effects of an ROW expansion on U.S. output. A sufficiently high degree of wage indexation produces the positive symmetric game of the next section, while the absence of full indexation and a sufficiently high elasticity of relative goods demand with respect to relative price produces the negative symmetric game of the next section. We consider next a version of our model that produces a transmission channel that does not operate symmetrically for the U.S. and ROW. The essential feature of this version is that the price of oil is set in dollars and is not indexed to any price or basket of prices during the adjustment period.

A Fixed Dollar Price of Oil

Equations (16) and (17) below give the solutions for U.S. and ROW output for the special case in which wage rates in both oil importing countries and the price of oil are fixed in nominal terms ($\gamma = \gamma^* = \gamma^0 = 0$).

(16) \[ \delta x = \rho_4 \delta g + \rho_5 \delta g^* - \rho_6 \delta q \]

(17) \[ \delta x^* = \rho_4 \delta g^* + \rho_5 \delta g - \rho_6 \delta q \]

where

\[ \rho_4 = (1/D_1 D_2) (1+\lambda)^2 (\theta + \phi) [\varepsilon A [2\tau (1+\lambda+\theta) + \lambda (\theta+2\phi) + 2\phi] + \lambda \varepsilon (\varepsilon v + \tau)] > 0 \]

\[ \rho_5 = (1/D_1 D_2) (1+\lambda)^2 (\theta + \phi) \lambda \varepsilon (\varepsilon A - \tau - \varepsilon v) \] (unsigned)

\[ \rho_6 = (1/D_1 D_2) 2D_1 \xi [(1+\lambda) \Delta \phi + \nu \lambda \theta] > 0 \]
\[ \rho^* = (1/D_1 D_2)(1+\lambda)^2 \{ \lambda \theta (\xi \Delta - \tau - \xi \nu) + 2 \xi \theta \tau [\lambda \nu (\theta + \phi) + \Delta (1+\lambda+\xi+\phi)] \} \] (unsigned)

\[ \rho^* = (1/D_1 D_2)(1+\lambda)^2 \{ \xi \Delta (\theta + \phi) [\lambda (\theta + \phi) + \phi (2+\lambda)] + 2 \xi \Delta \phi \tau (1+\lambda) + \lambda \theta (\theta + \phi) [\tau + \xi \nu (1-2\tau)] \} \] (unsigned)

and

\[ D_1 = \lambda (\theta + \phi + \tau) + \tau + \tau \theta + \phi > 0 \]
\[ D_2 = 2 \xi \Delta (1+\lambda+\theta) + 2 \lambda \theta > 0 \]

Other parameters are defined as before.

As before, an increase in the price of oil reduces output in both the U.S. and the ROW, and does so by the same amount in the two countries. An increase in the rate of growth of U.S. money has the same qualitative effect on U.S. output that it had in the previous case. However, for the same degree of wage indexation, this effect is stronger in the present case because the rise in the price of U.S. output associated with the monetary expansion lowers the real product price of oil as well as the real product wage.

The effect of an expansionary ROW monetary policy on ROW output may, in this case, be either positive or negative, whereas in the previous case it was unambiguously positive. The intuition behind this anomalous result is as follows: As before, the spillover effects of ROW monetary policy may be either positive or negative. If they are negative, the price of U.S. output declines, which can raise the real product price of oil for both the U.S. and the ROW. If the rise in the ROW real product price of oil is sufficiently large, it can dominate the positive effects normally associated with a monetary expansion and produce a decline in ROW output.
As equations (16) and (17) show, the spillover effects of monetary policy are not only ambiguous in sign in this case, they are also asymmetric. For both countries, the real interest rate and goods demand channels of the previous case are still present.\textsuperscript{11} As before, the real interest rate channel produces negative spillover, the goods demand channel produces positive spillover, and the spillover effects generated through these channels are symmetrical for the two countries. For simplicity, the degree of wage indexation has been set equal to zero for both countries. Accordingly, the wage indexation channel is not present in this version of our model.

The asymmetry of spillover effects in this case stems from the fact that the price of oil is set in dollars and is not adjusted in the short run for movements in the prices of U.S. or ROW output. The fourth channel of transmission considered in this paper is, then, the one associated with a less than fully indexed dollar price of oil. In making production decisions, U.S. firms are concerned with the real price of oil in terms of U.S. output, while ROW firms are concerned with the real price of oil in terms of ROW output. We will demonstrate the asymmetry of the spillover effects that are transmitted through our fourth channel by examining, in turn, the impact an expansionary U.S. monetary policy on the ROW real product wage and the impact of an expansionary ROW monetary policy on the U.S. real product wage.

An expansionary U.S. monetary policy lowers the real produce price of oil for the ROW. In understanding this result, it is useful to begin by noting that the expansion lowers the U.S. real product price of oil. This is because the expansion generates a rise in the price of U.S.
output. Given a fixed dollar price of oil, the real price of oil in terms of U.S. output necessarily falls. The effect on the price of oil in terms of ROW output is even stronger. The change in the ROW real product price of oil is equal to the change in the U.S. real product price less any change in the relative price of ROW output. Since the U.S. expansion results in a rise in the relative price of ROW output, any reduction in the price of oil in terms of U.S. output means an even greater reduction in terms of ROW output.

By contrast, an expansionary ROW monetary policy has an ambiguous effect on the U.S. real product price of oil. In motivating this result, we begin by noting that the fixed dollar price of oil does not provide a direct channel of transmission from ROW monetary policy to the U.S. real product price of oil and, therefore, U.S. output. A ROW expansion effects the U.S. real product price of oil indirectly through its impact on the price of U.S. output. The ROW expansion is transmitted to the price of U.S. output through the real interest rate and goods demand channels already discussed. The overall sign of the spillover effects transmitted through these two channels may be positive or negative — that is, the price of U.S. output as well as the level of U.S. output may either rise or fall. Accordingly, the real price of oil in terms of U.S. output may either rise or fall.

The one asymmetrical game examined in the next section involves the special case in which the spillover effects of U.S. monetary policy are positive while the spillover effects of ROW monetary policy are negative. In the version of our model considered here, this will occur for sufficiently large values of \( \tau \) and sufficiently small values of \( \alpha \).
Here $\delta x$ and $\delta x^*$ represent deviations of output from their full employment values in the U.S. and the ROW, and $\bar{\pi}$ and $\bar{\pi}^*$ are long-run inflationary expectations.

By assumption, all games are initiated by an unanticipated increase in the price of oil that will, in the absence of corrective policy, cause output to deviate from its full employment level. Such deviations are temporary; they persist for only one period which is referred to as the adjustment period. At the end of the adjustment period, labor contracts are renegotiated with the new price of oil in mind, and, absent further unanticipated shocks, output returns to its full employment level.

Monetary policy can be used to offset some or all of the output effects of an oil price shock, but only at the cost of changing the inflation rate expected to prevail in periods subsequent to the adjustment period. This is because private agents are assumed to expect the money growth rates established during the adjustment period to be permanent. This assumption is intended to capture, in a very simple way, the notion that policy makers are subject to a credibility constraint. The important implicit assumption is that the monetary authority cannot convince the private sector of a change in policy by simply announcing it. Agents expect past policy to continue until confronted with an actual policy change. Once a new policy has been in place for a period, it, then, is believed to be permanent. 12/

Thus, the tradeoff faced by policy makers following a shock such as an oil price increase is this: They can increase the rates of growth of their money supplies and increase employment during the period of
III. Policy Games

We turn now to some of the game theoretic aspects of monetary policy in an interdependent world. The formal analysis underlying the discussion of this section is contained in Canzoneri and Gray (1992b). Only a subset of the results of that paper are presented here.

Three policy games are examined in this section. Each game corresponds to one of the three types of policy spillover described in the preceding section: negative symmetric, positive symmetric, and asymmetric. Both the non-cooperative and the cooperative solutions to these games are presented. The implications of our results for measuring the size and distribution of the adjustment burden following an oil price shock are then discussed.

We begin by describing the social welfare function of the policy makers in each of the two oil importing countries. Policy makers use the one tool at their disposal--monetary policy--to maximize this objective function subject to the macroeconomic constraints developed in the last section. Consideration of OPEC's maximization problem is, by contrast, altogether omitted from our analysis; the price of oil is assumed to be exogenous.

The monetary authority in each oil importing country is assumed to be concerned only with domestic employment and long-run expectations of the domestic rate of inflation. Specifically, the policy makers' utility functions take the following form:

\[(18) \quad U = -(\delta x)^2 - \pi^2, \quad U* = -(\delta x*)^2 - \pi*^2.\]
adjustment to the oil price shock, but only at the price of higher expected rates of inflation in the period that follows. Policy makers have two choices in dealing with these expectations in the post-shock period. They can accommodate them, thereby achieving full employment output. Or they can lower them by lowering the rates of growth of their money supplies, thereby forcing their economies through a second adjustment period. The terms $\pi$ and $\pi^*$ enter the utility functions (1) because each of these choices has undesirable consequences in subsequent periods -- higher steady state inflation in the first case and additional periods of adjustment and unemployment in the second.

Under the assumptions discussed above, the long run expected rates of inflation in the U.S. and the ROW are simply

$$
(19) \quad \tilde{\pi} = g, \quad \tilde{\pi}^* = g^*,
$$

where $g$ and $g^*$ are the rates of growth of U.S. and ROW money supplies during the adjustment period. Note that

$$
(20) \quad \tilde{\pi} = \pi_0 + \delta g, \quad \tilde{\pi}^* = \pi_0^* + \delta g^*,
$$

where $\pi_0$ and $\pi_0^*$ are the rates of inflation in the full-information equilibrium that is assumed to have preceded the adjustment period. These are the inherited rates of inflation that each country enters the adjustment period with. For most of our analysis, we assume that the inherited rates in both countries are zero. Using equations (20), the utility functions (18) may be re-written as
(21) \[ U = -\delta x^2 - \mu (\pi_0 + \delta g)^2 \], \[ U^* = -\delta x^* - \mu^* (\pi_0^* + \delta g^*)^2 \].

The dependence of output levels in the U.S. and ROW on the two policy instruments, \( g \) and \( g^* \), is somewhat more complicated. As the analysis of the last section demonstrated, a variety of specifications are possible; we have selected three. In all three specifications, the own effects of monetary policy are assumed to be positive; an unanticipated increase in the rate of growth of a country's money supply raises that country's output. It is the cross effects of monetary policy that distinguish the three specifications and produce the three different game situations studied in this section. In the symmetric-negative specification, an expansionary policy in either country exports unemployment to the other. In the symmetric-positive case, an expansionary policy in either country increases employment in the other. In the asymmetric game, a U.S. expansion increases employment both at home and abroad, but a ROW expansion causes unemployment in the U.S.

At the beginning of the adjustment period, the U.S. and ROW monetary authorities see the exogenous oil price shock, and set the rates of growth of their money supplies to maximize the utility functions (21) subject to the relevant set of macroeconomic constraints. We now consider, in turn, the games corresponding to each of the three output specifications described above.

The Symmetric-Negative Game

With the symmetric-negative constraints, the spillover effects of monetary policy are negative; an expansion in one country causes a
contraction in the other. Reliance upon such "beggar thy neighbor" policies can, in a non-cooperative setting, result in an excessively inflationary outcome.

If \( \pi_0 \) and \( \pi_0^* \) are both zero, then both policy makers will want to adopt expansionary policies. The oil price shock would cause unemployment if they did nothing, and if \( \pi_0 \) and \( \pi_0^* \) are both zero the marginal utility of avoiding some of this unemployment outweighs the marginal disutility of the inflationary expectations engendered.\(^{14}\)

And when each policy maker realizes that the other is going to expand, he will want to adopt an even more expansionary stance to make up for the negative spillover of the other's policy.

The non-cooperative or Nash solution is the policy configuration from which neither player can move unilaterally to make himself better off. If both \( \pi_0 \) and \( \pi_0^* \) are zero, both players will adopt expansionary policies in the Nash solution.\(^{15}\)

To us, the interesting point to note is that the negative spillover effects of monetary policy give the Nash solution an inflationary bias. Each policy maker knows that the other will expand, and each must therefore run an even more inflationary policy to make up for the negative spillover of the other's policy. In fact, it turns out that there is a certain amount of needless competitive inflation (or devaluation) going on in the Nash solution. The Nash solution is always dominated by a range of cooperative solutions that are less inflationary and make both policy makers better off.
The intuition behind this result is fairly straightforward. At the Nash solution, each country has pushed its money growth rate to the point where its effect on domestic utility is zero at the margin. And yet, a decrease in either country's money growth rate will increase output -- and, therefore, utility -- in the other country.\footnote{16} Clearly, both policy makers would be better off if they could mutually agree to inflate less.

The trouble with cooperative solutions is that they require cooperation. In any cooperative solution, there is always an incentive to cheat. Suppose the U.S. and ROW policy makers agree to a less inflationary, Pareto Optimal solution.\footnote{17} In any such cooperative solution, at least one of the two countries can increase its welfare by increasing its money growth rate, provided the other country does not respond by altering its own money growth rate. Thus, at any cooperative solution, unemployment will seem high, and the risk of inflation low, to a public that does not fully understand the nature of the solution and the possibility of foreign repercussions. The political pressure to cheat on cooperative solutions could well be great.

More generally, it is difficult to see how two very independent policy makers would come to agree upon a cooperative solution in the first place. There is a continuum of cooperative solutions defined by a "contract" curve; some are more favorable to the U.S., while others are more favorable to the ROW.\footnote{18}

More likely, it seems to us, the outcome would be closer to the Nash solution, with each country attempting to induce the other to run a less expansionary policy. At the Nash solution the ROW, for example,
would have every incentive to maneuver the U.S. into running a less inflationary policy. If the U.S. acquiesed, ROW employment would be stimulated, but at the expense of U.S. employment. As a result, the ROW would find it desirable to reduce its own money growth rate, moving back along its own reaction curve in the direction of cooperative solutions more favorable to the ROW.

In general, the opportunities for such manipulation appear to be limited. Outright political pressure and the engineering of public opinion, whatever else they achieve, appear not to be very effective means of altering the policies pursued in other countries. An alternative strategy would be for one country to commit itself to a lower inflation rate in return for a similar promise by the other country. Unfortunately, such movements toward a Pareto Optimal solution require an effective means of precommitment. This requirement is extremely difficult meet in a world composed of politically sovereign players. Any policy maker attempting to strike such a bargain confronts the problem that his promises, as well as the promises of other policy makers, are, in the jargon of game theory, "incredible."

The Symmetric-Positive Game

With the symmetric-positive constraints, the spillover effects of monetary policy are positive; an expansion in one country causes an expansion in the other. The "locomotive" aspects of policy produce, in a non-cooperative setting, a solution that is not sufficiently inflationary.

Each policy maker knows that the other is going to inflate and, therefore, that he does not have to inflate as much to get a given
employment effect. The interesting point here is that the Nash solution does not exploit sufficiently the positive externalities embodied in the spillover effects of monetary policy. The basic arguments are the same as in the symmetric-negative game, only the signs of the spillover effects are reversed, as are the conclusions. At the margin, a more expansive U.S. policy would increase ROW employment without significantly affecting the U.S. policy maker's utility, and vice-versa. Both would be better off if they could mutually agree to inflate more.

As in the preceding case, it is not clear how a cooperative solution would be achieved in a world of decentralized policy making. There are many cooperative solutions and policy makers will not be indifferent among them. Further, in the absence of binding contractual arrangements, all cooperative solutions are subject to credibility problems. As before, we might expect a solution close to the Nash solution with each country attempting to influence the other's monetary policy. In this case, however, the pressure would be in the direction of more, rather than less, inflationary policies.

The Asymmetric Game

With the asymmetric constraints, the spillover effects of U.S. policy are positive, while the spillover effects of ROW policy are negative. This means that U.S. policy has a comparative advantage over ROW policy in combating the unemployment caused by an unanticipated rise in the price of oil. The Nash and cooperative solutions to this game are quite different from those of the two symmetric games already discussed.

If inherited inflation rates are zero, each policy maker knows that the other is going to inflate. However, in this case the two policy
makers do not react to that fact in the same way. The U.S. will inflate more to compensate for the negative spillover effects of ROW policy, while the ROW will inflate less because of the positive spillover effects of U.S. policy. So the Nash solution exploits the comparative advantage of U.S. monetary policy in this asymmetric game.

The interesting point, however, is that the Nash solution does not exploit the comparative advantage of U.S. policy enough. Both policy makers would be better off if the U.S. inflated more and the ROW inflated less. The argument should by now be familiar. At the Nash solution, there is no cost for either policy maker to making a marginal change in the rate of growth of his money supply. However, the ROW benefits from a more expansionary U.S. policy, and the U.S. benefits from a less expansionary ROW policy.

Once again, it is not clear how a cooperative solution would be achieved in a world of decentralized policy making. A solution close to the Nash might be expected, with each country attempting to influence the other's monetary policy. In this case, however, the ROW will attempt to induce the U.S. to inflate more, while the U.S. will attempt to induce the ROW to inflate less. Thus, the pressures exerted by the two countries as they strive for a better outcome might be interpreted to reflect differing objectives while, in fact, the objective functions of the two countries are identical.
IV. Measuring the Adjustment Burden

The analysis of the last section has one major implication for the size of the adjustment burden following an oil price shock. The absence of cooperation between policy makers unambiguously increases the size of the adjustment burden for all players, where the adjustment burden is measured by the reduction in policy makers' utility functions.\textsuperscript{19/}

Perhaps equally important and interesting, however, are the lessons to be derived from the present section concerning the appropriate measurement of the size and distribution of the adjustment burden.

The appropriate measure of each country's adjustment burden following an oil price shock is presumably the reduction in social welfare, somehow measured, experienced by that country following the shock. In the game framework of the present paper, that reduction is measured by the utility loss suffered by policy makers. This loss is a weighted average of the changes in output and inflationary expectations that follow the shock, and can not be directly observed. Clearly, an observable proxy for the actual adjustment burden -- a measure that is perfectly correlated with social welfare -- would be useful.

One candidate proxy for the adjustment burden born by any individual country following an oil price increase is the fall in its level of output. A casual reading of the popular literature suggests that this proxy is already in wide use. It appears to be common practice to gauge the overall impact of an oil shock by its aggregate effect on output levels, and the distribution of that impact by its effect on relative output levels. However, the analysis of this section suggests
that these measures are inadequate approximations of policy makers' utility in a wide range of circumstances.

Consider, for example, the problem of comparing the overall size of the adjustment burden in a non-cooperative game setting with the size of the burden in a cooperative setting. The move from a non-cooperative solution to a cooperative solution raises the utility of all players. However, if spillover is negative, this move is also associated with a greater output loss and lower long run inflation for all players. Accordingly, the size of the output loss does not, in this case, provide a correct indication of policy makers' utility. In general, the relationship between output losses and social welfare will depend on the nature of the spillover effects of policy which, in turn, depend on the structural and institutional features of the countries involved.

The difficulty of appropriately capturing the changes in social welfare following an oil price shock is even more evident when the question of the distribution of the adjustment burden is addressed. If one allows for differences in the relative weights assigned to output and inflation across countries, no systematic relationship between social welfare and output can be established, even for a given macroeconomic structure. A country with a very high aversion to inflation may, for example, suffer a substantial output loss compared to a country with a very high aversion to unemployment, and yet experience a lower overall utility loss.
Footnotes

1/ The game theoretic aspects of policy making in an interdependent world have been recognized by a number of earlier writers. See, for example, Bryant (1980), Hamada (1974, 1976 and 1979), and Johansen (1980).

2/ There is of course a voluminous literature devoted to the international transmission of monetary disturbances; however, explicit analyses of two (or more) country models are relatively scarce. We are unaware of any set of papers that present just the analyses we need here, although many of the individual pieces of our desired framework do appear in various other papers. Mundell (1963) for example, develops symmetric-negative constraints in a model with fixed prices and static expectations. Mussa (1979) presents an excellent discussion of macroeconomic interdependence which includes an exposition of Mundell's results. Argy and Salop (1979) present a model very like the two-good extension of the Canzoneri and Gray (1982a) model we discuss below, though they postulate static expectation formation. Daniel (1981) presents a rational expectations model that is also very like our two-good extension; her use of intermediate goods serves the function of our indexing. Bruno and Sachs's (1979) simulation model has all of the ingredients necessary to derive the three sets of constraints we employ.

3/ There exist a number of papers that explore the effects of an oil price shock in a single country context; see, for example, Blinder (1981), Bruno and Sachs (1979), and Findlay and Rodriguez (1977). A few
papers address this question in a multi-country setting that explicitly models OPEC's short run savings behavior. Examples include Caprio and Clark (1981), Krugman (1980), Sachs (1980), and Schmid (1976). Unfortunately, none of these papers provides the analysis we require of both an oil price shock and the spillover effects of monetary policy. For further discussion of this point, see footnote 1.

4/ In calculating the post shock equilibrium, which is necessary in solving the model, it is assumed that the real value of the original dollar increase in the price of oil is preserved. That is, the dollar price of oil is adjusted in the post shock period for any changes in the U.S. price level which occur subsequent to the pre shock equilibrium. Thus, the real value of the oil price increase can only be temporarily eroded during the adjustment period.

5/ The level of real income in each country is defined to be nominal net national income deflated by the domestic consumer price index. Accordingly, \( Y \) and \( Y^* \) are given by

\[
Y = \left(\frac{P}{PI}\right)X - \left(\frac{Q}{PI}\right)0 - rB
\]

\[
Y^* = \left(\frac{P^*}{PI^*}\right)X^* - \left(\frac{Q}{PI}\right)0^* - rB^*
\]

Because consumption patterns are identical in the two countries, and because the law of one price holds, \( \frac{Q}{PI} = \frac{Q^*}{PI^*} \); the real price of oil in the U.S. is equal to the real price of oil in the ROW. Log linearizing \( Y \) and \( Y^* \) around their no-shock equilibrium values yields equations (7).
6/ The demands for the two goods (in levels) are given by

\[ x^d = (PI/P)(P*E/P)^\psi(C+C^*) + \text{OPEC demand} \]
\[ x^*d = (PI^*/P^*)(P/P*E)^\psi(C+C^*) + \text{OPEC demand} \]

Equations (10) are obtained by log linearizing \( x^d \) and \( x^*d \) around their no-shock equilibrium values, setting the change in OPEC's demand for both goods equal to zero, and equating output demand to output supply. The \( \psi \) which enters equation (10), then, is the negative of the own relative price elasticity of the expenditure shares of the two goods. The term \( (1+2\psi) \) which enters equation (10) is the negative of the own relative price elasticity of the levels of expenditure on the two goods.

7/ An unitary income elasticity of money demand means that each country's nominal money balances are effectively deflated by the price of domestic output in the equations describing money market equilibrium. If this were not the case, valuation effects would be another source of spillover.

8/ The solution of equations (1) through (13) of the text requires calculating the one period ahead expectation of the price of U.S. and ROW output. This expectation appears in equation (12) as a determinant of the nominal interest rate. To find \( \delta p_{t+1} \), we begin by differencing equation (13), substituting the result, along with equation (12), into equation (11), and updating the result by one period.
\[ \delta m_{+1}^S - \delta p_{+1} = \delta x_{+1} - \lambda \delta r - \lambda (\delta p_{+2} - \delta p_{+1}) \]

Because of the way we have chosen to specify output supply and demand, \( \delta x \) and \( \delta r \) are expected to be equal to zero in the post shock equilibrium.

The term \( (\delta p_{+2} - \delta p_{+1}) \) is simply the change in the expected rate of inflation in the post shock equilibrium, which is simply equal to the change in the rate of growth of money, or \( \delta g \). The term \( \delta m_{+1}^S \) is the deviation of the money stock in the post shock period from the value it would have assumed in the absence of shocks. Since the post shock period is the second period in which altered money growth rate is applicable, this term is equal to \( 2 \delta g \). Substituting these values into the equation above and simplifying gives

\[ \delta p_{+1} = (2 + \lambda) \delta g \]

An exactly analogous expression for the post shock price of ROW output can be derived. Once these expressions are substituted into equation (11) of the text, the solution of the model is a matter of straightforward algebra.

9/ The nominal exchange rate also deprecitaes; that is,

\[ \delta e = (1 + \lambda)^2 (\theta + \tau + \phi) (1/D_1) (\delta g - \delta g^*) \]

This result is used in Canzoneri and Gray (1982b) to define the Fixed-Rate Regime.

10/ In fact, in the case of a fixed real price of oil, the ROW price level necessarily falls following a U.S. expansion.
Once again, the nominal exchange rate depends on the ratio of the money supplies; that is,

\[
\delta e = (1 + \lambda)^2 (\theta + \tau + \phi)(1/D_1)(\delta g - \delta g^*)
\]

This result is used in Canzoneri and Gray (1982b).

Recall that we are assuming that agents' expectations are "rational" given their assumptions about monetary policy. If their views about monetary policy (and the price of oil) are correct, then their price predictions will be realized.

It is clear from the money demand functions that in full employment equilibrium the rates of product price inflation are \(g\) and \(g^*\). Since the terms of trade is fixed in full equilibrium the rates of inflation of the general price levels (\(p_1\) and \(p_1^*\)) are also equal to \(g\) and \(g^*\).

With the utility functions (21),

\[
\frac{\partial u}{\partial \delta g} = -2(\delta x)(\delta x/\delta g) - 2(\pi_0 + \delta g) = 2\rho_3 \delta q_1 > 0
\]

\[
\frac{\partial u^*}{\partial \delta g^*} = 2\rho_3 \delta q_1 > 0
\]

when \(\pi_0 = \pi_0^* = 0\) and \(\delta g = \delta g^* = 0\).

It may be interesting to note that some unemployment will remain as long as any weight is given to inflation in the utility functions. The conditions \(U_{\delta g} = 0\) and \(U_{\delta g^*} = 0\) (which define the Nash equilibrium) imply
\[ \delta x = -(\mu/\rho_1)(\delta g + \pi_0) \]

\[ \delta x^* = -(\mu^*/\rho_1)(\delta g^* + \pi^*_0) \]

So if \( \delta g > -\pi_0 \) and \( \delta g^* > -\pi^*_0 \), then \( \delta x \) and \( \delta x^* \) are negative as long as \( \mu \) and \( \mu^* \) are positive.

16/ At the Nash solution, \( U_{\delta g^*} = 2\rho_2 (-\delta x) < 0 \) because \( \delta x < 0 \); see footnote 12. Similarly, \( U^*_{\delta g} = 2\rho_2 (-\delta x^*) < 0 \).

17/ See Canzoneri and Gray (1982b) for a demonstration that such a solution exists.

18/ See Canzoneri and Gray (1982b) for a derivation of the contract curve.

19/ Our model does have implications for the size of the adjustment burden that are independent of its game aspects. If, for instance, the oil price shock raises the price of domestic output in the two countries, it will be true that the adjustment burden is an increasing function of the degree of wage and oil indexation. In contrast to most other models in which indexing has been studied, however, the oil price shock does not necessarily raise output prices in our framework. Thus, the conclusions for indexing in the present paper are not clear cut. This point and other implications of our macroeconomic structure in the one good case are discussed in Canzoneri and Gray (1982a).
References


