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# THE OPTIMAL DEGREE OF COMMITMENT TO AN INTERMEDIATE MONETARY TARGET: INFLATION GAINS VERSUS STABILIZATION COSTS

by

Kenneth Rogoff

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## I. Introduction

Curiously, despite all the emphasis on thinking about monetary policy in terms of rules,[1/] no regime analyzed in the extant theoretical literature can be said to closely describe the monetary targeting regimes observed over the past decade.[2/] The real-world feature of these regimes which is absent from the theoretical models is the ability of the central bank to credibly commit itself to placing great weight on an intermediate monetary target (relative to direct social objectives), while retaining some scope to respond to new information. "Flexible" monetary targeting constrains the discretionary actions of the central bank but still leaves some room for the monetary authorities to use the money supply as an instrument of stabilization policy.

The present paper attempts to provide a rationale for flexible monetary targeting by synthesizing elements of the recent time consistency literature into a standard rational expectations *cum* wage contracting macroeconomic model.[3/] The possible need for monetary targeting arises as equilibrium under fully discretionary monetary policy is characterized by stagflation. Either because of monopolistic unions, or because income taxes and unemployment insurance distort the labor-leisure decision, the equilibrium level of employment is assumed to lie below the level that would arise in the absence of labor market distortions. As in Barro and Gordon (1983), inflation is generated because wage setters rationally fear that the central bank will inflate to try to drive down real wages to a level consistent with the socially desired rate of employment. By setting nominal wages high enough, they are able to place the monetary authorities in a position where the social cost of inflating beyond the level consistent with wage setters' desired real wages begins to outweigh the benefits of reduced unemployment. Thus, as in virtually all rational expectations models, the monetary authorities cannot systematically raise the level of employment. But since wage contracts are not fully indexed to the price level, monetary policy can still be used to improve social welfare by stabilizing employment and inflation around their market-determined levels.[4/]

By implementing monetary targeting, the central bank can convince wage setters that it is not as likely to inflate, but only at the cost of constraining its ability to respond to future unanticipated disturbances.[5/] The optimal degree to which the monetary authorities should credibly commit themselves to their intermediate monetary target trades off the inflation gains against the stabilization costs. Rigid targeting is appropriate only in certain very special cases. A number of types of targets for monetary commitment are considered: the money supply, nominal GNP, the inflation rate (price level) and the short-term nominal interest rate. Which one of these works best depends, of course, on the structure of the economy and the nature of the underlying disturbances. (Though the interest rate generally appears to be an unsatisfactory tool.)

It is important to stress that while flexible monetary targeting is preferable to either fully discretionary monetary policy or rigid monetary targeting, it is not necessarily the first-best solution to the problem of stagflation in this model. That depends on the source of the underlying labor market distortion which causes the market-determined average level of employment to be too low. If this distortion can be removed at low social cost, then it would be possible to both raise employment and lower inflation. A second-best solution, which does nothing to raise the average level of employment, would be for the central bank to somehow guarantee to wage setters that it will not systematically inflate for the purpose of raising employment to its socially optimal level. Such a guarantee would bring down the rate of wage inflation without limiting the scope of the central bank to use monetary policy to offset unanticipated disturbances. (Unfortunately, it is hard to envision how such a guarantee could be enforced.) Thus it is only when the first and second-best solutions are too costly or unachievable that monetary targeting should be used as a "third-best" solution to the problem of stagflation.

Section II of the text describes a stochastic rational expectations macroeconomic model in which, because of wage contracting, there is a well-defined role for central bank stabilization policy. Section III derives the ("time consistent") equilibrium under fully discretionary monetary policy. Section IV shows how society can make itself better off by appointing an agent to head the central bank whose dislike for inflation relative to unemployment is stronger than average. Section V reinterprets the formal analysis of Section IV as a model of inflation-rate targeting, and demonstrates how to extend the framework to encompass nominal GNP targeting, money supply targeting and nominal interest rate targeting. Section VI discusses comparisons across regimes, and the main results are summarized in the conclusions.

## II. The Macroeconomic Model

This section is devoted to developing a stochastic rational expectations IS-LM model.[6/] Monetary policy can have short-term real effects in this model because nominal wage contracts are set a period in advance. Due to high administrative and negotiation costs, these contracts are not indexed fully against all possible disturbances.[7/] But while the central bank can affect the variance of employment, it cannot systematically affect the average level of employment. The reason for this will become apparent when in Section III we study wage setters' expectations about future monetary policy.

### II.1. Aggregate Supply

Each of the large number of identical firms in the economy has a Cobb-Douglass production function. In the aggregate,

$$y_t = c_0 + \alpha\bar{k} + (1-\alpha)n_t + z_t, \tag{1}$$

where y is output, $\bar{k}$ is the fixed capital stock, n is labor, $c_0$ is a constant term and z is a serially uncorrelated aggregate productivity disturbance; $z \sim N(0,\sigma_z^2)$. Throughout, lower case letters denote natural logarithms and the period t value of a variable is denoted by a "t" subscript. All coefficients are non-negative.

Firms hire labor until the marginal product of labor equals the real wage:

$$c_0 + \log(1-\alpha) + \alpha\bar{k} - \alpha n_t^d + z_t = w_t - p_t, \tag{2}$$

where w is the nominal wage, p is the aggregate price level, and $n^d$ is the quantity of labor demand.

Labor supply, $n^s$, is an upward-sloping function of the real wage:

$$n_t^s = \overline{n} + \omega(w_t - p_t). \tag{3}$$

To simplify algebra without loss of generality, $\overline{n}$ is set equal to $\overline{k} + (1/\alpha)[\log(1-\alpha) + c_0]$.

The above labor supply curve (3) is assumed to embody some distortions which raise the real wage required to induce a given level of labor supply. Factors which might distort the labor-leisure decision towards leisure include income taxation and high levels of unemployment benefits.[8/] Monopolistic unions might also succeed in driving up real wage demands. The labor supply curve in the absence of distortions is given by

$$n_t^s = \overline{n} + d_0 + \omega(w_t - p_t), \tag{4}$$

where $-d_0$ is the distortion in the actual labor supply curve (3).[9/]

Price-indexed wage contracts for period t are negotiated at the end of period $t-1$. The base wage rate is $\overline{w}_t$, and the indexation parameter is $\beta$[10/]:

$$w_t = \overline{w}_t + \beta(p_t - \overline{w}_t), \quad 0 \leqq \beta \leqq 1. \tag{5}$$

The nature of the employment contract is that laborers agree to supply whatever amount of labor is demanded by firms in period t, provided firms pay the negotiated wage rate. The level of employment in period t is thus found by substituting the wage rate equation (5) into the labor demand equation (2):

$$n_t = \overline{n} + \nu(p_t - \overline{w}_t) + z_t/\alpha, \tag{6}$$

where $\nu \equiv (1-\beta)/\alpha$.

In choosing the base wage rate $\overline{w}_t$, wage setters seek to minimize $E_{t-1}(n_t - \overline{n}'_t)^2$, where $E_{t-1}$ denotes expectations based on period t-1 information and $\overline{n}'_t$ is the level of employment that would arise if contracts could be negotiated after observing the productivity disturbance $z_t$ and all other period t information. $\overline{n}'_t$ is found using the labor supply and demand equations (2) and (3):

$$\overline{n}'_t = \overline{n} + \omega z_t/(1+\alpha\omega). \tag{7}$$

From equations (6) and (7),

$$n_t - \overline{n}'_t = z_t/\eta + \nu(p_t - \overline{w}_t), \tag{8}$$

where $\eta \equiv \alpha(1+\alpha\omega)$. It is clear from equation (8) that $E_{t-1}(n_t - \overline{n}'_t)^2$ is minimized by setting $\overline{w}_t = E_{t-1}(p_t)$.[11/]

Using equations (1) and (6), together with the analytically convenient assumption (at no sacrifice in generality) that $-c_0 = \alpha\overline{k} + (1-\alpha)\overline{n}$ so that $E_{t-1}(y_t) = 0$, the aggregate supply equation can be written as

$$y^s_t = \theta(p_t - \overline{w}_t) + z_t/\alpha, \tag{9}$$

where $\theta \equiv (1-\alpha)(1-\beta)/\alpha$. Since $\overline{w}_t = E_{t-1}(p_t)$, equation (9) can be written in the standard rational expectations "price level surprise" form:

$$y^s_t = \theta[p_t - E_{t-1}(p_t)] + z_t/\alpha. \tag{9'}$$

It is very important to note from (9') and (6) that output and employment stabilization are not equivalent to price prediction error minimization in the presence of a productivity shock (z).

II.2. Aggregate Demand

Demand for the good which firms produce is a decreasing function of the real interest rate:

$$y_t^d = -\delta \{r - [E_t(p_{t+1}) - p_t]\} + u_t, \tag{10}$$

where r is the level of the nominal interest rate and $E_t(p_{t+1}) - p_t$ represents the rate of inflation expected by investors, based on period t information.[12/] The serially uncorrelated goods market demand disturbance $u_t \sim N(0,\sigma_u^2)$.

The demand for real money balances is a decreasing function of the nominal interest rate and an increasing function of output:

$$m_t - p_t = -\lambda r_t + \phi y_t + v_t, \tag{11}$$

where m is the logarithm of the nominal money supply and v is the money market disturbance term; $v \sim N(0,\sigma_v^2)$. To simplify exposition, the disturbances, v, u and z, are assumed to be independent.

II.3. The Social Loss Function

The principal differences between the present paper and previous rational expectations cum wage contracting analyses of monetary stabilization policy derive from the specification of the social objective function.[13/] Because it is generally agreed that the monetary authorities cannot systematically affect the level of employment when wage setters have rational expectations, the issue of whether or not the central bank wishes it could lower the average level of employment is commonly ignored. (It is ignored by assuming that the monetary authorities target the same level of employment as do the wage setters.) But this potential source of tension is fundamental to the interplay between stabilization policy and wage-setting. Indeed, if the monetary authorities do target the same level of employment as wage setters, then there is no reason to consider any regime other than fully discretionary monetary policy. (Though, for information-theoretic reasons, the optimal fully-discretionary monetary policy may involve using a feedback rule.)

Here is is assumed that the social loss function $\Lambda$ ~~depends on~~ deviations of employment and inflation from their optimal (socially-desired) levels:[14/]

$$\Lambda_t = (n_t - \tilde{n}_t')^2 + \chi(\pi_t - \tilde{\pi})^2, \tag{12}$$

where $\pi_t \equiv p_t - p_{t-1}$, $\tilde{\pi}$ is the socially-desired trend inflation rate, $\tilde{n}_t'$ is the socially-desired level of employment in period t, and $\chi$ is the relative weight society places on inflation stabilization versus employment stabilization. $\tilde{n}_t'$ is found by using the labor demand schedule (2) together with the "undistorted" labor supply schedule (4), and by assuming that wage setters have full period t information:[15/]

$$\tilde{n}_t' - \bar{n}_t' = d_0/(1 + \alpha\omega) \equiv \tilde{n} - \bar{n}. \tag{13}$$

We have already discussed why $\tilde{n}$ might be greater than $\bar{n}$.

It is somewhat difficult, in the context of a rational expectations model, to justify incorporating the level of the inflation rate directly into the social loss function. The costs of perfectly anticipated inflation are imperfectly understood.[16/] Some costs which are known include the administrative costs of posting new prices and the costs of adjusting the entire tax system to be fully neutral with respect to a variable but perfectly-anticipated rate of inflation. And of course, high rates of inflation force agents to economize on their holdings of non-interest-bearing money--the so-called "shoe leather cost of inflation." Despite the foregoing considerations, the optimal target inflation rate $\tilde{\pi}$ may be non-zero. It is sometimes optimal to use the seignorage tax at a non-zero level when alternative taxes also generate dead-weight costs through distortions.[17/]

## III. Time-Consistent Equilibrium Under Fully Discretionary Monetary Policy.

Here, stochastic equilibrium is derived under the assumption that the monetary authorities attempt to minimize the social loss function $\Lambda$, given by equation (12) above.[18/]

In the present model, as in most rational expectations monetary models, today's prices and interest rates depend in part on agents' expectations about the future path of the money supply. But here this path is not exogenously given; expectations about future money-supply growth depend endogenously on expectations about the monetary authorities' future short-run stabilization objectives. Agents will not, in fact, fully believe promised future paths for the money supply which are time inconsistent; i.e., promises on which the central bank will have a predictable incentive to reneg.[19/] Wage setters could protect themselves against systematic inflation by fully indexing wage contracts to the price level, but this would leave them with no insulation against supply shocks.[20/] Instead, as we demonstrate formally below, wage setters can set nominal wage increases at a sufficiently high level so that, in the absence of disturbances, the central bank will not choose to inflate the money supply beyond the point consistent with wage setters' desired real wage.[21] At this high level of inflation, the central bank finds that the marginal utility gain from the higher employment induced by a small further price increase is fully offset by the marginal disutility from still higher inflation.

By substituting equations (8) and (13) into equation (12), the central bank's objective function under fully discretionary monetary policy may be written as:

$$D_t = \Lambda_t = [z_t/\eta + \nu(p_t - \bar{w}_t^D) - (\tilde{n} - \bar{n})]^2 + \chi[p_t - p_{t-1} - \tilde{\pi}]^2. \tag{14}$$

("D" superscripts stand for "fully discretionary regime.")
The central bank maximizes social welfare by choosing a level of the money supply consistent with $p_t^D$, the period t price level which minimizes $\Lambda_t$.[22]

$$p_t^D = [\nu^2\bar{w}_t^D + \nu(\tilde{n}-\bar{n} - z_t/\eta) + \chi(p_{t-1} + \tilde{\pi})] / (\nu^2 + \chi). \tag{15}$$

Recall that (the logarithm of) wage setters' target real wage is zero. Thus wage setters select $\bar{w}_t^D$ by taking expectations across (15) and setting $\bar{w}_t^D = E_{t-1}(p_t^D)$:[23]

$$\bar{w}_t^D = E_{t-1}(p_t^D) = p_{t-1} + \tilde{\pi} + \nu(\tilde{n} - \bar{n})/\chi. \tag{16}$$

By choosing $\bar{w}_t^D$ according to (16), wage setters assure themselves that the monetary authorities will not attempt to systematically drive down the real wage.

Investors similarly use (16) to solve for $\bar{\pi}^D$, the expected rate of inflation under fully discretionary monetary policy:[24]

$$\overline{\pi}^D = \nu(\tilde{n}-\overline{n})/\chi + \tilde{\pi}. \qquad (17)$$

Thus, as Kydland and Prescott as well as Barro and Gordon have pointed out, the time consistent rate of inflation is too high when $\tilde{n} > \overline{n}$. Society suffers from stagflation.[25/]

We are now prepared to evaluate the social welfare function, $\Lambda$, under fully discretionary monetary policy. But to facilitate exposition in later sections, we shall first develop a notation for evaluating social welfare under any given monetary policy regime. Obviously, it does not make sense to evaluate $\Lambda$ for a particular realization of the disturbances, given that society must decide a period in advance which regime to choose. Rather, the relevant consideration is $E_{t-1}(\Lambda_t)$, the expected value of the social welfare function at the time the choice of regime is made. Defining $\Lambda_t^A \equiv E_{t-1}(\Lambda_t)$ under an arbitrary monetary policy regime "A",

$$\Lambda_t^A = (\tilde{n}-\overline{n})^2 + \chi\Pi^A + \Gamma^A, \qquad (18)$$

where $\Pi^A \equiv (\overline{\pi}^A - \tilde{\pi})^2$ and
$\Gamma^A \equiv E_{t-1}\{[z_t/\eta + \nu(p_t^A - E_{t-1}(p_t^A))]^2 + \chi[p_t^A - E_{t-1}(p_t^A)]^2\}$, where we have made use of the fact that $E_{t-1}(p_t) = \overline{w}_t$. The first component of $\Lambda_t^A$ is nonstochastic and invariant across monetary regimes. It represents the deadweight loss due to the labor market distortion. This loss cannot be reduced through monetary policy in a time consistent rational expectations equilibrium. The second term is also nonstochastic but does depend on the choice of monetary policy regime. The average (expected) inflation rate depends, in part, on the weight the monetary authorities place on inflation stabilization versus employment

stabilization. The final term, $\Gamma^A$, represents the "stabilization" component of the loss function. It measures how successfully the central bank offsets disturbances to stabilize employment and inflation around their average market-determined values.

We have already solved for the average level of inflation under fully discretionary monetary policy, $\bar{\pi}^D$; see equation (17). To derive $\Gamma^D$, first take t-1 expectations across (15) and use (16) to substitute for $\bar{w}_t^D$. One obtains

$$[p_t - E_{t-1}(p_t)]^D \equiv dp_t^D = -\nu z_t/\eta(\nu^2 + \chi) \equiv \rho^D z_t. \tag{19}$$

Note that u and v do not enter the expression for the price prediction error that the central bank allows to occur. The central bank offsets the price level effects of aggregate demand shocks to the best of its ability (here perfect, because of the complete information assumption), because offsetting these shocks is consistent with both employment stabilization and inflation-rate stabilization. Substituting (19) into the expression for $\Gamma^A$ given in (18), and simplifying, yields

$$\Gamma^D = (\sigma_z^2/\eta^2)[\chi/(\nu^2 + \chi)]. \tag{20}$$

## IV. Social Welfare under a "Conservative" Central Banker.

The main result of this section is the following: When $\tilde{n} > n$, society can make itself better off by selecting an agent to head the independent central bank who does not minimize the social loss function $\Lambda$, but instead prefers to minimize a loss function which places a greater weight on inflation relative to unemployment. But, within this class of regimes, it is never optimal to choose an individual who pays "too little" attention to unemployment, in a sense that will be made precise below.

Suppose, for example, that in period t-1 society selects an agent to head the central bank in period t who is known to have the following objective function (henceforth, time t subscripts are omitted where the meaning is obvious):

$$I = (n-\tilde{n}')^2 + (\chi+\varepsilon)(\pi-\tilde{\pi})^2, \quad \varepsilon > -\chi. \tag{21}$$

When $\varepsilon$ is strictly greater than zero, then this agent places a greater relative weight on inflation stabilization than society does.[26/]

The algorithm for deriving the time consistent equilibrium is exactly the same as in the previous section. Equations (22), (23), and (24) are the "I" regime counterparts of equations (17), (19) and (20) respectively:

$$\Pi^I = (\bar{\pi}^I - \tilde{\pi})^2 = [\nu(\tilde{n}-\bar{n})]^2/(\chi+\varepsilon)^2, \tag{22}$$

$$[p_t - E_{t-1}(p_t)]^I = dp_t^I = -\nu z_t/\eta(\nu^2 + \chi + \varepsilon), \tag{23}$$

$$\Gamma^I = (\sigma_z^2/\eta^2)[(\chi+\varepsilon)^2 + \nu^2\chi]/(\nu^2 + \chi + \varepsilon)^2. \tag{24}$$

(Note that $\Gamma^I$ is obtained by plugging $dp_t^I$ into $\Gamma^A$, as defined in equation (18).[27/]) The reader can confirm that these equations are the same as (17), (19) and (20) when $\varepsilon = 0$. The (expected value of the) social loss function under "inflation-rate targeting," as we shall term this regime is

$$\Lambda^I = (\tilde{n}-\bar{n})^2 + \chi\Pi^I + \Gamma^I. \tag{25}$$

To solve for the value of $\varepsilon$ which minimizes $\Lambda^I$, differentiate (25) with respect to $\varepsilon$:

$$\partial\Lambda^I/\partial\varepsilon = \chi(\partial\Pi^I/\partial\varepsilon) + \partial\Gamma^I/\partial\varepsilon , \tag{26a}$$

$$\partial\Gamma^I/\partial\varepsilon = 2(\sigma_z^2/\eta^2)[\varepsilon\nu^2/(\nu^2 + \chi + \varepsilon)^3], \tag{26b}$$

$$\partial\Pi^I/\partial\varepsilon = -2[\nu(\tilde{n}-\overline{n})]^2/(\chi+\varepsilon)^3. \tag{26c}$$

Define $\varepsilon^{min}$ as the value of $\varepsilon$ which minimizes $\Lambda^I$. We are now ready to prove:

Theorem 1. For $\tilde{n} > \overline{n}$, $0 < \varepsilon^{min} < \infty$ .

Proof: Note that $\varepsilon > -\chi$ by assumption. Thus, by inspection of (26c), $\partial\Pi^I/\partial\varepsilon$ is strictly negative. Note also, by inspection of (26b), that $\partial\Gamma^I/\partial\varepsilon$ is strictly negative for $-\chi < \varepsilon < 0$, zero when $\varepsilon = 0$, and positive for $\varepsilon > 0$. Therefore $\partial\Lambda^I/\partial\varepsilon$ is strictly negative for $\varepsilon \leqq 0$. $\partial\Lambda^I/\partial\varepsilon$ must change from negative to positive at some sufficiently large value of $\varepsilon$, since as $\varepsilon$ approaches positive infinity, $\partial\Gamma^I/\partial\varepsilon$ converges to zero at rate $\varepsilon^{-2}$, whereas $\partial\Pi^I/\partial\varepsilon$ converges to zero at rate $\varepsilon^{-3}$. Therefore $\varepsilon^{min} < \infty$. Q.E.D.

It follows immediately that for $\tilde{n} = \overline{n}$, $\varepsilon^{min} = 0$.

Theorem 1 states that in the presence of a labor market distortion, it is optimal to choose an agent to head the central bank who places a greater, but not infinitely greater, weight on inflation than society does. To intuitively interpret Theorem 1, consider the effects of raising $\varepsilon$ from zero. By increasing the central bank's commitment to fighting inflation, the time-consistent average rate of wage inflation is reduced. But the relative weight the central bank places on inflation versus employment stabilization is altered, and this distorts the monetary authorities' responses to unanticipated shocks. To see why the benefit outweighs the cost at $\varepsilon = 0$, it is suggestive to expand (26a) as follows:

$$\partial\Lambda^I/\partial\varepsilon = \chi(\partial\Lambda^I/\partial\Pi^I)(\partial\Pi^I/\partial\varepsilon) + (\partial\Lambda^I/\partial\Gamma^I)(\partial\Gamma^I/\partial\varepsilon). \qquad (27)$$

In the neighborhood of $\varepsilon = 0$, the monetary authorities are minimizing $\Gamma^I$ (they are stabilizing optimally), so that $\partial\Gamma^I/\partial\varepsilon$ is zero. But inflation is not being minimized, so that $\partial\Pi^I/\partial\varepsilon$ is not zero. We can argue similarly to suggest why $\varepsilon^{min} < \infty$. As $\varepsilon$ becomes large, $\bar{\pi}^I$ goes to $\tilde{\pi}$ and <u>both</u> of the terms in $\Pi^I$ go to zero in (27). But $\Gamma^I$ rises to an upper bound as $\varepsilon$ goes to infinity. (It is easy to calculate from (24) that this upper bound is $\sigma_z^2/\eta^2$). Therefore $\partial\Lambda^I/\partial\Gamma^I$ does <u>not</u> go to zero. Since in equation (27) only one of the terms in $\Gamma^I$ goes to zero and both the terms in $\Pi^I$ go to zero as $\varepsilon$ becomes large, it is intuitively plausible that the loss eventually outweighs the gain.[28/] In fact, Theorem 1 assures us this is true. Of course, when there is no labor market distortion, so that $\tilde{n} = \bar{n}$, then $\bar{\pi}^D = \tilde{\pi}$, and it does not pay to appoint a central banker who minimizes anything other than the social loss function.

## V. <u>Intermediate Monetary Targeting</u>.

In the previous section we demonstrated conditions under which society can make itself better off by appointing a central banker who places greater weight on inflation stabilization than society itself does. Here we reinterpret the formal analysis of Section IV as a model of intermediate monetary targeting.

Suppose society selects an individual to head the central bank who commits himself to achieving an intermediate monetary target, such as the money supply, the interest rate, nominal GNP, or the inflation rate. The commitment may be enforced directly through a system of rewards and punishments. Or it may derive credibility from the agent's known integrity and dedication as a public servant; the agent's reputation and therefore expected future income could be affected by his success in meeting his targets. The relevant degree of commitment should be measured, of course, only by these underlying incentives, and not by how loudly promises are proclaimed.

### V.1. Inflation - Rate (Price Level) Targeting

According to the above interpretation, the analysis of Section IV may be viewed as an analysis of inflation-rate (price-level) targeting.[29/] It is easiest to see this by rewriting the central banker's objective function (21) as

$$I = (n-\tilde{n}')^2 + \chi(\pi-\tilde{\pi})^2 + \varepsilon(\pi-\tilde{\pi})^2. \tag{28}$$

The first two terms in equation (28) comprise $\Lambda$, the social objective function (12). The final term represents the additional weight which the central bank places on the inflation rate because it has chosen the inflation rate (or price level) as an intermediate target. Unless the index parameter $\varepsilon$ is infinite, the central bank cannot be relied on to religiously hit its intermediate target, because that policy generally conflicts with the policy which would maximize the social utility function $\Lambda$. $\varepsilon$ measures the extra (artificial) incentives the central bank has for fulfilling its inflation rate target. These incentives are additional to the fact that the inflation rate enters directly into $\Lambda$. While the formalism embodied in equation (28) may appear somewhat contrived, it does capture the basic mechanism underlying the monetary targeting regimes implemented in numerous countries over the past decade.[30/]

Adopting an intermediate targeting interpretation of the analysis of Section IV, we observe that inflation-rate targeting succeeds in reducing the rate of wage inflation. But this gain is not without stabilization cost, as inflation-rate targeting causes the monetary authorities to place too much emphasis on reducing the variance of inflation and not enough emphasis on reducing the variance of employment. It is natural to ask whether the central bank can find other intermediate targets with which to bring down the inflation rate at lower stabilization cost. We shall now consider, in turn, nominal GNP targeting, money-supply targeting, and interest-rate targeting. Because the derivations of the results for these regimes are basically analogous to the algorithm of Sections III and IV, we shall omit some of the details.

## V.2. Nominal GNP Targeting

The drawback to inflation-rate targeting is that it does not allow real wages to properly adjust in response to supply shocks. Because a positive supply shock tends to raise output and lower the price level, one would expect there to be some circumstances in which nominal GNP targeting is more appropriate. Suppose then that the central bank adopts nominal GNP targeting so that its objective function is given by

$$G = (n-\tilde{n}^{t})^2 + \chi(\pi-\tilde{\pi})^2 + \tau(y_t + p_t - \bar{y} - p_{t-1} - \tilde{\pi})^2, \qquad (29)$$

where $\bar{y} = f(\bar{n})$ according to the production function (1).[31/]

G stands for "nominal GNP targeting regime," and the index parameter $\tau$ gives the weight that the central bank places on achieving its intermediate target relative to the weight it places on directly maximizing the social objective function $\Lambda$.

We can rewrite the objective function G by using equations (8), (9), (13), and (29):

$$G = [z_t/\eta + \nu(p_t - \overline{w}_t^G) - (\tilde{n}-\overline{n})]^2 + \chi[p_t - p_{t-1} - \tilde{\pi}]^2 + \tau[z_t/\alpha + \theta(p_t - \overline{w}_t^G) + p_t - p_{t-1} - \tilde{\pi}]^2, \ \tau \geqq 0. \tag{30}$$

To find the time consistent path of the economy under nominal GNP targeting, again follow the alogrithm of Section III.[32/] Equations (31), (32) and (33) below correspond to equations (17), (19) and (20) of Section III:

$$\Pi^G = (\overline{\pi}^G - \tilde{\pi})^2 = \nu^2(\tilde{n}-\overline{n})^2/[\tau(1+\theta) + \chi]^2, \tag{31}$$

$$dp_t^G = -z_t[\nu/\eta + \tau(1+\theta)/\alpha]/[\nu^2 + \tau(1+\theta)^2 + \chi] = \rho^G z_t, \tag{32}$$

$$\Gamma^G = \Gamma_n^G/\Gamma_d^G, \text{ where} \tag{33a}$$

$$\Gamma_n^G = \sigma_z^2\{[(\chi/\eta) + \tau(1+\theta)^2/\eta - \tau(1+\theta)\nu/\alpha]^2 + \chi[\nu/\eta + \tau(1+\theta)/\alpha]^2\}, \text{ and} \tag{33b}$$

$$\Gamma_d^G = [\nu^2 + \tau(1+\theta)^2 + \chi]^2. \tag{33c}$$

The expected value of the social loss function under nominal GNP targeting is

$$\Lambda^G = (\tilde{n}-\overline{n})^2 + \chi\Pi^G + \Gamma^G, \tag{34}$$

where $\Pi^G$ and $\Gamma^G$ are defined in equations (31) and (33) above.

Analogously to Theorem 1, we can prove that the value of $\tau$ which minimizes $\Lambda^G$, $\tau^{min}$, is in general greater than zero but less than infinity. There is, however, one special case where $\tau^{min} = \infty$. That is the special parameter configuration for which fully discretionary monetary policy and nominal GNP targeting imply exactly the same response to unanticipated supply shocks. (Note that nominal GNP targeting and fully discretionary monetary policy always imply the same response to aggregate demand disturbances; in both cases the central bank tries to stabilize the price level.)

The remainder of this subsection is devoted to discussing how to prove the propositions discussed in the paragraph above. Because the theorem proved here is, for the most part, conceptually identical to Theorem 1, the reader is advised to skip to the next subsection unless he is specifically interested in the small differences in the two proofs.

Theorem 2. For $\tilde{n} \geqq \bar{n}$, $0 < \tau^{min} < \infty$ unless $\tilde{n} = \bar{n}$, in which case $\tau^{min} = 0$, or unless $\rho^G = \rho^D$ for all $\tau \geqq 0$, in which case $\tau^{min} = \infty$.

Proof: It is possible, but algebraically cumbersome, to prove Theorem 2 by exactly the same method as was used to prove Theorem 1. A simpler though less direct method is discussed below:

In Section IV, it was easy to establish by simple inspection the fact that $\partial\Gamma^I/\partial\varepsilon$ is zero at $\varepsilon = 0$, and strictly positive for $0 < \varepsilon < \infty$. Here, $\partial\Gamma^G/\partial\tau$ has the same properties (except in one special case), but it is somewhat more work to establish this fact by direct differentiation of (33a). We can alternatively make use of the following Lemma: For an arbitrary regime "A", define $\Gamma^A$ as in equation (18) and

$$dp_t^A = \rho_1^A z_t + \rho_2^A u_t + \rho_3^A v_t. \tag{35}$$

<u>Lemma</u>. $\Gamma^A$ is minimized at $(\rho_1^A, \rho_2^A, \rho_3^A)^{min} = (\rho^D, 0, 0)$. Furthermore, $\Gamma^A$ is a strictly increasing function in all elements of $(|\rho_1^A - \rho^D|, |\rho_2^A|, |\rho_3^A|)$.

Proof: Substitute $dp_t^A \equiv p_t^A - E_{t-1}(p_t^A)$ into $\Gamma^A$ as defined in equation (18):

$$\Gamma^A = E_{t-1}\{[z_t/\eta + \nu dp_t^A]^2 + \chi(dp_t^A)^2\}. \tag{36}$$

Substituting (35) into (36), and making use of the fact that z, u, and v are independent by assumption yields

$$\Gamma^A = \sigma_z^2[(1/\eta + \nu\rho_1^A)^2 + \chi(\rho_1^A)^2] + [\sigma_u^2(\rho_2^A)^2 + \sigma_v^2(\rho_3^A)^2](\nu^2 + \chi^2). \tag{37}$$

By inspection of (37), $(\rho_2^A)^{min} = (\rho_3^A)^{min} = 0$. Differentiating the term multiplied by $\sigma_z^2$ and equating to zero confirms that $(\rho_1^A)^{min} = \rho^D$ as defined in equation (19). Note also that $\Gamma^A$ is quadratic in all elements of $(\rho_1^A, \rho_2^A, \rho_3^A)$ and that the second order conditions for a minimum are satisfied. Therefore $\Gamma^A$ is strictly increasing in all elements of $(|\rho_1^A - \rho^D|, |\rho_2^A|, |\rho_3^A|)$. Q.E.D.

The proposition underlying the Lemma is entirely sensible. Under fully discretionary monetary policy, the central bank is not encumbered by any commitment. Therefore it is free to respond to unanticipated disturbances in a way which minimizes the social loss function. The more that response is distorted by a commitment to a particular monetary target, the greater $\Gamma^A$, the weighted average variance of employment and inflation around their respective market-determined levels.

From the Lemma, it follows immediately that $\partial\Gamma^G/\partial\tau$ is zero at $\tau = 0$. To prove that $\Gamma^G$ is strictly increasing in $\tau$, the Lemma tells us that we only need to prove that $\rho^G - \rho^D$ is monotonically increasing or monotonically decreasing in $\tau$. Using the definition of $\rho^G$ given in equation (32), and differentiating yields

$$\partial\rho^G/\partial\tau = 2\{-[(1+\theta)/\alpha](\nu^2 + \chi) + \nu(1+\theta)^2/\eta\}/ [\nu^2 + \tau(1+\theta)^2 + \chi]^2. \tag{38}$$

Examining the right-hand side of (38), the denominator is positive since $\tau \geqq 0$, and the numerator is a constant which can be negative, zero, or positive.[33/] When the numerator is strictly negative or positive, $\partial\rho^G/\partial\tau$ is strictly monotonic and therefore according to the Lemma, $\Gamma^G$ is strictly monotonically increasing. In the special case where the numerator of (38) is zero, $\rho^G$ is independent of $\tau$ and, since $\rho^G = \rho^D$ at $\tau = 0$, $\Gamma^G$ equals $\Gamma^D$ for all $\tau$. In this special case, $\tau^{min} = \infty$ since that is the level of $\tau$ which minimizes $\Pi^G$ in equation (31).

This completes our discussion of the proof of Theorem 2. All other aspects of the proof are the same as for Theorem 1.[34/]

Comparing Theorem 1 and Theorem 2, we see that the qualitative difference between them is that totally rigid nominal GNP targeting is optimal in one special case. For one special parameter configuration, fixing nominal GNP is exactly equivalent to what the central bank would do in the absence of any commitments. Further comparison of the two regimes will be postponed until Section VI.

V.3. Money-Supply Targeting

Given the widespread adoption of money-supply targeting over the past decade, this case should require little motivation.

The central bank's objective function under money-supply targeting is assumed to be given by:

$$M = (n-\tilde{n}')^2 + \chi(\pi-\tilde{\pi})^2 + \mu(m_t - \tilde{m}_t)^2, \quad \mu \geqq 0, \tag{39}$$

where $\tilde{m}_t = \tilde{\pi} + p_{t-1} + \lambda\bar{\pi}^M$; the "M" superscript denotes the money-supply targeting regime. Employing the macro-model of equations (9), (10) and (11), one can easily demonstrate that the target level of the money supply, $\tilde{m}_t$, is the level of $m_t$ which would be consistent with society's desired inflation rate $\tilde{\pi}$ provided that (a) there are no disturbances in period t, and (b) $\bar{\pi}^M$ is the expected inflation rate in period t.

The choice of the money supply target requires some comment. If the central bank is going to select the money supply as an intermediate target, it is certainly logical for it to commit to a rate of money growth consistent with the socially-desired inflation rate. But because the demand for real balances is interest-elastic, the central bank needs to form an estimate at time t-1 of $E_t(p_{t+1} - p_t)$ in order to select the appropriate value of $\tilde{m}_t$. The time consistent value of $E_t(p_{t+1} - p_t)$ depends, in turn, on the policy regime expected to prevail in period t+1. Here we have assumed that everyone expects the money-supply targeting regime to remain in place. This assumption is not crucial to the results below. However, the assumption that the central bank chooses its money supply target based on a realistic assessment of next period's expected inflation rate, is quite important.

Employing the macro-model of equations (9), (10) and (11), one can obtain the equation that will be used to substitute out for $m_t$ in (39):[35/]

$$m_t = -\lambda\bar{\pi}^M + \xi p_t + (1-\xi)\bar{w}_t^M - (\lambda/\delta)u_t + (\xi-1)z_t/\theta\alpha + v_t, \quad (40)$$

where $\xi \equiv 1 + \theta\lambda/\delta + \theta\phi$. Substituting into (39) using equations (40), (8) and (13) yields:

$$\begin{aligned} M = {} & [z_t/\eta + \nu(p_t - \bar{w}_t^M) - (\tilde{n}-\bar{n})]^2 + \chi[p_t - p_{t-1} + \tilde{\pi}]^2 \\ & + \mu[\xi p_t + (1-\xi)\bar{w}_t^M - (\lambda/\delta)u_t + (\xi-1)z_t/\theta\alpha + v_t - p_{t-1} - \tilde{\pi}]^2. \end{aligned} \quad (41)$$

Once more following the algorithm of Section III for obtaining a time-consistent rational expectations equilibrium, one obtains[36/]

$$\Pi^M = (\bar{\pi}^M - \tilde{\pi})^2 = \nu^2(\tilde{n}-\bar{n})^2/[\chi + \mu\xi]^2 \; ; \quad (42)$$

$$dp_t^M = \{-[(\nu/\eta) + \mu\xi(\xi-1)/\theta\alpha]z_t + \mu\xi\lambda u_t/\delta - \mu\xi v_t\}/[\nu^2 + \mu\xi^2 + \chi], \text{ or} \quad (43a)$$

$$dp_t^M = \rho_1^M z_t + \rho_2^M u_t + \rho_3^M v_t \; ; \quad (43b)$$

$$\Gamma^M = \{\sigma_z^2[T_1^2 + \chi T_2^2] + T_3[\sigma_v^2 + (\lambda/\delta)^2\sigma_u^2]\}/T_4^2 \text{ , where} \quad (44a)$$

$$T_1 = \{\mu\xi[\nu(1-\xi)/\theta\alpha + (\xi/\eta)] + \chi/\eta\} \; , \quad (44b)$$

$$T_2 = \nu/\eta + \mu\xi(\xi-1)/\theta\alpha \; , \quad (44c)$$

$$T_3 = (\nu^2 + \chi)(\mu\xi)^2 \text{ , and} \quad (44d)$$

$$T_4 = \nu^2 + \mu\xi^2 + \chi. \quad (44e)$$

The expected value of the social objective function under monetary targeting is

$$\Lambda^{M} = (\tilde{n}-\overline{n})^{2} + \chi\Pi^{M} + \Gamma^{M}. \tag{45}$$

Defining $\mu^{min}$ as the value of $\mu$ which minimizes $\Lambda^{M}$, one can prove

Theorem 3. For $\tilde{n} \geqq n$, $0 < \mu^{min} < \infty$, unless $\rho_1^{M} = \rho^{D}$ for all $\mu$ and $\sigma_u^2 = \sigma_v^2 = 0$, in which case $\mu^{min} = \infty$, or unless $\tilde{n} = \overline{n}$ in which case $\mu^{min} = 0$.

Proof: The method of proof is exactly the same as for Theorem 2 and makes use of the Lemma developed in Section V.2. Note that the condition for $\rho_1^{M} = \rho^{D}$ for all $\mu$ is that[37/]

$$(\nu^2 + \chi)(\xi^2 - \xi)/\theta\alpha = \nu\xi^2/\eta. \tag{46}$$

Under full information, rigid money-supply targeting ($\mu^{min} = \infty$) is only optimal in the unlikely event that there are no aggregate demand shocks and condition (46) holds. However, the assumption that there are no aggregate demand shocks can be relaxed when the monetary authorities have incomplete contemporaneous information.[38/] Appendix C discusses how to extend the present analysis to the incomplete information case.

## V.4. Nominal Interest Rate Targeting

The monetary authorities cannot systematically raise or lower the average value of the real interest rate in the rational expectations model employed in this paper. But it might seem reasonable for the central bank to try to bring down the inflation rate by committing itself to achieving a low nominal interest rate.[39/] Here we argue that this method of intermediate monetary targeting is counterproductive.

Consider the objective function:

$$R = (n-\tilde{n}') + \chi(\pi-\tilde{\pi})^2 + \upsilon(r-\tilde{r})^2, \quad 0 \leqq \upsilon < \infty. \tag{47}$$

where $\tilde{r}$ is the nominal interest rate target. Using equations (8), (9), (10) and (13), the objective function (47) can be written as:

$$R = [z_t/\eta + \nu(p_t - \overline{w}_t^R) - (\tilde{n}-\overline{n})] + \chi(p_t - p_{t-1} - \tilde{\pi})^2 \tag{48}$$
$$+ \upsilon[\overline{\pi}_{t+1}^A + (\theta/\delta)(\overline{w}_t^R - p_t) + (u_t - z_t/\alpha)/\delta - \tilde{r}]^2.$$

As we shall see, the value of $\overline{\pi}_{t+1}^A \equiv E_t(p_{t+1}^A) - p_t$ is crucial to the analysis; it depends on the monetary policy regime expected to prevail in period t+1. Following the algorithm of Section III for deriving the time consistent equilibrium, one obtains:[40/]

$$\overline{\pi}^R - \tilde{\pi} = [\nu(\tilde{n}-\overline{n}) + (\upsilon\theta/\delta)(\overline{r}_{t+1}^A - \tilde{r})]/\chi, \tag{49}$$

where $\overline{r}_{t+1}^A$ has been substituted in for $\overline{\pi}_{t+1}^A$. (Recall that, given the assumptions we have made about the parameters of the macro-model, the average real interest rate under any monetary regime is zero.)

A comparison of equations (17) and (49) reveals that $\overline{\pi}^R \gtreqless \overline{\pi}^D$ as $\tilde{r} \lesseqgtr \overline{r}_{t+1}^A$. In other words, suppose a central banker is appointed for one period, but this appointment has no effect on future central bank regimes. Suppose this central banker announces that he is going to try to bring interest rates below their trend rate, so that $\tilde{r} < \overline{r}_{t+1}^A$. Then, instead of falling, the expected inflation rate and expected nominal interest rate rise. They rise because wage setters recognize that <u>once wages are set</u>, the central bank can lower interest rates through money growth. While it is true that the central bank could try to bring down inflation by setting $\tilde{r}$ greater than $\overline{r}_{t+1}^A$, the fact that this would indeed cause the market-determined interest rate $\bar{r}_t$ to be less than $\bar{r}^A_{t+1}$ suggests a serious credibility problem. The central bank has to target high interest rates if it wants low interest rates.

The underlying problem is that, given $E_{t-1}(p_{t+1} - p_t)$, announcing a target for $r_t$ is, in fact, tantamount to targeting the real interest rate. For the regimes analyzed earlier, targeting succeeds in at least temporarily lowering the inflation rate regardless of how long the targeting regime is expected to last. This is no longer true when the nominal interest rate is used as a target.[41/]

The nominal interest rate would not appear to be a suitable instrument for precommitment. This conclusion, of course, does not imply that the interest rate should not be used as a information variable in setting monetary policy, as in Poole (1970).

## VI. On Comparing Alternative Targeting Regimes

We have shown how the framework developed in this paper can be used to analyze a variety of widely-discussed intermediate monetary targeting regimes. The general result is that "flexible" intermediate monetary targeting almost always dominates both rigid targeting and fully discretionary monetary policy. The reader is unlikely to be stunned if he is now informed that the choice of intermediate monetary target (between nominal GNP, the money supply, etc.) should depend on all the parameters of the model as well as on the relative sizes of the disturbances; Poole's (1970) result is now standard.

The qualitative properties of the various intermediate targeting regimes are well known: Money-supply targeting works poorly when the monetary authorities have information on how aggregate demand shocks are affecting the price level. Inflation-rate targeting works poorly when supply shocks are large, etc. And we have demonstrated that while the nominal interest rate may be a useful information feedback variable, it is not useful for the monetary authorities to precommit to achieving a low nominal interest rate.

Algebraic comparisons of the alternative regimes, however, are complicated considerably by the fact that the optimally flexible regimes of Section IV and V do not have tractable closed-form solutions. This would not be such a problem if the following two-step procedure were entirely valid: First, calculate the values of the social objective function attained when each regime is employed rigidly; i.e., when the weight on the target goes to infinity in the central bank's objective function. The resulting closed-form expressions are reasonably easy to compare; (see Appendix F).[42/] Select the target which performs best. The second step would then be to find the optimal weight to place on the selected target. (This second step would require estimates of the parameters of the model.) Unfortunately, this procedure is not fool-proof because of the possibility of "rank-reversal." In Appendix D, we demonstrate that a comparison of rigid targeting regimes does not necessarily yield the same target rankings as a comparison of optimally flexible regimes. The problem of rank reversal is mitigated by the fact that it is reasonably easy to use a computer to quantitatively compare alternative optimally-flexible targeting regimes for any given set of parameter values.[43/]

## VII. Conclusions

This paper extends the standard rational expectations cum wage contracting model to an environment in which the central bank has explicit motivation for precommitting to an intermediate monetary target. We have demonstrated how this framework can be used to analyze money supply targeting, nominal GNP targeting, inflation-rate targeting and nominal interest rate targeting. The analysis can easily be extended to consider other targets, such as the exchange rate, or to encompass multiple intermediate targets.

A major conclusion of the analysis is that, except in certain special cases, the type of flexible monetary targeting used by many industrial-country central banks since the mid-seventies is preferable to both rigid commitment and to fully discretionary policy. The optimal weight to place on an intermediate monetary target (relative to direct stabilization goals) trades off the gain from a lower average inflation rate against the cost of reducing the central bank's capacity to stabilize employment and inflation around their market-determined average rates. One would expect that the best monetary target would be the one most highly correlated with the central bank's ultimate objective function.[44/] But while this is a useful rule of thumb, the situation is actually somewhat more complicated. If we compare how each of the targets would work if used rigidly, we do not necessarily get the same ranking as when we compare how each target works when the central bank gives it an optimal weight (relative to direct social objectives.)[45/]

Viewed from a broader perspective, the analysis of this paper demonstrates conditions under which society should appoint a "conservative" (but not too conservative) agent to head the central bank. In the presence of a labor market distortion, society can actually make itself better off by having a central bank which does not directly attempt to maximize the social welfare function.

Appendix A: Notation

$y$ = logarithm of output

$n$ = logarithm of employment

$p$ = logarithm of the price level

$w$ = logarithm of the wage rate

$m$ = logarithm of the nominal money supply

$r$ = level of the interest rate

$z$ = productivity (supply) disturbance

$v$ = money demand disturbance

$u$ = goods market demand disturbance

$\sigma_z^2$, $\sigma_u^2$, $\sigma_v^2$ = The variances of the three serially-uncorrelated, zero-mean disturbances.

$\bar{n}$ = level of employment targeted by wage setters

$\bar{n}'$ = level of employment which wage setters would target if they knew the aggregate productivity disturbance.

$\tilde{n}$ = average level of employment targeted by the monetary authorities.

$\tilde{n}'$ = level of employment targeted by the monetary authorities after receiving information about the productivity disturbance.

$\pi$ = inflation rate

$\tilde{\pi}$ = socially optimal inflation rate

$E_{t-1}$ = expectations operator, based on period t-1 information.

$dp_t \equiv p_t - E_{t-1}(p_t)$

$\Lambda$ = Social loss function

$\Gamma$ = A weighted-average of deviations of inflation and employment around their respective market-determined values; the weight is the same as in the social loss function.

$\Pi$ = squared deviation of the mean market-determined inflation rate from the socially-optimal inflation rate.

D = fully discretionary monetary policy regime

I = inflation-rate/price-level targeting regime

G = nominal GNP targeting regime

M = money-supply targeting regime

R = interest-rate targeting regime

A = arbitrary regime: D,I,G,R, or M.

A "$-$" above a variable denotes its average "market-determined" value and a "$\sim$" denotes its average socially-preferred value; a "t" subscript denotes the time t value of a given variable; the value of a variable corresponding to a given regime is superscripted by the letter corresponding to that regime (D,I,G,M,R or A).

$\varepsilon$, $\tau$, $\mu$, $\upsilon$ = weights placed on the intermediate monetary targets relative to direct employment and inflation-rate stabilization objectives in the central bank's loss function in regimes I,G,M, and R respectively.

$\omega$ = real wage elasticity of labor supply

$\alpha$ = coefficient on capital in the Cobb-Douglass production function

$\beta$ = wage indexation parameter

$-\delta$ = real interest rate semi-elasticity of aggregate demand

$-\lambda$ = nominal interest rate semi-elasticity of real money demand

$\phi$ = real income elasticity of real money demand

$\chi$ = the relative weight on squared inflation deviations relative to squared employment deviations in the social loss function.

The following coefficient definitions are used:

$\eta \equiv \alpha(1-\alpha\omega)$

$\nu \equiv (1-\beta)/\alpha$

$\theta \equiv (1-\beta)(1-\alpha)/\alpha$

$\xi \equiv 1 + \theta\lambda/\delta + \theta\phi$

All parameters are defined as positive numbers.

Appendix B: Proofs of Minor Propositions which are of Computational Convenience in Finding the Optimal Weight to Assign a Target.

In this Appendix, we prove the following three propositions:

Proposition B.1. $\varepsilon^{min}$ is the unique real positive root of $\partial\Lambda^I/\partial\varepsilon = 0$.

Proposition B.2. $\partial\Lambda^G/\partial\tau$ has at most three positive real roots; a sufficient condition for $\tau^{min}$ to be the only positive real root is that $(1-\beta)/\alpha(1-\alpha) > \chi$.

Proposition B.3. $\partial\Lambda^M/\partial\mu = 0$ has at most three positive real roots; a sufficient condition for $\mu^{min}$ to be the sole real positive root is that $(1-\beta)/\alpha(1-\alpha)[\phi + \lambda/\delta] > \chi$.

These three propositions are of no qualitative significance for any of the results in the text. They are relevant only for the choice of computational algorithms available to numerically calculate $\varepsilon^{min}$, $\tau^{min}$, and $\mu^{min}$ for specific values of the model's parameters. When the polynomial equations specified in the propositions have only one real positive root, one can use a local gradient algorithm to find the global minimum.(Since the objective function is then concave in the index variable.) Otherwise, it is necessary (but not much more difficult) to calculate all the positive roots of the polynomial and check to see which one is the minimum.

The proofs employ Descartes' Law of Signs, which states that the number of positive real roots of a polynomial is at most equal to the number of times that the coefficients of the polynomial alternate in sign.

Proof of Proposition B.1. From equations (26),

$$\partial\Lambda^{I}/\partial\varepsilon = [b\varepsilon(\chi + \varepsilon)^3 - a(h + \varepsilon)^3]/(\chi + \varepsilon)^3(h + \varepsilon)^3, \tag{B1}$$

where $b \equiv \sigma_z^2(\nu/\eta)^2$, $a \equiv \chi\nu^2(\tilde{n}-\bar{n})^2$, and $h \equiv (\nu^2 + \chi)$.
A necessary and sufficient condition for $\partial\Lambda^{I}/\partial\varepsilon$ to equal zero is for the numerator of (B1) to equal zero:

$$\varepsilon^4 + (3\chi - a/b)\varepsilon^3 + (3\chi^2 - 3ha/b)\varepsilon^2 + (\chi^3 - 3h^2a/b)\varepsilon - h^3a/b = 0. \tag{B2}$$

The fact that the first and last terms of (B2) are of opposite signs immediately implies that the coefficients of the polynomial alternate in sign at most three times. And in fact, because $h = (\chi + \nu^2) > \chi$, we can deduce that there is at most one sign change. (Since if $a/b \leqq \chi^2/h$, then $a/b < 3\chi > \chi^2/h$ and therefore the first three terms are positive. Since the final term is negative, there can only be one sign change. Or if instead $a/b > \chi^2/h$, then the fact that $h > \chi$ implies that $a/b > \chi^3/3h^2$ and therefore the last three terms are negative. Since the first term is positive, there can again be only one sign change.

Proof of Proposition B.2.

The fact that $\partial\Gamma^G/\partial\tau$ is of the form $b''\varepsilon/[\chi + \nu^2 + \tau(1+\theta)^2]^3$, with $b'' > 0$, can be confirmed by direct differentiation of equation (33) or can be deduced by noting that (a) any expression of the form $(d + e + f\tau^2)(h + j\tau)^{-2}$ has a derivative of the form $(q + r\tau)(h + j\tau)^{-3}$

and (b) The Lemma of Section V.2 implies that $\partial\Gamma^G/\partial\tau > 0$ for $0 < \tau < \infty$, and equals zero for $\tau = 0$. It is convenient to redefine $b' \equiv b''/(1+\theta)^3$ and $h' \equiv (\chi + \nu^2)/(1+\theta)$ so that $\partial\Gamma^G/\partial\tau$ is of the form $b'\varepsilon/[h' + \tau(1+\theta)]^3$. The fact that $\partial\Pi^G/\partial\tau$ is of the form $-a'/[\chi + \tau(1+\theta)]^3$ is confirmed by direct differentiation of (31).

The above discussion implies that $\partial\Lambda^G/\partial\tau = 0$ has exactly the form of equation (B2) with $\varepsilon$ replaced by $\tau(1+\theta)$, a/b replaced by a'/b' and h replaced by h'. (Note that $\theta > 0$.) Therefore $\partial\Lambda^G/\partial\tau$ has at most three positive roots and definitely has only one positive root if $h' = (\chi + \nu^2)/(1+\theta) \geqq \chi$. The foregoing condition can be rewritten, using the definitions of $\nu$ and $\theta$, as $(1-\beta)/\alpha(1-\alpha) \geqq \chi$.

Proof of Proposition B.3.

The same as the proof of Proposition B.2.

Appendix C: The Incomplete Contemporaneous Information Case.

In the text, the monetary authorities and investors are assumed to have full knowledge of the period t disturbances when making their period t money supply and investment decisions. It is extremely straightforward, though in some cases algebraically cumbersome, to extend the analysis to the case of incomplete contemporaneous information.[46/] All of the theorems can be generalized, but the ranking of regimes does depend on the characteristics of the information set.

To sketch an example, consider the case where the monetary authorities and investors observe price level and interest rate data contemporaneously, but only observe real output with a one-period lag. They can use their knowledge of the price level and the interest rate to make imperfect estimates of the three contemporaneous disturbance terms. Canzoneri et al. (1982) develop an algorithm for calculating these estimates and prove that the disturbance term prediction errors are independent of systematic monetary policy.[47/]

Using these results, it is possible to decompose the expected value of the social loss function $\Lambda_t$ under an arbitrary monetary policy regime "A" as follows (compare with equation (18) of the text):

$$E_{t-1}(\Lambda_t) = (\tilde{n}-\bar{n})^2 + (\sigma_z^2 - \sigma_{dz}^2)/\eta^2 + \chi\Pi^A + d\Gamma^A, \tag{18'}$$

where $\sigma_{dz}^2 = E_{t-1}(dz_t)^2$; $dz_t \equiv E_{t-1}^+(z_t) - E_{t-1}(z_t)$; $E_{t-1}^+$ stands for the expectations operator conditional on all period t-1 information plus the period t interest rate and price level. $\Pi^A$ is defined as in equation (18), and $d\Gamma^A$ is the same as $\Gamma^A$ in equation (18), except that $z_t$ is replaced by $dz_t$. In deriving (18'), we have used the fact that $dz_t$ is independent of $(z_t - dz_t)$ under rational expectations. The first two terms on the right-hand side of (18') are independent of monetary policy, so that monetary policy regimes should be compared on the basis of $\Pi^A$ and $d\Gamma^A$ only. For the information structure assumed in this appendix, all the results and theorems of the text on inflation-rate and nominal GNP targeting go through with the trivial modification that $z_t$ and $\sigma_z^2$ are replaced everywhere by $dz_t$ and $\sigma_{dz}^2$. While straightforward in principle, it is more work to modify the results on money supply targeting. The reason is that although $u_t$, $v_t$ and $z_t$ are uncorrelated by assumption, $du_t$, $dv_t$ and $dz_t$ are in general correlated.

Appendix D: Rank Reversal

This appendix gives both algebraic and numerical examples of "rank reversal." This is the phenomenon where the rankings of two targets (say, the inflation rate and nominal GNP) differ, depending on whether the intermediate targets are to be given infinite or optimal weights in the central bank's objective function.

To develop an algebraic example, we shall consider the special case where $\beta = \omega = 0$. In this case, equations (24) and (33) become

$$\Gamma^I = (\sigma_z^2/\alpha^2)[(\chi + \varepsilon)^2 + \chi/\alpha^2]/(1/\alpha^2 + \chi + \varepsilon)^2, \tag{24'}$$

$$\Gamma^G = (\sigma_z^2/\alpha^2)[\chi^2 + \chi(1+\tau)^2/\alpha^2]/(1/\alpha^2 + \tau/\alpha^2 + \chi)^2. \tag{33'}$$

It is easy to check that as $\varepsilon \to \infty$, $\Gamma^I \to \sigma_z^2/\alpha^2$; and as $\tau \to \infty$, $\Gamma^G \to \chi\sigma_z^2$. Using expressions (22) and (31) of the text, we see that $\Pi^I = \Pi^G$ when $\varepsilon = \tau/\alpha$. (Note that $1+\theta = 1/\alpha$ when $\beta = \omega = 0$). So by setting $\varepsilon = \tau/\alpha$ in (24'), we can confirm the possibility of rank reversal if we can show that $\Gamma^G - \Gamma^I$ changes signs as $\tau$ varies. This is sufficient because we know that we can generate any value of $\tau^{min}$ by varying the ratio of $(\tilde{n}-\bar{n})^2/\sigma_z^2$.

After dividing out the common factor $\sigma_z^2/\alpha^2$ from both (24') and (33'), form $\Gamma^G - \Gamma^I$. Note that the denominator of both expressions is positive. After multiplying the numerator of $\Gamma^G$ by the denominator of $\Gamma^I$, and visa versa, and after massive cancellation, we find that $\Gamma^G \gtreqless \Gamma^I$ as

$$\chi[\chi^2 + \chi/\alpha^2 + 2\tau\chi/\alpha + \tau^2/\alpha^2] \gtreqless 1/\alpha^4[1 + \chi\alpha^2 + 2\tau + \tau^2]. \tag{D1}$$

Both sides of (D1) are polynomials in $\tau$; we see that as $\tau$ becomes large, the inequality approaches $\chi \gtreqless 1/\alpha^2$. This is exactly what one would expect given the limits of (24') and (33') as $\varepsilon$ and $\tau$ go to infinity.

For $\chi > 1/\alpha^2$, a term by term comparison reveals that $\Gamma^G > \Gamma^I$ for all $\tau$.[48] However, for $\chi = 1/\alpha^2 - \Delta$, it is possible to choose $\Delta$ sufficiently small so that the first three terms on the left-hand side of (D1) are all larger than the first three terms on the right-hand side, although the term in $\tau^2$ on the right-hand side is larger than the term in $\tau^2$ on the left-hand side.[49]

This implies that although $\Gamma^G < \Gamma^I$ for large $\tau$, $\Gamma^G > \Gamma^I$ for small $\tau$. (For small enough $\tau$, the $\tau^2$ terms are almost irrelevant.)

Given propositions B.1, B.2 and B.3, it is easy to use a computer to find numerical examples of rank reversal. Rank reversal appears to occur frequently, <u>but</u> cases where it is very wrong to choose the target that would be best for a rigid regime are difficult to find.

Two numerical examples:

(1) Set $\alpha = .25$, $\tilde{n}-\bar{n} = .02$, $\chi = 1$, $\sigma_z^2 = .020$, $\omega = 0$ and $\beta = 0$. Then $\Pi^D = 6.4$, $\Gamma^D = 12.0$, $\Lambda^D = 18.4$; $\Pi^I = 1.4$, $\Gamma^I = 12.8$, $\Lambda^I = 14.2$, for $\varepsilon^{min} = 2.16$; $\Pi^G = 1.4$, $\Gamma^G = 13.0$, $\Lambda^G = 14.4$, for $\tau^{min} = .287$, where all numbers have been multiplied by 1000 except for $\varepsilon$ and $\tau$; the terms in $(\tilde{n}-\bar{n})^2$, which are independent of regime choice, are ignored. Note that $(\Lambda^I)^{min} < (\Lambda^G)^{min}$. However, $\Lambda^I|_{\varepsilon=\infty} = 200 > 32.8 = \Lambda^G|_{\tau=\infty}$.

(2) Set $\alpha = .25$, $\tilde{n}-\bar{n} = .03$, $\beta = 0$, $\chi = 1$, $\sigma_v^2 = .1$, $\sigma_u^2 = .00001$, $\sigma_z^2 = .02$, $\omega = 1$, $\lambda = 5$, $\delta = 3$, $\phi = 1$. Then $\Pi^D = 1.4$, $\Gamma^D = 1.2$, $\Lambda^D = 2.6$; $\Pi^I = .2$, $\Gamma^I = 1.3$, $\Lambda^I = 1.6$, $\varepsilon^{min} = 2.6$; $\Pi^M = .7$, $\Gamma^M = 1.4$, $\Lambda^M = 2.1$, $\mu^{min} = .043$, where all terms except $\varepsilon$ and $\mu$ have been multiplied by 100, and again the $(\bar{n}-\tilde{n})^2$ component of $\Lambda$ is ignored. Note that $(\Lambda^I)^{min} < (\Lambda^M)^{min}$. But $20.4 = \Lambda^I|_{\varepsilon=\infty} > \Lambda^M|_{\mu=\infty} = 9.6$.

Appendix E: <u>An Analysis of the Inefficient Form of Nominal GNP Targeting</u>.

In Section V.2 of the text, we discussed a potential implementation problem with nominal GNP targeting. A nominal GNP target has two components: price and output. Clearly there should be no problem announcing a price level consistent with the desired rate of inflation $\tilde{\pi}$. But it may be very difficult for the central bank to admit that its output target is consistent with the distorted market-determined level of employment $\bar{n}$ rather than the higher socially preferred level $\tilde{n}$. Announcing a nominal GNP target consistent with $\tilde{y} = f(\tilde{n})$ will have no effect on the average level of employment and will only serve to raise the average level of inflation. But, as we shall demonstrate below, there are cases where even the suboptimal form of nominal GNP targeting dominates money-supply and inflation-rate targeting.

A formal analysis of the case where the central bank targets the "suboptimal" $(\tilde{y} + p_{t-1} + \tilde{\pi})$ is essentially the same as the case considered in section V.2 of the text, where the target is $(\bar{y} + p_{t-1} + \tilde{\pi})$. But there are a few differences. Equation (29) becomes

$$Y = (n-\tilde{n}')^2 + \chi(\pi-\tilde{\pi})^2 + \zeta(y_t + p_t - \tilde{y} - p_{t-1} - \tilde{\pi})^2; \qquad (29')$$

equation (30) is modified similarly. Equations (32) and (33) are the same; $dp_t^G = dp_t^Y$, $\Gamma^G = \Gamma^Y$. But equation (31) becomes

$$\Pi^Y = (\bar{\pi}^Y - \tilde{\pi})^2 = [\nu + \zeta(1+\theta)(1-\alpha)]^2(\tilde{n}-\bar{n})^2/[\zeta(1+\theta) + \chi]^2. \qquad (31')$$

$\Pi^Y$ is always greater than $\Pi^G$; therefore $\Lambda^Y$ is always greater than $\Lambda^G$. In fact, the suboptimal form of nominal GNP targeting will not necessarily even produce a lower (average) inflation rate than fully discretionary monetary policy; $\Pi^Y \gtreqless \Pi^D$ as $(1-\alpha) \gtreqless \nu/\chi$. But it is not difficult to produce reasonable numerical examples where $\Lambda^G < \Lambda^Y < \Lambda^M$, $\Lambda^I$, $\Lambda^D$:

Example: Set $\alpha = .25$, $\tilde{n}-\bar{n} = .02$, $\beta = 0$, $\chi = 1$, $\sigma_v^2 = .01$, $\sigma_u^2 = .00001$, $\sigma_z^2 = .005$, $\omega = 0$, $\lambda = 5$, $\phi = 1$, $\delta = 3$. Then, at their respective minima, $\Pi^D = 6.4$, $\Gamma^D = 4.7$, $\Lambda^D = 11.1$; $\Pi^I = .9$, $\Gamma^I = 5.3$, $\Lambda^I = 6.2$, for $\varepsilon^{min} = 1.68$; $\Pi^Y = .2$, $\Gamma^Y = 5.0$, $\Lambda^Y = 5.2$, for $\zeta^{min} = 53$; $\Pi^G = .05$, $\Gamma^G = 4.85$, $\Lambda^G = 4.9$, for $\tau^{min} = 2.67$; $\Pi^M = 1.2$, $\Gamma^M = 5.9$, $\Lambda^M = 7.1$, for $\mu^{min} = .14$. All the numbers above are multiplied by $10^3$ except for $\varepsilon$, $\tau$, $\zeta$, and $\mu$; the common term $(\tilde{n}-\bar{n})^2$ is omitted from all the $\Lambda$'s.

Because the average inflation rate $\bar{\pi}$ does not converge to $\tilde{\pi}$ as $\zeta \to \infty$ under the suboptimal version of nominal GNP targeting, the argument used below equation (27) does not go through; it is possible for $\zeta^{min}$ to equal infinity under much weaker conditions than were necessary for $\tau^{min} = \infty$. For the special case, $\beta = \omega = 0$, it is possible to prove that $\zeta^{min} = \infty$ if

$$\chi^2\alpha^2\sigma_z^2 < [-(1-\alpha)^2\chi\alpha + (1-\alpha)](\tilde{n}-\bar{n})^2. \tag{E1}$$

(Note that when $\beta = 0$, $\chi(1-\alpha)\alpha < 1$ is the necessary condition for suboptimal nominal GNP targeting to be superior to fully discretionary monetary policy.) Condition (E1) implies that $\lim_{\zeta\to\infty} \chi[\partial\Pi^Y/\partial\zeta]/[\partial\Gamma^Y/\partial\zeta] > 1$.

Appendix F: <u>A Comparison of Rigid Targeting Regimes</u>

In general, it is difficult to derive tractable closed-form expressions for the social loss function under any of the monetary targeting regimes considered in Sections IV and V, provided that the targets are assigned optimal weights. But it is easy and illustrative to compare regimes when the central bank places an infinite weight on its target, and pays no direct attention to the social objective function, $\Lambda$.

Equations (F1) through (F4) give the expected value of the social loss function (12) under fully discretionary monetary policy, and rigid inflation-rate, nominal-GNP and money-supply targeting:[50/]

$$\Lambda^D = (\sigma_z^2/\eta^2)[\chi / (\nu^2 + \chi)] + \chi[\nu(\tilde{n}-\bar{n})/\chi]^2, \tag{F1}$$

$$\Lambda^I|_{\varepsilon=\infty} = \sigma_z^2/\eta^2, \tag{F2}$$

$$\Lambda^G|_{\tau=\infty} = \sigma_z^2/\eta^2 + \sigma_z^2[\nu^2 + \chi - 2\nu(1-\beta+\alpha\beta)/\eta]/(1 - \beta + \alpha\beta)^2, \tag{F3}$$

$$\Lambda^M|_{\mu=\infty} = \sigma_z^2/\eta^2 + \sigma_z^2[(\nu^2 + \chi)J^2 - 2\nu J/\eta] + (\nu^2 + \chi)[\sigma_v^2 + (\lambda/\delta)^2\sigma_u^2]/\xi^2, \tag{F4}$$

where $J \equiv (\xi-1)/\theta\alpha\xi$.

In (F2)-(F4), rigid targeting reduces the average rate of inflation, $\bar{\pi}$, to $\tilde{\pi}$; the common term $(\tilde{n}-\bar{n})^2$ has been omitted from all four equations, as it is independent of monetary policy and therefore does not affect comparisons across regimes.

A comparison of equation (F1) with equations (F2)-(F4) reveals that the larger the size of the labor market distortion, $\tilde{n}-\bar{n}$, and the smaller the variance of the supply shock, $\sigma_z^2$, the more likely it is that any given form of rigid targeting will be superior to fully discretionary monetary policy. Note also that the higher the variances of the two aggregate demand shocks, $\sigma_u^2$ and $\sigma_v^2$, the worse money supply targeting works relative to other regimes. The other regimes allow the monetary authorities to fully offset the combined effect of aggregate demand shocks on the price level; full neutralization is optimal because there is no trade-off between price stabilization and employment stabilization in the face of an aggregate demand shock.

In proving Theorem 2, we derived a condition under which the best form of nominal GNP targeting is rigid nominal GNP targeting. In this special case, rigid nominal GNP targeting brings the inflation rate down to its optimal level without any sacrifice in stabilization relative to fully discretionary monetary policy. Thus, nominal GNP targeting is clearly better than other forms of targeting for at least one special parameter configuration. Analogously, in proving Theorem 3, we found one special case where rigid money-supply targeting is optimal. (The necessary condition is given by equation (46); $\sigma_v^2 = \sigma_u^2 = 0$ is also required.) Examination of one further special case allows us to definitely conclude that any of the four regimes given by equations (F1)-(F4) may be best, depending on the structure of the economy. Setting the wage indexation parameter $\beta$ and real wage elasticity of labor supply $\omega$ equal to zero, (F2) and (F3) simplify to

$$\Lambda^I|_{\varepsilon=\infty} = \sigma_z^2/\alpha^2, \text{ and} \tag{F2'}$$

$$\Lambda^G|_{\tau=\infty} = \chi\sigma_z^2. \tag{F3'}$$

Inspection of (F2') and (F3') reveals that $\Lambda^I|_{\varepsilon=\infty} \gtreqless \Lambda^G|_{\tau=\infty}$ as $1/\alpha^2 \gtreqless \chi$. Thus when $\chi$ is very large, so that society places a great weight on inflation stabilization relative to employment stabilization, rigid inflation-rate targeting is superior to rigid nominal GNP targeting.[51]

Incidentally, the reason that (F3') gives such a simple expression in comparison with (F3), is that rigid nominal GNP targeting is exactly equivalent to rigid employment targeting when $\beta = \omega = 0$.[52]

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## Footnotes

* Board of Governors of the Federal Reserve System. Part of the work on this paper was completed while the author was on leave at the Research Department of the International Monetary Fund. The views expressed here are the author's own, and should not be interpreted as the official views of either institution. Mathew Canzoneri, David Folkerts-Landau, Maurice Obstfeld, Alessandro Penati, Franco Spinelli, Lawrence Summers and Clifford Wymer provided helpful comments on an earlier draft.

1/ See, for example, Sargent and Wallace (1976), and Fischer (1977).

2/ For a discussion of the early monetary targeting experience, see Organization for Economic Co-operation and Development (1979). For a discussion of more recent experiences, See Howard and Johnson (1982).

3/ The time consistency component of the analysis is based heavily on Barro and Gordon (1983), who in turn have extended the work of Kydland and Prescott (1977). An example of a standard rational expectations *cum* wage contracting analysis is Canzoneri, Henderson and Rogoff (1982), whose work is, in part, an update of Poole (1970).

4/ Fischer (1977), Phelps and Taylor (1977), Canzoneri (1980) and Taylor (1980) demonstrate that monetary policy can stabilize the level of output in the presence of nominal wage contracts, even when private agents have rational expectations.

5/ Section V contains a discussion of the underlying incentives which might work to make such a commitment credible (to a degree).

6/ The aggregate demand specification is the same as in Canzoneri, Henderson and Rogoff (1982). The aggregate supply specification is based on Gray (1976) and Fischer (1977).

7/ The model does incorporate price-level indexation.

8/ Barro and Gordon (1983) mention these as two possible factors which could distort the labor supply curve.

9/ Modeling the labor supply distortion as a shift in the intercept of the labor supply curve is analytically convenient; allowing the slope coefficient to change instead should not require altering any of the theorems proved later in an important way.

10/ Note that wages are indexed to the contemporaneous price level as in Gray (1976). The type of wage indexation considered here is not of the "catch-up" variety. The wage indexation parameter $\beta$ will be treated as a fixed parameter throughout the analysis. Modifying this assumption should not alter the main conclusions though it could, in principle, modify the comparison of regimes in Section VI. We shall return to this issue below.

11/ This is the first of many times throughout the paper where use is made of the fact that certainty equivalence holds when the loss function is quadratic; here the base nominal wage is set in exactly the same way as it would be if the price level had zero variance around its expected value. See Sargent (1979).

12/ Investors have "rational expectations." They choose $E_t(p_{t+1})$ to minimize $E_t[p_{t+1} - E_t(p_{t+1})]^2$. None of the main results would be affected in any important way if investors instead based their expectations on t-1 information. The assumption employed in the text implies that the monetary authorities have no information advantage whatsoever over investors.

13/ The social objective function used here is a slight generalization of the one employed by Kydland and Prescott (1977) in their deterministic model. Kydland and Prescott demonstrate how activist monetary policy can result in high average levels of inflation. Barro and Gordon (1983) also employ a similar specification. In contrast to the present paper, Barro and Gordon prefer not to incorporate Gray-Fischer-Taylor type nominal wage contracts into their model. For this reason, Barro and Gordon choose not to emphasize the potential use of activist monetary policy to offset random disturbances.

14/ The analysis below would be exactly the same if the social loss function were instead given by the present discounted value functional $\Lambda_t' = \sum_{s=t}^{\infty}[(n_s - \tilde{n}_s')^2 + \chi(\pi_s - \tilde{\pi})^2](1 + R)^{t-s}$.

15/ The value of $\tilde{n}_t'$ may be derived from equations (7) and (13). Note that $\tilde{n}_t' - \bar{n}_t'$ is independent of the productivity shock $z_t$.

16/ Fischer and Modigliani (1978) catalogue the economic costs of both anticipated and unanticipated inflation.

17/ See, for example, Phelps (1973).

18/ Throughout the text, the monetary authorities and investors are assumed to have complete contemporaneous information. The generalization of this analysis to the case of incomplete contemporaneous information is for the most part straightforward; see Appendix C.

19/ Kydland and Prescott (1977) develop the notion of time consistency in a macroeconomic context.

20/ Gray (1976) demonstrates that full price level indexation is suboptimal in the presence of supply shocks.

21/ Note that the individual groups of wage setters are, in fact, concerned with the aggregate inflation rate and not just with the level of employment at their own firms. But because each individual firm has only a small impact on the aggregate price level, an individual group of wage setters has little incentive to take into account the effects of their contract on the aggregate inflation rate.

22/ $p_t^D$ is found by setting $\partial D_t / \partial p_t = 0$. Note that the second order conditions for a mininum are met; given the quadratic form of D, the minimum is global.

23/ The reader is again reminded that quadratic form of (14) implies certainty equivalence.

24/ Assuming that the monetary authorities are expected to minimize the same objective function (14) in all future periods, the same algorithm can be applied repeatedly to derive a time consistent path for all future prices:

$$E_{t-1}(p_{t+s}^D) = \nu(\tilde{n}-\bar{n})(s+1)/\chi + (s+1)\tilde{\pi} + p_{t-1}, \quad s \geqq 0. \tag{f1}$$

Note that in the text, we treat the price level as if the monetary authorities controlled it directly, ignoring the fact that the central bank only directly controls the money supply. The anticipated future path of the money supply consistent with the time consistent path of prices (f1) can be found by working through the macro-model of equations (9), (10) and(11), and by imposing saddlepath stability:

$$E_{t-1}(m_t^D) = p_{t-1} + (1-\lambda)[\nu(\tilde{n}-\bar{n})/\chi + \tilde{\pi}], \tag{f2}$$

$$E_{t-1}(m_{t+s}^D) = E_{t-1}(m_t^D) + s[\nu(\tilde{n}-\bar{n})/\chi + \tilde{\pi}],\ s \geqq 0. \tag{f3}$$

(Obstfeld and Rogoff (1983) provide microeconomic justification for ruling out speculative hyperinflations and hyperdeflations unrelated to the underlying fundamentals.)

25/ Note that the time-consistent rate of inflation is a decreasing function of the indexation parameter $\beta$, since $\nu \equiv (1-\beta)/\alpha$. Obviously this result would not necessarily hold if indexation were of the "catch-up" variety. Also, one can imagine models in which indexation raises the time consistent rate of inflation. Suppose that the indexation scheme involved giving all workers the same absolute rather than percentage raise. (An example of such a scheme is the scala mobile in Italy.) Then a government interested in income redistribution might have a stronger incentive to inflate than it would in the absence of indexation.

26/ The assumption that $\varepsilon > -\chi$ implies that the agent gets disutility from inflation. We shall see in the next section how this regime may also be thought of as one in which the central bank targets the inflation rate.

27/ We want to evaluate the regime on the basis of the expected value of the social loss function, not the expected value of the central banker's loss function.

28/ Although it is extremely difficult to write down a closed-form solution for $\varepsilon^{min}$, we are able to prove in Appendix B that $\varepsilon^{min}$ is the unique real positive root of $\partial\Lambda^I/\partial\varepsilon = 0$, so that $\Lambda^I$ is concave in $\varepsilon$.

29/ Price-level targeting and inflation-rate targeting are equivalent here, since $p_{t-1}$ is known at the time the central bank commits itself to achieving a target for $p_t - p_{t-1}$.

30/ The standard literature on intermediate monetary targeting (Poole (1970), Friedman (1975) and Canzoneri *et al*. (1982), for example) takes limited information rather than credibility as the rationale for intermediate monetary targeting. The present model can incorporate both rationales; see Appendix C.

31/ The nominal GNP target embodied in the final term of equation (29) is consistent with the socially desired rate of inflation and the average market-determined level of employment. However, it may be politically difficult for the central bank to announce a nominal GNP target consistent with a level of employment lower than the level which would prevail in the absence of labor-market distortions. The case where $\bar{y}$ is replaced by $\tilde{y} = f(\tilde{n})$ will be discussed in Appendix E.

32/ To obtain equations (31) - (33), first differentiate equation (30) with respect to $p_t$; $p_t^G$ solves $\partial G_t/\partial p_t = 0$. Take t-1 expectations across the equation for $p_t^G$, and note that $\bar{w}_t^G = p_t^G$ in a time consistent equilibrium. (We have constructed the model so that wage setters' target (for the logarithm of the) real wage is zero.) Equations (31) and (32) follow immediately. Equation (33) is obtained by plugging (32) into the expression for $\Gamma^A$ in (18).

One check on the algebra is to consider the case of rigid nominal GNP targeting (see Appendix F). Equation (9') implies that

$p_t - E_{t-1}(p_t) = -z_t/\alpha(1+\theta)$ under rigid nominal GNP targeting (since u and v shocks are fully offset). It is easy to confirm that by plugging the above expression into $\Gamma^A$ in equation (18), one obtains the same expression as by taking the limit of (33) as $\tau \rightarrow \infty$.

33/ In the special case where $\omega = 0$, the numerator of (38) is negative, zero or positive as $\chi \gtreqless \nu\beta$. It is possible to show that the numerator of (38) is always negative when $\beta = 0$. In other words, rigid GNP targeting is never optimal in the absence of wage indexation.

34/ It is not possible to prove, in general, that $\tau^{min}$ is the sole real positive root of the polynomial equation $\partial\Lambda^G/\partial\tau = 0$. However, one can prove that $\partial\Lambda^G/\partial\tau$ has at most three positive real roots. A sufficient condition for $\tau^{min}$ to be the only positive real root (and thus for $\Lambda^G$ to be strictly concave in $\tau$) is that $(1-\beta)/\alpha(1-\alpha) > \chi$. See Appendix B for a proof. Note that the above uniqueness (concavity) condition holds as long as society does not place a very large weight on inflation-rate stabilization relative to unemployment rate stabilization. Recall that $\alpha$ is capital's share of output, when $\alpha = .25$, then $1/\alpha(1-\alpha) = 5.33$. Thus when there is no wage indexation (so that $\beta = 0$), uniqueness requires only that society not prefer a $\sqrt{5}$ percent fall in employment below its socially desired level to a one percent increase in the inflation rate above its desired level. In fact, it is likely that $\chi$ is much smaller than one in most societies. When the uniqueness condition fails, it is still possible to reliably compute $(\Lambda^G)^{min}$ by computing all the roots of $\partial\Lambda^G/\partial\tau = 0$, since for given values of the parameters, one can use a computer to solve for all the roots of a polynomial.

35/ In deriving (40), use is made of the fact that investors' inflation-rate expectations (in period t) depend only on the time-consistent solution to (39) and not on period t disturbances. It is implicitly assumed that investors expect that the same regime will be in place in period t+1 as in period t.

36/ Again, one can check the algebra in expression (44) for $\Gamma^M$ by considering the limiting case of rigid money supply targeting. It is straightforward to work through the model of equations (9), (10) and (11) to derive an expression for $p_t - E_{t-1}(p_t)$ under a fixed money rule. Plugging this expression into $\Gamma^A$ in equation (18) gives the same result as taking the limit of (44) as $\mu$ goes to infinity.

37/ For the special case $\beta = \omega = 0$, condition (46) reduces to $\chi\xi^2 - \xi[\chi + (1/\alpha^2)] = 0$. (Note from (40) that $\xi > 1$.) It is possible to prove that $\partial\Lambda^M/\partial\mu = 0$ has at most three positive real roots; a sufficient condition for $\mu^{min}$ to be the sole real positive root is that $(1-\beta)/\alpha(1-\alpha)[\phi + \lambda/\delta] > \chi$. See Appendix B for details.

38/ There are (special) cases where, although the authorities learn something each period about the aggregate demand shocks u and v (say, by observing the interest rate), it never pays to react. This happens when the knowledge about u and the knowledge about v are negatively correlated in such a way that nothing is known about price level movements. See, for example, Canzoneri et al. (1982). The reader will recognize that we are describing the special case where Poole's (1970) "combination" interest-rate feedback rule reduces to a fixed money supply rule.

39/ As Canzoneri et al. demonstrate, the central bank can only literally peg the nominal interest rate if it also announces at least one point on a mutually consistent money supply path; otherwise the price level is indeterminate.

40/ The expressions for $dp^R$ and $\Gamma^R$ are omitted as they are not essential to the main argument here. But it is relevant to note that the Lemma of Section V.2 assures that $\Gamma^R \geqq \Gamma^D$.

41/ It can be shown that low nominal interest-rate targeting is counterproductive when the regime is expected to last for any finite number of periods, if in the final period the expected inflation rate is the one consistent with a return to fully discretionary monetary policy. The regime fails because the central bank cannot systematically achieve a below-market real interest rate for any future period.

42/ "Step one" is executed in Appendix F, where rigid versions of the regimes are compared. That analysis makes even more obvious the fact that any of the targeting regimes considered (except interest-rate targeting) works best for some parameter configurations.

43/ In the absence of a fully-specified microeconomic model, one cannot be sure how much the parameters of the model change when policy regimes change; (Lucas 1976). Another caveat to the present analysis is that we have not considered the possibility of adopting more than one target; this is fodder for future research.

44/ This is the standard result; see Friedman (1975).

45/ Nor can one necessarily choose the best intermediate target solely on the basis of which target can be made the most credible. (This argument is sometimes given for why the monetary base is the ideal intermediate monetary target.) The stabilization properties of a target also need to be taken into account.

46/ The analysis can also be extended to the case where the monetary authorities have only a subset of the information available to investors, or visa-versa. Weiss (1980) and Canzoneri et al. (1982) show how the monetary authorities can systematically exploit any superior information available to investors by making promises of future money supply infusions contingent on variables which only investors can respond to contemporaneously (but which the central bank can respond to with a lag.) This "prospective feedback channel" does not function smoothly in a model where the monetary authorities and wage setters have different target employment rates. Investors will have no reason to believe contingent money supply promises which are not time consistent. But it may be possible to restore some role for prospective feedback rules by directly or indirectly introducing investors' inflation rate prediction errors into the social utility function. (The variance of these errors may affect the equilibrium level of the capital stock.)

47/ Strictly speaking, Canzoneri et al. derive these results for the case where the monetary authorities and investors observe only the contemporaneous interest rate. But their methods and results are easily extended to the case at hand. Also, it is necessary to assume that the monetary authorities in no case completely reduce the variance of prices or interest rates to exactly zero, as this would change the information structure. (Alternatively, we may assume that agents also observe the contemporaneous money supply.)

48/ Note that $\alpha$, the coefficient on capital in the Cobb-Douglas production function, is less than one.

49/ A more formal proof involves subtracting the right-hand side of (D1) from the left-hand side and equating to zero. It is then possible to choose $\chi$ and $\alpha$ so that the resulting quadratic equation in $\tau$ has two real roots, one positive and one negative. The positive root is the $\tau$ at which rank reversal occurs.

50/ Expressions (F2), (F3) and (F4) may be derived by taking the limits of equations (24), (33) and (44) with respect to the index variables. It is also fairly easy to derive them directly from the model, by alternately holding p, p+y, and m fixed. The second method, incidentally, provides an algebra check on expressions (24), (33) and (44).

51/ Rigid inflation-rate targeting is always better than fully discretionary monetary policy when the labor market distortion is relatively large, and is similarly always better than rigid money supply targeting when either or both of the aggregate demand shocks have large variance relative to the aggregate supply shock.

52/ Wage setters, who are concerned with employment stabilization, have no reason to index in this case anyway.