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This paper is a theoretical analysis of the effects of exchange rate variability on a firm's employment and output decisions. In section 1, two contracting models are developed and the static effects are explored. In the Gray-Fischer-Canzoneri model, only exchange rate prediction errors have effects, whereas in the Mussa-Taylor model anticipated exchange rate movements also have effects. In section 2, the same contracting models are used to describe the dynamic effects. The time series properties of employment and output depend upon the time series properties of the exchange rate, the cost of adjusting factor inputs, and the form of the labor contract. Also, exchange rate volatility can be profitably exploited by an individual firm that can adjust its output to price changes each period. In section 3, the implications of risk aversion are examined. Given an increase in exchange rate variability, a risk-averse firm will reduce its scale of operations if adjustment costs prohibit its responding to realizations of the exchange rate. However, if the firm is able to adjust its labor input in response to exchange rate movements, then it may well want to increase its scale of operations. Aversion to the increased risk has to be weighed against the expected profitability of being able to exploit wider fluctuations in the exchange rate.

The Effects of Exchange Rate Variability on Output and Employment

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The current regime of flexible exchange rates has focused attention upon the relationship between the volatility of exchange rate movements and the level of real economic activity. The concern is that exchange rate variability may decrease international trade flows, and may even affect employment and output at firms that are not directly involved in international markets.

In this paper, we survey the theoretical underpinnings of the relationship between exchange rate variability and a firm's employment and output decisions. Using simple models, we catalogue the different ways in which exchange rate movements affect the cost and revenue structure of the firm. (No attempt is made to integrate the financial structure of the firm into these models; such an extension would be most useful.)

Few of our results are really new. Rather, we hope that our survey will be helpful in developing empirical models of trade flows; we also think that our work points to certain measures of exchange rate volatility that are relevant to discussions of regime design. Some of these measures are already familiar from existing empirical work on trade flows and from the literature on rational expectations; others, particularly those suggested by section 2, are new. Care must be taken in the use of these measures. The implications

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of movements in one endogenous variable (here, the exchange rate) for movements in another (here, real economic activity) generally depend upon the source of the exogenous disturbance. We give examples of this problem in section 1.

The paper is organized as follows: The analysis focuses on the implications of different assumptions about labor contracts and the costs of adjustment. In section 1, we develop two contracting models and explore the static effects of exchange rate fluctuations on real economic activity. In the Gray-Fischer-Canzoneri model, exchange rate prediction errors cause fluctuations in employment and output, whereas in the Mussa-Taylor model anticipated exchange rate movements also affect the firm's employment and output decisions.

In section 2, we use the same contracting models to describe the dynamic effects of exchange rate fluctuations on employment and output. We show that the time series properties of employment and output depend upon the time series properties of the exchange rate, the cost of adjusting factor inputs, and the form of the labor contract. We also show that exchange rate volatility can be profitably exploited by an individual firm when the firm can adjust its output to price changes each period.

In section 3, we examine the implications of risk aversion. We illustrate the standard result that, given an increase in exchange rate variability, a risk averse firm will reduce its scale of operations if adjustment costs prohibit its responding to actual realizations of the exchange rate. However, we also show that if this last assumption is relaxed and the firm is able to adjust its labor input in response to exchange movements, then it may well want to increase its scale of operations. Aversion to the increased risk has to be weighed against the expected profitability of being able to exploit the wider fluctuations in the exchange rate.

In section 4 we summarize our results and describe different measures of exchange rate volatility that are suggested by the models presented in the first three sections. Prediction errors, time series properties, and variances are all measures of volatility that have implications for firms' employment and output decisions.

1. The Effects of Exchange Rate Prediction Errors

In this section we are concerned with the effects of unexpected exchange rate movements on a firm and its employees. Unexpected exchange rate movements can occur for a variety of reasons, and their effects, both positive and normative, will generally depend upon the source of the disturbance. Here we will assume that that source is external to the firm and the industry in which it operates.

Monetary disturbances would have no effect on employment or output were it not for some sort of price stickiness.[1] Here we attribute that stickiness to wage contracting. In section 1.a we develop the basic contracting framework that we will extend and use repeatedly throughout the rest of the paper to describe different ways in which exchange rate volatility affects unemployment, output and investment decisions.

In section 1.a we describe the simplest of contracting models and show how unexpected exchange rate movements lead to fluctuations in the price of the firm's product, and thus to fluctuations in employment and output. In section 1.b we extend the model to include wage indexing and imported intermediate goods, like oil; exchange rate fluctuations have had important cost implications in recent years via both of these channels. In section 1.c we extend the model to include long-term contracts, contracts that last for more than one period. There appear to be two relevant ways of doing this, both on theoretical and empirical grounds. The two approaches have very different implications for the way exchange rate volatility is transmitted to employment and output fluctuations, but empirical work has yet to differentiate strongly between them.

1.a A Simple Contracting Model

Figure 1.a illustrates the notional supply and demand for labor at the firm we are considering. The real wage ν is the market clearing wage; $\bar{n}$ is the equilibrium rate of employment, sometimes called the "natural" rate. For simplicity, we assume that labor supply is inelastic.[2] The labor demand curve is a graph of the marginal product of labor, which is appropriate for a price taking, profit maximizing firm.

More explicitly, if the firm's technology is Cobb-Douglas

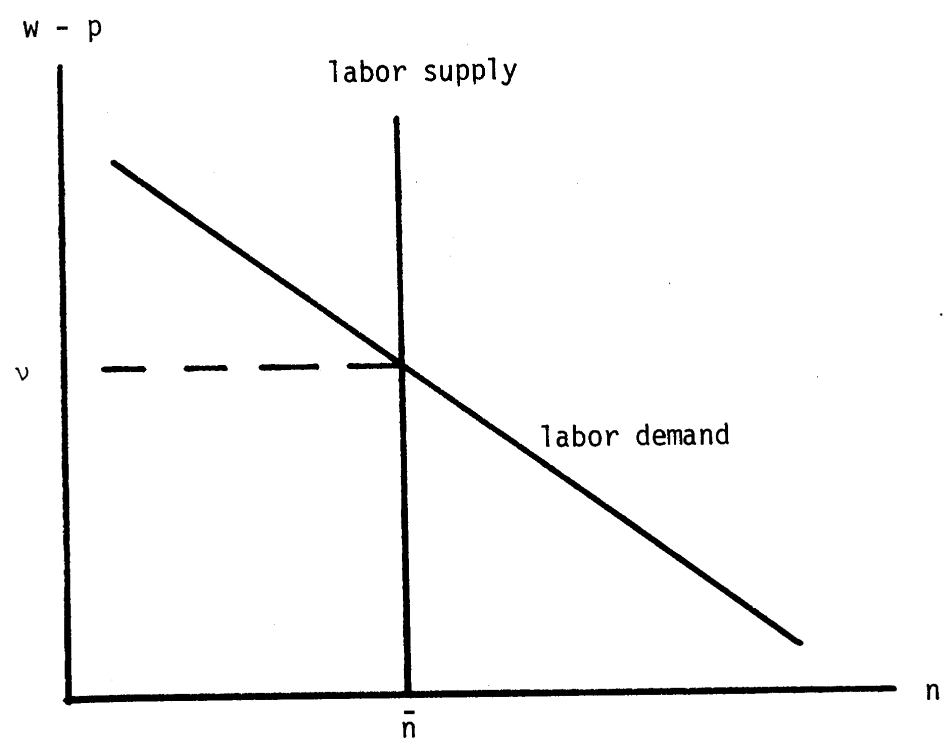
$$(1) \quad Y_t = K_t^{\alpha} N_t^{(1-\alpha)}$$

(where Y is output, N is employment, and K is the capital stock), and if the capital stock is fixed (at, say, $K_t = 1$) while there is no cost to adjusting employment, then the profit maximizing rate of employment is given by

$$(2) \quad W_t / P_t = (1-\alpha) N_t^{-\alpha}$$

(where W is the nominal wage rate and P_t is the domestic price of output). We will find it more convenient to work with linear models, and equations (1) and (2) have natural log-linear representations. Letting small letters denote the logarithms of the corresponding capital letters, (1) and (2) become

FIGURE 1.a



(3) $y_t = (1-\alpha)n_t$

and

(4) $w_t - p_t = \log\ (1-\alpha) - \alpha n_t$

$= [\log\ (1-\alpha) - \alpha\bar{n}] - \alpha(n_t - \bar{n})$

The bracketed term, $\log\ (1-\alpha) - \alpha\bar{n}$, is the market clearing wage ν in Figure 1.a.

The firm's product is sold on world markets, so the good's price p is equal to $p^* + s$, where p^* is "the" world price and s is "the" exchange rate. (For present purposes we are aggregating the "rest of world" into one entity.) Volatility in s that is not offset by movements in p^* is passed on to the domestic price of output.

Following Gray (1976), we simply postulate that the firm and its workers must get together at the beginning of each period, before goods prices and exchange rates are known, and set the nominal wage in a labor contract. Their goal is to achieve the market clearing solution; that is, they predict prices and exchange rates as best they can and set the nominal wage at a level that is expected to result in the market clearing real wage ν.[3] If $p^*_{t|t-1}$ and $s_{t|t-1}$ are the price and exchange rate expected to prevail, then the wage set in the contract is

(5) $w_t = \nu + p_{t|t-1} = \nu + (p^*_{t|t-1} + s_{t|t-1})$

The symbol "$|t-1$" denotes an expectation based upon information available at the end of period t-1.

Later, during the contract period, actual prices and exchange rates will become known, and wage setters' prediction errors will of course lead to a divergence between the actual real wage and ν; that is,

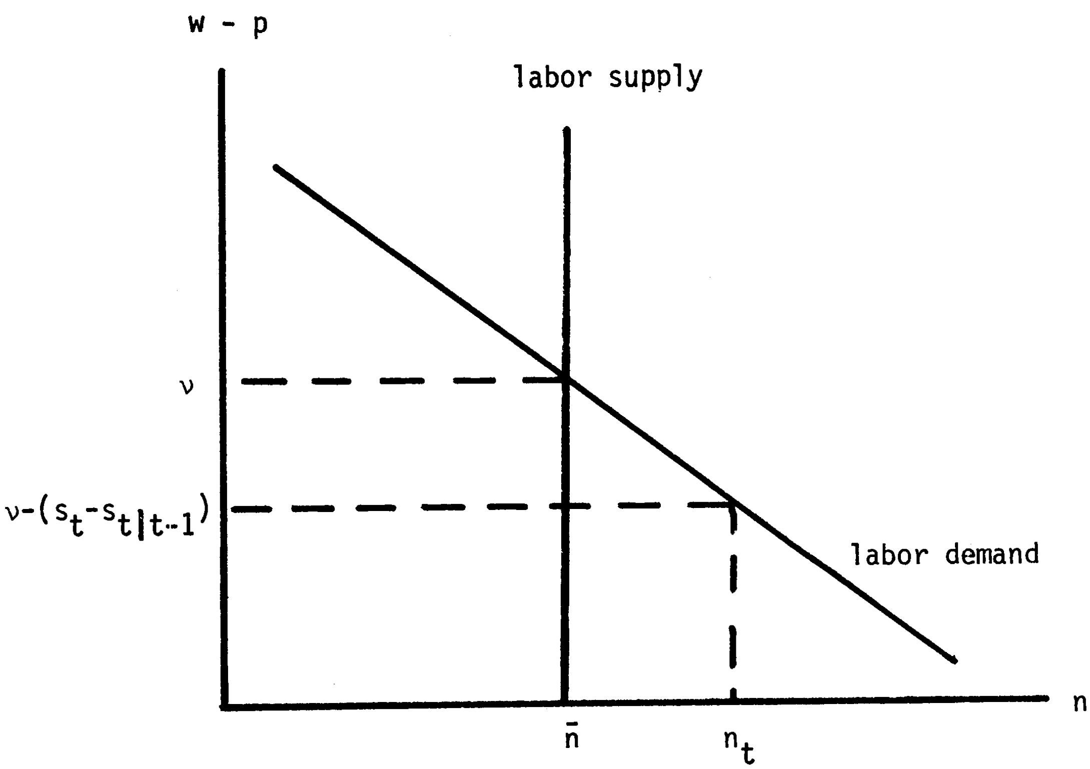
$$(6)\quad w_t - p_t = \nu - (p_t - p_{t|t-1})$$
$$= \nu - [(p_t^* - p_{t|t-1}^*) + (s_t - s_{t|t-1})]$$

Clearly, labor and management can not both be on their notional supply and demand curves in this situation, so the contract must also specify (perhaps implicitly) an employment rule stating how much labor is to be utilized at the actual real wage. One can imagine various employment rules that might be instituted; however, we will again follow Gray (1976) in assuming that essentially management wins out. During the contract period, the firm employs, and workers willingly supply, the profit maximizing amount of labor; the notional demand curve in Figure 1.a is also the employment rule.[4]

The practical import of all this is that unanticipated exchange rate movements cause fluctuations in employment and output. The effect of an unanticipated depreciation (a rise in s, with no change in p*) is depicted in Figure 1.b. The actual real wage, $\nu - (s_t - s_{t|t-1})$, is lower than was intended because the price of output, $p = p^* + s$, is higher than was anticipated. Consequently, the firm employs more labor than was orignally envisioned. In view of (4) and (6), the general result is

$$(7)\quad n_t = \bar{n} + (1/\alpha)\,(p_t - p_{t|t-1})$$

FIGURE 1.b



$$= \bar{n} + (1/\alpha)\ [(p^*_t - p^*_{t|t-1}) + (s_t - s_{t|t-1})]$$

Exchange rate prediction errors lead to fluctuations in employment about its "natural" rate.

It should be noted that an unanticipated increase in s will often be accompanied by an unanticipated movement in p*. Canzoneri and Underwood (1982) have shown that if the depreciation is due to an unanticipated increase in the rate of growth of the home money supply, both s and p will increase, but because of the wage stickiness, the increase in s will be larger [5] This means that p*, the foreign price of the firm's output, must be unexpectedly low, for

$$(8)\quad p^*_t - p^*_{t|t-1} = (p_t - p_{t|t-1}) - (s_t - s_{t|t-1})$$

In this case, just looking at $s_t - s_{t|t-1}$ in Figure 1.b will overstate the impact on employment. Indeed, suppose that the depreciation is due to an unanticipated decrease in the rate of growth of the foreign money supply. In this case, p will be unexpectedly low and employment will actually fall.[6] The fall in p* outweighs the rise in s and reverses the effect on employment illustrated in Figure 1.b.

These two examples illustrate an important caveat for studies such as this. There can be no fixed relationship between two endogenous variables such as the exchange rate and employment. Both are fluctuating in response to some truly exogenous disturbance (or group of disturbances), and the very sign of the covariance between the two will depend upon the source of the disturbance. The exchange rate is exogenous to the price taking firm, and we discuss the impact, ceteris paribus, of exchange rate volatility on the firm's employment. This is a standard partial equilibrium exercise, but we must recognize that, in the case of the exchange rate anyway, the "ceteris paribus" qualification is very important.

Fluctuations in employment will of course lead to fluctuations in output. In fact, our simple contracting model can be explained in terms of the price-marginal cost diagram illustrated in Figure 1.c. The (log of) the marginal cost curve for the firms is

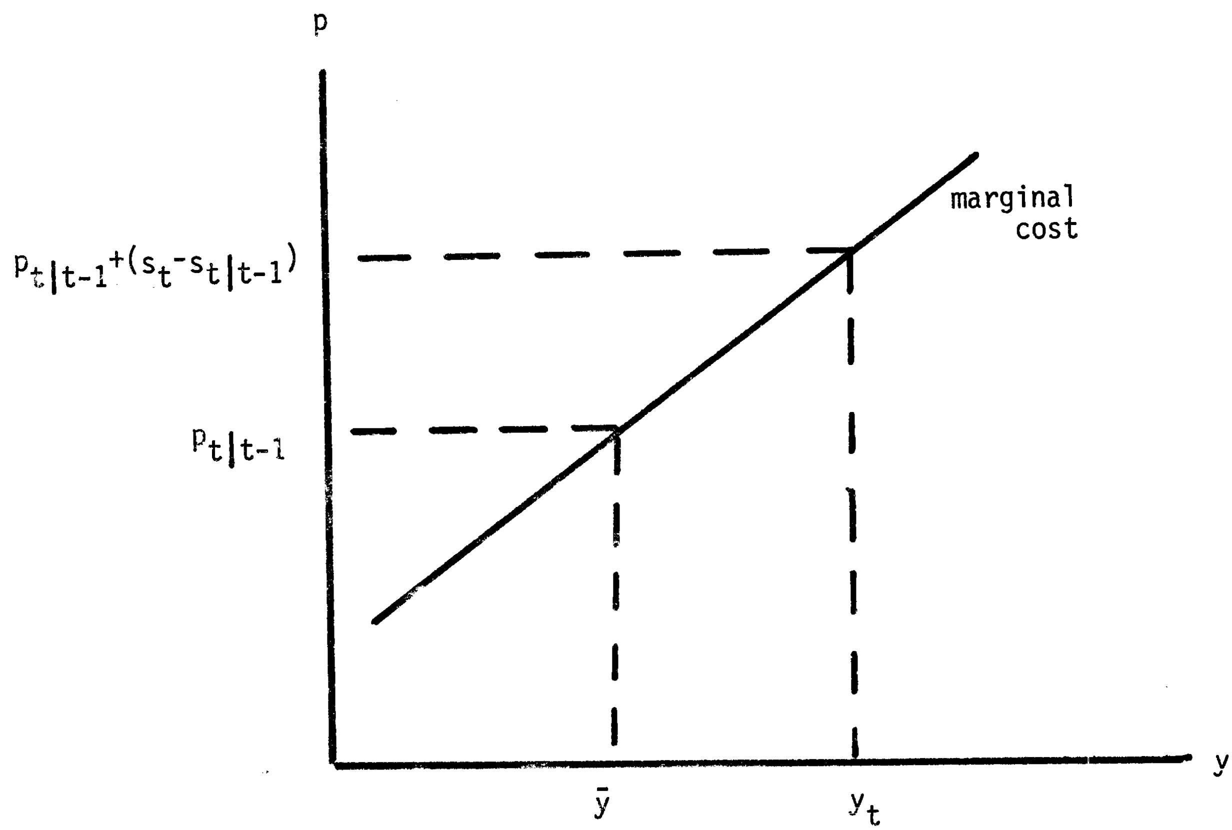
$$(9) \quad c'(y_t) = w_t - \log(1-\alpha) + [\alpha/(1-\alpha)]y_t$$
$$= (w_t - \nu) + [\alpha/(1-\alpha)](y_t - \bar{y})$$

where $\bar{y} = (1-\alpha)\bar{n}$ is the equilibrium or "natural" rate of output. The position of the marginal cost curve is determined by the w_t set in the contract. It is clear from (5) and (9) that w_t is set so that $c'(\bar{y}) = p_{t|t-1}$. In other words, w_t is set in the contract so that the marginal cost curve in Figure 1.c will intersect the expected price, $p_{t|t-1}$, at the equilibrium rate of output, $\bar{y}$. The unanticipated depreciation analyzed in Figure 1.b causes the actual price to be higher than expected and increases output. The general result is

$$(10) \quad y_t = \bar{y} + [(1-\alpha)/\alpha](p_t - p_{t|t-1})$$
$$= \bar{y} + [(1-\alpha)/\alpha][(p^*_t - p^*_{t|t-1}) + (s_t - s_{t|t-1})]$$

The key word here is unanticipated. If the depreciation had been accurately predicted, a higher nominal wage would have been set in the

FIGURE 1.c



contract, as shown in equation (5), and the market clearing solution would have obtained. This contracting model incorporates the "natural rate" hypothesis; perfectly anticipated movements in prices or exchange rates have no effect upon employment or output. Later we will describe a contracting model in which perfectly anticipated exchange rate fluctuations can have real effects.

1.b. Indexing and Imported Intermediate Goods

Wage indexing has become an important feature in the cost structure of many economies recently. When added to the contracting framework described above, it becomes a new channel for unanticipated exchange rate fluctuations to affect real wages and employment.

It will be recalled that anticipated inflation and exchange rate depreciation are already reflected in the base wage set in the contract; so, for example, indexing clauses designed to capture expected losses in purchasing power have already been accounted for. In the view of wage contracting taken here, this kind of indexing does not affect employment or output, though it may well be an important factor in inflation.

Instead, the indexing we want to discuss here is an explicit contract provision for modifying the base wage as actual price and exchange rate data become available.[7] The goal once again is to achieve the market clearing solution, and the basic idea is that movements in P or some index of prices, C, can act as signals for wage setting, much as interest rate movements are taken as indicators for monetary policy. Unfortunately, movements in P and C are both imperfect signals; neither tells one how to adjust the base wage to achieve the market clearing solution in all circumstances. The problem is analogous to Poole's

(1970). Just as Poole's optimal combination policy depends upon the relative sizes of disturbances in the IS and LM curves, so the optimal indexing scheme will depend upon the sizes of various disturbances affecting equilibrium in the labor market,[8] and just as pegging the interest rate is not generally optimal for monetary policy, so full indexation to P or C (which amounts to pegging W/P or W/C) is not generally optimal for wage policy.[9]

Suppose the labor contract calls for partial indexation of W to C, where

$$(11) \quad C = P^{\gamma_0}(I)^{\gamma_1}(I^*S)^{\gamma_2}, \; \gamma_0 + \gamma_1 + \gamma_2 = 1$$

I is an index of home goods, other than Y, and I* is an index of foreign goods. If (5) is modified to

$$(12) \quad w_t = \nu + p_{t|t-1} + \Omega(c_t - c_{t|t-1})$$

where Ω is an indexing parameter ($0 < \Omega < 1$), then the real wage is

$$(13) \quad w_t - p_t = \nu - (1-\Omega)\tilde{p}_t - \Omega[\gamma_1(\tilde{p}_t - \tilde{i}_t) + \gamma_2(\tilde{p}_t - \tilde{i}^*_t - \tilde{s}_t)]$$

where i is the log of I, a "~" denotes a prediction error (e.g. $\tilde{X}_t \equiv X_t - X_{t|t-1}$), and the general result for employment is

$$(14)\quad n_t = \bar{n} + (1-\Omega)(1/\alpha)\tilde{p}_t + \Omega(1/\alpha)[\gamma_1(\tilde{p}_t - \tilde{i}_t) + \gamma_2(\tilde{p}_t - \tilde{i}_t^* - \tilde{s}_t)] \quad .$$

As before, employment and output depend on own price prediction errors, $\tilde{p}_t$, but now they also depend upon the relative price prediction errors $\tilde{p}_t - \tilde{i}_t$ and $\tilde{p}_t - \tilde{i}_t^* - \tilde{s}_t$.

Consider once again the effect of an unanticipated depreciation. If the domestic price of output remained constant (that is, if a negative $\tilde{p}^*$ offset the positive $\tilde{s}$, so that $\tilde{p} = \tilde{p}^* + \tilde{s} = o$), then the only effect on the real wage and employment would be through the indexing clause. The price of foreign goods would rise, and this would be passed on to the nominal and then the real wage; employment and output would fall. If, at the opposite extreme, the domestic price of output rose by the full amount of the depreciation, then the real wage would have to fall. However, it would not fall in proportion with the rise in the domestic price, because the higher cost of domestic and foreign goods would be passed on to a higher nominal wage. Employment and output would rise, but not by as much as if there had been no indexing.[10]

Oil and other imported intermediate goods have also become an important factor in many firm's cost structure. Canzoneri and Gray (1983) provide a simple model in which oil and labor are used in fixed proportions.[11] One unit of oil and one unit of labor combine to form one unit of the variable composite input which the firm uses, along with its fixed capital stock, to produce output. This composite input replaces N in equation (1), and if q is the (log of) the domestic price of oil, then $\beta(w-p) + (1-\beta)(q-p)$ replaces w-p in equation (4) the real cost of a unit of the variable input; β is labor's share of the composite input bill at full employment. The base wage is set to clear the labor market, so

$$(15) \quad \beta w_t + (1-\beta)\, q_{t|t-1} = \gamma + p_{t|t-1}$$

and equation (5) is replaced by

$$(16) \quad w_t = \nu + p_{t|t-1} - [(1-\beta)/\beta]\,(q_{t|t-1} - p_{t|t-1})$$

and assuming no wage indexation, employment (of both oil and labor) will be

$$(17) \quad n_t = \bar{n} + (1/\alpha)[\tilde{p}_t - (1-\beta)\tilde{q}]$$

where once again a "~" denotes a one-period prediction error.

Canzoneri and Gray (1983) focus upon an important asymmetry in the pricing of oil. In the short-run, the price of oil is fixed in terms of U.S. dollars. For U.S. firms then, $\tilde{q}_t$ is simply an error in predicting the price setting behavior of oil producers. For other firms, the domestic price of oil, $\tilde{q}_t$, also includes a prediction error for the value of the dollar. In this way, the volatility of the dollar per se has become more important in the last decade. It has also been the source of international disputes; indeed some European policy maker's view the strong U.S. dollar as the "third oil shock".[12]

Summarizing, an unanticipated depreciation will, ceteris paribus increase the price the firm receives for its product if the good is traded internationally; however, it may also increase the price it has to pay for its labor and oil inputs. If indexing and the dollar pricing of oil play an important role in the firm's cost structure, then employment and output will not increase much. Again, the key word is unanticipated. If the depreciation had been foreseen, a higher base wage would have been specified in the contract, and employment and output would have been unaffected.

1.c. Long-term Contracting

In sections 1.a and 1.b, we asumed that contracts lasted only one period, or more precisely, we defined a period to be the length of a contract. If we were doing emperical work with quarterly (or even yearly) data, then this assumption would probably not be appropriate.[13] Most union contracts in the U.S. last for three years; most other workers are not covered by formal contracts, but are subject to annual salary and performance reviews.

In this section, we describe two of the ways long-term contracts have been modeled and discuss the implications of multiple period contracts for exchange rate volatility and employment fluctuations. We also discuss the relationship between exchange rate volatility and contract length.

The Gray-Canzoneri Model:

Suppose first that the length of the contract is given; we will return to the subject of contract length later. Perhaps the most natural extension of the model described above is to assume that a series of base wages is specified in the contract, a separate wage for each period covered by the contract. When the contract is negotiated, the time paths of prices and exchange rates are predicted, and nominal wages are set at levels that are expected to clear the market each period; the contract may also include indexing provisions. As before, price and exchange rate prediction errors will cause fluctuations in employment and output.

The basic difference here is in the information set that is reflected in the predictions. As we pass through the contract period the information on which the predictions were based becomes more and more dated; Canzoneri (1980) has shown that prediction errors (or more precisely, their variance) will get bigger, and fluctuations in employment and output will become larger. In addition, if the movements in prices and exchange rates are serially correlated, then the prediction errors and the associated fluctuations in employment and output will be serially correlated;[14] the model implies "cycles" or persistence.

Gray (1978) and Canzoneri (1980) have suggested that the length of contracts is determined by comparing the costs of more frequent negotiations with the benefits. The benefits of more frequent contracting accrue from more informed price and exchange rate predictions, smaller prediction errors (on average), and less deviation from the market clearing solution.[15]

According to this view, a volatile, hard to predict exchange rate would result in short contracts. A monetary policy that was successful in reducing this volatility would reduce the benefits of frequent recontracting and allow the firm and its workers to economize on the costs of negotiation. Contracts would become longer, and fluctuations in employment and output would be smaller, though more inertia ridden.

The Mussa-Taylor Model:

The Gray-Canzoneri model focuses on the bargaining costs of contract renegotiation; it assumes that there is little or no cost to specifying different wage settings for the various periods covered by the contract. The Mussa (1981) model, by contract, attributes the same cost to each and every change in wage setting. His is thus a model with one fixed wage for all of the periods in the contract. Given that restriction, his model is much like the Gray-Canzoneri model: The wage is set at the best level on the basis of expected inflation, etc., and the length of the contract is determined by weighing the costs of more frequent wage settings against the benefits.

It is not clear what employment rule Mussa himself would combine with his wage setting scheme.[16] However, we will continue to use the period by period profit maximizing labor demand curve in Figure 1.a. This model will of course have the same implications for the relationship between price and exchange rate prediction errors and employment and output fluctuations. What's new is that anticipated movements in prices and exchange rates will also affect employment and output; this wage setting model does not incorporate the "natural rate" hypothesis (though it is perfectly consistent with the "rational expectations" hypothesis). Consider for example the effect of a fully anticipated series of depreciations (with no change in P* or P**). Since just one nominal wage is specified for the duration of the contract, the real wage in Figure 1.b will start high (too high for market clearing) and be inflated away, period by period; employment will be low in the early periods, and high in later periods.[17]

Taylor's (1979,1980) contracting model also assumes that one wage is set for the life of the contract; indeed, this is the source (when combined with overlapping contracts) of the persistence effects that are thought to be a virtue of his model. Taylor uses a different employment rule, and his wage setters appear (on the surface anyway) to have different motivations; it is possible however that his model could be explained in a way that is quite consistent with Mussa (1981). Canzoneri and Underwood (1982) compare the effects of exchange rate volatility in Taylor's model with those in the Gray-Canzoneri model. Aside from the fact that Taylor's model does not incorporate the "natural rate" hypothesis, the basic difference is that exchange rate movements have longer lasting effects on employment and output. The effect persists even after all of the contracts have been negotiated.

2. Effects of Exchange Rate Dynamics and Adjustment Costs

In the previous section we looked at models of contracting and employment that are explicitly intertemporal: the firm and its employees make nominal wage rate and indexing decisions in one period and follow through with the consequences of those decisions for employment and output in subsequent periods. We saw how different contracting arrangement can lead, in the context of exchange rate variability, to different effects on output and employment. Implicit in the previous section is the assumption that after the contract is settled, the costs to the firm of adjusting its output and employment to unpredicted or predicted changes in the exchange rate are zero.

In this section we also investigate models which are explicitly intertemporal. We assume, for simplicity, a contracting arrangement in which one nominal wage rate is fixed over an indefinite horizon. This kind of assumption is more closely allied with the Mussa-Taylor model of contracting than it is with the Canzoneri-Gray model. With this simple kind of long-term contract, our model falls into the class of models for which perfectly predictable fluctuations in nominal magnitudes can generate real effects.

Instead of assuming zero costs of adjustment, we now explore a model of the firm in which the firm's decision-makers 1) consider explicitly the costs of adjusting their labor force and 2) view the exchange rate as a time series which, in the presence of the fixed nominal wage rate, generates a well-defined real wage rate process. The firm's decision-makers use their knowledge of this real wage process to choose the quantities of labor they employ in the periods over their planning horizon.

Two kinds of exchange rate processes are investigated here. One exhibits predictable and persistent cycling. With this kind of exchange rate variability we find that, even in the presence of adjustment costs, the firm will vary the quantity of labor it employs over the exchange rate cycle. Furthermore, we find that with finite adjustment costs, the firm's present value increases with increases in exchange rate variability. The other exchange rate process is stochastic, covariance stationary, and serially correlated. In this case we find that both the persistence of the exchange rate shocks and the size of the adjustment costs will affect the firm's demand for labor and supply of output.

In section 2a we present the assumptions common to the analysis in section 2b and 2c. In the later two sections we discuss the results of our investigations of the two exchange rate processes.

2a. Assumptions

To evaluate the results presented in section 2b and 2c it is useful to set out the assumptions which underly our model of the firm. As in the work of Sargent (1979) the firm is assumed to face a quadratic production function which can be expressed as

$$(18) \quad F(N_t) = f_0 N_t - 1/2 f_1 N_t^2$$

where f_0, f_1 are assumed to be greater than zero. The firm employs a single variable factor of production, labor (N_t), and has a fixed capital stock. Note that the marginal product of labor will be positive as long as $(f_0/f_1) > N_t$ and that the production function is concave in N_t. We adopt this specification primarily because it makes it less difficult to obtain an explicit solution to the firms dynamic optimization problem.

The firm is also assumed to be subject to real quadratic costs in adjusting its labor force which can be expressed as $d/2(N_{t+j} - N_{t+j-1})^2$ where $d > 0$. Two possible interpretations can be given to these costs. First, such costs could be interpreted as being part of the production function and therefore technological in nature. In this case, as in Sargent [1979], the cost is specified in terms of the firm's own output. Second, these adjustment costs might conceivably be related to unemployment insurance and financial costs other than wages which arise when the firm alters its demand for labor from period to period. Given this interpretation, the cost might be specified in terms of a basket of consumption goods. We have chosen the first interpretation.

The firm is assumed to maximize the expected present value of profits. Profits (π_{t+j}) are defined in terms of output[18] as:

$$(19) \quad \pi_{t+j} = f_0 N_{t+j} - 1/2 f_1 N_{t+j}^2 - 1/2 d(N_{t+j} - N_{t+j-1})^2 - (W_{t+j}/P_{t+j})N_{t+j}$$

The only new term introduced in (19) is the firm's real wage bill $(W_{t+j}/P_{t+j})N_{t+j}$ where W_{t+j} is the contract wage.

The assumption that both the firm's production function and costs of adjusting labor are quadratic result in a quadratic objective function. This is a convenient representation because when the firm maximizes the expected present value of profits with respect to its choice of employment (N_t), we obtain linear first order conditions which make it straightforward to obtain reduced-form expressions for the firm's employment rule. The quadratic nature of the firm's objective function also results in the "separation principle", applicable in the stochastic case, where we can first obtain optimal linear forecasts of the exogenous variables and then solve a nonstochastic optimization problem.[19]

In our model of the firm, assumptions about the nature of labor contracts and commodity arbitrage imply that exchange rate variability results in associated variability in real wages. In particular we assume that firms enter into long term contracts with labor in which laborers and firms agree on a nominal wage rate which is assumed to be fixed over the firm's entire planning horizon. Moreover, the firm's output is sold on domestic and world markets and is subject to commodity arbitrage so that $P_t = P_t^* S_t$ where P_t^* and S_t are defined as in section 1 above. Under these two assumptions the real wage rate can be written as:

$$(20) \quad (W_t/P_t) = (W/P^*)S_t^* = S_t^*,$$

where $(W/P^*) = 1$ and $S_t^* = (1/S_t)$.

For convenience we define the units of the nominal wage rate W and the foreign currency price of output, P*, such that their ratio equals one. Thus equation (20) implies that the real wage process will be generated by whatever assumptions are made about the process generating $S_t^* = (1/S_t)$.[20] Finally the firm is assumed to be a perfect competitor on both domestic and foreign output markets so that it views the real wage rate as an exogenous process which will be assumed to be either stochastic or purely deterministic.[21]

2b. Exchange Rate Variability and Adjustment Costs: The Certainty Case

In this section we investigate how perfectly predictable exchange rate variability affects a firm's output and employment decisions. We hypothesize a cyclical exchange rate path which the firm's decision-makers can predict without error, and consider their employment rule under three different adjustment cost assumptions: zero adjustment costs, positive but finite adjustment costs, and infinite adjustment costs. We find that the adjustment cost assumption made influences the output and employment responses of the firm to increased variability. In addition, we look at the effect of exchange rate variability on the firm's present value and find that, because of the convexity of the firm's profit function, the average present value over the exchange rate cycle may increase as the variability of the exchange rate increases.

Since we are interested in examining exchange rate paths which exhibit variability, we make the following simple assumption about the exchange rate path S^*_t

$$(21)\quad S^*_t = \overline{S}^* + a(-1)^t = W/P_t \qquad \overline{S}^* > a > 0 \qquad t = 0, 1, 2, \ldots$$

In this simple process S^*_t takes only two values as time progresses - a high value $(\overline{S}^* + a)$ when t is even and a low value $(\overline{S}^* - a)$ when t is odd. $\overline{S}^*$ represents some positive mean around which S^*_t cycles with amplitude (a) and a frequency of two periods.[22] Amplitude and frequency are the measures of exchange rate variability focused upon here. Given our assumptions about the nominal wage rate and the foreign currency price of output, the real wage rate path is thus identical to the exchange rate path.

We derive the firm's labor demand expression from the following maximization problem:

$$(22)\quad \max_{N_{t+j}} \; PV_t = \sum_{j=0}^{\infty} b^j \pi_{t+j}$$

$$\text{where } 0 \leq b < 1$$

π_{t+j} is the profit function discussed in section 2a. In this section, the firm's decision-makers know with certainty that the exchange rate S_t follows the process given in (21). Given this information, they must pick a sequence of N_{t+j} ($j = 0, 1, 2, \ldots$), and by so doing an output sequence, such that they maximize the firm's present value.

$$(23)\quad N_{t+j} = \frac{f_o - S_{t+j}}{f_1} = \bar{N} - \frac{a}{f_1} (-1)^{t+j}$$

$$j = 0, 1, 2, \ldots$$

$$\text{where } \bar{N} = \frac{f_o \bar{S}^*}{f_1}$$

In this case the firm chooses the quantity of labor it desires such that in each period labor's marginal product, $f_o - f_1 N_{t+j}$, equals the real wage rate S_{t+j}. This period-by-period profit maximization is what we would expect when the firm's decision-making in one period has no effect on profits in other periods. Using (23) and the fact that output, Y_t, is given by the firm's production function

$$(24)\quad Y_{t+j} = f_o N_{t+j} - 1/2 f_1 N^2_{t+j},$$

we can make the following observations. 1) The firm's output and employment vary with the same frequency as the exchange rate. 2) An increase in the amplitude of the exchange rate cycle brings about an increase in the amplitude of the output and employment cycle. 3) Because of the concavity of the production function, average output falls as exchange rate variability increases, even though average employment remains fixed at $\bar{N}$ as a increases.

By substituting the firm's employment rule (23) and the exchange (or real wage) rate process (21) into its profit function, we obtain:

$$(25)\quad \pi_t^* = \frac{(f_o-\bar{S}^*)^2 + a^2}{2f_1} - \frac{(f_o-\bar{S}^*)\, a(-1)^t}{f_1}\,,$$

where π_t^* is the maximized value of profits for the given wage rate path. We can see that the profits of the firm over the cycle, and average profits, increase with increases in a.

$$(26)\quad \frac{(\pi_t^* + \pi_{t+1}^*)}{2} = \frac{(f_o - \bar{S}^*)^2 + a^2}{2f_1}$$

The formal reason why average profits increase with increases in exchange rate variability is that the firm's profit function is convex in the real wage rate. Oi [1961] contains an early discussion of this property in connection with a firm's profit function. See also Samuelson [1972] and Chapter 1.9 in Varian [1978]. We will be able to discuss more clearly what this convexity means in terms of the behavior of the firm after we have presented the next two cases. Before we go on to the case of positive but finite adjustment costs, however, let us note that because the firm's profit function is convex in the real wage rate, the firm's present value function is also convex in the real wage rate.[23] Consequently, the average present value of the firm over the cycle increases with increases in a.[24]

$$(27)\quad \frac{(PV_t^* + PV_{t+1}^*)}{2} = \frac{(f_o - \bar{S}^*)^2 + a^2}{2f_1(1-b)}$$

Next consider the case in which adjustment costs are positive and finite (i.e. $0 < d < \infty$). In this case the firm's labor demand is given by (see Appendix A for derivation):

$$(28)\quad N_{t+j} = \frac{f_o - (q_o S_{t+j} + q_1 S_{t+j+1}) + c\lambda_1{}^{t+j}}{f_1}$$

$$j = 0, 1, 2, \ldots..$$

$$\text{where } q_o = \frac{f_1 + d(1+b)}{f_1 + 2d(1+b)}$$

$$q_1 = \frac{d(1+b)}{f_1 + 2d(1+b)}$$

$$q_o + q_1 = 1$$

$$0 < \lambda_1 < 1$$

Here the firm's employment rule is expressed in terms of a weighted average of the two values the real wage takes over the exchange rate cycle and is similar in form to (23). The term $c\lambda_1^{t+j}$ reflects the effects of initial conditions on the firm's adjustment problem: λ_1 is the stable root of the second-order difference equations generated by the maximization problem and c is a constant determined by initial conditions. By picking t+j large enough so that the effects of initial conditions are negligible, we can look at the firm in a kind of "steady-state," i.e., we can see how the firm responds to pure cyclical variation.

From (28) we can see that the firm's decision-makers choose the quantity of labor they desire such that in each period labor's marginal product equals a weighted average of the values the wage rate takes over the cycle. By substituting into (28) the expression for the exchange rate process (21) and by suppressing the term c_1^{t+j}, we can see more clearly the effect of increases in variability on the firm's employment rule in the steady state.

$$(29)\quad N_{t+j} = \frac{f_o - [\bar{S}^* + a(q_o - q_1)(-1)^{t+j}]}{f_1}$$

$$(30)\quad = \bar{N} - \frac{a(-1)^{t+j}}{f_1 + 2d(1+b)} \qquad j = 0, 1, 2, \ldots.$$

With adjustment costs the firm's production function is given by

$$(31)\quad Y_{t+j} = f_o N_{t+j} - 1/2 f_1 N^2_{t+j} - 1/2 d(N_{t+j} - N_{t+j-1})^2.$$

Comparing (30) with (23), we can make the following observations. 1) With adjustment costs the firm's output and employment still cycle with the same frequency as the exchange rate, but the amplitude of the cycling is diminished. 2) An increase in the amplitude of the exchange rate cycle brings about an increase in the amplitude of the output and employment cycles. 3) Again, because of the concavity of the production function, average output falls as the amplitude of the exchange rate cycle increases, even though average employment remains fixed at $\bar{N}$.

By substituting into the firm's present value function its employment rule (30) and the wage rate process, we can show that, even with adjustment costs, the average present value of the firm over the cycle increases with increases in a. Equation (32) shows the expression for the average present value over the cycle.

$$(32)\quad \frac{(PV_t^* + PV_{t+1}^*)}{2} = \frac{(f_0-\bar{S}^*)^2}{2f_1(1-b)} + \frac{a^2[f_1 + 4db]}{2(1-b)[f_1+2d(1+b)]^2}$$

Finally, consider the case in which adjustment costs are infinite. As d increases without bound, the second term of (30) vanishes and the firm's labor demand is given by:

$$(33)\quad N_{t+j} = \bar{N} \qquad j = 0, 1, 2, \ldots..$$

Since adjustment costs are infinite, the firm's decision-makers choose a quantity of labor, $\bar{N}$, and keep it constant throughout their planning horizon. They will pick $\bar{N}$ such that labor's marginal product, $f_0-f_1\bar{N}$, equals the average real wage rate over the cycle.

From (33) then, we can see that 1) the firm's output and employment do not vary over the exchange rate cycle and 2) an increase in the amplitude of the exchange rate cycle has no effect on output and employment. Furthermore, as d increases without bound, the second term of (32) vanishes and we can see that the average present value of the firm over the cycle neither increases nor decreases with increases in a. This case corresponds to the model of the firm commonly used in studies employing risk aversion to generate effects from increased variability. As is discussed in section 3, implicit in those studies is an assumption of infinite adjustment costs.

In this section we have seen how exchange rate cycling affects a firm's output and employment decisions under three different adjustment cost assumptions. We have found that as long as adjustment costs are zero or finite, the firm will adjust its output and employment to changes in the exchange rate. We have also seen that increases in the amplitude of the exchange rate cycle will, if the firm faces adjustment costs which are zero or finite, increase a firm's present value. All of these results are driven by the fact that the firm's profit function is convex in the real wage rate. A closer examination of what this convexity means in terms of the firm's behavior will reveal the relationships between this convexity and adjustment costs.

Consider first the case in which adjustment costs are infinite. In this case the firm does not vary its employment and output; it employs the quantity of labor it would have chosen had the amplitude of the exchange rate process been zero (i.e., $N_t = \overline{N}$). With employment and output fixed, the reduction in costs the firm enjoys when the real wage rate is low just offsets the increase in costs it bears when the real wage rate is high. Consequently, the sum of profits across the cycle equals the sum of profits the firm would have earned had the exchange rate stayed constant at $\overline{S}$.

Now consider the case in which adjustment costs are finite. When the real wage rate is low and the firm is employing $\overline{N}$, the marginal product of labor is greater than the real wage rate. If the firm can adjust at all, it can increase its profits by increasing employment and output. Since, with quadratic adjustment costs, the marginal cost of adjusting the labor force approaches zero as the adjustment gets smaller, the firm can always profit by adjusting its employment at least a small amount, no matter how large d is. Similarly, when the real wage rate is high and the firm employs $\overline{N}$, the marginal product of labor is below the real wage rate and the firm can reduce its losses if it can cut back on employment and output. Once again, if the firm has the ability to adjust, it can gain from adjusting its employment. Since the marginal costs of adjusting are arbitrarily small for small enough adjustments, firms can reduce their losses by reducing output and employment. Thus, if a firm has the ability to adjust, its profits over the exchange rate cycle are higher than the profits the firm would have earned had the exchange rate stayed constant. Put another way, if a firm has the ability to adjust, its profit function will be convex in the real wage rate, and its profits will increase as the amplitude of the real wage rate cycle increases.

In this section, then we have isolated another channel through which exchange rate variability affects firm behavior. Our analysis suggests that exchange rate variability is not necessarily costly to firms. Firms with any ability to adjust to changes in the exchange rate can gain from exchange rate variability in this model. While this analysis is a partial equilibrium result, there is nevertheless the possibility that in a general equilibrium analysis in which agents have varying degrees of flexibility, exchange rate variability may bring about transfers of income from those who find adjustment costly or impossible to those who find adjustment less costly.

2c. Exchange Rate Variability and Adjustment Costs: The Stochastic Case

In this section we examine the implications of relaxing several of the assumptions made in section 2b. First, we no longer assume that the exchange rate is characterized by a cyclical nonstochastic process. Instead we assume that the exchange rate follows a stochastic process which permits an examination of the effects of both unanticipated and anticipated exchange rate movements upon the firm's demand for labor and production. Second, we briefly consider the implications of relaxing the assumption that the firm's decision-makers maximize a quadratic objective function. Finally, we explore the implications of allowing firms and laborers to enter into wage contracts of the Canzoneri-Gray type, discussed in section 1, rather than engage in long term contracts in which the nominal wage rate is fixed for all periods in the contract.

Given our assumptions outlined in section 2a we now also assume that the exchange rate (or real wage rate) follows a covariance stationary first-order autoregressive process

$$(34) \quad (W_t/P_t) = S_t^* = \overline{S}^*(1-\rho) + \rho S^*_{t-1} + Z_t$$

where $0 < \rho < 1$. The mean of the exchange rate process is $\overline{S}^*$[25] and $\sigma_Z^2/(1-\rho^2)$ is the variance. Finally the error term (Z_t) is assumed to be a white noise process such that $E(Z_t) = 0$, $E(Z_e\ Z_j) = \begin{cases} 0, & j \neq e. \\ \sigma_Z^2, & j = e \end{cases}$

The simple stochastic process in (34) can be viewed as having both a predictable component based on the firm's knowledge of the autoregressive (ρ) and drift $(\overline{S}^*(1-\rho))$ parameters and an unpredictable component as represented by the error term (Z_t).

The firm maximizes

$$(35) \quad \max \quad V_t = {}_tE \sum_{j=0}^{\infty} b^j \pi_{t+j} \qquad (j = 0, 1, 2, \ldots)$$

where π_{t+j} is defined in equation (19), $0 < \underline{\underline{b}} < 1$, and ${}_tE = E[\,|\Omega_t]$, where the information set of the firm Ω_t is assumed to contain all the variables in the model dated t and earlier. In contrast to section 2b above the firm's decision-makers maximize the <u>expected</u> present value of profits. They do not know with certainty the future exchange rate (or the real wage rate), but they can form expectations conditioned on information at time t. Given their forecasts of the exchange rate, they must then pick a sequence of N_{t+j} (j=0, 1, 2, ...) and associated sequence of output such that they maximize the firm's present value. More formally this can be seen by representing the firm's optimal employment rule (see Sargent [1979]) as

$$(36) \quad N_{t+j} = \frac{\lambda_1 f_o}{d(1-\lambda_1 b)} + \lambda_1 N_{t+j-1} - \frac{\lambda_1}{d} \sum_{i=0}^{\infty} (\lambda_1 b)^i \, {}_{t+j}ES^*_{t+j+i}$$

$$(j = 0, 1, 2, \ldots)$$

where all parameters are defined as in section 2b above. Thus, given finite adjustment costs ($0 \leq d < \infty$) the firm's decision-makers will determine the level of employment and output by forming forecasts of the real wage rate based on information through the current period. If the firm's decision-makers are assumed to form linear least squares predictions [26] of the exchange rate we have (see Appendix B)

$$(37) \quad {}_tES^*_{t+j+i} = \overline{S}^*(1-\rho^i) + \rho^i S_{t+j}$$

The decision-makers expectation about future exchange rates (i.e. wage rates) will be influenced by the degree of serial correlation (ρ) in the exchange rate process. Moreover, as i (the number of steps in the forecast) becomes large the firms decision-maker's best forecast will tend to the asymptotic mean of the exchange rate process $\overline{S}*$. Substituting (37) into (36) and using the method of undetermined coefficients (see Appendix B)[27] results in a reduced form employment rule of the form

$$(38) \quad N_t = \overline{N} - C \sum_{q=o}^{\infty} \sum_{i=o}^{q} \lambda_1^{q-i} \rho^i Z_{t-q}$$

where $\overline{N} = \frac{1}{f_1}(f_0 - S*)$ is the steady state value of employment.

$C = \frac{\lambda_1}{d}\left[\frac{1}{1-(\lambda_1 b)\rho}\right]$, and λ_1 is defined as in section 2b.

Equation (38) suggests that the demand for labor depends upon an infinite moving average of innovations or shocks (as represented by the Z_{t-q}) in the real wage (or exchange rate) process. These shocks are discounted by ρ (the degree of persistence of the shock) which the firm's decision-makers are assumed to know and λ_1, the stable root, which is itself a function of the discount factor b, the real cost of adjusting labor d, and f_1, a parameter in the production function. Equation (38) also implies that both the anticipated component of the exchange rate process $(\overline{S}*(1-\rho) + \rho S^*_{t-1})$ and the unanticipated shocks (Z_t) will alter the firm's demand for labor and the level of output produced. The fact that anticipated changes in the exchange rate affect the firm's choice of employment and output stems from our assumption that wage earners and firms enter into wage contracts in which wages are fixed for all periods in the contract.

The effect on the firm's demand for labor of an unanticipated exchange rate (or real wage) shock is illustrated in figures 1-3 below. In figure 1, a positive real wage shock which is unanticipated at time t, would cause the firm to employ the level of labor at point b

Figure 1

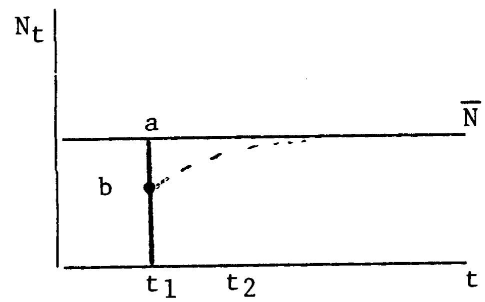


Figure 2

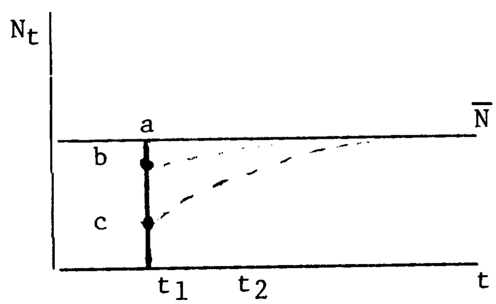
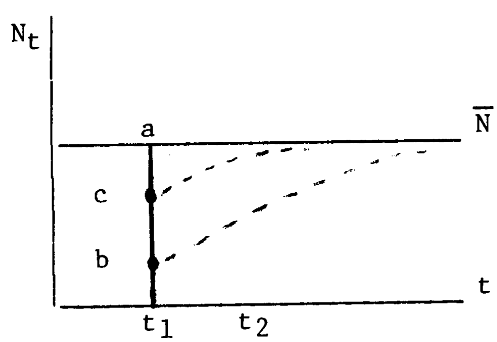


Figure 3



rather than at point a. Over time this shock would be discounted at a rate which would depend upon (1) the persistence of the shock (i.e., the size of ρ) and (2) upon the structural parameters affecting the root λ_1 in the model (as suggested above).

Figure 2 illustrates the effect of letting the persistence (or permanence) of a given unanticipated exchange rate (or real wage) shock increase (i.e., an increase in the size of ρ). In this case if there initially was a positive exchange rate (or real wage) shock (increase in Z_t) without any change in the persistence of the shock we would move from point a to b as in figure 1. However, suppose that given the same real wage shock, the firm perceives the shock to be more persistent or permanent (e.g., ρ has risen). In this case they will move from a to c so that their demand for labor will fall by more than it would have otherwise. Thus, as shocks are viewed as more permanent (thereby increasing the variance of the exchange rate process) firms will tend to adjust their demand for labor by larger amounts initially (e.g. at time t_1) in response to the unanticipated shock.

Figure 3 illustrates how the firm's decision makers will react to an exchange rate shock as adjustment costs rise. Note that for a given level of adjustment costs the firm's decision makers would reduce their demand for labor initially to b; however, if these costs were higher the firm would only reduce its employment to point c. Equation (38) also reveals that when adjustment costs are infinite ($d \to \infty$) the demand for labor is simply equal to $\overline{N}$ (the steady state level of employment). Alternatively as $d \to 0$ a case synonymous with that of section 1 the firm will only be concerned with the exchange rate shock in the current period.

The results presented above are heavily dependent upon the assumptions that (1) the firm's decision-makers maximize a quadratic objective function and (2) that labor contracts specify that the nominal wage be fixed at the same rate for all periods in the contract. The first of these two assumptions is critical because it limits the ways in which variability effects the firm's demand for labor. For example, changes in the variance of unanticipated exchange rate shocks (σ^2_Z) do not enter directly into the parameters or as an independent variable in the firms employment rule (see equations (36) and (38) above). Instead only changes in the extent of serial correlation in the exchange rate process effect the firm's decision-makers forecasts of the real wage rate and hence the level of employment and output. In the present context the variance of the exchange rate could enter into the firms optimal employment rule if either the production function or adjustment costs were of a higher order than quadratic (e.g., cubic). This is due to the fact that the first order conditions of the firm's maximization problem would no longer be linear, so that when conditional expectations were taken, conditional variances would enter the firm's employment rule. Relaxation of the quadratic assumption would allow higher moments to affect the firms employment rule without assuming that the firm's decision-makers are risk averse or that they maximize expected utility, two assumptions which have been prevelant in most of the literature explaining the relationship between exchange rate variability and trade flows.[28]

Finally having derived results for a simple model of the firm given a stochastic exchange rate process and long term wage contracts in which the nominal wage is indefinitely fixed at one level, we investigate the implications of adopting the sort of contracting scheme developed in

Canzoneri [1980] or Gray [1976] (see section 1). In the present context such a contracting scheme would allow laborers and firms to enter into contracts at the beginning of a given period [before exchange rate and other variables are realized] and predict the exchange rate so as to set nominal wages for each period at values expected to clear the labor market. Moreover, firms and laborers would be able to enter into new contracts at discrete intervals at which point they would repeat the process of setting nominal wages described above. Thus, under this type of contracting only changes in the exchange rate not anticipated at the time the contract is negotiated will affect the firms demand for labor.

In sum, this section has investigated how the firm's decision-makers will dynamically adjust employment and output given either an anticipated change in the exchange rate or an unanticipated exchange rate shock. In particular it was found that such decisions were affected importantly by the firm's perceptions about (1) the degree of serial correlation (or persistence) in the exchange rate process and (2) upon the costs of adjusting labor. Moreover, as in section 1 above, whether anticipated exchange rate changes or unanticipated exchange rate shocks will both affect the firms intertemporial chóices of employment and output depended critically on the assumptions made about the nature of firm-labor wage contracts.

3. Exchange Rate Variability and Expected Utility Maximization

In this section we take explicit account of the firm's attitude toward risk, and particularly, how its attitude affects the relationship between exchange rate variability and output and employment. We first assume, as in section 1b, that nominal wages are set for the duration of the period, i.e., we define the period to be the length of the wage contract. We also initially assume that over the period they are committed to a fixed nominal wage, firms are also committed both to the existing stock of capital and the level of labor employed because of high costs of adjusting both factors of production. The assumption of fixed labor inputs differs from that in section 1 and is the same as an extreme case considered in section 2. The firm is assumed to maximize the expected utility of profits, where the firm's attitude toward risk is reflected in whether marginal utility is decreasing, constant, or increasing. We illustrate the standard result in the literature--that an increase in exchange rate variability reduces output and employment if the firm is risk averse--in a simple model that embodies explicit assumptions about the form of the utility function (expotential) and the probability density function of the exchange rate (normal).

In the second part of this section we maintain the assumption of a fixed nominal wage but relax the assumption of fixed labor inputs and instead assume that the firm adjusts the number of workers employed in response to actual exchange rate realizations because costs of adjustment are not infinite. The approach taken here is similar to that in section 2 which explicitly considers alternative assumptions concerning the costs of adjusting labor inputs. As in that section, the firm's profit function is convex in the exchange rate and the expected value of profits is a positive function of the variance of profits. We find that if the firm is risk averse, an increase in exchange rate variability does not necessarily reduce output and employment, a result at variance with the standard one.

3a. Exchange Rate Variability and Expected Utility Maximization - The Standard Results

An exposition of the theory of the firm under uncertainty with expected utility maximization has been done along general lines by Sandmo (1971) and Batra and Ullah (1975) and Helpman and Razin (1978). A recent survey of the literature on the effects of uncertainty on international trade, with emphasis on the real side, has recently been provided by Pomery (1979). Specific applications of the expected utility maximization approach to the effects of exchange rate variability on the level of trade have been done by Clark (1973), Coes (1981), and Hooper and Kohlhagen (1978). Cushman (1983) and Justice (1983) also provide empirical evidence on the impact of exchange rate variability on trade flows.

Here we illustrate the connection between exchange rate variability and output and employment, rather than focusing specifically on the impact on the level of trade flows. We assume the same market structure described above that generates variability in the price of output, i.e.,

$$(39) \quad P = S \cdot P^*$$

where P^* = price of the same commodity in foreign currency and assumed to be fixed. As before, the firm is assumed to be a price taker on domestic and world markets, and commodity arbitrage ensures that the domestic-currency price varies in proportion to the exchange rate. We can without loss of generality define the unit of output such that $P^* = 1.0$, and consequently

P=S. Henceforth we shall make this substitution and define the price of output as equal to the exchange rate.

The firm is a price taker both with regard to its sale price and with respect to the prices, R and W, of the two inputs, K and N, respectively. The prices of the two inputs are assumed to be non-stochastic, and the labor-contracting assumptions are the same as in the first section of the paper. Uncertainty enters through the price of output being a random variable with the mean and variance known to the decisionmaker.[29]

Consistent with the standard approach using expected utility maximization, we assume initially that decisions concerning the volume of output and the hiring of both inputs are made prior to the realization of the market price.[30] We later partially relax this assumption by assuming that the firm adjusts output and the labor input each period in response to the observed exchange rate each period, but varies the capital stock only in the long run when there is a change in the mean and/or variance of the exchange rate. The assumption of short-run fixity of both labor and capital in the standard model appears to reflect an implicit assumption that the costs of adjusting capital and labor are so large in the short run that factor inputs are varied in some unspecifed manner only over some longer-run period as the average level and variability of the firm's price changes. The reasons for the short-run fixity of labor presumably include the explicit and implicit costs of hiring and firing labor, the need to train labor, and union rules specifying the conditions under which labor can be hired or fired. Because these costs of adjusting labor would appear to be lower than adjusting capital in the short run, and because there is a clear profit incentive to adjust output, and therefore, labor input in order to take advantage of price fluctuations, we consider below the case where the firm maximizes the expected utility of profits each period by varying the level of labor input.

We first use a general production function, $Y = f(K, N)$, where first and second derivatives with respect of each factor are positive and negative, respectively. The profit function of the firm is given by:

$$(40) \quad \pi = SY - WN - RK.$$

The firm maximizes the expected utility of profits. Rather than attempt to derive the most general result possible, which has already been done by, e.g. Batra and Ullah (1974), we illustrate our points and pay the price of some loss in generality for a gain in terms of concreteness and simplicity. We assume a given utility function--the expotential--and a specific probability density function for the exchange rate--the normal distribution. These two assumptions make it possible to relate the level of output and employment to explicit expressions involving the mean and variance of the exchange rate.

The utility function of the firm is given by

$$(41) \quad U(\pi) = - \exp(-\theta\pi),$$

where $\theta > 0$. This utility function exhibits positive but declining marginal utility because $U'(\pi) = \theta \exp(-\theta\pi) > 0$ and $U''(\pi) = -\theta^2 \exp(-\theta\pi) < 0$. Also, where $R_a(\pi)$ denotes the level of absolute risk aversion, the expotential utility function is characterized by constant absolute risk aversion = θ because $R_a(\pi) = -U''(\pi)/U'(\pi) = \theta$[31]

Taking the expectation of the utility of profits, where profits are distributed normally because the exchange rate is assumed to be normally distributed, yields:

(42) $E[U(\pi)] = (1/\sigma_\pi\sqrt{2pi}) \int_{\infty} - \exp(-\theta\pi) \exp[-(\pi-\pi)^2/2\sigma_\pi^2]d\pi.$

where pi = 3.1417...., $\overline{\pi}$ $E[\pi]$, and σ_π^2 = variance of profits. After some substitution [see Sargent 1979), p. 150], this reduces to:

(43) $E[U(\pi)] = -\exp[-\theta(\overline{\pi} - \theta\sigma_\pi^2/2)].$

To maximize the expected utility of profits it is then necessary for the decision-maker to maximize the term in parenthesis in equation (43)

(44) $V = \overline{\pi} - \theta\sigma_\pi^2/2.$

By making explicit assumptions about the form of the utility function and the probability density function of profits (or the exchange rate in the present context), one can therefore express the decision-maker's problem as the maximization of a function that is positively related to the expected level of profits and inversely related to the variance of profits.

The expected value of profits is given by:

(45) $E[\pi] = E[SY - WN - RK]$

$= E[Sf(K,N) - WN - RK]$

$= S\ f(\overline{K},N) - WN - RK$

because S, which is distributed as $N(\overline{S}, \sigma_S^2)$ is the only stochastic variable in the model. The variance of profits is therefore equal to a simple function of the variance of the exchange rate and all covariances are therefore zero:

(46) $\sigma_\pi^2 = E[\pi - \pi]^2 \overline{=} f^2\sigma_S^2.$

Substituting the right-hand side of (45) and (46) into equation (44) gives the following expression for the objective function of the firm:

$$(47)\quad V = \overline{S}\, f - WN - RK - \theta\, f^2\, \sigma_S^2/2$$

Maximizing V with respect to the quantity of labor employed, holding capital constant, gives:

$$(48)\quad \partial V/\partial N = \overline{S}\, f_N - W - \theta f\, f_N \sigma_S^2 = 0,$$

which can be re-arranged to yield an expression for the real wage.[32]

$$(49)\quad W/\overline{S} = (1-\theta f\, \sigma_S^2/\overline{S}) f_N.$$

Let $\beta = \theta f\, \sigma_S^2/\overline{S}$. Because $W/\overline{S} > 0$, the values of the variables that determine β are such that $\beta < 1$. The equilibrium employment condition given by equation (49) reduces to the standard non-stochastic real wage = marginal product-of-labor expression if the variance of the exchange rate is zero, i.e., $\sigma_S^2 = 0$, or if the decision-maker becomes risk neutral i.e., $\theta \to 0$.

The equilibrium condition given by equation (49) is depicted in Figure 3. The AA line gives the marginal product of labor, f_N, associated with a given capital stock. The BB line depicts the "adjusted" demand schedule for labor, where the adjustment factor is equal to $1-\beta$. The equilibrium quantity of labor demanded is N_1, which is determined by the intersection of the BB schedule and the given expected real wage.

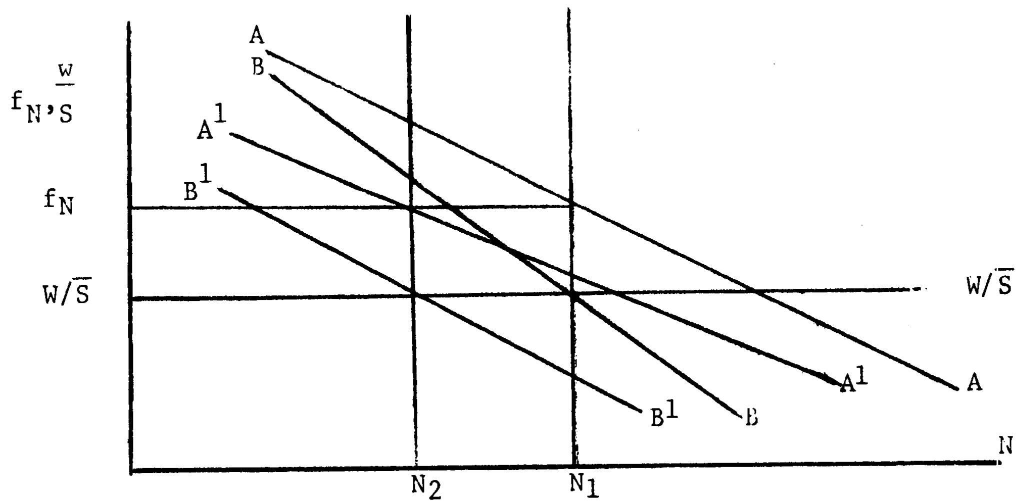


Figure 3. Effects of Exchange Rate Variability on Labor Demand.

Figure 3 depicts a specific example of the general result that if the decision-maker is risk averse, i.e., has a utility function characterized by decreasing marginal utility, the firm will produce at a lower level of output, and hire fewer factors of production, when the price of output is unpredictable compared to a situation where the price is known with certainty. When the firm must make its hiring and production decisions <u>before</u> the actual realization of the price of output, the equilibrium position of the firm is where production takes place such that the average, i.e., expected price, exceeds the marginal cost of production. This condition is shown in Figure 3 as the marginal product of labor exceeding the expected real wage, which implies that the expected exchange rate, $\overline{S}$, exceeds the marginal cost of labor.

Under uncertainty, a risk-averse firm will produce less than when the price is certain because by lowering output the firm reduces the variability of profits. Consider a risk-averse firm facing an initially certain price. Output will be determined where price equals marginal cost. If the price now becomes variable but the expected price and expected profits are unchanged, the firm will no longer be in equilibrium; expected utility will be reduced because the utility of the extra profits is less than the loss in utility from a reduction in profits of the same magnitude. To moderate the loss in expected utility, the firm reduces the level of output because this action will directly reduce the variability in profits [see equation (46)] and thereby increase the expected utility of profits relative to the position of higher output associated with the certain price.

Now let us consider what happens when there is an increase in the variance of the exchange rate, i.e., when there are larger exchange rate fluctuations. Here we assume that the firm adjusts both capital and labor in response to a perceived change in the variance of the exchange rate. The first-order condition for capital employment is similar to equation (48), and is given by:

$$(50) \quad \partial V/\partial K = \bar{S}f_K - R - \theta f\, f_K\, \sigma_S^2 = 0$$

The equilibrium condition for capital employment is:

$$(51) \quad R/\bar{S} = (1-\theta f\, \sigma_S^2/\bar{S})f_K.$$

If the production function is homogeneous of degree one, then because factor prices are assumed to be unchanged, it follows that the firm will continue to hire capital and labor in the same proportion and that the marginal products f_K and f_N will be functions of the unchanged capital-labor ratio. Consequently, for production functions homogeneous of degree one, f_k and f_N will be constant. Because the only two variables that change in equations (49) and (51) are f, i.e., output, and σ_S^2, it follows that an increase in the latter must reduce the former, i.e., greater exchange rate variability reduces output.

This result is depicted in Figure 3. An increase in σ_S^2 affects both the marginal-product-of-labor and "adjusted" demand-for-labor schedules, shifting both down and to the left, the former because the capital stock has declined, and the latter because of the argument in the above paragraph. The new equilibrium at the unchanged real wage and marginal product of labor is characterized by less labor employed, N_2. This effect is a specific example of the impact of a marginal increase in uncertainty. The more general result regarding the effects of an increase in variability or uncertainty is in terms of a mean preserving spread in the probability density function of the price of output. Using this technique, Batra and Ullah (1974) show that under the hypothesis of decreasing absolute risk aversion, increased variability leads to a decline in output and reduction in factor demand.

A major difference between the model and those in the first two sections of the paper is that there is no adjustment in the expected utility model to individual realizations of the exchange rate, whereas in the models in sections 1 and 2 there is full or partial adjustment by the firm to the exchange rate change when it occurs. In terms of Figure 3, deviations of S from $\overline{S}$ and the resulting changes in the real wage do not induce any change in the behavior of the firm; desired and actual labor demand remains at N_1. At the other extreme, in the approach taken in the first section of the paper, the firm always adjusts fully to the change in the price of its output generated by fluctuations in the exchange rate, and moves up and down its demand-for-labor schedule.

The extreme assumption in the expected utility approach of no adjustment to actual exchange rate movements (that do not change the mean value, $\overline{S}$) is consistent with the treatment of uncertainty. The idea is that because of the implicit assumption of very large adjustment costs, firms must make commitments in hiring capital and labor, and therefore output--these models do not allow for inventory changes--without knowing the actual prices at which output will be sold. The uncertainty is generated by the lag between the time factors are hired and the time the price level becomes known. The level of uncertainty in terms of the variability in the actual price relative to its expected value depends on the length of this lag, and therefore on the costs of adjustment. If the firm could adjust capital and labor costlessly in response to the price of output generated by the exchange rate each period, there would be no uncertainty of the kind that is assumed by the expected utility approach. A move in this direction is in fact described below.

3b. Adjustment of Labor Input to Realized Exchange Rates

In this section we relax the assumption that neither labor nor capital are adjusted to the current price, i.e., exchange rate realization. Here we assume instead that labor can be adjusted costlessly in the short run when the current exchange rate is observed. We still maintain the assumption that capital is a fixed factor that is adjusted only in the long run, i.e., the capital stock is chosen before the exchange rate is observed and is altered only in response to the mean and variance of the exchange rate.

Our approach is similar to that of Hartman (1976). We differ in that we use a specific utility function, again the exponential, and we include a nonlinear term for the cost of capital as a proxy for the cost of adjusting the capital stock. This term can be thought of as reflecting an upward sloping supply schedule for capital facing the firms. It is necessary in order to have the size of the firm well-defined. This nonlinear term is embodied in the profit function given below:

$$(52) \quad \pi = Sf(K,N) - WN - R_0K - R_1K^2.$$

We further simplify the exposition by assuming that the production function is Cobb-Douglas (Hartman (1976) uses a CES function):

$$(53) \quad f(K,N) = K^{\alpha} N^{1-\alpha} = K(N/K)^{1-\alpha}$$

The decision-making problem of the firm can be decomposed into two stages. First, the firm decides on a profit-maximizing employment rule which can be used to determine the optimum level of labor employed each period in response to the actual realization of the exchange rate. The level of employment is based on a given level of the capital stock. Second, the firm chooses the level of the capital stock that maximizes the expected utility of profits, where the employment rule is part of the second-stage process.

The profit-maximizing employment rule is the standard "marginal product equals real wage" condition because there is no uncertainty within the period; the firm knows the price in the current period, and because of the assumption of costless adjustment of labor, varys labor input so that equation (54) holds:

$$(54) \quad \partial f/\partial N = (1-\alpha)\ (K/N)^{\alpha} = W/\overline{S},$$

where $\overline{K}$ is the capital stock fixed in the short run, W is, as before, the fixed nominal wage, and S is the actual current-period exchange rate. The profit-maximizing employment rule derived from equation (54) is:

$$(55) \quad N = (1-\alpha)^{1/\alpha} (S/W)^{1/\alpha} \overline{K}$$

To further simplify the exposition we assume that $\alpha = 1/2$. Using equations (52), (53), and (55) give the following expression for profits:

$$(56) \quad \pi = S^2 (1/4\ W)\ \overline{K} - R_0 \overline{K} - R_1 \overline{K}^2$$

Expected profits are given by:

$$(57) \quad E[\pi] = \overline{\pi} = (\sigma_S^2 + \overline{S}^2)\ (1/4\ W)\ \overline{K} - R_0 \overline{K} - R_1 \overline{K}^2$$

In the risk-neutral case, the firm chooses the level of the capital stock that maximizes the expected value of profits. This gives:

$$(58) \quad d\ E[\overline{\pi}]/d\ \overline{K} = (\sigma_S^2 + \overline{S}^2)\ (1/4\ W) - R_0 - 2R_1\ \overline{K} = 0.$$

Solving for $\overline{K}$ yields:

$$(59) \quad \overline{K} = [(\sigma_S^2 + \overline{S}^2)/8\ R_1 W] - R_0/2R_1$$

Note that as $R_1 \to 0$, $\overline{K} \to \infty$. Also note that the optimal capital stock is <u>positively</u> related to the variance of the exchange rate.

Where the decision-maker is risk averse, we again assume that, as above, the exchange rate is distributed as $N(\overline{S}, \sigma_S^2)$ and that the utility function is expotential, so that it is necessary to maximize the objective function given by equation (44) above. Computing the variance of profits results, after some manipulation, in the following expression.

$$(60) \quad \sigma_{\pi}{}^{2} = E[\pi-\bar{\pi}]^{2} = (2\sigma_{S}^{4} + \bar{S}^{2}\,\sigma_{S}^{2})(\bar{K}/4W)^{2}$$

Using equations (57) and (60) to substitute for $\bar{\pi}$ and $\sigma_{\pi}{}^{2}$, respectively, in equation (44) yields the following expression for the optimal capital stock, $\bar{K}$:

$$(61) \quad \bar{K} = (\sigma_{S}^{2} + \bar{S}^{2} - 4R_{0}W)/\{8R_{1}W + \theta[(\sigma_{S}^{4}/2) + (\bar{S}^{2}\,\sigma_{S}^{2}/4]/W\}$$

Equation (61) shows that as the degree of risk aversion increases, the optimal capital stock declines. We know from equation (55) that labor employed, on average, will also decline, and therefore the average level of output will decline.

Of primary interest here is the impact of an increase in the exchange rate variability on the level of factors employed and on the level of output. Inspection of equation (61) shows that if θ is large, we get the standard result that an increase in exchange rate variability will reduce the optimal capital stock and thereby reduce the average level of output of the firm.

However, if θ is small, then greater exchange rate variability can increase the optimal capital stock and thus the firm's average output level.[33] Basically, what is going on is that there are two forces at work. On the other hand, if, as in the case here, the firm can adjust in part to both high and low prices, its expected profits will be larger with greater exchange rate variability; it can increase output, and therefore increase profits when the price is high, and vice versa.[34] When increased exchange rate variability raises both the expected value and the variability of profits, the effects on the optimum capital stock, and therefore on the level of output, depends on the size of θ. With relatively low risk aversion, i.e., small θ, the positive effect on utility of greater price variability on expected profits outweighs the negative impact of utility of greater variability of profits, and the firm will raise the average capital stock and level of output.

A similar result was derived by Hartman (1976). Using a CES production function, he showed that with low elasticity of substitution between capital and labor and returns to scale approaching unity, increased uncertainty could increase the expected value of profits. Using a general concave utility function, he shows that if $U''(\pi)$ is sufficiently small, the higher expected profits dominate the increased variability of profits, and greater price variability raises output. The advantage of the approach taken here is that it provides an explicit solution for the capital stock in terms of measurable parameters, the mean and variance of the exchange rate.

A recent contribution to the literature on the behavior of the firm under uncertainty [Pindyck (1982)] has also shown that under certain conditions, increased price variability can result in increased average investment and output. Pindyck assumes that the price of the firm's output follows a random walk and that it faces increasing marginal adjustment costs, i.e., $C'''(I) > 0$. Pindyck finds that with risk aversion, increased uncertainty will raise the target capital stock and output even if $C'''(I) = 0$. Increasing marginal adjustment costs reinforce the tendency to hold a higher capital stock.

4. Summary and Implied Measures of Volatility

Exchange rates can play an important role in a firm's price and cost structure. Consequently, exchange rate movements may affect the firm's short-run employment decisions, and exchange rate volatility may affect its scale of operation. In this paper, we have discussed a number of ways in which exchange rate fluctuations can be transmitted to real economic activity, and our examples suggest several measures of volatility. Some of these measures have been long recognized; others are new.

In section 1, we focused primarily on exchange rate prediction errors in contracting models like Gray's (1976). Nominal wages are set at levels that are expected to clear the labor market. An unanticipated depreciation will raise the price of an internationally traded good and result in an unexpectedly low real wage, causing the firm to employ more labor. Through its effects on the price structure, an unanticipated depreciation tends to be expansionary; however, the depreciation can also affect the firm's cost structure. If the nominal wage is indexed to a basket that includes foreign goods, then labor costs will rise; if the price of another factor of production, say oil, is fixed in terms of a foreign currency, then oil costs in domestic currency will also rise. These effects are both contractionary. Taken as a whole, these results suggests that prediction errors are an important measure of exchange rate variability, though a given prediction error may have different implications for different firms.

When using this measure of exchange rate variability (or any of the others suggested in this paper), it is important to remember that the results reported above are partial equilibrium in nature; it is assumed that the unanticipated depreciation is not accompanied by other price changes. In general equilibrium, this will not be the case; all prices and exchange rates will move in response to some exogenous shock, and these additional price movements may augment or conteract the exchange rate effects described above. Even if structural features like indexing and the dollar pricing of oil remain unchanged, in general equilibrium there is no fixed relationship between a firm's employment decisions and exchange rate prediction errors, such as given by the following equation:

$$(62) \quad n_t - \bar{n} = \rho(s_t - s_{t|t-1}).$$

The two-country model of Canzoneri and Gray (1984) provides a simple example of this problem. In their model, an unanticipated increase in the home money supply, m_t, depreciates the home currency and is expansionary both at home and abroad; similarly, an unanticipated increase in the foreign money supply, m_t^*, appreciates the home currency and is expansionary in both countries:

$$(63) \quad n_t - \bar{n} = \alpha_1(m_t - m_{t|t1}) + \alpha_2(m_t^* - m_{t|t-1}^*)$$

$$n_t^* - \bar{n} = \alpha_1(m_t^* - m_{t|t-1}^*) + \alpha_2(m_t - m_{t|t-1})$$

$$s_t - s_{t|t-1} = \alpha_3[(m_t - m_{t|t-1}) - (m_t^* - m_{t|t-1}^*)]$$

where $\alpha_1 > \alpha_2 > 0$ and $\alpha_3 > 0$. If a data sample is generated by home monetary disturbances, then the observed correlation in equation (62) will be $\rho = \alpha_1/\alpha_3$. If the data are generated by foreign monetary disturbances, then the observed correlation will be $\rho = -\alpha_2/\alpha_3$. Both the sign and the size of the coefficient ρ in equation (62) depend upon the source of the disturbance generating the fluctuations.

In section 2, we focused primarily on time series properties of exchange rate movements and their implications for employment decisions in firms facing fixed money wage rates and fixed costs of adjustment. If the firm's product is traded internationally, then exchange rate cycles produce cycles in the price the firm receives and in the real wage it has to pay. However, because the firm's profit function is convex in the real wage, it will actually benefit from this cycling even if there are (finite) costs of adjusting employment from period to period. If, for example, the amplitude of the exchange rate cycle increases, the firm can actually increase its average profits by allowing employment to adjust optimally. We found that an increase in the amplitude of the exchange rate cycle will lead to an increase in the amplitude of the employment cycle, and that an increase in the persistence of shocks to the exchange rate cycle will lead to bigger and longer lasting effects in the employment cycle. Here again, all of our results are partial equilibrium in nature.

In section 3, we focused primarily on variances of exchange rates and their implications for the investment decisions of a risk averse firm facing infinite short-run adjustment costs for its capital stock. Suppose the variance of exchange rate fluctuations increases, increasing with it the variance of real wage fluctuations. Here again, the convexity of the profit function will allow the firm to increase the expected value of its short-run profits by varying employment optimally. In the long run, this tends to make the firm want to increase its capital stock. However, the variance of profits is also increased, and this tends to make a risk averse firm want to contract its scale of operations. In theory, either effect can dominate. A high degree of risk aversion will lead the firm to contract in response to an increase in the variance of the exchange rate; a low degree of risk aversion will lead the firm to expand its scale of operations.

Our paper has focused on only a few of the channels through which variability in exchange rates affects the behavior of firms. We have, for example, deliberately ignored the financial structure of the firm. It would be useful to extend the work of this paper by integrating financial considerations into the models developed here to gain further insights into how firms make production and employment decisions in response to increased variability in exchange rates.

FOOTNOTES

1. The macroeconomic literature generally ascribes price stickiness to the "costs" of wage or price setting (see for example Gray (1978), Canzoneri (1980) or Mussa (1981)), or it simply asserts sluggish price adjustment (see Dornbusch (1976)) or the existence of "information islands" (see Lucas (1972) and Barro (1976)) or "contracts" (see Gray (1976), Fischer (1977), or Taylor (1979)). The positive implications of all of these views of price stickiness are similar: monetary disturbances can have transitory real effects. However, the normative implications can vary widely.

2. If labor supply were a function of a real wage defined in terms of a bundle of consumer goods, then the supply curve would be upward sloping, and it would in Figure 1.a shift with the terms of trade; the market clearing solution $(\nu,\bar{n})$ would also be a function of the terms of trade. See Salop (1974) Flood and Marion (1982) and Marston (1982) for discussions of this more complicated framework.

3. Fischer (1977) assumes the goal is to maintain a constant rate of employment. Canzoneri (1980) assumes a union imposed real wage target. In any case, the nominal wage is set on the basis of predicted prices and exchange rates, and the results, for our purposes anyway, are quite similar.

4. Fischer (1977) and Canzoneri (1980) make the same assumption. Barro (1977) has pointed out that there are other contracts that would probably be preferred by both the firm and its workers. For example, workers may be willing to trade a lower expected real wage for more employment stability (see Sargent's (1979) discussion of implicit labor

contracts); that is, the actual employment rule may lie "below" the labor demand curve in Figure 1.a and be "steeper". Our view is that modifications of this sort would probably not seriously alter the basic discussion above; however, this remains a controversial point in contract theory. The comments of Fischer (1980) and Waldo (1981) on Barro's criticism are also relevant.

5. This is essentially Dornbush's (1976) "overshooting" result in a wage contracting framework.

6. This need not be so, from a theoretical point of view anyway. The contractionary foreign monetary policy may actually be inflationary at home. Canzoneri and Gray (1983) describe the transmission mechanisms that tend to produce positive or negative spillovers for monetary policy.

7. In practice, it may be difficult to make this distinction when observing actual labor contracts.

8. Gray (1976) derived optimal indexing rules for closed economies. Marston (1982), Turnovsky (1982) and Flood and Marion (1982) have extended her work to open economies.

9. In the simple model described above, full indexation to P would always achieve the full employment solution. However, adding productivity shocks (which shift the labor demand curve in Figure 1.a) and an elastic labor supply curve (see footnote 2) would complicate the situation considerably. The equilibrium solution (ν, $\bar{n}$) would shift with both productivity disturbances and the terms of trade, and full indexation to either P or I would not be optimal.

10 . Again, one must remember the partial equilibrium nature of the exercise. Suppose, for example, that all contracts were fully indexed and that the depreciation was caused by an unanticipated increase in the home money supply. In this case, all prices (and the exchange rate) would increase proportionately, and there would be no effect on employment or output.

11 . See also Daniel (1981) where the fixed proportions assumption is relaxed.

12 . The phrase "third oil shock" appears to have originated with the French. In at least one instance, it is attributed to the French Minister of Economy and Finance, Delors, by Le Figaro (December 6, 1981).

13 . In addition, the length of the contract may have important implications for monetary policy; see Fischer (1977) and Canzoneri (1980).

14 . This is one reason why indexing provisions in long-term contracts may look like "catch-up" clauses.

15 . Christofides and Wilton (forthcoming) provide empirical support for their hypothesis.

16 . Mussa talks about wage setting, but most of his analysis is centered on the example of a price setting firm. We may be doing him a disservice in what follows.

17 . Begg (1982) discusses these rather anomalous results in some detail.

18 . Firms are concerned with real profits where nominal profits are deflated in each period by the price of the firm's output (P_t). Alternatively, we could have deflated nominal profits by a general price level (PI) which can be viewed as constant over time or it could be assumed to grow at some rate which would become part of the discount factor used to compute the present value of future profits.

19 . Relaxation of the assumption of a quadratic objective function is explored in section 2c below.

20 . Relaxing assumptions about world prices and nominal wages being fixed would lead to more complicated processes generating real wages. In this case the covariances between components of the real wage become important in affecting the behavior of the firm, as in Section 1. For our purposes, it is more convenient to look at the foreign currency price of the domestic currency, S_t^* , rather than its reciprocal, S_t.

21 . Blinder [1982] and Flood and Hodrick [1982] develop similar models to those considered below except that the firm is monopolist in output markets and is subject to demand rather than exchange rate shocks. In their models firms adjust inventories rather than labor intertemporally in response to exogenous shocks; however, the results of their analysis do not differ significantly from that presented below for the case of a stochastic real wage process (section 2c). Moreover, these authors do not consider how purely deterministic movements in exogenous variables will affect decision variables, as we do in section 2b.

22. In Appendix A we generalize the cyclical process of the exchange rate by allowing the period of the cycle to be any integer greater than zero (we have implicitly assumed a period of two for the exchange rate cycle presented in this section). The results obtained there correspond to those presented here.

23. The sum of two convex functions is convex.

24. See Appendix A for the derivation.

25. Assuming that the exchange rate is generated by a process like (34) allows us to investigate the effects of changes in the persistence (i.e., changes in ρ) and hence in the variability of the exchange rate without allowing the mean to change as in the work of Rothschild and Stiglitz [1970] on mean preserving spreads.

26. The reader should note that in general the conditional expectation $E[W_{t+j} | \Omega_t]$ is not a linear function in the variables contained in Ω_t. However, a sufficient condition for this to be the case is that the variates in W_t and those in Ω_t follow a multivariate normal distribution. In the present context given a simple first order Markov process generating the exchange rate where Ω_t contains contemporaneous and lagged values of the exchange rate (or real wages) one needs to assume that the exchange rate is distributed normally, implying in the present context that the real wage rate will also be distributed normally.

27. The method of undetermined coefficients is one method of solving a difference equation (see Appendix B), to obtain a particular solution.

28. See, for example, Hooper and Kohlhagen []978] and section 3 below.

29. Some authors, e.g., Batra (1975), introduce uncertainty by specifying a randon term in the production function.

30. These are referred to as "commitment models" by Pomery (1979).

31. Following Pratt (1964) and Arrow (1965), a decision-maker's attitude toward risk can be defined as:

$$R_a(\pi) = - U''(\pi) / U'(\pi)$$

where $R_a(\pi)$ is the absolute risk-aversion function. For risk-averse decision-makers, $R_a(\pi)$ is decreasing in π.

32. A similar expression can be derived for the real return on capital.

33. Hartman (1976, p. 681) derives the same result.

34. This result is similar to that shown in Appendix A, where it is demonstrated that the present value of the firm increases as long as it can adjust to some extent within the period in response to exchange rate flucuations. A recent contribution to the literature by Sikdar (1984) has also provided a result similar to that given here. Sikdar finds that if a firm postpones the choice of the input mix and level of output until the actual value of a random input price becomes known, then uncertainty may induce a risk-neutral or risk-averse firm to produce more than in the case of certainty, and increased risk in the form of a mean-preserving spread in a random input price may lead to an expansion of production by a risk-neutral firm.

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Appendix A

This model of the firm is taken directly from Sargent [1979]. His discussion begins on page 196. Since we are not interested in productivity shocks here we set his $a_{t+j} = 0$ for all t and j. Sargent states the problem of the firm as:

$$\text{(A1)} \quad \underset{N_{t+j}}{\text{Max}}\ v_t = \sum_{j=0}^{\infty} b^j \{f_0 N_{t+j} - 1/2 f_1 N_{t+j}^2 - 1/2d(N_{t+j} - N_{t+j-1})^2 - w_{t+j} N_{t+j}\}$$

where all variables are defined as in Section 2a and $w_t = (W_t/P_t)$ is the real wage as defined in equation (21).

Sargent demonstrates that the demand schedule for employment found by maximizing over the N_{t+j} can be written as

$$\text{(A2)} \quad N_{t+j} = \lambda_1 N_{t+j-1} - (\lambda_1/d) \sum_{i=0}^{\infty}(1/\lambda_2)^i (w_{t+j+i} - f_0) \qquad j = 0, 1, 2, \ldots$$

where λ_1 and λ_2 are the roots of the difference equation and the following conditions are known to hold:

(A3) $0 < \lambda_1 < 1 < \lambda_2$,

(A4) $1/\lambda_2 = \lambda_1 b$

(A5) $(f_1/d) + (1+b) = \lambda_1 b + 1/\lambda_1$.

Equation (A2) can be rewritten as

$$\text{(A6)} \quad N_{t+j} = \lambda_1 N_{t+j-1} + \lambda_1 f_0/d(1 - 1/\lambda_2) - (\lambda_1/d) \sum_{i=0}^{\infty} (1/\lambda_2)^i w_{t+j+i}.$$

Solving for N_{t+j}, finally, we get:

$$\text{(A7)}\quad N_{t+j} = \lambda_1 f_0/d(1 - 1/\lambda_2)(1-\lambda_1)$$

$$- (\lambda_1/d) \sum_{k=0}^{\infty} \lambda_1^k \sum_{i=0}^{\infty} (1/\lambda_2)^i \, w_{t+j+i-k}$$

$$+ c\lambda_1^{t+j}.$$

As in the text, we will henceforth assume that t+j is large enough so that the effect of any initial conditions on the firm's employment rule are negligible. To simplify (A7), we note that (A5) can be rewritten as:

$$\text{(A8)}\quad \lambda_1/d = (1- \lambda_1 b)(1- \lambda_1)/f_1.$$

Substituting from (A4) into (A8), we get:

$$\text{(A9)}\quad \lambda_1/d = (1 - 1/\lambda_2)(1 - \lambda_1)/f_1.$$

Using (A9) and the fact that

$1/(1 - 1/\lambda_2) = \sum_{i=0}^{\infty} (1/\lambda_2)^i$ and $1/(1- \lambda_1) = \sum_{i=0}^{\infty} \lambda_1^i$, we can rewrite

(A7) in the following form:

$$\text{(A10)}\quad N_{t+j} = f_0/f_1 = (1/f_1) \sum_{\ell=-\infty}^{\infty} Z_\ell \, w_{t+j+\ell}$$

where:

$$Z_\ell = \begin{cases} (1/s) \sum_{i=0}^{\infty} (\lambda_1)^{i-\ell} (1/\lambda_2)^i \text{ for } \ell < 0 \\[2ex] (1/S) \sum_{i=0}^{\infty} (\lambda_1)^i (1/\lambda_2)^{i+\ell} \text{ for } \ell > 0 \end{cases}$$

$$\sum_{\ell=-\infty}^{\infty} Z_\ell = 1$$

$$S = \sum_{n=0}^{\infty} \lambda_1^n \sum_{n=0}^{\infty} (1/\lambda_2)^n.$$

From (A10) we can see that labor's marginal product equals a weighted average of all past and future real wage rates, i.e.,

$$\text{(A11)} \quad f_0 - f_1 N_{t+j} = \sum_{\ell=-\infty}^{\infty} Z_\ell w_{t+j+\ell}.$$

If we examine the weights, Z_ℓ, more closely we see they can be written as:

$$\text{(A12)} \quad Z_\ell = \begin{cases} \lambda_1^{-\ell} Z_0 & \text{for } \ell < 0 \\ \\ (1/\lambda_2)^\ell Z_0 & \text{for } \ell > 0. \end{cases}$$

where $Z_0 = (1 - \lambda_1)[1 - (1/\lambda_2)]/[1 - \lambda_1/\lambda_2)]$.

Thus	$Z_0 > Z_i$	$\forall\ i \neq 0$
	$Z_j > Z_{j+1}$	$\forall\ j \geq 0$
	$Z_k > Z_{k-1}$	$\forall\ k < 0$
	$Z_k > Z_{-k}$	$\forall\ k < 0$

because $Z_k = \lambda_1^{-k} Z_0 > (b\lambda_1)^{-k} Z_k$

$$= (1/\lambda_2)^{-k} Z_0 = Z_{-k}$$

The weight on the current wage rate is the largest. More recent wage rates are weighted more heavily than those further into the future or past. And this last inequality implies that each weight over history is largest than its corresponding weight over the future.

If we let the wage rate path follow some periodic function, F, such that

$$\text{(A13)} \quad w_{t+j} = F(t+j) = F(t+j+P)$$

when P is the period of the function, then (A11) becomes

(A14) $f_0 - f_1 N_{t+j} = \sum_{\ell=-\infty}^{\infty} z_\ell \, F(t+j+\ell)$

Since $F(t+j+\ell)$ takes the same value every P time periods, we can partition the summation in (A14) into P summations, as follows:

(A15) $f_0 - f_1 N_{t+j} = \sum_{\ell=0}^{P-1} q_i \, F(t+j+\ell)$

where $q_i = \sum_{\ell=-\infty}^{\infty} Z_{\ell P+i}$ $\qquad i = 0, 1, 2, \ldots\ldots, P-1$

We can see, therefore, that if the wage rate follows a periodic function, the firm will hire labor such that the marginal product of labor equals a weighted average of the values the wage rate takes across its cycle.

If we look more closely at the weights, q_i, we can derive a further result: that for any function F of any period P, as λ_1 varies from zero to one, the values the weights take vary from $q_0 = 1$ and $q_{i\neq 0} = 0$ to $q_i = 1/P$ for all i. In other words, the weight shifts from being concentrated completely on q_0 to being divided equally across all weights. To see this, break up the summation in (A15) for q_i to obtain

(A16) $q_1 = z_0 \dfrac{(\lambda_1 b)^i}{-(\lambda_1 b)^P} + \dfrac{\lambda_1^{P-i}}{1 - \lambda_1^P}$

Now as $\lambda_1 \rightarrow 0$,

(A17) $z_0 = \dfrac{(1 - \lambda_1)(1 - \lambda_1 b)}{(1 - \lambda_1^2 b)} \rightarrow 1$

and

$$q_i \to \left\{ \frac{(0)^i}{1} + 0 \right\} \qquad \text{for } i=0,1,2,\ldots, P-1$$

Therefore, $q_0 \to 1$ and all other $q_i \to 0$.

To see what happens as $\lambda_1 \to 1$, we must rewrite (A16) substituting the expression for z_0 shown in (A17) and using the fact that

$$1-(\lambda_1 b)^P = (1-\lambda b)\left(\sum_0^{P-1} (\lambda_1 b)^\ell\right)$$

$$1-\lambda_1^P = (1-\lambda_1)\left(\Sigma_0^{P-1}\lambda_1^\ell\right).$$

These substitutions yield:

$$\text{(A18)} \quad q_i = \frac{(1-\lambda_1)(\lambda_1 b)^i}{(1-\lambda_1^2 b)(\Sigma(\lambda_1 b)^\ell)} + \frac{(1-\lambda_1 b)\lambda_1^{P-i}}{(1-\lambda_1^2 b)(\Sigma\lambda_1{}^\ell)}$$

From (A18), then, we can see that as $\lambda_1 \to 1$,

$$q_i \to \left\{ 0 + \frac{1}{P} \right\} \qquad \text{for } i=0,1,2,\ldots,P-1$$

This result can now be used to make a more general claim. Since as adjustment costs (d) approach zero, we can see that for any general periodic function, as $d \to 0$, the firm's decision-makers need not concern themselves with any but the current wage rate in the cycle. Similarly, as d increases without bound, λ, approaches unity and we can see that firms give equal weight to all the values of the wage rate that occur in the cycle.

To derive the specific labor demand rule discussed in the text, let us set

(A19) $F(t+j) = \overline{w} + a(-1)^{t+j}$

Then from (A15) we see that the firm's labor demand rule is given by:

(A20) $$N_{t+j} = \frac{f_0}{f_1} - \frac{1}{f_1} \sum_{\ell=0}^{P-1} q_\ell (\overline{w} + a(-1)^{t+j+\ell})$$

Since $\sum_{\ell=0}^{P-1} q_i = 1$ and since the period of this particular specification of F is two, (A20) can be rewritten as:

(A21) $$N_{t+j} = \frac{f_0 - \overline{w}}{f_1} - \frac{a}{f_1} \{q_0 - q_1\} (-1)^{t+j}$$

From (A16) we can see that

(A22) $$q_0 - q_1 = \frac{1/\lambda_1 - (1+b) + \lambda_1 b}{1/\lambda_1 + (1+b) + \lambda_1 b}$$

Using (A5) we can obtain:

(A23) $$q_0 - q_1 = \frac{f_1}{f_1 + 2d(1+b)}$$

Therefore,

(A24) $$N_{t+j} = \frac{f_0 - \overline{w}}{f_1} - \frac{a(-1)^{t+j}}{f_1 + 2d(1+b)}$$

This is the expression for labor demand given in the text. This expression for N puts the firm's employment rule in terms of clearly interpretable parameters.

To find the present value v_t^* that corresponds to the firm's choice of N_{t+j}, we can substitute (A24) the expression for w_{t+j} into (A1) to obtain

$$\text{(A25)}\quad v_t^* = \sum_{j=0}^{\infty} b^j \Big\{ [f_0 - \bar{w} - a(-1)^{t+j}][(f_0 - \bar{w})/f_1 - a(-1)^{t+j}/(f_1 + 2d(1+b)]$$

$$- \frac{f_1}{2}\left[\frac{f_0-\bar{w}}{f_1} - \frac{a}{f_1 + 2d(1+b)}(-1)^{t+j}\right]^2$$

$$- \frac{d}{2}\left[\frac{-2a\ (-1)^{t+j}}{f_1 + 2d(1+b)}\right]^2 \Big\}$$

$$= \sum_{j=0}^{\infty} b^j \Big\{ \frac{(f_0-\bar{w})^2}{f_1} - \frac{(f_0-\bar{w})a(-1)^{t+j}}{f_1 + 2d(1+b)} - \frac{a(f_0-\bar{w})}{f_1}(-1)^{t+j} + \frac{a^2}{f_1 + 2d(1+b)}$$

$$- \frac{f_1}{2}\left[\frac{(f_0-\bar{w})^2}{f_1^2} - \frac{2(f_0-\bar{w})a(-1)^{t+j}}{f_1[f_1 + 2d(1+b)]} + \frac{a^2}{[f_1 + 2d(1+b)]^2}\right]$$

$$- \frac{d}{2}\left[\frac{4a^2}{[f_1 + 2d(1+b)]^2}\right] \Big\}$$

which after some further manipulation can be written as:

$$v_t^* = \frac{(f_0-w)^2}{f_1(1-b)} - \frac{(f_0-w)a(-1)^t}{(1+b)[f_1 + 2d(1+b)]} - \frac{a(f_0-w)(-1)^t}{f_1(1+b)} + \frac{a^2}{(1-b)[f_1 + 2d(1+b)]}$$

$$- \frac{(f_0-w)^2}{2f_1(1-b)} + \frac{a(f_0-w)(-1)^t}{(1+b)[f_1 + 2d(1+b)]} - \frac{a^2 f_1}{2(1-b)[f_1 + 2d(1+b)]^2}$$

$$\frac{-2da^2}{(1-b)[f_1 + 2d(1+b)]^2}$$

$$= \frac{(f_0-\bar{w})^2}{2f_1(1-b)} + \frac{a^2}{2(1-b)[f_1 + 2d(1+b)]^2} \; 2[f_1 + 2d(1+b)] - f_1 - 4d]$$

$$- \frac{a(f_0 - \bar{w})(-1)^*}{(1+b)[f_1 + 2d(1+b)]} \left[1 + \frac{f_1 + 2d(1+b)}{f_1} - 1 \right]$$

$$\text{(A26)} \quad v^*_t = \frac{(f_0-\bar{w})^2}{2f_1(1-b)} + \frac{a^2[f_1 + 4db]}{2(1-b)[f_1 + 2d(1+b)]^2} - \frac{a(f_0-\bar{w})(-1)^t}{f_1(1+b)}$$

where

$$\frac{\partial v^*_t}{\partial a} = \frac{[f_1 + 4db]a}{(1-b)[f_1 + 2d(1+b)]^2} - \frac{(f_0-\bar{w})}{f_1(1+b)} \; (-1)^t$$

Thus, the value of the present value function depends on whether it is even or odd. When t is odd, the present value calculation begins in a period in which the wage rate is low and employment and output are high. In this case, variability (i.e., $a \neq 0$) can only increase the present value of the firm.

When t is even, however, it is possible that variability will decrease the present value. In the neighborhood of $a = 0$, an increase in a will decrease the present value. However, for large enough values of a, an increase in a will cause the present value to rise and it may be true that the present value at some big enough value of a will be larger than the present value at $a=0$.

One can see that, on average, the present value of the firm is higher with variability than without it:

$$\frac{v^*_{t+(t+1)}}{2} = \frac{(f_0-\bar{w})^2}{2f_1(1-b)} + \frac{a^2[f_1 + 4db]}{2(1-b)[f_1 + 2d(1+b)]^2}$$

Appendix B

In this appendix we use the model of the firm in Sargent [1979] to derive equations (37) and (38) in the text. The formal derivation of equation (36) in the text can be found in Sargent [1979] p. 156. Equation (37) in the text can be obtained by noting that the i- step ahead predictor for the real wage (or exchange rate) process in equation (34) can be written as

$$\text{(B1)}\quad {}_tES^*_{t+i} = \bar{S}^*(1-\rho)(1+\rho+\ldots+\rho^{i-2}+\rho^{i-1}) + \rho^i S^*_t$$
$$= \bar{S}^*(1-\rho^i) + \rho^i S^*_t$$

Moreover in the limit as the number of periods ahead (i) for which the forecast is being made becomes large

$\lim_{i \Rightarrow \infty} {}_tES^*_{t+1} = \bar{S^*}$ so that the mean of the real wage process will tend to become the optimal forecast for real wages when i is large. This result is due to the fact that we are considering a covariance stationary invertible first order autoregressive process where $0 < |\rho| < 1$. Note also that although changes in the autoregressive parameter ρ will not effect the mean of the exchange rate process (as $i \Rightarrow \infty$) the variance of the process can be written as $\text{var}\,(S^*_t) = \frac{\sigma_Z^{\;2}}{1-\rho^2}$ suggesting that changes in the autoregressive parameter will effect the variance of the exchange rate or the real wage rate.

Now to obtain equation (38) in the text we first write equation (36), letting j=0, as

$$\text{(B2)}\quad N_t = \frac{\lambda_1 f_0}{d(1-\lambda_1 b)} + \lambda_1 N_{t-1} - \frac{\lambda_1}{d} \sum_{i=0}^{\infty} (1/\lambda_2)^i\, {}_tES^*_{t+i}$$

Now substituting (B1) into (B2) yields

$$\text{(B3)}\quad N_t = \frac{\lambda_1 f_0}{d(1-\lambda_1 b)} + \lambda_1 N_{t-1} - \frac{\lambda_1}{d}\ [\bar{S}^* \sum_{i=0}^{\infty} (1/\lambda_2)^i$$

$$- \bar{S}^* \sum_{i=0}^{\infty} (1/\lambda_2)^i\ \rho^i + \sum_{i=0}^{\infty} (1/\lambda_2)^i \rho^i S_t^*]$$

$$= b_0 + \lambda_1 N_{t-1} - (\lambda_1/d) \cdot (1/(1-(1/\lambda_2)\rho)S_t^*$$

where,

$b_0 = (\lambda_1 f_0/d(1-\lambda_1 b) - (\lambda_1/d)[\bar{S}^*(1/(1-(1/\lambda_2)) - (1/(1-(\lambda_2)\rho)]$

since $0 < (1/\lambda_2) < 1$ and $0 < |\rho| < 1$. Now to obtain a solution for the difference equation in (B3) in terms of all past shocks to the exchange rate process we first note that the moving average representation of the exchange rate process can be written as

$$\text{(B4)}\quad S_t = \bar{S}^* + \sum_{i=0}^{\infty} \rho^i Z_{t-i}$$

substituting (B4) into (B3) yields

$$\text{(B5)}\quad N_t = C_1 + \lambda_1 N_{t-1} - C_2 \sum_{i=0}^{\infty} \rho^i Z_{t-i}$$

where $C_1 = b_0 - C_2 \cdot \bar{S}^*$

and $C_2 = \dfrac{\lambda_1}{d} \cdot \dfrac{1}{(1-(1/\lambda_2)\rho)}$

Now suppose our hypothesized solution to (B5) is

$$\text{(B6)}\quad N_t = \bar{N} + \sum_{i=0}^{\infty} \phi_i Z_{t-i}$$

substituting (B6) into (B5) for N_t, and N_{t-1} and solving for the ϕ's in terms of the structural parameters in the model using the method of undetermined coefficients yields

$$(B8)\quad N_t = N - C \sum_{q=0}^{\infty} \sum_{i=0}^{q} \lambda_1^{q-i} \rho^i Z_{t-q}$$

where $N = \frac{C_1}{1-\lambda_1} = \frac{1}{f_1}(f_0 - S^*)$

by using the fact that

$\frac{1}{f_1} = \frac{\lambda_1}{d(1-\lambda_1 b)(1-\lambda_1)}$ as indicated in equation (A8) of appendix A.

Also $C = \frac{\lambda_1}{d}\left[\frac{1}{1-(\lambda_1 b)\rho}\right]$ where it should be observed that

$|\lambda_1| < 1$ and we have assumed that the effects of any initial conditions on the firms employment rule are negligible. Note that equation (B8) corresponds to equation (38) in the text.

Some of the results regarding the effects of unanticipated exchange rate shocks on firm behavior described graphically in the text can be obtained more formally by deriving the variance of the demand for labor by using equation (B8). Thus we have

$$(B9)\quad \begin{aligned} Var(N_t) &= E[N_t - \overline{N}]^2 \\ &= E[\overline{N} - C \sum_{\ell=0}^{\infty} \phi_\ell Z_{t-\ell} - \overline{N}]^2 \\ &= \sigma_Z^2 (-C)^2 \sum_{\ell=0}^{\infty} \phi_\ell^2 \quad \text{since } E[Z_t Z_s] = 0 \text{ for } s \neq t. \end{aligned}$$

substituting for ϕ_ℓ we obtain:

$$[\sum_{\ell=0}^{\infty} \sum_{i=0}^{\ell} \lambda_1^{(\ell-i)} \rho^i]^2 = \sum_{j=0}^{\infty} \lambda_1^{j2} + \rho^2 \sum_{j=0}^{\infty} \lambda_1^{j2} + \rho^4 \sum_{j=0}^{\infty} \lambda_1^{j2}$$

$$+ \ldots\ldots\ldots] = 1/(1-\lambda^2)(1/(1-\rho^2)$$

so we have

$$\text{(B9')} \quad Var(N_t) = \sigma_Z^2(-C)^2(1/1-\lambda_1^2)(1/1-\rho^2)$$

where

$$\partial Var(N_t)/\partial\sigma_Z^2 = (-C)^2(1/1-\lambda_1^2)(1/1-\rho^2) > 0$$

∂ Var $(N_t)/\partial\rho > 0$ by inspection of (B9)'