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THE INTERNATIONAL TRANSMISSION OF OIL PRICE EFFECTS
AND OPEC'S PRICING POLICY

by

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Abstract

Analysis of oil-price effects generally maintain the assumption that oil-importers can be treated as small economies, which allows oil-price changes to be treated as exogenously set by OPEC. Analyses of oil-price determination rely on the assumption that the demand for oil is a stable function, which implies that real income of oil importers is unaffected by oil-price changes. Our analysis treats oil prices and economic activity as jointly determined. The effects of exogenous oil-price changes are studied in a simple theoretical world model. Hotelling's analysis is generalized to allow for both oil-price feedback effects and stabilization policies.
TO THE MEMORY OF

RAY LUBITZ
THE INTERNATIONAL TRANSMISSION OF OIL-PRICE EFFECTS AND OPEC'S PRICING POLICY

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Introduction

The effects of increases in oil prices on the world economy have stimulated research devoted to two concerns—namely, the macroeconomic consequences of the oil price increases and the underlying factors determining oil prices. To address the first of these concerns, most investigators have treated oil-price changes as supply shocks in the context of a small, open oil-dependent economy. In this framework, an increase in oil prices raises domestic prices and reduces income and employment, provided oil and labor (and other domestic factors) are not perfect substitutes for each other in production processes. Further analyses have examined the macroeconomic implications of higher oil prices under alternative policy responses and wage indexation schemes.

Although these analyses enhance our understanding of the effects of supply shocks, their usefulness in predicting the macroeconomic effects of an oil-price increase remains limited for several reasons. First, the effects of oil price increases on a particular economy depend on the repercussions of these increases on the rest of the world economy. For example, the transfer of resources from oil importers to the [countries of the] Organization of Petroleum-Exporting Countries generated by an oil-price hike is greatly diminished if OPEC increases its recycling of oil revenues through an increase in imports. In addition, higher oil prices result in higher export prices of industrialized countries with further implications for the growth prospects of developing countries. Yet most analyses do not incorporate these international linkages explicitly. Second, the few studies that do consider these linkages give little or no attention to the implications of higher oil prices for developing countries; therefore the effects of higher oil prices on a significant portion of the world's income and trade remain largely unexplained.
A third limitation of existing analyses is the assumption that oil importers can be treated as small economies. This is a very convenient assumption because it allows oil-price changes to be treated as exogenously set by OPEC. However, this price-taking assumption is of limited applicability to major oil importers. An increase in oil prices lowers their real income and oil consumption, and thus OPEC's revenues, thereby raising the question of why OPEC would increase oil prices. To assume that oil price changes are exogenous presumes that these changes are optimal, regardless of their consequences for oil producers. Thus the price shocks postulated in earlier analyses need not be observed in reality as they may be inconsistent with OPEC's pricing strategy.

In this regard, the large oil-price increases of the seventies also renewed the interest in oil-price determination. Typically, OPEC is characterized as a monopolist determining oil prices in such a way as to maximize the present value of future oil revenues, subject to a stable demand function and to a resource constraint. To assume a stable demand function amounts to assuming that real income—a demand determinant—can be considered exogenous and thus unaffected by the changes in oil prices, an assumption that stands opposed to the theoretical and empirical results. To the extent that higher oil prices affect the real income of oil importers, there is a feedback effect on the oil exports of OPEC, which should be recognized in setting an optimal price path.

Earlier analyses of oil-price effects and oil-price determination have proceeded in a dichotomous manner; what is assumed exogenous in one kind of analysis is endogenous in the other, and vice-versa. A realistic analysis should treat oil prices and economic activity as jointly
determined. In this context OPEC would face (in addition to the intertemporal allocation problem) a tradeoff between exploiting the price inelasticity of oil demand on the one hand, and avoiding the income feedback effects on the other. In particular, if the price path is too low, then OPEC is not exploiting the price inelasticity of oil demand and thus incurs revenue losses. If the price path is too high, then the real income of oil importers is adversely affected with a detrimental impact on OPEC's revenues.

The purpose of this paper is to examine the effects of oil-price changes and the determination of oil prices as one problem, by recognizing that oil-price changes affect the real income of oil importers and that changes in the real income of oil importers influence oil-price changes. The next section presents a simple theoretical model of the world economy to examine the different channels through which the effects of an exogenous oil-price increase are internationally transmitted.

The focus of the next two sections is on the comparative statics of an increase in the price of oil and on the sensitivity of the results to changes in both parameter values and policy reactions. We find that increases in oil prices are not necessarily associated with losses in output if one allows for the international repercussions of oil-price effects. The following section contains a generalization of Hotelling's analysis of oil-price determination to allow for the international transmission of oil price effects. The main result is that Hotelling's r-percent rule remains valid only in the extreme case in which oil-price changes have no feedback effects. Moreover, recognition of these feedback effects allows a closer examination of the influence of stabilization policies in developed
countries on optimal pricing strategies. Our main result is that optimal oil prices decline in response to a restrictive policy stance. Finally, the last section contains our conclusions.

A Theoretical Model Of Oil Price Effects

The world economy is divided into three country blocs: developed economies; oil-producing countries, OPEC; and non-OPEC developing countries. These country blocs export manufactures, oil, and primary products respectively. Financial asset holdings and nominal exchange rates are not modeled. Income is determined by demand in developed countries and by supply in both developing countries and OPEC. The structure of the model appears in Table 1.

Developed Economies

The real income of developed countries, $Y_d$, is determined from the demand side in equation (1), implying a situation of excess capacity in these countries. There are three demand components: private absorption, $C_d(Y_d)$; an exogenously given level of government purchases, $G_d$; and the trade account, $B_d$ from equation (2). Without financial assets the trade account equals the balance of payments; capital movements compensate for the flow of goods (Dornbusch and Fischer 1980 determine capital movements in a similar way). The trade balance is expressed in terms of manufactures, and thus the model incorporates the terms-of-trade effect of changes in oil prices, because oil imports are valued in terms of the manufactured goods needed to pay for them.
Exports of manufactures to both OPEC and developing countries are described in more detail below. Here it is simply indicated that they depend on the terms of trade and on the foreign exchange revenues of these countries. Oil imports from OPEC, \( M_o^d \) in equation (3), and primary-product imports from developing countries, \( M_p^d \) in equation (4), depend on the terms of trade and on the real income of developed countries. Finally, the price of manufactures, \( P_m \) in equation (5), is assumed to be a linear markup of average variable costs, as represented by the prices of primary products, \( P_p \), and oil, \( P_o \).

Non-OPEC Developing Countries

In developing countries, it is the scarcity of capital services, not of aggregate demand, that limits output growth in these economies. Thus real income of developing countries, \( Y^d \) in equation (6), is determined from the supply side by using a production function with capital, \( K^d \), and fixed labor, \( L^d \), as factors. Equation (7) defines the capital stock as accumulated net investment, \( I^d \), which in turn depends on imports of capital goods, \( M_m^d \), from equation (8). This latter relation can be derived by treating the economy-wide capital stock as an aggregate of a domestically made component and a foreign-made component, as suggested by McKinnon (1964) and Taylor (1979).

Foreign exchange constraints may limit output growth in developing countries. In this model, those constraints apply to imports of manufactures, \( M_m^d \) in equation (9). If they are binding, then imports of manufactures will be limited, dampening capital accumulation and income growth. These imports are financed by foreign exchange revenues (exports plus aid), net of oil payments to OPEC. This assumption implies that the
trade account of the developing countries is in balance except for external
credits and aid, R. Finally, oil imports from OPEC, $M_o$ in equation (10),
are determined as a function of the terms of trade and real income.\(^{13}\)

**OPEC**

The determination of OPEC's income is analogous to the case for
developing countries. The chief difference is that OPEC is assumed to
recycle a constant fraction $\beta$ of export earnings to purchase manufactures
from developed economies, as indicated by equation (14). As development
policies in OPEC evolve, with a corresponding impact on absorption of oil
revenues, the assumption of a constant value for $\beta$ appears to be a strong
one: it is used here to simplify the comparative-statics derivations. The
effects of relaxing it are studied below.

**The International Transmission Of Oil Price Effects**

**Analytical Solution of the Model**

Solving the model begins with totally differentiating the system of
equations (1) - (14); by using the equilibrium conditions (15)-(17), we
obtain a system of three differential equations:\(^{14}\)

$$
\begin{bmatrix}
Y_{11} & Y_{12} & Y_{13} \\
Y_{21} & Y_{22} & Y_{23} \\
Y_{31} & Y_{32} & Y_{33}
\end{bmatrix}
\begin{bmatrix}
dY^d/Y^d \\
dY^s/Y^s \\
dY^o/Y^o
\end{bmatrix}
= 
\begin{bmatrix}
\omega_{11} & \omega_{12} \\
\omega_{21} & \omega_{22} \\
\omega_{31} & \omega_{32}
\end{bmatrix}
\begin{bmatrix}
dP_o/P_o \\
dG^d/G^d
\end{bmatrix}
\cdot (18)
$$

The elements of the matrix $\Gamma_{ij}$ are

$$
Y_{11} = 1 - c^d + (1-\beta)\phi_o \eta_o > 0,
$$
\[ Y_{12} = (1-\beta) \phi_{o}^{d} \eta_{o}^{d} g_{d} > 0, \]
\[ Y_{13} = 0, \]
\[ Y_{21} = -f \ i \ \phi_{p}^{d} \eta_{p}^{d} g_{p}^{d} < 0, \]
\[ Y_{22} = 1 + \phi_{o}^{d} \eta_{o}^{d} f \ i > 0, \]
\[ Y_{23} = 0, \]
\[ Y_{31} = -F \ b \ \beta \ \phi_{o}^{d} \eta_{o}^{d} g_{o}^{d} < 0, \]
\[ Y_{32} = -F \ b \ \beta \ \phi_{o}^{d} \eta_{o}^{d} g_{o}^{d} < 0, \]
\[ Y_{33} = 1. \]

The elements of the matrix \( \Omega_{sk} \) are
\[
\omega_{11} = -(R/Y^{d})_{\pi_{o}} - (1-\beta)(g_{d}^{d} \phi_{o}^{d} (1+\varepsilon_{o}^{d} - \pi_{o}) + \phi_{o}^{d} (1+\varepsilon_{o}^{d}) (1-\pi_{o})) > 0, \\
\omega_{12} = \theta_{o}^{d} / Y^{d} > 0, \\
\omega_{13} = -f \ i (r/Y^{d}) g_{p}^{d} \pi_{o} + \phi_{p}^{d} g_{p}^{d} (1+\varepsilon_{o}^{d}) + \phi_{o}^{d} (1+\varepsilon_{o}^{d} - \pi_{o}) > 0, \\
\omega_{22} = 0, \\
\omega_{31} = F' b \beta (\phi_{o}^{d} g_{o}^{d} (1-\pi_{o}) (1+\varepsilon_{o}^{d}) + \phi_{o}^{d} g_{o}^{d} (1+\varepsilon_{o}^{d} - \pi_{o})) > 0, \\
\omega_{32} = 0,
\]

where the following notation has been used:
\[ \phi_{j}^{i} = M_{j}^{i} / Y_{j}^{i}, \eta_{j}^{i} = (1/\phi_{j}^{i}) (\partial M_{j}^{i} / \partial Y_{j}^{i}), \varepsilon_{j}^{i} = (P_{j}^{i} / M_{j}^{i}) (\partial M_{j}^{i} / \partial P_{j}^{i}), \]

\[ F' = \varepsilon / \varepsilon, \quad F' = \varepsilon / \varepsilon, \quad g_{s}^{i} = Y_{s}^{i} / Y^{s} \text{ for } s, i = d, o, l \text{ and } j = p, o, m. \]

An explicit solution to the equation system (18) requires inversion of the matrix \( \Gamma \), a task that is considerably simplified if this matrix is partitioned as
\[
\begin{bmatrix}
Y_{11} & Y_{12} & 0 \\
Y_{21} & Y_{22} & 0 \\
\vdots & \vdots & \vdots \\
Y_{31} & Y_{32} & 1
\end{bmatrix} =
\begin{bmatrix}
\Gamma_{11} & 0 \\
\Gamma_{21} & 1 \\
\vdots & \vdots \\
\end{bmatrix} = \Gamma.
\]

Consequently, the change in each region's real income can be expressed as a function of changes in both oil prices and purchases by the governments of developed countries:

\[
\begin{bmatrix}
dY^d/dY^d \\
dY^d/dY^d \\
dY^d/dY^d
\end{bmatrix} =
\begin{bmatrix}
\Gamma_{11}^{-1} & 0 \\
\Gamma_{21}^{-1} & 1 \\
\omega_{11} & \omega_{12} \\
\omega_{21} & \omega_{22} \\
\omega_{31} & \omega_{32} \\
\end{bmatrix}
\begin{bmatrix}
dP_o/P_o \\
dG^d/G^d
\end{bmatrix}.
\] (19)

After carrying out the matrix multiplication indicated by (19), the impact of an increase in the price of oil on the real income of developed countries is

\[
\tilde{\Gamma}^d/P_o = \left( (Y_{22} \omega_{11} - Y_{12} \omega_{21}) + \Gamma_{21} \omega_{12} (G^d/P_o) \right) / \det(\Gamma) > 0,
\] (20)

where the tilde '\(\tilde{\cdot}\)' over a variable denotes the percentage change of that variable (that is, \(\tilde{x} = \text{dx}/x\)). For developing countries this impact is

\[
\tilde{\Gamma}^d/P_o = \left( (Y_{11} \omega_{21} - Y_{21} \omega_{11}) - \Gamma_{21} \omega_{21} (G^d/P_o) \right) / \det(\Gamma) < 0.
\] (21)

Equations (20) and (21) constitute the first set of results of this paper. They reveal that the effect of an oil price increase on the real income of oil importers cannot be predicted a priori, unless further assumptions about the structural parameters and policy responses are provided. In this regard, if fiscal policy does not respond to the change in oil prices, and
if oil imports are price inelastic, then the income effect of an increase in
the price of oil is

\[ \tilde{Y}^d/\tilde{p} = \left( \begin{array}{cc} \gamma_{22} & \gamma_{12} \\ \gamma_{11} & \gamma_{21} \end{array} \right) \tilde{p} \geq 0, \]

(22)

and

\[ \tilde{Y}^e/\tilde{p} = \left( \begin{array}{cc} \gamma_{11} & \gamma_{21} \\ \gamma_{12} & \gamma_{22} \end{array} \right) \tilde{p} < 0. \]

(23)

Thus, as equations (22) and (23) indicate, in the absence of a fiscal-policy
response by developed countries, an increase in oil prices lowers the real
income of developing countries, but a definite conclusion cannot be obtained
for developed countries.

The International Transmission of Oil Price Effects

The ambiguity of the income effects of oil-price changes, as indicated by
equations (20) and (21), is due to the existence of direct and indirect
effects that tend to offset each other. Specifically, the direct effect of
an oil price hike is the transfer of real income from oil importing
countries to OPEC. In the case of the developed countries, this direct
effect takes the form of a deterioration of the real balance of payments,
given the increased cost of oil imports in terms of manufactured goods. In
the case of developing countries, the transfer takes the form of reduced
availability of foreign exchange, with subsequent indirect dynamic effects
on output growth through the influence on imports of capital goods and
investment.

The indirect effects of oil-price changes stem from both recycling
by OPEC and changes in the price of manufactures. An increase in oil prices
raises the oil revenues of OPEC, which in turn recycles a fraction \( \beta \) to
import manufactures from developed countries. This increase in OPEC's imports raises real income in developed countries—and could offset the direct negative effect. In turn, the stronger activity in developed countries stimulates imports from developing countries, and thus affords a higher volume of capital-goods imports and faster output growth in the latter countries.

An increase in the price of manufactures, induced by higher oil prices, has an ambiguous impact on the real income of developed countries. On the one hand, it mitigates the initial deterioration in the terms of trade, a process that reduces the real transfer to OPEC and thus raises the real income of developed countries. On the other hand, the increase in $P_m$ reduces the purchasing power of export revenues of both OPEC and developing countries with an adverse effect on the exports and GDP of developed economies. For developing countries, the increase in $P_m$ also has an ambiguous impact on their real income. On the one hand, it reduces the purchasing power of existing foreign exchange reserves with an adverse effect on capital accumulation and output growth. On the other hand, it stimulates exports of developing countries and thereby enlarges their foreign exchange reserves.

The Sensitivity of Oil Price Effects to Alternative Parameter Values

On the whole, the above discussion suggests that whether an increase in oil prices reduces real income or not depends, to a large extent, on the relative strengths of the direct and indirect effects. For developed economies, the direct effect of an oil-price increase—the transfer of real
income to OPEC—could be offset by the associated indirect effects—namely, increases in the price of manufactures and the recycling activities of OPEC. For developing countries, the strong and adverse direct effect—the increase in the real oil bill—could be aggravated by the associated increases in the price of manufactures, despite the beneficial effects associated with OPEC's recycling activities. In practice, whether the direct effects dominate the indirect effects depends on the relative magnitudes of key parameters such as the markup on oil prices, OPEC's recycling coefficient, demand price elasticities, and economic policy reactions.

Consequently, we examine the sensitivity of the comparative-static results for equations (22) and (23) with respect to changes in the values of $\pi_o$ and $\beta$. The results are summarized in Table 2. For developed economies, an increase in oil prices reduces real income if there is no markup in oil prices ($\pi_o = 0$) and if the value of the recycling coefficient is less than one ($\beta < 1$). In this case, the transfer of resources to OPEC is not fully recycled back to developed countries, and the value of this transfer is not deflated away since the price of manufactures does not change. If OPEC recycles all of its export revenues by importing manufactures from developed countries ($\beta = 1$), then oil-price increases do not affect the real income of the latter countries. In this case, the increase in the value of oil imports of developed countries (expressed in manufactures) is exactly matched by an increase in their exports of manufactures to OPEC, and thus the trade account is unaffected.

If the markup $\pi_o$ is positive, then an increase in oil prices raises the export price of manufactures with an ambiguous impact on the GDP of developed countries:
\( \frac{\partial (Y^d/P_o)}{\partial \pi_o} = O(\beta) < 0 \).

This outcome arises because increases in \( P_m \) have an ambiguous impact on the trade account of developed countries. If, however, OPEC recycles all of its export revenues, then an increase in the markup on oil prices does reduce exports and real income of developed countries and of their real income—-that is, \( O(\beta=1)<0 \).

In contrast to the case of developed economies, oil price increases reduce the GDP of developing countries for all values of \( \beta \) and \( \pi_o \) between zero and one. Furthermore, increases in the markup parameter \( \pi_o \) aggravate the already adverse effects of an oil-price increase:

\( \frac{\partial (Y^d/P_o)}{\partial \pi_o} < 0 \) for \( \beta \in [0,1] \).

This result is reasonable because a higher markup means that higher oil prices will be associated with even greater increases in the price of manufactures, reducing further the real foreign exchange reserves of developing countries and dampening their capital accumulation and output growth. These adverse effects could be partly offset by an increase in OPEC's recycling coefficient, \( \beta \). A higher \( \beta \) raises the real income of developed countries and stimulates exports of developing countries, allowing faster capital accumulation and output growth in the latter. However, as the negative entries in Table 2 indicate, any export gains of developing countries arising from an increase in OPEC's recycling are more than offset by the increase in their real oil bill.

**Oil Price Effects And Stabilization Policies**

The preceding analysis is simplified by the assumption that government purchases \( (G^d) \) do not respond to changes in oil prices. In order to examine
the consequences of relaxing such an assumption, suppose that fiscal policy responds to oil price shocks inasmuch as these shocks affect \( \frac{G^d}{\hat{P}_o} = \left( \frac{G^d}{\hat{P}_m} \right) \left( \frac{\hat{P}_m}{\hat{P}_o} \right) \).

If the fiscal policy response to changes in \( P_m \) is equal to \( \psi \Omega \), then the impact on \( \hat{Y}^d \) of a change in \( P_o \) is equal to

\[
\hat{Y}^d / \hat{P}_o = \left( (\gamma_{22} \omega_{11} - \gamma_{12} \omega_{21}) + \gamma_{22} \psi \omega_{12} \pi_o \right) / \det(\Gamma)
\]

\[
= \left( \hat{Y}^d / \hat{P}_o \bigg| \hat{\sigma}^d = 0 \right) + \gamma_{22} \psi \omega_{12} \pi_o / \det(\Gamma).
\] (24)

Thus the overall impact of a change in oil prices on the real income of developed countries can be decomposed into two terms. The first term—derived earlier as equation (22)—represents the impact of an oil-price change on real income assuming that fiscal policy does not respond to such a price change. The second term captures the direct and indirect effects on domestic output of the fiscal-policy response. The direct effect is the recessionary impact of a reduction in government purchases in a closed economy. The indirect effects stem from the impact of fiscal-policy changes on the imports of developed countries from both OPEC and developing countries with the subsequent feedback to exports and income of developed countries.

Equation (24) indicates that a restrictive fiscal policy could offset any positive impact of oil price increases on the income of developed countries (if the first term of the equation were positive). In particular, if
\[ \psi < -\left( \gamma_{22} \omega_{11} - \gamma_{12} \omega_{21} \right) / \left( \pi_0 \gamma_{22} \omega_{12} \right), \]

then an increase in the price of oil and a restrictive fiscal policy reduce real income of developed countries. 17

Oil Price Effects And OPEC'S Pricing Policy

The theory of depletion of nonrenewable resources (Hotelling, 1931, and Dasgupta and Heal, 1979) states that a monopolist's optimal price strategy should cause marginal profits to grow at the discount rate. An implicit assumption is that the monopolist faces a stable demand function, which implies that real income of oil importers can be taken as exogenously given and therefore unaffected by changes in oil prices. But price changes affect the real income of oil importers, especially given the possibility of restrictive policy responses. As a result, the demand for oil shifts and (if not taken into account in pricing decisions) could lower oil revenues. 18

Consequently, what matters for OPEC's price policy is the total price elasticity as seen by OPEC, which takes into account the effects of substitution, income feedbacks, and potential policy responses to an oil price increase.

In this section we extend the classical analysis of optimal pricing of nonrenewable resources to allow for the feedback effects of oil-price changes. Recognition of the feedback effects implies that price changes themselves shift the demand schedule in ways that are not necessarily known in advance. We develop the notion of the total price elasticity of oil demand, which measures the responsiveness of the demand for oil to changes in oil prices allowing for the international transmission
of oil-price effects. This notion of price elasticity is then used to characterize optimal oil-pricing policies.

**Total Price Elasticity of Oil Demand**

The demand for oil that OPEC faces is

\[ D^O = M^d_O + M^l_O. \]

Accordingly, the total effect of an increase in the price of oil on \( D^O \) is

\[ dD^O/dP_o = (\partial M^d/O \partial P_o + \partial M^l/\partial P_o) + ((\partial M^d_O/\partial Y^d)(dY^d/dP_o) + (\partial M^l_O/\partial Y^l)(dY^l/dP_o)), \]

where the first term on the right-hand side represents the substitution effect and the second term represents the income feedback effect. This total price effect can be expressed in elasticity form as

\[
\left( \frac{\Delta D^O}{\Delta P_o} \right) = \theta^d O \frac{d}{O} (1-\pi) + \theta^l \frac{e}{O} + \left[ \begin{array}{c}
\theta^d O N^d \theta^l O N^l \frac{Y^d}{P_o} \\
\theta^l O \frac{Y^l}{P_o}
\end{array} \right],
\]

(25)

where \( \theta^i O = M^i O / D^O \) for \( i = d, l \).

Substitution of equation (19) into equation (25) yields

\[
\left( \frac{\Delta D^O}{\Delta P_o} \right) = \left( \theta^d O \frac{d}{O} (1-\pi) + \theta^l \frac{e}{O} \right) + \\
+ \left( \theta^d O N^d \theta^l O N^l \right) \left[ \begin{array}{cc}
Y_{22} & -Y_{12} \\
-Y_{21} & Y_{11}
\end{array} \right] \left[ \begin{array}{cc}
\omega_{11} & \omega_{12} \\
\omega_{21} & \omega_{22}
\end{array} \right] \left[ \begin{array}{c}
\frac{1}{\Delta P_o} \Delta Y^d \\
\det(\Gamma)
\end{array} \right].
\]
which, after carrying out the matrix multiplication, can be expressed as

\[
\left( \frac{\tilde{D}^0}{\tilde{P}_0} \right) = \left( \theta^d \varepsilon^d_0 (1 - \pi^d_0) + \theta^d \varepsilon^d_0 \right) + \\
\left( \theta^o \eta^o_0 (Y_{22}w_{11} - Y_{12}w_{21}) + \theta^o \eta^o_0 (Y_{11}w_{21} - Y_{21}w_{11}) \right)\text{det}(\Gamma)^{-1} + \\
\left( \frac{\tilde{G}^d}{\tilde{P}_o} \right) \left( \theta^d \eta^d_0 (Y_{22}w_{12} - Y_{12}w_{22}) + \theta^d \eta^d_0 (Y_{11}w_{22} - Y_{21}w_{12}) \right)\text{det}(\Gamma)^{-1} \quad (26)
\]

Accordingly, three factors determine the total price elasticity of OPEC's oil exports. The first represents the substitution for alternative energy sources. The second represents the income feedback effect of oil-price changes in the absence of policy reactions. The third term captures the income feedback effect on oil exports arising from policy reactions to oil-price changes.

Implicit in equation (26) is the assumption that OPEC knows with certainty the policy reaction to an oil-price change. To relax this assumption, suppose only two policy reactions are possible:

---restrictive policy reaction \( \left( \frac{\tilde{G}^d}{\tilde{P}_o} \right)_r < 0 \), with probability \( \rho \),

---accommodative policy reaction \( \left( \frac{\tilde{G}^d}{\tilde{P}_o} \right)_a > 0 \), with probability \( 1 - \rho \),

with \( \left( \frac{\tilde{G}^d}{\tilde{P}_o} \right)_r < \left( \frac{\tilde{G}^d}{\tilde{P}_o} \right)_a \).

Recognition of the uncertainty surrounding policy responses suggests that the relevant notion of price elasticity is the expected total price elasticity:

\[
\zeta(\rho) \equiv \mathbb{E}\left[ \left( \frac{\tilde{D}^0}{\tilde{P}_o} \right) \right] = \mathbb{E} \left[ \left( \frac{\tilde{G}^d}{\tilde{P}_o} \right)_r - \left( \frac{\tilde{G}^d}{\tilde{P}_o} \right)_a \right] + \mathbb{E} \left[ \left( \frac{\tilde{G}^d}{\tilde{P}_o} \right)_a \right] < 0, \quad (27)
\]
where \( K \) and \( \alpha (\alpha > 0) \) are constants. As equation (27) indicates, the total price elasticity \( \zeta \) is, in principle, related to both \( \rho \) and the structural parameters of the model. Because of our interest in the relationship between stabilization policies and optimal oil price paths, we focus here on the role played by \( \rho 
abla \)

\[
\frac{\partial \zeta}{\partial \rho} = \alpha \left( \frac{\hat{G}_d}{\hat{P}_o} \right)_r - \left( \frac{\hat{G}_d}{\hat{P}_o} \right)_a < 0,
\]

which means that the (absolute) value of the total price elasticity increases in response to an increase in \( \rho \). Intuitively, increases in oil prices reduce oil exports not only because of the associated income and substitution effects, but also by the reduction in the expected real income of oil importers that arises from the increased likelihood of a restrictive fiscal policy.

Characterization of Optimal Pricing Strategies

The total price elasticity, equation (27), is a convenient way of summarizing how the international transmission of oil-price effects feeds back to the oil exports of OPEC. Our purpose now is to relate this price elasticity to the determination of optimal oil prices. To this end, OPEC is assumed to be a monopolist with constant extraction costs; it sets the price of oil to maximize the present value of expected future profits subject to a demand function and a resource constraint. Under these circumstances, the first-order conditions for profit maximization imply that marginal revenue, \( m \), grows over time at a rate equal to the discount rate, \( \delta \):

\[
\hat{m}_t = \delta \text{ for all } t.
\]
Since marginal revenue is defined as
\[ m_t = P_0 t (1 + 1/\zeta(\rho)), \]
logarithmic differentiation results in
\[ \delta = \tilde{m}_t = \tilde{P}_0 + \left( d(1+1/\zeta(\rho))/dt \right) / (1+1/\zeta(\rho)), \]
which yields the optimal growth rate of oil prices, \( \tilde{P}_0^* \):
\[ \tilde{P}_0^* = \delta - \left[ d(1+1/\zeta(\rho))/dt \right] / (1+1/\zeta(\rho)). \]  
\[ (28) \]

As equation (28) reveals, the optimal growth rate for oil prices is anchored to the discount rate, but departures from this rate are allowed as a response to changing market conditions arising from income effects and stabilization policies in oil importers. Specifically, if the total price elasticity is constant over time—that is, there are no feedback effects—then \( \tilde{P}_0^* = \delta \), which is Hotelling's r-percent rule. In addition, declines in oil prices are consistent with Hotelling's analysis of nonrenewable resources, provided changes in the price elasticity exceed the discount rate.\textsuperscript{21} Intuitively, oil prices could decline if favorable market conditions, such as strong growth in oil importers, were no longer present. Finally, optimal oil prices depend on stabilization policies in developed countries because of the association between \( \zeta \) and \( \rho, \zeta(\rho) \).

To examine this dependence further, assume that all parameters remain constant over time except \( \rho \), which experiences a once-and-for-all increase at time \( t \). The resulting impact on the optimal growth rate of oil prices is
$$\frac{\partial P_o^*}{\partial \rho} = \left\{ \left[ -\frac{1}{\zeta(\rho)} \right]^2 \left( \frac{\partial \zeta}{\partial \rho} \right) \right\} / \left[ 1 + \frac{1}{\zeta(\rho)} \right] < 0; \quad (29)$$

the associated optimal oil prices are shown in Table 3 and depicted in Figure 1. Equation (29) indicates that a once-and-for-all increase in the probability of a restrictive fiscal policy reduces the (contemporaneous) optimal growth rate of oil prices.\textsuperscript{22} Intuitively, a higher \( \rho \) means a greater probability that the oil-price increase will be followed by a restrictive fiscal policy, reducing the real income of oil importers and exacerbating the expected decline in oil exports and revenues. In effect, the increase in \( \rho \) raises the probability that the total price elasticity will be less than minus one implying that an increase in oil prices causes a decline in oil revenues. Thus it is optimal for OPEC to lower the rate of increase in oil prices.

**Conclusions**

The purpose of this paper has been to examine the interdependence between the effects of changes in oil prices and OPEC's price strategies. The analysis has relied on a highly specialized multicountry model, which allows us to determine the channels through which oil-price changes are internationally transmitted. Despite the relatively strong assumptions of the model, the impact of an oil-price change on the real income of oil importers cannot be predicted a priori without specific information about their policy responses and about the values for structural parameters. The ambiguity stems from the existence of offsetting effects. In particular, an increase in oil prices is associated with both a direct effect (the transfer of real resources to oil exporters) and indirect effects that operate
through both OPEC's revenue recycling and markups on oil prices. The paper characterizes the effects of oil-price changes using alternative values for the recycling coefficient of OPEC (β) and for the markup on the price of oil (π₀).

The paper also examines the influence of the income feedback effects of oil-price changes, as well as the induced policy reactions, on the optimal price of oil. For this purpose, the model of oil-price effects is combined with Hotelling's analysis of oil-price determination, and incorporates unstable demand functions. The motivation for this analysis stems from the possibility that increases in the price of oil might result in a reduction of oil revenues if the potentially adverse effects on the real income of oil importers is not taken into account. The centerpiece of the analysis is the notion of the total price elasticity of OPEC exports, which captures the influence of substitution effects, income effects, and the policy reactions of oil importers in measuring the response of oil exports to an increase in the price of oil.

The characterization of strategies for determining optimal oil prices relies on the behavior of the total price elasticity, which permits us to establish a formal association between optimal oil prices and policy stabilization in oil importers. We find that an increased likelihood of a restrictive policy in response to oil-price increases raises the probability of loss of oil revenues and thus reduces the optimal price of oil.

While the foregoing conclusions are intuitively plausible, they are conditioned on a number of simplifying assumptions that need to be relaxed in future research. First, a more general model of oil-price effects should allow not only for international trade flows, but also for
asset flows along with exchange rate determination. Similarly, the labor market and the role of expectations need to be considered in greater detail. Second, OPEC's interests extend beyond wealth maximization. Competing models of OPEC behavior, and the implications for the interaction between oil-price effects and oil-price determination, call for further analysis. Finally, the model in this paper could be extended by relaxing the assumption of only one producer in the oil market, and by recognizing that OPEC is not a coherent decision making bloc. Further analyses that relax or eliminate the assumptions made in this paper are certain to enhance our understanding of the interaction of oil-price effects and oil-price determination.
statistical and theoretical evidence points to a relationship between relative price changes (a change in oil prices) and the overall inflation rate (Cukierman 1983).

17 Fiscal policy may try to stabilize output instead of mitigating increases in prices. If this is the case, then one possible policy reaction would be

\[ 0 = (y_{22}\omega_{11} - y_{12}\omega_{21}) + y_{22}\omega_{12}(\hat{d}/\hat{P}_o), \]

or

\[ (\hat{d}/\hat{P}_o) = -(y_{22}\omega_{11} - y_{12}\omega_{21})/(y_{22}\omega_{12}). \]

For an empirical implementation of policy reactions using optimal control, see Marquez (1983).

18 This result has been verified empirically (in a dynamic context) by applying optimal control to a 3x3 medium size econometric model for the world economy; see Marquez (1983).

19 The values of \( K \) and \( \alpha \) are

\[ K = (\theta^d_{o}e^d_{o}(1-\pi_o) + \theta^p_{o}e^p_{o}) + \]

\[ + (\theta^d_{o}e^d_{o}(y_{22}\omega_{11} - y_{12}\omega_{21}) + \theta^p_{o}e^p_{o}(y_{11}\omega_{21} - y_{21}\omega_{11}))\text{det}(\Gamma)^{-1}, \]

and

\[ \alpha = (\theta^d_{o}e^d_{o}(y_{22}\omega_{12} - y_{12}\omega_{22}) + \theta^p_{o}e^p_{o}(y_{11}\omega_{22} - y_{21}\omega_{12}))\text{det}(\Gamma)^{-1}. \]

20 There are many other factors that influence OPEC's pricing decisions. First, the structure of the international oil market as reflected in the internal structure of OPEC and the appearance of non-OPEC oil suppliers clearly complicate the derivation of a single path for oil prices as indicated by Salant (1982), Newberry (1981), and Hyniulicz and Pindyck (1976). A second consideration influencing oil prices is the cost of extraction. As pointed out by Pindyck (1978), this cost is affected by the returns to scale, which have an impact on the rate of extraction and thus on oil prices. However, all these analyses presume the existence of either a stable demand function or one that shifts at an exogenously given rate. In other words, OPEC is assumed to ignore the possibility of a shift in the demand for oil induced by the oil-price change.

21 One of the first analyses showing that optimal oil prices could actually decline is that of Pindyck (1978). His empirical findings are supported by our analysis. In addition to the factors mentioned above, the total price elasticity is likely to increase over time in response to oil-price hikes because of supply-and-demand adjustments. These dynamic adjustments have been omitted from the paper because of the complexity involved in incorporating them in this analysis (for example one would need to couple a dynamic optimization model for oil consumers and a limit pricing model for OPEC with the model of table 1, which is already highly stylized). Increases in the total price elasticity arising from these dynamic adjustments clearly limit OPEC's ability to increase oil prices in the long run. What the results in the paper indicate is that even if these long run adjustments were not possible, the scope for oil price increases by
OPEC is reduced considerably by the existence of the feedback effects of oil price changes.

It is also possible to examine the response of optimal oil prices to changes in other parameters. In particular, suppose there is a one-time technological improvement that raises the (absolute) value of the price elasticity $\varepsilon_0^d$ for $i=d$, $\lambda$. Then the optimal response in oil prices is

$$\frac{\partial P^*_o}{\partial \varepsilon_0^d} = \begin{cases} \frac{-((-1/\xi)^2\delta_o^d(1-\pi_o))/(1+1/\xi)}>0 & \text{for } i=d; \\ \frac{-((-1/\xi)^2\delta_o^\lambda)/(1+1/\xi)}>0 & \text{for } i=\lambda. \end{cases}$$

A reduction in the $\varepsilon$'s (that is, an increase in their absolute value) reduces the growth rate of optimal oil prices.
Table 1
Theoretical Model of a Three Region World Economy

Developed Countries

\[ Y^d = C^d(Y^d) + G^d + B^d \]  \hspace{1cm} (1)

\[ B^d = X_m^d - [(P_o/P_m)M_o^d + (P_p/P_m)M_p^d] \]  \hspace{1cm} (2)

\[ M_o^d = M_o^d(P_o/P_m, Y^d) \]  \hspace{1cm} (3)

\[ M_p^d = M_p^d(P_p/P_m, Y^d) \]  \hspace{1cm} (4)

\[ P = \pi \cdot P_m + \pi \cdot P_p \]  \hspace{1cm} (5)

Non-OPEC Developing Countries

\[ Y_f^l = f(K^l, L^l) \]  \hspace{1cm} (6)

\[ K^l = K_{-1}^l + I^l \]  \hspace{1cm} (7)

\[ I^l = i_0 + iM_m^l \]  \hspace{1cm} (8)

\[ M_m^l = (R + P_p X_p^l - P_o M_o^l)/P_m \]  \hspace{1cm} (9)

\[ M_o^l = M_o^l(P_o/P_m, Y^l) \]  \hspace{1cm} (10)

OPEC

\[ Y_o^o = F(K_o^o) \]  \hspace{1cm} (11)

\[ K_o^o = K_{-1}^o + I_o^o \]  \hspace{1cm} (12)

\[ I_o^o = b_0 + b \times M_m^o \]  \hspace{1cm} (13)

\[ M_m^o = bP_o X_o^o/P_m \]  \hspace{1cm} (14)

Equilibrium Conditions

\[ X_m^d = M_m^o + M_m^l \]  \hspace{1cm} (15)

\[ X_o^o = M_o^d + M_o^l \]  \hspace{1cm} (16)

\[ X_p^l = M_p^d \]  \hspace{1cm} (17)
Table 1 (cont.)

Notation

<table>
<thead>
<tr>
<th>Variables</th>
<th>Superscripts</th>
<th>Subscripts</th>
</tr>
</thead>
<tbody>
<tr>
<td>N= Imports</td>
<td>(\kappa)</td>
<td>Non-OPEC LDCs</td>
</tr>
<tr>
<td>K= capital</td>
<td></td>
<td>OPEC</td>
</tr>
<tr>
<td>L= labor</td>
<td></td>
<td>DCs</td>
</tr>
<tr>
<td>Y= GDP</td>
<td></td>
<td>oil</td>
</tr>
<tr>
<td>P= prices</td>
<td></td>
<td>raw materials</td>
</tr>
<tr>
<td>I= investment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B= trade account</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G= government purchases</td>
<td></td>
<td></td>
</tr>
<tr>
<td>X= exports</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R= Resource transfers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C= absorption</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Parameters

- \(\phi_j^i\): GDP share of imports of commodity i in country j.
- \(\beta_o^i\): oil import share of country i in total world oil trade.
- \(\eta_j^i\): income elasticity of imports by country j of commodity i.
- \(\epsilon_j^i\): price elasticity of imports by country j of commodity i.
- \(g_s^i\): ratio of country i's income to country j's income.
- \(\beta\): OPEC's recycling coefficient
- \(i\): sensitivity of developing countries' investment to manufacture imports.
- \(b\): sensitivity of OPEC's investment to manufacture imports.
- \(\pi_i\): markup on the price of commodity i.
- \(c^d\): marginal propensity to spend.

Exogenous Variables

Oil prices, \(P_o\); primary product prices, \(P_n\); government purchases by developed countries, \(G_d^i\); aid to developing countries, \(R\); employment in developing countries, \(L^i\).

Endogenous Variables

Real income in each of the three regions, \((Y^d, Y^o, Y^i)\); imports of oil by developed and developing countries, \((M_o^d, M_o^i)\); imports of manufactures by OPEC and developing countries, \((M_o^m, M_o^i)\); imports of primary products by developed countries, \(M_o^d\); capital accumulation in OPEC and developing countries, \((I^o, I^i)\); trade account of developed countries, \(B^d\); export price of manufactures, \(P_m^i\).
Table 2

Effects of Oil Price Changes on Real Income: Sensitivity with Respect to Parameter Changes

<table>
<thead>
<tr>
<th>Developed Countries</th>
<th>Developing Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_o$</td>
<td>$\pi_o$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0&lt;\pi_o&lt;1</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>$\pm$</td>
</tr>
<tr>
<td>0&lt;\beta&lt;1</td>
<td>$\pm$</td>
</tr>
<tr>
<td>1</td>
<td>$\pm$</td>
</tr>
</tbody>
</table>

The signs shown in this table are obtained by evaluating equation (22) for the case of developed countries and equation (23) for the case of developing countries; for example, $\hat{\gamma}^{d}/P_o = f(\beta=1, \pi_o=0) = 0$. $\beta$ is the recycling coefficient of OPEC and $\pi_o$ is the markup on oil prices.
Table 3

Effect of a Permanent Increase in $\rho$ on the Optimal Growth Rate of Oil Prices

<table>
<thead>
<tr>
<th>Period</th>
<th>Probability of Restrictive Fiscal Policy $\rho$</th>
<th>Optimal Oil Price Growth Rate $\delta$</th>
<th>Optimal Oil Price $P_o^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t-2$</td>
<td>$\rho_o$</td>
<td>$\delta$</td>
<td>$P_o^{<em>,t-2} = P_o^{</em>,t-3} \exp(\delta)$</td>
</tr>
<tr>
<td>$t-1$</td>
<td>$\rho_o$</td>
<td>$\delta$</td>
<td>$P_o^{<em>,t-1} = P_o^{</em>,t-2} \exp(\delta)$</td>
</tr>
<tr>
<td>$t$</td>
<td>$\rho_t = \rho_o + d\rho$</td>
<td>$\delta \gtrless \delta - \zeta &lt; 0$</td>
<td>$P_{o,t}^* = P_{o,t-1}^* \exp(\delta - \zeta)$</td>
</tr>
<tr>
<td>$t+1$</td>
<td>$\rho_t$</td>
<td>$\delta$</td>
<td>$P_{o,t+1}^* = P_{o,t}^* \exp(\delta)$</td>
</tr>
<tr>
<td>$\ldots$</td>
<td></td>
<td></td>
<td>$\ldots$</td>
</tr>
</tbody>
</table>

Optimal oil prices could increase, remain constant, or decline following an increase in $\rho$. The effect of an increase in $\rho$ on optimal oil prices depends on the relationship between the discount rate, $\delta$, and the rate of change of the total price elasticity, $\zeta$:

$$
\frac{dP}{P} > 0 \text{ if } \delta \gtrless -(1/\zeta(\rho)^2)(\partial \zeta/\partial \rho)/(1+1/\zeta(\rho)).
$$
The top panel shows the time path of the probability of a restrictive policy reaction to an oil price increase, $\rho$. The bottom panel shows the effect of an increase in this probability on the optimal oil price path (expressed in logs for simplicity). The actual price path depends on the relative magnitudes of $\delta$ and $\zeta$; three paths are possible:

Path I: $\delta > \delta - \zeta < 0$; Path II: $\delta > \delta - \zeta = 0$; Path III: $\delta > \delta - \zeta > 0$. 
REFERENCES


