International Finance Discussion Papers

Number 257

July 1985

CURRENCY SUBSTITUTION AND THE NEW DIVISIA
MONETARY AGGREGATES: THE U.S. CASE

by

Jaime Marquez

NOTE: International Finance Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publications to International Finance Discussion Papers (other than an acknowledgment by a writer that he has had access to unpublished materials) should be cleared with the author or authors.
Currency Substitution and
The New Divisia Monetary Aggregates:
The U.S. Case

Jaime Marquez
International Finance Division
Federal Reserve Board
Washington, D.C. 20551

May 1985

The purpose of this paper is to examine the extent to which the behavior of aggregate money holdings is influenced by foreign exchange considerations, an influence that has been labeled as currency substitution. Knowledge of the extent to which monies of different countries can substitute for each other is important for the design and implementation of monetary policy. However, existing empirical analyses of currency substitution rest on official estimates of money holdings which imply an infinite elasticity of substitution between different monetary assets. Analyses of economic monetary aggregates do not impose the assumption of infinite elasticity of substitution, but no foreign exchange considerations are allowed. This paper combines both approaches into a unified explanation of money demand behavior.

I have benefitted from comments by William Barnett, Benjamin Friedman, Ronald McKinnon, Peter Tinsley, and Neil Wallace on an earlier version of this paper. This paper was presented at the conference on New Approaches to Monetary Economics, The University of Texas at Austin, May 23-24, 1985. I have also benefitted from the participants at the International Macro Workshop of the Federal Reserve Board. Suggestions and criticisms from Karen Johnson, Lois Stekler, Peter Isard, and Neil Ericsson are also gratefully acknowledged. I retain sole responsibility for remaining errors. This paper represents the views of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or other members of its staff.
Abstract

The purpose of this paper is to examine the extent to which the behavior of aggregate money holdings is influenced by foreign exchange considerations, an influence that has been labeled as currency substitution. Knowledge of the extent to which monies of different countries can substitute for each other is important for the design and implementation of monetary policy. However, existing empirical analyses of currency substitutions rest on official estimates of money holdings which imply an infinite elasticity of substitution between different monetary assets. Analyses of economic monetary aggregates do not impose the assumption of infinite elasticity of substitution, but no foreign exchange considerations are allowed. This paper combines both approaches into a unified explanation of money demand behavior.
1. INTRODUCTION

The purpose of this paper is to examine the extent to which the behavior of aggregate money holdings is influenced by foreign exchange considerations, an influence that has been labeled as currency substitution. Intuitively, one would expect foreign exchange considerations to influence domestic money holdings since, as a result of the greater integration of international financial markets, individuals generally face a choice not only between domestic money and domestic bonds, but also between foreign assets and domestic assets. Because holdings of these assets are interdependent, changes in either foreign interest rates or exchange rates would affect the optimal portfolio mix with a corresponding impact on domestic money holdings.

Knowledge of the extent to which monies of different countries can substitute for each other is important for the design and implementation of monetary policy. For example, it is conceivable that the intended effect of an open market operation would not materialize because of offsetting portfolio changes taking place through currency substitution.\(^1\) That individuals could rebalance their portfolio to offset a monetary policy decision has been recognized in the literature for some time.\(^2\) But the focus of much of the literature has been limited to substitutability between domestic money and other domestic assets without recognizing that foreign assets can also serve as substitutes for domestic money.\(^3\)

With the advent of both the flexible exchange rate system and the apparent instability of money demand behavior in the U.S., some investigators have argued that failure to recognize the influence of foreign exchange considerations in explaining money demand behavior might result in a misspecified money demand equation which could potentially be responsible for the large errors in forecasting aggregate money demand. Specifically, Hamburger
(1977a) studies the behavior of money demand in both Germany and the U.K., finding that this behavior is strongly influenced by movements in foreign interest rates. Relying on Chetty's work, Miles (1978) applies a production of monetary services model to Canada and finds that Canadians treat U.S. dollars as a good substitute for Canadian dollars; Brittain (1981) models the velocity of circulation as a function of domestic and foreign interest rates and concludes that their inclusion in the explanation for velocity eliminates part of the observed instability in its behavior; Arango and Nadiri (1981) include foreign interest rates and exchange rates as explanatory variables in modeling money demand behavior for several countries, and they find empirical support for the inclusion of these open economy variables in their specification. 4

Although these papers have examined the question of currency substitution in different ways, all of them rely on official estimates of money holdings which, as the work of Barnett indicates, imply an infinite elasticity of substitution between different monetary assets. In particular Barnett (1980, 1981) argues that for a monetary aggregate to be economically meaningful, it must rest on economic optimization principles. Consequently, the measurement of money balances ought to recognize that individuals minimize the cost of obtaining a given level of monetary services provided by a variety of assets by trading off the different attributes of several assets, given their associated opportunity costs. The resulting optimal asset holdings are (functionally) aggregated to obtain what Barnett denotes economic monetary aggregates. Only under very special circumstances the economic monetary aggregate is equal to a linear unweighted sum of asset values, which is the type of aggregation rule used for officially published estimates of money holdings. Barnett (1980, 1981) and Barnett, Offenbacher, and Spindt (1984) have investigated the behavior of money
demand using economic monetary aggregates by examining a number of issues including causality between money and income, structural parameter stability, and the dynamic specification of money demand behavior. However, these analyses rest on closed-economy explanations of money demand behavior with no allowance for foreign exchange considerations.

Even though the work on both economic monetary aggregates and currency substitution has enhanced our understanding of the behavior of money demand, the research effort in these two areas has proceeded independently of each other. On the one hand, proponents of the economic monetary aggregates give little or no consideration to the influence of foreign variables in modeling money demand. On the other hand, the literature on currency substitution treats monetary aggregates as if individuals' elasticities of substitution among different assets were infinite. This paper combines both approaches into a unified explanation of money demand behavior. To this end, section 2 applies the framework of economic monetary aggregates to develop a domestic money demand function allowing for foreign exchange considerations. Section 3 reviews the existing empirical work on money demand modeling for open economies. Section 4 presents empirical estimates of money demand behavior for the U.S. using economic monetary aggregates while recognizing open economy considerations. The empirical findings indicate that currency substitution is an important consideration in explaining U.S. money demand behavior. Finally, section 5 contains our conclusions.

2. DIVISIA MONETARY AGGREGATES AND CURRENCY SUBSTITUTION

The definition of money has been the subject of extensive research over the years. Specifically, issues such as what serves as money, what are different degrees of moneyness, and what are the alternative money substitutes play an important role in the design and implementation of monetary policy. In practice,
these issues are handled axiomatically—that is, based on certain properties (liquidity, interest earning, maturity), monetary instruments are classified as belonging to one monetary aggregate or to another. These components are then added up and the resulting total equals the estimate of money holdings for the aggregate.

The chief drawback of these estimates is that they rest on the assumption that individuals' preferences over assets possess an infinite elasticity of substitution, an unduly restrictive assumption for monetary policy analysis. To this end, Barnett (1980) has developed monetary aggregates that rest on individuals' optimizing economic behavior and thus eliminates the need for using the assumption of infinite elasticity of substitution in estimates of money holdings. Our purpose here is to apply Barnett's approach to deriving an open economy money demand function.

The point of departure of our analysis is the assumption that individuals optimize labor supply decisions, commodity purchases, and asset holdings subject to an intertemporal budget constraint under a weakly separable utility function. As Barnett (1981) has formally shown, this optimizing behavior yields an optimal level of monetary services, $M^*$, which is related to the available monetary assets via a functional aggregator $U$:

$$M^* = U(d_1, \ldots, d_s; c_1, \ldots, c_m; b_1, \ldots, b_n),$$

where

- $d_i = \text{domestic holdings of the ith domestic monetary asset, } i = 1, \ldots, s$;
- $c_j = \text{domestic holding of country j's non-interest bearing monetary asset},$;
- $b_j = \text{domestic holdings of country j's interest bearing monetary assets, where } j = 1, \ldots, n.$
The aggregate $M^*$ is not observed directly because the parameters associated with $U(.)$ are not known a priori; thus they need to be estimated. To this end, individuals are assumed to select their portfolio in a multistage decision process. In the first stage, individuals determine the aggregate levels of monetary services provided by both domestic and foreign asset holdings: subsequent optimizing stages determine the precise portfolio mix. In other words, optimizing behavior is assumed to begin with decisions about broad aggregates, such as domestic and foreign asset holdings, and to terminate with decisions about holding assets with detailed characteristics, such as a 90-day bond issued in country $j$ with an interest rate $r(j)$.

As the first step in this sequential process, it is assumed that the arguments of the function $U$ can be partitioned in such a way as to yield the following weakly separable functional aggregator:

$$
M^* = U(\delta(d_1, \ldots, d_s), \phi[c_1, b_1], \ldots, \phi[c_n, b_n]).
$$

where $\delta = \text{functional aggregator for monetary services provided by domestic assets}$, and

$\phi = \text{functional aggregator for monetary services provided by foreign assets}$.

To determine the optimal levels of monetary services provided by both domestic and foreign asset holdings, $\delta$ and $\phi$ respectively, individuals are assumed to minimize the cost associated with holding such assets, subject to the condition that the resulting aggregate of monetary services does not fall below $M^*$:

$$
\begin{align*}
\min & \pi_\delta \delta + \pi_\phi \phi \\
\text{subject to:} & \\
U(\delta, \phi) &> M^*
\end{align*}
$$
where $\pi_\delta = \text{user cost of monetary services provided by domestic assets, and}$

$\pi_\phi = \text{user cost of monetary services provided by foreign assets.}$

The solution to this problem yields the optimal level of monetary services of both domestic and foreign assets, $\delta^*$ and $\phi^*$ respectively, and the user cost of aggregate monetary services, $\pi_m$:

$$
\delta^* = \delta(\pi_\delta, \pi_\phi, M^*),
$$

$$
\phi^* = \phi(\pi_\delta, \pi_\phi, M^*),
$$

$$
\pi_m = \pi(\pi_\delta, \pi_\phi).
$$

If data on monetary services $\delta^*$ and $\phi^*$ were available, then one could estimate the parameters associated with these two functions to determine the extent of currency substitution. Although these monetary services are unobservable, the monetary aggregation framework of Barnett permits us to separate the problem of measuring these services from the problem of modeling their behavior.

Specifically, it is possible to apply Barnett's monetary aggregation framework while recognizing the influence of open economy considerations on the behavior of $\delta^*$.

To estimate $\delta^*$, individuals are assumed to find the minimum cost portfolio of domestic monetary assets subject to the constraint that the resulting aggregate level of monetary services is at least as high as $\delta^*$. Formally,

\[
\begin{align*}
\min_{\pi_d} \pi_d' \\
\text{subject to:} \\
\delta(\pi_d) > \delta^*,
\end{align*}
\]

where $\pi_d = \text{vector of opportunity costs for domestic assets,}$
\( D = (d_1, \ldots, d_g) \), and

\( \delta(D) \) = functional aggregator of monetary services of domestic assets.

The solution to this problem yields a set of domestic asset demand equations, 
\( d_i^* = d_i(\pi, \delta^*) \), the parameters of which can be estimated because data on \( \pi_d \) and \( D \) are available. These parameter estimates are then used in the functional aggregate, \( \delta(D) \), to obtain an estimated time series of \( \hat{\delta}(D^*(\pi_d)) = \hat{\delta}^* \), and to estimate the user cost of domestic monetary services, \( \pi_\delta = \pi(\pi_d) \).

Although estimates of \( \delta^* \) could be used to test the currency substitution hypothesis, they rely on the choice of a specific functional form for \( \delta(D) \). As a result, it might be difficult to establish whether the influence of foreign variables on domestic money holdings is due to recognition of individuals’ optimizing behavior or whether it is due to the choice of a specific functional form for \( \delta \). To avoid this “identification” problem, Barnett introduces the divisia monetary aggregate, which is a second approximation to any aggregate function in discrete time:

\[ \delta(D^*) = \Gamma(D^*, \pi_d), \]

where \( \Gamma(D^*, \pi_d) = \) divisia aggregate of domestic monetary services.

Data problems are particularly serious for monetary services rendered by foreign assets. In principle, an optimizing problem similar to (4) would yield a set of foreign asset demand equations which could be used to estimate the associated parameters. However, this approach requires time series for data on countries' bilateral holdings of foreign assets, which are not generally available; although the total (accounting) aggregates for domestic holdings of foreign assets are known, use of these would imply an infinite elasticity of substitution across all foreign assets.
To bypass this data problem, the present analysis relies on duality theory. Specifically, it is assumed that the economic aggregate of foreign assets obeys a Cobb-Douglas specification

\[ \phi = \prod_{i=1}^{m} \phi_i = \prod_{i=1}^{m} (c_i + b_i)^{Y_i}, \text{ with } \sum_{i=1}^{m} Y_i = 1, \]

which implies, by duality theory, that the user cost for monetary services of foreign assets is (Barnett 1981, p. 210)

\[ \pi_\phi = B \prod_{i=1}^{m} \left( \min(\pi_{c_i}, \pi_{b_i}) \right)^{Y_i}, \]

where

- \( \pi_{c_i} \) = user cost of holding non-interest bearing monetary assets of country i,
- \( \pi_{b_i} \) = user cost of holding interest bearing monetary assets of country i.

In other words, individuals are assumed to hold the asset with the lowest user cost, or alternatively, the asset with the highest return. Notice that the assumption of infinite elasticity of substitution between domestic holdings of country i's money and country i's bonds does not imply that assets of country i are perfect substitutes for assets of country j, or that domestic assets and any country's assets are perfect substitutes for each other. 9

3. MONEY DEMAND MODELING IN OPEN ECONOMIES

One of the key behavioral relationships of macroeconomic analysis is the willingness of individuals to hold domestic money balances. Because of its importance for the design of stabilization policies, the demand for money has
been the subject of considerable empirical research. However, it is only recently that foreign exchange considerations have gained significant attention in empirical analyses of money demand behavior.

Intuitively, the greater interdependency of the world economy and the advent of a flexible exchange rate system, has induced individuals to maintain a portfolio including more than one national currency in order to minimize the cost of carrying out international transactions. As a result, financial developments abroad affecting either foreign interest rates or exchange rates (and their expectations), will affect the optimal portfolio mix which includes holdings of domestic money. Consequently, in the presence of currency substitution, monetary policies abroad could influence domestic monetary policy not through fluctuations in the domestic money supply but rather by shifting the (closed-economy) money demand function. This point, first suggested by Miles (1978), has served as the basis of McKinnon's proposal for worldwide monetary policy coordination.

While the notion of currency substitution is intuitively plausible in principle, the empirical literature is sharply divided on its importance in explaining aggregate money demand behavior. This disagreement stems from the absence of a commonly accepted definition of currency substitution and by the different empirical implementations of whatever definition is adopted.

Specifically, the possibility of currency substitution has been examined in two different contexts. In the first, developed by McKinnon (1982), currency substitution is said to exist if the growth in world money supply is a better predictor of U.S. price inflation than is U.S. money growth. In the second, which is the focus of this paper, currency substitution is embedded in a portfolio balance model along with other domestic and foreign assets (see Cuddington 1983). In this context, currency substitution is said to exist if foreign exchange considerations—that is, foreign interest rates and
exchange rate expectations—contribute significantly towards explaining domestic money demand behavior. Accordingly, domestic money holdings are modeled as

\[ \Gamma/P = f(r, i^*, x, y), \]

where \( \Gamma \) = divisio monetary aggregate,
\( P \) = price level,
\( r \) = domestic interest rate,
\( i^* \) = foreign interest rate,
\( x \) = expected depreciation of domestic currency, and
\( y \) = scale variable, generally real income.

Although there is little disagreement in the literature about the choice of arguments in (5), there is considerably less agreement about the proper specification of their influences. Greatly simplified, two approaches have been developed to specify (5): a narrow approach and a broad approach. According to the former, money is defined as that asset earning no explicit interest, which implies that the only reason to hold foreign currency is an expected depreciation of the domestic currency. As a corollary, one must distinguish between currency substitution and capital mobility. Currency substitution is motivated only by depreciation of domestic currency—that is, a purely speculative motive for foreign currency holdings—whereas capital mobility is assumed to respond to changes in both foreign interest rates and exchange rate expectations. According to this narrow interpretation, the proper specification of (5) is

\[ \Gamma/P = f(r, i^* + x, x, y), \]
and currency substitution is said to exist if the coefficient associated with exchange rate expectations is significantly below zero.

Cuddington (1983) has tested the narrow version of the currency substitution hypothesis, equation (6), and has rejected this hypothesis for both the U.S and the U.K., while accepting it for Germany. However, Cuddington's empirical analysis suffers from serious drawbacks raising questions about the generality of his conclusions. First, expectations of future exchange rates are assumed to be equal to today's forward rate, which implies that the exchange risk is zero. In addition, the work of Hansen and Hodrick (1981) casts serious doubts on the validity of the forward exchange rate as an expectation model. Second, the estimated coefficient on the lagged dependent variable is equal to one; this estimate implies a zero speed of adjustment, or alternatively, that changes in money holdings are related to the levels of the explanatory variables. These findings are inconsistent with the underlying theory, even if we adopt the (very narrow) definition of money as a non-interest bearing asset.

Finally, the test for the currency substitution hypothesis involves a joint test for the hypotheses of no currency substitution and that the imposed exchange rate expectation mechanism is indeed valid. Cuddington's rejection of the hypothesis of currency substitution, seems not to recognize the joint nature of his tests. Recognition of the serious shortcomings of Cuddington's empirical analysis clearly limits the power of his conclusions.

Given the amount of research that has been devoted to defining money, it seems that using money in its narrowest possible sense (non-interest bearing asset) may be overly restrictive. There is a number of interest bearing assets which perform the same functions that money, in its narrowest sense, performs. Because of the restrictive nature of this definition of money, a broader view of
currency substitution has emerged in the literature which, in effect, assumes that interest and non-interest bearing assets of country i are highly substitutable from the domestic agent's point of view. Consequently, aggregate domestic money holdings are modeled as

\[(7) \quad \Gamma/P = f(r, \max(i^* + x, x), y) = f(r, i^* + x, y),\]

where \(i^* + x\) is the rate of return on foreign assets. In this context, currency substitution is said to exist if the influence of the foreign return on domestic money holdings is significantly below zero. \(^{14}\)

One common feature of all existing empirical analyses of currency substitutability, with the exception of Ewist and Fisher (1984), is their reliance on official estimates of money holdings—that is, unweighted sums of monetary assets. To the extent that the literature on currency substitution aims at establishing the extent to which domestic and foreign assets are substitutes, it seems inappropriate to impose the condition that all domestic assets are perfect substitutes for each other, but something less than perfect substitutes for foreign assets.

4. EMPIRICAL ANALYSIS

4.1 Functional Form

The simplest, and perhaps most convincing way of establishing the existence of currency substitution is to take a well-known functional form and estimate it allowing for foreign exchange considerations. Thus the empirical implementation of our analysis takes Goldfeld's (1973) specification of aggregate money demand behavior and extends it here to include foreign exchange considerations. One possible way to carry out this extension is to postulate that aggregate money holdings behave according to
\[(8) \quad \ln(\Gamma_{it}/P_t) = a_0 + a_1 \ln(Y_t/P_t) + a_2 r_t +
+ a_3 r* + a_4 E_t(e_{t+1})/e_t + a_5 \Delta \ln P_t + u_t,\]

where \(a_1 > 0, a_2 < 0, a_3 < 0, a_4 < 0, a_5 < 0;\)

\(\Gamma_{it}\) = monetary services provided by the \(i\)th (divisio) monetary aggregate,

\(P_t\) = price deflator for Gross National Product,

\(Y_t\) = Gross National Product in nominal terms,

\(r_t\) = nominal 90-day U.S. interest rate,

\(r^*_t\) = nominal 90-day foreign return,

\(i^*_t\) = foreign interest rate index,

\(e_t\) = exchange rate index (U.S.$/foreign currency),

\(E_t(e_{t+1})\) = one period ahead expected exchange rate based on today's information,

\(u_t = u_{t-1} + v_t,\)

\(u_t \sim N(0, \sigma_u^2), \quad E(u_t \cdot u_{t-1}) = \eta;\)

\(v_t \sim N(0, \sigma_v^2),\)

\(E(v_t \cdot v_{t-1}) = 2\eta - \sigma_u^2 = 0 \text{ if } \eta = \sigma_u^2/2.\)

According to (8), aggregate money demand responds to changes in real income and domestic interest rates, as the traditional formulation suggests; this demand also responds to changes in the foreign return from holding foreign assets and to the expected changes in the exchange rate. The inclusion of the inflation rate in the specification is less traditional but not less important. In particular, several arguments have made for this inclusion. First, it has been observed (Blejer 1981, Cukierman 1980) that an increase in the inflation rate is generally
associated with an increase in the dispersion of relative prices, which reflects greater price uncertainty and thus a higher financial risk associated with holding money. Second, if individuals adjust their money holdings according to an error correction mechanism, then the inflation rate will be part of the specification (Baba, Hendry, Starr 1985).

The inclusion of the inflation rate as an explanatory variable here rests on the application of an envelope aggregation theorem (Samuelson 1983) to individuals' money demand functions. The application of this theorem is developed in appendix 8 to this paper.

Three difficulties arise in estimating the parameters associated with equation (8). First, the estimation requires information on exchange rate expectations, which are not observable directly. Although the modeling of these expectations has been the subject of a considerable amount of research, both empirical and theoretical, an analysis of the existing literature reveals that little consensus exists as to what is the most appropriate expectation mechanism for exchange rates. As a result, the test for currency substitution involves a joint test for the hypotheses of currency substitution and for the validity of the model for exchange rate expectations.

Although the present analysis does not resolve this identification problem, it examines the sensitivity of the parameter estimates for two expectation mechanisms:  

(1) Forward market: $E_t(e_{t+1}) = e^f_{t+1}$, and

(ii) Rational Expectations: $E_t(e_{t+1}) = e_{t+1}$

where $e^f_{t+1}$ is the forward exchange rate index (U.S. $/foreign currency) for
delivery in \( t+1 \), and \( e_{t+1} \) is the realized spot exchange rate index in period \( t+1 \). According to the first expectation mechanism, the forward exchange rate \( e^f_{t+1} \) is the best predictor of future exchange rates. This approach has been criticized on the basis that it does not recognize the existence of risk premium, among other reasons (see Tryon 1983). Nevertheless, it is considered here because of its widespread use in the literature. The second mechanism rests on the rational expectations hypothesis, where today's expectations are unbiased predictors of future exchange rates. The rational expectation model of exchange rate determination has been empirically used by Symansky, Haas, and Hooper (1981) and Dooley and Isard (1982).

The second difficulty in estimating equation (8) is the sensitivity of the results to the spurious correlations characteristic of most time-series data. In addition, the construction of divisia indexes induces serial correlation in the error term. This difficulty is handled here by differencing equation (8), which yields

\[
\Delta \ln \left( \frac{R_{it}}{P_t} \right) = a_0 + a_1 \Delta \ln \left( \frac{Y_t}{P_t} \right) + a_2 \Delta r_t + a_3 \Delta \left( (1+i^*) (E_t (e_{t+1}) / e_t) \right) +
\]
\[
+ a_4 \Delta E_t (e_{t+1}) / e_t + a_5 \Delta^2 \ln P_t + \nu_t.
\]

Finally, earlier attempts at estimating an equation like (9) have encountered multicollinearity problems because of the intercorrelation between the three rates of return. To avoid this problem, the present analysis follows Daniel and Fried (1983) and imposes covered interest arbitrage to eliminate one rate of return.

The final specification for money demand behavior under the assumption that
money earns no explicit interest (narrow interpretation of currency substitution) is

\begin{equation}
\Delta \ln \left( \frac{r_{it}}{P_t} \right) = \alpha_0 + \alpha_1 \Delta \ln \left( \frac{y_t}{P_t} \right) + \alpha_2 \Delta r_t + \\
+ \alpha_4 \Delta \left( E_{t+1} / e_t \right) + \alpha_5 \Delta^2 \ln P_t + \nu_t,
\end{equation}

where \( \alpha_4 < 0 \) indicates the existence of currency substitution. If the definition of money is relaxed to include interest bearing assets (broad interpretation of currency substitution), then the specification is

\begin{equation}
\Delta \ln \left( \frac{r_{it}}{P_t} \right) = \alpha_0 + \beta_1 \Delta \ln \left( \frac{y_t}{P_t} \right) + \beta_2 \Delta r_t + \\
+ \beta_3 \Delta \max \left( \left( \frac{(1+i_*)}{E_{t+1}} \right), \left( \frac{E_{t+1}}{E_t} \right) \right) + \beta_4 \Delta^2 \ln P_t + \nu_t,
\end{equation}

and currency substitution is said to exist if \( \beta_3 \) is significantly less than zero. As indicated earlier, the tests for either interpretation of currency substitution are conditional on the validity of the expectation model assumed during estimation.

4.2 Data Construction

The construction of the ith divisia monetary aggregate rests on the work of Barnett (1980), and is computed according to the following expression:

\begin{equation}
\Gamma_{it} = \Gamma_{i,t-1} \prod_{j} \left( m_{jt} / m_{j,t-1} \right)^{(1/2)(s_{jt} + s_{j,t-1})},
\end{equation}
where $\Gamma_{it}$ = ith divisia monetary aggregate:

$$s_{jt} = \pi_j \cdot \pi_{kt} / \sum_k \pi_k \cdot \pi_{kt};$$

$\pi_j$ = current-period user cost of the jth component;

$$\pi_j = p^* (R_t - r_{jt}) (1-\tau_t) / [1+R_t (1-\tau_t)];$$

$p^*$ = true cost-of-living index;

$R_t$ = maximum available expected holding-period yield in the economy;

$r_{jt}$ = own current-period holding yield on component j;

$\tau_t$ = marginal tax rate; and

$m_{jt}$ = jth component of the ith monetary aggregate.

According to equation (12), the growth rate of the divisia monetary aggregate is a weighted-geometric average of the growth rates of the different components included in the aggregate; the weights change over time and they represent the expenditure share of the jth monetary component in the ith monetary aggregate.

The data for divisia monetary aggregates are available from the Federal Reserve Board on request; estimates of monthly divisia monetary aggregates for the period 1969-1980 can be found in Barnett and Spindt (1982). Data for nominal GNP, its deflator, and the 90-day U.S. CDs interest rate come from the data files of the Multi-Country Model of the Federal Reserve Board. The definition of foreign return used in this paper, $r^*$, is the foreign interest rate plus the expected depreciation of the domestic currency:

$$r^*_t = (1+i^*_t)[E_t(e_{t+1}/e_t)] - 1,$$

where
\( i^* \) = foreign interest rate index;

\( i^*_t \) = \( \Pi(i^*_t)^\gamma \); 

\( i^*_t \) = 90-day interest rate of country \( \ell \).

\( E_t(e_{t+1}^\ell) \) = one period ahead expected exchange rate based on today's information;

\( E_t(e_{t+1}^\ell) = \Pi(E(e_{t+1}^\ell)^\omega)\); 

\( E_t(e_{t+1}^\ell) \) = 90-day expected exchange rate (dollar/currency \( \ell \)) for country \( \ell \);

\( e^s_t \) = spot exchange rate index;

\( e^s_t = \Pi(e^s_t)^\omega \); 

\( e^s_t \) = spot exchange rate (dollar/currency \( \ell \)) index for country \( \ell \);

4.3 Testing of Error Properties

Nearly all econometric analyses of money demand behavior rest on the notion that the classical properties for the error term do indeed hold. While the hypothesis of serially independent errors is usually tested, the hypotheses of homoskedasticity and normality are generally taken for granted. This paper tests for the validity of the classical assumptions for ordinary least squares—namely, normality, serial independence, and homoskedasticity. In addition to these tests, the paper also presents Chow tests for the hypothesis of structural stability of the parameters.

Testing for the hypothesis that the errors behave according to the normal distribution relies on the Jarque-Bera statistic (Jarque and Bera 1980), which compares the extent to which the skewness and kurtosis of the distribution of estimated residuals represent significant deviations from the normal...
distribution. In particular,

\[ JB = T\left[ \mu_3/(6\mu_2) + (1/24)(\mu_4/\mu_2 - 3) \right] \sim \chi^2(2), \]

where \( T \equiv \) sample size, and \( \mu_j = \sum_{t=1}^{T} \hat{u}_t^j / T \); the first term of the JB statistic represents the skewness and the second represents departures of the estimated kurtosis from the kurtosis associated with the normal distribution; both measures are derived for the empirical distribution of the residuals, \( \hat{u} \).

The test for homoskedasticity rests on the work of Engle (1982) on autoregressive conditional heteroskedasticity (ARCH) disturbances. Specifically, the model used to test for homoskedasticity is

\[ E(\hat{u}_t^2 | \hat{u}_{t-1}^2) = \gamma_0 + \gamma_1 \hat{u}_{t-1}^2; \]

the null hypothesis for homoskedasticity cannot be rejected if \( \gamma_1 = 0 \), which is tested with a t-statistic. Finally, serial independence in the residuals is tested with an F-statistic for the null hypothesis that all the coefficients in an autoregressive form of order four are equal to zero.

4.4 Empirical Results

The parameters associated with equations (10) and (11) are estimated using ordinary least squares with quarterly data from 1974-1 to 1982-4 for the U.S., which includes the period of floating exchange rates. Table 1 displays the coefficients associated with the two interpretations of currency substitution—narrow and broad, for each of the three divisia monetary aggregates—\( T_1 \), \( T_2 \), and \( T_3 \), and for each of the exchange
Table 1
U.S. Money Demand
Estimate of Currency Substitution Parameter
Narrow and Broad Interpretation
Forward and Rational Expectation Models
(t-statistic)

<table>
<thead>
<tr>
<th>Statistical Diagnostics</th>
<th>Foreign Return</th>
<th>$\bar{R}^2$</th>
<th>J.B.</th>
<th>S.I.</th>
<th>ARCH</th>
<th>CHOW</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Narrow Interpretation</strong>&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Forward Market</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>-0.002</td>
<td>0.41</td>
<td>0.93</td>
<td>0.11</td>
<td>0.59</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>(-0.4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>0.002</td>
<td>0.60</td>
<td>0.42</td>
<td>0.76</td>
<td>0.34</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>(0.3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>0.002</td>
<td>0.62</td>
<td>0.61</td>
<td>0.49</td>
<td>0.34</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>(0.4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Rational Expectations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>-0.075</td>
<td>0.45</td>
<td>0.88</td>
<td>0.55</td>
<td>0.51</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>(-2.2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>-0.064</td>
<td>0.62</td>
<td>0.12</td>
<td>0.71</td>
<td>0.48</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>(-1.5)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>-0.058</td>
<td>0.64</td>
<td>0.06</td>
<td>0.38</td>
<td>0.31</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>(-1.5)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Broad Interpretation</strong>&lt;sup&gt;b&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Forward Market</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>-0.004</td>
<td>0.48</td>
<td>0.89</td>
<td>0.46</td>
<td>0.19</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>(-2.1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>-0.007</td>
<td>0.73</td>
<td>0.05</td>
<td>0.75</td>
<td>-0.30</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>(-4.0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>-0.006</td>
<td>0.73</td>
<td>0.01</td>
<td>0.41</td>
<td>-0.70</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>(-3.7)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Rational Expectations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>-0.003</td>
<td>0.42</td>
<td>0.91</td>
<td>0.44</td>
<td>0.05</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>(-1.8)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>-0.007</td>
<td>0.75</td>
<td>0.21</td>
<td>0.80</td>
<td>-0.21</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>(-4.4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>-0.006</td>
<td>0.76</td>
<td>0.20</td>
<td>0.64</td>
<td>-0.79</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>(-4.3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Notes

a Foreign return equals expected exchange rate depreciation.

b Foreign return equals expected exchange rate depreciation plus nominal interest rate.

c Jarque-Bera statistic ($\chi^2(2)$) for test of normality hypothesis; if it exceeds 0.95, then this hypothesis is rejected.

d F-statistic for serial independence hypothesis; if it exceeds 0.95, then this hypothesis is rejected.

e t-statistic for homoskedasticity; if it exceeds 1.645 then this hypothesis is rejected.

rate expectations models—forward and rational. Also shown in table 1 are the statistical diagnostics to test the maintained hypotheses associated with the error term—normality, serial independence, and homoskedasticity, and a Chow test for the hypothesis that the parameters have remained stable over the last two years of the estimation period. Tables A1 through A4 of appendix A display all the coefficient estimates associated with each specification shown in table 1.

According to the results of this table, the narrow view that foreign currency is held for speculative purposes is not supported by the data when the exchange rate expectations rely on the forward market. More specifically, the currency substitution coefficient is not significant and it has the wrong sign in two of the three cases. This lack of support for the currency substitution hypothesis is broadly consistent with Cuddington's (1983) findings. However, if exchange rate expectations are assumed to be rational, then the narrow interpretation of the currency substitution hypothesis receives strong support from the data; the sign of exchange rate expectations is negative for each divisia monetary aggregate, and quite significant for the aggregate \( \Gamma_1 \), which includes only currency and highly liquid assets. Furthermore, according to the values taken by the statistical diagnostics, the maintained hypotheses about the statistical properties of the error term cannot be rejected by the data; also, according to the Chow test, the specifications display structurally stable parameters.

The results for the broad interpretation of currency substitution, shown in table 1, are also highly supportive of the currency substitution hypothesis. Specifically, the coefficient for the foreign return variable is negative and highly significant for each divisia monetary aggregate and for each of the exchange rate expectation models considered here. Thus the empirical results are fairly robust. Furthermore, it is not possible to reject the hypothesis that the residuals in each equation behave according to the maintained assumptions of
classical least squares, which strengthens the validity of the statistical inferences made here. The results also indicate that in five out of six cases, the hypothesis of structural stability cannot be rejected.

Finally, limiting our attention to the rational expectation case, the results suggest that the proper interpretation of currency substitution—that is, narrow or broad—may depend on the monetary aggregate considered. To be specific, the narrow interpretation is not supported in the higher level divisia aggregates $\Gamma_2$ and $\Gamma_3$, but it is strongly supported for the divisia aggregate $\Gamma_1$, which includes only highly liquid components and thus it is the most appropriate characterization of currency holdings. On the other hand, the broad interpretation of currency substitution, generally associated with international capital mobility, is not strongly supported for the $\Gamma_1$ aggregate, but it is strongly supported for the higher level divisia aggregates $\Gamma_2$ and $\Gamma_3$. The bulk of these aggregates is made up of interest-sensitive components and therefore they are closely associated with the notion of capital mobility.

5. CONCLUSIONS

The purpose of this paper has been to determine whether the conduct of monetary policy in the U.S. should be influenced directly by the behavior of foreign interest rates and exchange rates. To address this issue, the present analysis studies the response of domestic money holdings to movements in both exchange rates and foreign interest rates—that is, whether currency substitution is an important consideration.

Although this issue has been treated in the literature, the existing analyses rest on official estimates of money holdings which assume perfect substitutability between different components. The present analysis removes this assumption and therefore extends the existing literature on currency substitution
by using the economic monetary aggregates developed by Barnett.

Relying on a traditional formulation of money demand holdings for open economies, our empirical analysis suggests that money demand behavior in the U.S. for the period 1974-1982 has been strongly influenced by foreign interest rates and expected movements in the value of the dollar. Thus it would seem that foreign exchange considerations could potentially influence the conduct of monetary policy to the same degree that domestic considerations do.

This conclusion, provocative as it may be, is preliminary because our analysis rests on a number of simplifying assumptions which need not hold in practice. Specifically, the assumed absence of simultaneity biases and the postulated within-one-quarter response of money holdings are restrictive enough to limit the generality of our findings; further research in these areas is clearly needed.
Footnotes

1. See McKinnon (1982); King, Putnam, and Wilford (1978); Boyer (1978); and Miles (1978). Although one would expect currency substitution to be intimately related to the effectiveness of official intervention in the foreign exchange market, this interconnection has not been established yet. An issue not investigated here is the extent to which substitution between domestic debt and domestic equity affects currency substitution. The former type of substitution has been the subject of analysis by Friedman (1978, 1985).

2. See Feige and Pearce (1977) who survey the work on the extent to which domestic assets are substitutes for each other; also, the literature on money demand instability, surveyed by Judd and Scadding (1982), argues that one of the reasons for the observed demand instability is the high degree of substitutability between money and the more recent financial instruments such as NOW accounts, Repurchase Agreements, etc.


4. Cuddington (1983) and Bordo and Choudri (1982) have examined the currency substitution hypothesis for a number of countries without finding empirical support for it. Goldstein and Haynes (1984) and Radcliffe, Warga, and Willet (1984) have also rejected the hypothesis of currency substitution for the U.S.

5. See Barnett (1980) and Hamburger (1977b).

6. There are exceptions to this procedure: Ewits and Fisher (1984), Chetty (1969), and Miles (1978).

7. Although the original formulation of the theory behind economic monetary aggregates excluded foreign monetary assets, the theoretical arguments are general enough to include these assets.

8. Notice that the aggregator of monetary services, δ(Q), depends only on quantities of monetary assets whereas Γ(Q, πd) depends on both prices and quantities. However, since Γ = δ and the latter follows from optimizing, then Γ will change only when the quantities of monetary assets change.

9. An implication of this assumption is that individuals' holdings of non-interest bearing monetary assets will be zero, since the returns to holding them (the expected depreciation of the domestic currency) is always smaller than the return to holding interest bearing assets. This assumption is also used by Dornbusch and Fischer (1980). The empirical evidence presented by Stekler and Isard (1985) reveals that banks' claims payable in foreign currency at the end of 1984 was $10.7 billion, which probably includes interest bearing assets, is the highest since 1981. This is a very small number when compared to the stock of financial assets.

10. This research has focused primarily on establishing the quantitative importance of income and relative rates of returns of domestic assets in
determining aggregate money holdings; another, and closely related area, of extensive research is the structural stability of money demand equations. Examples of this empirical work include the influential papers by Goldfeld (1973, 1976), Hamburger (1966, 1977b), Barnett, Offenbacher, and Spindt (1934), and Enzler, Johnson, and Paulus (1976).


12. For empirical tests rejecting McKinnon's currency substitution hypothesis, see Goldstein and Haynes (1984), and Radcliffe, Warga, and Willet (1984).


14. Clearly, differences in approaches that rest on either a narrow or a broad interpretation of money is not a satisfactory state of affairs. A preferable approach is to follow Ewias and Fisher (1984) or Johnson (1982). Both of these studies examine the behavior of specific asset demands allowing for the possibility of currency substitution. The former paper uses the framework of economic monetary aggregates with the stock of foreign assets defined as the sum of different foreign asset holdings. However, the rate of return to these assets is defined as the average rate, whereas it should have been the maximum rate; also, exchange rate changes are excluded from the return for foreign assets. The work of Johnson eliminates these difficulties, but it rests on official estimates of money holdings. The ultimate goal of the present research effort is to integrate both views.


16. A third mechanism is the static expectation model: \( \mathbb{E}(e_{t+1}) = e_t \). It has been observed that this expectation model has the largest predicting power among a variety of more sophisticated models (see Meese and Rogoff 1983). However, it is observationally equivalent to a model where the coefficient for expectations of the exchange rate is zero. Results not shown here, but available on request, indicate that foreign interest rates do exert a strong influence on the behavior of aggregate domestic money demand. Another complication arises for the rational expectations case. Specifically, if \( e_{t+1} = \mathbb{E}(e_{t+1}) + \omega_t \) where \( \omega_t \) is the forecasting error, then

\[
v_t = u_t - u_{t-1} + (\alpha_4/e_t)[\omega_t - (e_t/e_{t-1})\omega_{t-1}].\]

Clearly there is no a-priori reason to suspect that \( v_t \) obeys any of the properties of classical estimation. Of these, we test for normality, serial independence, and homoskedasticity leaving for the revised version of this paper the test for exogeneity.

17. For example, see Cuddington (1983) and Daniel and Fried (1983).

18. Differencing the equation is potentially troublesome. Specifically, the characterization of long run money demand is not unique since it will depend on the initial condition for the solution of the differential equation.
19. The construction of the \( l \)th divisia index includes only the monetary components of the \( l \)th official monetary aggregate. For example, \( \Gamma_l \) includes the same components that \( M_1 \) does. The construction of weighted averages for foreign variables rests on the methodology developed for the Multi-Country Model of the Federal Reserve Board. The weights used here are:

<table>
<thead>
<tr>
<th>( l )</th>
<th>Country</th>
<th>( y_l )</th>
<th>( \omega_{ly} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Canada</td>
<td>0.165</td>
<td>share of U.S. imports from Canada</td>
</tr>
<tr>
<td>2</td>
<td>England</td>
<td>0.215</td>
<td>share of U.S. imports from England</td>
</tr>
<tr>
<td>3</td>
<td>Germany</td>
<td>0.375</td>
<td>share of U.S. imports from Germany</td>
</tr>
<tr>
<td>4</td>
<td>Japan</td>
<td>0.165</td>
<td>share of U.S. imports from Japan</td>
</tr>
</tbody>
</table>
APPENDIX A

Parameter Estimates of Alternative Money Demand Specifications

This appendix provides a brief description of the parameter estimates for all the money demand equations presented in table 1. All of the parameters have the expected signs, except in table A1 where the sign of the coefficient associated with the foreign return is positive. The income elasticity ranges from 0.09 to 0.27, which differs significantly from Goldfeld's results. Indeed, it would be very surprising if using a fundamentally different monetary aggregate one were to obtain the same estimates as those using linear unweighted sums of monetary components. Moreover, our results are particularly close to those of Barnett, Offenbacher and Spindt (1984), who employ divisia monetary aggregates. Furthermore, using the same estimation period the latter encounter negative income elasticities, a result that is reversed here by the inclusion of foreign exchange considerations. Also, as pointed out by Benjamin Friedman, the coefficient associated with the foreign return generally exceeds the coefficient associated with the domestic return. This is because the computation of the domestic monetary divisia aggregate already internalizes substitution effects resulting from changes in interest rates. Consequently, the domestic interest rate effect could be smaller than the foreign interest rate effect.
TABLE A1

Parameter Estimates for U.S. Money Demand
Narrow Interpretation of Currency Substitution
Forward Market Expectation Model
1974.1 - 1982.4
(t-statistics)

<table>
<thead>
<tr>
<th>Explanatory Variables</th>
<th>Divisia Aggregates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_1$</td>
</tr>
<tr>
<td>Constant</td>
<td>0.021</td>
</tr>
<tr>
<td></td>
<td>(4.2)</td>
</tr>
<tr>
<td>Income</td>
<td>0.093</td>
</tr>
<tr>
<td></td>
<td>(0.7)</td>
</tr>
<tr>
<td>Domestic Return</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(-1.0)</td>
</tr>
<tr>
<td>Foreign return$^a$</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>(-0.4)</td>
</tr>
<tr>
<td>Inflation</td>
<td>-1.145</td>
</tr>
<tr>
<td></td>
<td>(-4.6)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.41</td>
</tr>
<tr>
<td>D.W.</td>
<td>2.25</td>
</tr>
<tr>
<td>S.E.R.</td>
<td>0.008</td>
</tr>
<tr>
<td>Normality: (JB)</td>
<td>0.93</td>
</tr>
<tr>
<td>Serial Independence: $F(4, 28)$</td>
<td>0.11</td>
</tr>
<tr>
<td>Homoskedasticity: ARCH-t</td>
<td>0.59</td>
</tr>
<tr>
<td>Chow Test: $F(8, 23)$</td>
<td>0.52</td>
</tr>
</tbody>
</table>

$^a$Foreign return equals the forward premium
TABLE A2

Parameter Estimates for U.S. Money Demand
Narrow Interpretation of Currency Substitution
Rational Expectation Model
1974.1 - 1982.3
(t-statistics)

<table>
<thead>
<tr>
<th>Explanatory Variables</th>
<th>Divisia Aggregates</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td></td>
<td>0.018</td>
<td>0.031</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.8)</td>
<td>(5.0)</td>
<td>(5.3)</td>
</tr>
<tr>
<td>Income</td>
<td></td>
<td>0.134</td>
<td>0.266</td>
<td>0.261</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.1)</td>
<td>(1.7)</td>
<td>(1.9)</td>
</tr>
<tr>
<td>Domestic return</td>
<td></td>
<td>-0.001</td>
<td>-0.003</td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-0.7)</td>
<td>(-2.6)</td>
<td>(-2.4)</td>
</tr>
<tr>
<td>Foreign return(^a)</td>
<td></td>
<td>-0.075</td>
<td>-0.064</td>
<td>-0.058</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-2.2)</td>
<td>(-1.5)</td>
<td>(-1.5)</td>
</tr>
<tr>
<td>Inflation</td>
<td></td>
<td>-1.044</td>
<td>-1.811</td>
<td>-1.672</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-4.4)</td>
<td>(-6.0)</td>
<td>(-5.9)</td>
</tr>
<tr>
<td>(R^2)</td>
<td></td>
<td>0.45</td>
<td>0.62</td>
<td>0.64</td>
</tr>
<tr>
<td>D.W.</td>
<td></td>
<td>2.49</td>
<td>1.24</td>
<td>1.45</td>
</tr>
<tr>
<td>S.E.R.</td>
<td></td>
<td>0.007</td>
<td>0.009</td>
<td>0.008</td>
</tr>
</tbody>
</table>

Normality: (JB) 0.88 0.12 0.06
Serial Independence: F(4, 27) 0.55 0.71 0.38
Homoskedasticity: ARCH-t 0.51 0.48 0.31
Chow Test: F(8, 22) 0.51 0.77 0.48

\(^a\)Foreign return equals the (rational expectation) realized depreciation of the dollar.
TABLE A3
Parameter Estimates for U.S. Money Demand
Broad Interpretation of Currency Substitution
Forward Market Expectation Model
1974.1 - 1982.4
(t-statistics)

<table>
<thead>
<tr>
<th>Explanatory Variables</th>
<th>Divisia Aggregates</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Constant</td>
<td>0.019</td>
<td>0.026</td>
<td>0.024</td>
</tr>
<tr>
<td></td>
<td>(4.0)</td>
<td>(5.3)</td>
<td>(5.4)</td>
</tr>
<tr>
<td>Income</td>
<td>0.091</td>
<td>0.250</td>
<td>0.250</td>
</tr>
<tr>
<td></td>
<td>(0.7)</td>
<td>(2.0)</td>
<td>(2.2)</td>
</tr>
<tr>
<td>Domestic return</td>
<td>-0.004</td>
<td>-0.002</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>(-0.4)</td>
<td>(-2.0)</td>
<td>(-1.8)</td>
</tr>
<tr>
<td>Foreign return&lt;sup&gt;a&lt;/sup&gt;</td>
<td>-0.004</td>
<td>-0.007</td>
<td>-0.006</td>
</tr>
<tr>
<td></td>
<td>(-2.1)</td>
<td>(-4.0)</td>
<td>(-3.7)</td>
</tr>
<tr>
<td>Inflation</td>
<td>-1.061</td>
<td>-1.600</td>
<td>-1.480</td>
</tr>
<tr>
<td></td>
<td>(-4.5)</td>
<td>(-6.5)</td>
<td>(-6.7)</td>
</tr>
<tr>
<td>R&lt;sup&gt;2&lt;/sup&gt;</td>
<td>0.48</td>
<td>0.73</td>
<td>0.73</td>
</tr>
<tr>
<td>D.W.</td>
<td>2.08</td>
<td>1.34</td>
<td>1.57</td>
</tr>
<tr>
<td>S.E.R.</td>
<td>0.008</td>
<td>0.008</td>
<td>0.007</td>
</tr>
</tbody>
</table>

| Normality: (JB)       | 0.89               | 0.05  | 0.01  |
| Serial Independence: F(4,28) | 0.46              | 0.75  | 0.41  |
| Homoskedasticity: ARCH-t | 0.19             | -0.30 | -0.70 |
| Chow Test: F(8,23)    | 0.33               | 0.98  | 0.77  |

<sup>a</sup>Foreign return is defined here as the covered interest rate.
### TABLE A4

Parameter Estimates for U.S. Money Demand  
Broad Interpretation of Currency Substitution  
Rational Expectation Model  
1974.1 – 1982.3  
(t-statistics)

<table>
<thead>
<tr>
<th>Explanatory Variables</th>
<th>Divisia Aggregates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td><strong>Constant</strong></td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td>(3.5)</td>
</tr>
<tr>
<td><strong>Income</strong></td>
<td>0.126</td>
</tr>
<tr>
<td></td>
<td>(1.0)</td>
</tr>
<tr>
<td><strong>Domestic return</strong></td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(-0.5)</td>
</tr>
<tr>
<td><strong>Foreign return&lt;sup&gt;a&lt;/sup&gt;</strong></td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>(-1.8)</td>
</tr>
<tr>
<td><strong>Inflation</strong></td>
<td>-0.985</td>
</tr>
<tr>
<td></td>
<td>(-4.1)</td>
</tr>
<tr>
<td><strong>R&lt;sup&gt;2&lt;/sup&gt;</strong></td>
<td>0.42</td>
</tr>
<tr>
<td><strong>D.W.</strong></td>
<td>2.26</td>
</tr>
<tr>
<td><strong>S.E.R.</strong></td>
<td>0.008</td>
</tr>
<tr>
<td><strong>Normality: (JB)</strong></td>
<td>0.91</td>
</tr>
<tr>
<td><strong>Serial Independence: F(4,28)</strong></td>
<td>0.44</td>
</tr>
<tr>
<td><strong>Homoskedasticity: ARCH-t</strong></td>
<td>0.05</td>
</tr>
<tr>
<td><strong>Chow Test: F(8,23)</strong></td>
<td>0.20</td>
</tr>
</tbody>
</table>

<sup>a</sup>Foreign return is defined here as the (rational expectation) realized depreciation of the dollar plus the foreign interest rate.
Appendix B

Aggregation of Micro Money Demands: An Application of Envelope Aggregation

Assume for simplicity that the individual's demand function is

\[ \ln \Gamma_{jit} = a_0 + a_1 \ln Y_{jt} + (1-a_1) \ln P_{jt} \]  \hspace{1cm} (B.1)

where \( j \) subscript refers to individuals.

Aggregation across \( N_t \) individuals yields

\[ \frac{1}{N_t} \sum_j \ln \Gamma_{jit} = a_0 + a_1 \left( \frac{1}{N_t} \right) \sum_j \ln Y_{jt} + (1-a_1) \left( \frac{1}{N_t} \right) \sum_j \ln P_{jt} \]  \hspace{1cm} (B.2)

However, the aggregate money demand equation subject to estimation is

\[ \frac{1}{N_t} \ln (\bar{\Gamma}_{jt}) = a_0 + a_1 \left( \frac{1}{N_t} \right) \ln (\bar{Y}_{jt}) + (1-a_1) \left( \frac{1}{N_t} \right) \ln (\bar{P}_t) \]  \hspace{1cm} (B.2)

One possible way to reconcile both specifications is by assuming that \( \Gamma_{jit}, Y_{jt}, \) and \( P_{jt} \) are lognormally distributed. The resulting aggregate money demand equation is

\[ \ln \left( \frac{\bar{\Gamma}_{jt}}{P_t} \right) = [a_0 + (1-a_1) \ln N_t + \left( \sigma_{yt}^2 / 2 \right) - a_1 \left( \sigma_{yt}^2 / 2 \right)] + \]  \hspace{1cm} (B.3)

\[ + a_1 \ln \left( \frac{\bar{Y}_{jt}}{P_t} \right) - (1-a_1) \sigma_{pt}^2 / 2, \]
where $\sigma^2_{mt}$ = variance of the log of monetary holdings across individuals,  
$\sigma^2_{yt}$ = variance of the log of nominal income across individuals, and  
$\sigma^2_{pt}$ = variance of the log of the price level across individuals.

It seems plausible to postulate that changes in $\sigma^2_{pt}$ are positively correlated with the variance of inflation, and the latter is in turn positively correlated with the inflation rate (Marquez and Vining 1984). Thus if $\sigma^2_{pt} = \beta \Delta \ln P_t$, then

$$\Delta \ln \left( \frac{\sum_{j} Y_{jt}}{P_t} \right) = \alpha_0 + a_1 \Delta \ln \left( \frac{\sum_{j} Y_{jt}}{P_t} \right) - a_2 \Delta \ln P_t,$$

where $\alpha_0 = (1-a_1)n + \Delta \sigma^2_{mt}/2 - a_1 \sigma^2_{yt}/2$; $a_2 = \beta(1-a_1) > 0$ provided that $a_1 < 1$;

and $n$ = growth rate of population.
References


Leamer, E., and R. Stern, 1970, Quantitative International Economics, (Allyn and
Bacon, Boston).


