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by

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ABSTRACT

This paper articulates a model of the small, open economy in which the stock market, rather than the bond market, determines domestic aggregate demand. It resembles in many respects the widely adopted dynamic Mundell-Fleming approach, but can, in some circumstances, exhibit output and asset price dynamics that differ in economically illuminating ways from that more standard framework. In particular, if the stock market effects are important enough, then a monetary expansion can result in real exchange rate appreciation, rather than depreciation. Anticipated fiscal expansion can, if the favorable effects on future productivity lead to strong enough stock market effects, lead to an output expansion, rather than a contraction as in, for example, Burgstaller (1983), Blanchard (1984) and Branson, Fraga and Johnson (1985). Furthermore, if the delay between announcement and implementation of the fiscal expansion is long enough, an anticipated fiscal expansion can lead to exchange rate depreciation, rather than appreciation.

The Stock Market and Exchange Rate Dynamics

by

Michael K. Gavin*

I. Introduction

This paper articulates a model of the small, open economy in which the price of shares in the stock market, rather than the real interest rate, determines domestic aggregate demand. The model builds upon Blanchard's 1981 analysis, and apart from the introduction of stock-market effects on aggregate demand, is also in the spirit of the dynamic, Mundell-Fleming analyses that are well-established in the international macroeconomics literature. Indeed, the model includes as a special case one version of this "standard" framework. It differs from Blanchard's analysis in the assumption that the economy is open. Despite this close kinship to the two strands of the literature, I show below that the open-economy stock-market model can, in some circumstances, exhibit output and asset-price dynamics that differ significantly, and in an economically illuminating way, from those that are in the literature.

There are two motivations for introducing the stock price into a macroeconomic model. The first derives from the observation that movements in current and expected future profitability, as well as interest rates, are widely believed to affect investment and consumption, through both relative price and wealth effects. The stock market, broadly interpreted as all marketable claims on the future

profits of firms, forms the link between future profit and interest rate fluctuations, and current investment and consumption decisions.¹ This theoretical link between the stock market and the macroeconomy has an empirical counterpart. For example Fischer and Merton (1984) document a strong empirical link between movements in the stock market and subsequent movements in output, and numerous studies of both investment and consumption find significant stock market effects. One motivation for this paper, then, is to determine how the introduction of a stock market alters the output and asset price dynamics of an otherwise conventional macroeconomic model of the open economy.

A second motivation is more directly related to the literature on asset preferences and exchange rate determination. In recent years, it has become apparent that models of exchange rate risk premia that focus on the currency denomination of outside assets (usually defined, for empirical purposes, as the stock of government debt plus high-powered money) have not performed well. [See Krugman (1981) and Frankel (1982).] One response to these failures has been to place more emphasis on models in which the location, not just the currency denomination, of assets matters. [See especially Dooley and Isard (1986) and Isard (1986).]

One such asset is marketable claims on the productivity of physical assets; for short, the stock market. Returns on capital are country-specific for a variety of reasons, including the uncertain prospect of changes in tax and repatriation rules, and country-specific fluctuations in the macroeconomic environment. In the following, nonstochastic, model I do not analyse the determination of risk premia

on capital stocks located in different countries. I do, however, present a model that is capable of answering questions of a macroeconomic nature related to changes in the expected profitability or riskiness of a country's capital stock, and which accounts for the link between output, profitability and asset prices.

The reader is forewarned that the following analysis becomes somewhat taxonomic at times, with the results depending upon which of several cases are being considered. I try to avoid simply listing a series of cases, but to some extent it is necessary to distinguish between cases that lead to qualitatively different behavior. The reader may find it helpful to bear in mind that the multiplicity of distinct cases results primarily from two sources. First, the effect of an increase in output on the stock market is uncertain, because higher output leads to both high profits and high interest rates, so we must distinguish between cases in which output is "good" for the stock market, and others in which it is "bad". Second, we must occasionally distinguish between those cases in which the stock market or the real exchange rate are more influential (in a sense that is defined more precisely below).

The paper is organized as follows. In Section II the model is outlined, and its general properties discussed. In Sections III and IV the implications of the model are further explored by using it to analyze the impact of changes in monetary and fiscal policy, respectively. In Section V the analysis is given some empirical content by simulating the model for parameter values that might roughly

correspond to those of the U.S. economy. Section VI presents the conclusions of the study.

II. The Model

As noted above, the model follows Blanchard (1981) very closely, including, wherever possible, notation. It focusses on the joint determination of output and asset prices. Asset prices determine aggregate demand, and therefore output. Output, through both profitability and interest-rate channels, is a key determinant of asset prices. The price level is sticky, and output is demand determined in the short run. In the long run it is assumed that output converges to an exogenously-determined full-employment level.^{2/} There are four assets held by risk neutral investors: money, domestic bonds, foreign bonds, and shares in the stock market, the last of which are claims on the economy's profits.

Aggregate demand: Equation (1) relates the log of real aggregate demand, d , to the real value of the stock market, q , the log of real output, y , the log of the real exchange rate, θ , and a measure of fiscal policy, g . (All parameters in the following structural equations are positive.)

$$(1) \quad d = \alpha q + \beta y + \gamma \theta + g$$

The stock market affects aggregate demand both because it is part of wealth, and because it determines the market valuation of capital relative to replacement cost, determining investment demand. Output influences aggregate demand to the extent that workers are liquidity

constrained, so that current income, and not just the capitalized value of lifetime income, determines consumption. The real exchange rate determines net exports through relative prices and, conceivably, a wealth effect, the sign of which would depend upon the currency denomination of assets held by domestic investors relative to the weight of foreign goods in the consumption basket. Here I simply assume that an exchange rate depreciation, an increase in θ , raises aggregate demand.

Output dynamics: Output is assumed to adjust to the discrepancy between aggregate demand and output.

$$(2) \quad \dot{y} = \sigma(d - y) = \sigma(aq - by + \gamma\theta + g)$$

where $b = 1 - \beta$, and a dot on top of a variable denotes a time derivative. Blanchard (1981) offers two rationalizations for this formulation: that output might adjust slowly to aggregate spending with the residual demand being satisfied with inventory fluctuations, or that aggregate spending itself may adjust to the target level, d , with a lag. In some instances below I consider the special case in which σ is very large, so that output always equals aggregate demand.^{3/}

Money demand: I adopt the standard liquidity formulation of money demand, solved for the nominal interest rate, i . Money supply is exogenously given.

$$(3) \quad i = cy - h(m-p)$$

$$(4) \quad r = i - \dot{p}$$

Note that real balances are deflated with the price of home output (value-added), rather than an expenditure deflator which would, given wages, depend upon the real exchange rate. The consequences of making the alternative assumption are well established in the literature, [see, for example, Branson and Buiter (1983) or Henderson (1983)], so for now I make the simpler assumption embodied in (3).

Price dynamics: Equation (5) gives the price adjustment equation, and equation (6) defines the steady state price level:

$$(5) \quad \dot{p} = -\delta(p - \bar{p})$$

$$(6) \quad \bar{p} = m + (\bar{r} - c\bar{y})/h$$

The steady state price level is that which secures money market equilibrium at the steady state interest rate and level of output.^{4/} This formulation was chosen for analytical tractability. Alternative specifications, such as a more standard Phillips curve framework, can be explored in a simulation model.

Asset market equilibrium: Assuming perfect foresight, the equilibrium conditions are:

$$(7) \quad \dot{q}/q + \pi/q = r$$

$$(8) \quad r = r^* + \dot{\theta}$$

$$(9) \quad \pi = \alpha_0 + \alpha_1 y$$

Equation (7) requires that the expected real return on a share of the stock market, which consists of both capital gains and profits (π), equal the real return on domestic bonds. ^{5/} Equation (8) is the open interest parity condition, where r^* is the foreign real interest rate. r^* is exogenously given, and as discussed below it is the steady state interest rate to which the small economy must converge. Henceforth, I will therefore use r^* and \bar{r} interchangeably. Equation (9) gives real profits as a function of real output.

For future discussion, it is useful to note that equation (7) can be solved forward, under the appropriate transversality condition, to obtain an expression for the stock price as the present value of anticipated future profits, discounted at the real interest rate.

$$(7a) \quad q(t) = \int_t^{\infty} \pi(\tau) e^{-\int_t^{\tau} r(s) ds} d\tau$$

Similarly, (8) can be solved forward for the current value of the real exchange rate as follows:

$$(8a) \quad \theta(t) = \bar{\theta} - \int_t^{\infty} (r(\tau) - \bar{r}) d\tau$$

So the current real exchange rate equals the steady state real exchange rate minus the difference between the home-currency and the foreign-currency interest rate, where the interest rate is the real rate on a very long zero-coupon bond.

Note that when α_1 is zero, the "stock" looks exactly like an indexed perpetuity, so that the model reduces to a standard open-economy IS-LM analysis, in which the interest rate that affects aggregate demand is the real rate on an indexed consol. Thus, α_1 is a natural measure of how different the present model is from those that have been analysed in the past. Consequently, it will be a key parameter in the analysis that follows.

The steady state: Steady state output is by assumption exogenously given, and the steady state interest rate is the foreign interest rate, which is also exogenously given. From (7), the steady state stock market is the present value of steady state profits, discounted at the steady state (foreign) rate of interest.

$$(10) \quad \bar{q} = \bar{\pi}/\bar{r} = (\alpha_0 + \alpha_1\bar{y})/\bar{r}$$

The steady state price level was defined in equation (6). Note that there is no allowance in this simple framework for non-zero monetary growth and inflation; extending the model to incorporate this possibility would be straightforward. Finally, the steady state real exchange rate is determined by imposing equilibrium in the goods market. In steady state, output and aggregate demand must be equal to the full-employment level of output. Imposing these equalities, and rearranging (1), we obtain:

$$(11) \quad \bar{\theta} = (b\bar{y} - a\bar{q} - g)/Y$$

As in the static Mundell-Flemming model, a permanent fiscal expansion has no effect on output or the interest rate, it merely induces an exchange rate appreciation sufficient to crowd out enough net exports to undo the expansionary impact of the fiscal stimulus.^{6/}

Dynamics: Equation (12) is a linearized version of equations (1) to (9), which I will use to examine the dynamic properties of the model in the neighborhood of the steady state.

$$(12) \quad \begin{bmatrix} \dot{y} \\ \dot{q} \\ \dot{\theta} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} -b\sigma & a\sigma & \gamma\sigma & 0 \\ (c\bar{q} - \alpha_1) & \bar{r} & 0 & (h + \delta)\bar{q} \\ c & 0 & 0 & (h + \delta) \\ 0 & 0 & 0 & -\delta \end{bmatrix} \begin{bmatrix} y - \bar{y} \\ q - \bar{q} \\ \theta - \bar{\theta} \\ p - \bar{p} \end{bmatrix}$$

The characteristic equation for the Jacobian in (12) is:

$$(13) \quad (\delta + \lambda) \{ \lambda [(b\sigma + \lambda)(\bar{r} - \lambda) + a\sigma(c\bar{q} - \alpha_1)] - c\gamma\sigma(\bar{r} - \lambda) \} = 0$$

where λ is an eigenvalue of the dynamic system.

Because of the way that the price dynamics were specified, (12) is block recursive, so (13) factors naturally into a cubic and a linear term. By inspection, therefore, one root of (13) is $-\delta$, which is, of course, negative.

The cubic part of (13) can be rearranged as follows:

$$(14) \quad \lambda^3 + (b\sigma - \bar{r})\lambda^2 - \sigma[a(c\bar{q} - \alpha_1) + b\bar{r} + c\gamma]\lambda + c\gamma\sigma\bar{r} = 0$$

$$\text{If: } (15) \quad a(c\bar{q} - \alpha_1) + b\bar{r} + c\gamma > 0$$

then the coefficients of the polynomial change sign twice, and we are guaranteed two positive roots. This condition is similar to the one found in Blanchard (1981), which is, in turn, satisfied if the IS curve cuts the LM curve from above. I assume that the slightly weaker condition (15) is satisfied. Thus, equation (13) has two positive roots and two negative roots. The negative roots will be denoted $-\lambda$ and $-\delta$ for the remainder of the paper.

The two positive and two negative roots, combined with the assumption that output and the price level are predetermined while the stock price and exchange rate can jump, ensures a unique equilibrium to (12).

The characteristic vectors that correspond to the negative roots $-\lambda$ and $-\delta$ are proportional to:

$$\begin{array}{c}
 \underline{-\lambda} \\
 \left[\begin{array}{c} 1 \\ -(c\bar{q}-\alpha_1)/(\bar{r}+\lambda) \\ -c/\lambda \\ 0 \end{array} \right]
 \end{array}
 \quad
 \begin{array}{c}
 \underline{-\delta} \\
 \left[\begin{array}{c} \gamma(\bar{r}+\delta)+a\delta\bar{q} \\ \delta\bar{q}(b-\delta/\sigma)+\gamma\alpha_1 \\ (\bar{r}+\delta)(b-\delta/\sigma)-a\alpha_1 \\ [-\delta/(h+\delta)][(\bar{r}+\delta)(b-\delta/\sigma+c\gamma/\delta)+a(c\bar{q}-\alpha_1)] \end{array} \right]
 \end{array}$$

The first eigenvector is easy to interpret, as it corresponds exactly to the fixed-price analysis in Blanchard (1981), with the addition of a real exchange rate. The sign of the second element depends upon the sign of $(c\bar{q}-\alpha_1)$. Because this term crops up repeatedly in the remainder of the paper, I denote it Δ . If Δ is greater than zero, we are in the "bad news case," in which an increase in output raises interest rates proportionately more than it raises profits, so that an increase in output causes a fall in the stock market. If Δ is less than zero, we are in the "good news case," in which an increase in output raises the value of the stock market.^{7/}

The third element of the eigenvector, which applies to the real exchange rate, is necessarily negative. This is the slope of the saddle path in (θ, y) space, so its negativity indicates that there is necessarily real exchange rate overshooting in the fixed-price version of this model. This is easy to understand: with prices fixed, output

below the steady state implies that the home interest rate is below the world interest rate. The interest rate differential must be offset by anticipated appreciation. But, because there is only one stable root in the fixed-price version of the model, the convergence to the steady state is necessarily monotonic, which means that the exchange rate can only converge to its steady state if it starts out by a jump depreciation that is larger than the steady state depreciation. That is, the real exchange rate must overshoot. We shall see below that this familiar chain of reasoning can break down in a model with slightly more complicated dynamics.

III. Monetary Policy

The first application of the model developed above is to analyze a change in the money supply. We suppose that at time zero the economy is in steady state, and is disturbed by an unanticipated increase in the money supply of \hat{m} . The new steady state is identical to the old, except that the price level is increased by \hat{m} . The dynamics are as follows:

The solution for prices is trivial, because of the simple price adjustment equation (5). The solution is:

$$(16) \quad [p(t)-p(0)] = \hat{m}(1-e^{-\delta t})$$

So, the steady state price level adjusts monotonically from its old steady state to its new steady state regardless of output dynamics or, for that matter, anything else.

Output is "humpbacked," rising from the initial steady state, to a maximum at time t^* , where:

$$(17) \quad t^* = \frac{\log(\delta/\lambda)}{(\delta-\lambda)}$$

then falling asymptotically and monotonically toward the unchanged steady state.⁸ This is not the truly cyclical behavior that would arise from a multiplier-accelerator framework, or a more conventional Phillips curve equation; however, there is a family resemblance.

The solution for the stock price dynamics can be briefly summarized. The steady state value of the stock market is unchanged by the monetary expansion, but the stock price necessarily increases after the monetary expansion. This is intuitively appealing, since profitability necessarily increases, and the interest rate declines at least transitorily. However, as will be discussed below, it is possible that the long real interest rate increases after the monetary expansion, so this result is less obvious than it might appear at first glance. In the longer version of the paper I discuss other aspects of stock price dynamics in more detail; they are not, however, of central interest so I forego the discussion in this version of the paper.

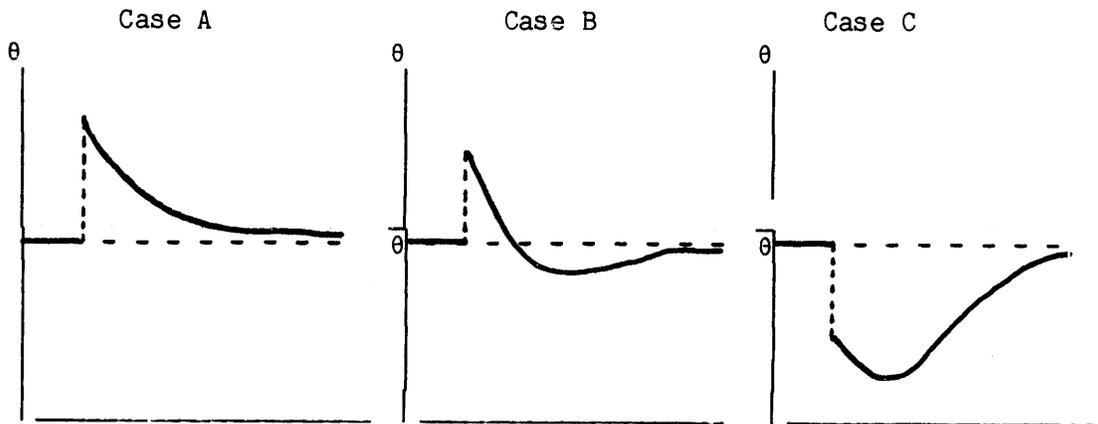
So far the model has exhibited no surprising properties. The exchange rate dynamics, however, are somewhat more interesting. The steady state real exchange rate is unchanged by the more expansionary monetary policy, although the nominal exchange rate would, of course, depreciate in proportion to the increase in the money supply. I begin the discussion of dynamics with two observations: first, the solution

is the sum of two declining exponentials, so the time path for the real exchange rate can have at most one "hump," that is, one point at which the time derivative switches sign. Second, because both output and the price level are "sticky," the home interest rate necessarily falls below the world rate just after the increase in the money supply, which implies that the rate of change of the real exchange rate at time zero is negative.

These two observations restrict the possible time paths for $\theta(t)$ to three, which are illustrated in Figure One:

Figure One

Possible Time Paths for the Real Exchange Rate



In cases A and B, the exchange rate's initial jump is a depreciation, the standard case. They are discussed in more detail below, but for now consider case C. In this case, the increased money supply results in a jump appreciation of the exchange rate. These diagrams alone do not establish that case C is possible, but I show in appendix A that if:

$$(21) \quad \bar{r}(b+c\gamma/\lambda) - a\alpha_1 < 0$$

then there is some δ^* such that if δ (the price adjustment parameter) is less than δ^* , the real exchange rate jump appreciates after a monetary expansion, as in Figure One (C) above.

This counterintuitive result is clearly linked to the presence of the stock market: it requires that $a\alpha_1$ be a big number, where a is the impact of the stock market on aggregate demand, and α_1 is the impact of output on the stock market. In particular, in the case in which the return on the domestic asset is insensitive to output (the "indexed bond" case, with $\alpha_1=0$) this result would be impossible.

The intuitive explanation for this apparently perverse "reverse overshooting" is as follows. In this scenario, the monetary expansion leads to a transitory decline in the short real interest rate, both because of liquidity and inflation effects. Aggregate demand and output increase, as a result of a large increase in the value of the stock market. The increase in the stock market is large, and the impact aggregate demand important, because a and α_1 are large. As output and the price level increase, the demand for real balances increases while the supply declines, requiring an equilibrating increase in the nominal interest rate. This combines with a gradually declining rate of inflation to generate a sharp increase in the short real interest rate from its initial position below the world rate, to a level above the world rate.

If this reversal of the interest differential is large, and occurs quickly enough, then immediately after the monetary expansion, the long real interest rate actually rises, even though the short rate necessarily falls. As summarized in equation (8a), with no change in the steady state real exchange rate, this increase in the long, real rate to a level above the foreign interest rate generates a jump exchange rate appreciation. Hence, if (21) holds and price adjustment is slow enough, the rise in the long, real interest rate induced by an expansionary monetary policy creates an exchange rate appreciation. (In Section V the plausibility of condition (21) being satisfied is examined.)

This "reverse overshooting" scenario thus requires a rapid, large increase in output which, because the real exchange rate has appreciated, requires an increase in the stock market that is both large and effective in increasing aggregate demand. It is easy to see how these requirements relate to condition (21).

I have already noted that a large α_1 implies that output expansion will have a large effect on the stock price, and that a large value for "a" implies that an increase in the stock market will have a large effect on output. (21) is also more likely to hold if λ is large, which is to say, if output can adjust rapidly. This is necessary to ensure that the real interest differential is quickly reversed. A small b , which is to say a large β , also increases the likelihood that (21) will hold. This is because, through the simple Keynesian multiplier process, it reinforces the positive effect of output on aggregate demand that is at the center of the intuitive explanation offered above.

Condition (21) is unlikely to hold if γ is large, because in that case an exchange rate appreciation would have an important dampening effect on output, reducing the likelihood of the strong, rapid output increase needed to bring about reversal of the interest rate differential.

Finally, the "reverse overshooting" scenario requires slow price adjustment. This is also easy to understand. If price adjustment were very rapid, then money would be approximately neutral and monetary expansion would have only a small effect on output and interest rates. There would not be a substantial effect on the stock price, because output would not increase enough, or for a long enough period of time, to significantly raise the discounted value of a claim on future profits, thus eliminating the strong stock price-output-stock price feedback that generates the "reverse overshooting" scenario.

The difference between the real exchange rate's dynamics in cases A and B in Figure One is in the convergence to steady state. In Case A, the real exchange rate converges monotonically, and never falls below the steady state. In Case B, the real exchange rate initially depreciates, and gradually appreciates over time until it falls below the steady state level. After a while, the real exchange rate stops appreciating, and gradually depreciates, approaching the steady state from below. The reason for Case B's overshooting the steady state is that, in that case, output increase fast relative to the increase in the price level. This leads to an increase in the interest rate, which eventually rises above the world rate. When this happens, asset market equilibrium requires expected depreciation of the home currency. Thus, the real exchange rate must converge to the steady state from below.

IV. Fiscal Policy

I turn now to a discussion of the effects of fiscal policy in the open-economy stock-market model. I am particularly interested in analysing anticipated, and/or transitory fiscal disturbances, for which the analysis quickly becomes algebraically cumbersome. To preserve analytical tractability, I limit myself in this paper to the case in which σ is very large, so that output adjusts instantaneously to aggregate demand. This permits graphical analysis of the model. My earlier paper contains some results for the more general case, and the simulation results in Section V also examine the case in which σ is finite.

Fiscal policy has no effect on the price level in this model, because, as in the static Mundell-Fleming model, steady state internal balance is achieved through exchange rate fluctuations with no changes in the real money supply.^{9/} The inflation equation (5) then implies that the price level is fixed even in the short run.

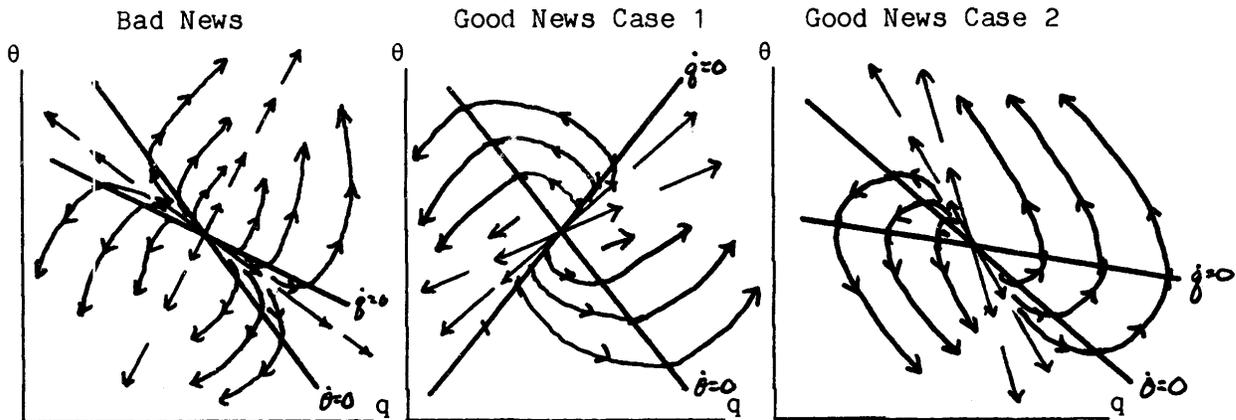
With prices fixed and output equal to aggregate demand, the economy can be represented with two, rather than four dynamic equations:

$$(22) \begin{bmatrix} \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \bar{r} + a\Delta & \gamma\Delta \\ ac & \gamma c \end{bmatrix} \begin{bmatrix} q - \bar{q} \\ \theta - \bar{\theta} \end{bmatrix}$$

where all of the parameters are defined as in Section II, except that the parameters a and γ should be interpreted as the "a" and "γ" in

equation (1) divided by "b." This redefinition is not substantive, it merely simplifies the notation. I show in Appendix B that, under condition (15), there are two positive roots to the characteristic equation for the Jacobian in (22), so that the saddle "path" is a point in (θ, q) space. The dynamics are shown graphically in Figure Two. 10/

Figure Two



The $\dot{\theta}=0$ schedule is also the locus of points for which output equals steady-state output. This is because the $\dot{\theta}=0$ schedule is the locus of points for which $r=r^*$. With a constant price level and money supply, the domestic real interest rate depends only upon real output. Thus, the interest rate is above the world rate when output is above the steady state level, and conversely.

An increase in the real exchange rate raises net exports, and aggregate demand. To keep output and the real interest rate at their steady state, an increase in θ must therefore be offset by a decrease in the stock price. Thus, the $\dot{\theta}=0$ schedule is downward sloping. Above it,

output is high, and the domestic interest rate is above the world rate; therefore asset market equilibrium requires that the real exchange rate be depreciating ($\dot{\theta} > 0$). Conversely, below the $\dot{\theta} = 0$ schedule, the real exchange rate must be appreciating.

The slope of the $\dot{q} = 0$ schedule depends upon the size of Δ . If Δ is positive (the bad news case) then the schedule is a negatively sloped curve that is steeper than the $\dot{\theta} = 0$ curve. In the "good news" case, when Δ is negative, the schedule is either negatively sloped and less steep than the $\dot{\theta} = 0$ schedule, or it is positively sloped. Above the $\dot{q} = 0$ schedule, the real exchange rate depreciation leads to higher output, raising both the interest rate and profits. In the bad news case, the adverse interest rate effect dominates; thus, asset market equilibrium requires expected capital gains ($\dot{q} > 0$). In the good news case, the favorable profitability effect dominates, so that asset market equilibrium requires expected capital losses in the stock market ($\dot{q} < 0$).

The remainder of this section uses these phase diagrams to consider the following experiment. At time T_0 , it is announced that there will be a fiscal expansion beginning at time T_1 and lasting until T_2 . After T_2 fiscal policy returns to the original setting. (There is nothing restrictive about the assumption that the fiscal expansion is "transitory" or "anticipated." A permanent fiscal expansion is the special case in which T_2 is infinite, and an unanticipated fiscal expansion is the case in which $(T_1 - T_0)$ is zero.)

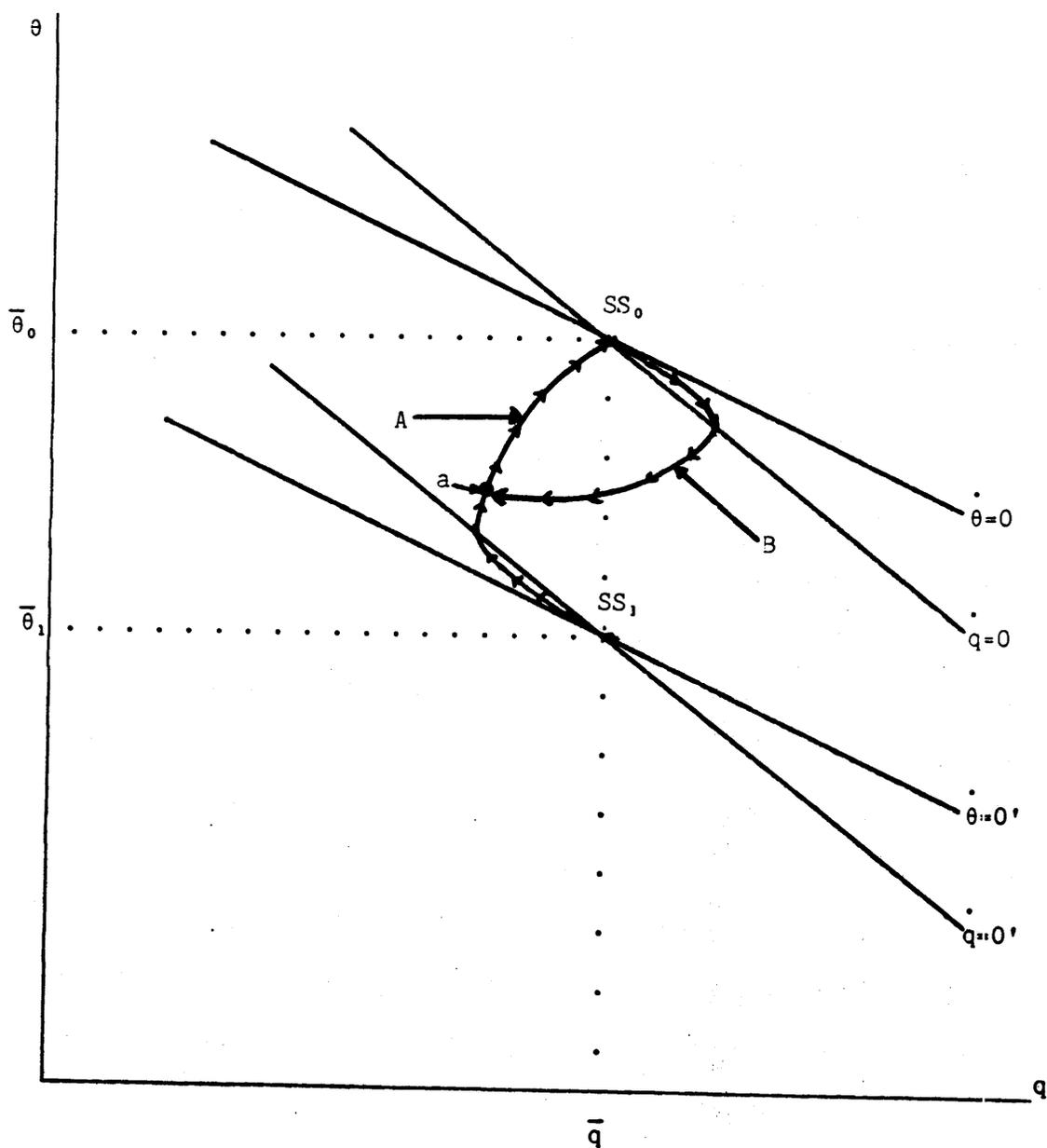
I begin by discussing the bad news case, and show that in that case the predictions of the model are essentially identical to those from a standard, dynamic Mundell-Fleming analysis. I then consider the

good news case, and show that some of the propositions that have become commonplace in the literature can, under some circumstances, be reversed.

Bad news case: Figure Three illustrates the dynamic adjustment path to an anticipated, transitory fiscal expansion. Before and after the fiscal expansion, the economy is governed by dynamics summarized in the upper schedules, and during the fiscal expansion it is governed by the dynamics summarized by the lower schedules. In the case of an unanticipated, permanent fiscal expansion, the economy jumps from SS_0 to SS_1 , and remains there. Thus, the only effect is on the real exchange rate and, therefore, the composition of aggregate demand.^{11/} Output, interest rates and the stock price are unaffected.

Anticipated or transitory fiscal expansions lead to more interesting asset price dynamics, which we now explore. By ruling out anticipated asset price jumps, we know that at time T_2 , when the fiscal expansion ends, the economy must be at point SS_0 . This means that during the fiscal expansion, (between T_1 and T_2), the economy must be somewhere on the curve labelled (A). This is the locus of points for which the economy's dynamics (as influenced by the fiscal expansion) lead through the point SS_0 . Where on this trajectory the economy will be at time T_1 depends upon the length of the fiscal expansion, $(T_2 - T_1)$. If the expansion lasts for a very long time, the economy must be very close to SS_1 at time T_1 . If the expansion is very short, the economy must be somewhere near SS_0 at time T_1 .

Figure Three
 Dynamic Adjustment to a Future, Transitory Fiscal Expansion
 Bad News Case



For illustrative purposes, let us suppose that $(T_2 - T_1)$ is such that at time T_1 the economy is at point (a) in Figure Three. The economy's behavior before the fiscal expansion, (between T_0 and T_1), is determined by the requirement that there be no anticipated jump in asset prices at time T_1 . Thus, between T_0 and T_1 , the economy must be somewhere on the trajectory labelled (B). To which point on this trajectory the economy must jump at time T_0 depends upon how long is the delay between announcement and implementation of the fiscal expansion, $(T_1 - T_0)$. If the delay is very long, then the economy must jump to a point near SS_0 after the fiscal policy change is announced. If the delay is very short, it must jump to a point on (B) near (a).

We can use Figure Three to deduce the following characteristics of the equilibrium: (These assertions are proven in Appendix B.)

1. Output always falls in anticipation of the fiscal expansion, and rises above the steady state level during the fiscal expansion. We know that this is so because (B) lies entirely below the $\dot{\theta}=0$ schedule, and (A) lies above the $\dot{\theta}=0'$ schedule.

The size of the recession between T_0 and T_1 is increasing in the length of the fiscal expansion and decreasing in the delay between the announcement and the implementation of the fiscal expansion. The size of the boom during the fiscal expansion depends upon the length of the fiscal expansion. Long expansions lead to small output effects, because they generate larger exchange rate appreciations, while short fiscal expansions generate small exchange rate appreciations, and therefore have a larger effect on output. (In the limit, as the length of the

fiscal expansion goes to infinity, the effect on output goes to zero, as in the static Mundell-Fleming model.)

2. The real exchange rate necessarily appreciates relative to the original steady state both before and during the fiscal expansion. The exchange rate appreciation is increasing in the length of the fiscal expansion, and decreasing in the delay between announcement and implementation of the fiscal expansion.

3. The stock price is necessarily below the steady state during the fiscal expansion (between T_1 and T_2). This is because during the fiscal expansion output is above the steady state, leading to higher profits and interest rates. In the bad news case, the interest rate effect dominates, so the increase in output leads to a fall in the stock price.

Before the fiscal expansion the stock price is above the steady state if the delay is long, and below it if the delay is short. This is because there are two offsetting effects on the stock price. Between T_0 and T_1 output and interest rates are low, which is good for the stock market. But stock-holders must also look forward to the period after T_1 during which output and interest rates will be high. If the period of time between announcement and implementation of the fiscal expansion ($T_1 - T_0$) is long, the favorable effect of the anticipatory recession will dominate, and the stock market will rise. But, if delay is short, the recession will be short and the subsequent boom is imminent. Consequently, the adverse second-period effects will dominate, and the stock price will fall upon announcement of the future fiscal expansion.

Good news case: We can characterize the "bad news" economy's reaction to an anticipated fiscal expansion as follows. The real exchange rate must appreciate during the future fiscal expansion. Because asset markets are forward-looking, it jump-appreciates part of the way upon announcement of the fiscal expansion. This reduces net exports and, because the fiscal stimulus has not yet arrived, results in a recession until the fiscal expansion is implemented. When the future fiscal expansion is announced, the stock price can either fall (intensifying the anticipatory recession) or rise (offsetting the fiscal expansion). It cannot, however, rise enough to reverse the impact on aggregate demand of the exchange rate appreciation. (This is easy to understand. The stock price only rises when the favorable impact of the initial recession overwhelms the unfavorable effect of the future boom. Thus, an increase in the stock market is inconsistent with an expansion of output between period T_0 and T_1 .)

All of these results can be reversed in the "good news" case. A future fiscal expansion can lead to a transitory appreciation, as in the "bad news" case, or a jump depreciation. Output can either fall or rise in anticipation of the fiscal expansion, and in the case in which output rises it can be due either to an investment boom associated with high stock prices, or to an increase in net exports associated with real exchange rate depreciation. Unlike in the "bad news" case, there can be a temporary period after the fiscal expansion is implemented in which the economy goes into recession.

The task of the following discussion is to bring some order into this list of possibilities, to provide greater insights into the

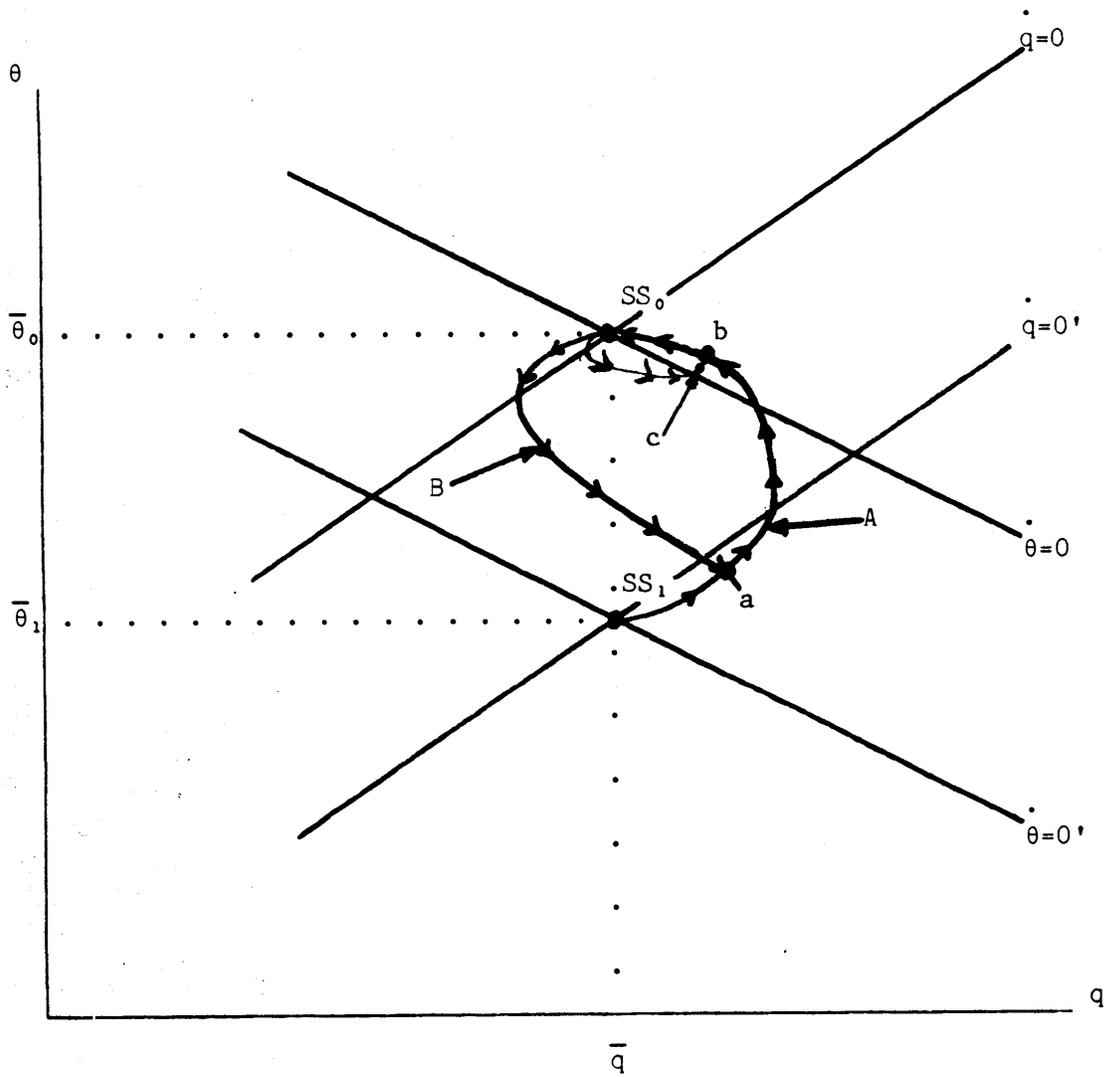
interaction of stock prices and exchange rate movements. To do so, I eschew a comprehensive listing of all the possible outcomes, and focus on two interesting cases; one in which the stock market is more "influential" than is the real exchange rate, and one in which the reverse is true. In particular, I do not discuss cases in which the economy's dynamics are cyclical.^{12/} A more complete taxonomy of possible cases is given in Appendix B.

Figure Four illustrates the dynamic adjustment of the economy to an anticipated, transitory fiscal expansion in the strong-stock-market case. In this case, $a\Delta + c\gamma < 0$.^{13/} Thus, the effect of output on the stock market (Δ) and the stock market on output (a) must be large relative to the effect of output on the exchange rate (c) and the effect of the exchange rate on output (γ). As in Figure Three, the economy must be on the curve (A) between T_0 and T_1 , and on a curve like (B) between time T_1 and T_2 .

We can see from Figure Four that, in this case as in the "bad news" case, the exchange rate must always appreciate relative to the original steady state, both in anticipation of and during the fiscal expansion. Output is always above the steady state during the fiscal expansion.

Whether output rises or falls in anticipation of the fiscal expansion depends upon the length of the expansion, and the delay between announcement and implementation. Suppose the expansion is expected to last for a fairly long time, so that the economy lands at a point like (a) at time T_1 . Then between T_0 and T_1 the economy must be

Figure Four
 Dynamic Adjustment to a Future, Transitory Fiscal Expansion
 Good News Case 1



somewhere on curve (B), which lies entirely below the $\dot{\theta}=0$ schedule. Thus, if the fiscal expansion is expected to last long enough, output necessarily falls before implementation of the expansion.

On the other hand, suppose that the fiscal expansion is expected to last for a shorter period of time, so that at time T_1 the economy lands at point (b). Then the economy must be somewhere on curve (C) between T_0 and T_1 . If the delay between announcement and implementation is long, then at time T_0 the economy jumps to a point on (C) that is close to SS_0 . These points lie below the $\dot{\theta}=0$ schedule, which indicates that output falls at time T_0 . If, however, the delay between announcement and implementation is short enough, the economy jumps at T_0 to a point on (C) like (c), which is above the $\dot{\theta}=0$ schedule, indicating that there is an increase in output at time T_0 . Thus, if the fiscal expansion is expected to be fairly short, the delay between announcement and implementation is short, and $a\Delta + cY < 0$, then output increases upon announcement of a future fiscal expansion.^{14/}

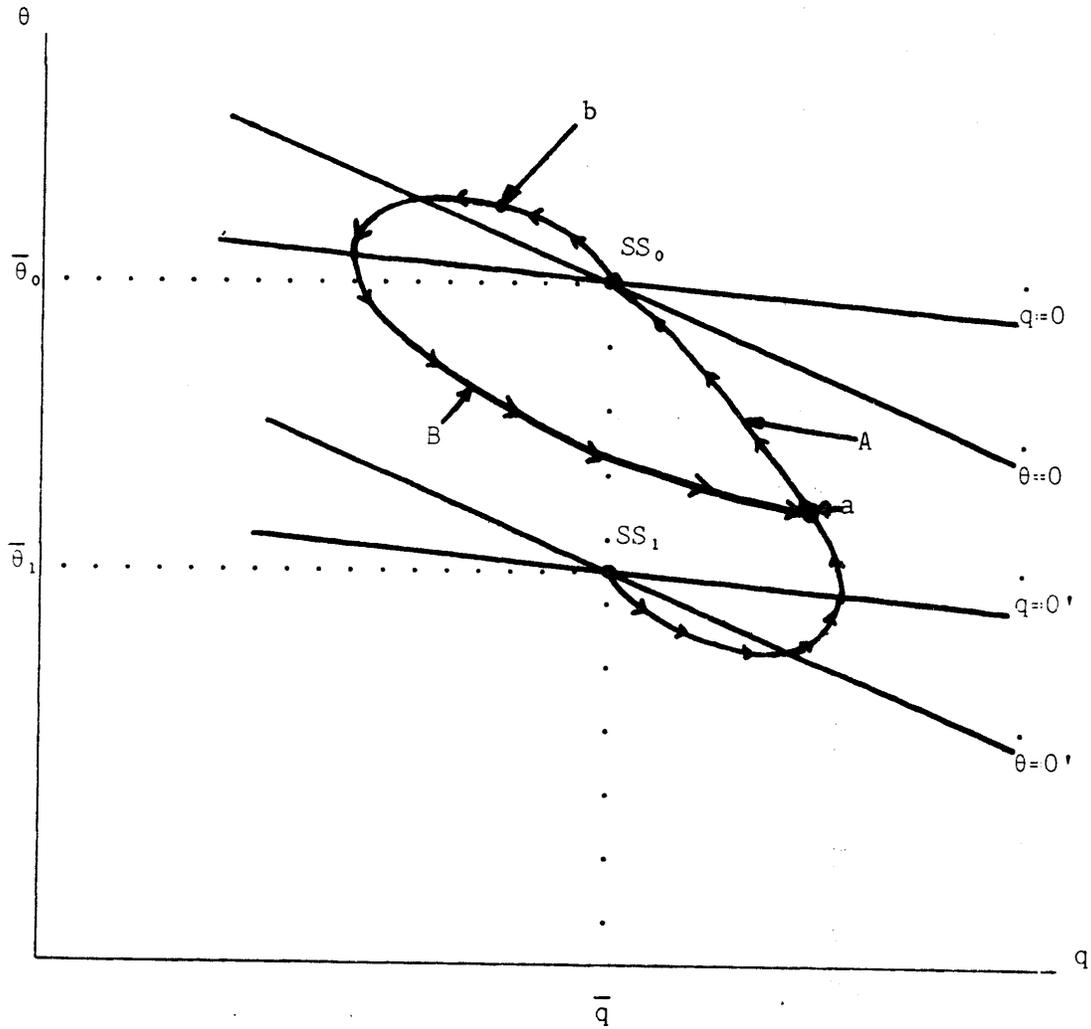
This reversal of the "anticipatory recession" result occurs for a perfectly intuitive reason. Stock-holders look forward to the increase in output during the fiscal expansion, which is good for the stock market. The shorter the fiscal expansion, the larger the output increase, and the closer the fiscal expansion is, the more it counts in investors' present-value calculations. In addition, if the expansion is short, then the real interest rate only increases for a short period of time, which implies that the long real interest rate does not increase much, and therefore that the real exchange rate does not appreciate much.

With a large increase in the stock price and a small exchange rate appreciation, output expands immediately.

A less intuitive possibility is illustrated in Figure Five, which depicts one case in which $a\Delta + cY > 0.15$. This is the case in which the exchange rate is more "influential" than the stock price. Note that (A), the curve upon which the economy must be during the fiscal expansion, lies entirely below the $\dot{\theta} = 0$ schedule. This means that, if the delay between announcement and implementation of the fiscal expansion is short enough, then output must fall upon announcement of the future fiscal expansion. However, consider a point like (b), which is where the economy must jump if the delay between announcement and implementation is long. This segment of curve (B) lies above the $\dot{\theta} = 0$ schedule, which implies that output is above steady state. Thus, as in Figure Four, output may increase upon announcement of a future fiscal expansion, but only if the delay between announcement and implementation is long enough. The output expansion is, unlike in Figure Four, characterized by a decline in the stock market, and a real exchange rate depreciation.

The intuition behind this outcome is as follows. When the fiscal expansion is implemented, the economy will travel from a point like (a) back to SS_0 . During that period of time, the short, real interest rate will be above the world rate. This period of high real interest rates tends to increase the time- T_0 long, real interest rate, thus providing a force for exchange rate appreciation. There is, however an offsetting effect. Before the fiscal expansion is implemented, the economy must go through a recession in which the real interest rate is below the world

Figure Five
 Dynamic Adjustment to a Future, Transitory Fiscal Expansion
 Good News Case 2



rate. If this recession lasts long enough, its depressing effect on the time- T_0 long real interest rate will be larger than the effect of high real interest rates between time T_1 and T_2 , so upon announcement of the future fiscal expansion the long real interest rate will fall, rather than increase. Thus, by equation (8a), the real exchange rate must depreciate. We are, by the assumption that $a\Delta + c\gamma > 0$ (and $c\gamma > r + a\Delta$, see footnote 15), in a case in which the effect of the exchange rate on output is strong, so this exchange rate depreciation leads to a transitory increase in output. As time passes, the stock price declines, and eventually the real exchange rate starts to appreciate, and the economy passes through a period of recession until the fiscal expansion is implemented.

V. Simulation results

The previous sections explored the theoretical properties of the open-economy stock-market model. It was demonstrated that the qualitative behavior of the model depends, inter alia, upon the relative importance of interest rate versus stock market effects on equity valuation, exchange rate versus stock price effects on aggregate demand, and relative speeds of adjustment of output and the price level.

This section makes a preliminary attempt to determine the empirical importance of the theoretical possibilities delineated in sections III and IV. The intention is not to put forth a realistic forecasting model of the United States economy; no four equation dynamic system can fulfil such an ambitious goal. Rather it is to see if the introduction of a stock market significantly alters the response to

various shocks, in a very stylized model that resembles in certain important respects the U.S. economy. The hope is that this will contribute to our understanding of the more difficult question of how the stock market would affect a more fully articulated model of the economy.

The section is organized as follows. First I discuss empirical estimates of the model's parameters. Then I simulate the model for a monetary and a fiscal policy shock, using the solution algorithm developed by Austin and Buiter (1982) and Johnson (1985). For each shock, I consider three cases: one in which stock market effects are negligible (the "indexed bond" case); one in which stock market effects are given by my best estimate (the base case), and one in which stock market effects are somewhat larger than my best estimate.

Parameter estimates: I choose units in what follows so that steady state real output is equal to one. Thus \bar{y} , the log of steady state real output, is zero. In the following empirical discussion, I will use capital letters to correspond to the level of the variable that is expressed in logs in the model. Thus, $y = \log(Y)$, and so on. We need to make assumptions about money demand, the sensitivity of profits to cyclical fluctuations in output, the impact of asset prices on goods markets, the speed of adjustment of the price level, and the speed of adjustment of output.

A simple average of the money demand estimates surveyed in Judd and Scadding (1982, Table 1) implies a long-run income elasticity of 0.725, and an interest elasticity of -0.275. These elasticities imply that "c" in equation (3) is .225, and "h" is $.310.16/$

In equations (25) and (26) I present estimates of the sensitivity of real corporate profits (deflated with the GNP deflator) to cyclical fluctuations in real output, where Y is real GNP and \bar{Y} is potential real GNP:

$$(25) \quad \log(\pi) = .31 + .65 \log(\bar{Y}) + 4.5 \log(Y/\bar{Y}) \quad 1948:1-1985:3$$

$$(0.3) \quad (4.1) \quad (14.5) \quad \rho = .935$$

$$(26) \quad \log(\pi) = -7.5 + 1.61 \log(\bar{Y}) + 4.5 \log(Y/\bar{Y}) \quad 1960:1-1985:3$$

$$(1.2) \quad (2.1) \quad (11.3) \quad \rho = .976$$

The estimated relationship between real profits and potential output is somewhat sensitive to the time-period of estimation, inclusion of a trend and lagged variables, and estimation in differences. However, the estimated elasticity of profits with respect to cyclical fluctuations in output, which is the parameter of interest, was very robust to all the variations I tried.

With $\bar{y}=0$, it follows that α_0 is the share of steady state profits in steady state output. Corporate profits in late 1985 were slightly over 7.5 percent of output.^{17/} Equations (25) and (26) suggest, then, that the steady state share of profits in output, α_0 , is roughly .085. The elasticity of profits with respect to output is α_1/α_0 , which assuming the elasticity is 4.5, implies an estimate for α_1 of about 0.4. This will be my "base case," below. In the "indexed bond" case, I will set α_1 equal to 0. In my "strong stock" case, I will set α_1 equal to 0.5

I assume that the required real rate of return on equity is about .085. This is above the historical rate of return on equity, but real

interest rates have, for the past several years, also been substantially above historical levels. Thus, the steady state value of the stock market, \bar{q} , is $(.085/.085)$ or 1.0 times steady state real output.

A reasonable rule of thumb for the U.S. is that it takes a 12 percent real depreciation to improve the trade balance by one percent of GNP. (This is consistent with Dornbusch (1986), Dornbusch and Frankel (1985), and the results of the Federal Reserve Board's multi-country model.) This implies that a one percent increase in the real exchange rate increases net exports, and therefore aggregate demand, by roughly .085 percent of GNP, or \$3.5 billion. So, γ in equation (1) is set at .085.

There is less empirical evidence on the impact of the stock market on aggregate demand. With no pretensions that these estimates are definitive, I offer the following approximations. Summers (1981) estimates that a 10 percent increase in the stock market raises the ratio of investment to the capital stock by about .009. The ratio of the capital stock to output is about 0.8, so a 10 percent increase in the stock market raises investment by about 0.72 percent of GNP.

In our model, the stock market is about 1.0 times steady state output, so a 10 percent increase in the stock market would increase wealth by .10 times output. If we assume that the marginal propensity to consume out of wealth is about .05, the implied change in consumption demand is about 0.50 percent of output. Thus, a 10 percent increase in the stock market is assumed to raise aggregate demand by roughly 1.2 percent of output.

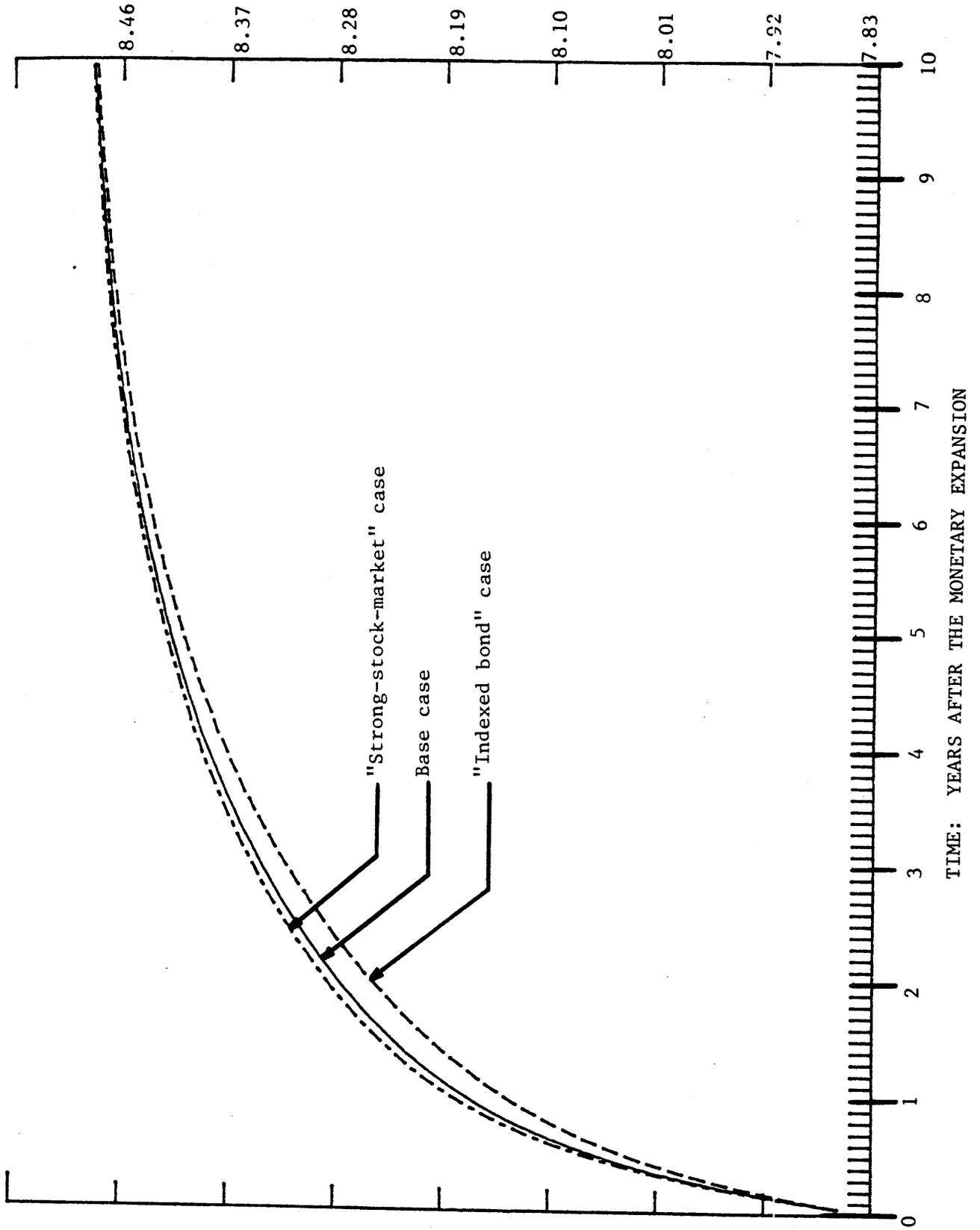
In our model, \bar{q} is about 1.0, so a 10 percent increase in q corresponds to a change in the level of about .10. Thus, "a" in equation (1) which is the percentage change in aggregate demand from a unit change in the level (not the log) of the stock market, is $(.012/.10) = .12$.

Finally, we need to make assumptions about the dynamic adjustment of prices and output. I assume that δ , the price adjustment parameter, is about 1/3. This implies, for example, that a one percent increase in the money supply leads to an increase in the price level of .3 percent at the end of one year, and .5 percent at the end of the second year. Thus, it is assumed that money exhibits substantial non-neutrality in the short run. I assume that σ is about 6, which implies fairly rapid adjustment of output to aggregate demand. This parameter was chosen rather arbitrarily and after some experimentation. It results in an output peak about one year after the beginning of a monetary expansion.

Application to a Monetary Expansion: With these parameters, I simulate the model for an unanticipated monetary expansion of one percent. Figures Six through Nine depict the results. They are qualitatively very similar. In each case, the sudden increase in the money supply leads to a drop of about 0.7 percentage points in the real interest rate. The decline in interest rates leads to a stock price increase and depreciation of the real exchange rate. Output increases transitorily, but as the price level rises, real balances fall, the interest rate rises, and the economy returns to its original steady state.

FIGURE SIX UNANTICIPATED MONETARY EXPANSION (REAL INTEREST RATE)

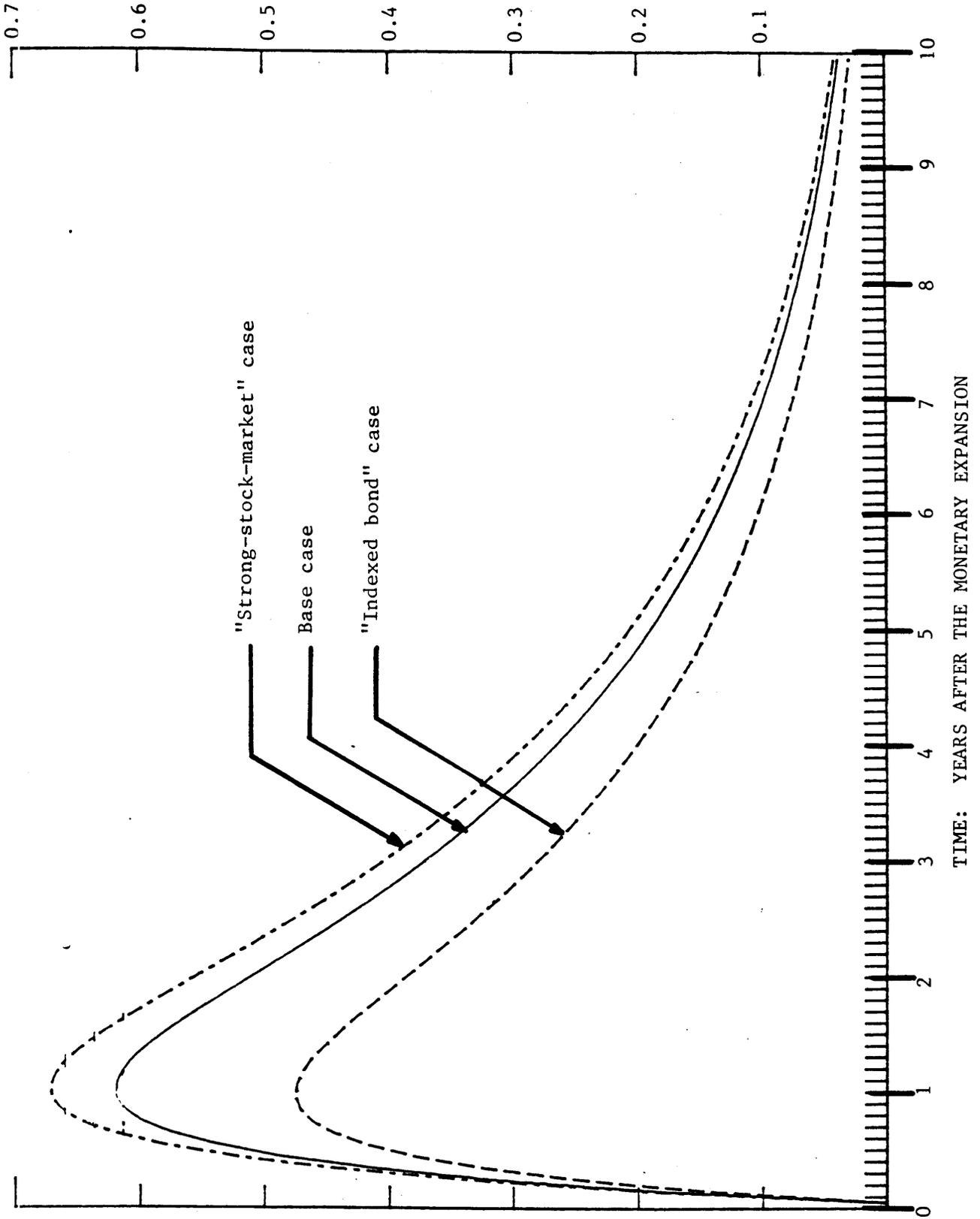
(PERCENTAGE POINTS: STEADY STATE IS 8.5%)



TIME: YEARS AFTER THE MONETARY EXPANSION

FIGURE SEVEN UNANTICIPATED MONETARY EXPANSION (REAL OUTPUT)

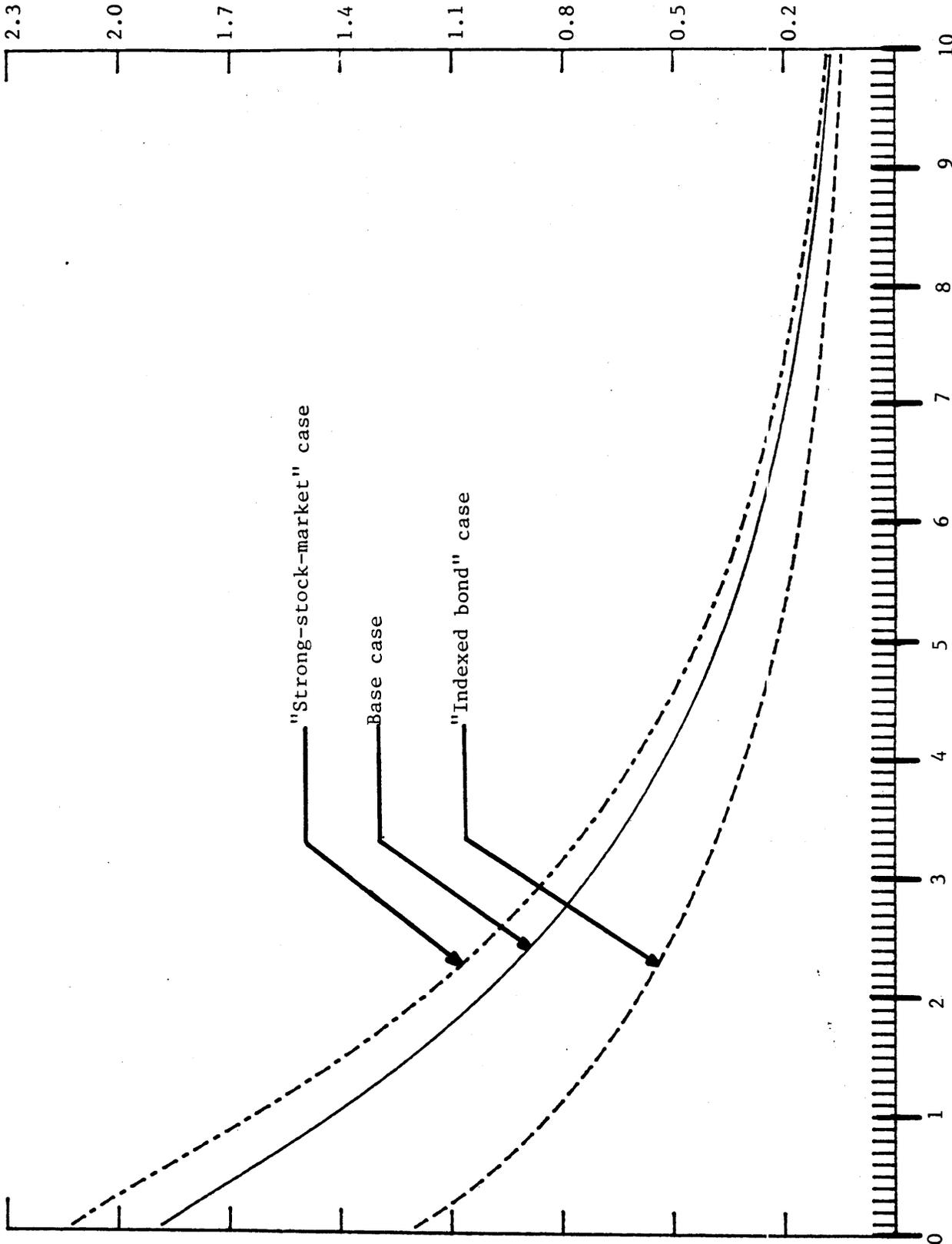
(PERCENT DEVIATION FROM THE STEADY STATE)



TIME: YEARS AFTER THE MONETARY EXPANSION

FIGURE EIGHT UNANTICIPATED MONETARY EXPANSION (STOCK PRICE)

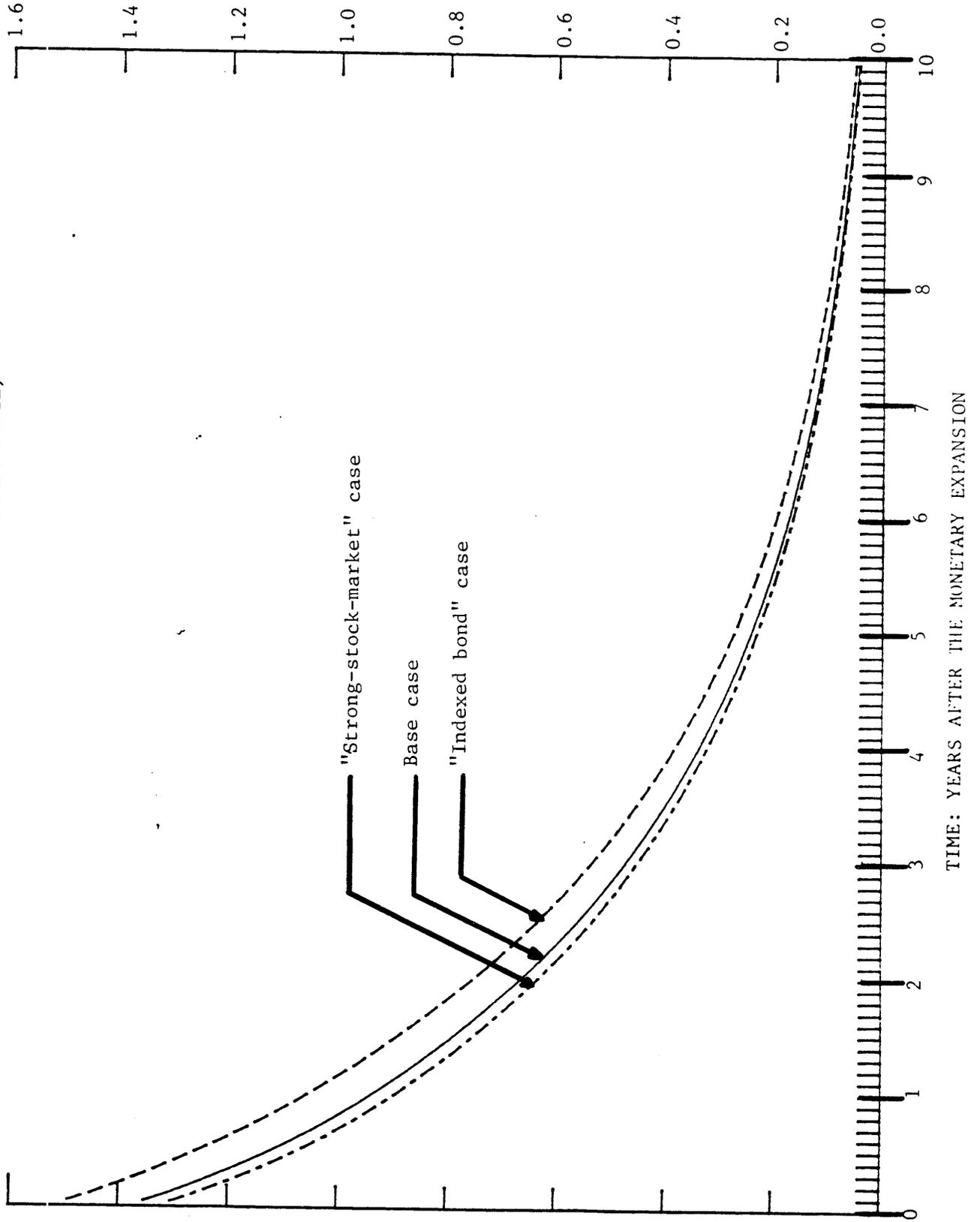
(PERCENT DEVIATION FROM THE STEADY STATE)



TIME: YEARS AFTER THE MONETARY EXPANSION

FIGURE NINE UNANTICIPATED MONETARY EXPANSION (REAL EXCHANGE RATE)

(PERCENT DEVIATION FROM THE STEADY STATE)



TIME: YEARS AFTER THE MONETARY EXPANSION

In the base case, output peaks about 0.62 percent above the steady state, roughly one year after the monetary shock. The increase is only 0.47 percent in the "indexed bond" case, and it is 0.67 percent in the "strong stock market" case. Thus, the stronger the impact of output on the stock market, the larger the impact of monetary policy on output.

This is because the cases with stronger output-profit effects result in larger stock price increases. In the "indexed bond" case, the "stock price," rises only 1.2 percent. (This should be interpreted as a decrease in the long real interest rate on an indexed perpetuity of 1.2 percent, or about .10 percentage points.) In the "base case", the stock price jumps 1.9 percent, and in the "strong stock" case, the stock price jumps 2.1 percent.

The higher is output, the higher is the real interest rate and therefore the less is the exchange rate depreciation induced by the monetary expansion. In the "indexed bond" case the real exchange rate depreciates 1.5 percent, and then gradually appreciates to the steady state. In the "base case," the initial depreciation is 1.35 percent, and in the "strong stock" case, it is 1.30 percent.

Thus, for the chosen parameter values, the impact of introducing a stock market is quantitative, not qualitative. In particular, while the stock market tends to reduce the extent of the real exchange rate overshooting after a monetary expansion, it does not lead to the "reverse overshooting" possibility indicated in the theoretical section. It is somewhat interesting to note that this is not because the stock market is too unimportant, in particular, condition (21) does hold for

these parameter values. However, the critical price adjustment parameter, δ^* , below which the actual price adjustment parameter must lie to obtain the "reverse overshooting," is very low indeed, on the order of .05. Thus, the empirical results are qualitatively similar to the "indexed bond" case not because stock market effects are too small, but rather because price adjustment is too fast, or, put another way, money is too neutral.

Application to an Anticipated Fiscal Expansion: I now consider the simulated impact of an anticipated, transitory fiscal expansion. At time zero, it is learned that in six months there will be an increase in g equal to 2 percent of GNP, an expansion that will last for a year. After that year, fiscal policy returns to its original setting.

Figures 10 through 13 summarize the results of this experiment. In the "indexed bond" case, output falls upon announcement of the future fiscal expansion, as a result both of exchange rate appreciation, and a decline in the "stock market," which is in this context an increase in the long, real interest rate. Output continues to fall until the arrival of the fiscal stimulus, hitting a trough of about 0.33 percent below steady state. In the "strong stock market" case, output actually rises in anticipation of the fiscal stimulus. The real exchange rate appreciates in this case, but its effect on aggregate demand is more than offset by an increase in the value of the stock market. In this case the stock market, looking forward to the high profits to be made after the arrival of the fiscal stimulus, increases immediately. In the "base case," the stock price increases but not enough to completely

FIGURE TEN
ANTICIPATED FISCAL EXPANSION
REAL INTEREST RATE
(PERCENTAGE POINTS: STEADY STATE IS 8.5%)

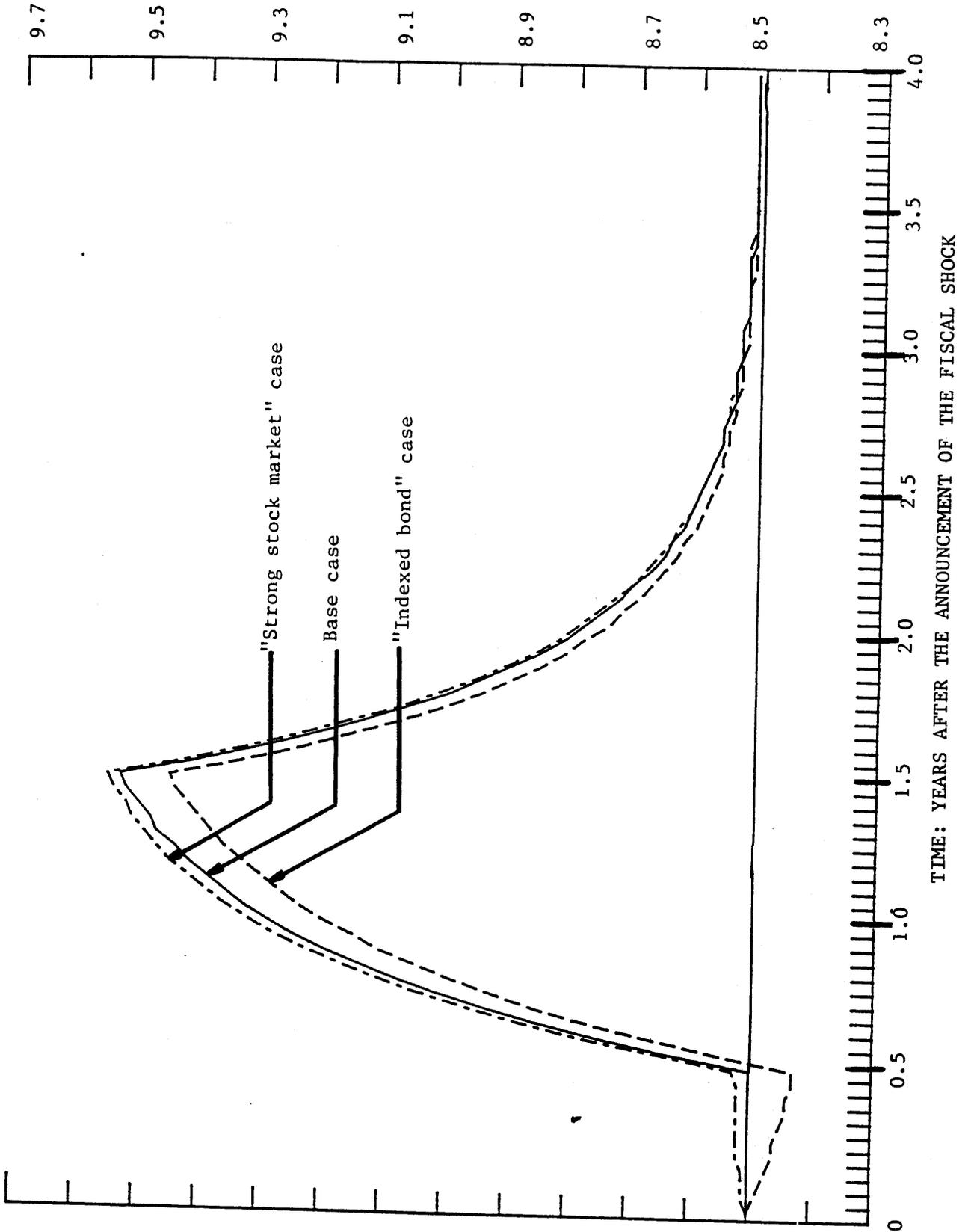


FIGURE ELEVEN
ANTICIPATED FISCAL EXPANSION
REAL OUTPUT
(PERCENT DEVIATION FROM STEADY STATE)

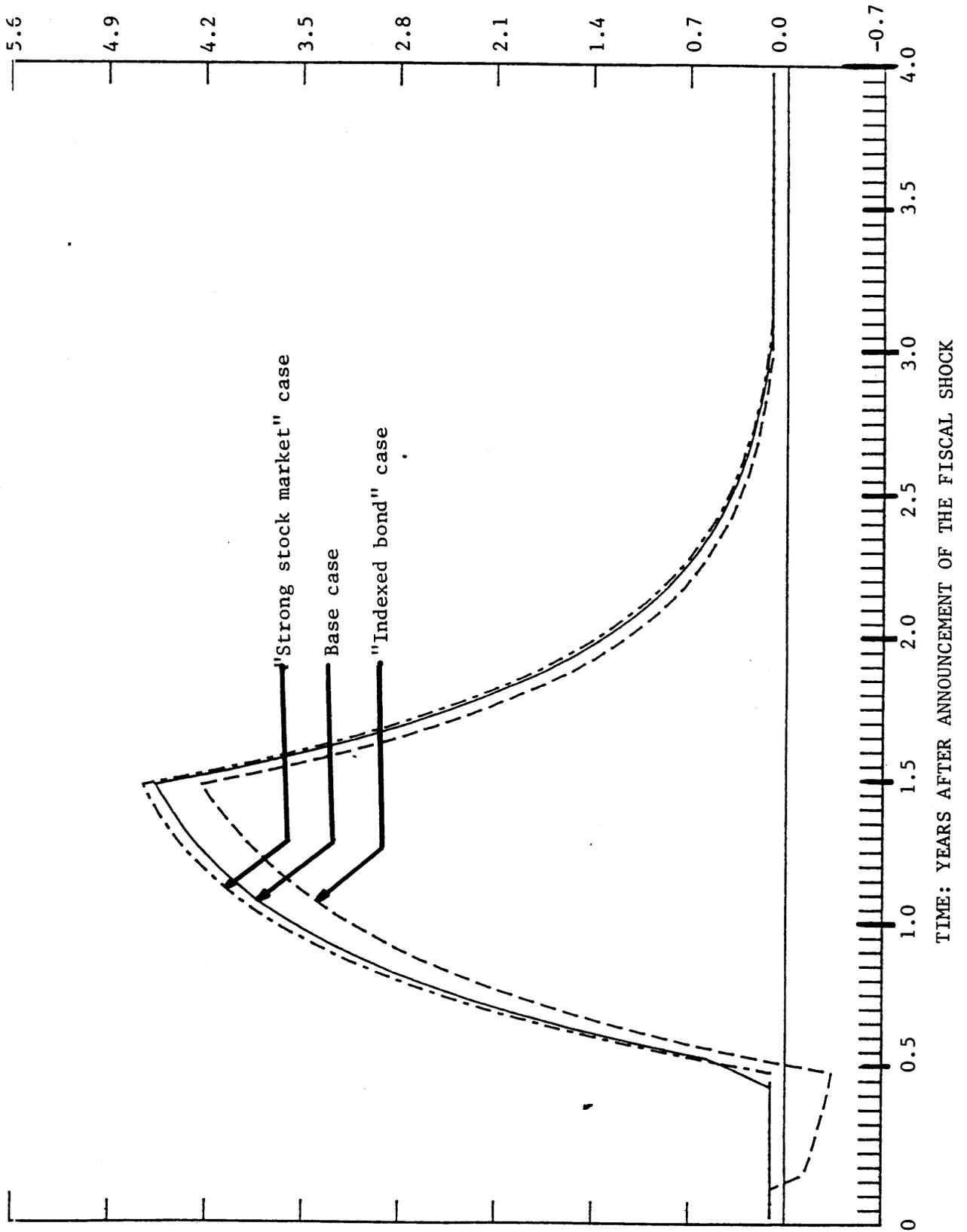
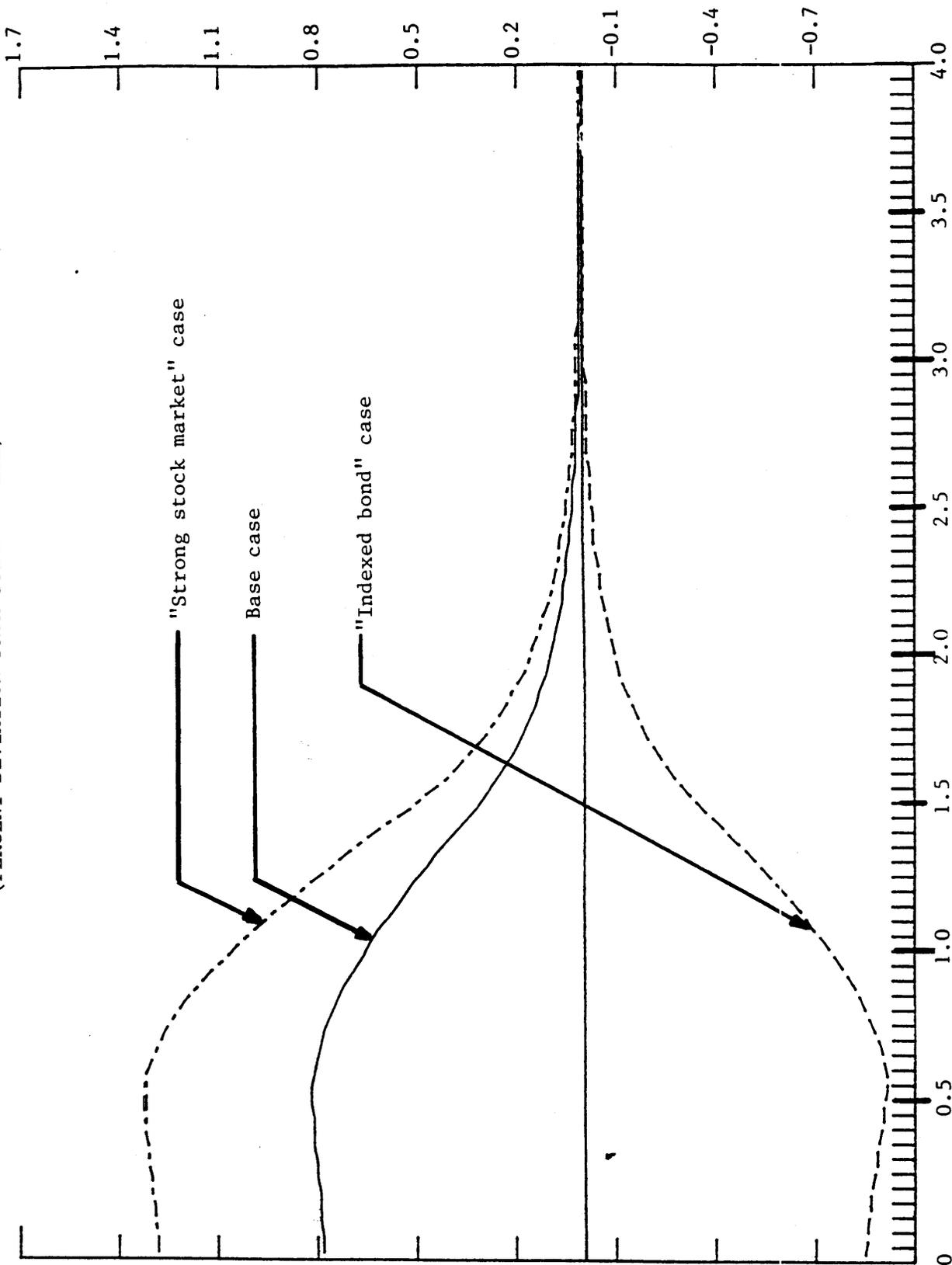


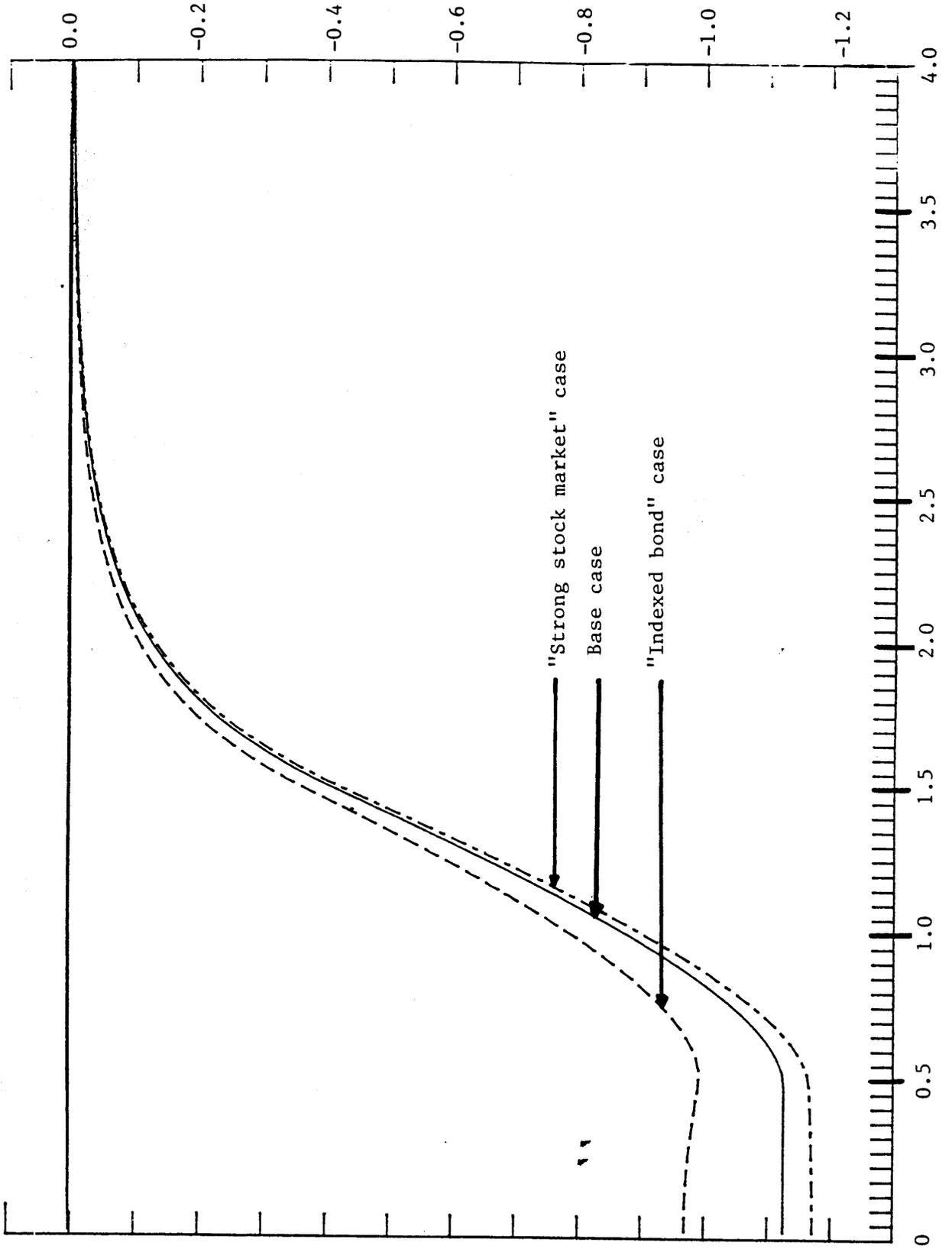
FIGURE TWELVE ANTICIPATED FISCAL EXPANSION STOCK PRICE

(PERCENT DEVIATION FROM STEADY STATE)



TIME: YEARS AFTER ANNOUNCEMENT OF THE FISCAL SHOCK

FIGURE THIRTEEN
ANTICIPATED FISCAL EXPANSION
REAL EXCHANGE RATE
(PERCENT DEVIATION FROM THE STEADY STATE)



TIME: YEARS AFTER ANNOUNCEMENT OF THE FISCAL SHOCK

offset the unfavorable impact on aggregate demand of the exchange rate appreciation. In fact, to a first approximation output is unchanged between announcement and implementation of the fiscal expansion.

After the fiscal expansion arrives, output increases in all cases. The stronger the impact of output on profits, the larger the increase in output, which makes intuitive sense.

The real exchange rate initially appreciates in all three cases. The extent of the appreciation depends upon the strength of the stock market effects: strong stock market effects imply higher future output and interest rates, and therefore generate larger appreciations.

The stock price initially falls in the "indexed bond" case, because the fiscal expansion means higher real interest rates with, by assumption, no impact on profitability. The 0.9 percent decline in the "stock market" can be interpreted as a 0.75 percentage point increase in the internal rate of return on an indexed bond. In the other two cases, the value of the stock market increases. In these cases, the higher profits due to higher future output more than compensate for the higher real interest rates.

VI. Conclusion

This paper analysed a model of the small, open economy in which the stock market, rather than the bond market, determines aggregate demand. It was shown that the asset price and output dynamics can differ in interesting ways from more conventional dynamic, Mundell-Fleming models. In particular, if the stock market effects are large enough, and if money is not "too neutral," then expansionary

monetary policy can lead to real exchange rate appreciation, rather than depreciation. Unlike in models with only a bond market, anticipated fiscal policy can lead to an anticipatory expansion, rather than contraction. Furthermore, if the delay between announcement and implementation of the fiscal expansion is long enough, the anticipated fiscal policy can lead to exchange rate depreciation, rather than appreciation.

The model was simulated for plausible parameter values. The results indicated that, for monetary policy, the behavior of the model was qualitatively similar to that of models without a stock market. The stronger the stock market effects, the larger the impact of the monetary expansion on output, and the smaller the real exchange rate depreciation. Simulations for fiscal policy shocks suggest that stock market effects may be more important in this case. The reversal of the "anticipatory recession" result in, for example, Branson, Fraga and Johnson (1985) appears to be plausible. This is buttressed by the presumption that certain sensible modifications to the model would tend to strengthen the positive impact of an expected future fiscal expansion on current output.

A variety of extensions to the model would be worth considering, although they would likely require simulation methods. It would be useful to deflate nominal money with an expenditure deflator, like the CPI, and to recognize the lag between changes in the real exchange rate and net exports. Both of these would tend to increase the expansionary influence of transitory fiscal expansions, making it more likely that an anticipated fiscal expansion would lead to an anticipatory output

expansion, rather than recession. Working against this is the fact that investment, too, responds to changes in q with a lag.

There are several long-run issues that could be addressed. The effect of different rates of capital accumulation on full-employment output is a potentially important consideration. In addition, the wealth effects of alternative current account and capital accumulation paths should be modelled.

Finally, it would be possible to relax the small-country assumption, by creating a two-country version of the model. And, it would be possible to add risk premia to equations (7) and (8), which would allow the exploration of "safe-haven," or "location" vs. "currency-denomination" risk.

Footnotes

* Division of International Finance, Federal Reserve Board. This paper reflects the views of the author, and should not be construed as reflecting the views of the Board of Governors, or of any other members of its staff. I am especially indebted to Rudiger Dornbusch and Dale Henderson for encouragement and many useful comments. Helpful conversations with Olivier Blanchard and Stanley Fischer are also gratefully acknowledged. I am, of course, solely responsible for any remaining errors.

1/ The standard references are Tobin (1978) and Hayashi (1982). Unlike these papers, I do not make the important distinction between average q , which affects wealth, and marginal q , which affects the investment decision.

2/ The small country assumption, which fixes the real interest rate and, therefore, given the profitability function, the level of investment, makes this assumption more palatable than it might be in a closed economy. However, for experiments in which the steady-state capital stock is potentially affected, (for example, changes in the profitability of capital or the steady state real interest rate), the exogeneity of potential output would be a questionable assumption.

3/ It is noteworthy that, while output is "sticky," it is assumed that the composition of output can be shifted instantaneously between, for example, net exports and government consumption. It would be more realistic to make investment adjust slowly to changes in q (see Fischer (1984)) and net exports respond slowly to changes in the real exchange rate (see the long literature on net exports and the real exchange rate, one example of which is Dornbusch (1986)). Such dynamics would alter the model's properties significantly. They would also make the model analytically intractable, although more realistic models can be explored with simulation methods. Branson, Fraga, and Johnson (1985) includes some of these dynamic complications.

4/ Money demand depends upon the nominal, not the real interest rate. However, for tractability I have assumed that in the steady state monetary growth and inflation are zero, so that the steady nominal interest rate is equal to the steady state real interest rate.

5/ In equations (7) and (8) I have assumed risk neutrality. However, it would be very easy to introduce separate risk premia in equation (8), corresponding to currency-risk, and equation (7), corresponding to "safe-haven" or "country-risk," considerations.

6/ This result holds even in the short run because of the money demand specification that I have adopted. It is well understood (see for example Branson and Buiter (1983) or Henderson (1983)) that deflating nominal balances with an expenditure deflator like the CPI introduces a

dependence between the real exchange rate and real balances. In this case, a fiscal expansion leads to a transitory increase in output because the induced appreciation reduces the CPI, and raises real balances. Another factor that would lead to short-run output effects from a permanent, unanticipated fiscal expansion is sluggish adjustment of net exports to the real exchange rate. However, in any case equation (11) holds as a steady state relation.

7/ This distinction is discussed in more detail in Blanchard (1981) and an earlier version of this paper.

8/ I discuss the analytical solutions for output, the real exchange rate, and the stock price in Appendix A, available from the author.

9/ This result depends upon the money demand assumption that I have adopted. See footnote 6.

10/ In Figure Two I have considered only those cases in which the positive roots are not complex. Appendix B discusses the assumptions under which the roots are real. Section V shows that, for plausible parameter estimates, the qualitative properties of the model are not much affected when the roots are complex.

11/ See footnote 6 for appropriate qualifications.

12/ See Appendix B for a more comprehensive discussion. In the case of real roots, there are actually three distinct subcases, of which I describe only two in the text of the paper. In the third subcase described more fully in Appendix B, an anticipated fiscal expansion is necessarily contractionary, as in the bad news case. The dynamics for this case look like those of Figure Four, except that the curve labelled (A) cuts through the SS_0 curve from below the $\theta=0$ curve.

13/ To interpret this condition, consider a one-period, unit increase in output. The stock price would, through interest rate and profit effects, increase by $-\Delta$, which would, in turn, raise aggregate demand by $-a\Delta$. The one-period, unit increase in output would raise the real interest rate, and lower the real exchange rate, by c . This, in turn, would lower aggregate demand by cY . If $-a\Delta > cY$, then the favorable effect on aggregate demand from the stock market increase would be larger than the unfavorable impact of the real exchange rate appreciation.

14/ In the closed economy case analysed by Blanchard (1981), an anticipated fiscal policy expansion was necessarily expansionary in the "good news" case. There are two sources of difference between his analysis and that of this paper. First, he analysed fiscal policy shocks under the assumption of fixed prices, so that output and profits were permanently increased by the fiscal expansion. Second, with no foreign sector, there was in his model no possibility of crowding out through net exports that is an important part of this model.

15/ There are actually two subcases to consider when $a\Delta + cY > 0$, depending upon whether $(\bar{r} + a\Delta)$ is greater than or less than cY . In the case discussed in the text, $(\bar{r} + a\Delta) < cY$. When the reverse is true, an anticipated fiscal expansion is necessarily contractionary. See Appendix B for further discussion.

16/ To see this, note that the money demand function can be written:

$$\begin{aligned}(m-p) &= .725y = .275\log(i) + \text{const} \\ \log(i) &= (.725/.275)y = (1/.275)(m-p) + \text{const}\end{aligned}$$

Linearizing about the steady state:

$$\begin{aligned}\hat{i}/i &= (.725/.275)\hat{y} = (1/.275)(\hat{m}-\hat{p}) \\ i &= \bar{i} + \bar{i}[(.725/.275)\hat{y} = (1/.275)(\hat{m}-\hat{p})]\end{aligned}$$

17/ It would also be possible to assume that non-corporate capital income is capitalized and considered part of "the stock market." For that matter, in a model with forward-looking wage earners who are not liquidity constrained, it would be appropriate to assume that labor income is capitalized as well. These broader definitions of "the stock market" would presumably make the stock-market transmission mechanism more powerful than my narrow definition.

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Appendix A: Monetary Expansion:

We suppose at time zero the economy is in steady state, when there is an unanticipated and permanent increase in the money supply of \hat{m} . The only change in the steady state is an increase in the price level equal to \hat{m} . This appendix provides analytical justification for the discussion of dynamics in Section III of the main text of this paper. The solution for (12) can be written:

$$\begin{bmatrix} y(t)-\bar{y} \\ q(t)-\bar{q} \\ \theta(t)-\bar{\theta} \\ p(t)-\bar{p} \end{bmatrix} = \begin{bmatrix} 1 \\ -\Delta/(\bar{r}+\lambda) \\ -c/\lambda \\ 0 \end{bmatrix} k_1 e^{-\lambda_1 t} + \begin{bmatrix} \gamma(\bar{r}+\delta)+a\delta\bar{q} \\ \delta\bar{q}(b-\delta/\sigma)+\gamma\alpha_1 \\ (\bar{r}+\delta)(b-\delta/\sigma)-a\alpha_1 \\ -\Omega \end{bmatrix} k_2 e^{-\delta t} \quad (A1)$$

$$\text{where } \Omega = \delta/(\bar{r}+\delta)[(\bar{r}+\delta)(b-\delta/\sigma-c\gamma/\delta)+a(c\bar{q}-\alpha_1)] \quad (A1a)$$

$$\text{and } \Delta = (c\bar{q}-\alpha_1) \quad (A1b)$$

We know that the constants associated with the positive eigenvalues are zero by the assumption that the economy converges to a steady state, ie, that it is on its saddle path. The constants k_1 and k_2 are determined by the requirement that neither the price level nor output can jump at time zero. That is, $y(0) = \bar{y}$, and $p(0) = \bar{p}$. Solving for k_1 and k_2 , we have:

$$k_1 = (\hat{m}/\Omega) \{\gamma(\bar{r}+\delta)+a\delta\bar{q}\} \quad (A3)$$

$$k_2 = \hat{m}/\Omega \quad (A4)$$

With (A1) through (A4) we can discuss the dynamics in more detail

1. Price dynamics

These dynamics are trivial, because of the simple price adjustment equation (5). The solution is simply:

$$[p(t)-\bar{p}] = -\hat{m} e^{-\delta t} \quad (A5)$$

where \bar{p} is the new steady-state price level. So the price level adjusts monotonically from its old steady state, $p(0)$, toward its new steady state, \bar{p} , regardless of output dynamics or, for that matter, anything else.

2. Output dynamics

The solution for output can be written:

$$[y(t)-\bar{y}] = \frac{\hat{m}}{\Omega} [\gamma(\bar{r}+\delta)+a\delta\bar{q}](e^{-\delta t} - e^{-\lambda t}) \quad (A6)$$

Assuming $\lambda \neq \delta$, output is "hump-backed," starting at \bar{y} , rising to a maximum at time t_{\max} where:

$$t_{\max} = \frac{\log(\delta/\lambda)}{(\delta - \lambda)} \quad (A7)$$

then falling asymptotically and monotonically toward full-employment output.

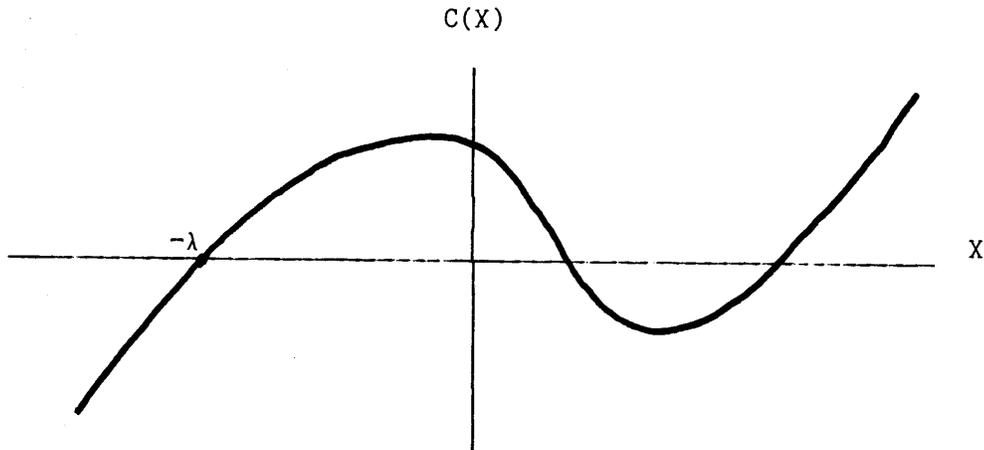
From (A6), it is not immediately obvious whether output rises or falls after the monetary stimulus. Observe that:

$$\dot{y} \Big|_{t=0} = \frac{[\gamma(\bar{r}+\delta)+a\delta\bar{q}](\lambda-\delta) \hat{m}}{\Omega} \quad (\text{A8})$$

Therefore, if $(\lambda-\delta)$ and Ω have the same sign, output rises after the monetary expansion. However, if $(\lambda-\delta)$ and Ω are of opposite signs, then output initially declines after the monetary expansion. However, it is possible to show that $\lambda > \delta$ implies $\Omega > 0$, and the converse. This implies that, in this model, a monetary expansion necessarily results in a transitory increase in output.

To see this, recall that $-\lambda$ is the negative root of the cubic part of the characteristic equation (14). I showed above that there are two positive roots and one negative root to this equation. Denote the polynomial in (14) as $C(X)$. By inspection, $C(0) > 0$, so the polynomial looks like the following:

Figure A1



This shows that, for positive X , $-X < -\lambda$ implies that $C(-X) < 0$ and conversely. This matters because it turns out that Ω , viewed as a function of δ , has the same sign as $C(-\delta)$. It follows that $\lambda > \delta$, implies (and is implied by) $\Omega > 0$, and the converse. This allows us to conclude that output necessarily rises after the monetary stimulus.

3. Exchange rate dynamics

As noted above, the steady state real exchange rate is unchanged by the more expansionary monetary policy. From (A1), (A3), and (A4), we obtain the solution for the real exchange rate's dynamics:

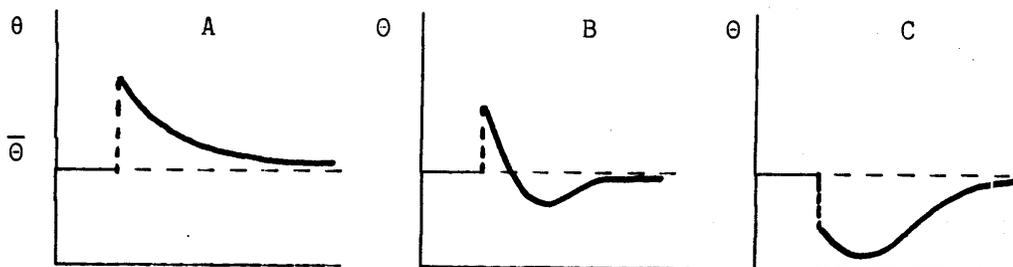
$$[\theta(t) - \bar{\theta}] = \frac{\hat{m}}{\Omega} \left[\frac{[\gamma(\bar{r} + \delta) + a\delta\bar{q}]ce^{-\lambda t}}{\chi} + [(\bar{r} + \delta)(b - \delta/\sigma + c\gamma/\delta) - a\alpha_1]e^{-\delta t} \right] \quad (A9)$$

We begin the analysis of (A9) by making two observations; first, the solution is the sum of two declining exponentials, so that the time path for the real exchange rate can have at most one "hump", that is, one point at which the time derivative equals zero. Second, because both output and the price level are sticky, the home interest rate necessarily falls below the world rate at time zero, which means that just after the monetary expansion (and after the exchange rate jumps) the exchange rate has to be appreciating. Formally:

$$\dot{\theta} \Big|_{t=0} = (r - \bar{r}) \Big|_{t=0} = -\hat{m} (h + \delta) < 0 \quad (\text{A10})$$

These two facts allow us to narrow the possible time paths for $\theta(t)$ down to three, illustrated below:

Figure A2



We begin to expand upon these general observations by figuring out what happens at time zero, just after the increase in the money supply; does the exchange rate initially depreciate (the standard result) or does it appreciate? From (26), we have:

$$[\theta(0) - \bar{\theta}] = \frac{\hat{m}}{\Omega} \left[(\bar{r} + \delta) \left[b - \delta / \sigma + \frac{cY}{\delta} (\delta / \lambda) \right] + a [c\bar{q}(\delta / \lambda) - \alpha_1] \right] \quad (A11)$$

$$\equiv \frac{\hat{m}}{\Omega(\delta)} Q(\delta) \quad (A11')$$

$Q(\delta)$ is defined implicitly in (A11), and for the rest of this discussion I am viewing Ω (equation (A1a)) as a function of δ .

From (A11') we see that when $\Omega(\delta)$ and $Q(\delta)$ are the same sign, the exchange rate's initial jump is a depreciation. When they are of opposite sign the exchange rate initially appreciates. I now investigate conditions under which $Q(\delta)$ and $\Omega(\delta)$ are of opposite sign, to see if there are any conditions under which a monetary expansion leads to a jump appreciation as in Figure A2-C.

There are three facts about the functions $Q(\delta)$ and $\Omega(\delta)$ that will help us.

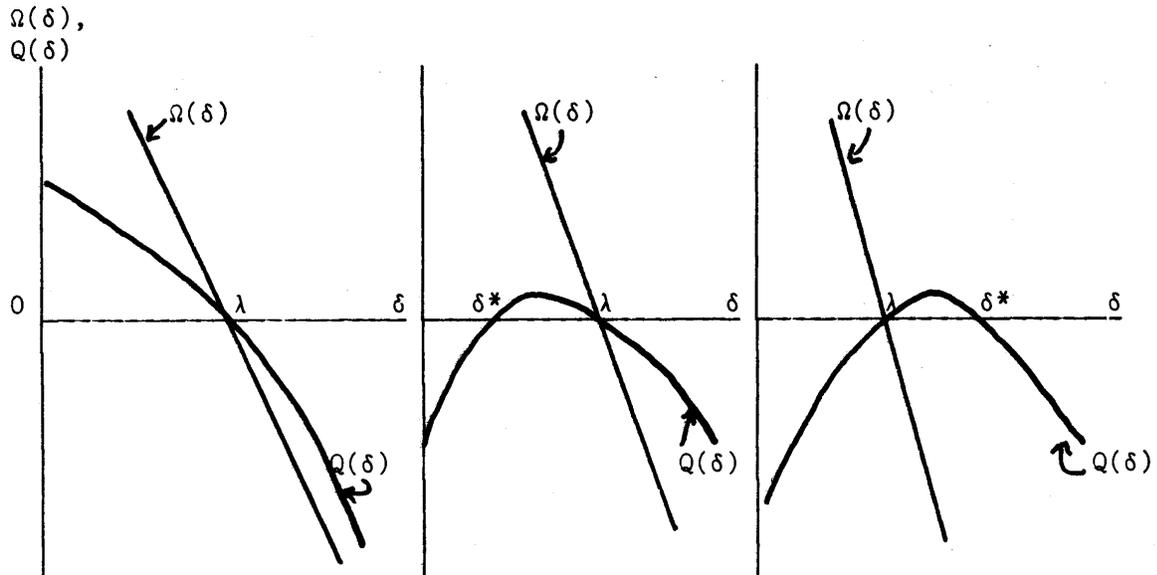
$$1. \quad \Omega(\lambda) = Q(\lambda) = 0 \quad (A12a)$$

$$2. \quad \Omega(\delta) \begin{matrix} > 0 \\ < 0 \end{matrix} \text{ as } \lambda \begin{matrix} > \\ < \end{matrix} \delta \quad (A12b)$$

$$3. \quad Q''(\delta) < 0 \quad (A12c)$$

These facts narrow the list of possibilities to three, described in the following diagrams:

Figure A3:
 Ω and Q as Functions of δ



Note that $\Omega(\delta)$ is always greater than zero when δ is less than λ , and vice versa, and that when δ equals λ , both $Q(\delta)$ and $\Omega(\delta)$ are equal to zero. This is the diagrammatic correspondent of facts (1) and (2) above. Fact (2) was discussed in the section on exchange rate dynamics, and also implies that $\Omega(\lambda)=0$. That $Q(\lambda)=0$ is trivial to show; note that $\Omega = Q (=0)$ when $\lambda=\delta$.

I have also drawn all of the $Q(\delta)$ schedules concave, which is fact (3). This fact is easily verified by differentiating $Q(\delta)$ twice.

Now, consider case (A). In this case $Q(0) > 0$, and there is no value of δ for which Q and Ω are of opposite signs. This means that the exchange rate necessarily depreciates at the moment that the money supply is increased. The condition for case (A) is:

$$Q(0) = \bar{r}(b+c\gamma/\lambda) - a\alpha_1 > 0 \quad (\text{A13})$$

Now consider case (B). Here things are more interesting; when δ is less than δ^* , $Q(\delta)$ and $\Omega(\delta)$ have opposite signs. This means that, in this case, there is a critical price adjustment parameter δ^* such that if $\delta < \delta^*$, the exchange rate initially appreciates when the money stock is increased. The conditions for case (B) are:

$$Q(0) = \bar{r}(b+c\gamma/\lambda) - a\alpha_1 < 0 \quad (\text{A14a})$$

$$Q'(\lambda) = -(2\lambda + \bar{r} - b\sigma) + c\sigma(\gamma + a\bar{q})/\lambda < 0 \quad (\text{A14b})$$

Case (C) is qualitatively the same as case (B), except that the critical speed of price adjustment is in this case greater than λ . The conditions for case (C) are:

$$Q(0) = \bar{r}(b+c\gamma/\lambda) - a\alpha_1 < 0 \quad (\text{A15a})$$

$$Q'(\lambda) = -(2\lambda + \bar{r} - b\sigma) + c\sigma(\gamma + a\bar{q})/\lambda > 0 \quad (\text{A15b})$$

So, if price adjustment is slow enough, and condition (A14a) holds, then the exchange rate initially jump appreciates after a monetary expansion. On the other hand, if price adjustment is fast, or if condition (A14a) does not hold, then the exchange rate must jump depreciate after an unanticipated monetary expansion.

A question is whether case (B) or (C) is indeed possible. The issue here is that λ is a complicated function of the various structural

parameters that enter into condition (A14a), and it may be that λ depends on them in some way that makes (A14a) impossible. The easiest way to prove that (B) or (C) can indeed occur is to construct an example of an economy for which (A14a) holds. This turns out to be possible, and in fact holds for the base case of the empirical section of the text, so the result is of at least theoretical interest.

4. Stock price dynamics

Using (A1), (A3), and (A4), we can derive the solution for the time path of stock prices. It is:

$$[q(t) - \bar{q}] = \frac{\hat{m}}{\Omega} \left[\frac{(c\bar{q} - \alpha_1)[\gamma(\bar{r} + \delta) + \alpha\delta\bar{q}]e^{-\delta t}}{(\bar{r} + \lambda)} + [\delta\bar{q}(b - \delta/\sigma) + \gamma\alpha_1]e^{-\delta t} \right] \quad (\text{A16})$$

This is not particularly revealing, so we begin by establishing the direction of the jump at $t=0$. Using (A16), we obtain:

$$[q(0) - \bar{q}] = \frac{\hat{m}}{\Omega} \left[\frac{(c\bar{q} - \alpha_1)[\gamma(\bar{r} + \delta) + \alpha\delta\bar{q}]}{(\bar{r} + \lambda)} + [\delta\bar{q}(b - \delta/\sigma) + \gamma\alpha_1] \right] \quad (\text{A17})$$

$$\equiv \frac{\hat{m} W(\delta)}{\Omega(\delta)} \quad (\text{A17}')$$

where $W(\delta)$ is defined implicitly in (A17).

As with the exchange rate, we have three bits of information about $\Omega(\delta)$ and $W(\delta)$.

$$1. \quad \Omega(\lambda) = W(\lambda) = 0 \quad (\text{A18a})$$

$$2. \quad \Omega(\delta) \begin{matrix} > 0 \\ < \end{matrix} \text{ as } \lambda \begin{matrix} > \\ < \end{matrix} \delta \quad (\text{A17b})$$

$$3. \quad W''(\delta) < 0 \quad (\text{A18c})$$

There are again three cases, which are depicted in Figure A3. (Simply relabel the $Q(\delta)$ curve as $W(\delta)$). To see which of the three are relevant, evaluate $W(\delta)$ at $\delta=0$:

$$W(0) = \gamma \left[\frac{(c\bar{q} - \alpha_1)\bar{r}}{(\bar{r} + \lambda)} + \alpha_1 \right] \quad (\text{A19})$$

$$= \gamma \left[c\bar{q} \left[\frac{\bar{r}}{\bar{r} + \lambda} \right] + \alpha_1 \left[\frac{\lambda}{\bar{r} + \lambda} \right] \right] > 0 \quad (\text{A19'})$$

Unlike in the case of the real exchange rate, $W(0)$ is necessarily positive. That means that only case (A) in Figure A3 is relevant, which is to say that the price of shares in the stock market must jump upward just after the increase in the money supply.

Appendix B: Fiscal Policy

Preliminaries:

This appendix provides analytical support for the intuitive and graphical arguments in Section IV. I consider the case in which σ is very large, so that output equals aggregate demand. Then, equation (1) of the paper becomes:

$$B1) \quad (y - \bar{y}) = a(q - \bar{q}) + \gamma(\theta - \bar{\theta})$$

where the a and γ in equation (B1) equal the a and γ in equation (1) divided by $(1-\beta)$.

As explained in the text of the paper, the price level is constant. Without loss of generality, I set $p=m$, in which case money market equilibrium can be written:

$$B2) \quad (r - \bar{r}) = c(y - \bar{y})$$

The equations for asset market equilibrium are:

$$B3) \quad \dot{\theta} = (r - \bar{r}) = c(y - \bar{y})$$

$$B4) \quad \dot{q} = rq - \pi$$

Linearizing (B4) around the steady state, and substituting the expression for π , we have:

$$B4') \quad \dot{q} = (\bar{r}+a\Delta)(q - \bar{q}) + \gamma\Delta(\theta - \bar{\theta})$$

where $\Delta = (c\bar{q} - \alpha_1)$

(B1)-(B4') can be summarized as follows:

$$B5) \quad \begin{bmatrix} \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \bar{r}+a\Delta & \gamma\Delta \\ ac & c\gamma \end{bmatrix} \begin{bmatrix} q-\bar{q} \\ \theta-\bar{\theta} \end{bmatrix}$$

The characteristic equation for (B5) is:

$$B6) \quad \phi^2 - \phi(\bar{r}+a\Delta+c\gamma) + \bar{r}\gamma c = 0$$

If $(\bar{r}+a\Delta+c\gamma) > 0$, then (B6) has two positive real roots, or a pair of complex roots with a positive real part. (In either case, the saddle point stability of the system is guaranteed.) I assume this condition throughout, and denote the roots ϕ_1 and ϕ_2 , with $\phi_1 < \phi_2$. The roots are:

$$B7a) \quad \phi_1 = \frac{1}{2} [(\bar{r}+a\Delta+c\gamma) - \sqrt{(\bar{r}+a\Delta+c\gamma)^2 - 4\bar{r}c\gamma}]$$

$$B7b) \quad \phi_2 = \frac{1}{2} [(\bar{r}+a\Delta+c\gamma) + \sqrt{(\bar{r}+a\Delta+c\gamma)^2 - 4\bar{r}c\gamma}]$$

The roots are real if:

$$B8) \quad (\bar{r}+a\Delta+c\gamma)^2 - 4\bar{r}c\gamma > 0$$

If $\Delta > 0$ (the bad news case) then (B8) necessarily holds, so the roots are necessarily real. In the good news case, the roots might be complex. To keep the analysis tractable, I rule out complex roots. Writing (B8) as a

quadratic in \bar{r} , we can determine conditions on \bar{r} such that the roots are real.

$$\text{B8')} \quad \bar{r}^2 + 2(a\Delta - c\Upsilon)\bar{r} + (a\Delta + c\Upsilon)^2 > 0$$

By inspection it is obvious that this holds for very small and very large values of \bar{r} . Specifically, the roots are real in the good news case if:

$$\text{B8a)} \quad \bar{r} > (c\Upsilon - a\Delta) + 2\sqrt{-a\Delta c\Upsilon}$$

or:

$$\text{B8b)} \quad \bar{r} < (c\Upsilon - a\Delta) - 2\sqrt{-a\Delta c\Upsilon}$$

It will be assumed in what follows that one of these conditions is met. When it matters, I will specify whether I am assuming (B8a) or (B8b). This is a restrictive assumption, but it makes the analysis much easier. The simulations in Section V of the text showed that complex roots are possible, but also that the dynamic behavior of the model was, at least for that set of parameters, well described by the following analysis.

We will need some basic relations between the eigenvalues and the structural parameters of the model, which I now establish.

In the bad news case, we have the following important facts:

$$\text{B9a)} \quad \phi_1 < \bar{r} < \phi_2$$

$$\text{B9b)} \quad \phi_1 < (\bar{r} + a\Delta) < \phi_2$$

$$\text{B9c)} \quad a\Delta + c\Upsilon > 0$$

$$\text{B9d)} \quad \text{Both } \phi_1 \text{ and } \phi_2 \text{ are necessarily real.}$$

To prove (B9a), use (B7a), and rearrange the term under the radical to obtain:

$$B10a) \quad \phi_1 - \bar{r} = \frac{1}{2} [(a\Delta + c\gamma - \bar{r}) - \sqrt{(a\Delta + c\gamma - \bar{r})^2 + 4ra\Delta}]$$

In the bad news case, Δ is positive, so the radical is larger in absolute value than is $(a\Delta + c\gamma - \bar{r})$. Thus, $(\phi_1 - \bar{r})$ must be negative. As for ϕ_2 , we have:

$$B10b) \quad \phi_2 - \bar{r} = \frac{1}{2} [(a\Delta + c\gamma - \bar{r}) + \sqrt{(a\Delta + c\gamma - \bar{r})^2 + 4ra\Delta}]$$

Again, the radical is larger in absolute value than the first term in parentheses, so $(\phi_2 - \bar{r})$ is necessarily positive.

(B9b) is proved by exactly the same logic, where:

$$B11a) \quad \phi_1 - (\bar{r} + a\Delta) = \frac{1}{2} [(c\gamma - (\bar{r} + a\Delta)) - \sqrt{(c\gamma - (\bar{r} + a\Delta))^2 + 4c\gamma a\Delta}]$$

$$B11b) \quad \phi_2 - (\bar{r} + a\Delta) = \frac{1}{2} [(c\gamma - (\bar{r} + a\Delta)) + \sqrt{(c\gamma - (\bar{r} + a\Delta))^2 + 4c\gamma a\Delta}]$$

(B9c) holds trivially, because in the bad news case $\Delta > 0$. (B9d) holds because the term under the radical in equation (B10) is by inspection positive.

Turning now to the good news case, we will need the following facts:

B12a) ϕ_1 and ϕ_2 are less than $(\bar{r} + a\Delta)$ if $(\bar{r} + a\Delta) > c\gamma$, and conversely

B12b) ϕ_1 and ϕ_2 are less than \bar{r} if $(a\Delta + c\gamma) < \bar{r}$, and conversely.

B12c) The slope of the equilibrium trajectory in (q, θ) space as it passes through the original steady state, SS_0 , is greater than the slope of the $\dot{\theta} = 0$ schedule if $(a\Delta + c\gamma) < 0$

To prove (B12a) and (B12b), consider equations (B10) and (B11). With Δ less than zero, the term in parentheses is larger in absolute value than is

the radical. Thus, the signs of both ϕ_1 and ϕ_2 are unchanged by the addition or subtraction of the radical. (B12a) and (B12b) follow immediately.

I defer a proof of (B12c) until after I have derived analytical solutions for the time paths of $\theta(t)$ and $q(t)$, immediately below.

Finally, we need to discuss the eigenvectors of the system. I normalize so that the first element of the eigenvector, that which multiplies the stock price, is 1. Denote the second element of the eigenvector corresponding to the small eigenvalue s_1 , and the second element of the eigenvector corresponding to the large eigenvalue s_2 . Simple algebra then establishes that:

$$B13) \quad s_1 = (\phi_1 - (\bar{r} + a\Delta)) / \gamma\Delta, \quad s_2 = (\phi_2 - (\bar{r} + a\Delta)) / \gamma\Delta$$

Analytical Solution for the Anticipated, Transitory Fiscal Expansion

At time zero the economy is in steady state. It is announced that, at time T_1 , there will be a fiscal expansion of amount g , lasting until time T_2 , when fiscal policy will go back to its original setting. If T_1 is very close to zero, then this is the same as an unanticipated fiscal policy change. If $(T_2 - T_1)$ is very large, then the analysis is the same as for a permanent fiscal policy change. Thus, there is nothing restrictive about either the "transitory" or "anticipated" assumptions.

The only change in the stationary state is a reduction in the steady state real exchange of g/Y between time T_1 and T_2 . Thus, equation (B5) holds, except that between T_1 and T_2 the steady state exchange rate is the

original steady state minus g/γ . So, we can write the solution to the problem as follows:

$$B14) \quad [q(t) - \bar{q}] = \begin{cases} k_1 e^{\phi_1 t} + k_2 e^{\phi_2 t} & t < T_1 \\ k_3 e^{\phi_1 t} + k_4 e^{\phi_2 t} & T_1 < t < T_2 \end{cases}$$

$$B15) \quad [\theta(t) - \bar{\theta}] = \begin{cases} s_1 k_1 e^{\phi_1 t} + s_2 k_2 e^{\phi_2 t} & t < T_1 \\ s_1 k_3 e^{\phi_1 t} + s_2 k_4 e^{\phi_2 t} - g/\gamma & T_1 < t < T_2 \end{cases}$$

where the barred values denote original steady states, ie, the steady state in the absence of the fiscal expansion. After T_2 , when the fiscal expansion has ended, both $q(t)$ and $\theta(t)$ return to their original steady state values.

Four conditions determine the four constants, k_1 through k_4 : neither the stock market nor the exchange rate can jump at time T_1 , when the fiscal expansion begins, nor can either jump at T_2 , when the fiscal expansion ends. These conditions imply:

$$\text{B16a)} \quad k_1 e^{\phi_1 T_1} + k_2 e^{\phi_2 T_1} = k_3 e^{\phi_1 T_1} + k_4 e^{\phi_2 T_1}$$

$$\text{B16b)} \quad s_1 k_1 e^{\phi_1 T_1} + s_2 k_2 e^{\phi_2 T_1} = s_1 k_3 e^{\phi_1 T_1} + s_2 k_4 e^{\phi_2 T_1} - g/\gamma$$

$$\text{B16c)} \quad k_3 e^{\phi_1 T_2} + k_4 e^{\phi_2 T_2} = 0$$

$$\text{B16d)} \quad s_1 k_3 e^{\phi_1 T_2} + s_2 k_4 e^{\phi_2 T_2} - g/\gamma = 0$$

Solving (B16a) through (B16d) for k_1 through k_4 , and substituting these constants into (B1), (B14), and (B15), we obtain solutions for the time path of output and asset prices:

$$\text{B17)} \quad \hat{q}(t) = \begin{cases} \left[\frac{g\Delta}{\Lambda} \right] \left[(1-e^{-\phi_1 D})e^{-\phi_1(T_1-t)} - (1-e^{-\phi_2 D})e^{-\phi_2(T_1-t)} \right] & t < T_1 \\ \left[\frac{g\Delta}{\Lambda} \right] \left[-e^{-\phi_1(T_2-t)} + e^{-\phi_2(T_2-t)} \right] & T_1 < t < T_2 \end{cases}$$

$$\text{B18)} \quad \theta(t) = \begin{cases} \left[\frac{g}{\gamma\Lambda} \right] \left[(\phi_1 - (\bar{r} + a\Delta))(1-e^{-\phi_1 D})e^{-\phi_1(T_1-t)} - (\phi_2 - (\bar{r} + a\Delta))(1-e^{-\phi_2 D})e^{-\phi_2(T_1-t)} \right] & t < T_1 \\ \left[\frac{g}{\gamma\Lambda} \right] \left[-(\phi_1 - (\bar{r} + a\Delta))e^{-\phi_1(T_2-t)} + (\phi_2 - (\bar{r} + a\Delta))e^{-\phi_2(T_2-t)} - \Lambda \right] & T_1 < t < T_2 \end{cases}$$

$$\text{B19)} \quad \hat{y}(t) = \begin{cases} \left[\frac{g}{\Lambda} \right] \left[(\phi_1 - \bar{r})(1-e^{-\phi_1 D})e^{-\phi_1(T_1-t)} - (\phi_2 - \bar{r})(1-e^{-\phi_2 D})e^{-\phi_2(T_1-t)} \right] & t < T_1 \\ \left[\frac{g}{\Lambda} \right] \left[-(\phi_1 - \bar{r})e^{-\phi_1(T_2-t)} + (\phi_2 - \bar{r})e^{-\phi_2(T_2-t)} \right] & T_1 < t < T_2 \end{cases}$$

where Λ is the radical in equations (B7a) and (B7b), D is the length of the fiscal expansion ($T_2 - T_1$), and the carat denotes a deviation from the original steady state.

Now we are in a position to prove (B12c). Using (B17) and (B18), we can write the slope of the equilibrium trajectory in (q, θ) space as follows:

$$\left. \frac{d\theta}{dq} \right|_{t=T_2} = \frac{\dot{\theta}}{\dot{q}} \Big|_{t=T_2} = \frac{[g/\gamma\Lambda][-\phi_1(\phi_1 - (\bar{r} + a\Delta)) + \phi_2(\phi_2 - (\bar{r} + a\Delta))]}{[g\Delta/\Lambda][-\phi_1 + \phi_2]}$$

Using the fact that $\phi_2 = \phi_1 + \Lambda$, this expression simplifies to:

$$\left. \frac{d\theta}{dq} \right|_{t=T_2} = \frac{c}{\Delta}$$

This is greater than the slope of the $\dot{\theta} = 0$ schedule if $(c/\Delta) > (-a/\gamma)$, which, with Δ negative, occurs if $(a\Delta + c\gamma) < 0$.

Discussion of the Analysis in the Text:

We can use these results to explain the graphical analysis and formally prove some key assertions made in the text of the paper. The discussion is simple for the "bad news" case, but in the "good news" case there are three subcases to be considered separately. We begin with the "bad news" case.

Bad News Case:

First, I discuss why Figure Three is drawn as it is. The key features of Figure Three are that the $\dot{q} = 0$ schedule is negatively sloped and steeper than the $\dot{\theta} = 0$ schedule, that the eigenvector corresponding to the

large eigenvalue (for which I will henceforth use the shorthand phrase "large eigenvector") is positive, and that the eigenvector corresponding to the small eigenvalue is negative, and lies between the $\dot{\theta}=0$ schedule and the $\dot{q}=0$ schedule.

The slope of the $\dot{\theta}=0$ schedule is $-a/\gamma$. The slope of the $\dot{q}=0$ schedule is $-a/\gamma - \bar{r}/\gamma\Delta$. With Δ positive, it follows immediately that the $\dot{q}=0$ schedule is negatively sloped and steeper than the $\dot{\theta}=0$ schedule. The large eigenvector has slope $(\phi_2 - (\bar{r} + a\Delta))$. I showed above that, in the good news case, this is positive (B9b). The small eigenvector has slope $(\phi_1 - (\bar{r} + a\Delta))$, which is negative (B9b). It can also be written $(-a/\gamma + (\phi_1 - \bar{r})/\gamma\Delta)$. We have shown (B9a) that $(\phi_1 - \bar{r})$ is negative, so the small eigenvector is steeper than the $\dot{\theta}=0$ schedule. Since ϕ_1 is positive, it is also obvious that the small eigenvector is less steep than the $\dot{q}=0$ schedule.

This justifies the way I have drawn Figure Three. Now I use (B17)-(B19) to prove some of the key conclusions that I drew from that graphical analysis.

In the bad news case: output is always below steady state between the announcement and implementation of the fiscal expansion, and is always above the steady state during the fiscal expansion.

It was demonstrated above that $(\phi_1 - \bar{r}) < 0 < (\phi_2 - \bar{r})$. It follows by inspection of equation (B19) that output is below steady state between time zero and T_1 , and above the steady state between time T_1 and time T_2 .

In the bad news case: the real exchange rate is appreciated relative to the original steady state both in anticipation of and after implementation of the fiscal expansion.

It was demonstrated above that $(\phi_1 - (\bar{r} + a\Delta)) < 0 < (\phi_2 - (\bar{r} + a\Delta))$.

The negativity of the real exchange rate for $t < T_1$ then follows from inspection of equation (B18). For $T_1 < t < T_2$, first define:

$$z_i = (\phi_i - (\bar{r} + a\Delta)) \quad i = 1, 2$$

$$T_i = (T_i - t) \quad i = 1, 2$$

In this case, $z_1 < 0$, $z_2 > 0$. Differentiate (B18) with respect to T_2 to obtain:

$$\frac{d\theta(t)}{dT_1} = \left[\frac{g}{\gamma\Delta} \right] [\phi_1 z_1 (e^{-\phi_1 T_2} - e^{-\phi_2 T_2}) - \lambda \gamma c] < 0$$

where I used the fact that $z_2 = z_1 + \lambda$, and $\phi_2 = \phi_1 + \lambda$. So, as $T_2 - t$ increases, the real exchange rate decreases (a fact that is also apparent from Figure Three in the text.) So, if the real exchange rate is below the steady state when T_2 approaches t , then it will be even further below if for T_2 larger than t . So, consider the limit of (B18) as $T_2 \rightarrow 0$. This is:

$$\lim_{T_2 \rightarrow 0} \frac{\theta(t)}{1 - e^{-\phi_1 T_2}} = \frac{g}{\gamma\Delta} [\phi_1 z_1 - \phi_2 z_2] / \phi_1 < 0$$

where I have used l'Hospital's rule and omitted some algebra. This means that, for very small $(T_2 - t)$ the real exchange rate is below the steady state which, combined with the derivative condition above, establishes that the real exchange rate is below the steady state for all t less than T_2 .

In the bad news case: (A) The stock market is necessarily below its steady state after implementation of the fiscal expansion. (B) The stock market is necessarily below the steady state before implementation of the fiscal policy if there is a short delay before the fiscal expansion will take

place. (C) The stock market is necessarily above the steady state before implementation of the fiscal expansion if there is a long delay before the expansion will happen.

(A) In the bad news case, $\Delta > 0$. The negativity of (B17) for $T_1 < t < T_2$ follows from this and the fact that $\phi_2 > \phi_1$.

(B) For given D , consider the case in which $(T_1 - t)$ goes to zero. Then, we can write (B17) as follows:

$$\hat{q}(t) = \left[\frac{g\Delta}{\Lambda} \right] [(1 - e^{-\phi_1 D}) - (1 - e^{-\phi_2 D})] = \left[\frac{g\Delta}{\Lambda} \right] [e^{-\phi_2 D} - e^{-\phi_1 D}]$$

With $\Delta > 0$ and $\phi_2 > \phi_1$, this is necessarily negative.

(C) For given D , consider the case in which $(T_1 - t)$ goes to infinity. Then, \hat{q} goes to zero, (which just implies that a fiscal expansion that's very far into the future has essentially no impact on today's equilibrium.) To determine the sign of $\hat{q}(t)$ as $(T_1 - t)$ approaches infinity, write (B17) as follows:

$$\hat{q}(t) = \left[\frac{g\Delta}{\Lambda} \right] e^{-\phi_1(T_1 - t)} [(1 - e^{-\phi_1 D}) - (1 - e^{-\phi_2 D - (\phi_2 - \phi_1)(T_1 - t)})]$$

As $(T_1 - t)$ goes to infinity, the second term in the square brackets goes to zero, and it is obvious that $\hat{q}(t)$ is positive.

Good News Case:

I turn now to the good news case. I confine myself to the case in which the positive roots ϕ_1 and ϕ_2 are not complex. This requires that \bar{r} is either "big enough" or "small enough," as specified in (B8a) and

(B8b). In terms of the phase diagrams, it requires that the eigenvectors be steep enough, whether positively or negatively sloped.

I begin by splitting the parameter space into four regions, depending upon the sign of $(a\Delta+c\gamma)$, and of $(\bar{r}+a\Delta-c\gamma)$. I denote the cases as follows:

	<u>$a\Delta+c\gamma < 0$</u>	<u>$a\Delta+c\gamma > 0$</u>
<u>$(\bar{r}+a\Delta) > c\gamma$</u>	Case 1	Case 3
<u>$(\bar{r}+a\Delta) < c\gamma$</u>	Case 4	Case 2

Case 4 will be ruled out by the assumption of real roots and saddlepath stability. Case 1 is the one in which the possibility of an investment-led anticipatory expansion exists. Case 2 is the one in which the possibility of a net export-led anticipatory boom exists. In Case 3, (not discussed in the text of the paper), neither possibility exists.

First, I demonstrate that Case 4 can be ruled out. With $\bar{r} < (c\gamma - a\Delta)$, the assumption of real roots implies (B8b) that

$$B20) \quad \bar{r} < (c\gamma - a\Delta) - 2\sqrt{-a\Delta c\gamma}$$

Suppose that $(a\Delta+c\gamma) < 0$. Then, adding $a\Delta+c\gamma$ to both sides of (B20), we obtain:

$$B21) \quad \bar{r} + a\Delta + c\gamma < 2(c\gamma - \sqrt{-a\Delta c\gamma})$$

By assumption, $-a\Delta > c\gamma$, which implies that the right side of (B21) is negative which, in turn, implies that the left side is negative. But this is

inconsistent with the assumption of saddlepoint stability, so we arrive at a contradiction. Thus, Case 4 is inconsistent with the assumptions of saddlepath stability and real roots, so I rule it out.

Now I examine cases 1-3 individually. The purpose of the discussion is to justify the graphical analysis used in the paper, and to prove some of the key assertions made in the paper.

Good News Case 1: [$a\Delta + c\gamma < 0$, $\bar{r} + a\Delta > c\gamma$] This is the Case 1 of the paper, and corresponds to Figure Four. The key features of Figure Four are (i) the eigenvectors are positive, and (ii) the trajectory labelled (A) approaches SS_0 after passing above the $\dot{\theta} = 0$ schedule. From the geometry, it's obvious that (ii) can happen only if the slope of the equilibrium trajectory at SS_0 is greater than the slope of the $\dot{\theta} = 0$ schedule. It was shown above that this can only happen if $a\Delta + c\gamma < 0$. Since that is assumed in this case, it was correct to draw the trajectory (A) as in Figure Four. As for the positive eigenvectors, that is guaranteed by the assumption that $\bar{r} + a\Delta > c\gamma$.

Now I prove some of the key assertions derived from Figure Four in the text of the paper.

In the good news case 1: (A) If the delay between announcement and implementation of the fiscal expansion is large enough, then output falls in anticipation of the fiscal stimulus, (B) If the expected duration of the fiscal stimulus is long enough, then output falls in anticipation of the fiscal stimulus, (C) If the expected duration of the expansion is short enough, and the delay between enactment and implementation is short enough, then output increases in anticipation of the stimulus, (D) After the fiscal expansion is implemented, output is above the steady state level.

(A) Take the limit as $T_1 = (T_1 - t)$ goes to infinity. Then, from (B19), output is:

$$B22) \quad \lim_{(T_1-t) \rightarrow \infty} \hat{y}(t) = (g/\Lambda) e^{-\phi_1 T_1} \left[(\phi_1 - \bar{r})(1 - e^{-\phi_1 D}) - (\phi_2 - \bar{r})(1 - e^{-\phi_2 D - (\phi_2 - \phi_1) T_1}) \right]$$

The second term in square brackets approaches zero as (T_1-t) becomes large, and the first term is negative by the assumption that $a\Delta + c\gamma < 0$ (B12a). Thus, for large enough (T_1-t) , output is below the steady state.

(B) Take the limit as (T_2-T_1) goes to infinity. Then, from (B19):

$$B23) \quad \lim_{D \rightarrow \infty} \hat{y}(t) = (g/\Lambda) \left[(\phi_1 - \bar{r}) e^{-\phi_1 (T_1-t)} - (\phi_2 - \bar{r}) e^{-\phi_2 (T_1-t)} \right]$$

$$= (g/\Lambda) \left[(\phi_1 - \bar{r}) (e^{-\phi_1 (T_1-t)} - e^{-\phi_2 (T_1-t)}) - \Lambda e^{-\phi_2 (T_2-t)} \right] < 0$$

(C) Now we examine the behavior of output as (T_2-T_1) and (T_1-t) become small. Taking the limit of (B19) as (T_1-t) goes to zero, we obtain:

$$B24) \quad \hat{y}(t) = (g/\Lambda) (1 - e^{-\phi_1 D}) \left[(\phi_1 - \bar{r}) - (\phi_2 - \bar{r}) (1 - e^{-\phi_2 D}) / (1 - e^{-\phi_1 D}) \right]$$

where $D = (T_2 - T_1)$. Using l'Hospital's rule, and taking the limit as D goes to zero, we see that the term in square brackets goes to:

$$B25) \quad \lim_{D \rightarrow 0} [\bullet] = \left[(\phi_1 - \bar{r}) - (\phi_2 - \bar{r}) (\phi_2 / \phi_1) \right]$$

Using the facts that $\phi_2 = \phi_1 + \Lambda$, and that $\Delta < 0$, it is easy to show that this is positive if $a\Delta + c\gamma < 0$, which we have assumed to be true in this case.

(D) Using (B19), we can show that for $T_1 < t < T_2$ output is:

$$B26) \quad \hat{y}(t) = (g/\Lambda) \left[-(\phi_1 - \bar{r}) (e^{-\phi_1 (T_2-t)} - e^{-\phi_2 (T_2-t)}) + \Lambda e^{-\phi_2 (T_2-t)} \right]$$

Noting that $\phi_1 - \bar{r} < 0$ and $\phi_1 < \phi_2$, (B26) is positive by inspection.

In the good news case 1: The real exchange rate is appreciated relative to the steady state both in anticipation of and after the implementation of the fiscal expansion.

First consider $t < T_1$. For notational simplicity, define:

$$z_i = (\phi_i - (\bar{r} + a\Delta)) \quad i = 1, 2$$

$$T_i = T_i - t, \quad i = 1, 2$$

In this case, $z_i < 0$. Differentiate (B18) with respect to D to obtain:

$$\begin{aligned} \text{B27) } \frac{d\theta}{dD} &= \phi_1 z_1 e^{-\phi_1 T_2} - \phi_2 z_2 e^{-\phi_2 T_2} \\ &= \phi_1 z_1 (e^{-\phi_1 T_2} - e^{-\phi_2 T_2}) - c\gamma\lambda < 0 \end{aligned}$$

where I have used the fact that $T_2 = T_1 + D$. Since $d\theta/dD < 0$, we can consider the case in which $D \rightarrow 0$: if in that case the exchange rate deviation is negative, it must be even more negative for D larger than zero. Using (B18), we can compute the exchange rate deviation for very small D . Using l'Hospital's rule, we have:

$$\text{B28) } \lim_{D \rightarrow 0} \theta(t) = \frac{g(1 - e^{-\phi_1 D})}{\gamma\lambda} [z_1 e^{-\phi_1 T_1} - z_2 (\phi_2 / \phi_1) e^{-\phi_2 T_1}]$$

The term in square brackets can be written:

$$\text{B29) } [\bullet] = \phi_1 z_1 (e^{-\phi_1 T_1} - e^{-\phi_2 T_1}) - \lambda c \gamma < 0$$

where the inequality holds because in this case $z_i < 0$. This proves that between time zero and T_1 , the real exchange rate is necessarily below the original steady state. Now consider $T_1 < t < T_2$. Differentiating (B18) with respect to T_2 , and rearranging, we have:

$$B30) \quad \frac{d\theta}{dT_2} = \frac{g}{\gamma\Lambda} [\phi_1 z_1 (e^{-\phi_1 T_2} - e^{-\phi_2 T_2}) - \Delta c\gamma] < 0$$

So, as T_2 decreases, the real exchange rate increases. (This is apparent from Figure Four.) So we consider the case in which $T_2 \rightarrow t$. If the real exchange rate is below the steady state in this case, it must be below the steady state in all cases. We have from equation (B18):

$$B31) \quad \lim_{T_2 \rightarrow 0} \frac{\theta(t)}{1 - e^{-\phi_1 T_2}} = \frac{g}{\gamma\Lambda} [\phi_1 z_1 - \phi_2 z_2] / \phi_1 < 0$$

where I have used l'Hospital's rule and omitted some of the algebra. This establishes that for all t between time T_1 and T_2 , the real exchange rate is necessarily below the original steady state.

In the good news case 1: (A) Before the fiscal expansion arrives, the stock price is above the steady state if the length of time until the fiscal expansion is implemented is not too long, and it is below the steady state if the delay is long, (B) After the arrival of the fiscal stimulus, the stock price is necessarily above the steady state.

(A) Consider first the case in which $T_1 - t$ is large. (B17) can be written as follows:

$$B32) \quad \hat{q}(t) = [g\Delta/\Lambda] e^{-\phi_1 T_1} [(1 - e^{-\phi_1 D}) - (1 - e^{-\phi_2 D - (\phi_2 - \phi_1) T_1})]$$

As $(T_1 - t) = T_1$ gets large, the second term in square brackets goes to zero. With $\Delta < 0$, $\hat{q}(t)$ becomes negative. Now consider the case in which $(T_1 - t)$ is very small. Taking the limit of (B17) as $T_2 \rightarrow t$, we have:

$$B33) \quad \lim_{T_1 \rightarrow t} = \frac{g\Delta}{\Lambda} (e^{-\phi_2 D} - e^{-\phi_1 D}) > 0$$

Thus, if the delay until the fiscal expansion takes place is short enough, then the stock price is necessarily above the steady state.

(B) When $T_1 < t < T_2$, $\hat{q}(t)$ is positive by inspection of equation (B17).

Good News Case 2: [$a\Delta + cY > 0$, $\bar{r} + a\Delta < cY$] This is Case 2 of the text of the paper, and corresponds to Figure Five. The important assumption in Figure Five is that the eigenvectors are negatively sloped, and steeper than the $\dot{\theta} = 0$ curve. I first prove that $a\Delta + cY > \bar{r}$ in this case:

1. $a\Delta + cY > 0 \rightarrow cY > -a\Delta$
2. $\bar{r} < cY - a\Delta \rightarrow \bar{r} < cY - a\Delta - 2\sqrt{-a\Delta cY}$ by the assumption of real roots. Adding $2a\Delta$ to both sides and rearranging, this implies:
3. $a\Delta + cY > \bar{r} + 2(cY + a\Delta) > \bar{r}$

It was shown above that this implies that $(\phi_1 - \bar{r}) > 0$ which in turn implies that $\phi_1 - (\bar{r} + a\Delta) > 0$. The fact that $(\phi_1 - \bar{r}) > 0$ implies that the eigenvectors are not only negatively sloped, but also steeper than the $\dot{\theta} = 0$ schedule.

Now I prove some of the assertions made in the text about this case.

In the good news case 2: If there is a long enough period of time before the anticipated fiscal expansion will be implemented, then output increases. If the delay before implementation is short, then output must be below steady state.

First take the case in which $(T_1 - t)$ is very large. Then, (B19) can be written:

$$B34) \quad \hat{y}(t) = (g/\Lambda) e^{-\phi_1 T_1} [(\phi_1 - \bar{r})(1 - e^{-\phi_1 D}) - (\phi_2 - \bar{r})(1 - e^{-\phi_2 D}) e^{-(\phi_2 - \phi_1) T_1}]$$

As $(T_1 - t) = T_1$ goes to infinity, this becomes positive because $(\phi_1 - \bar{r}) > 0$.

If instead we take the limit of (B34) as $T_1 \rightarrow 0$, we obtain:

$$B35) \quad \hat{y}(t) = (g/\Lambda) [(\phi_1 - \bar{r})(1 - e^{-\phi_1 D}) - (\phi_2 - \bar{r})(1 - e^{-\phi_2 D})]$$

$$= (g/\Lambda) [(\phi_1 - \bar{r})(e^{-\phi_2 D} - e^{-\phi_1 D}) - \Lambda(1 - e^{-\phi_2 D})] < 0$$

So, if the delay between announcement and implementation is short, then output is below the steady state.

In the good news case 2: If the length of time before the fiscal expansion is to be implemented is very long, then the exchange rate depreciates in anticipation of the fiscal expansion. If it is short, then the real exchange rate must appreciate.

(B18) can be written as follows:

$$B36) \quad \theta(t) = (g/\gamma\Lambda) e^{-\phi_1 T_1} [(\phi_1 - (\bar{r} + a\Delta))(1 - e^{-\phi_1 D}) - (\phi_2 - (\bar{r} + a\Delta))(1 - e^{-\phi_2 D}) e^{-(\phi_2 - \phi_1)T_1}]$$

As T_1 gets very large, this becomes positive. As T_1 goes to zero, this is:

$$B37) \quad \theta(t) = (g/\gamma\Lambda) [(\phi_1 - (\bar{r} + a\Delta))(1 - e^{-\phi_1 D}) - (\phi_2 - (\bar{r} + a\Delta))(1 - e^{-\phi_2 D})]$$

$$= (g/\gamma\Lambda) [(\phi_1 - (\bar{r} + a\Delta))(e^{-\phi_2 D} - e^{-\phi_1 D}) - \Lambda(1 - e^{-\phi_2 D})] < 0$$

I forego a discussion of the stock price dynamics because they are qualitatively the same as in the good news case 1. (This can be seen by inspection of equation (B17), or by comparing Figures Four and Five in the text of the paper.

Good News Case 3: [$a\Delta + c\gamma > 0$, $(\bar{r} + a\Delta) > c\gamma$] This third and final case is the same as case 1, except that $a\Delta + c\gamma > 0$. For comparison, see Figure B-One. The eigenvectors look much like in case 1. The difference is that, while in case 1 the equilibrium trajectory approaches SS_0 after crossing above the $\dot{\theta} = 0$ schedule, in case 3 the trajectory passes through SS_0 from below the $\dot{\theta} = 0$ schedule. This implies that the possibility of an anticipatory boom that is present in case 1 is not present in case 3. In fact, output

must fall in anticipation of the fiscal expansion. The exchange rate must appreciate, and the stock price dynamics are qualitatively the same as in cases 1 and 2.

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