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INTERNATIONAL DUOPOLY WITH TARIFFS

by

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and

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Abstract

This paper analyzes the effects of a tariff on price-setting duopolists who cannot segment geographically distinct markets; hence, commercial policy has effects in domestic and foreign markets. Although each firm's payoff function is discontinuous, there is a unique equilibrium for an arbitrary tariff. We find that a tariff serves to increase the profits of both the domestic and foreign producer. Moreover, the profits of both firms rise monotonically with the tariff.

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1. Introduction

In the March 24, 1986, issue of Auto News, the automobile industry trade journal, the lead article stated that several Japanese automobile manufacturers were beginning to direct their sales effort to capturing larger shares of their own domestic market. The article explained that this effort was in response to the rising value of the yen and the threat of increased trade barriers in Japan's export markets.

In 1986, there were also reports of automobile brokers buying American cars in Canada and re-exporting them to the United States' domestic market, thereby avoiding the Canadian excise tax and undercutting the wholesale price of such cars in the United States. This occurred presumably

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because the markup for domestic automobiles was larger than the two-way transportation cost.

These two phenomena underscore two increasingly salient aspects of the current environment for international trade. First, because many international markets are oligopolistic, trade policies instituted in one market may have an influence in all national markets. Second, there is a limit to the degree of geographic market segmentation that a producer can create when the goods he sells in different countries are near perfect substitutes.

The model we present is meant to provide a simple framework in which to analyze the effects of commercial policy in an oligopoly. We examine an international duopoly selling a homogeneous good and choosing price as the strategic variable. We assume that there is a home firm and a foreign firm, each of which can produce the good at zero marginal cost. There is a home market and a foreign market; hence, the world market is the aggregation of the two geographically distinct markets. Trade is unrestricted in the foreign country, but the home country imposes a specific tariff on imports of the good produced by the foreign firm.

Almost all of the research conducted until now has focussed upon the effects of commercial policy only in the market where the policy is imposed. A notable exception, which is not an strictly an analysis of international trade, is that of Bulow, Geanakoplos, and Klemperer [2]; they examine firms' choices as strategic complements or substitutes in a more general framework. In this paper, in a model that is chosen for its applicability to trade, we show that there are international repercussions to the imposition of a tariff in a domestic market which is a part of a larger world market.

Without a tariff, the model reduces to the standard Bertrand model without capacity constraints. In this case, the only Nash equilibrium is for both firms to set price equal to marginal cost. Even with a tariff, a similar result obtains if market segmentation is permitted. This paper examines the case where market segmentation is not allowed. This is equivalent to the assumption that there is no dumping by the home firm; in particular, if the home firm charges a price above marginal cost for sales in its own market, it must charge the same export price. This implies that the home firm can benefit from the tariff only from charging a positive price in both markets. This in turn provides the foreign firm with the possibility of making profits in its own market.

We are interested, then, in characterizing the pricing strategies for each firm which are best responses against one another. As is frequently the case in such models, equilibria in pure strategies do not exist. Under assumptions which are not too restrictive, we are able to characterize fully a unique mixed strategy equilibrium for each tariff. Our model serves as an example of an equilibrium in a game with discontinuous payoffs; see Dasgupta and Maskin [3] for a full treatment of this issue.

The strategies we are going to describe are complex, but the intuition behind them is not. We will be describing actions which by their very nature are probabilistic, and it is useful to interpret the firms' strategies as marketing choices. The protected firm's strategy can be thought of as one which places some weight on being content to reap the rents accruing to it in its own domestic market and some weight on undercutting the foreign firm in order to capture a larger world market. The foreign firm's marketing strategy is one which takes full advantage of the home firm's tendency to shade price above marginal cost. It is perhaps initially surprising that higher tariffs lead to higher profits for the foreign firm, but the intuition is that the profitability of a protected home market for the domestic firm may redound in part to the benefit of the foreign firm.

The analysis is unabashedly within the framework of partial equilibrium; this may irk the trade theorist, who will find solace in Dixit's descriptions of the shortcomings of this approach for the study of commercial policy [4]. Fisher [5] solved this problem for the particular case of a prohibitive tariff, and Krishna [6] showed that trade restrictions placed on one firm can benefit a competitor in its own market.

The rest of the paper is structured as follows. In the second section we present the model one initial lemma. In the third section, we derive a series of lemmata which characterize the equilibrium for an arbitrary tariff. In the fourth section we prove that the equilibrium exists and is unique. In the fifth section we present an example of an equilibrium for a simple linear demand function, and in the sixth section, we make our conclusions.

2. The Model

There are two firms, the home firm and the foreign firm. They both produce a homogeneous good at zero marginal cost. The demand for the good in the foreign country is D(p). We assume that D(p) has a choke-off price, is bounded, is non-increasing, and has a downward-sloping marginal revenue function. In the home country, demand is kD(p), where $0 < k \leq 1$. The foreign country imposes no tariff, but the home country imposes a specific tariff of t. Each firm must choose a price at which it will sell its good. We require that each firm must charge a single f.o.b. price regardless of the market in which it sells. Thus if the home firm charges price p, its product sells at price p in both countries. If the foreign firm charges price p, then its product sells at price p in its domestic market but sells at price p+t in the home market. The firm with the lowest after-tariff price in any country captures that market. If both firms charge the same price in some market, then they split the market equally. It will become clear later that our results are not sensitive the specifics of the sharing rule.

In order to obtain a Nash equilibrium when firms compete in prices, it will be necessary to permit firms to randomize their choice of p. Let G be the <u>decumulative</u> distribution function of the price charged by the home firm. That is, for each price p, G(p) is the probability that the home firm charges a price <u>greater than or equal to</u> p. Let q(p) denote the probability mass of G at price p. Let S denote the support of the distribution G, and define $p_- = \sup\{p: G(p) = 1\}$ to be the bottom and $\bar{p} = \inf\{p: G(p) = 0\}$ to be top of the support of G. Let G^*, q^*, S^*, p_-^*, and $\bar{p}^*$ denote the corresponding function and values for the foreign firm. We will suppose that firms may only charge non-negative prices so that $S, S^* \subset \Re_+$.

Turning to the payoff functions, let $\pi(p) = pD(p)$ be the profit to the home firm from selling in the foreign market at price p and $k\pi(p)$ its profit from selling in its own market at price p. The profit to the foreign firm from selling in its own domestic market at price p is also $\pi(p)$, but, because of the tariff, its profit from selling abroad at price p is $k\pi^*(p) \equiv kpD(p+t) = k[p/(p+t)]\pi(p+t)$. Then

$$\Pi(p,G^*) = [kG^*(p-t)+G^*(p)]\pi(p) - [kq^*(p-t)+q^*(p)]\pi(p)/2$$

is the expected profit to the home firm from selling at price p given that the foreign firm follows strategy G^*, and

$$\Pi^*(p,G) = kG(p+t)\pi^*(p) + G(p)\pi(p) - [kq(p+t)\pi^*(p)+q(p)\pi(p)]/2$$

is the expected profit to the foreign firm from selling at price p given that the home firm follows strategy G.

The payoff functions are illustrated in Figure 1, appearing on page 43, for the case where each firm concentrates all of its mass on one price. Let p be the price charged by the home firm and p^* the price charged by the foreign firm. If $p^* > p$, then the home firm captures both markets and the foreign firm earns zero profits. If $p^* < p < p^*+t$, then each firm captures its domestic market. If $p^* < p-t$, then the foreign firm captures both markets. If the price of the foreign firm is exactly equal to the price of the home firm, then they split the foreign market while if the price of the foreign firm is equal to the price of the home firm minus the tariff, they split the home market. The profit function of each firm is consequently discontinuous on both the diagonal 0A and the parallel line tB.

Given that the foreign firm follows strategy G^*, let $v = \sup_p \Pi(p,G^*)$ be the highest possible expected profit that the home firm can attain. Similarly, given the strategy G by the home firm, let $v^* = \sup_p \Pi^*(p,G)$ be the highest possible expected profit that the foreign firm can attain. Then a pair of strategies (G,G^*) is a <u>Nash Equilibrium</u> if

$\int \Pi(p,G^*)dG(p) = v$ and $\int \Pi^*(p,G)dG^*(p) = v^*$. That is, both firms are following strategies which are optimal, given the strategy of the other firm.

Let $p_m = \text{argmax}\ \pi(p)$. The assumptions we have made about D(p) imply that

A1: π is an increasing, continuous, differentiable, concave function on $[0,p_m]$ and $\pi(p) < \pi(p_m)$ for all $p > p_m$.

Note that $\pi(0) = 0$ and that p_m is the price an unencumbered monopolist would charge in either market. We have imposed this structure in order to guarantee that the profit function satisfies the following property.

<u>Lemma 1</u>: (a) $\pi^*(p)$ is increasing in p on $[0,p_m-t]$; and

(b) $\pi(p+t)-\pi(p)$ is non-increasing in p on $[0,p_m-t]$.

<u>Proof</u>:

Since $\pi(p+t)$ is increasing and $D(p+t)$ is non-increasing for $p < p_m-t$, it follows immediately that $\pi^*(p) = [p/(p+t)]\pi(p+t)$ is increasing for $p < p_m-t$. This proves (a). Part (b) follows immediately from the definition of concavity. Q.E.D.

3. <u>Derivation of Nash Equilibrium</u>

In this section, we will establish a series of results which imply the existence and uniqueness of a Nash equilibrium pair of strategies. For the remainder of the paper, (G,G^*) will refer to a Nash equilibrium pair of strategies.

Our first result establishes that the expected profits to both firms must be positive in equilibrium.

Lemma 2: $v > 0$ and $v^* > 0$.

Proof:

Since, by assumption, $G^*(p) = 1$ for $p < 0$, it follows that $\Pi(p,G^*) \geq k\pi(p)$ for $p \leq t$. Therefore, $v \geq \sup\{k\pi(p) : p \leq t\} > 0$.

Define $\hat{p} < \bar{p}$ by $(1+k)\pi(\hat{p}) = k\pi(t)$. Then since $\Pi(p,G^*) \geq (1+k)\pi(\hat{p})$, Assumption A1 implies that $p\_ \geq \hat{p} > 0$. Therefore, $\Pi^*(p,G) \geq \pi(p)$ for $p < \hat{p}$ which implies that $v^* \geq \pi(\hat{p}) > 0$. Q.E.D.

Since the presence of the tariff guarantees a positive expected profit to the home firm, the support of distribution of prices chosen by the home firm must be bounded away from zero. But this in turn guarantees a positive profit to the foreign firm.

Our next step is to establish that the equilibrium distribution of prices must be continuous, except possibly at the monopoly price for the home firm. To establish this result, we require the following implications for the equilibrium price distributions.

Lemma 3: (a) $G^*(\bar{p}-t) > 0$;

(b) $\bar{p}^* \leq \bar{p} \leq p_m$;

(c) $p\_^* \leq p\_ \leq p\_^*+t$.

Proof:

Suppose $G^*(\bar{p}-t) = 0$. Then $\Pi(\bar{p},G^*) = 0 < v$, a contradiction of Lemma 2. This establishes (a).

To establish that $\bar{p}^* \le \bar{p}$, suppose the contrary. Then there is a $p > \bar{p}$ such that $\Pi^*(p,G) = v^*$. However, $p > \bar{p}$ implies that $\Pi^*(p,G) = 0$, again a contradiction.

To show that $\bar{p} \le p_m$, again suppose the contrary. Let $p \in (p_m, \bar{p})$. Then since G^* is non-increasing, (a) implies

$$\begin{aligned}\Pi(p,G^*) - \Pi(p_m,G^*) &= \Big[[kG^*(p-t)+G^*(p)]\pi(p) - [kq^*(p-t)+q^*(p)]\pi(p)/2\Big] \\ &\quad - \Big[[kG^*(p_m-t)+G^*(p_m)]\pi(p_m) - [kq^*(p_m-t)+q^*(p_m)]\pi(p_m)/2\Big] \\ &\le \Big[[kG^*(p_m-t)+G^*(p_m)-[kq^*(p_m-t)+q^*(p_m)]/2]\Big][\pi(p)-\pi(p_m)] \\ &< 0.\end{aligned}$$

This contradiction establishes (b).

To show that $\underline{p}^* \le \underline{p}$, suppose the contrary. Let $p \in (\underline{p}, \underline{p}^*)$. Since $\underline{p}^* \le \bar{p}^* \le p_m$, we have

$$\Pi(p,G^*) - \Pi(\underline{p},G^*) = (1+k)[\pi(p)-\pi(\underline{p})] > 0$$

which contradicts the definition of $\underline{p}$.

To show that $\underline{p} \le \underline{p}^*+t$, again suppose the contrary. Then $\underline{p}^* < \underline{p}-t$. Let $p \in (\underline{p}^*, \underline{p}-t)$. Since $\underline{p} \le \bar{p} \le p_m$, we have

$$\Pi^*(p,G) - \Pi^*(\underline{p}^*,G) = [\pi(p)+k\pi^*(p)] - [\pi(\underline{p}^*)+k\pi^*(\underline{p}^*)] > 0$$

which contradicts the definition of p_-^*. This establishes (c). Q.E.D.

Part (a) of Lemma 3 states that there is a positive probability that the foreign firm will charge some price so high that it will be unable to sell in the protected market. Part (b) states that neither firm will charge a price above the monopoly price, and the foreign firm will never charge a price above all the prices the protected firm might charge. Part (c) states that the foreign firm may charge some price at least as low as the lowest price the protected firm might charges; it says also that the protected firm will charge some price at least as low as the lowest after-tariff price it faces.

Lemma 4: (a) If $q(p) > 0$, then there is an $\delta > 0$ such that $G^*(p)-G^*(p+\delta) = 0$ and $G^*(p-t)-G^*(p-t+\delta) = 0$.
(b) If $q^*(p) > 0$, then there is a $\delta > 0$ such that $G(p)-G(p+\delta) = 0$ and $G^*(p+t)-G^*(p+t+\delta) = 0$.

Proof:

Suppose that $q(p) > 0$. Lemma 2 implies that $\pi(p) > 0$ and hence that $p > 0$. For any $\bar{\epsilon} \in (0,p]$, we can choose an $\epsilon \in (0,\bar{\epsilon}]$ such that $p-\epsilon \in S$ and $q(p-\epsilon) = q(p-\epsilon+t) = 0$. Then for any $\delta > 0$

$$\begin{aligned}\Pi^*(p+\delta,G) - v^* &\le \Pi^*(p+\delta,G) - \Pi^*(p-\epsilon,G)\\ &= \Big[[G(p+\delta)-q(p+\delta)/2]\pi(p+\delta) + k[G(p+\delta+t)-q(p+\delta+t)/2]\pi^*(p+\delta)\Big]\\ &\quad - \Big[G(p-\epsilon)\pi(p-\epsilon) + kG(p-\epsilon+t)\pi^*(p-\epsilon)\Big]\end{aligned}$$

$$= [G(p+\delta)-q(p+\delta)/2]\pi(p+\delta) - G(p-\epsilon)\pi(p-\epsilon)$$

$$+ k[G(p+\delta+t)-q(p+\delta+t)/2]\pi^*(p+\delta) - kG(p-\epsilon+t)\pi^*(p-\epsilon)$$

(then, because G(p) is a decumulative distribution, we know)

$$\leq [G(p)-q(p)/2]\pi(p+\delta) - G(p-\epsilon)\pi(p-\epsilon)$$

$$+ k[G(p+t)-q(p+t)/2]\pi^*(p+\delta) - kG(p-\epsilon+t)\pi^*(p-\epsilon)$$

(again, using the fact that G(p) is decumulative, we have)

$$\leq [G(p)-q(p)/2]\pi(p+\delta) - G(p)\pi(p-\epsilon)$$

$$+ k[G(p+t)-q(p+t)/2]\pi^*(p+\delta) - kG(p+t)\pi^*(p-\epsilon)$$

$$= [G(p)-q(p)/2][\pi(p+\delta)-\pi(p-\epsilon)] - \pi(p-\epsilon)q(p)/2$$

$$+ k[G(p+t)-q(p+t)/2][\pi^*(p+\delta)-\pi^*(p-\epsilon)] - k\pi^*(p-\epsilon)q(p+t)/2$$

(and, finally, because $G(p)-q(p)/2 \leq 1$, we have)

$$\leq [\pi(p+\delta)-\pi(p-\epsilon)] - \pi(p-\epsilon)q(p)/2$$

$$+ k[\pi^*(p+\delta)-\pi^*(p-\epsilon)] - k\pi^*(p-\epsilon)q(p+t)/2$$

Then since π and π^* are continuous functions, omitting the term $k\pi^*(p-\epsilon)q(p+t)/2$ and letting $\epsilon \to 0$, we obtain,

$$\Pi^*(p+\delta,G) - v^* \leq [\pi(p+\delta)-\pi(p)] - q(p)\pi(p)/2 + k[\pi^*(p+\delta)-\pi^*(p)]$$

for sufficiently small ϵ. It follows, again from the continuity of π and π^*, that $\Pi^*(p+\delta,G) < v^*$ for $\delta > 0$ sufficiently small. This establishes that there is an $\delta > 0$ such that $G^*(p)-G^*(p+\delta) = 0$.

The remainder of the Lemma is established by similar arguments.

Q.E.D.

Suppose the home firm chooses to set some price p with positive probability. Then the foreign firm can significantly increase the probability of capturing its own market by slightly lowering its price. Similarly, by lowering its price slightly below p-t, it can significantly increase its chance of capturing the protected home market. Consequently, the foreign firm will never charge any price equal to or slightly above p or p-t. Similar arguments obtain for the home firm.

Lemma 5: (a) G^* is a continuous function.

(b) $q(p) > 0$ implies $p = p_m$.

Proof:

To establish (a) suppose that $q^*(p) > 0$ for some p. Then Lemma 4 implies that there is an $\delta > 0$ such that $G(p)-G(p+\delta) = 0$ and $G(p+t)-G(p+t+\delta) = 0$. First, suppose $p \geq p_m$. Then since $\bar{p} \leq p_m$, it follows that $G(p) = 0$ and hence that $\Pi^*(p,G) = 0 < v^*$. Second, suppose $p < p_m$ and $G(p) > 0$. Then Lemmata 2 and 4 imply that there is a $\delta \in (0,\bar{p}-p)$ such that

$$
\begin{aligned}
v^* - \Pi^*(p,G) &\geq \Pi^*(p+\delta,G) - \Pi^*(p,G) \\
&= G(p)[\pi(p+\delta)-\pi(p)] + kG(p+t+\delta)[\pi^*(p+\delta)-\pi^*(p)] \\
&> 0
\end{aligned}
$$

In either case, therefore, $q^*(p) = 0$. These two contradictions imply that there is no mass point in the density of the foreign firm's equilibrium strategy, and thus they establish (a).

To establish (b), we may suppose that $q(p) > 0$ for some $p < p_m$. By definition, $p \leq \bar{p}$. Then Lemma 4 again implies that there is an $\delta \in (0,\bar{p}-p]$ such that $G^*(p)-G^*(p+\delta) = 0$ and $G^*(p-t)-G^*(p-t+\delta) = 0$.

Furthermore, Lemma 3 implies that $G^*(p-t) > 0$. Therefore, using Assumption A1, we have

$$v - \Pi(p,G^*) \geq \Pi(p+\delta,G^*) - \Pi(p,G^*) = [G^*(p)+kG^*(p-t)][\pi(p+\delta)-\pi(p)] > 0,$$

which implies that $q(p) = 0$. This contradiction proves the Lemma. Q.E.D.

The argument behind Lemma 5 is as follows. From Lemma 2, we know that it is never optimal for either firm to set a price above the monopoly price, p_m. On the other hand, if either firm sets a price <u>less</u> than the monopoly price with positive probability, then Lemma 4 implies that the other firm will never set a price at or just above this price. But then it would be optimal for the first firm to raise its price, since the probability of capturing either market remains unchanged. The only price which a firm might set with positive probability, therefore, is the monopoly price. But if the foreign firm charges price p_m with positive probability, then the home firm will charge a lower price with probability 1 and hence the profits to the foreign firm at p_m will be zero. It is possible, however, that the home firm can charge price p_m and earn positive profits since, in order to capture the home market, the foreign firm must charge a price less than or equal to p_m-t. We conclude, therefore, that the distribution function of the foreign firm will be continuous and the price distribution of the home firm can have a mass point only at p_m.

One important implication of Lemma 5 is that the expected profit function for the home firm must be continuous in prices and the expected profit of the foreign firm must be continuous except possibly at $\bar{p}$ and

$\bar{p}$-t.

With the preceding lemmata in hand, we are able to characterize in more detail the support of the equilibrium price distributions.

Lemma 6: $\bar{p} \leq (k+1)t$.

Proof:

There are two cases to consider. First, if $\bar{p} < t$, the lemma follows immediately from the assumption that $k > 0$. Second, suppose that $\bar{p} \geq t$. Then Lemma 3 implies that

$$\begin{aligned} 0 &\leq v - \Pi(\bar{p}-t,G^*) = \Pi(\bar{p},G^*) - \Pi(\bar{p}-t,G^*) \\ &= kG^*(\bar{p}-t)\pi(\bar{p}) - [G^*(\bar{p}-t)+kG^*(\bar{p}-2t)]\pi(\bar{p}-t) \\ &\leq G^*(\bar{p}-t)[k\pi(\bar{p})-(k+1)\pi(\bar{p}-t)]. \end{aligned}$$

Further, it follows again from Lemma 3 that

$$0 \leq k\pi(\bar{p})-(k+1)\pi(\bar{p}-t) = k[\pi(\bar{p})-((k+1)/k)\pi((\bar{p}-t)].$$

The concavity of π and the fact that $(k+1)/k > 1$ imply that

$$0 \leq [\pi(\bar{p})-((k+1)/k)\pi(\bar{p}-t)] \leq \pi(\bar{p}) -\pi((\bar{p}-t)(k+1)/k).$$

Therefore, $k\bar{p} \geq (\bar{p}-t)(k+1)$ which implies the Lemma. Q.E.D.

Lemma 6 establishes an upper bound on the support of the price distribution of the home firm in terms of the relative size of its domestic market and the tariff. Given Lemma 3, we know that if the home firm charges price $\bar{p}$, it will never capture the foreign market, but may capture its own market with some probability. Now suppose the home firm were to cut its price by the tariff t. Then it would capture the foreign market with exactly the same probability that it captured its own market. In addition, the probability with which it captures its own market could only increase. Furthermore, since demand increases with a lower price, the firm's expected level of sales would increase by at least a factor of (k+1)/k. It could only be profitable to charge the higher price $\bar{p}$, then, if it exceeded $\bar{p}$-t by at least a factor of (k+1)/k.

In the next lemma we exploit the concavity of the profit function to generate a crucial restriction on the price distribution of the foreign firm. Our characterization of the equilibrium depends critically on this result. The importance of the restriction that k be less than or equal to 1 is that it implies

<u>Lemma 7</u>: Suppose there exist p' and p'' such that $p' < p'' \le \min\{p_m - t, p\_^* + t\}$. If $\Pi(p', G^*) \ge \Pi(p'', G^*)$ and $\Pi(p'+t, G^*) \le \Pi(p''+t, G^*)$ then $G^*(p''+t) > 0$.

<u>Proof</u>:

Lemma 3 and the hypothesis of this Lemma imply that $G^*(p''-t) = 1$ and $G^*(p'-t) = 1$. Further, we have assumed that $\Pi(p', G^*) \ge \Pi(p'', G^*)$. Hence,

$$
\begin{aligned}
0 &\ge \Pi(p'', G^*) - \Pi(p', G^*) \\
&= [k+G^*(p'')]\pi(p'') - [k+G^*(p')]\pi(p') \\
&= [k+G^*(p'')][\pi(p'')-\pi(p''+t)] - [k+G^*(p')][\pi(p')-\pi(p'+t)]
\end{aligned}
$$

$$+ \ [k+G^*(p")]\pi(p"+t) \ - \ [k+G^*(p')]\pi(p'+t)$$

(then, since $G^*(p") \le G^*(p')$, and since $p' < p_m-t$ implies $\pi(p')-\pi(p'+t) < 0$)

$$\ge [k+G^*(p")]\Big[[\pi(p")-\pi(p"+t)] \ - \ [\pi(p')-\pi(p'+t)]\Big]$$

$$+ \ [k+G^*(p")]\pi(p"+t) \ - \ [k+G^*(p')]\pi(p'+t)$$

(and, since the concavity of π implies that $\pi(x)-\pi(x+t)$ is decreasing for $x \le p_m-t$)

$$\ge [k+G^*(p")]\pi(p"+t) \ - \ [k+G^*(p')]\pi(p'+t)$$

$$= [kG^*(p"+t)+G^*(p")]\pi(p"+t) \ - \ [kG^*(p'+t)+G^*(p')]\pi(p'+t)$$

$$+ \ k[\pi(p"+t)-\pi(p+t)] \ - \ k[G^*(p"+t)\pi(p"+t) \ - \ G^*(p'+t)\pi(p'+t)]$$

(and, again, since $p' < p" \le p_m-t$ implies $\pi(p'+t)-\pi(p"+t) < 0$)

$$> [kG^*(p"+t)+G^*(p")]\pi(p"+t) \ - \ [kG^*(p'+t)+G^*(p')]\pi(p'+t)$$

$$-k[G^*(p"+t)\pi(p"+t) \ - \ G^*(p'+t)\pi(p'+t)]$$

$$= [\Pi(p"+t,G^*)-\Pi(p'+t,G^*)] \ - \ kG^*(p"+t)\pi(p"+t) \ + \ kG^*(p'+t)\pi(p'+t)].$$

Therefore, if $\Pi(p"+t,G^*) - \Pi(p'+t,G^*) \ge 0$, then $G^*(p"+t) > 0$. Q.E.D.

Suppose that the home firm captures its own protected market with certainty when it charges a relatively low price. Suppose further that its expected profits do not decrease when it lowers its price even more, while its expected profits do not increase when it is considering the same move at a higher price. Lemma 6 says that there must be some chance that it will capture the foreign market even at that high price. The argument depends, of course, on the concavity of the profit function.

Lemmata 5 and 7 and the assumption that $k \le 1$ imply that the length of the support of the distribution of prices charged by the home firm must be less than or equal to the size of the tariff.

<u>Lemma 8</u>: $p_- \geq \bar{p}-t$.

<u>Proof</u>:

Suppose not. Then since $k \leq 1$, Lemma 6 implies that $\bar{p} \leq 2t$, and we have $p_- < \bar{p}-t \leq t \leq p_-^* + t$. Since G^* is continuous, it then follows that $\Pi(p_-,G^*) = \Pi(\bar{p},G^*) = v$. Therefore,

$$\Pi(\bar{p}-t,G^*) - \Pi(p_-,G^*) \leq 0 \quad \text{and} \quad \Pi(\bar{p},G^*) - \Pi(p_-+t,G^*) \geq 0.$$

But then Lemma 7 implies that $G^*(\bar{p}) > 0$, which violates Lemma 3. Q.E.D.

We have also

<u>Lemma 9</u>: $S = [p_-,\bar{p}]$.

<u>Proof</u>:

If the Lemma is false, then there is a pair p',p'' such that $p_- < p' < p'' < \bar{p}$ and $0 < G(p') = G(p'')$ and $G(p'-\epsilon) > G(p')$ for all $\epsilon > 0$. Then for any $p \in [p',p'')$, Lemma 8 implies that $G(p'+t) = 0$ and hence that

$$\Pi^*(p'',G) - \Pi^*(p,G) = G(p')[\pi(p'')-\pi(p)] > 0.$$

Similarly, Lemma 8 implies that $G(p'-t) = 1$ and hence that

$$\Pi^*(p''-t,G) - \Pi^*(p-t,G) = kG(p')[\pi^*(p''-t)-\pi^*(p-t)] + [\pi(p''-t)-\pi(p-t)] > 0.$$

Therefore, the foreign firm's best response must satisfy $G^*(p'') = G^*(p')$ and $G^*(p''-t) = G^*(p'-t)$. This implies in turn that

$$\Pi(p'',G^*) - \Pi(p',G^*) = [kG^*(p'-t)+G^*(p')][\pi(p'')-\pi(p')] > 0.$$

But since $\Pi(\cdot,G^*)$ is a continuous function, we may conclude that if G is a best response, there must be $\delta > 0$ such that $G(p'-\delta) = G(p'')$. This contradiction establishes the Lemma. Q.E.D.

Lemmata 8 and 9 imply that the support of the price distribution of the home firm is a connected interval of width no greater than the tariff. If there were a gap in the home firm's support, then the foreign firm would also have a gap in its support. Since the foreign firm's density is continuous and the home firm's revenue function is increasing, we may conclude that the home firm will never charge a price that is even slightly lower than the bottom of the gap in its support. But this line of reasoning obviously unravels as price approaches 0.

We turn our attention now to the support of the price distribution of the foreign firm. Using the fact that the support of the home firm is contained in an interval of length less than or equal to t, we first establish the analog of Lemma 7.

<u>Lemma 10</u>: Suppose there exist p',p'' such that $\bar{p}-2t < p' < p'' \le \bar{p}-t$. If $\Pi^*(p',G) \ge \Pi^*(p'',G)$, then $\Pi^*(p'+t,G) > \Pi^*(p''+t,G)$.

Proof:

Assume $\bar{p}-2t < p' < p'' \leq \bar{p}-t$. Then, by our hypothesis,

$0 \geq \Pi^*(p'',G) - \Pi^*(p',G)$

(and since Lemma 8 and Lemma 5(b) imply $G(p') = G(p'') = 1$)

$= [\pi(p'')+kG(p''+t)\pi^*(p'')] - [\pi(p')+kG(p'+t)\pi^*(p')]$

(also, since $\pi(p'') > \pi(p')$)

$> kG(p''+t)\pi^*(p'') - kG(p'+t)\pi^*(p')$

(and using the fact that $\pi^*(p) = [p/(p+t)]\pi(p+t)$)

$= kG(p''+t)[p''/(p''+t)]\pi(p''+t)-kG(p'+t)[p'/(p'+t)]\pi(p'+t)$

(further, since $p'/(p'+t) < p''/(p''+t)$

$\geq k[p'/(p'+t)][G(p''+t)\pi(p''+t) - G(p'+t)\pi(p'+t)]$

(and since, if $k \leq 1$, Lemma 6 implies that $G(p'+2t) = G(p''+2t) = 0$)

$= k[p'/(p'+t)][\Pi^*(p''+t,G) - \Pi^*(p'+t,G)]$.

which implies the Lemma.

Q.E.D.

We see that this lemma is the analog for the foreign firm of what Lemma 7 was for the domestic firm. Because of the concavity of the profit function, if the foreign firm stands to gain by undercutting the home firm's prices significantly, then it has some chance of capturing the protected home market even at relatively high prices.

Lemma 11: $p_-^* \geq \bar{p}^*-t$.

Proof:

Suppose $\underline{p}^* < \overline{p}^*$-t. Then Lemma 3(b) implies that $\underline{p}^* < \overline{p}$-t. Furthermore, since

$$\Pi^*(\overline{p}^*-t,G) - \Pi^*(\underline{p}^*,G) = \Pi^*(\overline{p}^*-t,G) - v^* \le 0,$$

Lemma 10 implies that $\Pi^*(\overline{p}^*,G) - \Pi^*(\underline{p}^*+t,G) < 0$, contradicting the definition of $\overline{p}^*$. Q.E.D.

Lemma 11 states that all of the prices charged by the foreign firm are contained within an interval of length t. Lemmata 8 and 11 imply that each firm's equilibrium strategy is contained within an interval of width t. This fact will be very useful in characterizing the equilibrium strategies. We shall see below that for small tariffs, the supports are often staggered intervals.

Lemma 12: (a) $G^*(\underline{p}) = G^*(\overline{p}-t)$.

(b) If $\overline{p} < p_m$, then $\underline{p} = \overline{p}-t$.

(c) If $\overline{p} = \overline{p}^*$, then $S^* = S$. Otherwise $S^* = [\overline{p}^*-t,\overline{p}-t] \cup [\underline{p},\overline{p}^*]$.

Proof:

(a) Let p′ and p″ be such that $\overline{p}-t < p' < p'' < \overline{p}*$. Then

$$\Pi^*(p'',G) - \Pi^*(p',G) = \pi(p'') - \pi(p') > 0.$$

Therefore, in order for G^* to be a best response, Lemma 5(a) implies that $G^*(\bar{p}-t) = G^*(p\_)$. This establishes (a).

(b) Suppose $p\_ \neq \bar{p}-t$. Then Lemma 8 implies $p\_ > \bar{p}-t$. Part (a) implies then that there is a $p > \bar{p}$ such that $G^*(p-t) = G^*(\bar{p}-t)$. Then for this p

$$0 \geq \Pi(p,G^*) - v = \Pi(p,G^*) - \Pi(\bar{p},G^*) = kG^*(\bar{p}-t)[\pi(p)-\pi(\bar{p})].$$

Hence, $0 \geq \pi(p)-\pi(\bar{p})$. But π is increasing on $[0,p_m]$, and since $p > \bar{p}$, we may conclude that $\bar{p} \geq p_m$, contradicting Lemma 3(b). This establishes (b).

(c) Note that $\bar{p}^* > p\_$. Otherwise, it follows from part (a) and Lemma 5(a) that $\bar{p}^* \leq \bar{p}-t$, which contradicts Lemma 3(a).

We show first that $[p\_, \bar{p}^*] \subset S^*$. Choose p', p'' such that $p\_ \leq p' < p'' < \bar{p}^*$. Then Lemma 11 implies that $p''-t < p\_^*$. Therefore, $G^*(p'-t) = G^*(p''-t) = 1$. It then follows from Lemmata 3(b), 5(a), and 9 that

$$\begin{aligned}
0 = v - v &= \Pi(p'',G^*) - \Pi(p',G^*) \\
&= \pi(p'')[kG^*(p''-t)+G^*(p'')] - \pi(p')[kG^*(p'-t)+G^*(p')] \\
&= k[\pi(p'') - \pi(p')] + \pi(p'')G^*(p'') - \pi(p')G^*(p') \\
&> \pi(p'')G^*(p'') - \pi(p')G^*(p') \\
&> \pi(p'')[G^*(p'')-G^*(p')]
\end{aligned}$$

which implies that $G^*(p'') < G^*(p')$ and hence that $[p', p''] \subset S^*$.

We show second that, if $\bar{p}^* < \bar{p}$, then $[\bar{p}^*-t, \bar{p}-t] \subset S^*$. Choose

p', p'' so that $\bar{p}^*-t < p' < p'' < \bar{p}-t$. Since $p'+t > \bar{p}$ and $p''+t > \bar{p}$, $G^*(p'+t) = G^*(p''+t) = 0$. Also, since $p'' < \bar{p}-t$, Lemma 3(a) implies that $G^*(p'') > 0$. Furthermore, since $p\_ < \bar{p}^*$, we know that $p'+t \in S$ and $p''+t \in S$. This implies that

$$0 = v - v = \Pi(p''+t, G^*) - \Pi(p'+t, G^*) = k[G^*(p'')\pi(p'') - G^*(p')\pi(p')]$$
$$> k\pi(p'')[G^*(p'')-G^*(p')]$$

which implies that $G^*(p'') < G^*(p')$ and again that $[p', p''] \subset S^*$

We have now established that $[\bar{p}^*-t, \bar{p}-t] \cup [p\_, \bar{p}^*] \subset S^*$.

It follows from Lemma 11 and part (a) of this Lemma that $S^* \subset [\bar{p}^*-t, \bar{p}-t] \cup [p\_, \bar{p}^*]$. This establishes that $S^* = [\bar{p}^*-t, \bar{p}-t] \cup [p\_, \bar{p}^*]$. Finally, if $\bar{p}^* = \bar{p}$, $\bar{p}^*-t = \bar{p}-t$. Hence, Lemma 5(a) implies that $S^* = S$. Q.E.D.

Lemma 12(a) states that the foreign firm will not charge any prices between the $\bar{p}$-t and $p\_$. Since the home firm's support is contained in an interval of width t, $\bar{p}-t \le p\_$. Then the only reason the foreign firm charges any prices less than $p\_$ is that it is competing for the protected market. In order to do so, it must charge prices at least as low as $\bar{p}$-t. Lemma 12(b) states that if the home firm does not charge the monopoly price, its support is an interval of full length t. Lemma 12(c) states that the foreign firm charges prices in two intervals. In the lower interval, it charges prices low enough to compete for the protected domestic market. In the higher interval, it charges prices high enough to capture its own market profitably. One surprising implication of 12(c) is that the foreign firm's support need not be connected. These ideas are illustrated in Figure 2, which appears on page 44.

Before proceeding with our derivation of the equilibrium, we require a more complete description of the equilibrium for the case where $\bar{p}^* = \bar{p}$. First, define $\tilde{p}$ by $(1+k)\pi(\tilde{p}) = k\pi(p_m)$. Note that $\tilde{p}$ is the price at which the home firm is indifferent between capturing the world market with certainty and reaping monopoly rents in its own protected market.

Lemma 13: (a) $\bar{p}^* = \bar{p}$ if and only if $\bar{p}^* = p_m$;
(b) If $\bar{p}^* = p_m$, then $p\_^* = p\_ = \tilde{p} \geq p_m - t$ and $G(p_m) = k/(1+k)$.

Proof:

(a) If $\bar{p}^* = p_m$, then it follows from Lemma 3(b) that $\bar{p} = p_m$.

Conversely, suppose that $\bar{p}^* = \bar{p}$. Then it follows from Lemma 5(a) and Lemma 2 that

$$0 < v^* = \lim_{\epsilon \downarrow 0} \Pi^*(\bar{p}^* - \epsilon, G) = \pi(\bar{p})G(\bar{p}),$$

which implies that $G(\bar{p}) > 0$. It then follows from Lemma 5(b) that $\bar{p} = p_m$.

(b) If $\bar{p}^* = p_m$, then part (a) and Lemma 12(c) imply that $p\_^* = p\_$. It follows then from Lemma 11 that $G^*(p_m - t) = 1$ and hence from Lemma 5(a) that

$$\begin{aligned} v &= \Pi(p_m, G^*) = k\pi(p_m) \\ &= \Pi(p\_, G^*) = (k+1)\pi(p\_). \end{aligned}$$

Therefore, $p\_^* = p\_ = \tilde{p}$.

Next, note that Lemma 6 implies that $p_m \leq (1+k)t$. Therefore, since $\pi(\cdot)$ is concave on $[0,p_m]$ and $(1+k)\pi(\tilde{p}) = k\pi(p_m)$, it follows that

$$\tilde{p} \geq p_m[k/(1+k)] \geq p_m[k/(1+k)] + p_m/(1+k) - t = p_m - t.$$

Finally, using Lemmata 5(a) and 8, note that

$$\begin{aligned} v^* &= \lim_{\epsilon \downarrow 0} \Pi^*(p_m - \epsilon, G) = \pi(p_m)G(p_m) \\ &= \lim_{\epsilon \downarrow 0} \Pi^*(p\_ + \epsilon, G) = \pi(p\_) \\ &= \pi(\tilde{p}) = [k/(1+k)]\pi(p_m) \end{aligned}$$

from which it follows that $G(p_m) = k/(1+k)$. Q.E.D.

Lemma 13 is essentially the description of an equilibrium where the tariff is large enough to be prohibitive. In this case the home firm charges the monopoly price with positive probability, and the foreign firm charges price in the interval from $\tilde{p}$ to p_m. The lemma states that the tariff need not be as large as the monopoly price to be prohibitive. These lemmata have placed enough restrictions on the nature of equilibrium that we may proceed to a proof of its existence and uniqueness.

4. Existence of a Unique Equilibrium

Using the restrictions implied by Lemmata 8, 9, and 12, we are now prepared to establish the existence of a unique equilibrium. In this section, we shall present a series of formulae that will enable us to characterize the equilibrium values of the game for both firms for an arbitrary tariff.

Given the continuity of G^*, Lemmata 9 and 12 imply that

$$(1)\qquad \begin{aligned} v &= \Pi(\bar{p},G^*) = kG^*(\bar{p}-t)\pi(\bar{p}); \\ &= \Pi(\bar{p}^*,G^*) = k\pi(\bar{p}^*); \\ &= \Pi(p\_,G^*) = [k+G^*(p\_)]\pi(p\_). \end{aligned}$$

Likewise, Lemmata 12 and 13 imply

$$(2)\qquad \begin{aligned} v^* &= \lim_{p\uparrow\bar{p}^*}\Pi^*(p,G) = G(\bar{p}^*)\pi(\bar{p}^*); \\ &= \Pi^*(p\_,G) = \pi(p\_); \\ &= \Pi^*(p\_^*,G) = \pi(p\_^*) + kG(p\_^*+t)\pi^*(p\_^*) \end{aligned}$$

We will use equations (1) and (2) to construct an equilibrium. We then show that a pair of equilibrium strategies can be constructed which yield that value to the home firm. This will establish the existence of an equilibrium.

Let $X = [0,\pi(\tilde{p})]$. Then for $v^* \in X$, define $p\_(v^*)$ by

$$(3)\qquad \pi(p\_(v^*)) = v^*$$

and let

$$(4)\qquad \bar{p}(v^*) = \min\{p\_(v^*)+t,\ p_m\}.$$

If v^* is the equilibrium level of expected profits for the foreign firm, then equation (2) and Lemma 12 imply that $p\_(v^*)$ and $\bar{p}(v^*)$ must be respectively the upper and lower bound of the support of the price distribution of the home firm. Lemma 12 implies that $G^*(p\_) = G^*(\bar{p}-t)$. Therefore, equation (1) implies that

$$(5) \qquad kG^*(\bar{p}-t)\pi(\bar{p}) = kG^*(p\_)\pi(\bar{p}) = \Pi(\bar{p},G^*) = v = \Pi(p\_,G^*) = [k+G^*(p\_)]\pi(p\_).$$

Solving for $G^*(p\_)$ and using the fact that $\pi(p\_) = v^*$, it follows that if v^* is the equilibrium expected return to the foreign firm, then

$$(6) \qquad G^*(p\_) = kv^*/[k\pi(\bar{p}(v^*))-v^*] \qquad \text{if } k\pi(\bar{p}(v^*))-v^* \neq 0.$$

Substituting this value of $G^*(p\_)$ into equation (1) then yields the equilibrium value of the expected profits for the home firm as a function of v^*

$$(7) \qquad v(v^*) = [k+G^*(p\_)]\pi(p\_(v^*)) = kv^*[1 + [v^*/[k\pi(\bar{p}(v^*))-v^*]]$$
$$= k^2v^*\pi(\bar{p}(v^*))/[k\pi(\bar{p}(v^*))-v^*] \qquad \text{if } k\pi(\bar{p}(v^*))-v^* \neq 0.$$

Using equations (3) and (4), we can write a convenient representation of (7) as a function of $x \in X$. In particular, we have

$$(8) \qquad f(x) = \begin{cases} [k^2x\pi(\pi^{-1}(x)+t)]/[k\pi(\pi^{-1}(x)+t)-x] & \text{if } x \leq \pi(p_m-t) \\ [k^2x\pi(p_m)]/[k\pi(p_m)-x] & \text{if } \pi(p_m-t) < x \end{cases}$$

For $t \in (0,p_m]$, we note the following properties of $f(\cdot)$:

Lemma 14: (a) $k\pi(p_m) \le \pi(p_m - t)$ if and only if there is an x_o, with $0 < x_o \le k\pi(p_m)$, such that $\lim_{x\uparrow x_o} f(x) = \infty$ and $f(x) < 0$ for all $x > x_o$;

(b) $f(\cdot)$ is a continuous and strictly increasing function on the domain on which it is non-negative; and

(c) $f(0) = 0$ and $f(x)/x$ is increasing on the domain on which it is non-negative.

Proof:

(a) First, note that if $k\pi(p_m) \le \pi(p_m - t)$, then $p_m - \pi^{-1}(k\pi(p_m)) \ge t$. Since $k^2x\pi(p) > 0$ for all $p > 0$, it suffices to show that there is an x_o, with $0 < x_o < \pi(p_m)$ such that $k\pi(\pi^{-1}(x)+t)-x \le 0$ for $x \ge x_o$. This is equivalent to $\pi^{-1}(x/k) - \pi^{-1}(x) \ge t$. Since π is increasing, continuous, and concave on $[0,p_m]$, π^{-1} is increasing, continuous, and convex on $[0,\pi(p_m)]$. Since $k \in (0,1]$, $\pi^{-1}(x/k) - \pi^{-1}(x)$ is increasing in x on $[0,k\pi(p_m)]$. For $x = 0$, we have $\pi^{-1}(0/k) - \pi^{-1}(0) = 0$. We know that, for all $x \in [0,k\pi(p_m)]$, $p_m - \pi^{-1}(k\pi(p_m)) \ge \pi^{-1}(x/k) - \pi^{-1}(x)$, with strict equality for $x = k\pi(p_m)$. The continuity of π^{-1}, then, implies that there is an x_o, with $0 < x_o \le k\pi(p_m)$, such that $\pi^{-1}(x_o/k) - \pi^{-1}(x_o) \ge t$ for all $x \ge x_o$.

Conversely, if there is an x_o, with $0 < x_o < k\pi(p_m)$, such that $\lim_{x\uparrow x_o} f(x) = \infty$, we may infer that $\pi^{-1}(x_o/k) - \pi^{-1}(x_o) \ge t$ for all $x \ge x_o$. Since $\pi^{-1}(x/k) - \pi^{-1}(x)$ is continuous and increasing in x on $[0,k\pi(p_m)]$, we may conclude that $p_m - \pi^{-1}(k\pi(p_m)) \ge t$, which is equivalent to the fact that $k\pi(p_m) \le \pi(p_m - t)$.

(b) The continuity of f follows from the facts that π is continuous and that π^{-1} is a homomorphism of π. Since $k^2x\pi(\pi^{-1}(x)+t)$ and

$[k^2 x\pi(p_m)]/[k\pi(p_m)-x]$ are both increasing when $k\pi(p_m) - x$ is positive, in order to show that f is increasing, it is suffices that, for sufficiently small x, $[k\pi(\pi^{-1}(x)+t) - x]$ is non-increasing in x. Using the differentiability of π and the fact that $k \in (0,1]$, we see that this follows immediately from the concavity of π.

(c) The fact that $f(0) = 0$ follows from algebraic substitution. Of course, for $x \leq \pi(p_m-t)$, $f(x)/x = [k^2\pi(\pi^{-1}(x)+t)]/[k\pi(\pi^{-1}(x)+t)-x]$. Since $k^2\pi(\pi^{-1}(x)+t)$ is increasing for sufficiently small x, the fact that $f(x)/x$ is increasing for $x \leq \pi(p_m-t)$ is an immediate consequence of (b). Finally, the fact that $f(x)/x$ is increasing for $\pi(p_m-t) < x$ follows immediately from an evaluation of the second line of (8). Q.E.D.

Similarly, let $Y \equiv [k\pi(t), k\pi(p_m)]$. For $v \in Y$, define $\bar{p}^*(v)$ by

$$\pi(\bar{p}^*(v)) = v/k \tag{9}$$

and define

$$p\_^*(v) = \begin{cases} \bar{p}^*(v)-t & \text{if } v < k\pi(p_m); \\ \tilde{p} & \text{if } v = k\pi(p_m). \end{cases} \tag{10}$$

If v is the equilibrium level of expected profits for the home firm, then equation (1) and Lemma 12 imply that $p\_^*(v)$ and $\bar{p}^*(v)$ must be respectively the upper and lower bound of the support of the price distribution of the foreign firm.

Suppose $v < k\pi(p_m)$. Then it follows from equation (8) that $\bar{p}^* < p_m$. Lemma 12 then implies that $p\_^*+t = \bar{p}^*$ and hence that $G(p\_^*+t) = G(\bar{p}^*)$. This fact and equation (2) imply that

$$(11) \qquad G(\bar{p}^*)\pi(\bar{p}^*) = \Pi^*(\bar{p}^*,G) = v^* = \Pi^*(p\_^*,G)$$
$$= \pi(p\_^*) + kG(p\_^*+t)\pi^*(p\_^*).$$

Solving for $G(\bar{p}^*)$, it follows that, if the equilibrium expected value of the home firm's profits v is less than $k\pi(p_m)$, then

$$(12) \qquad G(\bar{p}^*) = \pi(p\_^*(v))/[\pi(\bar{p}^*(v))-k\pi^*(p\_^*(v))].$$

Substituting this value of $G^*(\bar{p}^*)$ back into equation (2) and using the definition of π^* then yields the equilibrium value of the expected profits for the foreign firm as a function of v:

$$(13) \qquad v^*(v) = \pi(\bar{p}^*(v))G(\bar{p}^*(v))$$
$$= \pi(\bar{p}^*(v))\pi(p\_^*(v))/[\pi(\bar{p}^*(v))-k\pi^*(p\_^*(v))]$$ [1]

Using equations (9) and (10) and the definition of π^*, we write another representation of (13) as a function of $y \in Y$. In particular, we have

$$(14) \qquad h(y) = \begin{cases} [\pi^{-1}(y/k)\pi(\pi^{-1}(y/k)-t))]/[(1-k)\pi^{-1}(y/k) + kt] & \text{if } y < k\pi(p_m) \\ [k\pi(p_m)]/[1+k] & \text{if } y = k\pi(p_m). \end{cases}$$

For $t \in (0,p_m]$, we note the following properties of $h(\cdot)$:
****

1 Since $k \le 1$, as long as $t > 0$, $v^*(v)$ is well defined for all values of $v \in V$.

<u>Lemma 15</u>: (a) $h(\cdot)$ is a continuous increasing function on $[k\pi(t), k\pi(p_m))$;

(b) $\lim_{y\uparrow k\pi(p_m)} h(y) = \pi(p_m - t)p_m/[(1-k)p_m + kt]$; and

(c) $h(k\pi(t)) = 0$ and $h(y)/y$ is increasing on $[k\pi(t), k\pi(p_m))$.

<u>Proof</u>:

(a) The continuity of h follows from that of π and from the fact that π^{-1} is a homomorphism. To see that h is increasing, note that $\pi^{-1}(y/k)-t$ is positive and increasing for $y > k\pi(t)$. Using the differentiability of π^{-1}, it is easy to check that $[\pi^{-1}(y/k))]/[(1-k)\pi^{-1}(y/k) + kt]$ is increasing for $t \in (0,p_m)$. Hence h is increasing on $[k\pi(t), k\pi(p_m))$.

(b) This follows from the continuity of h.

(c) The fact that $h(k\pi(t)) = 0$ follows from algebraic substitution. We know that

$$
\begin{aligned}
h(y)/y &= [\pi^{-1}(y/k)\pi(\pi^{-1}(y/k)-t))]/[y((1-k)\pi^{-1}(y/k)+kt)] \\
&= [(\pi^{-1}(y/k))/((1-k)\pi^{-1}(y/k)+kt)]\ [\pi(\pi^{-1}(y/k)-t)/y]
\end{aligned}
$$

We have already established that $[(\pi^{-1}(y/k))/((1-k)\pi^{-1}(y/k)+kt)]$ is increasing. All we need to show is that $r(y) \equiv [\pi(\pi^{-1}(y/k)-t)/y]$ is non-decreasing. Differentiating $r(y)$, we have

$$
r'(y) = [y\pi'(\pi^{-1}(y/k)-t)/\pi'(\pi^{-1}((y/k)) - \pi(\pi^{-1}(y/k)-t)]/[y^2]
$$

Since π is concave and $\pi(\pi^{-1}(y/k)-t) \le \pi^{-1}(\pi(y/k))$, we know that

$$r'(y) \geq [y - \pi(\pi^{-1}(y/k)-t)]/[y^2]$$

It will suffice to show that $[y - \pi(\pi^{-1}(y/k)-t)] > 0$. This is equivalent to

$$\pi^{-1}(y) - \pi^{-1}(y/k) + t > 0$$

which in turn is equivalent to

$$\pi^{-1}(y/k) - \pi^{-1}(y) < t,$$

again an expression which is increasing in y. Since part (b) of this Lemma implies that h(y) is bounded, Lemma 14(a) implies that there is no $y_o < k\pi(p_m)$ such that $\pi^{-1}(y_o/k) - \pi^{-1}(y_o) = t$. Since $\pi^{-1}(0/k) - \pi^{-1}(0) = 0 < t$, we can conclude that $r'(y) > 0$ and hence that r(y) is non-decreasing. Q.E.D.

We shall use the properties of f(x) and h(y) to show that there is a unique equilibrium.

Let $T = \{t: 0 \leq t \leq p_m\}$. Without loss of generality, we restrict ourselves to T because this is the set of economically interesting tariffs. Any tariff larger than p_m is, of course, surely prohibitive. We can now conclude this section with a statement and proof of the existence theorem.

Theorem: For any $t \in T$, there is a unique equilibrium.

Proof:

If $t = 0$, then this problem reduces to that of pure Bertrand competition, and $v = v^* = 0$. Henceforth, assume that $t > 0$.

Let $T_1 = \{t \in T: k\pi(p_m) \leq \pi(p_m-t)\}$. Since $\pi(p_m-t) \geq k\pi(p_m)$, only the first line of equation (8) is germane in the evaluation of f(x). Also, by Lemma 14, we know that f(x) is continuous and strictly increasing on an interval which is a subset of $[0,k\pi(p_m)]$. Consider

$$\lim_{x\uparrow\pi(\tilde{p})} f(x) = [k^2\pi(\tilde{p})\pi(\tilde{p}+t)]/[k\pi(\tilde{p}+t)-\pi(\tilde{p})].$$

If $k\pi(\tilde{p}+t)-\pi(\tilde{p}) \leq 0$, then Lemma 14(a) implies that $x_o \in [0,\pi(\tilde{p})]$ and $\lim_{x\uparrow x_o} f(x) = \infty$. Since Lemma 15 implies that h(y) is bounded, there is a pair $(x,y) \in (0,\pi(\tilde{p})) \times (k\pi(t),k\pi(p_m))$ such that $y = f(x)$ and $x = h(y)$. Otherwise, if $k\pi(\tilde{p}+t)-\pi(\tilde{p}) > 0$, then $\pi(\tilde{p}) < x_o$, and since $\pi(p_m-t) > \pi(\tilde{p})$ implies that $p_m > \tilde{p} + t$, we can infer that

$$\begin{aligned}
\lim_{x\uparrow\pi(\tilde{p})} f(x) &= [k^2\pi(\tilde{p})\pi(\tilde{p}+t)]/[k\pi(\tilde{p}+t)-\pi(\tilde{p})] \\
&> [k^2\pi(\tilde{p})\pi(\tilde{p}+t)]/[k\pi(p_m)-\pi(\tilde{p})] \\
&= [k^2(k/1+k)\pi(p_m)\pi(\tilde{p}+t)]/[k\pi(p_m)-(k/1+k)\pi(p_m)] \\
&= [k^2(1/1+k)\pi(\tilde{p}+t)]/[1-(1/1+k)] \\
&= k\pi(\tilde{p}+t).
\end{aligned}$$

Now consider $\lim_{y\uparrow k\pi(\tilde{p}+t)} h(y)$. Using Lemma 15, we have

$$\lim_{y\uparrow k\pi(\tilde{p}+t)} h(y) = [\pi(\tilde{p}+t)(\tilde{p}+t)]/[(1-k)(\tilde{p}+t) + kt]$$
$$= [\pi(\tilde{p}+t)(\tilde{p}+t)]/[(\tilde{p}+t) - k\tilde{p}]$$
$$> \pi(\tilde{p}+t)$$
$$> \pi(\tilde{p})$$

Since f and h are continuous on $(0,\pi(\tilde{p}))$ and $(k\pi(t),k\pi(p_m))$ respectively, we may again infer that there is a pair $(x,y) \in (0,\pi(\tilde{p})) \times (k\pi(t),k\pi(p_m))$ such that $y = f(x)$ and $x = h(y)$.

Now let $T_2 = \{t \in T: \pi(\tilde{p}) \le \pi(p_m-t) < k\pi(p_m)\}$. Again, since $\pi(p_m-t) \ge \pi(\tilde{p})$, only the first line of equation (8) is germane in the evaluation of f(x) when restricted to $[0,\pi(\tilde{p})]$. Since $\pi(\tilde{p}) \le \pi(p_m-t)$, Lemma 14(a) implies that f(x) is well defined on $(0,\pi(\tilde{p}))$. Using the same line of reasoning as above, we know that $\lim_{x\uparrow\pi(\tilde{p})} f(x) \ge k\pi(\tilde{p}+t)$. Since $\lim_{y\uparrow k\pi(\tilde{p}+t)} h(y) > \pi(\tilde{p})$, we may infer exactly analogously that there is a pair $(x,y) \in (0,\pi(\tilde{p})) \times (k\pi(t),k\pi(p_m))$ such that $y = f(x)$ and $x = h(y)$.

Finally, let $T_3 = \{t \in T: \pi(0) \le \pi(p_m-t) < \pi(\tilde{p})\}$. T_3 is the set of tariffs for which we must evaluate the second line of the definition of f(x) in equation (8); these tariffs are large enough so that the supremum of the home firm's support may be the monopoly price p_m. Since $\pi(p_m-t) < \pi(\tilde{p})$, by evaluating the definition of f(x) at $\pi(p_m-t)$, we know that f(x) is continuous and increasing on $[0,\pi(\tilde{p})]$ and that $f(\pi(\tilde{p}))=k\pi(p_m)$. Consider $\lim_{y\uparrow k\pi(p_m)} h(y)$; Lemma 15 states that

$$\lim_{y\uparrow k\pi(p_m)} h(y) = [p_m\pi(p_m-t)]/[(1-k)p_m + kt],$$

an expression which is continuous and decreasing in t. For $t \in T_3$, with t sufficiently near $p_m - \tilde{p}$, we know that $[p_m\pi(p_m-t)]/[(1-k)p_m + kt > \pi(\tilde{p})$ and hence that $\lim_{y\uparrow k\pi(p_m)} h(y) > \pi(\tilde{p})$. Therefore, using analogous arguments as above, we can conclude again that there is a pair $(x,y) \in (0,\pi(\tilde{p})) \times (k\pi(t),k\pi(p_m))$ such that $y = f(x)$ and $x = h(y)$. Moreover, since $[p_m\pi(p_m-t)]/[(1-k)p_m + kt]$ is continuous in t, there is a $t_o \in T_3$ such that $[p_m\pi(p_m-t_o)]/[(1-k)p_m + kt_o] = \pi(\tilde{p})$. (In fact, using Lemma 13(b), one can check that t_o defines the smallest prohibitive tariff.) For all $t \geq t_o$, the definition of h(y) is such that $h(k\pi(p_m))=\pi(\tilde{p})$. Indeed, when the tariff is prohibitive, we can take $x = \pi(\tilde{p})$ and $f(x) = k\pi(p_m)$. Hence, for $t \in T_3$ there is a pair such that $y = f(x)$ and $x = h(y)$.

Note that $T = \{0\} \cup T_1 \cup T_2 \cup T_3$, and we have shown that there is an $x \in X$ such that $x = h(f(x))$ for all $t \in T$. The uniqueness of this x follows from the fact that Lemmata 14(c) and 15(c) imply that both $f(x)/x$ and $h(y)/y$ are increasing over the relevant domains. In the rest of the proof, we will focus our attention on the pair $(x,f(x))$ such that $h(f(x)) = x$

To demonstrate that $v^*=x$ and $v=f(x)$ are values of the game in equilibrium for the foreign and home firms respectively, we construct the two firms' equilibrium strategies. In particular, recall that

(15.1) $$v = k\pi(p)G^*(p-t) + \pi(p)G^*(p) \text{ and}$$

(15.2) $$v^* = k\pi^*(p)G(p+t) + \pi(p)G(p).$$

Further, following Lemmata 9 and 12, we know that the supports of the firms' strategies are given by

$$S = [p\_(v^*),\bar{p}(v^*)] \text{ and}$$

$$S^* = [\bar{p}^*(v)-t,\bar{p}(v)-t] \cup [p\_(v),\bar{p}^*(v)]$$

if $v^* < \pi(\tilde{p})$. If $v^* = \pi(\tilde{p})$, Lemma 13 implies that

$$S = S^* = [\tilde{p}, p_m].$$

Equations (15.1) and (15.2) define implicitly the mixed strategies that are equilibrium best responses for both firms. Indeed, for $v^* < \pi(\tilde{p})$, we have

$$G(p) = \begin{cases} v^*/\pi(p) & \text{if } p \in [p\_(v^*),\ \bar{p}^*(v)] \\ [v^*-\pi(p-t)]/[k\pi^*(p-t)] & \text{if } p \in (\bar{p}^*(v),\ \bar{p}(v^*)] \end{cases}$$

and

$$G^*(p) = \begin{cases} v/[k\pi(p+t)] & \text{if } p \in [p^*\_(v),\ p\_(v^*)] \\ [v-k\pi(p)]/[\pi(p)] & \text{if } p \in (p\_(v^*),\ \bar{p}^*(v)]. \end{cases}$$

Otherwise, for $v^* = \pi(\tilde{p})$, we have

$$G(p) = \begin{cases} v^*/\pi(p) & \text{if } p \in [\tilde{p}, p_m) \\ k/[1+k] & \text{if } p = p_m \end{cases}$$

and

$$G^*(p) = [v-k\pi(p)]/[\pi(p)] \qquad \text{if } p \in [\tilde{p}, p_m].$$

Given these definitions of the firms' strategies, it is easy to check for any $p \in S$ that $v = \Pi(p, G^*) = f(x)$; likewise, for any $p \in S^*$,

$v^* = \Pi^*(p,G) = x$. Further, consider $p \notin S$. If $p < p\_(v^*)$ and $v^* < \pi(\tilde{p})$, then

$$\begin{aligned}\Pi(p,G^*) &= k\pi(p)G^*(p-t)+\pi(p)G^*(p)\\ &= k\pi(p)+[v\pi(p)]/[k\pi(p+t)]\\ &< f(x)\end{aligned}$$

where the inequality follows from equation (7) and the definition of $f(x)$. Analogously, if $\bar{p}(v^*) < p$ and $v^* < \pi(\tilde{p})$, then

$$\begin{aligned}\Pi(p,G^*) &= k\pi(p)G^*(p-t)\\ &= k\pi(p)[v-k\pi(p-t)]/[\pi(p-t)]\\ &< f(x)\end{aligned}$$

where again the inequality follows from (7) and the definition of $f(x)$. Checking the cases for $p \notin S^*$ are analogous and use Lemma 15. Finally, the cases where $v^* = \pi(\tilde{p})$ are again exactly analogous.

Q.E.D.

The intuition behind this theorem is the central to our discussion. First, notice that the value of the game for the home firm increases with the tariff and the relative size of the domestic market; in particular, the lower bound of Y shifts upward with t and k. Second, since h(y) is increasing, the value of the game for the foreign firm is also increasing in t and k. Although a larger tariff makes it more difficult for the foreign firm to sell in the protected market, it makes for a less aggressive domestic firm and thus increases the expected profits of the foreign firm from sales outside the protected market. Third, notice that, in almost every step of the derivation of the equilibrium, prices played two roles; any price charged by either firm competes in the protected market at the after-tariff price and in the world market at the before-tariff price. Weighing the costs and benefits of pricing for export versus pricing solely for domestic sales is inherent in every strategy played by either firm. In many oligopolistic international markets, commercial policy creates this innate tension.

5. Two Examples

In the preceding section we say that an equilibrium existed for an arbitrary tariff. In this section, we will derive two such equilibria using numerical methods for a simple demand function. In particular, we consider demand in the foreign market given by

$$D(p^*) = 1 - p^*$$

where the variables are obvious. Recall that marginal costs are zero by assumption; hence, this demand curve implies that profits for the foreign firm are given by

$$\pi(p) = p\ (1 - p)$$

which satisfies assumptions A1. Note in particular that the monopoly price $p_m = 1/2$, and $\pi(p)$ is increasing on $[0, 1/2]$. By assumption, we have demand in the home market given by

$$D(p) = k\ (1 - p) \qquad 0 < k \le 1$$

where k, again, is the relative size of the home market as compared with the foreign market. Further, we can define the profits to the foreign firm from selling in the tariff-ridden home market; these are given by

$$k\pi^*(p) = k\ p\ (1 - t - p)$$

where t is the level of the specific tariff.

We will now calculate an equilibrium. Lemma 12 implies that, for a small tariff t, the widths of the supports for the home and foreign firm's strategies are just equal to the tariff. It also implies that each support is connected. Of course, $p\_^* < p\_ < \bar{p}^* < \bar{p}$, and hence the two supports are staggered intervals. Let v^* be the value of the game for the foreign firm. Using the function $\pi(p)$, equation (2) and the quadratic formula, we have

$$p\_ = [1 - \sqrt{(1-4v^*)}]/2$$

where we have chosen the root which corresponds to a price less than 1/2. Note that $v^* \le 1/4$ in order that the root be real. Since 1/4 is the monopoly profit from a perfectly protected market, the tariff cannot create greater than monopoly profits for the foreign firm. We know that

$$\bar{p} = p\_ + t$$

which enables us to calculate the value of the tariff for the domestic firm. Using equations (2) and (7), we have

$$v = [k^2\ \pi(p\_)\ \pi(\bar{p})]/[k\ \pi(\bar{p})\ -\ \pi(p\_)]$$

which enables us to calculate v. This implies a specific value for the supremum of the support of the foreign firm's strategies. In particular, using equation (9) and the quadratic formula, we see that

$$\bar{p}^* = [1 - \sqrt{(1-4(v/k))}]/2.$$

Lemma 12 implies that

$$p\_^* = \bar{p}^* - t$$

which allows us to state the value of the game to the foreign firm. Using equation (13), we have

$$v^* = [\pi(p\_^*)\ \pi(\bar{p}^*)]/[\pi(\bar{p}^*)\ -\ k\pi^*(p\_^*)],$$

which brings us back to the original value of the game for the foreign firm. We have followed the algebra of the lemmata leading up to the existence proof used in the preceding section.

The examples we present below were solved by using numerical methods. For concreteness, let $k = 1$ and $t = .2$. Hence, from the point of view of the protected domestic firm, the home and export markets are equally important; also, the tariff is forty percent of the monopoly price. Solving this series of equations numerically, we have

$$v = .2021, \quad S = [.1188, .3188]$$
$$v^* = .1047, \quad S^* = [.0811, .2811]$$

where these values and these supports define implicitly the densities which are the two firms' strategies in equilibrium. Note that a tariff that is forty percent as large as the monopoly price generates more than eighty percent of the monopoly rent inherent in the domestic market. As in the previous section, we construct these strategies by using

$$v = k\pi(p)G^*(p-t) + \pi(p)G^*(p) \text{ and}$$
$$v^* = k\pi^*(p)G(p+t) + \pi(p)G(p)$$

which imply that

$$G(p) = \begin{cases} .1047/[p(1-p)] & \text{if } p \in [.1188, .2811] \\ [.1047-(p-.2)(1.2-p)]/[(p-.2)(1-p)] & \text{if } p \in (.2811, .3188] \end{cases}$$

and

$$G^*(p) = \begin{cases} .2021/[(p+.2)(.8-p)] & \text{if } p \in [.0811, .1188] \\ [.2021-p(1-p)]/[p(1-p)] & \text{if } p \in (.1188, .2811]. \end{cases}$$

It is straightforward to check that these are equilibria; each firm's mixed strategy is displayed in Figure 3, appearing on page 46. For ease of exposition, we have displayed the conventional <u>cumulative</u> probability distribution.

We present also the equilibrium strategies for a prohibitive tariff. Again, letting $k = 1$ and $t = .4114$, we have

$$v = .25, \quad S = [.1188, .5]$$
$$v^* = .125, \quad S^* = [.0811, .5]$$

It is interesting to note that a prohibitive tariff is less that the full monopoly price. It guarantees the domestic firm monopoly profits in its own market, but it also gives the foreign firm its highest expected profits. This occurs of course because the value of the tariff for the domestic firm defines implicitly how aggressive it will be in international markets. We illustrate the prohibitive tariff in Figure 4, again on page 45. This is also the conventional cumulative probability distribution. Note the spike for the domestic firm at the monopoly price.

Conclusion

We have presented a model of an international duopoly which is a ready extension of that of Bertrand. By assuming that there was no dumping, we show that the imposition of a tariff in one market has effects on the strategies of both firms in both markets. Even though the payoff functions for each duopolist are not continuous, we show that an equilibrium exists for an arbitrary tariff.

The properties of the equilibrium are such that the value of the game increases for the home firm to the extent that its domestic market is large or that the tariff is large. Further, the value of the game for the foreign firm increases with that for the domestic firm. This model serves, then, as another example of how trade restrictions which seemingly discriminate against one firm may actually serve to increase its profits.

The importance of this analysis for the policy-maker is that it is a reminder that commercial policy in oligopolistic markets may have untoward effects. In particular, to the extent that producers make strategic choices involving sales in international markets, the imposition of a trade restriction aimed at one firm in one market has influences on the choices of all firms in all markets. As we saw, the tariff served to raise the profits of the foreign firm. Although we have chosen not to explore the welfare effects of the tariff, we would be remiss not to conclude with the ironic observation that the imposition of a tariff by the domestic government can raise the profits of a foreign firm at the expense of foreign consumer surplus!

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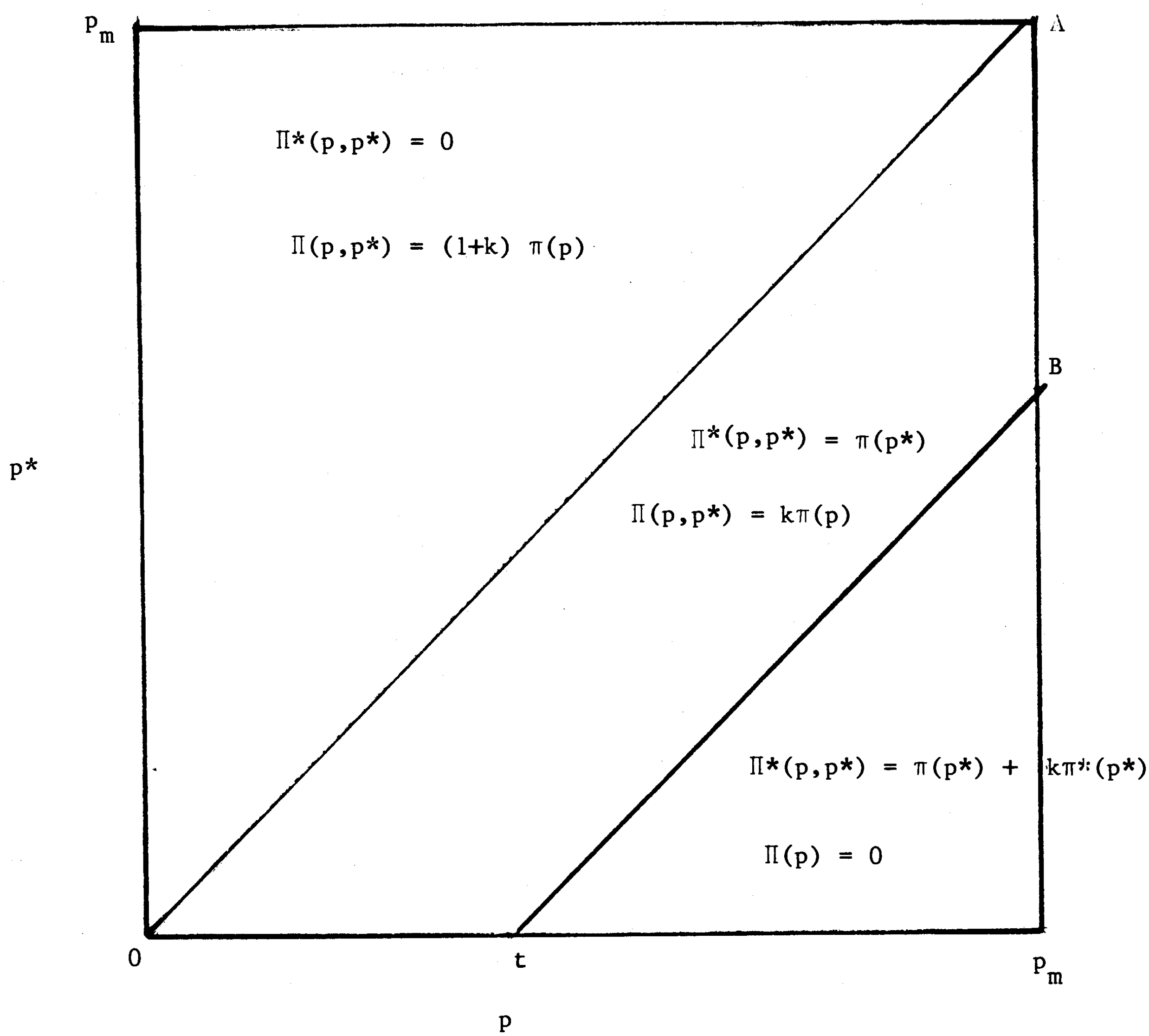
Figure 1



For $p = p^*$: $\Pi(p,p^*) = k\pi(p) + \pi(p)/2$

$\Pi^*(p,p^*) = \pi(p)/2$

For $p = p^* + t$: $\Pi(p,p^*) = k\pi(p)/2$

$\Pi^*(p,p^*) = k\pi^*(p^*)/2 + \pi(p^*)$

Figure 2

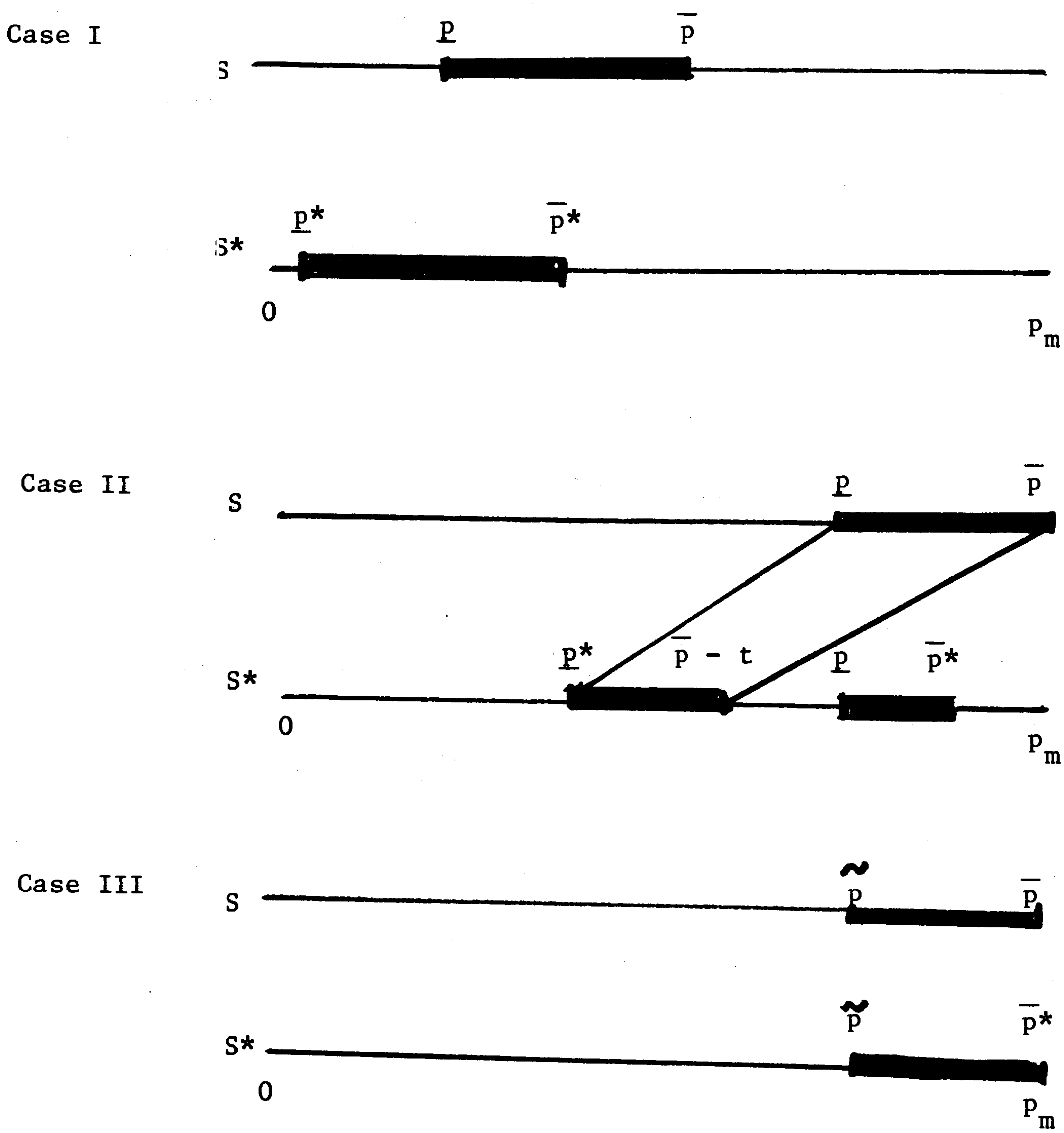

Case I
S
$\underline{p}$
$\overline{p}$
S*
$\underline{p}^*$
$\overline{p}^*$
0
p_m
Case II
S
$\underline{p}$
$\overline{p}$
S*
$\underline{p}^*$
$\overline{p} - t$
$\underline{p}$
$\overline{p}^*$
0
p_m
Case III
S
$\tilde{p}$
$\overline{p}$
S*
$\tilde{p}$
$\overline{p}^*$
0
p_m

Figure 3

Tariff = .2

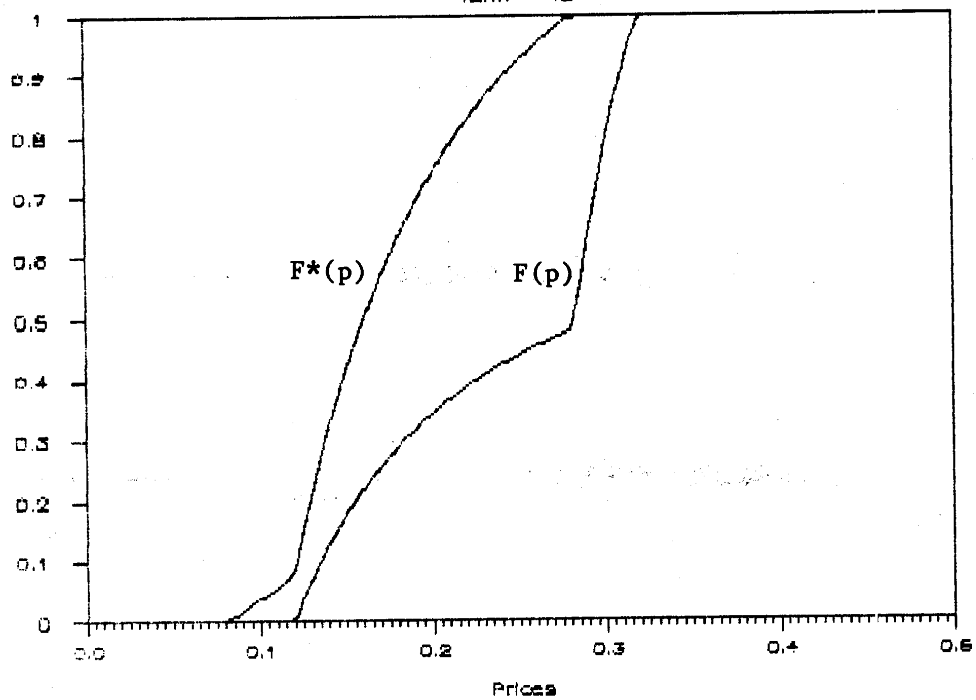
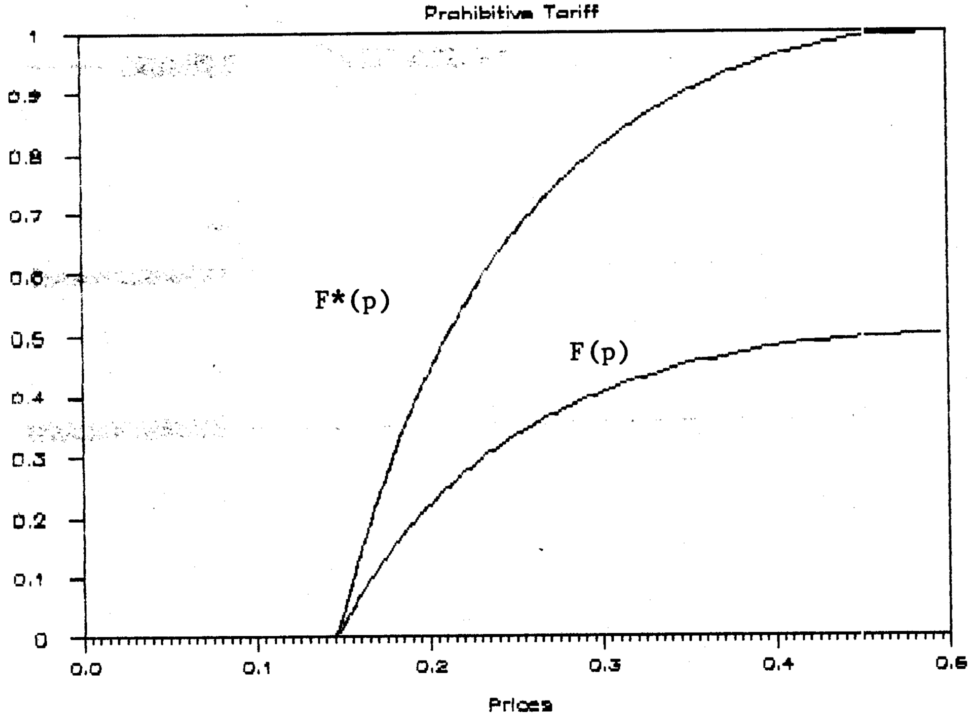


Figure 4

Prohibitive Tariff



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