ADJUSTMENT COSTS AND INTERNATIONAL TRADE DYNAMICS

Joseph E. Gagnon
ABSTRACT

This paper develops a model of trader behavior that is characterized by quadratic adjustment costs, imperfect competition, and rational expectations. The model is fitted to data on aggregate trade flows between the United States and three of its largest trading partners. Tests against alternative specifications confirm the importance of imperfect competition and adjustment costs. The hypothesis of rational expectations cannot be rejected. The estimated price elasticities of trade flows are generally in the range reported by previous researchers, but the activity elasticities are significantly higher.
Adjustment Costs and International Trade Dynamics

Joseph E. Gagnon¹

The most salient international economic phenomenon of the 1980s has been the unprecedented growth of the U.S. trade deficit. At first this deficit seemed to be the natural consequence of ongoing macroeconomic developments: Between 1981 and 1985 the U.S. dollar appreciated steadily. In 1982 the U.S. fiscal deficit more than doubled, and it continued to grow thereafter, despite the quick recovery from the 1982 recession. Under these circumstances, most economists were not surprised at the emergence of a large U.S. trade deficit.

However, the substantial depreciation of the dollar during 1985 convinced many economists that a turnaround in the trade deficit would occur during 1986. For example, the Economic Report of the President [February 1986, p. 60] predicted that real net exports would improve in 1986 even if nominal net exports did not.² One year later, the Council of Economic Advisors had to report that both real and nominal net exports declined

¹ The author is a staff economist in the Division of International Finance. This research was conducted when the author was a doctoral student at Stanford University, and was supported by the National Science Foundation and the Alfred P. Sloan Foundation. The author would like to thank Tam Bayoumi, Steve Durlauf, Neil Ericsson, David Howard, Ron McKinnon, Tom Sargent, Ralph Tryon, and especially John Taylor for helpful advice and comments. This paper represents the views of the author and should not be interpreted as reflecting those of the Board of Governors of the Federal Reserve System or other members of its staff.

² It is important to note the distinction between nominal and real trade flows. If traded goods prices are sticky in the exporter's currency, then a depreciation of the exchange rate will cause an immediate deterioration of a country's nominal trade balance even as it begins to improve the real trade balance. This so-called "J-curve" effect is discussed in Magee (1973) and McKinnon (1979). This paper is concerned primarily with real trade flows.
sharply during 1986.³

The economics profession is now divided in its attempts to explain this
development. Some researchers claim that the lags in trade adjustment are
much too long to have created a turnaround in 1986, especially since the
dollar continued to rise through early 1985. Other researchers believe that
trade is simply not sensitive to relative prices and that the trade deficit
is more closely related to the fiscal deficit and aggregate demand in the
U.S. and its trading partners.⁴

Over the past 20 years numerous empirical studies have sought to answer
precisely these questions.⁵ However, the theoretical justification for the
data and functional forms being estimated in these studies was always
lacking, especially for the use of lagged variables. The goal of this paper
is to establish a more rigorous theoretical framework within which to answer
questions about the lags and elasticities in international trade.

Generally speaking, researchers believe that foreign trade is slow to
adjust to economic circumstances because of information lags, transportation
lags, and special contracting costs unique to international business rela-
tionships. These contracting costs may arise from language barriers, the
unfamiliarity of foreign business practices and regulations, and the expense
of assembling and flying a negotiating team to a far-away locale. Even
simply expediting or delaying the transportation of imports is likely to be
more costly than expediting or delaying domestic goods shipments because

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⁴ For a brief introduction to this debate and its major protagonists, see
Sylvia Nasar, "Fortune Forecast: Is the Dollar Too High--Or Too Low?"

⁵ See Junz and Rhomberg (1973), Magee (1973, 1974), Clark (1974), Kravis
and Lipsey (1978), Hooper and Kohlhagen (1978), Artus and Young (1979),
McPeters and Stronge (1979), and Witte (1980), to name but a few.
transport costs are a larger fraction of import prices. Due to all of these added costs, foreign firms will generally find it more difficult to adapt to changing market conditions than their domestic competitors.

The standard procedure in most empirical studies and macro models has been to include a lagged dependent variable, distributed lags of the independent variables, or both, when estimating export and import equations. The coefficient estimates on these lagged variables have usually been large and significant. However, if one takes the costly adjustment hypothesis seriously as an explanation for slow trade adjustment, then economic theory also indicates that expected future market conditions should be important factors in the determination of current trade flows. That is, a permanent improvement in the terms of trade should have a larger immediate impact than a temporary improvement. Despite the apparent importance of adjustment costs, no one has yet developed a formal model of the optimal behavior of individual traders when trade adjustment is costly, or examined the empirical implications of such a model.  

The first section of this paper develops a model of international trade flows based on a representative trading agent who maximizes a discounted sum of expected future operating profits. This model assumes quadratic costs of trade adjustment, imperfect competition, and rational expectations. Section II discusses econometric and empirical issues, and Section III estimates the model using data on bilateral trade between the U.S. and three of its largest trading partners—Japan, West Germany, and the United Kingdom. Section III also conducts various specification tests of the model. The restrictions implied by perfect competition are strongly rejected in favor of

6. Kollintzas and Husted (1984) construct a theoretical model with inventory and transportation costs that leads to a distributed lag specification for trade flows. They do not contemplate costly adjustment in the level of trade, however.
imperfect competition. The restrictions implied by rational expectations cannot be rejected. The importance of adjustment costs is confirmed.

Section IV presents the trade elasticities implied by the estimation results. The price elasticities of trade flows are in the range reported by other researchers, but they are sometimes insignificantly different from zero. On the other hand, the activity elasticities are substantially higher than most previously reported estimates, and they are quite significant.

I. The Representative Trader

Suppose $X$ represents the quantity of country 1 goods purchased by an importer based in country 2. Let $P_1$ be the country 1 price of the imported goods converted to country 2 currency at the market exchange rate. Let $P_2$ be the sales price of the imports in country 2, and $P_D$ the domestic price level in country 2. If the importer is risk neutral, his maximization problem at time $t$ can be expressed in real terms as:

$$
\underset{(X)}{\text{Max}} \ E_{t-1} \sum_{i=0}^{\infty} \theta^i \left[ \left( \frac{P_{2t+i}}{P_{Dt+i}} \right)^X_{t+i} \left( \frac{P_{1t+i}}{P_{Dt+i}} \right)^X_{t+i} \right] - d_{t+i} e^{\left( X_{t+i} - X_{t+i-1} \right)^2}.
$$

The transportation cost is a linearly increasing function of the volume of trade given by the parameter $d$. The adjustment and recontracting cost is modeled as quadratic in the first difference of the trade flow with parameter $e$. If trade flows do not change from one period to the next, there

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7. The following model analyses the decision to export or import separately from production and distribution decisions. The paradigm is that of the trader-arbitrager who purchases goods on the wholesale market in one country and sells them immediately after transport to wholesalers in another country.

8. Quadratic costs are an analytically tractable form of convex cost. Convex costs capture the notion that a given amount of adjustment is more costly the quicker it is undertaken.
will still be a transportation cost, but no adjustment or recontracting cost. The real discount factor, $\theta$, is assumed constant.

If there are no information or gestation lags, then the expectation operator, $E(\cdot)$, is conditional on period $t$ information. However, if period $t$ imports are contracted for in a previous period or there are communication lags, then the expectation operator may be conditional on period $t-1$ or $t-2$ information. Given the presumed importance of adjustment costs and lags, it seems reasonable to assume that the decision to import is based on data from the previous quarter. The next section presents evidence to support the hypothesis that period $t$ trade is determined conditional on period $t-1$ information. (See footnote 17.)

If the domestic market for the imported good is not perfectly competitive, then the trader will not be able to set $X$ independently of $P_2$. He will instead face a downward-sloping demand curve:

$$P_{2t} = aP_{Dt} - bP_{Dt}X_t + cP_{Dt}Y_t.$$  

(2)

Here $Y$ represents aggregate demand in the domestic market. By placing $P_D$ on the right-hand-side, equation (2) assumes that imports are substitutes for domestic products in the aggregate. The case of perfect competition is nested in equation (2); it implies that $a=1$ and $b=c=0$.

If country 2 imports are large relative to country 1 production, then one would expect to find an upward-sloping supply function in terms of $P_1$ analogus to the downward-sloping demand function in terms of $P_2$. For simplicity, the following model is developed under the assumption that the purchase price of the imported good is fixed at the country 1 price level.\(^9\)

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9. This approximation is valid when the market for each good is largest in the country where it is produced and trade is only a fringe activity. One test of this assumption is to include aggregate demand in country 1 as an explanatory variable for country 2 imports from country 1. In practice this term was generally insignificant and often had the wrong sign.
The importer's maximization problem can now be written:

$$\text{Max } E_{t-1} \sum_{i=0}^{\infty} \theta^i \left[ (a-bX_{t+i} + cY_{t+i} - P_{t+i} - d)X_{t+i} - e(X_{t+i} - X_{t+i-1})^2 \right]. \quad (3)$$

Here $P$ is the ratio of the foreign price level to the domestic price level.

Equation (3) can be solved using standard dynamic programming techniques to yield:

$$X_t = \Gamma + \alpha X_{t-1} + E_{t-1} \sum_{i=0}^{\infty} \theta^i \left[ \delta P_{t+i} + \gamma Y_{t+i} \right]. \quad (4)$$

It can be shown that $\alpha$ is defined implicitly by:

$$1 - [(b/e + 1 + \theta)/\theta]L + (1/\theta)L^2 = (1-\alpha L)(1-(\alpha \theta)^{-1}L) . \quad (5A)$$

The remaining coefficients of (4) can also be expressed in terms of the original parameters:

$$\delta = -\alpha/2e, \quad \gamma = \alpha c/2e, \quad \Gamma = \alpha(a-d)/(1-\alpha\theta)^2e. \quad (5B)$$

Equations (5) demonstrate that the transportation cost, $d$, cannot be identified separately from the constant term in the demand curve, $a$. Imports will respond positively to domestic price and aggregate demand. The speed of trade adjustment depends on the cost of adjustment, the discount factor, and the slope of the demand curve.

II. Econometric Issues

The primary econometric issues in estimating an import supply function like (4) are that the expectations are not directly observable and there are other causal variables such as government trade policies and consumer tastes for imports. Furthermore, equation (4) is derived from the individual

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10. The transversality condition for (4) to be an optimum requires that the sequences of future relative price, trade flow, and aggregate demand be of exponential order less than the square root of the reciprocal of the discount factor.
trader's optimization problem, yet the objective of this paper is to describe the behavior of aggregate trade flows. Some care must therefore be taken in choosing and transforming the data to be fitted to equation (4).

The trade flow data used in this paper are real merchandise exports from the United States to the United Kingdom, West Germany, and Japan, as well as merchandise exports from each of these three countries to the United States. In 1981, these three countries were among America's five largest trading partners. (The other two were Canada and Mexico.) Canadian and Mexican exports are not analysed here because the model of the previous section is more theoretically relevant for manufactures than for commodities, and Canadian and Mexican exports are dominated by commodities.

The index of market demand is taken as total private consumption and aggregate investment. The price ratio is the ratio of GNP deflators after converting the foreign country's price level into domestic currency using

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11. Since bilateral trade is measured in nominal terms only, it is necessary to use a multilateral price deflator in order to convert the data into real trade flows. This paper uses exports instead of imports because most countries' exports are more specialized than their imports, and the composition of exports varies less across trading partners. This property minimizes the error introduced in the conversion to real trade flows.

12. Commodities tend to be traded in auction-type markets, which lessen the importance of contracting and other adjustment costs. Furthermore, both demand and supply of commodities are inelastic, leading to erratic price movements that are not well-captured by aggregate price indices. Finally, even manufactured goods trade among the United States, Canada, and Mexico is likely to be characterized by insignificant costs of adjustment compared with overseas trade due to the high degree of integration of the North American market.

13. Government consumption is excluded because it consists chiefly of labor services and non-imported defence goods. If imports are primarily finished goods, then aggregate private demand is an appropriate index of market demand for importables. GNP is a better measure of market demand for importables if imports are basic inputs into the production process. Some trial estimations were conducted using GNP, and the results were generally similar. The most significant difference is that the estimated activity elasticities are even larger when GNP is used in place of aggregate demand.
the contemporaneous exchange rate. The GNP deflator captures the overall price of each country's output.

One salient characteristic of aggregate trade and demand is that both series grow exponentially over time. In order to convert the aggregate data into the relevant supply and demand quantities faced by the representative trader, it is appropriate to divide by the total number of traders. Unfortunately, there are no reliable data on the true number of economic decision agents in bilateral trade. Moreover, technological growth is increasing the value of goods that can be traded with given transportation and adjustment costs. In order to recover the agent's objective function, the aggregate flows must also be scaled by this technological growth.

If one assumes that the representative agent's real profit flow is stationary and that all the growth in trade volumes is due to smoothly growing population and technology, then the representative agent's quantity variables can be recaptured, up to a constant multiplicative factor, by the ratio of actual to trend trade volume.\(^4\),\(^5\)

Stationary omitted variables are modelled by allowing for an autocorrelated disturbance term. The basic model is estimated both with and without autocorrelated omitted variables.

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\(^{14}\). The estimation in this paper fits a deterministic exponential trend to aggregate trade and demand. To the extent that population and technology are not smoothly growing over time, it would be appropriate to model trade and demand as first-difference-stationary and conduct stochastic detrending. However, stochastic detrending would greatly increase the computational intensity of the estimation. Moreover, Dickey-Fuller tests can reject a unit root in three of the six trade series at the 10 percent level, thus lending mild support to a trend-stationary representation.

\(^{15}\). The normalization by an unknown factor of the data used for estimation implies that, even though optimization problem (3) is the correct functional form, the estimated parameters can no longer be interpreted as the true representative agent's parameters.
The final issue is the identification of the traders' expectations of future relative price and aggregate demand. The estimation in this paper employs the assumption of rational expectations. The forcing variables are hypothesized to follow autoregressions, which are estimated simultaneously with the trade flow equations. Expectations of future variables in the trade flow equations are exactly those implied by the estimated autoregressive parameters of these variables.¹⁶

When specifying the forcing variable processes, it is important to include lags of all variables that Granger-cause (i.e. help to predict) each forcing variable, as long as the traders are assumed to have had access to this information. The working hypothesis of the following estimation is that exports do not Granger-cause relative prices and aggregate demand. This hypothesis allows one to model the relative price and aggregate demand series as a vector autoregression that does not include exports, and it simplifies the imposition of the cross-equation restrictions on trade that are required by the hypothesis of rational expectations.

In order to test the hypothesis that trade flows do not help to predict the other variables, vector causality tests were conducted for each of the two-country models. The null hypothesis is that the vector of relative prices \( P \), U.S. demand, \( U_Y \), and foreign demand, \( F_Y \), is not caused by the vector of U.S. exports, \( U_X \), and foreign exports, \( F_X \).¹⁷ This hypothesis could not be rejected at the 5 percent level for any of the three models.

¹⁶. This paper thus employs a strong form of rational expectations. Agents are assumed to know the structure and parameters of the model throughout the estimation period, even though the econometrician cannot estimate these parameters until after the data has been collected. The alternative of using rolling regressions to proxy for expectations is prohibitively expensive in the maximum likelihood framework used here.

¹⁷. The test uses four lags of each variable and is based on quarterly data from 1973:1 to 1985:4. Nominal exports are from the IMF's Direction of Trade Statistics. The remaining data are from national sources.
(U.S.-Germany, U.S.-Japan, and U.S.-U.K.). The opposite test for Granger-causality of P, UY and FY on UX and FX does find significant evidence of causality in two of the three models. These findings are also upheld by a battery of single-equation tests on Granger-causality for each variable individually.

The data also present little evidence of causality among P, UY, and FY. Indeed, it would be surprising if this were not so, since trade is the most likely transmission mechanism for one country’s demand to affect another’s, or for relative prices to affect demand. A joint test of interaction among P, UY, and FY was conducted by estimating an unrestricted vector autoregression of the three variables and testing the significance of restricting all the off-diagonal coefficients to zero. The test statistic is significant at the 10 percent level for only one of the three models.

Of course, most economists would describe all of these variables as endogenous and interrelated in the context of a well-specified structural macro model. In particular, there are good reasons to expect that trade flows do affect domestic demand and relative prices. Some justification for the causal ordering of this estimation is provided by noting that there are many factors influencing exchange rates, prices, and aggregate demand. The effect of trade flows on these variables may be overwhelmed by the effects

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18. The likelihood ratio tests in this paper use a small-sample correction that has been proposed by Whittle (1953) and employed by Sims (1980). The correction reduces the likelihood ratio statistic by the factor \((T-p/q)/T\), where \(T\) is the number of observations, \(p\) is the number of parameters estimated by the unrestricted model, and \(q\) is the number of equations in the system.

19. The contemporaneous effect of P, UY and FY on UX and FX was also insignificant. This finding lends support to the assumption that traders' expectations are based on period t-1 data.

20. There was significant evidence of contemporaneous correlation between these variables, however.
of monetary and fiscal policy and the animal spirits of businessmen. On the other hand, prices and demand may be the primary determinants of trade. By analyzing trade with each partner country separately one hopes to dilute further the effect of exports on aggregate demand and prices without diluting the effect of demand and prices on exports.

III. Estimation and Testing

There are three separate bilateral models, one for each of Germany, Japan, and the United Kingdom. Each model is estimated with quarterly data, not seasonally adjusted, from 1973:2 to 1985:4. The sample period does not extend earlier because of the regime shift associated with the breakdown of Bretton Woods in 1971. All the data are expressed in logarithms. 21

The P process is modeled as a second-order autoregression with seasonal dummies. The UY and FY series are modeled as third-order autoregressive processes about trend with seasonal dummies. The structural equation (4) for UX and FX is fitted using the ratio of actual to trend data, where the trend includes the seasonal effect. The expectations of future P, UY, and FY in equation (4) are exactly the values implied by their estimated autoregressions divided by the estimated trend and seasonal components.

Table 1 presents maximum likelihood estimates of the five variable system for bilateral trade between the United States and Germany, Japan, and the United Kingdom. 22 The $R^2$ statistic is quite high for each equation.

21. The use of logarithms is justified by two properties: first, an exponential growth trend is linear in logarithms, and second, $x=\log(x)+1$ for $x$ close to 1 (as is the case with the ratio data used here).

22. The multivariate normal log likelihood function was maximized using the DFP method in GQOPT. A very limited number of trials at different starting values of the parameters always converged to the same optimum. In the neighborhood of the optimum, use of the Gradx method did not lead to noticeably different parameter values, but the Gradx standard errors were much smaller than the DFP standard errors.
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Note: Rho(1) and rho(4) are the first and fourth order autocorrelations of the estimated residuals. They are not parameters of the model.
First- through fifth-order autocorrelations of the residuals never exceeded 0.5, and 85 percent of these autocorrelations were in the range -0.2 to 0.2. The discount factor $\theta$ was fixed at 0.98 for all of the estimations in this paper.\(^{23}\) In order to simplify the presentation of Table 1, the trend and seasonal estimates are omitted.

A number of specification tests have been performed on the models in Table 1. Because of the computational intensity of the estimation, several of the tests employ the Wald or Rao lagrange multiplier statistics rather than the likelihood ratio statistic.\(^{24}\) The remainder of this section summarizes the results of each test briefly. A complete listing of the test statistics is available from the author upon request.

Various alternative hypotheses have been compared to the basic model. In order to maximize the power of each test and conserve the total degrees of freedom, each alternative model either relaxes one specific set of restrictions implied by the basic model or imposes one specific set of restrictions onto the basic model. The first test examines the restriction $\theta=0.98$. The likelihood function appears to have a very gentle slope in the $\theta$-dimension, so that the restriction is never rejected at the 10 percent level.

Several tests were conducted to examine the dynamic structure of the forcing variables. The restriction that the relative price has no trend is rejected by only one of the three models. Reducing the price dynamics to a first order autoregression is strongly rejected by all three models. However, allowing a third order process in relative prices is not necessary for

\(^{23}\) When $\theta$ is free it sometimes tends to drift below zero or above one. Other researchers, including Sargent (1978), have been compelled to restrict the discount factor to some a priori value when conducting similar estimation. Since the forcing variables are characterized by a high degree of autocorrelation, there is very little information available in a small sample to distinguish between different weights on the future values.

\(^{24}\) See Amemiya (1985) Chapter 4.
any of the models. None of the UY and FY processes can reject the restriction from fourth to third order. The UY process and two of the FY processes do reject the restriction to second order. Overall, the estimated forcing variable structure appears satisfactory.

One major concern with the UX and FX processes is that the trade equation developed in the previous sections may not include every variable of interest to the traders. One may also think of an omitted variable as a disturbance in the demand curve—equation (2)—for imports. To model the effect of omitted variables or taste shifts, rewrite the importer's maximization problem to include a random variable, \( u \), which is observable only to the importer:

\[
\max \ E_{t-1} \sum_{i=0}^{\infty} \theta^i \left[ (a + d - b x_{t+i} + c y_{t+i} - p_{t+i} + u_{t+i}) x_{t+i} - e \left( x_{t+i} - x_{t+i-1} \right)^2 \right].
\]

(6)

The solution for \( x \) in period \( t \) is

\[
x_t = \Gamma + a x_{t-1} + E_{t-1} \sum_{i=0}^{\infty} (a \theta)^i \left[ \delta p_{t+i} + \gamma y_{t+i} - \delta u_{t+i} \right].
\]

(7)

If \( u_j \) is independently and identically distributed for all \( j > 0 \), then \( E_{t-1} u_{t+i} = 0 \) for all \( i \geq 0 \) and (7) reduces to (4). In general, the more likely case involves autocorrelated \( u \)'s. Most economic time series are positively autocorrelated, and \( u \) is intended to proxy for an unobserved time series.

When the disturbance \( u \) in (7) follows a first-order autoregression, a simple transformation is necessary to create an equation with white noise residuals. Multiply both sides of the equation by \( \rho L \), where \( L \) is the lag operator and \( \rho \) is the autoregressive coefficient of \( u \) (i.e., \( u_t = \rho u_{t-1} + \epsilon_t \)). Subtract the transformed equation from the original to obtain:

25. Consistent estimation of (8) requires that the lagged unobservables innovation, \( \epsilon \), be uncorrelated with lagged relative prices and aggregate demand. Otherwise, the coefficients on price and demand will be biased.
\[ X_t = (1-\rho)\Gamma + (\alpha+\rho)X_{t-1} + \alpha\rho X_{t-2} + E_t \Sigma_{i=0}^{\infty} (\alpha \theta)^i \delta \left( \frac{\epsilon_t}{P_{t+1}} \right) + \rho E_{t-2} Y_{t+1} + \gamma [Y_{t+1} - \rho E_{t-2} Y_{t+1} - 1] + \epsilon_{t-1}. \] (8)

Estimation of the model with autocorrelated trade residuals—equation (8)—yields negative values of \( \rho \) for every model. Negative autocorrelation coefficients are highly implausible if \( u \) is proxying for unobserved time series. Fortunately, the improvement in the likelihood function associated with the addition of these two new parameters is not great. A likelihood ratio test cannot reject the restriction \( \rho = 0 \) at the 10 percent level for any model.

The final few tests deal with some of the more fundamental assumptions of the model. The first assumption is that it is costly to adjust export flows. If export adjustment were costless, then the coefficient on lagged exports would be zero and each quarter's exports would depend only on the best prediction of that quarter's relative prices and demand. To see this result, rewrite the importer's objective function with no adjustment costs:

\[ \max_{(X)} E_{t-1} \sum_{i=0}^{\infty} \theta^i \left[ \left( a - d - b X_{t+i} + c Y_{t+i} - P_{t+i} \right) X_{t+i} \right]. \] (9)

The problem is now time separable, and the solution is

\[ X_t = E_{t-1} \left( a - d + c Y_t - P_t \right) / 2b + \epsilon_t. \] (10)

Expected future prices and demand are irrelevant. These zero restrictions are rejected at the 1 percent level for every model, however.

An alternative test of the significance of adjustment costs keeps the above zero restrictions, but allows an autocorrelated disturbance consistent with omitted variables. The resulting trade equation is

\[ X_t = T + \rho X_{t-1} + E_{t-1} \left[ c Y_t - \rho E_{t-2} c Y_{t-1} - P_t + \rho E_{t-2} P_{t-1} \right] / 2b + \epsilon_{t-1}. \] (11)

Neither equation (11) nor equation (4) is nested in the other, and they both use the same number of degrees of freedom. When the whole system is reestimated using (11) instead of (4) the likelihood value drops in every model,
sometimes greatly. This result suggests that lags in trade flows arise primarily from adjustment costs and not from autocorrelated taste shocks or other variables besides aggregate demand and relative price.

A final test of the adjustment cost specification concerns the hypothesis that only one lag of trade enters the trader's decision rule. A more general formulation of the adjustment process might allow two or more lags of trade into equation (7). However, the restriction of a zero coefficient on a second lag of trade is rejected at the ten percent level by only one bilateral model.

A second fundamental assumption is that of imperfect competition (versus perfect competition). The model with perfect competition implies coefficients of unity on lagged trade and zero on aggregate demand. These restrictions are rejected extremely strongly, both individually and when they are tested jointly.

One possible reason for the rejection of a unit coefficient on lagged trade is that the system under estimation is essentially a partial equilibrium framework. The real world almost certainly has mechanisms by which continuing trade imbalances can influence relative prices and aggregate demand and thus damp themselves down over time. In the case of perfect competition these mechanisms are the only way to keep trade flows from behaving like random walks.

If tradable goods markets are characterized by perfect competition and trade does not follow a random walk, then the data should provide evidence of these equilibrating forces at work. Yet, Section II found no evidence of

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26. Eichenbaum (1983) constructs a costly adjustment, perfectly competitive model in which both market demand and price help to predict an individual firm's output. Demand enters only through its ability to predict future price, however. The Granger tests of Section II found that demand does not help to predict future price in this case.
causality running from trade to aggregate demand or relative prices. Such a finding is consistent with imperfect competition, however. The model with imperfect competition implies a tendency for trade to return to trend without relying on outside equilibrating forces. All that is required is that the forcing variables are stationary.

The final test concerns the assumption of rational expectations. If market expectations are not equal to those generated by the model, or if agents are not forward-looking, then the lags of price and demand may affect trade flows directly rather than simply as predictors of future price and demand. In this case the unrestricted model calls for separate coefficients on each lag of price and demand in the trade equation, for a total of five coefficients. The restricted model (presented in Table 1) uses the estimated autoregressive parameters from the price and demand equations to compute expected discounted price and demand, thereby estimating only two new coefficients in the trade equation. A likelihood ratio test cannot reject these rational expectations restrictions at the 10 percent level for any model.

Altogether then, the basic model stands up quite well to a wide range of specification tests.

IV. Interpretation of Results

As discussed in Section II, the use of aggregate data prevents the identification of the true parameters of the representative trader's optimization problem. However, since the data were transformed to ratios with a mean value of unity, the parameters b and c in equation (2) may be interpreted as elasticities of the import price with respect to the quantity of
imports and domestic demand, respectively. The first part of Table 2 presents the implied values of b and c using equation (5) and the estimates in Table 1.  

For U.S. exports to the United Kingdom and for German and Japanese exports to the United States, the estimates of b and c are almost identical across models. The standard errors on the remaining sets of estimates are generally much higher, and they are never significantly different from the estimates with smaller standard errors. These results suggest that import prices are much more sensitive to domestic demand than to import supply.

The elasticities of bilateral trade flows with respect to prices and aggregate demand are easily computed. The short-run elasticity with respect to an anticipated one-period jump in P is simply the coefficient δ. The short-run elasticity with respect to a permanent jump in P is δ/(1-αθ).  

The long-run elasticity with respect to a permanent jump in P is equal to δ/[(1-αθ)(1-α)]. The formulas for computing the elasticities with respect to Y are similar to those for P, except that γ is substituted for δ. The long-run elasticity estimates are presented in the second part of Table 2 next to the heading "Expectations". Table 2 also presents the elasticities implied by an unrestricted reduced form regression of trade on lagged prices.

27. The standard errors associated with all of the parameters and elasticities in Table 2 are computed using the estimated covariance matrix of the coefficients in Table 1 and a second-order Taylor approximation to the true standard errors based on the mapping from the coefficients of Table 1 to the parameters of interest in Table 2.

28. Recall that future prices are discounted at the rate αθ in equation (4). It is important to recognize that these trade elasticities capture the effect of a hypothetical suspension of the estimated forcing variable processes. An alternative approach is to plot the response of trade over time to an innovation in one of the forcing variables.
Table 2  
Parameters and Elasticities

U.S.-Germany  U.S.-Japan  U.S.-U.K.

Model Parameters

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Long-Run Elasticities

|----------|-------|-----------|-------|-----------|-------|-----------|
| Relative Price
| UX      | Expectations  | -0.19 | 0.15   | -0.33 | 0.30   | -1.78 | 0.80   |
|         | Unrestricted  | -0.20 | 0.11   | -0.13 | 0.14   | -1.14 | 0.32   |
| FX      | Expectations  | -1.10 | 0.21   | -1.21 | 0.31   | -0.60 | 0.24   |
|         | Unrestricted  | -1.04 | 0.15   | -0.78 | 0.18   | -0.31 | 0.19   |
| Aggregate Demand
| UX      | Expectations  | 2.61  | 0.68   | 5.33  | 1.35   | 8.14  | 3.90   |
|         | Unrestricted  | 2.06  | 0.41   | 3.86  | 0.55   | 3.67  | 1.56   |
| FX      | Expectations  | 4.42  | 1.04   | 5.92  | 1.59   | 1.16  | 0.99   |
|         | Unrestricted  | 3.47  | 0.63   | 1.50  | 0.60   | 0.84  | 0.85   |
and demands, as is typical in the literature. These estimates are presented next to the heading "Unrestricted".

A striking characteristic of Table 2 is that unrestricted reduced form estimation using this data implies consistently smaller estimates of the long-run price and demand elasticities in trade. Although the difference between the two sets of estimates is not statistically significant, it is often large in economic terms. This result is especially notable because the previous section of this paper shows that these restrictions cannot be rejected in the context of multivariate model estimation. If the theoretical model of Section I is a good approximation to the typical trader's environment, then imposing the model's structure onto the estimation process will enable one to obtain more accurate estimates of the parameters of interest. Because they extract information from the data more efficiently, the elasticities labeled "Expectations" are preferable to those labeled "Unrestricted".

29. One remaining difference between these unrestricted elasticities and most previous research is that trade and demand are allowed to have separate growth trends in the models reported here. Many researchers have implicitly assumed that growth in trade is dependent on growth in demand (or income) and that improving transportation and communication technology cannot lead to trade growth when aggregate demand is stagnant. Taking the separate trend out of the trade equations is rejected for all three models used here. (Because of the spurious regressions problem when both regressor and regressand are growing, the true significance level of these tests is higher than the nominal level. It is somewhat reassuring, however, that the nominal level was below 1 percent.)

30. Indeed, if the restricted estimates were more than two standard deviations away from the unrestricted estimates, the likelihood ratio test would probably have rejected the rational expectations restrictions. In addition, one must be cautious in comparing the standard errors attached to the two sets of estimates because the associated elasticities are conceptually different. The unrestricted elasticity estimates are not explicitly forward-looking; they simply sum the coefficients on lagged price or demand and divide by \((1-\alpha)\). This procedure leads to a very different second-order Taylor approximation when computing standard errors. The standard errors on the directly estimated parameters (shown in Table 1) are almost uniformly larger when the rational expectations restrictions are relaxed.
V. Conclusion

Previous research on trade flows has generally sought to estimate reduced forms directly, as if they were structural models. A drawback to estimating unrestricted reduced forms is that the dynamic response of trade flows to changing market conditions is inextricably meshed with the dynamics of the market conditions.\(^{31}\) By imposing valid theoretical restrictions and estimating the model in a full information framework, one should be able to disentangle the forcing variable dynamics from the trader's decision rule, and therefore obtain more accurate estimates of the lags and elasticities in international trade.

This paper estimates a model of aggregate international trade flows based on an infinitely-lived representative trader assumption. The model is characterized by quadratic adjustment costs, imperfect competition, and rational expectations. The data generally support all of these hypotheses. The alternative hypotheses of perfect competition and costless adjustment are strongly rejected.

Table 3 presents the long-run trade elasticities of this paper for comparison with two other sets of elasticities estimated at the Federal Reserve Board.\(^{32}\) The price elasticities of trade estimated here are generally in the low range of the MCM and Marquez estimates. The activity elasticities, however, are much larger than the MCM and Marquez estimates.

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31. For example, if government policy alters the behavior of prices and demands, a rational trader should adapt to these changes optimally. This reaction by the trader will generally change the observed relationship between trade flows and lagged prices and demands.

32. For a description of the data and methodology used to estimate the MCM and Marquez elasticities, see Edison, Marquez, and Tryon (1987) and Marquez (1988). For other estimated elasticities see Houthakker and Magee (1969), Artus and Young (1979), and McPheters and Stronge (1979).
Table 3
Long-Run Elasticity Comparisons

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Note: The MCM elasticities are taken from Edison, Marquez, and Tryon (1987). The Marquez elasticities are from Marquez (1988). Standard errors are presented below each estimate in parentheses. There are no standard errors available for the MCM price elasticities.
The coefficients on lagged trade (not shown) estimated by Marquez average 0.51, which is very close to the average of 0.55 in this paper.\footnote{33}

In light of the continuing strength of U.S. aggregate demand and the weakness of foreign aggregate demand through the end of 1987, the estimation results of this paper are consistent with the growth of the U.S. trade deficit in 1986 and 1987. This research does not support the argument that lags in trade adjustment can explain the small response of the U.S. trade deficit to the marked depreciation of the dollar. The main culprit appears to be the very large effects of aggregate demand, coupled with moderately low price elasticities.

\footnote{The MCM does not include lagged trade in its regressions. The speed of adjustment in all three models also depends on the distribution of lag coefficients on the independent variables. Both the MCM and Marquez models have longer lags on relative price and demand than the model of this paper.}
REFERENCES


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