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ABSTRACT

According to conventional wisdom, if deficits are inflationary then current deficits should predict subsequent movements in money growth. This paper uses a general equilibrium model fit to data to: (1) explore the policy behavior underlying this accepted viewpoint; (2) examine alternative equilibrium deficit policies ranging from an exclusive reliance on direct lump-sum taxes to a mix of direct and inflation taxes; and (3) evaluate the empirical trade-offs implied by the various financing schemes. The results suggest that reduced-form analyses of whether "deficits matter" can lead to seriously misleading conclusions by mistakenly attributing fiscal effects to monetary policy.

I demonstrate that simple monetary and tax policy rules and plausible assumptions about when private agents learn of fiscal actions can produce a classical economy whose nominal equilibrium depends on the process for lump-sum taxes and whose time series contradict the view that monetized deficits predict inflation. I assess the fit of versions of the model to U.S. data and reinterpret existing reduced-form studies in light of the results.

Policy Rules, Information, and Fiscal Effects in a "Ricardian" Model

Eric M. Leeper*

1. Introduction

"[I]f there is no predictive content of current deficits [for subsequent movements in money creation], then there can be no rational expectational link from deficits to interest rates and inflation." King and Plosser's (1985, p.148) assertion encapsulates accepted wisdom about the connections between fiscal policy and inflation and has been applied to justify existing reduced-form analyses of whether "deficits matter." This paper: (1) explores the policy behavior underlying this accepted viewpoint; (2) examines alternative equilibrium deficit policies ranging from an exclusive reliance on direct lump-sum taxes to a mix of direct and inflation taxes; and (3) evaluates the empirical trade-offs implied by the various financing schemes. The results suggest that reduced-form regressions may lead to seriously misleading conclusions about the effects of fiscal policy by mistakenly attributing fiscal effects to monetary policy.

I demonstrate that simple monetary and tax rules and plausible assumptions about when private agents learn of fiscal actions can produce a classical economy whose nominal equilibrium depends on taxes but whose time series contradict the view that monetized deficits predict future inflation. Deficit shocks may be fully monetized yet deficit realizations will not predict current or future inflation (or money growth). I construct a general equi-

*The author is a staff economist in the International Finance Division. This paper represents the views of the author and should not be interpreted as reflecting those of the Board of Governors of the Federal Reserve System or other members of its staff. The author's Ph.D. dissertation at the University of Minnesota forms the basis for this paper. I thank Chris Sims for many discussions and insightful suggestions at the early stages of this research. The paper has also benefited from the comments of John Coleman, Mark Gertler, Ross Levine, and especially David Gordon and Dale Henderson.

librium model whose parameters are chosen by an informal method of moments procedure to produce simulations that mimic some of the correlations implied by a six-variable vector autoregression for post-World War II U.S. time series. The regression of inflation on current and past deficits that emerges from U.S. data can be reproduced by an equilibrium in which the pricing function depends on taxes and private agents are given foreknowledge of fiscal variables.

The model is a stochastic growth economy with infinitely lived consumers that is constructed to ensure a classical dichotomy. Deficit shocks do not influence nominal outcomes through real interest rates or other departures from a competitive Ricardian environment. I derive analytical results for a linearized version of the model and generate simulations for the nonlinear structure. The simulations are produced using Sims's (1989a) "backsolving" algorithm, which is employed to find a solution consistent with the nonlinear Euler equations and budget constraints.

The role of taxes in determining equilibrium prices hinges on assumptions about monetary and fiscal behavior. Policy authorities obey rules specifying feedback from observable endogenous variables to policy variables. When the space of feedback parameters is divided into four disjoint regions, existence and uniqueness of monetary equilibria depend on which one of the regions the monetary and tax parameters are drawn from. After establishing for the linearized model that two of the regions imply unique equilibria, I characterize the pricing functions produced by parameters from these two regions.

Although all the model's equilibria rely on some degree of direct lump-sum taxation, policy behavior in the two regions is differentiated by the

extent to which deficit innovations are monetized. In the first region no monetization takes place so deficits are financed entirely by direct lump-sum taxes. Consequently, the equilibrium process for prices is unrelated to the process for taxes. The second region requires that some fraction of deficit shocks be monetized, with the degree of monetization indexed by the responsiveness of taxes to lagged debt. Given the extent of monetization required by fiscal behavior, the monetary authority's sensitivity to inflation determines whether policy resorts to distorting inflation taxes or one-time changes in the price level to maintain government intertemporal budget balance.

Distorting inflation taxes spread the monetization over time and reproduce the conventional view expounded by King and Plosser (1985). In equilibrium, deficits predict current and future money growth, nominal interest rates, and inflation. When deficit shocks induce a one-time jump in prices, however, all monetization of current and expected future deficits occurs immediately and the dynamic correlations between deficits and money creation are forced into a contemporaneous correlation alone.

To eliminate this contemporary correlation an alternative assumption is made about when private agents learn of fiscal variables. King and Plosser's (1985) perspective comes out of the usual rational expectations environment in which private agents make decisions using observations on only current and past policy variables. In this case private agents and the econometrician have equivalent information sets.

In actual economies fiscal decisions are debated at length, usually publicly, before any legislation is enacted. Throughout the legislative debate news-conscious taxpayers may alter their behavior in light of expected future tax changes. I model the public debate by giving private agents today

exact knowledge of the realizations of fiscal variables tomorrow. Now private agents have strictly more information than the econometrician who attempts to infer from observable time series whether deficits are monetized.

Combining immediate monetization with advanced knowledge of fiscal variables eliminates any tendency for deficits to lead money growth. In this case the empirical tests of reduced-form regressions employed by Dwyer (1982), Joines (1985), and King and Plosser (1985) will fail to uncover evidence of deficits causing inflation even though deficit shocks are monetized completely. This outcome breaks down, of course, when the monetization is postponed.

The conventional view has been influenced by Sargent and Wallace's (1981) illustration of an economy in which fiscal deficits are exogenous and the money stock must expand with deficit shocks to ensure the government's intertemporal budget constraint is satisfied. In their setup higher current deficits require higher money growth eventually, supporting the quotation from King and Plosser (1985).

The present work builds on Aiyagari and Gertler's (1985) insight that tax realizations may affect prices even though they do not affect allocations. The price effects depend on how monetary and tax policies interact to ensure the government's intertemporal budget is balanced. Sims (1988) elaborates on this theme in a partial equilibrium setup by modeling policy interactions in terms of explicit policy rules. I blend Sims's approach to modeling policy rules with a variant of Aiyagari and Gertler's general equilibrium framework to study the equilibrium of a monetary growth economy.

Section 2 lays out the model. Section 3 analytically derives the properties of a linearized version of this model and employs phase diagrams to

characterize the dependence of the nominal equilibria on policy parameters. The correlations that appear in U.S. data, which are used to judge the model's fit, are discussed in Section 4. Section 5 presents simulations from two qualitatively different sets of policy rules assuming agents observe only current and past fiscal variables. The next section explores a simulation in which agents have advanced knowledge of realizations of taxes and spending and where policy authorities immediately monetize current and expected future deficits. Both Sections 5 and 6 assess the fit of simulated data to U.S. time series. Section 7 contains concluding remarks.

2. The Model

The equilibria come from a stochastic monetary growth model that is stationary. A representative consumer lives forever, acts competitively, and discounts utility at a positive rate. The unified policy authority is a large agent that chooses a mixture of direct taxes, inflation taxes, and debt obligations to finance an uncertain stream of government expenditures according to a feasibility constraint. Three assets circulate in this economy: one real asset — capital — and two nominal assets — interest-bearing government debt and non-interest-bearing fiat currency.

The per capita capital stock at time $t-1$, k_{t-1} , produces consumption goods with a technology subject to diminishing returns and a net depreciation rate of $(1-\delta)$ each period. Distinct exogenous shocks allow the marginal and average products of capital to fluctuate randomly and possibly independently. Total output equals production plus the time t realization of a mean zero additive shock, θ_{2t} , so output in period t is:

$$y_t = \theta_{1t}f(k_{t-1}) + \theta_{2t}$$

where $f(k_{t-1}) = Ak_{t-1}^\nu$ with $A > 0$ and $0 < \nu < 1$, and θ_{1t} is an exogenous productivity shock that is always positive.

Preferences are separable at each date in consumption and real balances and utility is subject to a random shock:

$$u(c_t, m_t; \theta_{3t}) = \theta_{3t}[\varphi \log(c_t) + (1-\varphi)\log(m_t)]$$

where c_t is real per capita consumption at t and $m_t = M_t/p_t$ is real per capita cash balances at t ; M_t is the nominal money stock and p_t , the price level, is the price of consumption goods in terms of currency. Assume that $0 < \varphi < 1$ and $\{\theta_{3t}\}$ is a stationary stochastic process that is positive with probability one.

Private agents own the technology and receive the total output each period. At each date t they pay real lump-sum taxes of τ_t and choose consumption, capital, and savings in the form of money balances and one-period government bonds. Nominal bonds, B_t , earn a gross sure nominal return of R_t from t to $t+1$, and have real value $b_t = B_t/p_t$.

The private economy is imagined as having started in the infinite past, while policy authorities begin to follow fixed rules at time 1. Private agents base their time t decisions on an information set that consists of at least all the variables dated t and earlier. They carry k_{t-1} units of capital into period t , as well as government liabilities whose nominal value is $M_{t-1} + R_{t-1}B_{t-1}$. Agents solve the following problem:

$$\max_{(c_t, k_t, m_t, b_t)} E_1 \sum_{t=1}^{\infty} \beta^t u(c_t, m_t)$$

subject to

$$k_t + c_t + m_t + b_t + \tau_t = \tag{2.1}$$

$$\theta_{1t} f(k_{t-1}) + \theta_{2t} + \delta k_{t-1} + \pi_t^{-1} m_{t-1} + R_{t-1} \pi_t^{-1} b_{t-1}$$

where $\pi_t = p_t/p_{t-1}$ is the gross inflation rate from time $t-1$ to t .

The first order necessary conditions for an interior solution to this problem reduce to:

$$-\theta_{3t}/c_t + \beta E_t \{ (\theta_{3t+1}/c_{t+1}) [\theta_{1t+1} f'(k_t) + \delta] \} = 0 \tag{2.2}$$

$$-\theta_{3t}/c_t + \beta R_t E_t [\theta_{3t+1}/(c_{t+1} \pi_{t+1})] = 0 \tag{2.3}$$

$$m_t = [(1-\varphi)/\varphi] c_t [1 - R_t^{-1}]^{-1} \tag{2.4}$$

Each equation holds for $t \geq 1$. E_t is the mathematical expectations operator conditional on information known at t . Equilibrium conditions (2.2) and (2.3) are the Euler equations for capital and government debt. Condition (2.4) is the economy's money demand schedule, which is obtained by combining the Euler equations for real balances and debt.

The government finances an exogenous random stream of real spending $\{g_t\}$ through a combination of direct taxes and seignorage. The choice of financing must imply a process for debt $\{b_t\}$ that satisfies the government budget constraint:

$$b_t + m_t + \tau_t = g_t + \pi_t^{-1} m_{t-1} + R_{t-1} \pi_t^{-1} b_{t-1} \tag{2.5}$$

for $t \geq 1$. The government inherits liabilities with nominal value $M_0 + R_0 B_0$.

Policy follows simple rules that are variations on those used by Sims (1988). These rules are rich enough to highlight several important aspects of monetary and fiscal interactions. The rules for the monetary, tax, and spending authorities are:

$$R_t = \alpha_1 + \alpha_2 \pi_t + \theta_{4t} \quad (2.6)$$

$$\tau_t = \gamma_1 + \gamma_2 b_{t-1} + \psi_1 \theta_{5t-1} + (1-\psi_1) \theta_{5t} \quad (2.7)$$

$$g_t = \bar{g} + \psi_2 \theta_{6t-1} + (1-\psi_2) \theta_{6t} \quad (2.8)$$

where $\{\theta_{4t}, \theta_{5t}, \theta_{6t}\}$ are stationary stochastic processes. They have mean zero and may be serially correlated. Each of these rules holds for $t \geq 1$. The parameters $\psi_i \in [0,1]$ index the extent to which realizations of fiscal shocks at time $t-1$ reveal the time t values of taxes and government spending.

The random components to the policy rules emerge from supposing that policy behavior depends on aspects of the environment that are not perfectly predictable to private agents. For example, if the technology of policy selection is imperfectly understood by private agents (and the econometrician), although it is known exactly to the policy authority, individual decisionmakers will treat this as a random part of policymaking that is unrelated to the economic variables they observe contemporaneously. From the viewpoint of private agents (and the econometrician), these additive policy shocks appear as exogenous stochastic processes.

The monetary authority manipulates the nominal money stock to set R according to (2.6). R 's dependence on current inflation represents the monetary authority's attempts to "lean against inflationary winds." The

specification of policy draws an empirical distinction between two sorts of monetary contractions: those induced by efforts to combat inflationary pressures, which are R's contemporary reaction to π , and those designed to pursue alternative objectives, which are disturbances to the random term θ_4 .

The coefficient on lagged debt, γ_2 in equation (2.7), reflects the extent to which the fiscal authority ratifies increases in either deficits or debt service with higher future taxes. This specification of the process for lump-sum taxes ensures a positive level of expected direct taxation. Setting $\gamma_2 = 0$ reflects an approach to fiscal financing that ignores interest payments on the debt and makes the net deficit, $g_t - \tau_t$, unresponsive to economic conditions. Finally, government spending has a mean of \bar{g} and is determined exogenously according to equation (2.8).

Changes in ψ_1 and ψ_2 in (2.7) and (2.8) represent alternative assumptions about the timing of fiscal news. The conventional informational assumptions set $\psi_1 = \psi_2 = 0$, so current realizations of the tax and spending shocks help to predict future taxes and spending only through the shocks' serial correlation properties. The limiting case where $\psi_1 = \psi_2 = 1$ implies that the consumer (but not the econometrician) knows fiscal variables dated at $t+1$ when the consumer makes his time t decisions.

In addition to satisfying the equilibrium conditions (2.2) - (2.5), an optimal solution must meet the government's intertemporal budget constraint.¹ The transversality condition for government debt ensures that the real value of debt held by the private sector equals the present value of future surplus-

¹McCallum (1984) demonstrates the necessity of the transversality condition for debt in a deterministic version of this model.

es plus seignorage less interest payments on the debt. This intertemporal constraint is:

$$\frac{B_t}{P_t} = \sum_{s=0}^{\infty} \left\{ \prod_{j=0}^s \pi_{t+j+1} R_{t+j}^{-1} \right\} [\tau_{t+s+1} - g_{t+s+1} + m_{t+s+1} - (1/\pi_{t+s+1})m_{t+s}] \quad (2.9)$$

It is convenient to define the vector stochastic process $\{\theta_t\}$, which private agents treat as exogenous: $\theta_t = [\ln(\theta_{1t}), \theta_{2t}, \ln(\theta_{3t}), \theta_{4t}, \theta_{5t}, \theta_{6t}]'$. Let ϵ_{it} be the innovation in θ_{it} . Assume $\{\theta_t\}$ for $t \geq 0$ is a first-order autoregressive process with a diagonal covariance matrix among the innovations, the ϵ_{it}' s.

3. Equilibrium: A Qualitative Example

I now derive the dynamic properties of a version of the model that is linearized around the deterministic steady state of the nonlinear model.² The steady state used is one for which the real variables are constant through time and the nominal variables grow at the constant rate of inflation. To simplify matters, I assume that capital depreciates completely each period, so $\delta = 0$, and that the exogenous stochastic processes are serially uncorrelated.³ Agents are initially assumed to observe only current and past values of the variables, so $\psi_1 = \psi_2 = 0$.

The separability of consumption and real balances in preferences ensures the model displays a classical dichotomy. This allows the real equilibrium to

²A description of this derivation and a more complete set of results are contained in Leeper (1989).

³In the deterministic steady state, the multiplicative technology shock and the shock to preferences are set at unity and the remaining shocks are fixed at zero.

be characterized first. The nominal equilibrium's dependence on the monetary and tax parameters (α_2, γ_2) is then explored, treating equilibrium allocations as given.

The Real Sector

The real sector is summarized by the Euler equation for capital and feasibility. Convert the dynamic Euler equation (2.2) to a stochastic difference equation by introducing a forecast error:

$$-\theta_{3t}/c_t + \beta(\theta_{3t+1}/c_{t+1})\theta_{1t+1}f'(k_t) = \eta_{1t+1} \quad (3.1)$$

where $E_t \eta_{1t+1} = 0$. Combine the consumer's and the government's budget constraints — equations (2.1) and (2.5) — to yield the feasibility condition:

$$c_t + k_t + g_t = \theta_{1t}f(k_{t-1}) + \theta_{2t} \quad (3.2)$$

The linearized system can be represented generally as:

$$\begin{bmatrix} \tilde{k}_{t+1} \\ \tilde{c}_{t+1} \end{bmatrix} = H_0 \begin{bmatrix} \tilde{k}_t \\ \tilde{c}_t \end{bmatrix} + H_1 \xi_{t+1} + H_2 \xi_t, \quad t = 0, 1, 2, \dots \quad (3.3)$$

where \tilde{x}_t denotes x_t 's deviation from steady state, and the vector including the forecast error and exogenous shocks to technology, tastes, and government spending is $\xi_t = (\eta_{1t}, \tilde{\theta}_{1t}, \tilde{\theta}_{2t}, \tilde{\theta}_{3t}, \tilde{\theta}_{6t})'$.

Since (ξ_t) is a stationary stochastic process, the system's dynamics are determined by the eigenvalues of the transition matrix H_0 . Appendix A shows that with positive consumption the eigenvalues satisfy $\lambda_1 < 1 < 1/\beta < \lambda_2$.⁴

Equilibrium requires that the transversality condition for capital is satisfied, which implies:

$$\tilde{c}_t = w_1 \tilde{k}_t + w_2 \tilde{\theta}_{3t}, \quad t = 0, 1, 2, \dots \quad (3.4)$$

where the w_1 's are constants. A unique equilibrium also provides a linear mapping from the innovations in the exogenous processes, the ϵ 's, to the forecast error in the Euler equation for capital, η_1 :

$$\eta_{1t+1} = -P_1 \epsilon_{1t+1} - P_2 \epsilon_{2t+1} + P_3 \epsilon_{3t+1} + P_4 \epsilon_{6t+1}, \quad t = 1, 2, \dots \quad (3.5)$$

where the coefficients are positive.

The decision rules are obtained by substituting the equilibrium condition (3.4) into feasibility (3.2) to yield a nonlinear first-order difference equation in capital. The equilibrium consumption and investment functions depend on the four exogenous processes in technology, preferences, and government spending. According to (3.5), negative surprises in productivity shocks and positive innovations in preference or government spending shocks increase the forecast error and the real interest rate, inducing private agents to reduce current consumption.

⁴According to Blanchard and Kahn (1980), since the Euler equation contains one future-dated endogenous variable, the single unstable eigenvalue is sufficient for the existence of a unique equilibrium allocation.

The Nominal Sector

The remaining equations of the model can be combined to obtain a bivariate system in inflation and real debt. Convert the Euler equation for debt (2.3) to a stochastic difference equation by adding the forecast error η_{2t+1} with the property $E_t \eta_{2t+1} = 0$:

$$-\theta_{3t}/c_t + \beta R_t \theta_{3t+1}/(c_{t+1} \pi_{t+1}) = \eta_{2t+1} \quad (3.6)$$

Combine the three policy rules with (3.6), the money demand schedule (2.4), and the government budget constraint (2.5) to yield two nonlinear stochastic difference equations in (π_t, b_t) . The linearized system is:

$$\begin{bmatrix} \tilde{\pi}_{t+1} \\ \tilde{b}_{t+1} \end{bmatrix} = \begin{bmatrix} \alpha_2 \beta & 0 \\ \Gamma & \beta^{-1} - \gamma_2 \end{bmatrix} \begin{bmatrix} \tilde{\pi}_t \\ \tilde{b}_t \end{bmatrix} + H_1 \xi_{t+1} + H_2 \xi_t, \quad t = 0, 1, 2, \dots \quad (3.7)$$

where $\Gamma = \frac{\alpha_2(1-\varphi)\bar{c}}{\varphi(\bar{R}-1)^2} [\alpha_2 \beta - \beta^{-1}]$

and (ξ_t) is the vector stochastic process containing the forecast error from the Euler equation for debt, consumption, the preference shock, and the shocks from the three policy equations, $\xi_t = (\eta_{2t}, \tilde{c}_t, \tilde{\theta}_{3t}, \tilde{\theta}_{4t}, \tilde{\theta}_{5t}, \tilde{\theta}_{6t})'$.⁵ The triangularity of the transition matrix implies immediately that the eigenvalues are $\alpha_2 \beta$ and $\beta^{-1} - \gamma_2$.

We can now see that existence and uniqueness of nominal equilibria are determined by the extent to which monetary policy reacts to inflation (through

⁵With the equilibrium allocations determined in the real sector, consumption can be treated as a forcing variable for the system.

α_2) and fiscal policy reacts to lagged debt (through γ_2). Figure 1 defines four regions of the (α_2, γ_2) policy parameter space. In Region I, when R_t is made to react strongly to inflation, monetary policy stabilizes prices so only direct lump-sum taxes are used to finance the deficit. This implies the complete irrelevance of revenue shocks. The monetary authority's resistance to inflation, coupled with the fiscal authority's ratification of deficits with future tax hikes, determines prices and ensures the expected discounted value of government debt is zero.

When tax increases are insufficient to cover debt service, as in Region II, transversality requires that the monetary authority stabilize the nominal interest rate by expanding base money in the face of a deficit shock. This reliance on inflation taxes undermines the irrelevance of deficit shocks for the nominal equilibrium.⁶ The rigidity of fiscal behavior combines with an accommodating monetary stance to produce a unique nominal equilibrium.

In Region III both monetary and tax policy accommodate fiscal expansions. Intuitively, authorities apply two instruments to achieve the single objective of intertemporal budget balance, leaving the active role of price determination unfilled. This redundancy satisfies transversality, but since any inflation-debt trajectory is an equilibrium, these parameter combinations are uninteresting for policy analysis.⁷

The opposite situation occurs in Region IV, where both authorities aggressively pursue price stability. Their behavior destroys the equilibrium:

⁶A Region II equilibrium could arise when it is politically costly to raise lump-sum taxes, but relatively cheap to finance deficits through inflation.

⁷The indeterminacy takes the form of multiple equilibria indexed by the initial price level.

either inflation "explodes" (asymptotically driving the economy out of a monetary equilibrium) or real debt grows "too rapidly" (violating transversality).

The tension that must exist between monetary and fiscal behavior if there is to exist a unique monetary equilibrium characterizes the sense in which policies must be coordinated. The coordination required is summarized by the policy parameters (α_2, γ_2) . This underscores that fundamentally both monetary and fiscal policy "matter," regardless of the effects of their individual shocks on equilibrium time paths.

Since this delicate tension between monetary and fiscal policy exists only in Regions I and II, I now draw on parameters from these regions to illustrate three equilibrium policies for financing deficit shocks.⁸ The first scheme comes from policy parameters in Region I and relies exclusively on direct lump-sum taxes. Within Region II deficits may be financed either by distorting inflation taxes or by a non-distorting one-time jump in the price level.

Region I - Irrelevance of Tax Shocks

This policy region is characterized by $\alpha_2\beta > 1$ and $|\beta^{-1} - \gamma_2| < 1$. The monetary authority adjusts the money stock to make the nominal interest rate very responsive to contemporary inflation and the fiscal authority acts to raise future taxes by enough to cover the higher real interest payments produced by increases in debt.

⁸Regions I* and II* are included in the figure for completeness but will not be considered in the analysis that follows. Leeper (1989) explores these regions.

The qualitative nature of equilibria in this region is summarized by the phase diagram in Figure 2, where real debt's expected deviations from steady state are plotted on the horizontal axis and inflation's expected deviations are on the vertical axis. The $\dot{x} = 0$ lines depict the loci of points along which $E_t x_{t+1} = x_t$ at each date, and the steady state lies at the origin. The sensitivity of monetary policy to inflation generates unstable vertical forces that are counterbalanced by the stable horizontal forces induced by fiscal policy behavior. This tension produces a saddle point equilibrium whose path of convergence, as depicted by the heavy line, coincides with the $\dot{\pi} = 0$ locus. A one-time unanticipated shock in an exogenous process may cause contemporary realizations of inflation and debt to deviate from this convergent path, but the economy moves to the stable arm the period after the shock and converges toward equilibrium along this path.

The stable arm in Figure 2 implies the equilibrium condition:

$$\bar{R}_t = -w_1 \bar{k}_t + w_2 \bar{c}_t - w_3 \bar{\theta}_{3t} \quad , \quad t = 0, 1, 2, \dots \quad (3.8)$$

With the (\bar{k}_t, \bar{c}_t) sequences solved from the real economy, the equilibrium nominal interest rate depends on the same four exogenous processes that determine allocations: technology, taste, and government spending shocks. In this region the monetary authority controls inflation by adjusting the nominal interest rate to induce consumers to demand a price stabilizing level of real balances. Thus, equilibrium nominal interest rate movements arise solely to offset the money demand effects of real rate disturbances. Policy behavior precludes trajectories in which expected inflation deviates from steady state.

Since the monetary policy shock has no effect on the real rate, it does not affect equilibrium nominal rates.⁹

In equilibrium, the error in forecasting the (inverse) price level one-step-ahead is given by:

$$\eta_{2t+1} = -P_1\epsilon_{1t+1} - P_2\epsilon_{2t+1} + P_3\epsilon_{3t+1} + P_4\epsilon_{4t+1} + P_5\epsilon_{6t+1} \quad , \quad t = 1, 2, \dots \quad (3.9)$$

Unexpectedly small productivity shocks or large preference or government spending shocks generate unanticipated price increases. With \bar{R}_t determined by (3.8), positive disturbances to the interest rate rule must generate offsetting unanticipated declines in the price level.

The response of the system to serially uncorrelated policy shocks is obtained by using the stable arm, $E_t \tilde{\pi}_{t+1} = 0$, to replace the first equation in (3.7). A positive transitory monetary policy disturbance at time T increases the forecast error η_{2T} and reduces the price level at T. With condition (3.8) fixing the nominal interest rate, expected inflation is forced to remain at steady state. The one-time drop in prices, combined with the open market sale of nominal debt, increases real debt and the economy jumps to point A in Figure 2, but moves along the stable arm from T+1 onward, as shown by points A' and A''. According to the second equation in (3.7), real debt converges toward steady state at rate $\beta^{-1} - \gamma_2$, as future direct taxes rise by enough to satisfy the government's intertemporal budget constraint. A similar adjustment to equilibrium is triggered by an unanticipated increase in government spending.

⁹When monetary shocks display serial correlation, current realizations help forecast future prices, making R_t depend on the current shock.

As seen from (3.9), surprise cuts in direct taxes at time T , however, produce no such change in the price level. The tax reduction is financed by new issuances of nominal debt that are converted one-for-one into real debt expansions, pushing the economy to point B in the figure. In the next period direct taxes rise by γ_2 and the level of real debt declines toward steady state, as depicted by point B' .

When $\psi_1 = \psi_2 = 1$, so that both r_{t+1} and g_{t+1} are known to private agents when they make their date t decisions, the government spending shock has a positive effect on the equilibrium nominal interest rate in (3.8). This reflects the certainty that a positive spending shock at t will increase g_{t+1} one-for-one, raising the real interest rate and generating inflationary pressures. Naturally, if tax shocks are irrelevant, advanced knowledge of taxes is also irrelevant, so the tax rule disturbance maintains its zero coefficient in the equilibrium condition.

Region II - Monetized Deficits

Now suppose that $|\alpha_2\beta| < 1$ and $\gamma_2 < \beta^{-1} - 1$. The money stock is manipulated to make the nominal interest rate fairly insensitive to inflationary pressures. With the fiscal authority adjusting taxes weakly to debt changes, the intertemporal budget constraint requires that shocks that increase interest payments on the debt must be met with monetary expansions.

Figure 3 depicts a situation in which policy behavior generates equilibrium time series that are consistent with King and Plosser's (1985) quotation. The monetary authority leans lightly against the wind ($0 < \alpha_2 < \beta^{-1}$) and fiscal policy raises direct taxes by less than the future value of the change in real debt ($0 < \gamma_2 < \beta^{-1} - 1$). These assumptions about policy

imply the $\dot{b} = 0$ locus has a slope of $(1+\gamma_2-\beta^{-1})/\Gamma$ and the stable arm has a slope equaling $(\alpha_2\beta+\gamma_2-\beta^{-1})/\Gamma$, which in absolute value exceeds that of $\dot{b} = 0$. The insufficient responsiveness of taxes to debt produces unstable horizontal forces, while monetary policy's languid reaction to inflation generates stable vertical forces.

The equilibrium condition implied by the stable arm is:

$$\bar{b}_t = -w_1\bar{k}_t + w_2\bar{c}_t - w_3\bar{m}_t + w_4\bar{R}_t - w_5\bar{\theta}_{3t} \quad , \quad t = 0,1,2,\dots \quad (3.10)$$

Factors that raise marginal utility of consumption and, therefore the real interest rate, lower outstanding real debt. This is analogous to Region I where the monetary authority controlled prices by adjusting R_t in the face of real interest rate fluctuations. In Region II the control is exerted by the fiscal authority's manipulation of real debt.

The equilibrium mapping from exogenous disturbances to the error in forecasting prices is:

$$\eta_{2t+1} = -P_1\epsilon_{1t+1} - P_2\epsilon_{2t+1} + P_3\epsilon_{3t+1} + P_4\epsilon_{4t+1} + P_5\epsilon_{5t+1} + P_6\epsilon_{6t+1}, \quad t = 1,2,\dots \quad (3.12)$$

With time $t+1$ consumption determined by real shocks, unexpectedly higher taxes decrease p_{t+1} . The negative coefficient on government spending surprises implies an unambiguous inflationary effect from positive innovations in spending.

The positively sloped stable arm in Figure 3 suggests that policy now relies on distorting taxes that alter nominal interest rates and real balances. An unexpected negative realization of the monetary disturbance at time T moves the economy to point A at impact. The resulting lower nominal

interest rate at T, however, reduces future inflation and pushes the economy to the stable arm at point A' at T+1, from where it converges to the steady state at rate $\alpha_2\beta$. Real debt follows inflation's convergent path, according to the slope of the stable arm.¹⁰

Unanticipated tax cuts or government spending increases drive both inflation and real debt higher to a point like B upon impact. From that point the variables move to B' and decrease monotonically toward the steady state along the stable arm. In contrast to Region I equilibria, policy shocks induce distorting inflation taxes because the direct tax response to changes in real debt is too weak to balance the government's budget, while at the same time the monetary authority adopts a stance that spreads deficit financing over time.

Even within Region II the policy authorities need not resort to distorting taxes, however. When both the nominal interest rate and direct taxes are unrelated to economic variables ($\alpha_2 = \gamma_2 = 0$), the stable arm and the $\dot{b} = 0$ locus coincide with the vertical axis as in Figure 4. A negative monetary shock at time T has no initial effect on prices, but the reduction in R_T results in lower expected inflation at time T+1, moving the system to point A' at T+1. By period T+2 the economy settles down to its steady state at the origin. Thus, when the interest rate is pegged, monetary shocks generate a one-period distorting inflation tax that causes real balances to deviate from steady state at the time of the shock. Since the real interest rate from T to

¹⁰The impact effect of the monetary disturbance depends on the responsiveness of direct taxes to lagged real debt. When $\gamma_2 = 0$ the economy remains at the steady state upon impact and when $\gamma_2 < 0$ it jumps to a point to the southeast of A' initially. After the first period the pattern of convergence is qualitatively the same regardless of the sign of γ_2 . These results are explored in Leeper (1989).

T+1 remains constant and future surpluses are fixed, real debt stays at its steady state level.

Surprise changes in fiscal variables produce a one-time jump in contemporaneous prices and no change in the pegged interest rate. Nominal debt and prices at time T move proportionally to keep real debt at the steady state. Since money demand is also unaffected by the fiscal disturbances, nominal money balances must move proportionally with prices, implying an instantaneous monetization of the deficit shock. Thus, following a tax cut or spending increase at T the economy jumps to point B in Figure 4 and returns to the origin by period T+1. When fiscal shocks are serially correlated, a current deficit realization portends lower future surpluses, which decrease current real debt. Essentially, all current and future deficits are monetized immediately, so the price level must rise by more than the increase in nominal debt. The fixity of real balances, in turn, implies the monetary base must expand along with the higher prices to clear the money market.

When agents have advanced knowledge of fiscal variables, both current spending and tax shocks enter the equilibrium real debt relationship in (3.10). This contemporaneous dependence arises from the government's intertemporal constraint, which makes the real value of debt a function of expected future net surpluses.

4. U.S. Time Series and the Simulation Strategy

The impulse response functions discussed in this section and the next orthogonalize the contemporaneous covariance matrix of the innovations in the order that the variables appear, so the first variable is treated as predetermined for the second through sixth variables, the second is predetermined for

the third through sixth series, and so on. The impulse response functions depicted in the figures run the length of the page and are read by shifting the page 90 degrees clockwise so the heading is on the right. The $(i,j)^{th}$ cell of the graphs depicts the 16-quarter response of variable i to a one standard deviation innovation in variable j .

*Moving Average Representation of U.S. Data*¹¹

Figure 5 shows the moving average representation (MAR) estimated from actual U.S. data, which will be used to evaluate the model's fit. The U.S. series are orthogonalized in the order: total real government purchases (g), the real deficit exclusive of interest payments on the debt (d), the three-month Treasury bill rate (R), the inflation rate (π), the real monetary base (m), and real GNP (y).¹²

In the first column purchases innovations are followed by slightly lower interest rates and inflation, steadily increasing real balances, and a weak positive output response.¹³ As shown in the second column, shocks to the net deficit holding purchases fixed generate negative responses in interest rates and inflation, higher real balances, and sharply lower output.

¹¹The U.S. series are discussed in Appendix B.

¹²The interest rate and inflation are at net rates, multiplied by 100. All variables are fit in levels. The VAR was estimated using a Kalman filter updating algorithm and a loose Bayesian prior. (In RATS nomenclature, the estimation uses a symmetric prior, an overall tightness of .1, a relative tightness of .5, a harmonic decay rate of 1.0, and a tightness on the constant term of 15.0.) The responses of variable i are scaled by the largest (in absolute value) response of i to innovations in each of the variables in the system, giving the responses a range of ± 1.0 . The scaling factors are listed in the figure's heading.

¹³Sims (1988) discusses the small multiplier effects of government purchases in systems that include an interest rate and prices. Garcia-Mila (1987) finds these effects are enhanced in systems that include only components of GNP.

After an interest rate innovation in the third column, real balances and output respond strongly negatively. The decline in real balances, when coupled with the lethargic reaction of inflation, suggests the nominal money stock is falling.¹⁴ Net deficits increase at the same time that purchases decline, implying an even stronger drop in revenues. The real balance response, while large after two or more years, is very gradual with no contemporaneous correlation with the interest rate.

This pattern of responses appears inconsistent with the equilibria discussed in the previous section under conventional informational assumptions. While the increase in the deficit following an interest rate shock suggests a Region II equilibrium, the inflation and interest rate reactions to fiscal disturbances are inconsistent.

Producing and Evaluating Simulations

The model's parameters are chosen to produce a deterministic steady state equilibrium that matches the mean values of corresponding U.S. post-World War II time series. The innovation variances and serial correlation properties of the exogenous processes are selected to allow the simulated data to mimic the time series properties of analogous U.S. data. Table 1 presents the U.S. statistics and the model parameters used in the simulations. Table 2 reports the policy parameters and exogenous processes used in the simulations.

The model is solved and simulated using Sims's (1989a) "backwards" method, which derives solution paths for the model's variables that exactly

¹⁴Sims (1989b) obtains similar responses in a VAR system consisting of real GNP, the nominal interest rate, prices, and the nominal money stock.

solve the Euler equations and budget constraints.¹⁵ The algorithm replaced the equations for the two exogenous technology shocks with the two equilibrium conditions implied by the transversality conditions for the linearized model. This ensures that the processes for the policy shocks in the simulations are exactly those posited by the theoretical structure. The induced technology shocks, however, will approximate their original specifications.

The MARs of simulated data are reported for the analogous set of six variables appearing in Figure 5. One simulated time path is 160 quarters long to match the length of the time series from U.S. data. An unrestricted VAR with four lags is estimated and the MAR is calculated over a 16-quarter horizon. After repeating this procedure 100 times the variance of the MAR coefficients is calculated. The simulated MARs are scaled by the same factors used for U.S. responses.

Table 3 summarizes the fit of the simulations using various breakdowns of the number of U.S. impulse response functions lying within a two-standard-deviation band for the simulated responses. The breakdown in the first part of the table records the number of functions within the bands over specified forecast horizons. The table also separately reports the responses to fiscal and price innovations.

5. Simulations with Usual Informational Assumptions

This section presents two simulations assuming private agents observe only current and past fiscal variables (so $\psi_1 = \psi_2 = 0$), and evaluates their ability to mimic the correlations observed in U.S. data in Figure 5. The

¹⁵The simulations start the system at its deterministic steady state.

first comes from policy behavior that implies the irrelevance of tax shocks. In the second equilibrium the monetization of deficit shocks is spread over time, reproducing the conventional view reflected in King and Plosser's (1985) remarks. The comparison of actual to simulated MARs illustrates the empirical trade-offs of studying equilibria in which only direct lump-sum taxes finance deficits or equilibria in which some distorting inflation taxes are also used to satisfy the government's intertemporal constraint.

Simulation 1: Irrelevance of Tax Shocks

In the first simulation the fiscal authority makes taxes strongly responsive to lagged debt by setting $\gamma_2 = .35$. The monetary authority fixes $\alpha_2 = 5.0$ to lean against the wind. Figure 6 overlays this simulation's MAR with the same system fit to U.S. data in Figure 5. The U.S. responses appear as solid lines and the two-standard-deviation bands of the simulated responses appear as dashed lines.

Column one of Figure 6 shows that higher government spending forecasts initially higher net deficits. The response of deficits is self-correcting and turns to a surplus after a few quarters. The nominal interest rate and inflation responses are tightly centered on zero. Column two depicts the irrelevance of tax shocks endemic to Region I policy parameters. The deficit's own response dies out quickly as taxes respond in the next period to the real debt expansion. The responses of all the variables are concentrated around zero.

Columns three and four point out the pitfalls that may arise if the econometrician fails to distinguish systematic monetary reactions from exogenous monetary initiatives. An econometrician who treats the R innovation

in the third column as an exogenous monetary contraction might mistakenly conclude from the sizeable deficit response that the data were generated by an equilibrium where deficit shocks are monetized (that is, Region II parameters with $0 < \gamma_2 < \beta^{-1} - 1$). Column four clears up this misunderstanding by demonstrating that when the interest rate is fixed, higher prices induced by a negative monetary policy surprise are followed by higher net deficits.

The fit of simulation 1 is summarized in Table 3. Thirty percent of the simulated bands in Figure 6 include the U.S. responses over the full forecast horizon and nearly 60 percent encompass U.S. responses for at least half the horizon. Overall, 55 percent of the actual impulse response points are likely to be produced by this parameterization. As shown in the lower portion of Table 3, the simulation does better at reproducing responses to interest rate and inflation innovations (64 percent) than at mimicking fiscal effects (41 percent).

Whereas in U.S. data interest rate disturbances do not predict inflation, under the simulation's assumptions on monetary policy behavior, nominal rates are likely to be high because of contemporary inflationary pressures. Cell (4,3) shows that π rises for over a year before settling down to its steady state path. The model implies a very large immediate response of money demand to an R shock, rather than the smooth but strong pattern displayed by actual data. In Region I we also expect the higher net deficit path following an exogenous monetary expansion that shows up in cell (2,4). Although the U.S. data response does not fall within the error bands for the full 16-quarter horizon, it remains close in the out years. The model also produces a large initial decline in output following an innovation in inflation. This

appears in actual data as a less pronounced impact effect and a gradual cumulation over the forecast horizon.

Simulation 2: The Conventional View of Monetization

The second simulation draws policy parameters from Region II to illustrate the conventional view that deficits should predict current and future money growth if deficits cause inflation. The model produces this result when monetary policy exerts some effort to stabilize prices ($\alpha_2 = .3$) and fiscal policy raises taxes somewhat when the government's level of real indebtedness increases ($\gamma_2 = .01$). The MAR appears as Figure 7.

With the monetary authority acting to alleviate inflationary pressures, both government spending and deficit shocks raise the nominal interest rate and suppress money demand in the first period. Inflation is sharply higher. Distributed lag regressions of interest rates or inflation on current and past net deficits will successfully recover the true theoretical relationship.¹⁶ In addition, neither interest rate innovations (column three) nor exogenous monetary policy disturbances (column four) help to predict future fiscal variables, so the Granger-causality runs only from fiscal variables to inflation.

Table 3 summarizes Figure 7's performance. Its overall fit is approximately as good as the equilibrium where taxes shocks are irrelevant, with 58 percent of the U.S. impulse response points within the simulation's two-standard-deviation band. The simulation encompasses fewer entire response functions (14 percent) than did the first simulation, but about the same number of U.S. responses have all but one quarter included in the bands (28

¹⁶Evans (1987) is an example of such an interest rate regression.

percent). This simulation marginally outperforms the first in matching responses to fiscal shocks and prices.

6. Simulation with Foreknowledge of Fiscal Variables

In this section I assume private agents know tomorrow's fiscal variables exactly when they make today's consumption and portfolio decisions (so that $\psi_1 = \psi_2 = 1$). Underlying this redating of shocks in the fiscal rules is the notion that U.S. spending and tax decisions are made through a legislative process that evolves slowly. The simulation adopts the perspective of an econometrician who lacks data on the unobservable fiscal shocks $(\theta_{5t}, \theta_{6t})$ and naively identifies fiscal policy actions at time t with innovations in observable series (g_t, τ_t) .

The alternative informational assumption is coupled with a tax policy that sets direct taxes independently of debt ($\gamma_2 = 0$) and a monetary policy that adjusts the monetary base passively to maintain a stochastically pegged nominal interest rate ($\alpha_2 = 0$). This policy mix implies that shocks to interest payments on the debt bring forth an accommodating expansion of base money, but no expectation of higher future direct taxes or inflation taxes.

Simulation 3: Immediate Monetization of Deficits When $\psi_1 = \psi_2 = 1$

Figure 8 presents simulation 3's MAR. When private agents observe realizations of fiscal variables one period before the econometrician does, the fiscal shocks in columns one and two lose their positive correlation with inflation. Cells (4,1) and (4,2) of the figure show that the simulated error bands are concentrated on zero and encompass the U.S. responses. This contradicts King and Plosser's (1985) claim that when deficit shocks are

monetized the deficit innovations necessarily predict subsequent movements in inflation. It also demonstrates that the simulation's assumptions about policy behavior and agents' information sets mimic this aspect of U.S. data better than does the alternative depicted in Figure 6.

The monetary authority's pegging of nominal rates monetizes all current and expected future deficits immediately so interest rate innovations do not predict fiscal variables. However, inflation disturbances do anticipate the fiscal expansions that reach a peak in period two in cells (1,4) and (2,4), reversing the Granger-causal ordering of deficits and inflation.¹⁷ Inflation does not appear to forecast future deficits as strongly in U.S. data.

Foreknowledge of fiscal variables also allows the model to reproduce the output response to π innovations in cell (6,4). The final noteworthy improvement compared to the other simulations is that U.S. output's response to its own innovations is included in the simulated band for the full forecast horizon. The simulation misses most of the correlations between spending and deficits, and it continues to fail to produce the decline in nominal interest rates recorded in cells (3,1) and (3,2).

Table 3 reports the fit of the simulation. The simulation's fit is nearly as good as simulation 2, with 68 percent of all U.S. responses encompassed by simulated bands. In all the categories listed in the table the simulation outperforms all the simulations except number 2. It outperforms 2 by encompassing the entire path of inflation following a fiscal disturbance.

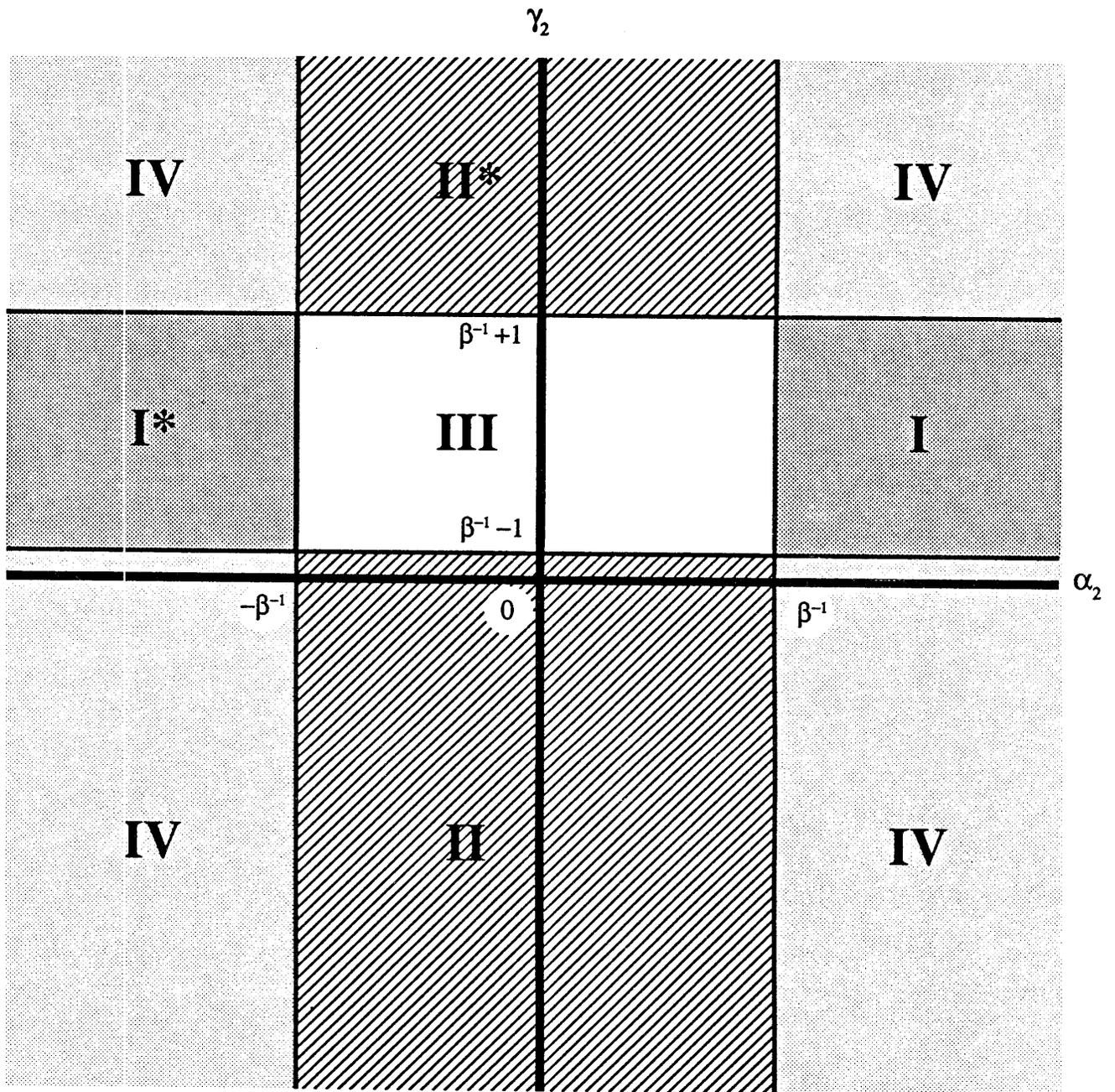
¹⁷Under the policy assumptions in simulation 2 ($\alpha_2 = .3$ and $\gamma_2 = .01$), interest rate innovations forecast future deficit expansions when agents have advanced knowledge of fiscal variables.

7. Concluding Remarks

This work has exploited the insights of Aiyagari and Gertler (1985) and Sims (1988) to argue that even if deficit shocks are fully monetized, single-equation reduced forms may lead to the conclusion that only direct non-distorting taxes have been used to finance deficits. I demonstrated that coupling minor variations in policy behavior with reasonable assumptions about when private agents learn about fiscal shocks can reverse the Granger-causal ordering between deficits and money creation.

This setup generates a dependence of the equilibrium pricing function on taxes that cannot be recovered from reduced-form regressions of inflation (or money growth) on current and past fiscal variables. In particular, surprises in inflation forecast higher future net deficits, but deficit innovations have no predictive value for subsequent movements in inflation. Although U.S. data are not entirely consistent with the full set of predictions from the model, the results nonetheless cast doubt on the inferences about fiscal effects that have been drawn from reduced-form analyses of the sort performed by King and Plosser (1985), among others.

Figure 1. Regions of Monetary and Tax Policy Parameters



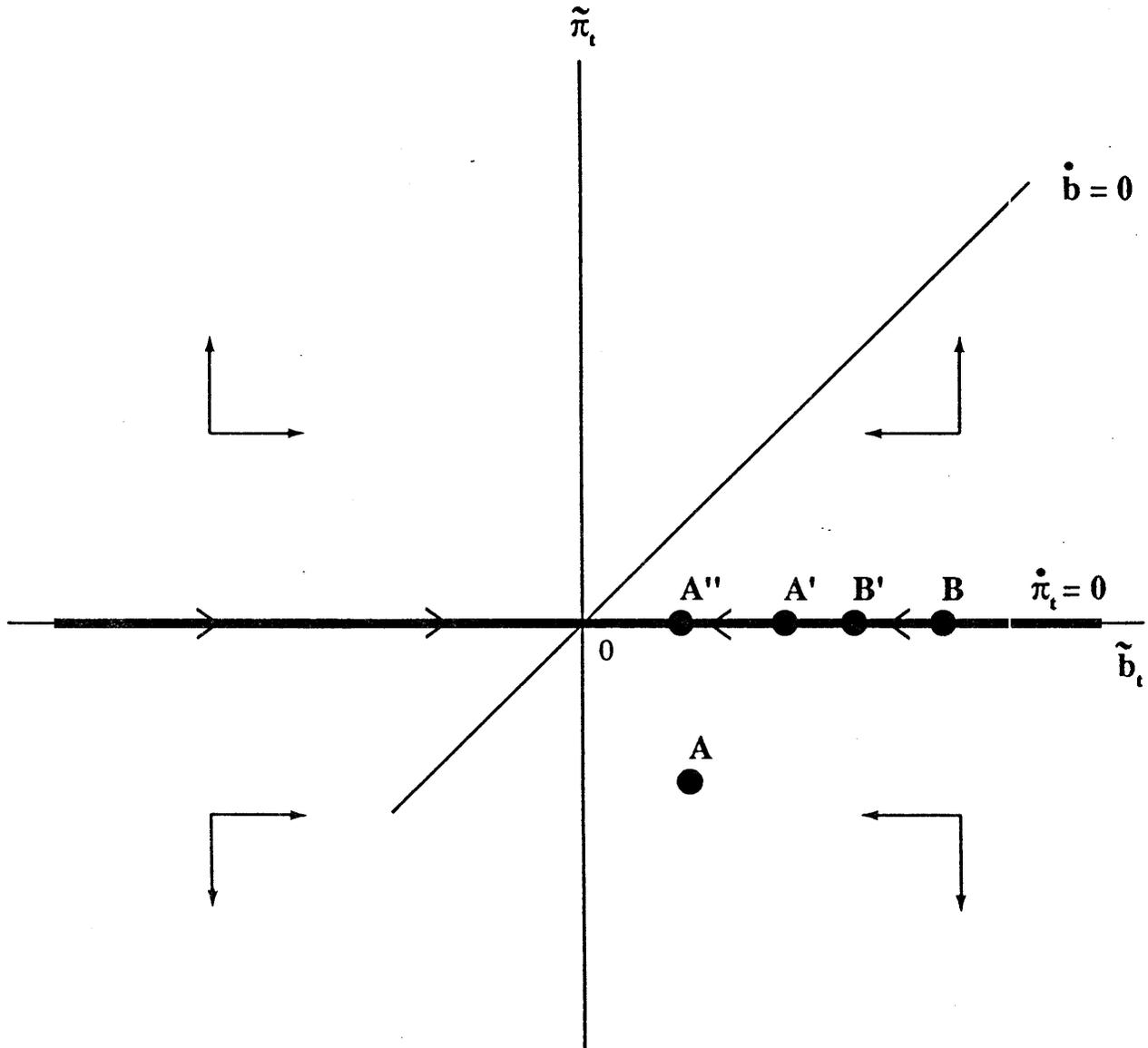
Region I : $|\alpha_2 \beta| > 1$ and $|\beta^{-1} - \gamma_2| < 1 \Rightarrow$ unique nominal equilibrium

Region II : $|\alpha_2 \beta| < 1$ and $|\beta^{-1} - \gamma_2| > 1 \Rightarrow$ unique nominal equilibrium

Region III : $|\alpha_2 \beta| < 1$ and $|\beta^{-1} - \gamma_2| < 1 \Rightarrow$ multiple nominal equilibria

Region IV : $|\alpha_2 \beta| > 1$ and $|\beta^{-1} - \gamma_2| > 1 \Rightarrow$ non-existence of nominal equilibrium

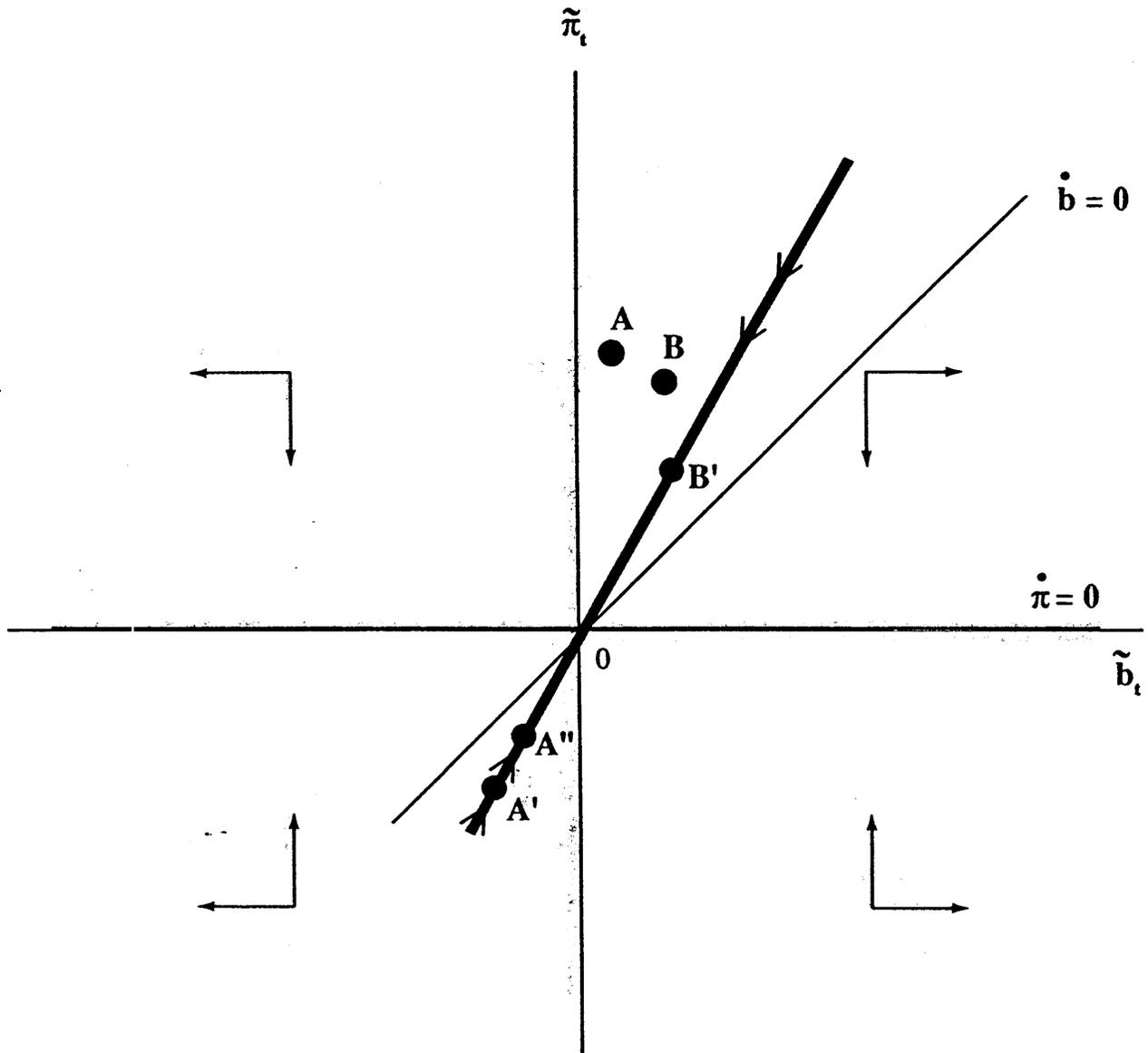
Figure 2. Region I: $\alpha_2 > \beta^{-1}$ and $\beta^{-1} - 1 < \gamma_2 < \beta^{-1} + 1$



$\tilde{\pi}_t$ and \tilde{b}_t are deviations of expected inflation and real debt from steady state, conditional on information available at time T , when the unanticipated transitory policy shock is realized; points A and B hold at time T , A' and B' hold at time $T+1$, and so on.

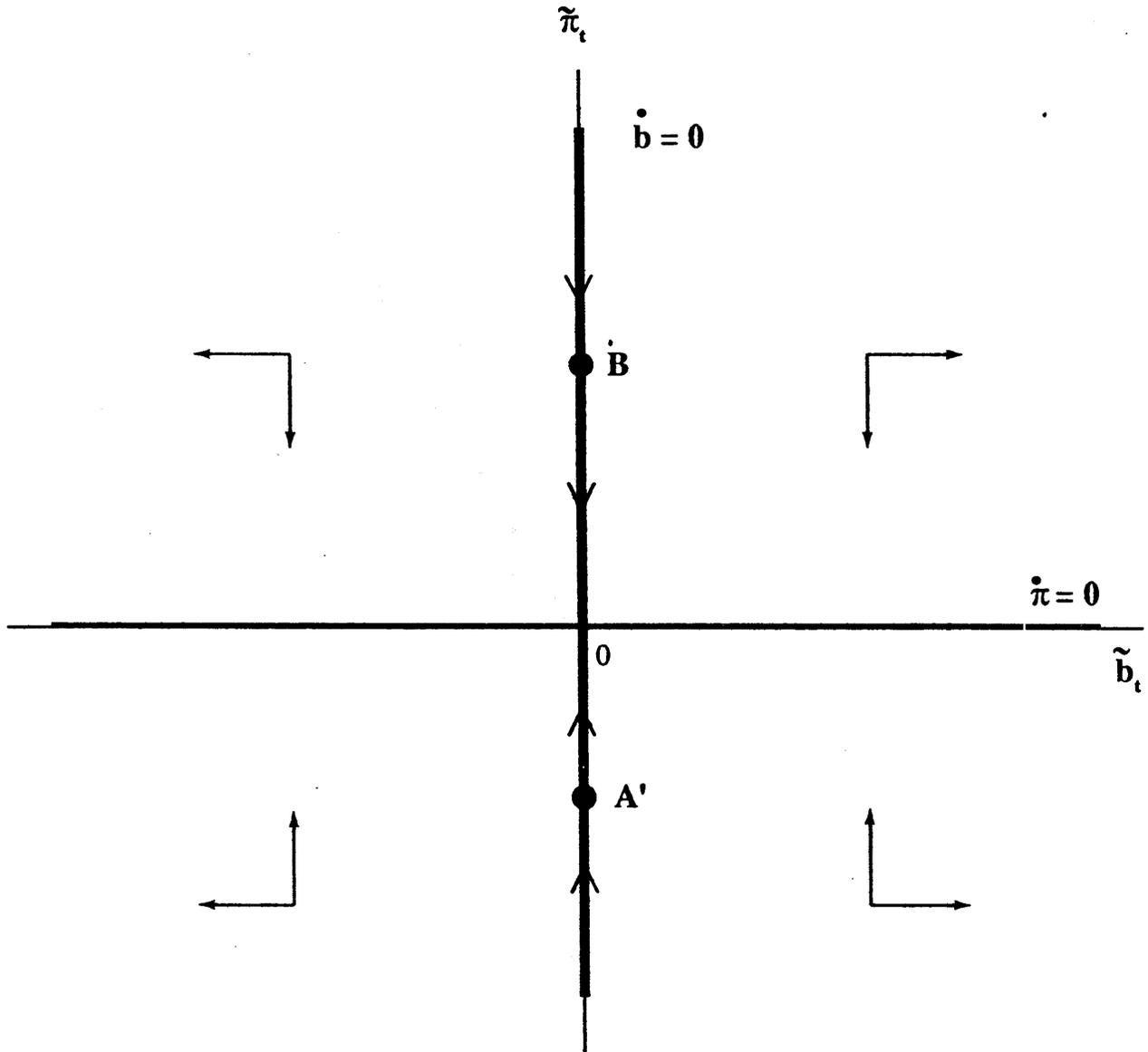
Note: $\dot{b} = 0$ locus drawn for $\alpha_2 > \beta^{-2}$

Figure 3. Region II: $0 < \alpha_2 < \beta^{-1}$ and $0 < \gamma_2 < \beta^{-1} - 1$



$\tilde{\pi}_t$ and \tilde{b}_t are deviations of expected inflation and real debt from steady state, conditional on information available at time T, when the unanticipated transitory policy shock is realized; points A and B hold at time T, A' and B' hold at time T+1, and so on.

Figure 4. Region II: $\alpha_2 = 0$ and $\gamma_2 = 0$



$\tilde{\pi}_t$ and \tilde{b}_t are deviations of expected inflation and real debt from steady state, conditional on information available at time T , when the unanticipated transitory policy shock is realized; points A and B hold at time T , A' and B' hold at time $T+1$, and so on.

Figure 5. U.S. Quarterly Data, 1948Q2 - 1987Q3

maximum responses: .59e-1, .93e-1, .74, .54, .74e-2, .16

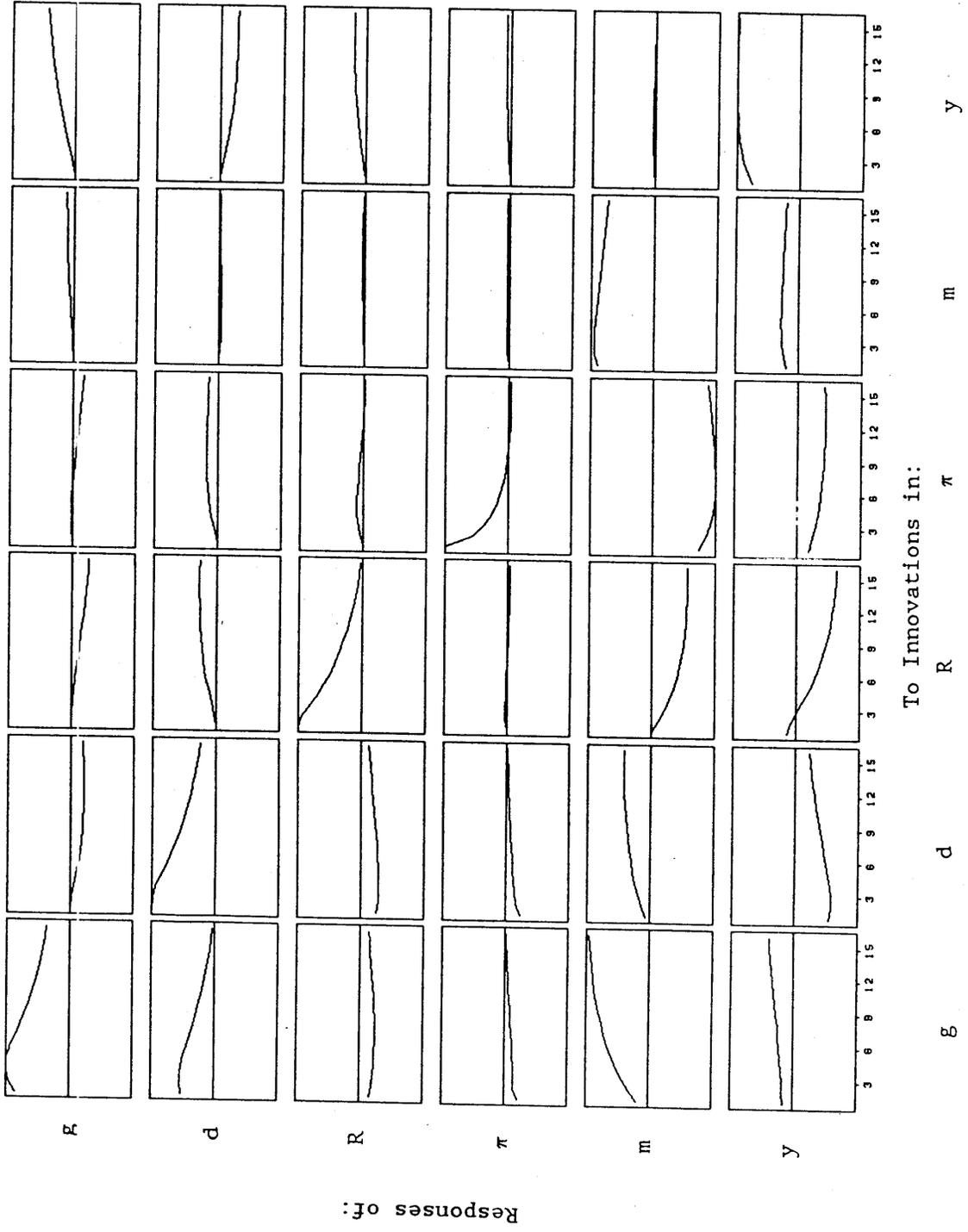


Figure 6. Simulation 1: $\alpha_2 = 5.0$, $\gamma_2 = .35$, $\psi_1 = \psi_2 = 0$
 range: $g, y = \pm 1.2$, $m = \pm 12.0$, others = ± 1.0

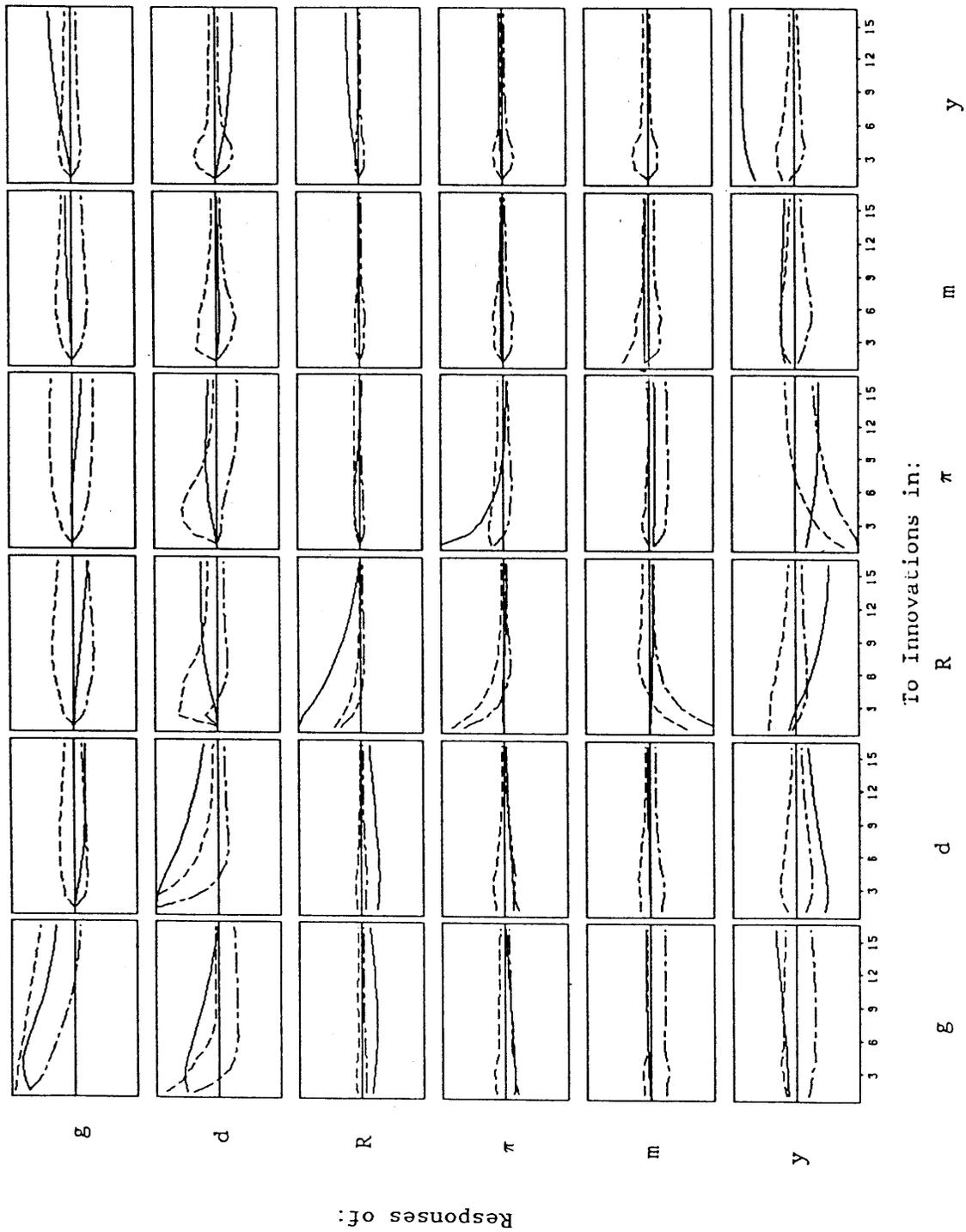


Figure 7. Simulation 2: $\alpha_2 = .3$, $\gamma_2 = .01$, $\psi_1 = \psi_2 = 0$
 range: $g, y = \pm 1.2$, $\pi = \pm 1.5$, $m = \pm 50$, others = ± 1

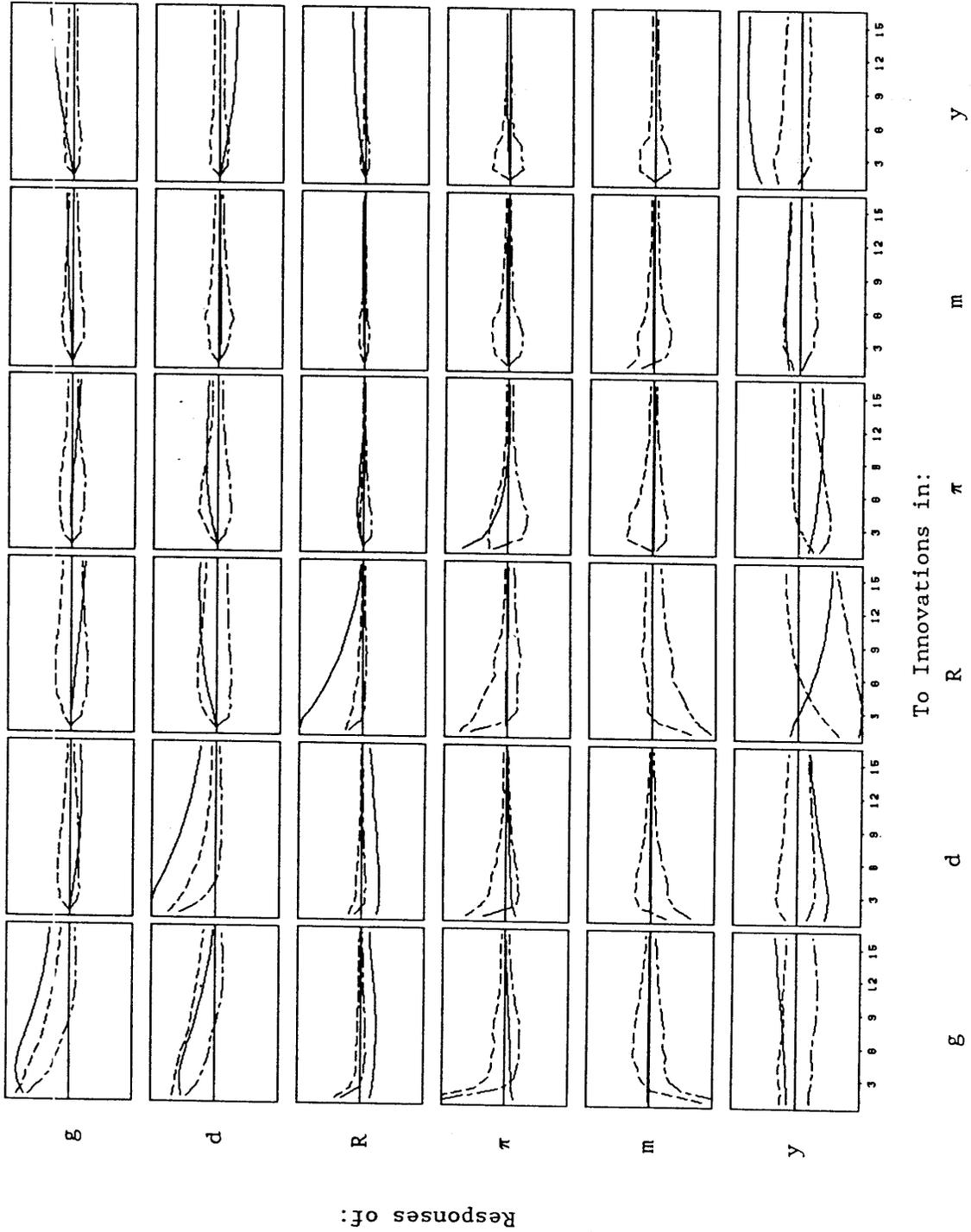
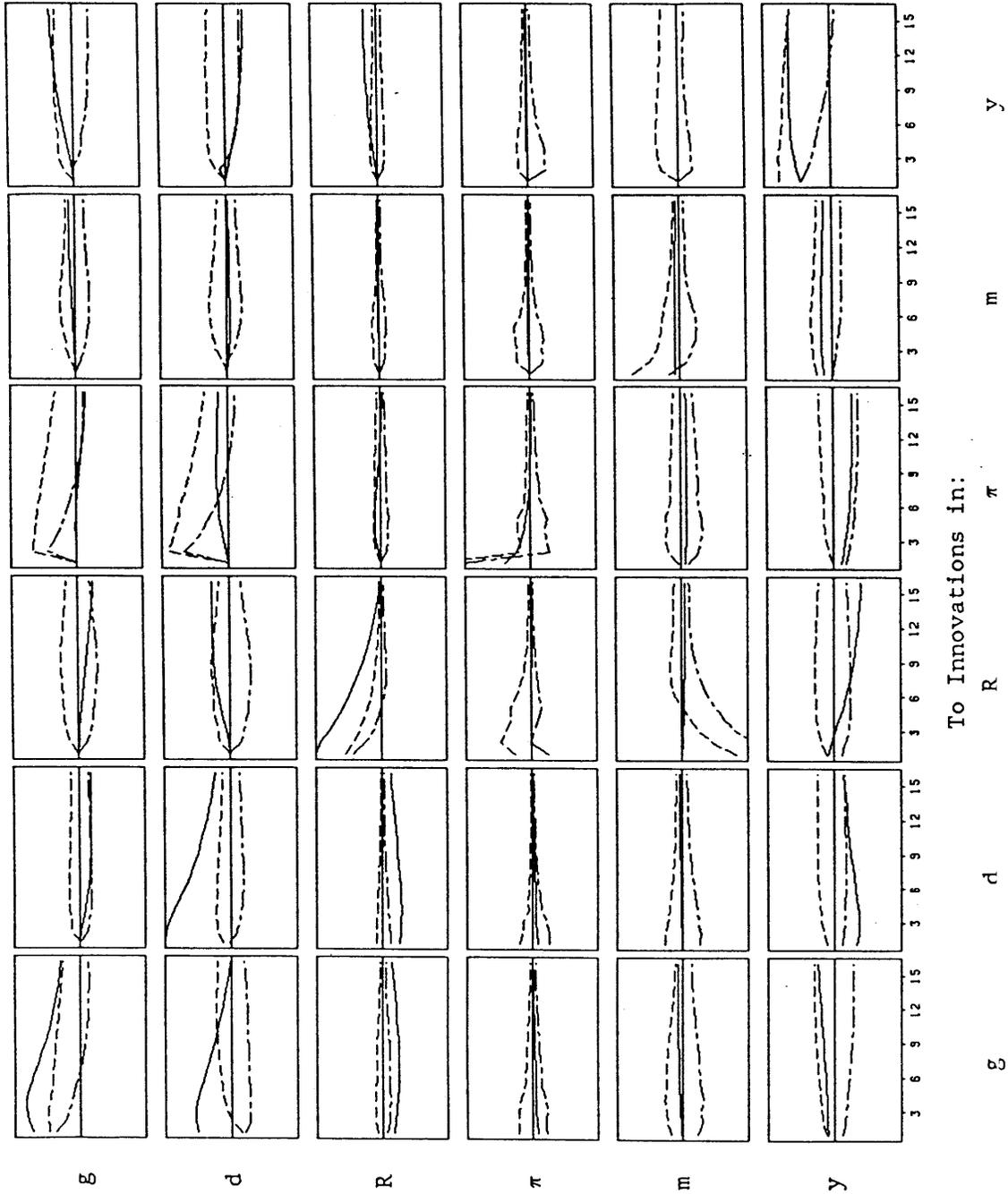


Figure 8. Simulation 3: $\alpha_2 = \gamma_2 = 0$, $\psi_1 = \psi_2 = 1$

range: $g = \pm 1.2$, $\pi = \pm 2.6$, $m = \pm 12$, $y = \pm 1.6$, others = ± 1



Responses of:

Table 1: U.S. Data and Deterministic Steady State of Model
(at quarterly rates, 1948Q2-1987Q3)

For model parameters: $\beta = .98$, $\nu = .26$, $\delta = .965$, $A = 3.832$, $\varphi = .99647$
and appropriate settings of policy parameters.

<u>Series</u>	<u>U.S. Mean</u>	<u>Steady State</u>
Real GNP	10.58	10.58
Consumption/GNP	0.64	0.63
Investment/GNP	0.16	0.16
GNP/Monetary Base	13.24	13.25
Purchases/GNP	0.20	0.20
Revenues/GNP	0.29	0.29
Nominal Interest Rate	1.012	1.031
Inflation Rate	1.01	1.01

Table 2: Policy Parameters and Exogenous Processes

Policy parameters:

	α_2	γ_2	ψ_1	ψ_2
Simulation 1:	5.0	.35	0	0
Simulation 2:	0.3	.01	0	0
Simulation 3:	0	0	1	1

<u>Exogenous Process</u>	<u>Simulation 1</u>		<u>Simulation 2</u>		<u>Simulation 3</u>	
	σ_1	ρ_1	σ_1	ρ_1	σ_1	ρ_1
θ_1	1.0e-2	.93	1.65e-2	.90	1.65e-2	.95
θ_2	2.0e-3	.90	1.0e-3	.90	1.0e-3	.90
θ_3	5.0e-3	.00	2.0e-3	.00	3.0e-3	.00
θ_4	1.85e-2	.65	7.0e-4	.75	4.0e-3	.75
θ_5	9.0e-2	.97	6.7e-2	.70	6.3e-2	.90
θ_6	6.5e-2	.95	5.8e-2	.85	5.6e-2	.93

The exogenous shocks are: θ_1 is multiplicative technology, θ_2 is additive technology, θ_3 is preference, θ_4 is interest rate rule, θ_5 is tax policy rule, θ_6 is government spending. σ_1 is the standard deviation of the innovation in θ_1 and ρ_1 is the coefficient on lagged θ_1 in the first-order autoregression. Simulation 2 sets $\varphi = .98$.

Table 3: Fit of Model Responses to U.S. Responses

All impulse responses

Simulation \ k	Number of Impulse Response Functions Within Simulated Bands for $\geq k$ Quarters (as a % of 36 impulse response functions)				Total Number of Quarters Within Simulated Bands (as a % of 576)
	≥ 16	≥ 15	≥ 12	≥ 8	
1	11 (30%)	11 (30%)	12 (33%)	21 (58%)	319 (55%)
2	5 (14)	10 (28)	18 (50)	22 (61)	334 (58)
3	14 (39)	18 (50)	21 (58)	25 (69)	390 (68)

Responses to (g,d) innovations

Total Number of Quarters Within Simulated Bands (as a % of 192)

Simulation

1	78 (41%)
2	89 (46)
3	97 (51)

Responses to (R, π) innovations

Total Number of Quarters Within Simulated Bands (as a % of 192)

Simulation

1	122 (64%)
2	126 (66)
3	133 (69)

Appendix A: Dynamics in the Real Sector

The transition matrix of the linearized system for the real sector in equation (3.3) is given by:

$$H_0 = \begin{bmatrix} -\bar{c}(\nu-1)\bar{k}^{-1} + \nu A \bar{k}^{\nu-1} & -1/\beta \nu A \bar{k}^{\nu-1} \\ \bar{c}(\nu-1)\bar{k}^{-1} & 1/\beta \nu A \bar{k}^{\nu-1} \end{bmatrix}$$

where \bar{c} and \bar{k} are the levels of consumption and capital in the non-stochastic steady state. The roots of this matrix must satisfy:

$$\lambda_1 \lambda_2 = 1/\beta$$

and

$$\lambda_1 + \lambda_2 = \phi + (1 + 1/\beta)$$

where $\phi = (1-\nu)(\beta\nu A)^{1/(\nu-1)}\bar{c}$. Since $0 < \nu < 1$, $0 < \beta < 1$, and $A > 0$, $\phi \geq 0$ for all $\bar{c} \geq 0$. The roots are real when $\phi + (1 + 1/\beta) \geq 2\beta^{-1/2}$ and they are distinct when the inequality is strict. Let λ_1 be the smaller root. When \bar{c} is positive, the roots are real and distinct and $\lambda_1 + \lambda_2 > 1 + 1/\beta$. This observation leads immediately to the conclusion that λ_1 is bounded above by unity and λ_2 is bounded below by $1/\beta$.¹⁸

The unique stable solution to system (3.3) in the text is obtained from the left eigenvector, μ , associated with the unstable eigenvalue, λ_2 . Stability is ensured by forcing the solution to lie on the stable manifold given by:

¹⁸One proof is along the lines of Sargent (1979, pp. 197-198). Alternatively, one can plot (3.4) and (3.5) in (λ_1, λ_2) -space to draw the same conclusions.

$$\mu z_t + \mu H_2 \xi_t = 0, \quad t = 0, 1, 2, \dots$$

This expression implies equilibrium condition (3.4) in the text.

A stable solution also requires that the model's forecast errors be exact functions of the disturbances to the underlying exogenous processes.

This mapping is given by:

$$\mu H_1 \xi_{t+1} = 0, \quad t = 1, 2, \dots$$

This expression yields equation (3.5) in the text.

Appendix B: U.S. Statistics and Model Parameterization

The U.S. data recorded in Table 1 were converted to real per capita terms by dividing by the civilian non-institutional population (from the Department of Labor, Bureau of Labor Statistics) and deflating by the personal consumption expenditures price index. The series that exhibit growth were then scaled by the exponential growth rate of real GNP over the sample period 1948, second quarter, to 1987, third quarter.¹⁹ For this sample period the growth rate is 4.78e-3. All data except interest rates and population were seasonally adjusted at the source.

The data series are: real GNP; investment = gross private domestic investment plus personal consumption expenditures on durables; real federal plus state and local government purchases (not including interest payments on outstanding debt); real federal plus state and local government tax receipts; consumption = GNP - investment - government purchases; the gross quarterly

¹⁹This procedure is justified by introducing exogenous growth into the theoretical structure. See King and Rebelo (1986) for an example of this approach.

inflation rate in personal consumption expenditures deflator (all from the Department of Commerce, Bureau of Economic Analysis, National Income and Produce Accounts); the gross nominal three-month Treasury bill rate, at a quarterly rate (from the Board of Governors of the Federal Reserve System).

The spread between the nominal interest rate and inflation reported in Table 1 implies a quarterly real interest rate of 1.002, which requires $\beta = .998$. After matching other aspects of U.S. data, this real rate leads to a debt-output ratio of 43.0, which is unlike the U.S. post-war experience. Consequently, the real rate was increased to make the debt-output ratio more reasonable.

The steady state inflation rate was set by altering the constant term in the monetary authority's interest rate rule appropriately. The revenue-GNP and purchases-GNP ratios were maintained similarly by changing the intercept terms in the fiscal authority's tax and spending rules.

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International Finance Discussion Papers

<u>IFDP NUMBER</u>	<u>TITLES</u> <u>1989</u>	<u>AUTHOR(s)</u>
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