MODELING THE DEMAND FOR NARROW MONEY
IN THE UNITED KINGDOM AND THE UNITED STATES

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ABSTRACT

Despite the importance of well-specified empirical money-demand functions for inference, forecasting, and policy, problems in modeling have arisen concerning the economic theories of money demand, the data, institutional frameworks, financial innovation, and econometric implementation. By developing constant, data-coherent M₁ demand equations for the UK and the US, we investigate these issues and explain such puzzles as "missing money", the great velocity decline, and the recent explosion in M₁. The endogeneity of money, the Lucas critique, and the non-invertibility of our M₁ models are also discussed.

Key words and phrases: conditional models, encompassing, error-correction, exogeneity, feedback, feed-forward, invariance, Lucas critique, money demand, predictive failure.
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1. Introduction

A. An Overview

Empirical econometric models of the demand for money, of which \( M_1 \) equations in both the UK and the US are now classic examples, have been a major focus of interest since the early 1970s.\(^1\) Despite its importance for inference, forecasting, and policy, empirical parameter constancy has proved elusive, as documented by the predictive failure of many estimated money-demand equations during the periods of "missing money", "great velocity decline", and the recent explosion in \( M_1 \). The magnitude of the task confronting the applied econometrician is well illustrated by Figures 1a and 2a (for the UK) and 1b and 2b (for the US), which graph real \( M_1 \) over samples excluding and including 1984–1989. The large variability in Figures 1a and 1b is dwarfed by the end-of-sample increases in real \( M_1 \) for both countries in Figures 2a and 2b, and puts into perspective the missing-money episode of 1974–1976. The main objective of this paper is to develop constant-parameter models of \( M_1 \) characterizing the entire sample for both countries.

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Figure 1a. The log of the real money stock \([(m-p)_t]\) in the United Kingdom over 1963(1)-1983(4).

Figure 1b. The log of the real money stock \([(m-p)_t]\) in the United States over 1959(1)-1983(4).
Figure 2a. The log of the real money stock \([(m-p)\_t]\) in the United Kingdom over 1963(1)-1989(2).

Figure 2b. The log of the real money stock \([(m-p)\_t]\) in the United States over 1959(1)-1988(3).
Nonconstant empirical equations do not preclude a constant underlying money-demand function. Thus, in the absence of a constant-parameter econometric model, an unresolved issue is whether observed predictive failure is due to shifts in the demand function or simply mis-specification in econometric models thereof. Mis-specification is necessary but not sufficient for parameter nonconstancy. When marginal processes are subject to regime shifts (such as the movement from fixed to floating exchange rates, the "oil crisis", and changes in monetary policy), both valid conditioning and correct dynamic specification are critical for parameter constancy. Consequently, it is essential to examine the legitimacy of conditioning and to investigate general dynamic specifications. To that end, we implement recently proposed tests for super exogeneity and parameter invariance. Further, we adopt a less restrictive lag structure than the conventional partial adjustment model and instead use an error-correction mechanism with a term representing past disequilibria, related to the theory of cointegration.

Economic theory and the correspondence of theory variables to the available data also play central roles in empirical modeling. Most empirical models are derived from theoretical frameworks involving many dynamic latent variables which have to be proxied in practice. For example, the general form of a money-demand equation relates real money to real income, a vector of interest rates, inflation, and other potential determinants which vary between investigators. Presently, the latent variable most in doubt is the measure of money itself, but the measures of income, inflation, and the opportunity cost of holding money pose problems as well. Additionally, other unobservables like "risk", "financial innovation", and "learning" may matter.

Below we present congruent error-correction models of \( M_1 \) demand in the UK and the US over the past three decades. We are not concerned here with how these models were developed, but focus on their properties, especially their constancy over recent data. Once established as congruent, these models serve to explain several puzzles in the literature, and provide insights into the role of money demand for policy.

The remainder of this introduction touches upon the underlying economic theories, recent financial innovation, monetary policy, and econometric implementation. We take
the economic theory as well established and have no new contributions to make, except to point out (in later sections) some aspects of empirical relevance that theory models might incorporate. We take the institutional framework as known and only comment on features immediate to our work. We comment on policy only as it bears on our empirical analysis or is implied by our findings. See Goodhart (1989) for a recent international overview of institutions and policy.

Section 2 considers data and its measurement for the UK and the US. Section 3 raises various issues and puzzles apparent in money-demand studies, focusing on the non-constancy of conventional equations. Section 4 develops a congruent empirical model for the UK data, and establishes the super exogeneity of prices, incomes, and interest rates for the parameters in our constant money-demand equation. Several testable implications follow: the Lucas critique is refuted, the money-demand function is not invertible to obtain a constant equation for prices, and the parameters of the money-demand equation are invariant to changes in the processes for prices, incomes, and interest rates. A data-based forward-looking re-interpretation of error-correction models is suggested. Section 5 analyzes the US data in a similar fashion, drawing heavily upon results in Baba, Hendry, and Starr (1990). Section 6 concludes.

B. Economic Theories of Money Demand

Two conceptually distinct justifications for holding money commonly appear in the literature: transactions demand, with money held as a medium of exchange; and asset (or portfolio balance or speculative) demand, with money as one of several possible assets in which wealth may be held. In practice, agents’ money demand probably depends on both factors.

The transactions-demand theory is based on the need for money to even out the differences between income and expenditure streams. Thus, the aggregate real quantity of money demanded (i.e., nominal money demanded ($M_d$) divided by an appropriate price level ($P$)) is an increasing function of some measure of the volume of real transactions ($Y$). Real Gross National Product (GNP) and Total Final Expenditure (TFE) are two common measures of the volume of transactions, albeit each with caveats; cf. Section 2. Further,
money demand declines as the opportunity costs of holding money increase, with the latter depending upon the returns to the alternative forms in which wealth might be held.\footnote{See Baumol (1952) and Tobin (1956) for detailed development of the transactions demand theory. Miller and Orr (1966), Clower (1967), Ando and Shell (1975), Akerlof (1979), Gale (1982, 1983), Milbourne (1983), and Smith (1986) inter alia provide further developments, some including "cash-in-advance" models which give rise to a transactions demand. Laidler (1984) has a related discussion on money as a "buffer stock".} Other assets typically include various less liquid financial instruments (e.g., bonds, deposit accounts in banks, money market mutual funds, etc.), so the cost of holding money is the interest foregone. Thus, we have:

\begin{align}
(1a) \quad M^d/P &= h(Y,R),
\end{align}

where $R$ is a vector of interest rates on the alternatives to money, and $h(\cdot, \cdot)$ is increasing in $Y$ and decreasing in the elements of $R$. Often, much more specific functional forms are adopted, e.g.,

\begin{align}
(1b) \quad m^d-p &= \delta \cdot y + \gamma R,
\end{align}

where (here and elsewhere) variables in lower case are in logarithms. The parameters in $\gamma$ are negative, and $\delta=0.5$ in Baumol and Tobin’s transactions demand theory or $\delta=1$ in Friedman’s quantity theory of money.\footnote{The difference in the value of $\delta$ is only one of many contrasts between these two theories.} With nonzero costs to moving between assets, (1b) may require returns on all relevant alternative assets, rather than some summary measure. If components of the measure of money bear interest, the associated interest rates should also appear in $R$ and the corresponding elements in $\gamma$ should be positive.

Additionally, credit facilities may affect the transactions demand for money.

In the portfolio balance approach, money is one of many alternative forms of holding wealth, and each form has its own explicit return (i.e., interest and capital gains) and implicit return (i.e., nonpecuniary). For money, transactions services are presumably

\footnote{Friedman (1956) and Friedman and Schwartz (1982) describe Friedman’s modern version of the quantity theory of money. Money is treated as an ordinary commodity demanded both by producers (to improve the efficiency of their financial transactions) and by consumers (to smooth out differences in timing between expenditure and income streams and to reduce risk). Friedman’s analysis brings him to a money-demand equation much like (1a). Cf. Desai (1981).}
included in the latter. Individuals choose the composition of their portfolio of assets (including money) so as to maximize the returns they expect from that portfolio. Different assets have different expected returns and different degrees of uncertainty associated with their return, and individuals choose their portfolio to balance more certain but lower returns with higher but riskier ones. The resulting money-demand function is similar to (1a), but wealth rather than income or expenditure is the "scale" variable, and some measure of the volatility of alternative assets' returns enters in addition to their expected returns; cf. Tobin (1958) and Walsh (1984). In some theories, money is a dominated asset, but this result need not hold if borrowing and lending interest rates differ; cf. Baba, Hendry, and Starr (1985).

Dynamic adjustment is characterized by a contingent planning model of the form:

\[
\Delta (m-p)_t = \mu_0(L)\Delta (m-p)_{t-1} + \mu_1(L)\Delta p_t + \mu_2(L)\Delta y_t + \mu_3(L)\Delta R_t \\
+ \mu_4[(m^d-p)_{t-1} - (m-p)_{t-1}] + \epsilon_t,
\]

where \(\mu_i(L)\) (\(i=0,\ldots,3\)) are finite polynomials in the lag operator \(L\), \(\epsilon_t\) is the deviation of the outcome from the plan, and \(m^d_{t-1}\) denotes the long-run target value \(m^d\) in (1b) evaluated at \([p,y,R]=[p_{t-1},y_{t-1},R_{t-1}]\). Equation (2) is an error-correction model, and so is a re-parameterization of an autoregressive-distributed lag model of \([m,p,y,R]\) (i.e., in levels). It generalizes the conventional partial-adjustment model, allows separate rates of reaction to the different determinants of money demand (reflecting potentially different costs of adjustment and of disequilibrium), yet via the error-correction term in square brackets ensures that the long-run target (1b) is achieved in steady state. Economically, (2) is related to a theory of money adjustment in which the short-run factors determine money movements given desired bands, and the longer-run factors influence the levels of the bands themselves: see Miller and Orr (1966), Akerlof (1979), Milbourne (1983), and Smith (1986). To be interpretable as a demand equation, \(\mu_1(1)\leq 0, \mu_2(1)\geq 0, \mu_3(1)\leq 0\) for \(R_j\) on assets outside \(M_1\), and \(\mu_3(1)\geq 0\) for \(R_j\) on assets inside \(M_1\); and for cointegration \(\mu_4<0\). Whether or not the equality holds for a given polynomial has implications for a forward-

\[5\text{Individuals also may have different attitudes towards risk }\text{per se.}\]
looking interpretation of (2); see Section 4.C below. The sign, magnitude, and number of individual lag polynomial coefficients must be data-based since economic theory is generally uninformative on those aspects of (2).

While the determinants of money demand are central to modeling observed money holdings, the role of the "money supply" is also an issue. Often in macroeconomic models, an observed money stock (supposedly set by policy makers) is equated to money demand, from which prices (or interest rates, or income) are determined by "inverting" the money-demand function. For example, Barro (1987, pp. 128ff, 195ff) inverts to obtain a price equation in a simple theoretical supply and demand model; and Edison, Marquez, and Tryon (1987, pp. 130–131) and Fair (1984, pp. 319–323) invert to obtain interest-rate equations in large empirical macro-models. Alternatively, a policy reaction function may make some interest rate endogenous. The empirical validity of these structures may be investigated via tests of super exogeneity, as discussed below.

To summarize, money may be demanded for at least two reasons, as an inventory to smooth transactions and as one of several assets in a portfolio.

C. Financial Innovation

The United Kingdom and the United States have witnessed numerous financial innovations changing the meaning of money; cf. Desai and Low (1987) and Hall, Henry, and Wilcox (1989). In the UK during the 1980s, both building societies and commercial banks introduced checkable interest-bearing accounts, with accounts at the former being outside of M₁ and those at the latter within. In the US, interest-bearing checking accounts appeared first as NOW accounts (nationally around 1981) and then as SuperNOW accounts (1983). New liquid assets were introduced outside of US M₁ as well, such as small certificates of deposit (CDs) by commercial banks in 1965 and money market mutual funds (MMMF) in 1974. All of these innovations, and the ways in which agents learn and adapt to use them, potentially affect the specification of money-demand functions.

D. Monetary Policy

Monetary policy in both countries has changed substantively over the last few decades, with the factors determining policy agency behavior of possible importance to
some models of money demand. The UK has seen the introduction of Competition and Credit Control (1971, breaking up the commercial banks' cartel), the corset's removal, tight monetary control and targeting under Thatcher, and de facto abandonment of such targeting by the mid-1980s. The US has experienced the enactment and removal of the New Operating Procedures. Money itself has become "internationalized", more clearly so in the UK with removal of exchange controls. Central banks have taken an active concern in the deregulation of financial markets. See Hawtrey (1938) and Goodhart (1984, 1989) for comprehensive analyses of UK and US monetary policy in an historical perspective.

E. Econometric Implementation

Empirical modeling usually aims to characterize data properties in simple, reasonably constant, parametric relationships which account for the findings of preexisting studies and are interpretable in the light of the economic theory at hand. Testing helps determine how well empirical models meet those ends. Since a congruent model must have constant parameters, weakly exogenous regressors, and innovation errors, and must encompass rival explanations, we will consider the following issues.

(i) Parameter constancy. The lack of constancy in conventional money-demand models is well-documented, with "explanations" including structural change, regime shifts, and the Lucas critique. Conversely, a conditional money-demand model which remained constant in spite of structural change elsewhere in the economy would be both economically and statistically appealing.

(ii) Exogeneity. If the money-demand coefficients are constant but the process of the conditioning variables changes (e.g., due to regime shifts), then super exogeneity of the conditioning variables, invariance of the associated parameters, and the endogeneity of money are implied. Cf. Engle, Hendry, and Richard (1983) and Engle and Hendry (1989).

(iii) Expectations formation. If agents' behavior depends upon forward-looking model-based expectations, super exogeneity generally is precluded. In the literature, prices, incomes, and interest rates have all been proposed as candidates for appearing in money-demand equations as expectations rather than observed values; cf. Laidler (1985, pp. 86ff) and Goldfeld and Sichel (1990, pp. 335–336, 345ff).
(iv) **Lucas critique.** Further, if the expectations formation process is ignored in modeling, the Lucas critique potentially applies, linking (i) to (iii). However, it is also possible to refute the Lucas critique empirically, in which case the role of model-based expectations formation in agents' decision-taking is ruled out. Cf. Lucas (1976), Cuthbertson (1988), Hendry (1988), Favero and Hendry (1989), and Ericsson and Hendry (1989).

(v) **Invertibility of the money-demand function.** This is precluded by super exogeneity. Cf. Hendry (1985).

(vi) **Identifiability.** Super exogeneity can identify parameters by establishing uniqueness via invariance. Coefficient estimates of (e.g.) the own rate can help identify whether the model being estimated is of supply or demand, thereby aiding interpretation of the results.

(vii) **Nonstationarity of the data.** This may exist in at least three nonexclusive forms: integratedness, such as random walks; regime shifts; and inherent (non-ergodic). The first leads to issues of cointegration; the latter two play roles in testing for super exogeneity.

(viii) **Cointegration and error correction.** Integration of data series raises the possibility of cointegration between series. Error-correction models such as (2) map one-for-one with cointegration, with implied long-run relationships between the variables of concern. If the cointegration vector appears in more than one of the equations determining the set of cointegrated variables, weak (and so super) exogeneity is lost, with implications for (ii). Also, for sets of three or more series, more than one cointegration vector may exist. Cf. Sargan (1964), Davidson, Hendry, Srba, and Yeo (1978), Salmon (1982), Hendry (1986), Engle and Granger (1987), Johansen (1988), Phillips (1988), Phillips and Loretan (1989), Hylleberg and Mizon (1989), and Johansen and Juselius (1990).

(ix) **Ceteris paribus clauses of theories.** The economic theory upon which an empirical model is based often contains many *ceteris paribus* conditions which may not
hold in practice. Thus, one of the tasks of the econometrician is to choose an empirical model which both embodies the economic theory (in order to interpret the model itself) and allows for the presence of any significant factors not fully specified by the economic theory. Such factors include dynamic specification, functional form, and an extended menu of observed variables. Cf. Hendry, Pagan, and Sargan (1984) on the first of these.

(x) Model testing and design. It is important to check that an empirical model satisfies the assumptions made at the outset; otherwise, interpreting parameter estimates according to the underlying theory may be misleading. Conversely, it may be possible to design models to have certain desirable properties. Cf. Hendry (1983). White (1988, 1990) provides a statistical theoric basis for using test statistics as design criteria.


2. Data
   A. Issues

The measurement of all the data in the money-demand function has come under considerable scrutiny. The actual choice in any particular empirical study reflects the theory(s) discussed and the data available. The definition of money itself may be rather narrow (as for transactions demand) or broader (e.g., including relatively liquid substitutes like savings accounts, which are not excluded by the asset approach). And, substitutability may exist between different types of money. Cf. Barnett (1980) and Simpson and Porter (1980). Prices may reflect the consumers' or the producers' point-of-view, or both, depending on the money holdings which one is trying to model. Further, aggregate indices

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6For instance, many economic theory models assume known and constant stochastic processes for the uncontrolled variables (e.g., for income, interest rates, and prices in a money demand equation). Such an assumption in effect removes the uncertainty from the problem, and can be interpreted as narrowing the applicability of the resulting theory to an equilibrium world: that is, one which is stationary, essentially certain, and devoid of problems like evolving seasonality, changes in tastes or governmental policy, and so on. Nevertheless, the resulting equations help constrain the equilibrium solutions of the empirical model as well as to indicate relevant variables and the parameterizations of interest.
may suffer from changes in relative prices. The scale variable is either *income* (or *expenditure*) or *wealth*, depending upon the theory. However, given the paucity of data on wealth in most Western countries, income is often used as a proxy for wealth in portfolio balance models. Income series are not flawless either and may be a poor measure of transactions, as could occur if firms became more vertically integrated. The opportunity cost of holding money is typically one or more *interest rates*. For transactions demand models, it may be the rate for a short-term security which is the closest alternative to holding money (such as a Treasury bill). The asset approach suggests using the returns on a wider set of short- to longer-term financial assets (or even nonfinancial ones) as they are potential substitutes for money as an asset. Clearly, the measure of *volatility* as a proxy for risk will depend upon the measured opportunity costs.

The choice of data reflects the corresponding economic notions involved (which depend upon the theory itself) and the data available. The latter cannot be overemphasized as economists, unlike (e.g.) many natural scientists, usually do not gather their own data, and almost invariably those responsible for collecting it do not have the economists' theories in mind when doing so.

B. *The Data Series and Data Transformations*

Now we consider the data explicitly. To match various previous studies, all data are quarterly and seasonally adjusted. For simplicity, we use the same notation for parallel series in the two countries in so far as possible, even though the precise definitions may differ.

For the UK, the data are $M_1$ (denoted $M$, and including interest-bearing sight deposits), 1985 price real total final expenditure ($Y$), its deflator ($P$), the three-month local authority interest rate ($R_3$, regarded as the dominant short-term interest rate in the secondary market), and the $M_1$ retail sight-deposit interest rate ($R_r$, i.e., the rate on checkable interest-bearing accounts at commercial banks). Money and expenditure are in £ million, the deflator is unity for 1985, and interest rates are in fractions. The data begin in 1963(1), the earliest date for which the Bank of England calculated detailed monetary
statistics, and end in 1989(2), due to a major change in the institutions included as banks.\footnote{In July 1989, the Abbey National Building Society converted to a public limited company (plc) and so was classified as a bank, increasing M\textsubscript{1} by 16\% overnight. In light of this, and because other building societies may convert subsequently, the Bank of England stopped reporting total M\textsubscript{1}. Still, total M\textsubscript{1} can be constructed from its components if desired. Non-interest bearing M\textsubscript{1} continues to be reported, but it clearly is not an adequate measure of transactions demand for money. See the Bank of England Quarterly Bulletin (August 1989, pp. 352–353) and Healey, Mann, Clews, and Hoggarth (1990) for further details.} For details on the UK data, see Appendix A.

For the US, the data are M\textsubscript{1} (M, including NOW and SuperNOW accounts, when available), 1982 price GNP (Y), its deflator (P), the 20-year T-bond yield to maturity (R\textsubscript{T}), the one-month Treasury bill rate (R\textsubscript{t}), the passbook rate (R\textsubscript{p}), the commercial bank small CD rate (R\textsubscript{c}), the money market mutual fund rate (R\textsubscript{m}), and NOW and SuperNOW account rates (R\textsubscript{n} and R\textsubscript{su}). Money and GNP are in $ billion (10\textsuperscript{9}) and $ trillion (10\textsuperscript{12}) respectively, the deflator is 100 for 1982, and interest rates are in fractions. The data begin in 1959(1) and end in 1988(3). See Baba, Hendry, and Starr (1985, 1990) for details on the US data.

We turn to examining the data, doing so with pairs of graphs for the two countries, with the first of the pair (e.g., Figure 1a) generally being for the UK and the second (e.g., Figure 1b) for the US. As will be seen, the data for both countries are remarkably similar in many respects, albeit with data fluctuations for the UK usually being twofold or even threefold larger than those for the US. Thus, the data will be described generically, only explicitly referring to countries when necessary. Capital letters denote both the generic name and the level; logs of scalars are in lower case.

Figures 3, 4, and 5 respectively show the levels of m and p (adjusted so their means are equal), the growth rates \Delta m and \Delta p cross-plotted over the whole sample, and those growth rates plotted over the 1980s. From Figure 3, m and p deviate from each other for long periods, but do cross occasionally, with strong upward trends in both series and so strong positive correlations between the series. Figure 4 presents a contrasting short-run picture. We have fitted regressions to each of ten approximately equal-length subsamples of the data, and virtually every possible correlation between the growth rates of money and
Figure 3a. The log of the nominal money stock ($m_t$) and the log of the implicit price deflator ($p_t$) in the United Kingdom.

Figure 3b. The log of the nominal money stock ($m_t$) and the log of the implicit price deflator ($p_t$) in the United States.
Figure 4a. Cross-plot of $\Delta m_t$ against $\Delta p_t$ for the United Kingdom over 1963(2)-1989(2) with ten sub-sample regression lines.

Figure 4b. Cross-plot of $\Delta m_t$ against $\Delta p_t$ for the United States over 1959(2)-1988(3) with ten sub-sample regression lines.
Figure 5a. Cross-plot of $\Delta m_t$ against $\Delta p_t$ for the United Kingdom over 1980(1)-1989(2) with one regression line.

Figure 5b. Cross-plot of $\Delta m_t$ against $\Delta p_t$ for the United States over 1980(1)-1988(3) with one regression line.
prices can be observed over some subsample. From Figure 5, the growth rates for the 1980s are negatively correlated. Such contrasts in short- and long-run behavior immediately preclude certain classes of models from explaining the data, e.g., partial adjustment models with equal (unit) short- and long-run price elasticities.

From Figure 6, the persistent high positive growth rates of real money in the 1980s are themselves unprecedented. While sometimes regarded as the bane of econometric modelers, such dramatic changes in data properties serve to filter out unsatisfactory models, leaving more robust ones for further analysis. We will use the data in this manner below.

Income is required to create velocity: Figure 7 graphs the growth rate of the former, and Figure 8 the (log) inverse level of the latter. From Figure 7a, several episodes in the UK are evident: Heath's push for growth (1973), the subsequent decline from the first oil price shock (1974–1976), and Thatcher's monetarist "experiment" (1980). Effects from both oil price shocks are evident in the US data. From Figure 8, the money-income ratio in each country falls by approximately 50% from the beginning of the sample until 1982. In the UK, it increases thereafter and especially rapidly after 1985, returning to pre-1973 levels by the end of the sample. In the US, the increase is smaller and is temporarily offset by a fall in 1983–1984.

Figures 9a and 9b graph the annual inflation rate and the short-term interest rate of the UK and US respectively. Noting the approximate factor of two between the two figures, the inflation rates have similar patterns, with the two oil price increases evident, and the subsequent falls to single-digit (UK) or lower single-digit (US) rates in the mid- to late-1980s. Short-term interest rates are less similar, with high nominal rates in the 1980s persisting much longer in the UK. Figures 9a and 9b also reveal a considerably changing covariance between inflation and interest rates over time, which could be interpreted as large negative ex post real interest rate during the mid-1970s and a sustained historically large positive real rate during much of the 1980s. Figure 10a graphs the UK local authority interest rate and M_1 retail sight-deposit interest rate, the latter being introduced in 1984(3) and mimicking the former with a relatively constant spread. Figure 10b shows
Figure 6a. The annual growth rate of the real money stock $[\Delta_4(m-p)_t]$ in the United Kingdom.

Figure 6b. The annual growth rate of the real money stock $[\Delta_4(m-p)_t]$ in the United States.
Figure 7a. The annual growth rate of real income ($\Delta_4 y_t$) in the United Kingdom.

Figure 7b. The annual growth rate of real income ($\Delta_4 y_t$) in the United States.
Figure 8a. The log of the ratio of $M_1$ to nominal income in the United Kingdom.

Figure 8b. The log of the ratio of $M_1$ to nominal income in the United States.
Figure 9a. The annual inflation rate ($\Delta_4 p_t$) and the three-month local authority rate ($R3$) in the United Kingdom.

Figure 9b. The annual inflation rate ($\Delta_4 p_t$) and the one-month Treasury bill rate ($R1$) in the United States.
Figure 10a. The three-month local authority rate (R3) and the M1 retail sight-deposit rate (Rr) in the United Kingdom.

Figure 10b. The one-month Treasury bill rate (R1), the NOW account rate (Rn), and the SuperNOW account rate (Rsu) in the United States.
the corresponding graph for the US based on the one-month T-bill rate and the interest rates on NOW and SuperNOW accounts.

To model the adaptation following financial innovation, we adopt the approach proposed in Baba, Hendry, and Starr (1985, 1990) of learning adjustment on own interest rates as well as interest rates on non-transactions M₂. Each new interest rate is multiplied by an ogive-shaped weighting function \( \{w_t\} \) to represent agents’ learning about the corresponding asset. The function is \( w_t = (1 + \exp[\alpha - \beta(t-t^*+1)])^{-1} \) for \( t \geq t^* \) and zero otherwise, where \( t \) is time, \( t^* \) is the date of introduction of the account, and \( \alpha \) and \( \beta \) correspond to initial knowledge and rate of learning. Interest rates weighted by this function are denoted with a suffix "a" for adjusted. As a first attempt at capturing the effects of learning, Baba, Hendry, and Starr (1990) set \( \alpha = 7 \) and \( \beta = 0.8 \) for the US, implying \( w_t = 0.50 \) after two years and \( w_t = 0.99 \) after 3\( \frac{1}{2} \) years. Additionally, they find that the learning-adjusted NOW and SuperNOW rates entered their equations with equal coefficients, so we use the average of those rates, denoted Rnsa and shown in Figure 11b. Because interest-bearing retail sight deposits appeared in the UK subsequent to the introduction of similar US accounts, we set \( \alpha = 5 \) and \( \beta = 1.2 \) for the UK, implying higher initial knowledge and more rapid learning, with \( w_t = 0.50 \) after one year and \( w_t = 0.99 \) after two years. Figure 11a plots the resulting interest rate Rra. Empirically, the models appear insensitive to the choice of \( \alpha \) and \( \beta \); see Appendix B for estimates of \( \alpha \) and \( \beta \) for the UK and the US.

Finally, before turning to empirical puzzles and issues, many of which stem from properties of the data described above, we consider the primary feedback variables in the error-correction mechanisms (Figure 12). In light of our and others’ studies, these differ for the two countries, following Baumol and Tobin’s "square-root" law in the US to give \( (m-p-\frac{1}{2}y) \) and the quantity-theory unit income coefficient for the UK to give \( (m-p-y) \). Although the two countries’ velocities are not so similar, these feedback variables are very much alike. Baba, Hendry, and Starr (1985, 1990) further discuss the learning adjustment and error-correction term for the US, as do Hendry (1979, 1985) and Hendry and Mizon (1989) the error-correction term for the UK.
Figure 11a. The learning-adjusted own interest rate on $M_1$ retail sight deposits in the United Kingdom (Rra).

Figure 11b. The average of learning-adjusted NOW and SuperNOW account interest rates in the United States (Rnsa).
Figure 12a. The primary feedback variable \((m-p-y)_{t-1}\) in the model of demand for \(M_1\) in the United Kingdom.

Figure 12b. The primary feedback variable \((m-p-y)_{t-2}\) in the model of demand for \(M_1\) in the United States.
3. **Empirical Puzzles and Issues**

A. **Puzzles and Issues**

Goldfeld and Sichel (1990) neatly summarize the key puzzles and issues.

As has been widely documented, especially for the United States but elsewhere as well, matters have been considerably less satisfactory since the mid-1970s. First, there was the episode of the "missing money" when conventional money demand equations systematically overpredicted actual money balances. Moreover, attempts to fit conventional demand functions to a sample that included the missing money period invariably produced parameter estimates with some quite unreasonable properties. Second, in the 1980s, U.S. money demand functions, whether or not fixed up to explain the 1970s, generally exhibited extended periods of underprediction as observed velocity fell markedly. (p. 300)

In a somewhat different categorization, we distinguish the following five salient issues.


(ii) **The exogeneity or endogeneity of money.** Cf. Howe (1980, pp. 68, 75), Judd and Scadding (1982), and Friedman and Schwartz (1982) *inter alia*. This is discussed in many papers and is a major focus of Engle and Hendry (1989) and Hendry and Ericsson (1990) for the UK. The solution we adopt depends upon (iii).
(iii) The (non-)invertibility of existing models. Since inversion of money-demand equations to obtain price equations is commonplace in macroeconomics but would be precluded if prices were super exogenous for the parameters of the money-demand function, the validity of such procedures should be examined. Cf. Laidler (1985), Barro (1987), Friedman and Schwartz (1982), Hendry (1985), and Hendry and Ericsson (1990).

(iv) Long-run and short-run determinants. As Hendry (1979), Rose (1985), and Gordon (1984) demonstrate, dynamic specification can be critical to the constancy of money-demand models. Relatedly, the speed of adjustment to changes in the environment is of independent economic interest. Finally, both the total and partial interest rate elasticities of money demand are of potential policy consequence.

(v) Causal links. Money, prices, incomes, interest rates, and exchange rates may be causally linked, possibly in several directions. Understanding such linkages is central to overall economic policy, and generally requires a systems approach; cf. Hendry and Ericsson (1986), Johansen (1988), and Johansen and Juselius (1990). Here we focus on money-demand equations alone, given the evidence on weak exogeneity in the cointegration analysis of Hendry and Mizon (1989) for the UK.

Although these issues are discussed in an economic context, they parallel the econometric issues in Section 1.E, albeit being more condensed. Thus, the economic implications of econometric issues (and the converse) form the centerpiece of this paper.

In summary, the major issues in the 1980s are: the introduction of interest-bearing accounts in M1, dramatic increases in real M1, increased interest-rate volatility, falling inflation, switches in monetary control policy, deregulation in financial markets, the nonconstancy of (some) existing models, the role of expectations, identifiability and invertibility of M1 demand models, and the policy role of M1.

B. Solutions

The most satisfactory way to resolve the puzzles and issues summarized above is with a congruent model, that is, one which captures the salient features of the existing data and is interpretable in light of available economic theory. With such a model, puzzles are explained as (and implied by) the mis-specification of other models, i.e., the models from
which the puzzles arose. Although the quote above from Goldfeld and Sichel suggests that no such models exist, we disagree, and seek to establish our case in the next two sections.

Beginning with Hendry's (1979, 1988) model of UK money demand, Section 4 below develops a congruent model of money demand for the UK, examines its properties, and considers how that model explains various anomalies apparent in other models of money demand. Following a parallel structure, Section 5 reports a model of US money demand based upon that in Baba, Hendry, and Starr (1985).

4. An Empirical Model of UK Money Demand

Hendry (1979) develops a constant, parsimonious error-correction model of UK money demand over 1964(1)–1977(4), using 1970 price Total Final Expenditure (TFE) as the scale variable. Subsequent money-demand models for the UK by Trundle (1982), Hendry (1985), Davidson (1987), Cuthbertson (1988), and Hendry (1988) are similar in form and in numerical parameter values, with the main differences arising from using different data sets. For instance, in the last case, Hendry (1988, equation (26)) re-estimates his 1979 model, slightly revised and simplified in light of a new (1980 price) TFE series, and finds that the model's coefficients are still constant over the extended sample of 1964(3)–1979(4).

A. Replication, Specification, and Constancy

We begin by replicating his 1988 model with the most recent (1985 price) TFE series.

\[
\Delta (m-p)_t = 0.28 \Delta y_{t-1} - 0.80 \Delta p_t - 0.31 \Delta (m-p)_{t-1} \\
- 0.63 R^3_t - 0.102 (m-p-y)_{t-2} + 0.022 \\
\]

\[
T = 62 \quad [1964(3)–1979(4)] + 12 \text{ forecasts} \quad R^2 = 0.69 \quad \hat{\sigma} = 1.401\% \quad DW = 2.05
\]

Chow F[12, 56] = 0.42 Forecast \( \chi^2[12]/12 = 0.45 \)

Normality \( \chi^2[2] = 2.33 \quad \text{AR} 1-4 \quad F[4, 52] = 0.71 \quad \text{ARCH} 1-4 \quad F[4, 48] = 0.65 \)

\( X^2 F[10, 45] = 0.97 \quad \text{RESET} F[1, 55] = 0.15 \)

[\cdot\cdot\cdot] denotes heteroscedasticity-consistent estimated standard errors; see White (1980),
Table 1. Some Criteria for Evaluating and Designing Econometric Models

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Statistic</th>
<th>Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td>first-order residual autoregression</td>
<td>DW</td>
<td>Durbin and Watson (1950, 1951)</td>
</tr>
<tr>
<td>q^{th}-order residual autoregression</td>
<td>AR 1-q $\chi^2[q]$; AR 1-q $F[q,T-k-q]$</td>
<td>Box and Pierce (1970); Godfrey (1978), Harvey (1981, p. 173)</td>
</tr>
<tr>
<td>q invalid parameter restrictions; non-innovation errors</td>
<td>INN $F[q,T-k-q]$</td>
<td>Johnston (1963, p. 126)</td>
</tr>
<tr>
<td>q^{th}-order ARCH</td>
<td>ARCH 1-q $\chi^2[q]$, ARCH 1-q $F[q,T-k-2q]$</td>
<td>Engle (1982)</td>
</tr>
<tr>
<td>q^{th}-order RESET</td>
<td>RESET $F[q,T-k-q]$</td>
<td>Ramsey (1969)</td>
</tr>
<tr>
<td>parameters not constant over subsamples</td>
<td>COV $F[k,T-2k]$</td>
<td>Fisher (1922), Chow (1960, pp. 595ff)</td>
</tr>
<tr>
<td>predictive failure over a subset of q observations</td>
<td>Forecast $\chi^2[q]$</td>
<td>Hendry (1979); Chow (1960, pp. 594-595)</td>
</tr>
<tr>
<td>Chow $F[q,T-k-q]$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes.

a. There are T observations and k regressors in the model under the null. The value of q may differ across statistics, as may those of k and T across models and samples.

b. *abbrev* $\chi^2[q]$ and *abbrev* $F[q,r]$ denote statistics with abbreviated name *abbrev* and which have central $\chi^2[q]$ and $F[q,r]$ distributions respectively under a common null and against the ostensible alternative for that test statistic. Thus, AR 1-q $\chi^2[q]$ and AR 1-q $F[q,T-k-q]$ both test for q^{th}-order residual autocorrelation.

c. We note the importance of the Chow statistic Chow $F[q,T-k-q]$ both in relationship to the issue of constancy in the substantive debate on monetary behavior and because of its crucial role as an indirect test of weak exogeneity through testing the conjunction of hypotheses embodied in super exogeneity. The covariance test statistic COV $F[k,T-2k]$ is often (and confusingly) referred to as the "Chow statistic" although Chow (1960, p. 592) was well aware of its presence in the literature.
Nicholls and Pagan (1983), and MacKinnon and White (1985). See Table 1 for definitions of the statistics; Engle (1984) provides a framework for their derivation. The corresponding coefficients and equation standard error (\( \hat{\sigma} \)) in Hendry (1988, (26)) are 0.33, -0.79, -0.34, -0.71, -0.10, 0.030 and \( \hat{\sigma} = 1.38\% \) respectively, matching closely those in (3) and indicating that there are no substantive differences between the new and old data for this sample.

When the data include the first half of the 1980s, equation (3) fits equally well and the coefficient estimates remain virtually unchanged.

\[
\Delta(m-p)_t = 0.27 \Delta y_{t-1} - 0.77 \Delta p_t - 0.27 \Delta(m-p)_{t-1} - 0.59 \text{R3}_t - 0.093(m-p-y)_{t-2} + 0.021 \\
T = 84 [1964(3)–1985(2)] + 16 forecasts \quad R^2 = 0.67 \quad \hat{\sigma} = 1.409\% \quad DW = 2.03 \\
\text{Chow F}[16, 78] = 4.72 \quad \text{Forecast } \chi^2[16]/16 = 10.04 \\
\text{Normality } \chi^2[2] = 2.73 \quad \text{AR 1–4 } F[4, 74] = 1.15 \quad \text{ARCH 1–4 } F[4, 70] = 0.52 \\
X_{1}\chi^2 F[10, 67] = 1.44 \quad X_{1}*X_{1} F[20, 57] = 0.79 \quad \text{RESET } F[1, 77] = 0.39
\]

This closely matches Hendry's (1988, p. 147) regression with \( \hat{\sigma} = 1.33\% \) for the same sample period, but with the coefficient on current inflation constrained to be minus unity.

In spite of its historical constancy, (4) exhibits predictive failure over the remainder of the 1980s. Figure 13a plots the actual and fitted values over the estimation period, and the actual and forecast values over the sixteen observations 1985(3)–1989(2).\(^8\) Figure 13b details the latter period, adding bands of plus-or-minus twice the forecast standard error (denoted \( \pm 2\sigma \) in the graphs) to each forecast for individual approximate 95% confidence intervals. The corresponding Chow (1960) statistic is \( F[16,78]=4.72 \), and reflects the massive under-prediction by (4), averaging 4.3% per quarter.

This predictive failure has a simple explanation, and one derivable from the economic theory on which the model was based: the local authority interest rate R3 no

\(^8\)Here and elsewhere, forecasts are "ex post". The forecast for period s is \( \hat{y}_s = x_s' \hat{\beta} \) in a standard notation, where \( x_s \) is the observed value of \( x \) for period s, \( \hat{\beta} \) is estimated from the first t observations of data, and s>t.
Figure 13a. Equation (4): actual, fitted, and forecast values of $\Delta(m-p)_t$ in the United Kingdom.

Figure 13b. Equation (4): one-step ahead forecasts of $\Delta(m-p)_t$ in the United Kingdom, with $\pm 2$ forecast standard errors.
longer represents the opportunity cost of holding money. Rather, with the introduction of interest-bearing \( M_1 \) retail sight deposits, the corresponding interest rate \( R_r \) now plays a role. In fact, inclusion of the (learning-adjusted) \( M_1 \) retail sight-deposit interest rate \( R_{ra} \) in (4) is sufficient to explain the rapid growth of \( M_1 \) in the forecast period, as the following equation indicates.\(^9\)

\[
(5) \quad \Delta (m-p)_t = \frac{0.25 \Delta y_{t-1}}{[0.12]} - \frac{0.70 \Delta p_t}{[0.15]} - \frac{0.30 \Delta (m-p)_{t-1}}{[0.07]} - \frac{0.63 R_{3t}}{[0.07]} + \frac{0.74 R_{ra_t}}{[0.33]} - \frac{0.094 (m-p-y)_{t-2}}{[0.009]} + \frac{0.023}{[0.005]}
\]

\( T = 84 \) [1964(3)–1985(2)] + 16 forecasts \( R^2 = 0.69 \) \( \sigma = 1.374\% \) DW = 2.12

Chow \( F[16, 77] = 0.59 \) Forecast \( \chi^2[16]/16 = 0.76 \)

Normality \( \chi^2[2] = 1.88 \) AR 1–4 \( F[4, 73] = 1.87 \) ARCH 1–4 \( F[4, 69] = 0.64 \)

\( X_{t^2} F[12, 64] = 1.28 \) \( X_t^* X_j F[25, 51] = 0.73 \) RESET \( F[1, 76] = 0.46 \)

Actual, fitted, and forecast values of \( \Delta (m-p)_t \) appear in Figures 14a–b, showing no tendency to mis-predict. The Chow statistic reflects this, being insignificant at 0.59 with a p-value of 0.88. Actual, fitted, and forecast values of the level of real money \( (m-p)_t \) appear in Figures 15a–b, and demonstrate both the accuracy and precision with which (5) forecasts over a period with historically unprecedented levels of \( M_1 \).\(^{10}\)

Goodhart (1986, p. 84), citing work by J. Wilcox at the Bank of England, notes that Trundle's error-correction model for \( M_1 \) (similar to (4)) exhibits predictive failure starting in 1984. That coincides with the rapid growth of interest-bearing \( M_1 \) retail sight deposits. Economic theory predicts qualitatively why predictive failure should occur, and (5) provides a quantitative explanation.

---

\(^9\)Because \( R_{ra_t} \) is nonzero for only a short portion of the estimation sample, its conventional estimated standard error (of 0.33) is reported, rather than White's heteroscedasticity-consistent standard error (of 1.10), noting the latter's empirical tendency towards spurious values with dummies or variables behaving like dummies (such as \( R_{ra_t} \) in this sample). Conventional standard errors are denoted by parentheses (·). A few dummies in equations below appear with conventional rather than White's standard errors for similar reasons.

\(^{10}\)Although forecasts for the level \( (m-p)_t \) could be derived from (5), the forecasts in Figures 15a and 15b were obtained directly by re-expressing the dependent variable as \( (m-p)_t - (m-p)_{t-1} \) and estimating rather than imposing the unit coefficient on \( (m-p)_{t-1} \).
Figure 14a. Equation (5): actual, fitted, and forecast values of $\Delta(m-p)_{t}$ in the United Kingdom.

Figure 14b. Equation (5): one-step ahead forecasts of $\Delta(m-p)_{t}$ in the United Kingdom, with $\pm 2$ forecast standard errors.
Figure 15a. Equation (5): actual, fitted, and forecast values of \((m-p)_t\) in the United Kingdom.

Figure 15b. Equation (5): one-step ahead forecasts of \((m-p)_t\) in the United Kingdom, with ±2 forecast standard errors.
Although interpretable as a generalization of (4), (5) is also obtained directly by simplifying from a general fourth-order autoregressive distributed lag model of \([m, p, y, R_3, R_{ra}]\), for which \(\hat{\sigma} = 1.306\%\) and \(\text{INN } F[18,75] = 1.15\) over the whole sample. Interestingly, (5) can be validly simplified further, noting that the coefficients on \(\Delta(m-p)_{t-1}\) and \(\Delta y_{t-1}\) have virtually equal magnitude, oppositely signed coefficients, resulting in the single term \(\Delta(m-p-y)_{t-1}\). This restriction, which is readily apparent in Hendry (1988, (26)) and (3) and (4) above but was not imposed, has the additional advantage that the resulting model is invariant to whether the error-correction term is at the first or second lag. To simplify interpretation, the first lag is chosen. Finally, the coefficients on \(R_{3t}\) and \(R_{rat}\) in (5) are oppositely signed and approximately equal in magnitude: that suggests reformulating the model in terms of the spread or net opportunity cost \((R_3-R_{ra})_t\), denoted \(R_t^*\).

We now consider the properties of (5) with these changes when estimated over the full sample, 1964(3)–1989(2).

\[
\Delta(m-p)_t = -0.69 \Delta p_t - 0.17 \Delta(m-p-y)_{t-1} \\
-0.630 \hat{\sigma}^{*} - 0.093 (m-p-y)_{t-1} + 0.023 \\
T = 100 \text{ [1964(3)–1989(2)]} \quad R^2 = 0.76 \quad \hat{\sigma} = 1.313\% \quad DW = 2.18
\]

Normality \(\chi^2[2] = 1.53\) \(\text{AR 1–4 F}[4, 91] = 1.94\) \(\text{ARCH 1–4 F}[4, 87] = 0.74\) \(X_t^2 F[8, 86] = 1.36\) \(X_t^{*} X_t^* F[14, 80] = 1.05\) \(\text{RESET F}[1, 94] = 0.08\)

Economically, the coefficients in (6) satisfy the sign restrictions on the short-run dynamics to be interpretable as a money-demand function; cf. Miller and Orr (1966), Milbourne (1983), and Smith (1986). Their values imply large immediate responses to changes in inflation and interest rates, but slow adjustment to remaining disequilibria via the error-correction term, possibly reflecting minimal costs to being out of equilibrium; cf. Akerlof (1979, p. 170). Current inflation enters with a near minus unit coefficient, implying that (6) could be rewritten with \(\Delta m_t\) as the dependent variable and current inflation mattering little for determining nominal money in the short-run, as in Hendry (1979). Income enters only at a lag, and interest rates only currently: these are data-based, data-accept able
restrictions first noted by Trundle (1982), but they need not hold in general; see (2). The regressors correspond to nearly orthogonal decision variables, with none of the six correlations between regressors exceeding two-thirds. That is consistent with (6) representing a contingent plan of agents who partition available information into conceptually separate entities.

Equation (6) has numerous desirable statistical properties. Its residuals are white noise (AR test) and also an innovation process against the information set generated by (2) as a fourth-order autoregressive-distributed lag in the variables used (INN F[20,75]=1.05). Tests of residual ARCH, RESET, and heteroscedasticity (X_i^2 and X_i^*X_j) are insignificant; and the residuals are approximately normally distributed (Normality).

Constancy is an additional, crucial statistical property, particularly in the context of money-demand equations, and will play a role in the related issue of exogeneity. To investigate constancy, we use recursive least-squares, since sequences of constancy tests are easily constructed from the associated one-step innovations and because the sequences of coefficient estimates are both intuitive and informative; cf. Brown, Durbin, and Evans (1975) and Dufour (1982). Graphs efficiently summarize the large volume of output. Figure 16a records the one-step residuals and the corresponding calculated equation standard errors, i.e., \{y_{i t} - \hat{\beta}_i x_{i t}\} and \{0.0\pm 2\hat{\sigma}_i\} in a standard notation, with the latter denoted \pm 2\sigma(t) in the graphs. The equation standard error \hat{\sigma} varies little, and none of the "break-point" Chow statistics for the sequence \{1968(3)\text{--}1989(2), 1968(4)\text{--}1989(2), 1969(1)\text{--}1989(2), ..., 1989(1)\text{--}1989(2), 1989(2)\} is significant at even the 5\% level; see Figure 16b. Figures 16c\text{--}16g show the numerical values of all the coefficients, together with plus-or-minus twice their sequentially estimated standard errors (denoted \beta(t) and \beta(t)\pm 2SE(t) respectively in the graphs) which provide an approximate 95\% confidence interval at each t. Aside from a minor fluctuation in 1969, the coefficients vary by only a fraction of their \textit{ex ante} standard errors. All coefficients are highly significant for all samples extending beyond the early 1970s, and the accrual of information is apparent from the confidence intervals narrowing over time. To summarize, (6) is a constant, data-
Figure 16a. Equation (6): one-step residuals and the corresponding calculated equation standard errors for a model of $\Delta(m-p)_t$ in the United Kingdom.

Figure 16b. Equation (6): sequence of break-point Chow statistics over 1968(3)-1989(2) for a model of $\Delta(m-p)_t$ in the United Kingdom, with the statistics scaled by their one-off 5% critical values.
Figure 16c. Equation (6): recursive estimates of the coefficient of $\Delta p_t$ for a model of $\Delta (m-p)_t$ in the United Kingdom, with $\pm 2$ estimated standard errors.

Figure 16d. Equation (6): recursive estimates of the coefficient of $(m-p_\gamma)_{t-1}$ for a model of $\Delta (m-p)_t$ in the United Kingdom, with $\pm 2$ estimated standard errors.
Figure 16e. Equation (6): recursive estimates of the coefficient of $R_t^*$ for a model of $\Delta(m-p)_t$ in the United Kingdom, with $\pm 2$ estimated standard errors.

Figure 16f. Equation (6): recursive estimates of the coefficient of $\Delta(m-p-y)_{t-1}$ for a model of $\Delta(m-p)_t$ in the United Kingdom, with $\pm 2$ estimated standard errors.
Figure 16g. Equation (6): recursive estimates of the coefficient of the constant term for a model of $\Delta(m-p)_t$ in the United Kingdom, with ±2 estimated standard errors.
coherent model of money demand in the UK, in spite of large changes in the properties of the data.

The constancy of Hendry's (1979) equation, and so of (6), may appear remarkable when contrasted with "missing money" in Goldfeld's and others' partial-adjustment equations. Thus, we examine whether such problems would have appeared for a partial-adjustment model of UK money demand. Paralleling Goldfeld's (1973) equation, we obtain the following estimates on UK data over 1964(3)—1972(4).

\[
\begin{align*}
(m-p)_t &= \frac{0.869}{0.081} (m-p)_{t-1} + \frac{0.055}{0.028} y_t \\
&\quad - \frac{0.66}{0.18} R3_t + \frac{0.86}{1.03} - \frac{0.33}{0.18} \hat{u}_{t-1} \\
T &= 34 [1964(3)—1972(4)] + 12 \text{ forecasts} \quad \hat{\sigma} = 1.535% \\
\text{Chow F}[12, 29] &= 1.89 \quad \text{Forecast } \chi^2[12]/12 = 2.89
\end{align*}
\]

The coefficient on \( \hat{u}_{t-1} \) is the estimated parameter of the (modeled) AR(1) disturbance. Although the forecasts are numerically inaccurate, as evidenced by the \( \chi^2 \) forecast statistic, the Chow statistic has a p-value of only 0.079. The actual and forecast values in Figures 17a–b clarify how closely this model tracks the fall in \( M_1 \) during the early 1970s, only substantially over-predicting in 1975, near the bottom of the fall. When (7) is estimated through 1979 and forecast over the 1980s, a similar picture develops, with the Chow statistic being \( \text{F}[38,57] = 1.57 \) (p-value of 0.060) but the forecast \( \chi^2[38]/38 \) statistic being 5.62 (0.81 and 2.54 respectively if \( R3_t^* \) replaces \( R3_t \) in (7)). Both sets of results contrast with the massive predictive failure of partial-adjustment models on US data over the mid-1970s and the 1980s, as documented by Goldfeld (1976) and Goldfeld and Sichel (1990) and replicated below with recent, revised data.

Even though (7) appears only moderately nonconstant, (6) substantially variance-dominates (7), and (6) parameter-encompasses (7) with ENC \( \text{F}[3,92] = 0.90. \) By contrast, (7) does not parameter-encompass (6): ENC \( \text{F}[3,92] = 11.36. \) These results identify mis-specification in (7) and highlight the low power of Chow tests in poorly fitting, mis-specified models.
Figure 17a. Equation (7): actual, fitted, and forecast values of $(m-p)_t$ in the United Kingdom by a Goldfeld-type specification.

Figure 17b. Equation (7): one-step ahead forecasts of $(m-p)_t$ in the United Kingdom by a Goldfeld-type specification, with ±2 forecast standard errors.
B. \textit{Exogeneity and Endogeneity}

Our analysis has taken contemporaneous income, prices, and interest rates as if they were weakly exogenous so that it is valid to condition upon them for purposes of statistical inference, interpreting the coefficients of the resulting model as those of a money-demand equation. If the conditioning variables are weakly exogenous and their associated demand parameters are invariant to changes in the process generating the conditioning variables (i.e., their marginal process), then those variables are said to be super exogenous. Super exogeneity has empirical consequences for cointegration, the Lucas critique, invertibility of the estimated model, and invariance of the associated parameters. We consider these four consequences and their corresponding testable hypotheses, both in principle and as applied to the money-demand equation (6).

First, Hendry and Mizon (1989) investigate one of the necessary conditions for the weak exogeneity of income, prices, and interest rates for the parameters of the money-demand equation in (6) over a shorter sample. Using the Johansen (1988) system cointegration approach, they establish that there are two cointegrating vectors, one of which corresponds to the long-run money-demand function. That cointegrating vector does not enter the other three equations, so no cross-equation restrictions arise involving the corresponding cointegration parameters.

Second, super exogeneity implies that the Lucas critique does not hold for the relevant class of interventions. An implied testable hypothesis is that the parameters of the conditional model remain constant even while those of the marginal processes change; cf. Hendry (1988) and Favero and Hendry (1989). Under super exogeneity, the constant, conditional money-demand model (6) is not interpretable as a re-parameterized expectations model in which the re-parameterization involves functions of the underlying structural parameters and the time-dependent parameters of the marginal process for the exogenous variables. The proof is by contradiction. If the conditional model were interpretable in this way, then the coefficients in it ought to change as the parameters of the marginal process change; but the former are constant. In effect, refuting the Lucas critique is a \textit{non}-encompassing implication from an expectations theory confronted by a
constant conditional model and a nonconstant marginal model; cf. Ericsson and Hendry (1989). We have already demonstrated the constancy of the conditional money-demand model (6), so we turn to showing the nonconstancy of the marginal processes for inflation and the net interest rate \( R^* \). A model for \( \Delta y \) was not estimated because \( \Delta y \) enters (6) only at a lag.

Starting with univariate fourth-order autoregressive processes for \( \Delta p_t \) and \( R_t^* \) and simplifying, we obtain the following specifications, which are similar to those reported in Cuthbertson (1988, Table 1) and Hendry (1988, pp. 142–146) over a shorter sample.

\[
\begin{align*}
\Delta p_t & = 0.62 \Delta p_{t-1} + 0.23 \Delta p_{t-2} + 0.0232 \text{DV793}_t + 0.0030 \\
\text{[0.15]} & \text{[0.15]} \text{[0.0015]} \text{[0.0014]} \\
T &= 100 \quad [1964(3)–1989(2)] \quad R^2 = 0.72 \quad \hat{\sigma} = 0.768\% \quad DW = 2.07 \\
\text{Normality } & \chi^2[2] = 19.53 \quad \text{AR 1–4 } F[4, 92] = 0.40 \quad \text{ARCH 1–4 } F[4, 88] = 5.16 \\
X_i^2 & F[5, 90] = 0.64 \quad X_j^*X_j \quad F[6, 89] = 4.17 \quad \text{RESET } F[1, 95] = 0.75
\end{align*}
\]

\[
\begin{align*}
\Delta R_t^* & = -0.095 R_{t-1}^* + 0.0085 \\
\text{[0.034]} & \text{[0.0028]} \\
T &= 100 \quad [1964(3)–1989(2)] \quad R^2 = 0.06 \quad \hat{\sigma} = 0.01358 \quad DW = 1.74 \\
\text{Normality } & \chi^2[2] = 2.20 \quad \text{AR 1–4 } F[4, 94] = 0.70 \quad \text{ARCH 1–4 } F[4, 90] = 3.86 \\
X_i^2 & F[2, 95] = 4.48 \quad \text{RESET } F[1, 97] = 1.74
\end{align*}
\]

The dummy DV793 is unity for 1979(3) and zero elsewhere and aims to capture the one-off effect of the increase in VAT on inflation. Figures 18a and 18b graph the one-step residuals and the sequence of break-point Chow statistics for (8), and likewise Figures 19a and 19b for (9). Constancy is easily rejected for each equation at the 1% critical level, with nonconstancy apparent inter alia in the sequentially estimated equation standard errors.\(^{11}\)

These nonconstancies, paired with the constancy of (6), imply that the Lucas critique

\[^{11}\text{Results such as these reveal that Chow tests can have high power for detecting nonconstancy, even for models in differences. Note that the results in Hendry and Neale (1989) and Goldfeld and Sichel (1990) reveal the low power of Chow tests for shifts in the mean of the level when a differenced-data model is fitted. An alternative interpretation is that such models are more robust to shifts in the mean. Models like (6), while they have a differenced variable as the regressand, are actually expressible in levels due to the error-correction term. Also, nothing precludes the Chow test from having power against shifts in the slope coefficient in either levels or differenced models.}\]
Figure 18a. Equation (8): one-step residuals and the corresponding calculated equation standard errors for a time-series model of $\Delta p_t$ in the United Kingdom.

Figure 18b. Equation (8): sequence of break-point Chow statistics over 1968(3)-1989(2) for a time-series model of $\Delta p_t$ in the United Kingdom, with the statistics scaled by their one-off 1% critical values.
Figure 19a. Equation (9): one-step residuals and the corresponding calculated equation standard errors for a time-series model of $\Delta R_t^*$ in the United Kingdom.

Figure 19b. Equation (9): sequence of break-point Chow statistics over 1968(3)-1989(2) for a time-series model of $\Delta R_t^*$ in the United Kingdom, with the statistics scaled by their one-off 1% critical values.
cannot apply to (6). Surprisingly, this implication holds even if (8) and (9) ignore some information relevant to the processes generating $\Delta p$ and $R^*$; cf. Hendry (1988, pp. 137–138).

However, refutation of the Lucas critique is tempered by nonzero Type I and Type II errors when testing for constancy with finite samples, so the power of this procedure is an issue. The empirical nonconstancy of the inverted money-demand equation (below) is consistent with high finite-sample power of this test. Also, for a given expectations process, Hendry (1990) provides an analytical framework in which the approximate finite-sample power can be calculated from estimated parameters.

Third, super exogeneity is not invariant to re-normalization, implying that the "inverted" money-demand equation must be nonconstant when the conditional and marginal models are as described above for the Lucas critique analysis. That is, the constant, conditional money-demand model cannot be inverted to obtain a constant model of (e.g.) prices given money. Statistically speaking, such an inversion is equivalent to re-factorizing the joint distribution of money and prices, noting that the joint distribution always can be written as the product of a conditional and a marginal distribution. To simplify exposition, we ignore lags and additional variables.

The joint density of $m_t$ and $p_t$ always factorizes into $D(m_t | p_t; \theta_{1t}) \cdot D(p_t; \theta_{2t})$ where $D(m_t | p_t; \theta_{1t})$ is the model of money demand conditional on prices, $D(p_t; \theta_{2t})$ is the marginal process for prices, and $\theta_{1t}$ and $\theta_{2t}$ are their associated parameters. Equally, the joint density factorizes into $D(p_t | m_t; \lambda_{1t}) \cdot D(m_t; \lambda_{2t})$ where $D(p_t | m_t; \lambda_{1t})$ is the model of prices given money, $D(m_t; \lambda_{2t})$ is the marginal model for money, and $\lambda_{1t}$ and $\lambda_{2t}$ are these densities' parameters. By simple analytics, $\lambda_{1t}$ and $\lambda_{2t}$ each are a function of both $\theta_{1t}$ and $\theta_{2t}$. In our empirical analysis, $\theta_{1t}$ is shown to be constant over time ($\theta_{1t} = \theta_1$), whereas $\theta_{2t}$ is nonconstant. Thus, $\lambda_{1t}$, the parameter vector for the model of prices conditional on money, cannot be constant, i.e., the "inverted" money-demand equation is not constant.

From such evidence, results in Hoover (1990) would imply that prices cause money, but that money does not cause prices. Empirically, non-invertibility can be demonstrated by estimating the inverted equation and testing (and rejecting) its constancy.
To invert the money-demand equation (6), we note first that (6) is statistically unchanged by adding $\Delta p_t$ to both sides of the equation, in which case the dependent variable becomes the growth rate of nominal money $\Delta m_t$ and the coefficient on $\Delta p_t$ becomes +0.31. Switching the positions of $\Delta m_t$ and $\Delta p_t$ and re-estimating results in the following equation.

\begin{equation}
\Delta p_t = \frac{0.197}{[0.083]} \Delta m_t - \frac{0.02}{[0.06]} \Delta (m-p-y)_{t-1}
+ \frac{0.338}{[0.063]} R^*_t + \frac{0.017}{[0.010]} (m-p-y)_{t-1} - \frac{0.0050}{[0.0030]}
\end{equation}

$T = 100 \ [1964(3)-1989(2)] \quad R^2 = 0.48 \quad \hat{\sigma} = 1.041\% \quad DW = 0.84$

Normality $\chi^2[2] = 17.07 \quad AR 1-4 F[4, 91] = 24.73 \quad ARCH 1-4 F[4, 87] = 5.76$

ENC F[3, 92] = 34.95 \quad ENC F[2, 93] = 43.20

Figures 20a, 20b, and 20c respectively graph the one-step residuals, the sequence of breakpoint Chow statistics, and the recursively estimated coefficient of $\Delta m_t$ for (10). From Figures 20a and 20b, constancy is rejected at the 1% critical level, with the estimated equation standard error more than doubling over the sample. From Figure 20c, the coefficient on money growth is highly nonconstant, with the 95% confidence interval of the final estimate lying virtually entirely outside the initial estimate's 95% confidence interval, in spite of the noted increase in the estimated equation standard error. Further, (10) cannot encompass the simple time-series model (8) for prices, whether with or without the dummy DV793: that also follows because (8) variance-dominates (10).

Economically, "non-invertibility" implies that policy implications do not follow directly from a constant money-demand equation such as (6). Rather, we would require additional information, e.g., about the form of a a well-specified (marginal) price equation and/or interest rate equation.

Fourth, super exogeneity implies that the (constant) parameters of the conditional model are invariant to the parameters in the marginal processes, so determinants of those processes' nonconstancies should be statistically insignificant if added to the conditional model. Specifically, because the (conditional) money-demand parameters $\theta_1$ are invariant
Figure 20a. Equation (10): one-step residuals and the corresponding calculated equation standard errors for the inverted money-demand model explaining Δp_t in the United Kingdom.

Figure 20b. Equation (10): sequence of break-point Chow statistics over 1968(3)-1989(2) for the inverted money-demand model explaining Δp_t in the United Kingdom, with the statistics scaled by their one-off 1% critical values.
Figure 20c. Equation (10): recursive estimates of the coefficient of $\Delta m_t$ in the inverted money-demand model explaining $\Delta p_t$ in the United Kingdom, with $\pm 2$ estimated standard errors.
to changes in $\theta_{2t}$, variables helpful in explaining the in-sample nonconstancy of $\theta_{2t}$ should be unimportant if added to the conditional money-demand model. Testing for their significance is the basis for Engle and Hendry's (1989) test of super exogeneity.

To apply this test, constant models of $\Delta p$ and $R^*$ must be developed, and to do so, we use dummy variables to capture "regime shifts" affecting these variables. The significance in (6) of these dummies is tested, as are other aspects of these auxiliary models, such as functions of their residuals.

The following models were obtained for $\Delta p$ and $R^*$, starting from fourth-order autoregressive processes with several dummies, with the dummies entering additively and interactively. These models and the associated tests of super exogeneity parallel those in the empirical section of Engle and Hendry (1989), with slight modifications made due to the re-basing of expenditure series and the longer sample.

\begin{equation}
\hat{\Delta p}_t = 0.031 p_{t-1} + 0.017 D_{V793_t} - 0.069 D_{73(4)_t} + 0.010 D_{79(3)_t} \\
[0.009] [0.007] [0.017] [0.005] \\
+ 0.067 D_{p_{t-1} \cdot D_{73(4)_t}} + 0.37 \Delta p_{t-1} \cdot D_{73(4)_t} \\
[0.016] [0.012] [0.11] \\
T = 100 [1964(3)–1989(2)] \quad R^2 = 0.80 \quad \hat{\sigma} = 0.648\% \quad DW = 2.09
\end{equation}

Normality $\chi^2[2] = 0.61 \quad AR 1–4 F[4, 89] = 0.48 \quad ARCH 1–4 F[4, 85] = 2.39

$X^2 F[12, 80] = 1.78 \quad X^*_i X_j F[15, 77] = 2.05 \quad RESET F[1, 92] = 0.05$

\begin{equation}
\hat{\Delta R^*}_t = -0.111 R^*_{t-2} + 0.009 + 0.0230 \Delta_{D79(3)_t} + 0.040 \Delta_{D73(3)_t} \\
[0.033] [0.003] [0.0042] [0.013] \\
T = 100 [1964(3)–1989(2)] \quad R^2 = 0.20 \quad \hat{\sigma} = 0.0127 \quad DW = 1.88
\end{equation}

Normality $\chi^2[2] = 1.67 \quad AR 1–4 F[4, 92] = 0.24 \quad ARCH 1–4 F[4, 88] = 3.61

$X^2 F[6, 89] = 1.40 \quad X^*_i X_j F[7, 88] = 1.18 \quad RESET F[1, 95] = 1.06$

ARCH 1 F[1, 94] = 13.37

$D73(4)$ and $D79(3)$ are zero/one shift dummy variables beginning at the indicated quarters to capture OPEC's increase in oil prices and the policy switches of the Thatcher government. $D73(3)$ is the one-period lead of $D73(4)$. Both equations are reasonably constant relative to the simpler marginal models (8) and (9). Thus, the dummies in (11)
and (12) are one way of capturing the parameter nonconstancy of (8) and (9), and so the dependence of $\theta_{2 \ell}$ on time (in the notation from the invertibility discussion).

Invariance implies that these determinants should not affect $\theta_1$, and that may be tested directly by adding the dummies to (6).

$$
\Delta(m-p)_t = -0.69 \Delta p_t - 0.17 \Delta(m-p-y)_{t-1} \\
\quad - 0.624 R^*_t - 0.091 (m-p-y)_{t-1} + 0.024 \\
\quad - 0.008 \text{DV793}_t + 0.001 \text{D73(4)}_t - 0.000 \text{D79(3)}_t \\
\quad - 0.006 \Delta \text{D73(3)}_t + 0.003 \Delta \text{D79(3)}_t
$$

$$
T = 100 [1964(3)--1989(2)] \quad R^2 = 0.76 \quad \hat{\sigma} = 1.346\% \quad \text{DW} = 2.16
$$

Individually and jointly, the dummies are insignificant, with the corresponding joint F statistic F[5,90] being 0.08.

Such variable-addition tests of super exogeneity may use aspects of the marginal models other than the dummies, with the choice of variables depending upon likely sources of induced nonconstancy in (8) and (9). Functions of the residuals from (11) and (12) are an obvious choice, especially for (12), in which substantial ARCH is evident. Thus, some alternative variables are the residuals themselves (denoted $\hat{u}_t(\Delta R^*)$ and $\hat{u}_t(\Delta p)$, which by themselves would give the Wu-Hausman test), four-period moving standard deviations of $\hat{u}_t(\Delta R^*)$ and $\hat{u}_t(\Delta p)$ (denoted $\sqrt{\hat{\nu}}(\Delta R^*)$ and $\sqrt{\hat{\nu}}(\Delta p)$), and the predictable and unpredictable ARCH components of $\hat{u}_t^2(\Delta R^*)$ (denoted $\hat{a}_t(\Delta R^*)$ and $\hat{\text{dev}}_t(\Delta R^*)$, where the latter is $\hat{u}_t^2(\Delta R^*)-\hat{a}_t(\Delta R^*)$).
(14)  \[ \Delta(m-p)_t = -0.60 \Delta p_t - 0.18 \Delta (m-p-y)_{t-1} \]
\[ 0.06 \]
\[ 0.17 \]
\[-0.676 R^*_t - 0.099 (m-p-y)_{t-1} + 0.025 \]
\[ 0.010 \]
\[ 0.057 \]
\[ 0.06 \]
\[ 0.25 \hat{u}_t(\Delta p) - 0.20 \sqrt{N}_t(\Delta R^*) \]
\[-0.11 \]
\[ 0.28 \]
\[ 0.01 \]
\[ 0.11 \]
\[ 0.05 \]
\[ 0.45 \]
\[-0.03 \sqrt{N}_t(\Delta p) - 7.6 \hat{a}_t(\Delta R^*) - 5.7 \hat{\text{dev}}_t(\Delta R^*) \]
\[ 0.88 \]
\[ 20.9 \]
\[ 5.3 \]
\[ 0.28 \]
\[ 0.05 \]
\[ 0.45 \]
\[ T = 97 [1965(2)-1989(2)] \]
\[ R^2 = 0.77 \]
\[ \hat{\sigma} = 1.339\% \]
\[ DW = 2.16 \]

As with the dummies, these variables are individually and jointly insignificant; the corresponding joint F statistic F[6,86] is 0.79, noting that three observations are lost in creating the standard deviations of the residuals.

As with tests of the Lucas critique, the power of these super exogeneity tests is of interest. Engle and Hendry (1989) provide some evidence by intentionally mis-specifying their conditional money-demand equation, and testing the significance of residual-based variables in it. Even if the parameters \( \theta_1 \) in the original conditional model are invariant to changes in \( \theta_2 \), the parameters of the mis-specified conditional model generally are not, so tests of super exogeneity in the mis-specified model provide some measure of power. Engle and Hendry (1989) consider two mis-specifications of Hendry (1988, (26)), one without \( \Delta p_t \) and the other with \( \Delta p_t \) replaced by \( \Delta p_{t-1} \). In both cases, strong rejection is obtained. Using (6), parallel tests for the dummies obtain F[5,91]=2.54 and F[5,90]=1.21, the first being highly significant. Further, for (6) without \( R^*_t \) or with \( R^*_t \) replaced by \( R^*_{t-1} \), super exogeneity tests for the residual-based variables are F[6,87]=1.05 and F[6,86]=3.26 and for the dummies are F[5,91]=3.21 and F[5,90]=0.85, with the second and third of these tests being highly significant. Although open to additional study, the power of these tests appears considerable.

That the constancy tests used for the Lucas critique differ from the invariance tests above helps clarify the difference between the two underlying concepts. Indeed, most of the mis-specified models for the invariance tests appeared constant when evaluated by the various sequences of Chow statistics. Simply put, the tests of constancy and of invariance
are evaluating the conditional model against different sources of information. For constancy, the model is evaluated across different subsamples, given a set of variables (i.e., those in the conditional model). For invariance, the sample length is fixed and the model is evaluated across different data sets, one including and the other excluding information from the marginal model. However, constancy and invariance often are bundled together as desirable properties of estimated parameters from an empirical model.

C. Forward-looking Behavior

Given these results on the super exogeneity of prices, incomes, and interest rates for the parameters in the money-demand equation (6), it may appear puzzling that agents do not bother forming expectations about future values of these variables. One explanation is that (6) (and every error-correction model) may be rewritten as a forward-looking model, one in which the forward-looking aspects arise from data-based predictors rather than model-based (expectations-type) predictors; cf. Campos and Ericsson (1988). This may be seen by re-estimating (6) with all long-run determinants (here, Δp and R*) entering the lagged error-correction term explicitly. By construction, those determinants now enter the equation differenced one order higher, with (approximately) their original coefficients.

\[
\Delta (m-p)_t = -0.87 \Delta^2 p_t - 0.18 \Delta (m-p-y)_{t-1} - 0.50 \Delta R^*_t \\
- 0.094 [(m-p-y) - \gamma_1 R^* - \gamma_2 \Delta p]_{t-1} + 0.0237 \\
T = 100 [1964(3)–1989(2)] \quad R^2 = 0.77 \quad \hat{\sigma} = 1.299% \quad DW = 2.17 \\
\text{Normality } \chi^2[2] = 2.20 \quad \text{AR 1–4 } F[4, 91] = 2.07 \quad \text{ARCH 1–4 } F[4, 87] = 0.60 \\
X_t^2 F[8, 86] = 0.64 \quad X_t X_j F[14, 80] = 0.60 \quad \text{RESET } F[1, 94] = 0.09
\]

\(\gamma_1\) and \(\gamma_2\) are the solved long-run coefficients of \(R^*\) and \(\Delta p\) from (6), with values of approximately \(-6.8\) and \(-7.4\) respectively.

The interpretation of (15) is as follows. Suppose that, due to information costs, etc., agents forecast by data functions rather than by models. For an integrated process \(x_t\) which is \(I(d)\), a simple and effective forecast is obtained from \(\Delta^{d+1} x_{t+1} \approx 0\). For \(d=2\), that implies \(\Delta^2 \hat{x}_{t+1} = \Delta^2 x_t\) or that \(\hat{x}_{t+1} = x_t + \Delta x_t + \Delta^2 x_t\). Several properties of \(\hat{x}_{t+1}\)
(i) It is unbiased if $\Delta^d x_t$ is AR(q) with a symmetrical error process.

(ii) No parameters are required other than the order of integration $d$.

(iii) The error variance of $(x_{t+1} - \hat{x}_{t+1})$ need not enter the conditional model.

(iv) $\Delta^{d+1} x_{t+1} \approx 0$ implies $\Delta^{d+2} x_{t+1} \approx 0$, but not conversely. So:

(a) if the level of $d$ chosen by the econometrician ($d^*, \text{say}$) is too small, the resulting econometric model may well fail because of the associated omitted variable; and

(b) if $d^* > d+1$ and $d$ alters, mis-specification may be apparent only by outliers at the time that $d$ changes.

(v) If the order of integration of $x_t$ increases but the $d$ for agents remains the same, agents may well experience predictive failure (e.g., systematic under- or over-prediction), providing the basis for agents' revision of their $d$.

(vi) Such data-based predictions could be "rational" if information is costly.

Flemming's (1976, pp. 62ff) synthesis of rational and adaptive expectations into a "change of gear" model presages our analysis.

In terms of Chow statistics, $\Delta^2 p_t$ does reasonably well in the UK as an empirical predictor for $\Delta^2 p_{t+1}$, except during 1973–1974, when $d$ (or at least the mean of $\Delta p_t$) appears to change. Likewise, $\Delta R^*_t$ predicts $\Delta R^*_{t+1}$, reasonably well, except for 1973–1974 and 1977–1979, at which times changes in the structure of interest rates are apparent. This is consistent with the orders of differencing of these variables in (6) and (15).

To summarize, Hendry's (1979, 1988) model of UK money demand remains constant over the 1980s when the opportunity cost is adjusted to account for financial innovation. Correct dynamic specification and inclusion of the relevant interest rates are central to obtaining a congruent empirical model. Prices, incomes, and interest rates are super exogenous for the parameters of the conditional money-demand equation. That refutes the Lucas critique for changes in the parameters of expectations processes, and precludes either inverting the money-demand equation to obtain a constant model of inflation in terms of money growth or interpreting the error-correction model as derived from a forward-looking
expectations-based theory model. However, we can provide a forward-looking interpretation of the error-correction model using data-based predictors. We now examine a model of the US money demand.

5. **An Empirical Model of US Money Demand**

A. *Replication, Specification, and Constancy*


\[
\Delta (m-p)_{t} = -0.63 \Delta p_{t} + 0.89 \text{Vol}_{t} + 0.391 \Delta (m-p)_{t-2}
\]
\[
- 1.78 (\Sigma 0_{t}S)_{t} - 0.56 (\Delta (m-p)_{t-1}/4)
\]
\[
- 0.95 (\Delta 4p_{t-2}/4) + 6.3 \text{SVol}_{t} + 12.9 \Delta \text{SVol}_{t-1}
\]
\[
+ 0.0126 \text{DM}_80_{t} + 0.466 \text{Rnsa}_{t-1} + 0.47 (\Delta y/2)_{t}
\]

\[\text{T = 113 [1960(3)–1988(3)] R}^2 = 0.88 \chi^2[16] = 1.95\]

Chow F[16, 83] = 0.83 \chi^2[16]/16 = 1.95

Normality \chi^2[2] = 1.21 AR 1–4 F[4, 95] = 0.67 ARCH 1–4 F[4, 91] = 0.30

\[\text{RESET F[1, 98] = 0.50 AR 8–8 F[1, 98] = 4.21}\]

The variables are defined as follows. Rma is the maximum over Rp and the learning-adjusted non-transactions M2 interest rates (Rc, Rm), where learning adjustment is with respect to the excess over Rp. Also, A1Rma is the normalized, tail-constrained, first-order Almon of Rma (= 2/3Rma_{t-1} + 1/3Rma_{t-1}; cf. Sargan (1980a)). Rnsa is the average of learning-adjusted NOW and SuperNOW interest rates. S and s are spreads, the first between Rl and R1 and the second between R1 and Rma. \Sigma 0_{t}S and \Sigma 0_{t}s are two-period moving averages of the respective spreads. Vol is a nine-quarter moving average of the one-year moving standard deviation of the 20-year Treasury bond yield (at quarterly
rates), SVol is the product of Vol with the maximum of zero and the spread S, and DM80 is a dummy for credit control (=−1 in 1980(2), =+1 in 1980(3), zero otherwise). See BHS (1990), who explain these variables' relation to the underlying economic theory and obtain a simpler, more intuitive version of (16).

The diagnostic tests indicate residuals which are white noise, homoscedastic, and approximately normally distributed, other than possible eighth-order autocorrelation which BHS (1985, 1990) ascribe to separate seasonal adjustment of the individual data series.

When estimated through only 1984(3), (16) shows no tendency to mis-predict, as is apparent from actual, fitted, and forecast values of Δ(m−p)t in Figures 21a and 21b, and reflected by the Chow statistic above. Actual, fitted, and forecast values of the level of real money (m−p)t appear in Figures 22a—b, and demonstrate both the accuracy and precision with which (16) forecasts over a period with historically unprecedented levels of M1.12

To investigate constancy in greater detail, we turn to recursive estimation of (16). Because of computer program limitations for recursive least-squares, four coefficient restrictions have been imposed: the coefficients on DM80_t, Rnsa_t−1, and Δ2y_t are taken as estimated in (16), and SVol* is defined as as SVol_t+2·ΔSVol_t−1. For sequences of Chow statistics, these restrictions bias the results in favor of rejection because there are fewer degrees of freedom for coefficients to adjust to new data, and because the estimate of σ is biased downward. Defining Δ(m−p)_t* as Δ(m−p)_t less the effects of DM80_t, Rnsa_t−1, and Δ2y_t, we obtain the following.

\[
\Delta(m-p)_t^* = -0.63 \Delta p_t + 0.89 \text{Vol}_t + 0.391 - 0.275 (m-p-4y)_{t-2} \\
- 1.14 (A_1Rma)_t - 1.78 (\Sigma_0S)_t - 1.11 (\Sigma_0S)_t \\
- 0.56 (\Delta_4[m-p]_{t-1}/4) - 0.95 (\Delta_4p_{t-2}/4) + 6.5 SVol^*_t \\
\]

T = 113 [1960(3)—1988(3)] R² = 0.84  σ = 0.395%  DW = 1.75

12 The slight over-predictions at the end of the sample are due to estimating rather than imposing the unit coefficient on (m−p)t−1 from Δ(m−p)_t in (16).
Figure 21a. Equation (16): actual, fitted, and forecast values of $\Delta(m-p)$, in the United States, estimated over 1960(3)-1984(3) and forecast over 1984(4)-1988(3).

Figure 21b. Equation (16): one-step ahead forecasts of $\Delta(m-p)_t$ in the United States, with $\pm 2$ forecast standard errors, estimated over 1960(3)-1984(3) and forecast over 1984(4)-1988(3).
Figure 22a. Equation (16): actual, fitted, and forecast values of \((m-p)_t\) in the United States, estimated over 1960(3)-1984(3) and forecast over 1984(4)-1988(3).

Figure 22b. Equation (16): one-step ahead forecasts of \((m-p)_t\) in the United States, with \(\pm 2\) forecast standard errors, estimated over 1960(3)-1984(3) and forecast over 1984(4)-1988(3).
From Figure 23a, the equation standard error of (17) is very constant, and none of the "break-point" Chow statistics in Figure 23b is significant at even the 5% level. Figure 23c shows the recursive estimates of the coefficient on $\Delta p_t$, together with plus-or-minus twice its sequentially estimated standard error. The estimated coefficient varies by only a fraction of its ex ante standard error, with the latter shrinking markedly over time. To summarize, (16) is a constant, data-coherent model of money demand in the US, despite large swings in velocity, inflation, and interest rates.

From an economic perspective, (16) cannot be interpreted as a money-supply function, given the positive coefficient on the own rate of return (Rnsa) and the negative coefficient on the yield to non-M1 M2 ($A_1Rma$). Therefore, we identify (13) as a money-demand in the sense of "interpret". Its identification in the sense of uniqueness is shown below via the nonconstancy of a marginal model for Rma. This contrasts sharply with Cooley and Leroy (1981).

In addition to its own statistical and economic merits, (16) resolves several existing and potential puzzles arising from the data and from other money-demand models. To clarify the discussion, in each case we consider the puzzle giving rise to predictive failure in more classical specifications, the cause of the puzzle (often associated with institutional change), and the econometric solution offered in (16).

(i) Missing money of the early- to mid-1970s. Rose (1985) shows that an overly restrictive dynamic specification imposing a unit short-run elasticity of nominal money with respect to prices leads to this predictive failure. Inflation needs to enter the empirical demand function as a separate variable. BHS (1993) show the importance of Vol to their specification over this period in that omitting it induces predictive failure. Thus, $Rt$ needs to be adjusted by some measure of risk. In general, marginalizing any constant-parameter model with respect to a variable which has nonconstant parameters in its marginal process will lead to the resulting equation being nonconstant.

(ii) Financial innovation, learning, and adaptation. Predictive failure results if only the passbook rate is used, rather than the maximum of the learning-adjusted rates on competitive assets, or if the maximum rate is not learning-adjusted.
Figure 23a. Equation (17): one-step residuals and the corresponding calculated equation standard errors for a model of $\Delta(m-p)_t$ in the United States.

Figure 23b. Equation (17): sequence of break-point Chow statistics over 1965(4)-1988(3) for a model of $\Delta(m-p)_t$ in the United States.
Figure 23c. Equation (17): recursive estimates of the coefficient of $\Delta p_t$ for a model of $\Delta (m-p)_t$ in the United States, with ±2 estimated standard errors.
(iii) The great velocity decline, the New Operating Procedures, and their removal. The New Operating Procedures introduced in October 1979 induced higher volatility in interest rates, making bonds especially risky and thereby \( M_1 \) more attractive. Even so, the importance of volatility is statistically detectable before October 1979; cf. BHS (1990). See Fischer (1989, pp. 430ff) for institutional background of the Fed’s policy shifts.

(iv) Explosion in \( M_1 \) after the introduction of NOW and SuperNOW accounts. Interest rates corresponding to both accounts are learning-adjusted (as with \( R_{ma} \)), but they appear to enter as an average rather than a maximum, perhaps reflecting the different clienteles of the two accounts. As noted in Section 4, interest-bearing \( M_1 \) retail sight deposits provide the parallel explanation for the recent explosion of \( M_1 \) in the UK.

While more formal discussion of (i)–(iv) appears in BHS (1990), along with corresponding encompassing-based tests, we will consider missing money (i) here in somewhat more detail, as was done for the UK. Estimating Goldfeld’s (1973) equation on the re-based data yields results similar to his original ones; see Goldfeld and Sichel (1990). By itself, accounting for financial innovation in \( M_2 \) as modeled by BHS (1990) via \( R_{ma} \) does not explain missing money, as the following equation demonstrates.

\[
(18) \quad (m-p)_t = \frac{0.54}{0.23} (m-p)_{t-1} + \frac{0.176}{0.072} v_t - \frac{0.16}{0.15} R_{1t} - \frac{0.69}{0.39} R_{ma,t} + \frac{0.48}{0.25} \hat{u}_{t-1} + \frac{0.64}{0.32}
\]

\( T = 54 \) [1960(3)–1973(4)] + 12 forecasts \( \hat{\sigma} = 0.516\% \)

Chow \( F[12, 48] = 2.76 \) Forecast \( \chi^2[12]/12 = 17.35 \)

The \( \chi^2 \) predictive failure and Chow statistics are highly significant, with the systematic over-predictions evident in Figures 24a and 24b. With \( \Delta p_t \) entering (18) unrestrictedly (as in Rose (1985)), the Chow becomes insignificant at \( F[12,47]=1.19 \), but the forecast \( \chi^2[12]/12 \) statistic equals 8.30, with eleven out of the twelve forecast errors being negative.

Predictive failure of this extension of Goldfeld’s (1973) equation is not restricted to the 1970s. Even with \( \Delta p_t \) included in (18), estimation over the full sample yields the following.
Figure 24a. Equation (18): actual, fitted, and forecast values of \((m-p)_t\) in the United States by a Goldfeld-type specification.

Figure 24b. Equation (18): one-step ahead forecasts of \((m-p)_t\) in the United States by a Goldfeld-type specification, with \pm 2\ forecast standard errors.
\[
(19) \quad (m-p)_t = 0.853 (m-p)_{t-1} + 0.082 y_t - 0.93 \Delta p_t \\
(0.032) \quad (0.013) \quad (0.14) \\
+ 0.09 R_1_t - 0.55 R_{ma_t} + 0.66 \hat{u}_{t-1} + 0.199 \\
(0.12) \quad (0.12) \quad (0.09) \quad (0.045) \\
T = 113 \ [1960(3) - 1988(3)] \quad \hat{\sigma} = 0.641\% 
\]

Nonconstancy is apparent for the 1980s, with the Chow statistic \(F[35,71]\) being 4.58 and the forecast \(\chi^2[35]/35\) being 7.30, both significant at any reasonable level.

Although sometimes heralded for their predictive accuracy, time-series models can suffer from predictive failure as well. A first-order autoregressive process for \(\Delta (m-p)_t\) results in the following.

\[
(20) \quad \Delta (m-p)_t = 0.56 \Delta (m-p)_{t-1} + 0.0013 \\
(0.11) \quad (0.0010) \\
T = 113 \ [1960(3) - 1988(3)] \quad R^2 = 0.31 \quad \hat{\sigma} = 0.927\% \quad DW = 2.14 
\]


Figure 25 gives the sequential break-point Chow statistics, revealing (20) to be highly nonconstant. This nonconstancy is an encompassing implication of the constant conditional money-demand model (16) and a nonconstant marginal process for one or more of the variables conditioned upon. Because of that implication, and because of its importance in discussing exogeneity, we now estimate a marginal process for \(R_{ma}\) to demonstrate that it too is nonconstant.

B. \textit{Exogeneity, Endogeneity, and Forward-looking Behavior}

Beginning with a fourth-order autoregressive process for \(R_{ma}\), we obtain the following simpler model.

\[
(21) \quad \Delta R_{ma_t} = -0.056 R_{ma_{t-2}} + 0.004 \\
(0.058) \quad (0.003) \\
T = 113 \ [1960(3) - 1988(3)] \quad R^2 = 0.04 \quad \hat{\sigma} = 0.00897 \quad DW = 1.91 
\]

Normality \(\chi^2[2] = 554.90\) \(AR 1-4 F[4, 107] = 1.03\) \(ARCH 1-4 F[4, 103] = 47.84\) \(X_t^2 F[2, 108] = 18.10\) \(RESET F[1, 110] = 2.88\)
Figure 25. Equation (20): sequence of break-point Chow statistics over 1965(4)-1988(3) for a time-series model of $\Delta(m-p)_t$ in the United States, with the statistics scaled by their one-off 1% critical values.

Figure 26. Equation (21): sequence of break-point Chow statistics over 1965(4)-1988(3) for a time-series model of $\Delta R_{ma}_t$ in the United States, with the statistics scaled by their one-off 0.001% critical values.
Constancy is easily rejected at even the 0.001% level: Figure 26 graphs the break-point Chow statistics at that critical level. As with the UK, the Lucas critique is rejected for an expectations interpretation of (16).

Rather than duplicating results on invertibility and super exogeneity from BHIS (1990), we summarize. Empirically, if (16) is inverted to obtain inflation, given the growth rate of money, the resulting model is nonconstant. Further, (16) satisfies various invariance tests based upon variable addition. Finally, because (16) is an error-correction model, it may be rewritten to obtain a representation which can be interpreted as forward-looking with data-based predictors, as in (15).

6. Conclusions and Discussion

Two conditional models of money demand in the UK and the US have remained remarkably constant and otherwise well-specified in the presence of substantial data revisions and financial innovations, and with the accrual of new data which differs greatly from the previous within-sample observations. Even so, the historical development of these models highlights that econometric models must be adaptive to the environment in the same way that agents are. Although it would be difficult to predict the quantitative effects on $M_1$ of institutional changes or circumstances which have never occurred before (e.g., of interest-bearing accounts before they existed), related experience from other times and places can help. For example, BHIS (1985, 1990) were able to model the effects of NOW accounts from those of money-market mutual funds, and of SuperNOW accounts from those of NOW accounts. In turn, we modeled the effects of interest-bearing $M_1$ retail sight deposits in the UK by adapting Baba, Hendry, and Starr's results from the US experience as a whole.

Most of the puzzles extant in the literature appear to be resolved by the congruent models above; cf. Goldfeld and Sichel (1990) and Judd and Scadding (1982). In the US, the "missing money" appears to be due to mis-specified dynamics and omitted interest-rate volatility, not financial innovation; the underlying $M_1$ demand function remained constant.

---

13 Similar results obtain using $R_1$ in place of $Rma$. 
in spite of the Fed's New Operating Procedures; and the very large increases in $M_1$ witnessed in the mid- to late-1980s can be seen as lagged adjustment to falling interest rates and inflation and the introduction of interest-bearing checking accounts. A similar story applies to the UK although earlier results had already established that a properly specified dynamic model of $M_1$ demand did not experience predictive failure over the equivalent missing-money episode and Thatcher's monetary "experiment". In addition, merely including a learning-adjusted own interest rate for $M_1$ maintains parameter constancy through the 1980s. From an economic perspective, no modification of the extant model for $M_1$ was required. Rather, the data measurement changed because the opportunity cost of holding money no longer was the three-month local authority interest rate.

One explanation for $M_1$ demand equations being constant is that the stock of $M_1$ is determined by private sector behavior with the relevant policy agency in effect determining the interest rate. If so, a key aspect of monetary policy is the determination of a baseline interest rate rather than of any particular nominal magnitude of $M_1$. Conversely, had the Bank of England or the Fed raised interest rates on the basis that the large increases in $M_1$ recorded in the second half of the 1980s presaged an upturn in inflation, they would have significantly misunderstood the economic behavior of the private sector, and in doing so unnecessarily reduced output via higher interest rates. Money may feed back onto income, prices, and interest rates, and it is found to do so by Hendry and Ericsson (1986) and Hendry and Mizon (1989) for the UK. However, the evidence on the money-demand equations above precludes using those equations by themselves for inferring prices or interest rates.
### APPENDIX A. Data Definitions for the UK

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<th>Variable</th>
<th>Definition</th>
<th>Source</th>
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<tr>
<td>GDP</td>
<td>Gross domestic product (expenditure-based) at market prices [£ million, current prices, seasonally adjusted]</td>
<td>RBGBGB01.Q</td>
</tr>
<tr>
<td>GDP85</td>
<td>Gross domestic product (expenditure-based) at market prices [£ million, 1985 prices, seasonally adjusted]</td>
<td>RHGBGB01.Q</td>
</tr>
<tr>
<td>IMP</td>
<td>Imports of goods and services at market prices [£ million, current prices, seasonally adjusted]</td>
<td>RFWBGB01.Q</td>
</tr>
<tr>
<td>IMP85</td>
<td>Imports of goods and services at market prices [£ million, 1985 prices, seasonally adjusted]</td>
<td>RLWBGB01.Q</td>
</tr>
<tr>
<td>M</td>
<td>Monetary aggregate M1; notes and coin in circulation with the public plus UK private sector sterling sight bank deposits (both non-interest-bearing and interest-bearing) [£ million, current prices, seasonally adjusted, financial year constrained]</td>
<td>ABBBGB91.Q</td>
</tr>
<tr>
<td>P</td>
<td>Implicit deflator for total final expenditure (constructed) = (GDP+IMP)/(GDP85+IMP85)</td>
<td>-</td>
</tr>
<tr>
<td>R3</td>
<td>Interest rate on deposits with local authorities, for a minimum of three months and thereafter at seven days’ notice (quarterly average of the rate on the last Friday of each month) [fraction]</td>
<td>AJOI</td>
</tr>
<tr>
<td>R*</td>
<td>Learning-adjusted net interest rate = R3 - Rra [fraction]</td>
<td>-</td>
</tr>
<tr>
<td>Rr</td>
<td>Interest rate on (M1) sterling retail sight deposits at banks [fraction]</td>
<td>Unpublished: see below.</td>
</tr>
<tr>
<td>Rra</td>
<td>Learning-adjusted interest rate on retail sight deposits at banks = w_t . Rf_t [fraction]</td>
<td>-</td>
</tr>
<tr>
<td>X</td>
<td>Total final expenditure at market prices: see Appendix B [£ million, current prices, seasonally adjusted]</td>
<td>DIAB</td>
</tr>
<tr>
<td>X85</td>
<td>Total final expenditure at market prices: see Appendix B [£ million, 1985 prices, seasonally adjusted]</td>
<td>DIAU</td>
</tr>
<tr>
<td>w_t</td>
<td>Weighting function representing agents’ learning about interest-bearing retail sight deposits = (1+exp[α·β(t-t*+1)])^{-1} for t≥t*, zero otherwise; t* = 1984(3). α and β are defined in the text, estimated in Appendix B.</td>
<td>-</td>
</tr>
<tr>
<td>Y</td>
<td>Total final expenditure at market prices (constructed) = GDP85 + IMP85 [£ million, 1985 prices, seasonally adjusted]</td>
<td>-</td>
</tr>
</tbody>
</table>
Sources. The data sources are: Bank of England Quarterly Bulletin, various issues (BEQB); Bank of International Settlements data tape, April 1990 (BIS); Economic Trends Annual Supplement, 1990 Edition, No. 15 (ETAS); and Financial Statistics, various issues (FS). The first is a publication of the Bank of England, London; the second is a tape prepared by the Bank of International Settlements (Basel, Switzerland) with data from various central banks; and the last two are published by the Central Statistical Office (CSO), Her Majesty's Stationary Office, London. An alphanumeric sequence ending in .Q is a tape code for the BIS. A four-character sequence is a CSO databank series number for the BEQB, ETAS, and FS. Because the BIS data which we use originate from the Bank of England, their corresponding CSO databank series numbers are given in parentheses for ease of retrieval.

GDP, GDP85, IMP, IMP85 are from the BIS, and correspond to series in ETAS (Table 3). M is from the BIS, and corresponds to the M1 series in FS (January 1989, Supplementary Table S32, Column 7) and BEQB (November 1989, Table 11.1, Column 14). R3 is from various issues of the BEQB (e.g., May 1989, Table 9.2) and FS (e.g., February 1990, Table 13.14). X and X85 are from the ETAS (Table 3).

All data are quarterly and span 1963(1)—1989(2), unless otherwise noted.

Adjustments. As Topping and Bishop (1989) document, numerous breaks exist in the series for M1. We account for the four primary breaks in M1, proportionately rescaling data before the break to match the post-break value of M1 for the quarter in which the break occurred. Adjusting the data for these breaks is critical, statistically as well as economically, noting that the magnitude of the breaks ranges from 1.5% to 6.3%, but that \( \sigma \) in (6) is only 1.3%. The breaks are given in Table A.1 below, and although the actual values of the breaks are for data not seasonally adjusted, the breaks for seasonally adjusted data (which we use) should be the same; cf. Topping and Bishop (1989, p. 11).

Acknowledgments. We are grateful to Stephen Hall at the Bank of England for providing the interest rate series for retail sight deposits. This series is zero prior to 1984(3), and as listed in Table A.2 thereafter.
### Table A.1. The Four Primary Breaks in M₁

<table>
<thead>
<tr>
<th>Date</th>
<th>Break</th>
<th>(M_1) (s.a.) after break</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1971(4)</td>
<td>+403 (+3.9)</td>
<td>10765</td>
<td>A break occurs &quot;... due to the incorporation of new information collected from the London clearing banks ... on the sector split of current and deposit accounts ...&quot;. (p. 25)</td>
</tr>
<tr>
<td>1975(2)</td>
<td>+618 (+4.0)</td>
<td>15929</td>
<td>&quot;New, more comprehensive, statistical returns introduced in May 1975 further reduced the estimation necessary to calculate (M_1) ...&quot;. (p. 26)</td>
</tr>
<tr>
<td>1976(1)</td>
<td>-266 (-1.5)</td>
<td>17588</td>
<td>&quot;This is due to the incorporation of data on public corporations' holdings of notes and coin ...&quot;, i.e., which are not included in (M_1). (pp. 26-27)</td>
</tr>
<tr>
<td>1981(4)</td>
<td>+2081 (+6.3)</td>
<td>35257</td>
<td>&quot;... the 'monetary sector' was introduced in place of the 'banking sector'; amongst others, this brought the [Trustee Savings Banks] into the monetary sector.&quot; (pp. 12, 28)</td>
</tr>
</tbody>
</table>

*Source for quotes and breaks: Topping and Bishop (1989); see their Table 2(a) for breaks.*

*Units: £ million (% in parentheses).*

### Table A.2. The Series for the Interest Rate \(R_r\)

<table>
<thead>
<tr>
<th>Year</th>
<th>(Q1)</th>
<th>(Q2)</th>
<th>(Q3)</th>
<th>(Q4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1984</td>
<td>0.0</td>
<td>0.0</td>
<td>7.4167</td>
<td>7.5000</td>
</tr>
<tr>
<td>1985</td>
<td>8.8333</td>
<td>9.1667</td>
<td>8.4583</td>
<td>8.2500</td>
</tr>
<tr>
<td>1986</td>
<td>8.7500</td>
<td>7.3750</td>
<td>7.0000</td>
<td>7.3333</td>
</tr>
<tr>
<td>1987</td>
<td>7.5000</td>
<td>6.3750</td>
<td>6.2083</td>
<td>6.3333</td>
</tr>
<tr>
<td>1988</td>
<td>5.8750</td>
<td>5.4167</td>
<td>6.5667</td>
<td>7.5750</td>
</tr>
<tr>
<td>1989</td>
<td>8.3000</td>
<td>8.3000</td>
<td>8.3000</td>
<td>8.9000</td>
</tr>
</tbody>
</table>


*Units: Per cent per annum. However, note that computations are with \(R_r\) as a fraction.*
APPENDIX B. Additional Regression Results

This appendix presents regression results for the money-demand equations (6), (16), and (17) with the learning parameters $\alpha$ and $\beta$ of the weighting function $w_t$ estimated, noting that these estimates are conditional on the selected specification of the rest of the model. Also, (6) is estimated using the ETAS measures of total final expenditure rather than the constructed series.

In (6) (for the UK), the parameters $\alpha$ and $\beta$ were set at 5 and 1.2. Estimating $\alpha$, $\beta$, and the regression coefficients in (6) by nonlinear least-squares, we obtain the following.\footnote{Equations (B.6), (B.16), and (B.17) were estimated by nonlinear least-squares with the computer package TROLL Version 13; cf. Intex Solutions (1989).}

\[
\begin{align*}
\Delta (m-p)_t &= -0.69 \Delta p_t - 0.17 \Delta (m-p-y)_{t-1} - 0.094 (m-p-y)_{t-1} \\
&\quad - 0.643 \left\{ R3_t - Rr_t / (1 + \exp [3.2 - 0.75 \cdot (t-t^*+1)]) \right\} + 0.024 \\
T &= 100 \left[ 1964(3)-1989(2) \right] \quad \hat{\sigma} = 1.319\% 
\end{align*}
\]

$\alpha$ and $\beta$ change somewhat (but not statistically significantly so), with $\hat{\beta} = 0.75$ closely matching the assumed US value and $\hat{\alpha} = 3.2$ indicating greater initial knowledge. The estimated values of $\alpha$ and $\beta$ imply $w_t = 0.50$ after one year and $w_t = 0.99$ after 2½ years: their arbitrary values imply virtually the same times. All the other estimated coefficients, their estimated standard errors, and $\hat{\sigma}$ are virtually unchanged.

In (16) and (17) (for the US), the interest rates $Rn$, $Rsu$, $Rm$, and $Rc$ have separate weighting functions, each with an $\alpha$ and $\beta$. Estimating the four pairs of $(\alpha, \beta)$ unconstrainedly results in very imprecisely estimated values for them (especially those for $Rm$ and $Rc$), but little reduction in the likelihood function. When $\alpha$ is constrained to be equal across interest rates, and likewise $\beta$, we obtain the following.
(B.16) \[ \Delta(m-p)_t = -0.62 \Delta p_t + 0.89 \text{ Vol}_t + 0.390 - 0.274 (m-p-ty)_{t-2} \]
\[ - 1.15 (\Sigma_{01} Rma)_t - 1.78 (\Sigma_{01} S)_t - 1.13 (\Sigma_{01} S)_t - 0.55 (\Delta_4 [m-p]_{t-1}/4) \]
\[ - 0.92 (\Delta_4 p_{t-2}/4) + 6.1 \text{ SVol}_t + 12.9 \Delta S\text{Vol}_{t-1} \]
\[ + 0.0127 \text{ DM80}_t + 0.474 \text{ Rnsa}_{t-1} + 0.47 (\Delta_2 y/2)_t \]
\[ T = 113 [1960(3)-1988(3)] \quad \hat{\sigma} = 0.407\% \quad \hat{\alpha} = 5.5 \quad \hat{\beta} = 0.65 \]

(B.17) \[ \Delta(m-p)^*_t = -0.62 \Delta p_t + 0.89 \text{ Vol}_t + 0.391 - 0.275 (m-p-ty)_{t-2} \]
\[ - 1.15 (\Sigma_{01} Rma)_t - 1.79 (\Sigma_{01} S)_t - 1.12 (\Sigma_{01} S)_t \]
\[ - 0.55 (\Delta_4 [m-p]_{t-1}/4) - 0.94 (\Delta_4 p_{t-2}/4) + 6.4 \text{ SVol}^*_t \]
\[ T = 113 [1960(3)-1988(3)] \quad \hat{\sigma} = 0.399\% \quad \hat{\alpha} = 5.7 \quad \hat{\beta} = 0.67 \]

The estimated values of \( \hat{\alpha} \) and \( \hat{\beta} \) suggest a slightly higher initial knowledge but slower rate of learning than implied by their values in (16) and (17). Even so, the numerical values of coefficient estimates in both (B.16) and (B.17) are virtually unchanged relative to those in (16) and (17). Further, the values of \( \hat{\sigma} \) in (B.16) and (B.17) are slightly higher than those in (16) and (17), implying that a test of \( \alpha=7, \beta=0.8 \) would not be rejected.\(^5\)

Equation (B.6*) below reproduces (6), but using the ETAS measures of total final expenditure (X and X85, with implied deflator X/X85) rather than total final expenditure constructed from GDP and imports (Y-P and Y, with implied deflator P).

(B.6*) \[ \Delta(m-[x-x85])_t = -0.67 \Delta(x-x85)_t - 0.18 \Delta(m-p-x85)_{t-1} \]
\[ \quad \text{[0.14]} \quad \text{[0.06]} \]
\[ - 0.633 R^*_t - 0.093 (m-p-x85)_{t-1} + 0.023 \]
\[ \text{[0.052]} \quad \text{[0.008]} \]
\[ T = 100 [1964(3)-1989(2)] \quad R^2 = 0.76 \quad \hat{\sigma} = 1.308\% \quad \text{DW} = 2.18 \]

\(^5\)Although TROLL permits estimation (i.e., function minimization) using numerical derivatives, currently it calculates the covariance matrix only with analytical derivatives. Because (B.16) and (B.17) include parameters within the "max" function, no analytical derivatives are available for those equations, so no estimated standard errors are given.
The estimates in equations (6) and (B.6*) are virtually identical, reflecting the presence of only minor differences between the series.

The ETAS measure for real TFE differs from the constructed series for years up to and including 1982 because of the method used to re-base the series on 1985 prices. Because the components are available from the BIS on a more timely basis than the TFE series is from ETAS, we chose to construct total final expenditure from its components and use that series for the computations in the text.
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