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PURCHASING POWER PARITY AND UNCOVERED INTEREST RATE PARITY: THE UNITED STATES 1974 - 1990

Hali J. Edison and William R. Melick

ABSTRACT

This paper examines the factors behind long-run movements of the dollar. Most recent work has concluded that structural exchange rate models explain only a small proportion of exchange rate movements. However, many economists still find the theory that links exchange rates and interest rates persuasive. We investigate the relationship between exchange rates, prices, and interest rates using multivariate maximum likelihood cointegration tests. In particular, we explicitly test for purchasing power parity and uncovered interest rate parity when using nominal exchange rates, and implicitly test for these two hypothesis when using real exchange rates. The conclusion that emerges from this study is that we almost always identify at least one cointegrating vector among the variables, but we can not verify the theoretical models that show how exchange rates and interest rates are linked.

Purchasing Power Parity and Uncovered Interest Rate Parity: the United States 1974 - 1990

Hali J. Edison and William R. Melick[1]

I. Introduction[2]

Since the beginning of the 1980s most of the economics profession has concluded that structural exchange rate models explain only a small proportion of the dollar's movement (see Meese and Rogoff (1983)). Furthermore, the building blocks for many of the exchange rate models, uncovered interest rate parity and purchasing power parity, have been shown to have been violated. These points were re-emphasized in Isard (1987) and Meese (1990). Nevertheless, many economists still find the theory that links exchange rates and interest rates persuasive. Instead of trying to explain short-run fluctuations of the exchange rate, the goal of this paper is to examine the factors behind the permanent or long-run movements of the dollar. In particular, we investigate the relationship between exchange rates, prices, and interest rates exploiting maximum likelihood cointegration tests.

The motivation behind this study can be found in Figure 1, which plots the CPI-adjusted value of the dollar against a measure of the real long-term interest rate differential. This figure suggests that these two

1. The authors are staff economists in the Division of International Finance. We would like to thank Neil Ericsson, Joe Gagnon, Bill Helkie, Dale Henderson, David Howard, Dianne Pauls, Charles Thomas and participants in the IF Monday Workshop. David Eiler provided valuable research assistance. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as those reflecting the Board of Governors of the Federal Reserve System or other members of its staff.

2. This paper is an extension of the work described in Edison and Pauls (1991). In that paper, the authors focus on testing the relationship between real exchange rates and real interest rate differentials only, using the Engle-Granger single equation cointegration test. They fail to find a cointegrating vector and thus conclude that the data do not support the real uncovered interest rate parity relationship.

time series move together. To establish this econometrically we focus on the long run relationship between exchange rates and other variables.

This paper relates to an extensive body of literature. It is closely aligned to investigations of long-run purchasing power parity (PPP) and uncovered interest rate parity (UIP), particularly those papers which study whether these two propositions hold jointly. This first, or nominal, strand of literature contains a large number of papers that focus only on PPP, but very few that focus on both propositions jointly.[3] The results of these studies are mixed. Those papers that consider both propositions tend to find one cointegrating vector between nominal exchange rates and relative prices, but at least one study (Johansen and Juselius (forthcoming)) reports finding two cointegrating vectors. Our paper is also closely related to those papers that investigate the long-run relationship between real exchange rates and real interest rate differentials.[4] The results in this second, or real, strand of literature are also mixed. Most conclude that they cannot reject the null hypothesis of non-cointegration between long-term interest rates and real exchange rates.

This paper bridges these two strands of literature, treating the real strand as a restricted version of the nominal. The paper is organized as follows. Section II gives the model framework and the basic relationships that are examined empirically. Section III discusses the data and the econometric methodology. Section IV presents the cointegration

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3. For the recent literature on testing PPP see, for example, Edison and Klovland (1987), Lothian (1991), Abuaf and Jorion (1990), Hakkio and Joines (1990), Mark (1990), Edison and Fisher (1991), and Patel (1991). For the literature that jointly tests PPP and UIP, see, for example, Juselius (1990), Johansen and Juselius (forthcoming), and Adams and Chadha (1991).

4. See, for example, the earlier papers by Campbell and Clarida (1987), Meese and Rogoff (1988) and the more recent papers by Coughlin and Koedijk (1990), Amano and van Norden (1991), Blundell-Wignall and Browne (1991), and Edison and Pauls (1991).

results for nominal exchange rates, prices, and nominal interest rates and section V describes the cointegration results for real exchange rates and real interest rates. Section VI concludes the paper.

II. The Model

The empirical investigation of the long run behavior of the exchange rate is based on uncovered interest rate parity (UIP) and purchasing power parity (PPP). The uncovered interest rate parity condition is defined as

$$(1) \qquad s_t = E(s_T) + i_{t,T} - i^*_{t,T} ,$$

where:

- s = log of spot exchange rate (foreign currency per dollar)
- E(x) = the expected value of any future variable x based on information at time t
- i, i^* = nominal own rates of interest on assets denominated in home and foreign currencies, as compounded over horizon T-t

Purchasing power parity and hence the real exchange rate is defined as:

$$(2) \qquad q = s + p - p^*,$$

where:

- q = log of the real exchange rate
- p, p^* = log of domestic, foreign price levels

Combining (1) with an expression for $E(s_T)$ derived from (2):

$$(3) \qquad s_t = E(q_T) + E(p^*_T) - E(p_T) + i_{t,T} - i^*_{t,T}.$$

It is convenient to rewrite the expected future logarithmic price levels in terms of current prices and expected inflation, using the approximation:

$$(4) \qquad E(p_T) = p_t + E(\pi),$$

$$E(p_T^*) = p_t^* + E(\pi^*).$$

Replacing expected future prices in equation (3) using equation (4) gives:

$$(5) \qquad s_t = E(q_T) + p_t^* + E(\pi^*) - p_t - E(\pi) + i_{t,T} - i_{t,T}^* .$$

To empirically implement equation (5) two assumption are made. First, we assume that expected inflation is stationary.[5] The assumption for expected inflation may be too stringent, so it is relaxed when testing real exchange rates. Second, we assume that the expected long-run real exchange rate is stationary. While some might find this assumption unappealing, it is made because previous attempts to model the long-run real exchange rate have been unsuccessful. Furthermore, work by Frankel (1986) and Diebold, Husted, and Rush (1991) have shown that over a long time horizon the real exchange rate is stationary or mean reverting. The other variables in equation (5) are considered to be non-stationary as is explained in detail in the following sections. Thus, the final equation for the nominal exchange rate used in the cointegration tests is as follows:

$$(6) \qquad s_t = p_t^* - p_t + i_{t,T} - i_{t,T}^* .$$

5. A stationary variable is said to be integrated of order zero and is denoted by I(0). A non-stationary variable that is rendered stationary by first differences is denoted by I(1).

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(6) $$s_t = p_t^* - p_t + i_{t,T} - i_{t,T}^* .$$

5. A stationary variable is said to be integrated of order zero and is denoted by I(0). A non-stationary variable that is rendered stationary by first differences is denoted by I(1).

To examine the real exchange rate-real interest rate relationship, we again start with equation (5). Using the definition for the real exchange rate in (2) yields:

$$(7) \qquad q_t = i_{t,T} - E(\pi) - i^*_{t,T} + E(\pi^*) + E(q_T).$$

Equation (7) describes the relationship between real exchange rates, nominal interest rates, inflation rates, and the expected long-run real exchange rate. Note that the expected real rates of interest may be written as:

$$(8) \qquad E(r_{t,T}) = i_{t,T} - E(\pi),$$

$$E(r^*_{t,T}) = i^*_{t,T} - E(\pi^*).$$

Thus, equation (7) can be viewed as the relationship between real exchange rates and real interest rate differentials. Implementing (7) requires modelling the expected future real exchange rate. As in the nominal case, we simplify matters by assuming that the expected real exchange rate is stationary and the other variables are treated as nonstationary.

III. The Data and the Econometric Methodology

The data are quarterly observations for 1974 - 1990. Exchange rates are the Federal Reserve Board staff's trade-weighted value of the U.S. dollar against the other G-10 currencies, and four bilateral rates against the U.S. dollar: the Japanese yen, the German mark, the British pound sterling, and the Canadian dollar. Nominal interest rates are the 10-year constant maturity rate on Treasury bonds for the United States and yields on bellwether government bonds for the foreign G-10 countries.[6] Prices are measured by CPIs. The weighted average value of the dollar in real terms is calculated by adjusting the nominal value by the ratio of the U.S. to the foreign CPI. For the analysis of the trade-weighted dollar, the foreign variables are similarly trade weighted. The measure of expected inflation is a 12-quarter centered moving average of CPI inflation rates, where forecasts are used when published data are not available.

We do not present a detailed analysis of the univariate statistical properties of the time series. The data are the same as those used in Edison and Pauls (1991) and they report a number of unit root tests. In general, they conclude that all the time series are integrated of the same order -- I(1). Appendix I gives details of the data and sources.

To test the long-run relationships, we use cointegration tests based on the maximum likelihood estimates from a VAR specification as described in Johansen (1988), Johansen and Juselius (1989) and Johansen (1991).[7] The Johansen procedure analyzes the relationship among p I(1) variables using the following VAR system:

$$(9) \qquad \Delta X_t = \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_{k-1} \Delta X_{t-(k-1)} - \Pi X_{t-k} + \mu + \sigma D_t + \epsilon_t,$$

where:

X_t = a (p,1) vector of observations on the p variables at time t

D_t = a (p,3) matrix of centered, seasonal, dummy variables[8]

6. In most of the foreign G-10 countries, the liberalization of financial markets is a fairly recent phenomena. Previously, 10-year bonds did not exist in many of these countries. For the early part of our sample, we used the best available proxy -- often an average yield on a set of bonds of intermediate maturity.

7. Edison and Pauls use the Engle Granger (1987) cointegration tests for real exchange rates and real interest rates. Because they do not find cointegration using this technique, it is not used in this paper.

8. A centered, seasonal dummy variable sums to zero over a year's time.

μ = a (p,1) vector of constant terms for each equation

Γ_i = a (p,p) matrix of short-run dynamic coefficients

Π = a (p,p) matrix of long-run dynamic coefficients

ϵ = a (p,1) vector of error terms

The matrix Π captures the long-run relationships between the p variables, and there are three possibilities for it:

1. Rank of $\Pi = p$, the vector process X is stationary.
2. Rank of $\Pi = 0$, Π is the null matrix and hence ΔX is stationary.
3. Rank of $\Pi = r < p$, there are r linear combinations of X that are stationary -- r cointegrating vectors.

The rank of Π is determined by calculating its p eigenvalues and determining if they are statistically different than zero. Johansen has set out two tests to determine the number of cointegrating vectors. The first is an unconditional test based on the trace of the eigenvalues, while the second is a conditional test based on a sequential comparison of the eigenvalues (maximum eigenvalue statistic). The number of non-zero eigenvalues provides an estimate of r, the number of cointegrating vectors. Note that the statistical distributions of these test-statistics are non-standard.

To interpret the results it is possible to decompose Π into two matrices α and β such that

(10) $\Pi = \alpha\beta'$.

The α and β matrices are not unique, but can be useful in interpreting the results. The β matrix contains the coefficients of the r (p,1) cointegrating vectors, while α contains the coefficients for the cointegrating vectors in each of the p equations. Johansen has developed a set of procedures which enables one to test linear restrictions imposed on the β matrix, that is restrictions suggested by long-run economic theories.

These tests will be considered in the following sections to help us determine what relationship may be generating the cointegrating vectors.

IV. Cointegration Tests: Nominal Rates

This section applies the Johansen procedure to the data set containing the nominal exchange rate, domestic and foreign prices, and domestic and foreign nominal interest rates (equation (6)).[9] In terms of equation (9), $X' = (e, p, p^*, i, i^*)$, and a cointegrating vector would be written as

$$(11) \qquad (e_t,\ p_t,\ p_t^*,\ i_{t,T},\ i_{t,T}^*) * \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \end{bmatrix}.$$

Table 1 reports the number of cointegrating vectors that are statistically significant, based on the maximum eigenvalue statistic, for the trade-weighted value of the dollar and the four bilateral exchange rates against the dollar.[10] For the trade-weighted value of the dollar and for Japan the results indicate that there are two cointegrating vectors, while for the other bilateral exchange rates only one cointegrating vector was identified.

In an attempt to determine whether these cointegrating vectors are consistent with PPP and/or UIP, we reestimate the model and impose the restrictions on the cointegrating vectors. A general formulation for testing PPP would set $\beta_1 = \beta_2 = -\beta_3$ in equation (11), allowing the two interest

9. The methodology and the hypothesis testing employed in this section follows closely the paper of Johansen and Juselius (forthcoming).

10. Note that prior to applying the Johansen procedure to the data it was necessary to determine the appropriate length of the vector autoregression. We estimate VARs from one to seven lags and choose the lag length of the VAR which gives white noise errors based on LM tests.

rates to enter the cointegrating vector(s) without any restrictions. The results of the PPP test are reported in the column marked T1. In all cases with the exception of Germany the general restriction of PPP is rejected.[11] A more restricted test for PPP was also applied to Germany, setting the coefficients on interest rates equal to zero ($\beta_4=\beta_5=0$), this restriction was rejected.

The second hypothesis of interest is UIP. As above, a general formulation of this test would restrict $\beta_4= -\beta_5$, placing no restrictions on the other coefficients. This test is reported in column T2 and the restriction is rejected for all exchange rates with the exception of the trade-weighted dollar.[12] By way of completeness, we present a third column, T3, where we test PPP and UIP jointly. Not surprisingly, considering our previous results, these restrictions are rejected for all exchange rates.

Given that the results are somewhat tentative for PPP, we test whether the PPP relation is a stationary process itself following Johansen and Juselius (forthcoming). This is done first by excluding nominal interest rates from the X_{t-k} while still including them in the ΔX_{t-1} .... $\Delta X_{t-(k-1)}$ matrices (see equation (10)) and second by excluding nominal interest rates altogether.[13] These tests are linked closely to the mainstream PPP literature, although not of particular interest to the main

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11. Appendix Two, Table A1 shows the unrestricted coefficient estimates. The coefficients on the exchange rates have been normalized to one. The coefficients on the two prices are of opposite sign and much larger than one. The reason we do not reject the PPP hypothesis for Germany may be that the coefficients are very imprecisely estimated.
12. Table A1 shows that, except for Germany, the coefficients on the two interest rates are almost of equal and opposite sign.
13. In Table 1 these test refer to the columns labelled vector 2 and 3. Excluding the interest rates from the X_{t-k} matrix while still including them in the ΔX_{t-1} .... $\Delta X_{t-(k-1)}$ matrices allows interest rates to affect the short-run dynamics of the system but not the system's long-run equilibrium.

question of the paper. All the results were the same across currencies and for both tests, one cointegrating vector was identified in every instance, but the PPP restriction of proportionality is always rejected. This result says that nominal exchange rates and prices may be cointegrated, but not with a unit coefficient as suggested by PPP. Alternatively, the two price vectors may be cointegrated -- that is, relative prices may be stationary.

Our results differ somewhat from Johansen and Juselius (forthcoming) who also test jointly for UIP and PPP using the trade-weighted value of the British pound sterling. They find that PPP holds for the trade-weighted pound in the five variable vector case, equation 11. In contrast, we find support for PPP only for the German-U.S. bilateral exchange rate. These different findings appear to be the result of different sample sizes. Johansen and Jusulieus (forthcoming) use data from 1972:1 through 1987:3. When we extended data quite similar to theirs through 1990:4 the PPP hypothesis for the trade-weighted pound was rejected.[14] Moreover, when we estimated over the period 1974:3 to 1990:4 (our sample period) the PPP hypothesis for the trade-weighted pound was again rejected. On balance, these findings are consistent with exchange market participants who may give more weight to PPP considerations at different times.

The results uncovered thus far seem to suggest that although at least one cointegrating vector is identified, the restrictions associated with PPP or UIP are not consistent with the data. Thus we cannot verify the theoretical models that show how exchange rates and interest rates are linked. In the next section, we relax the assumption that expected

14. Soren Johansen kindly provided us with their original data set. We replicated their results and constructed an almost identical set of series that covered the time period 1972:1 through 1990:4.

inflation is stationary and examine the real exchange rate-real interest rate relationship.[15]

V. Cointegration Tests: Real Rates

This section investigates the relationship between real exchange rates and real interest rates as described in equation (7) in section II. Ignoring the expected long-run real exchange rate and using a proxy for expected inflation leaves an equation with five I(1) variables that can be generalized to

$$\beta_1 q_t = \beta_2 i_{t,T} + \beta_3 i^*_{t,T} + \beta_4 \pi + \beta_5 \pi^*. \tag{12}$$

In terms of equation (12), real uncovered interest rate parity (RUIP) imposes the restrictions that $\beta_2 = -\beta_4 = -\beta_3 = \beta_5$.

Table 2 contains the results for the five variable system for five currencies vis-a-vis the U.S. dollar. In two out of the five cases, one cointegrating vector was found. For these two cases (Germany and Canada), columns 3 through 6 (labeled T1-T4) attempt to determine what coefficient restrictions can be placed on the single cointegrating vector. The results of the tests of the restrictions imposed by RUIP are found in the column labelled T4. This restriction is rejected for both countries. Given this rejection, tests T1-T3 are conducted in an attempt to determine what combination of the variables is generating the single cointegrating vector. For Canada, it appears that the inflation differential is the cointegrating vector.[16] For Germany, the single cointegrating vector is not generated by

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15. By making the assumption that <u>expected</u> inflation is I(1) we are close to making the assumption that prices are really I(2) variables. The unit root tests do not show prices being I(2), however, they do show <u>expected</u> inflation as being I(1). This result may be due to the low power of such unit root tests.

16. This result can also be seen in Table A2. The coefficients on inflations for Canada appear to be of equal and opposite sign.

the inflation differential, the nominal interest rate differential or the real interest rate differential.

The Johansen procedure identified two cointegrating vectors for the remaining three countries. The fact that any linear combination of two cointegrating vectors is also a cointegrating vector makes it difficult to determine what lies behind the cointegrating vectors for these three currencies.[17] The columns labelled T5-T8 test what combinations of the variables might have generated the two cointegrating vectors. For the trade-weighted dollar, the United Kingdom, and Japan, the two cointegrating vectors are not consistent with any of the hypotheses advanced in T5-T8.

Even though we cannot find which long-run theory accounts for the cointegrating vectors in the five variable system, for the sake of completeness, we consider two smaller systems reported in Table 3. The first of these is a three variable system, which contains real exchange rates and real interest rates, and the second is a two variable system, which contains only real exchange rates and real interest differentials. In general, the overall results do not support RUIP. However, in the three variable system for Germany we cannot reject RUIP and in the two variable case for the trade-weighted dollar we find a cointegrating vector, which implies RUIP. This finding is consistent with the visual evidence presented in Figure 1.

Our results for Canada are consistent with those of Amano and van Norden (1991) who also use the same multivariate approach for the bilateral

17. This is true in any system when there is more than one cointegrating vector, but especially so in this instance. Further complications arise in the present case because there are sensible economic hypothesis that overlap. For example, RUIP and the inflation differential might be stationary or RUIP and the nominal interest rate differential might be stationary (hypothesis T7, Table 2). This difficulty did not arise in the nominal system because two sensible, competing hypothesis did not exist and were not tested.

Canadian dollar/ U.S. dollar exchange rate. In general, our results are also consistent with Adams and Chadha (1991) who find some limited number of cointegrating relationships for the four bilateral exchange rates using the Johansen procedure. However, our results are in contrast to those found when using the single equation approach as applied by Edison and Pauls (1991) and Meese and Rogoff (1988). In these studies the authors cannot reject the null of non-cointegration. In a similar single equation approach, Blundell-Wignall and Browne (1991) find cointegration. They acheive this by adding the share of the cumulated crrent account relatve to GNP.

The missing story in our investigation for both the nominal and the real exchange rates is what variables form the cointegrating vector(s) and how they link to economic theory. While we have tried to test various hypotheses, our testing has not been totally exhaustive. Our permutations have, however, allowed us to test those hypotheses that relate specifically to purchasing power parity, uncovered interest rate parity, and real uncovered interest rate parity. The many rejections of each of these propositions might be explained by the low power of the tests. It could also be related to our sample period. This has been demonstrated with our reconstruction of the Johansen Juselius (forthcoming) results for the trade-weighted pound. Many of the studies that have successfully identified PPP use long data sets spanning the century. Perhaps expanding the data set (although difficult to do on a multilateral basis when long term interest rates are involved) and exploring other techniques may fill in our unanswered questions.

VI. Conclusion

The objective of this paper is to examine the factors behind long-run movements of the dollar. Although most of the economics profession has concluded that structural exchange rate models explain only a small proportion of exchange rate movements, some economists still hold onto the belief that exchange rates and interest rates are intrinsically related to each other as theory suggests. This paper investigated the relationship among exchange rates, prices, and interest rates using multivariate maximum likelihood cointegration tests following the Johansen procedure. In particular, we explicitly tested for purchasing power parity and uncovered interest rate parity when using nominal exchange rates, and implicitly tested for these two hypothesis when using real exchange rates. The conclusion that emerges from this study is that we find at least one cointegrating vector amongst exchange rates, prices and interest rates, but we can not verify the theoretical models that show how exchange rates and interest rates are linked.

Figure 1
The Dollar and Real Interest Rates

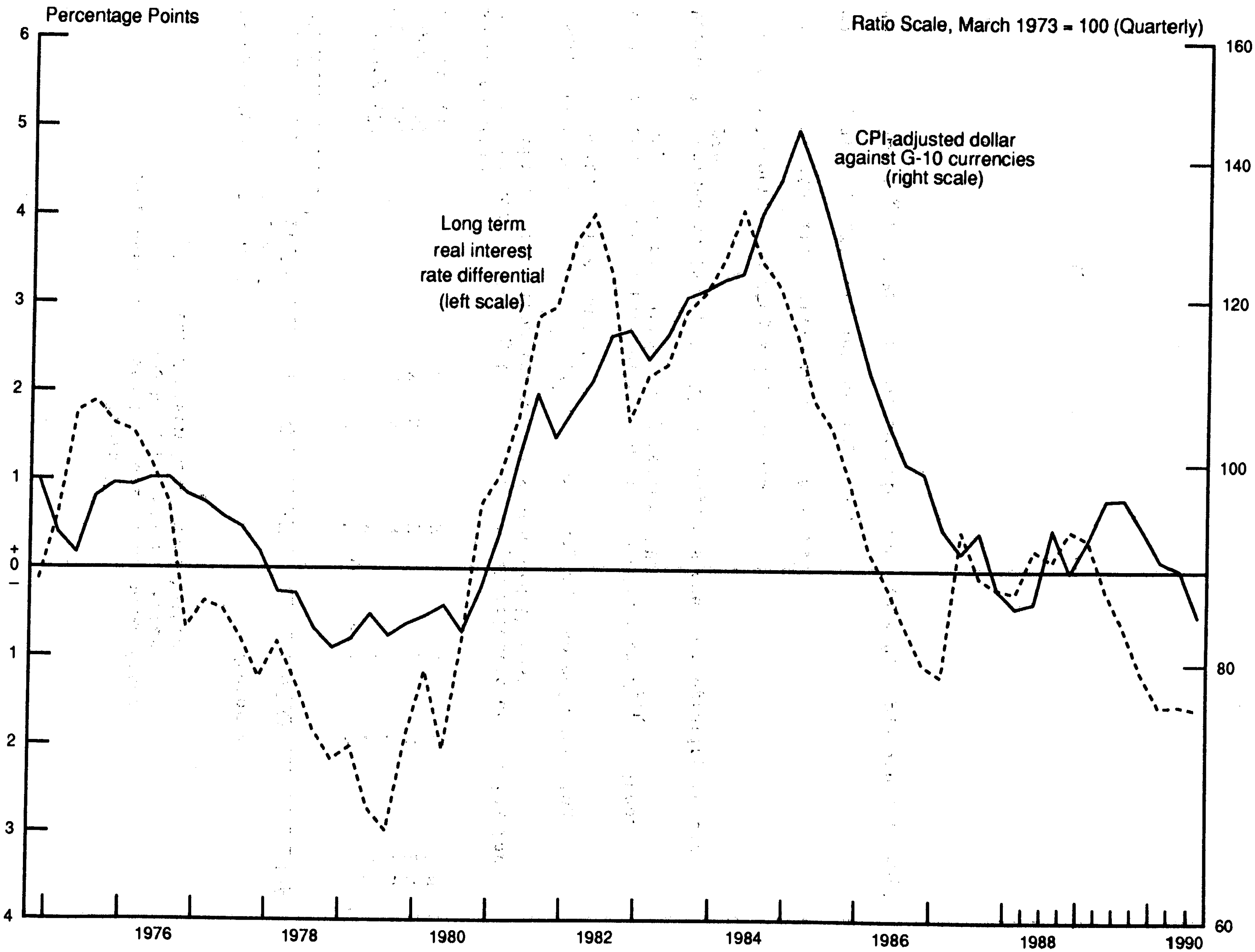

Table 1
Cointegration Tests for Nominal Exchange Rates,
Prices, and Nominal Interest Rates
Johansen Maximum Likelihood Method

Country	(VL)	Vector 1				Vector 2		Vector 3	
		# of CV	T1	T2	T3	# of CV	T1	# of CV	T1
TW	(3)	2	N	Y	N	1	N	1	N
Germany	(3)	1	Y	N	N	1	N	1	N
UK	(3)	1	N	N	N	1	N	1	N
Japan	(4)	2	N	N	N	1	N	1	N
Canada	(4)	1	N	N	N	1	N	1	N

Notes to Table:

VL : Defined as VAR lag length. Lag length selected on testing for white noise error in VAR using standard LM test.

Vector 1: Defined as (e,p,p^*,i,i^*). Theory suggests two cointegrating vectors: PPP and UIP.

Vector 2: Defined as $(e,p,p^*$ with i and i^* excluded from $X_{t-k})$. Theory suggests one cointegrating vector: PPP.

Vector 3: Defined as (e,p,p^*). Theory suggests one cointegrating vector: PPP.

# of CV : Number of Cointegrating Vectors.

T1 : Testing PPP restriction: $(\beta_1 = \beta_2 = -\beta_3)$

T2 : Testing UIP restriction: $(\beta_4 = -\beta_5)$

T3 : Testing both PPP and UIP restrictions: $(\beta_1 = \beta_2 = -\beta_3)$ and $(\beta_4 = -\beta_5)$

Table 2

Cointegration Tests for Real Exchange Rates,
and Real Interest Rates
Johansen Maximum Likelihood Method

Country	VL	# of CV	Vector 1 T1	T2	T3	T4	T5	T6	T7	T8
Germany	4	1	N	N	N	N	-	-	-	-
Canada	4	1	N	Y	N	N	-	-	-	-
TW	2	2	-	-	-	-	N	N	N	N
UK	3	2	-	-	-	-	N	N	N	N
Japan	4	2	-	-	-	-	N	N	N	N

Notes to Table:

VL : Defined as VAR lag length. Lag length selected on testing for white noise error in VAR using standard LM test.

Vector 1: Defined as (q,i,i^*,π,π^*). Theory suggests two cointegrating vectors: RUIP and UIP.

# of CV : Number of Cointegrating Vectors.

T1 : Testing Nominal Interest Differential stationarity: $(\beta_2 = -\beta_3)$.

T2 : Testing Inflation Differential stationarity: $(\beta_4 = -\beta_5)$.

T3 : Testing Real Interest Rate stationarity: $(\beta_2 = -\beta_4 = -\beta_3 = \beta_5)$.

T4 : Testing RUIP Stationarity: equation (8).

T5 : Tests T1 and T2.

T6 : Test each real interest rate for stationarity: $\beta_4 = -\beta_2$ and $\beta_5 = -\beta_3$

T7 : RUIP and either T1 or T2.

T8 : RUIP and either domestic real interest rate stationarity or foreign real interest rate stationarity.

Table 3

Cointegration Tests for Real Exchange Rates,
and Real Interest Rates
Smaller Systems: Johansen Method

Country	Vector 2			Vector 3	
	VL	# of CV	T1	VL	# of
Germany	4	1	Y	3	0
Canada	4	1	N	2	0
TW	5	2	N	3	1
UK	2	0	-	2	0
Japan	4	0	-	3	0

Notes to Table:

VL : Defined as VAR lag length. Lag length selected on testing for white noise error in VAR using standard LM test.

# of CV : Number of Cointegrating Vectors.

Vector 2: Defined as (q,r,r^*). Theory suggests one cointegrating vector: RUIP.

Vector 3: Defined as (q,r^+). Theory suggests one cointegrating vector: RUIP.

T1 : RUIP, for TW also the real interest rate differential.

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Appendix I

The data for the trade-weighted value of the dollar are quarterly from 1974 Q3 to 1990 Q3 (65 observations). The data for the bilateral exchange rates are quarterly from 1974 Q3 to 1990 Q4.

Exchange Rate:
Trade-Weighted Value of the dollar (FRB Bulletin).
German mark/ U.S. dollar (FRB Bulletin).
Japanese yen/ U.S. dollar (FRB Bulletin).
British pound sterling/ U.S. dollar (FRB Bulletin).
Canadian dollar/ U.S. dollar (FRB Bulletin).

Interest Rate:[1]
10-year constant maturity rate on Treasury bonds (FRB Bulletin).
Trade-Weighted average of yields on bellwether government bonds for foreign G-10 countries (various publications)
German bellwether government bonds (Bundesbank Monthly Report).
Japanese bellwether government bonds (Toyko Stock Exchange).
British bellwether government bonds (Bank of England Quarterly Report).
Canadian bellwether government bonds (Bank of Canada Review).

Prices:
U.S. CPI price index (FRB Bulletin)
Trade-weighted average of CPIs. for the foreign G-10 countries
Germany (Bundesbank Monthly Report).
Japan (Bank of Japan, Economic Statistics).
U.K. (CSO, Employment Gazette).
Canada (Bank of Canada Review).

Expected Inflation:
(Created from CPI price indices)
12-quarter center moving average of CPI inflation rates.

[1] The interest rate data are also available from FRB publication: "Selected Interest and Exchange Rates - Weekly Series of Charts".

Appendix II
Table A1
Cointegration Tests for Nominal Exchange Rates

A. 5 - Variable System

Coefficient[1]	e	p	p*	i	i*	Test of Restriction.[2] LR	pvalue
Germany							
Vector 1	1.0	157.0	-338.0	.19	7.79		
T1						3.0	(.221)
T2						13.8	(.0002)
T3						22.0	(0)
Canada							
Vector 1	1.0	74.7	-65.6	.46	-1.20		
T1						43.0	(0)
T2						43.2	(0)
T3						59.2	(0)
Trade-Weighted							
Vector 1	1.0	5.3	-5.6	-.175	.23		
	1.0	-20.0	22.8	-.22	.12		
T1						22.7	(.0001)
T2						4.7	(.097)
T3						34.8	(0)
United Kingdom							
Vector 1	1.0	6.21	-4.4	-.53	.135		
T1						45.3	(0)
T2						9.9	(.0016)
T3						48.2	(0)
Japan							
Vector 1	1.0	-13.4	24.7	-.61	.45		
	1.0	-.3	1.5	.07	-.25		
T1						14.1	(.007)
T2						7.0	(.03)
T3						26.1	(.0002)

Table A1 continued

B. Smaller Systems

Coefficient[1]	e	p	p^*	i	i^*	Test of Restriction.[2] LR	pvalue
Germany							
Vector 2	1.0	6.7	-13.1				
T1						18.1	(.0001)
Vector 3	1.0	6.9	-13.4				
T1						16.0	(.0003)
Canada							
Vector 2	1.0	-27.2	24.9				
T1						16.9	(.0002)
Vector 3	1.0	17.5	15.8				
T1						19.2	(0)
Trade-Weighted							
Vector 2	1.0	-16.2	18.1				
T1						21.6	(0)
Vector 3	1.0	15.4	-17.1				
T1						18.1	(.0001)
United Kingdom							
Vector 2	1.0	-27.6	21.4				
T1						29.4	(0)
Vector 3	1.0	26.1	-20.2				
T1						31.8	(0)
Japan							
Vector 2	1.0	5.9	-9.48				
T1						14.0	(.0009)
Vector 3	1.0	8.1	-15.1				
T1						13.8	(.001)

Notes to Table
1. The coefficient estimates come from the unrestricted model for each vector.
2. The LR column displays the likelihood ratio test statistic while the p-value column displays the significance level. (For a definition of the restrictions being tested see Table 1.)

Table A2
Cointegration Tests for Real Exchange Rates

Coefficient[1]	q	i	i^*	π	π^*	Test of Restriction.[2] LR pvalue
Germany						
Vector 1	1.0	-.003	-.219	-.089	.277	
T1						28.6 (0)
T2						15.8 (.001)
T3						21.5 (.0003)
T4						21.1 (.0003)
Canada						
Vector 1	1.0	-.029	-.295	3.54	-3.47	
T1						34.6 (0)
T2						1.4 (.85)
T3						17.4 (.0005)
T4						18.1 (.001)
Trade-Weighted						
Vector 1	1.0	-.220	-.062	.391	-.378	
	1.0	-.134	.170	.045	-.058	
T5						27.5 (.0005)
T6						30.7 (.0002)
T7						27.3 (.0006)
T8						29.7 (.0002)
United Kingdom						
Vector 1	1.0	.780	-7.35	-1.61	3.27	
	1.0	-.125	.024	.13	-.07	
T5						46.6 (0)
T6						38.5 (0)
T7						44.3 (0)
T8						36.2 (0)
Japan						
Vector 1	1.0	.041	-.19	.001	.14	
	1.0	-.067	-.111	.048	-.01	
T5						35.7 (0)
T6						34.7 (0)
T7						36.8 (0)
T8						38.0 (0)

Notes to Table
1. The coefficient estimates come from the unrestricted model for each vector.
2. The LR column displays the likelihood ratio test statistic while the p-value column displays the significance level. (For a definition of the restrictions being tested see Table 1.)

Table A3
Cointegration Tests for Real Exchange Rates
For Smaller Systems

Coefficient[1]	q	r	r*	Test of Restriction.[2] LR pvalue
Germany				
Vector 2	1.0	-.089	.162	
T1				3.7 (.16)
Vector 3[3]	1.0	-.079	.079	
Canada				
Vector 2	1.0	-.372	-.253	
T1				8.8 (.01)
Vector 3[3]	1.0	-.052	.052	
Trade-Weighted				
Vector 2	1.0	-.102	.149	
T1				22.3 (.0001)
Vector 3[3]	1.0	-.091	.091	
United Kingdom				
Vector 2	1.0	-1.53	1.07	
T1				na
Vector 3[3]	1.0	-.677	.677	
Japan				
Vector 2	1.0	4.64	-7.18	
T1				na
Vector 3[3]	1.0	-.384	.384	

Notes to Table
1. The coefficient estimates come from the unrestricted model for each vector.
2. The LR column displays the likelihood ratio test statistic while the p-value column displays the significance level. (For a definition of the restrictions being tested see Table 1.)
3. No restrictions are tested in this instance.

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