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UNDERSTANDING THE EMPIRICAL LITERATURE ON PURCHASING POWER  
PARITY: THE POST-BRETTON WOODS ERA

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# Understanding the Empirical Literature on Purchasing Power Parity: The Post-Bretton Woods Era

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## I. Introduction

Purchasing Power Parity (PPP) in the post-Bretton Woods era by and large has failed to stand up to empirical scrutiny. This paper sheds light on this failure by documenting the low small-sample power of existing empirical tests. We also implement new, more powerful tests that find moderate evidence in favor of PPP.

The PPP hypothesis states that the ratio of domestic to foreign prices determines the "fundamental" or "equilibrium" exchange rate,

$$E = \lambda \frac{P^*}{P} \quad (1)$$

where  $E$  denotes the exchange rate (that is, the foreign price of one unit of domestic currency),  $P^*$  denotes an index of foreign prices,  $P$  denotes an index of domestic prices, and  $\lambda$  is a constant. In its strictest form, PPP is always rejected empirically because equation (1) does not hold exactly for any pair of countries over any time period. However, PPP may be said to hold in the long run if deviations from PPP are not permanent.<sup>2</sup> Since the early 1980s, it has been noted that exchange rates and prices are non-stationary; thus, a necessary

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<sup>2</sup>An alternative approach to the study of PPP is to search for auxiliary variables that can explain the empirical failure of equation (1). See, for example, Edison (1987), Edison and Klovland (1987) and Johansen and Juselius (1992). Other studies have looked for structural breaks in equation (1), for example, Perron and Vogelsang (1992). In this paper we shall focus only on exchange rates and prices, and we shall not consider the possibility of structural breaks or additional variables to equation (1).

condition for finding PPP is that  $E$ ,  $P$ , and  $P^*$  are cointegrated. PPP also implies two additional properties: (1) *symmetry* between domestic and foreign prices, and (2) *proportionality* between relative prices and the exchange rate.

Two classes of econometric methods have been developed for the analysis of non-stationary data. In the early 1980s, univariate and single-equation methods were formulated (e.g., Dickey and Fuller (1979), Engle and Granger (1987)); more recently, systems methods have been developed (e.g., Johansen (1991), Stock and Watson (1993)). With regard to tests of PPP on post-Bretton Woods data, the univariate and single-equation methods generally are unable to find cointegration between the three variables, while systems methods often find cointegration but reject the proportionality and symmetry conditions.

Our results indicate that previous studies have failed to find evidence for PPP in the post-Bretton Woods era largely because they do not have sufficient power given the short sample period. Univariate and single-equation methods fail because they do not efficiently model the interaction between, and the different dynamic behavior of, prices and exchange rates. Systems methods better model these interactions and dynamics, but they typically do not impose the symmetry and proportionality restrictions implied by PPP. By testing against the diffuse alternative of cointegration in general, rather than the specific alternative of PPP, they lose power to reject the null hypothesis that PPP does not hold. Many of the apparent findings of cointegration by multivariate methods in the literature are due to inappropriate critical values. Most studies use asymptotic critical values, but we show that the appropriate small-sample critical values are much larger.

Using a newly devised test based on Horvath and Watson (1993), which imposes the symmetry and proportionality restrictions and tests for cointegration in a multi-equation setting, we often can reject the null hypothesis that PPP does not hold. Using Monte Carlo techniques we show that the Horvath-Watson test has more power than the Johansen tests commonly used in the existing systems literature.

The rest of the paper is organized as follows: Section II briefly describes the results of previous studies of PPP, focusing on the post-Bretton Woods period. Section III presents our tests of PPP using the Johansen methodology. Section IV reports on tests using the Horvath-Watson methodology. Finally, section V contains our concluding remarks.

## **II. Recent Literature**

When interest in the empirical testing of PPP resurfaced in the mid-1970s, conventional econometric methods were employed to test the coefficient restrictions implied by PPP. Since the early 1980s these studies have been criticized on the grounds that it is inappropriate to apply conventional techniques because exchange rates and prices are nonstationary series. The simplest approach to testing for PPP in a framework that allows for nonstationarity is to impose the symmetry and proportionality conditions by defining the real exchange rate,  $R$ , as follows:

$$R = E \frac{P}{P^*} \quad (2)$$

Standard unit-root tests are then applied to determine if the null hypothesis of a unit root in the real exchange rate can be rejected. Rejection of the null hypothesis would be evidence in favor of long-run PPP, since it would imply that deviations of the real exchange rate from its

mean value are only temporary. However, nearly all unit-root studies have concluded that the null hypothesis of a nonstationary real exchange rate cannot be rejected for most countries in the post-Bretton Woods era.<sup>3</sup>

A second group of studies uses single-equation cointegration and error-correction techniques. Instead of imposing the symmetry and proportionality conditions, these studies freely estimate a cointegrating vector between the exchange rate, the domestic price level, and the foreign price level. On post-Bretton Woods datasets these studies almost always fail to reject the null hypothesis of non-cointegration of prices and the exchange rate, and hence they find no evidence in favor of long-run PPP.<sup>4</sup>

Recently, newer procedures based on systems estimation techniques (e.g., the procedures of Johansen and Stock-Watson) have been used to examine PPP. These new approaches often find cointegration in the post-Bretton Woods era, but the estimated cointegrating vectors typically violate the symmetry and proportionality conditions implied by PPP.<sup>5</sup>

Several authors have noted that the evidence for PPP is weaker when using dollar exchange rates than when using other exchange rates. This dollar/non-dollar dichotomy was first noted by Frenkel (1981). More recently, Fisher and Park (1991) find almost no evidence

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<sup>3</sup>See, for example, Corbae and Ouliaris (1988), Edison and Fisher (1991), and Grilli and Kaminsky (1991).

<sup>4</sup>See, for example, Baillie and Selover (1987), Taylor (1988), Mark (1990), Patel (1990), Ardeni and Lubian (1991), and Cheung and Lai (1993b).

<sup>5</sup> See, for example, Patel (1990), Fisher and Park (1991), Johansen and Juselius (1992), Crowder (1992), and Cheung and Lai (1993b).

that U. S. dollar bilateral exchange rates are cointegrated with the relevant price indices, whereas German mark exchange rates often are cointegrated with the relevant price indices.

In contrast to studies on the post-Bretton Woods era, a large number of studies that use datasets spanning most of the century provide evidence in favor of PPP. Edison and Klovland (1987), Ardeni and Lubian (1991) and Kim (1990) find evidence of PPP using Engle-Granger cointegration tests, while Pippenger and Steigerwald (1993) find evidence of PPP using an error-correction model.<sup>6</sup> We believe that these results using larger datasets are consistent with our main finding that tests of PPP limited to the post-Bretton Woods era have very low power due to the small sample size.<sup>7</sup>

### III. Testing for PPP using the Johansen Procedure

The Johansen procedure analyzes the relationship among  $q$  quarterly nonstationary (I(1)) or stationary (I(0)) variables using the following VAR system:

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \dots + \Gamma_{k-1} \Delta X_{t-(k-1)} + \Pi X_{t-k} + \mu + \eta D_t + \varepsilon_t \quad (3)$$

$X_t$  is a  $(q,1)$  vector of observations on the  $q$  variables at time  $t$ .  $D_t$  is a  $(3,1)$  matrix of centered, seasonal dummies,  $\mu$  is a  $(q,1)$  vector of constant terms for each equation, and  $\varepsilon_t$  is a  $(q,1)$  vector of error terms.  $\Gamma_i$  and  $\Pi$  ( $q,q$ ) and  $\eta$  ( $q,3$ ) are matrices of coefficients.

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<sup>6</sup>There is also a large literature on testing PPP during the 1920s floating exchange rate episode; results of these studies have been mixed. The first study of the 1920s was carried out by Frenkel (1978) using traditional OLS methods and it finds PPP. Edison (1985) estimates a general error-correction model and fails to reject non-cointegration for two of the three exchange rates. Taylor and McMahon (1988) find PPP over a similar sample using the Engle-Granger cointegration method. However, Ardeni and Lubian (1991) and Ahking (1990) do not find PPP using this same method. The differences in results are due to different sample sizes and lag lengths.

<sup>7</sup> Hakkio and Rush (1991) use Monte Carlo simulations to show that the power of standard unit root tests increases as the span of the data increases. Lothian and Taylor (1993) also show that standard unit root tests have low power over sample sizes corresponding to the recent float, but the power improves as the sample size increases.

The long-run relationships in the data set are captured in the  $\Pi$  matrix. If the rank of  $\Pi$  is between 0 and  $q$ , (denoted  $z$ ), then there are  $z$  linear combinations of the variables in the system that are  $I(0)$  (cointegrated). In this situation,  $\Pi$  can be decomposed into two  $(q,z)$  matrices  $\alpha$  and  $\beta$  such that  $\Pi = \alpha\beta'$  where  $\beta$  contains the coefficients of the cointegrating vectors and  $\alpha$  is the matrix of coefficients on the cointegrating vectors (speed-of-adjustment coefficients) in each equation.

Johansen (1991) presents two tests for determining the rank of  $\Pi$ , the "trace" test, and the "maximum eigenvalue" test. Johansen and Juselius (1990) present tables of asymptotic critical values for the two test statistics. In addition, Johansen (1991) demonstrates that tests of restrictions on the coefficients of  $\beta$  have chi-squared asymptotic distributions conditional on the order of cointegration being correct. This test is used to test the PPP restrictions of symmetry and proportionality.

For the application to PPP we define  $X_t = (p_t, e_t, p_t^*)$  where lower case letters denote logarithms. This study uses quarterly data from 1974:1 through 1992:4, with observations from 1972:1 through 1973:4 available for presample lags as needed. The price series are consumer price indexes, not seasonally adjusted, published by national statistical agencies for the last month of each quarter. The exchange rates are monthly-average market exchange rates for the last month of each quarter taken from the International Monetary Fund's *International Financial Statistics*. The countries and their abbreviations are Australia (AU), Belgium (BE), Canada (CA), France (FR), Germany (GE), Italy (IT), Japan (JA), the Netherlands (NE), Spain (SP), Sweden (SD), Switzerland (SZ), Turkey (TU), the United Kingdom (UK), and the United States (US). Although not presented here, augmented

Dickey-Fuller tests indicated that the levels of these variables are nonstationary, and that in most cases we can reject the hypothesis that the first differences are nonstationary.

Implementation of the Johansen procedure is complicated by our small sample size (76 observations). Cheung and Lai (1993a) calculate small-sample corrections to the trace and maximum eigenvalue asymptotic critical values using response-surface regressions, but they consider, for our dataset, an uninteresting maintained hypothesis.<sup>8</sup> To generate appropriate critical values we conducted 10,000 trials on the VAR given by equation (3) with  $\Pi = 0$ , the appendix contains details on the Monte Carlo experiments. A comparison of the critical values for the maximum eigenvalue and trace test statistics are found in the table below. Table 1 shows that the use of the asymptotic or Cheung and Lai (1993a) size-adjusted critical values would result in more rejections of the hypothesis of non-cointegration (that is, would lend more support to PPP) than if our Monte Carlo critical values are used.

Size	Trace Test			Maximum Eigenvalue Test		
	Asymptotic	Cheung/Lai	Monte Carlo	Asymptotic	Cheung/Lai	Monte Carlo
.01	37.29	na	44.76	26.15	na	31.69
.05	31.26	36.50	38.33	21.28	24.95	26.35
.10	28.44	33.18	35.08	18.96	22.18	23.68
.20	25.45	na	31.33	16.52	na	20.65

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<sup>8</sup>The null hypothesis for the Johansen trace and maximum eigenvalue tests is non-cointegration. Therefore, determining the appropriate critical values requires sampling from a data generating process (DGP) that does not have a long-run relationship between the variables. Cheung and Lai use  $X_t = X_{t-1} + \epsilon_t$  as their DGP. This process does not capture the short-run dynamics of, or interactions between, the variables in our dataset.



To implement the Johansen test, we estimated equation (3) with  $k = 4$  for each of the 13 country pairs.<sup>9</sup> The top panel of Table 2 presents results of the estimation when Germany is treated as the foreign country, while the bottom panel presents results when the United States is treated as the foreign country.

If PPP held, each country would have at least one cointegrating vector. With Germany as the foreign country (Table 2 top panel), and using the trace test (row TT), for six (Australia, Belgium, France, Japan, Netherlands, and Spain) of the 13 countries there is a significant cointegrating vector ( $TT > 38.33$ ) with a test size of 5 percent. If the maximum eigenvalue test is used (row MT), six significant cointegrating vectors ( $MT > 26.35$ ) are again identified with a slight change in countries (as above, replacing Spain with Switzerland). With the United States as the foreign country (bottom panel) and using the trace statistic (row TT) five (Belgium, Italy, Japan, Netherlands, and Spain) of the 13 countries have a significant cointegrating vector with a test size of 5 percent. If the maximum eigenvalue test is used (row MT) Italy and the Netherlands no longer have a significant cointegrating vector.<sup>10</sup> We conclude that a necessary condition for PPP is evident for fewer than half of the countries in our dataset, regardless of the particular test used or choice of the foreign country.

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<sup>9</sup>This choice of lag length is typical of other empirical studies of PPP that use quarterly data. Lagrange multiplier serial correlation tests indicated that in almost all cases  $k = 3$  or  $4$  was required to ensure white noise errors in equation (3). Given that the Johansen procedure is known to be sensitive to the choice of  $k$ , we also estimated the system with  $k = 2$  and  $3$ ; differences in the results were not substantial.

<sup>10</sup>The use of a different lag length ( $k = 2$  and  $3$ ) would have led us to reach slightly different conclusions about the number of countries with a significant cointegrating vector. The different conclusions are especially noticeable for the United States when  $k = 2$ . However, when  $k = 2$  the equations in (3) exhibit significant serial correlation, invalidating any hypothesis test.

Table 2  
Johansen Results, k = 4, 1974:Q1 - 1992:Q4

Germany as Foreign Country													
	AU	BE	CA	FR	US	IT	JA	NE	SP	SD	SZ	TU	UK
E	0.37	0.50	0.17	0.38	0.18	0.22	0.34	0.39	0.27	0.18	0.34	0.14	0.20
TT	48.16 *	60.75 *	20.97	42.26 *	21.12	31.67	50.78 *	54.36 *	39.63 *	21.50	38.28 +	16.70	26.58
MT	33.58 *	50.29 *	13.34	34.77 *	14.39	17.68	30.34 *	35.73 *	22.75	14.40	30.45 *	11.15	16.52
$\beta_1$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\beta_2$	1.25	0.84	0.74	1.56	-1.86	-0.96	5.46	3.03	1.88	1.71	0.73	0.97	0.12
$\beta_3$	-2.74	-0.82	-1.11	-0.56	-4.41	-5.39	-6.44	-0.72	0.40	-0.04	-1.35	-0.90	-2.18
$\alpha_1$	0.00	-0.10	-0.01	0.00	0.01	0.03	0.00	-0.03	-0.00	-0.01	-0.05	-0.01	-0.05
$\alpha_2$	-0.10	-0.13	-0.12	-0.27	0.03	0.14	-0.06	-0.15	-0.16	-0.16	-0.66	-0.22	0.15
$\alpha_3$	-0.02	0.02	-0.01	-0.00	0.00	0.00	-0.00	0.00	0.01	0.00	0.04	0.01	0.01
$\alpha_s$	-0.08	-0.25	-0.12	-0.25	0.03	0.16	-0.05	-0.18	-0.18	-0.18	-0.75	-0.23	0.10
TS	15.35 +	36.50 *	na	15.61 +	na	na	19.38 *	12.42	5.17	na	19.96 *	na	na

United States as Foreign Country													
	AU	BE	CA	FR	GE	IT	JA	NE	SP	SD	SZ	TU	UK
E	0.22	0.42	0.22	0.24	0.18	0.25	0.33	0.26	0.33	0.24	0.29	0.21	0.25
TT	31.85	57.96 *	30.65	35.01	21.12	40.01 *	43.80 *	39.04 *	44.90 *	29.77	33.22	31.49	37.52 +
MT	18.23	39.78 *	17.45	19.55	14.39	20.27	28.81 *	22.12	29.17 *	20.00	24.66 +	16.93	20.50
$\beta_1$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\beta_2$	1.25	0.08	-0.07	0.38	-0.42	0.15	0.04	-0.15	0.06	-0.29	-0.41	-0.39	-37.18
$\beta_3$	-1.34	-0.72	-1.08	-0.99	-0.23	-1.68	-0.48	-0.26	-2.00	-1.41	-0.07	3.68	9.42
$\alpha_1$	-0.01	-0.17	-0.11	-0.02	-0.02	-0.12	-0.15	-0.03	0.02	-0.05	0.02	0.01	-0.00
$\alpha_2$	-0.11	-0.05	0.46	-0.54	0.14	0.30	0.22	-0.05	0.44	0.56	0.24	-0.00	0.00
$\alpha_3$	-0.00	-0.07	0.04	0.02	-0.03	0.03	-0.03	-0.04	0.06	0.01	-0.05	-0.00	-0.00
$\alpha_s$	-0.12	-0.15	0.32	-0.57	0.15	0.16	0.09	-0.04	0.41	0.50	0.31	0.01	0.00
TS	na	31.45 *	na	na	na	11.61	23.58 *	12.53	22.34 *	na	18.53 *	na	11.76

Notes to Table:

E = Largest calculated eigenvalue  
 MT = Maximum Eigenvalue Test Statistic, Ho: non-cointegration  
 $\beta_2$  = Second coefficient from the cointegrating vector  
 $\alpha_1$  = Adjustment coeff. for cointegrating vector in the p equation  
 $\alpha_2$  = Adjustment coeff. for cointegrating vector in the e equation  
 $\alpha_3$  = Adjustment coeff. for cointegrating vector in the p\* equation  
 + = Significant at 10 percent level

TT = Trace Test Statistic, Ho: non-cointegration  
 $\beta_1$  = First coefficient from the cointegrating vector  
 $\beta_3$  = Third coefficient from the cointegrating vector  
 $\alpha_s = \alpha_1 + \alpha_2 - \alpha_3$   
 TS = Test statistic for PPP, (test if  $\beta_1 = \beta_2 = -\beta_3$ )  
 \* = Significant at 5 percent level  
 na = not applicable

Cheung and Lai (1993b) examine a similar data set (monthly exchange rates against the U.S. dollar for Canada, France, Germany, Switzerland, and the United Kingdom) and, using asymptotic critical values, find a significant cointegrating vector for all five of the countries.<sup>11</sup> That result is essentially duplicated with our dataset; when the asymptotic critical value of 31.26 for the trace test is used, all of the countries studied by Cheung and Lai (1993b) except Germany have a significant cointegrating vector vis-a-vis the United States. In fact, use of the asymptotic critical values would have led us to conclude that eight of 13 countries have a significant cointegrating vector vis-a-vis Germany and ten of 13 countries vis-a-vis the United States. As noted above, when our Monte Carlo critical values are applied to our quarterly dataset neither Canada, France, Germany, Switzerland nor the United Kingdom have a significant cointegrating vector vis-a-vis the United States at the 5 percent level.

Under proportionality and symmetry, the coefficients of the cointegrating vector  $\beta$  should satisfy  $\beta_1 = \beta_2 = -\beta_3$ , or  $\beta = (1,1,-1)$  when the coefficient on the domestic price has been normalized to one. In contrast to the trace and maximum eigenvalue tests, rejection of the null for the tests of these restrictions is evidence *against* PPP. To gauge the effect of sample size on these tests, another Monte Carlo experiment was conducted, in which the data generating process (DGP) contained a cointegrating vector with coefficients (1,1,-1), see appendix for more details. Table 3 compares the critical values taken from this experiment to the chi-squared (2 degrees of freedom) critical values appropriate for asymptotic results. Use

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<sup>11</sup> However, if one applies the small-sample critical values of Cheung and Lai (1993a) to the results in Cheung and Lai (1993b), then Canada and Germany no longer have a significant cointegrating vector.

of the asymptotic critical values would lead to many more rejections of proportionality and symmetry.

Size	Chi-Square	
	Asymptotic	Monte Carlo
.01	9.21	20.71
.05	5.99	15.61
.10	4.61	13.04
.20	3.22	10.05

For three (Australia, the Netherlands, and Spain) of the six countries that have a significant cointegrating vector with Germany (Table 2, top panel, trace test at 5 percent level) the restrictions imposed by symmetry and proportionality cannot be rejected with a test size of 5 percent ( $TS < 15.61$ ). For two (Italy and the Netherlands) of the five countries that have a significant cointegrating vector with the United States (bottom panel, trace test at 5 percent level) the restrictions imposed by symmetry and proportionality cannot be rejected (at the 5 percent level). At first these findings of proportionality and symmetry seem somewhat at odds with previous work (e.g., Cheung and Lai (1993b) reject proportionality and symmetry in almost every case); however, the difference is explained by the small-sample adjustments. If asymptotic critical values were used ( $TS < 5.99$ ), proportionality and symmetry would be rejected for every country with a significant cointegrating vector (regardless of the foreign country) with the exception of Spain vis-a-vis Germany. Given the small-sample adjustments, the evidence against symmetry and proportionality is not as overwhelming as previously reported.

Altogether then, out of the 26 country pairs examined, we found significant evidence for PPP at the 5 percent level in only five cases (using the trace test). In another six cases we found significant evidence that exchange rates and prices are cointegrated, but the cointegrating vectors were significantly different from those implied by PPP. We are unable to provide an economic theory that would explain the coefficients in these latter cases, especially since there is no discernible pattern to the cointegrating vectors. Use of a 10 percent significance level yields only two additional cointegrating vector (Switzerland vis-a-vis Germany and the United Kingdom vis-a-vis the United States) with only the UK satisfying the restrictions of symmetry and proportionality.

The relatively weak support for PPP in the above results may be due to low power of the various test statistics in discriminating between the null and alternative hypotheses. Previous work (Cheung and Lai (1993b) and Gonzalo (1994)) suggests that the trace and maximum eigenvalue tests have better power than univariate tests of cointegration.<sup>12</sup> In these studies, however, the short-run dynamics and variable interactions are not as rich as those in our data set, and sample sizes are longer. Moreover, there has been no work on the power of the test of the restrictions on the coefficients of the cointegrating vector. Two Monte Carlo

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<sup>12</sup>Kremers, Ericsson, and Dolado (1992) attribute the low power of the univariate ADF tests to inappropriate implicit common factor restrictions.

experiments were used to analyze the power of all three tests, with results presented in table 4 below, see the appendix for details.

Size	Trace Test	Maximum Eigenvalue Test	Restriction on $\beta$
.01	.017	.017	.068
.05	.068	.064	.230
.10	.129	.125	.368
.20	.242	.254	.578

The results for the trace and maximum eigenvalue tests are disheartening. Given our sample size and the complicated dynamics in our three variable system, the power of these two tests is about equal to their size. This means that the probability of rejecting non-cointegration among the three variables when there is no cointegration is roughly equal to the probability of rejecting non-cointegration when the variables are cointegrated. A coin toss will be about as accurate as these tests for deciding whether or not the variables are cointegrated. The power of the test of restrictions on  $\beta$  is better, about 3 to 5 times the size. After accepting the "coin toss" results of either the trace or maximum eigenvalue test, there is more reason to be confident about testing restrictions imposed on  $\beta$ .

In short, the results of the Johansen procedure provide only weak empirical support for PPP. Given the low power of the procedure, this conclusion is not surprising. We are surprised, however, by the six cases (out of 26 studied) in which there was a significant cointegrating vector that also rejected the symmetry and proportionality conditions implied by PPP. This finding suggests that our statistical models may be missing some characteristics of

the true DGP, despite the lack of serial correlation in our residuals. In contrast to previous studies that also use systems methods on post-Bretton Woods datasets, our small sample critical values find fewer significant cointegrating vectors but fewer violations of proportionality and symmetry.

#### IV. Testing for PPP with the Horvath-Watson Procedure

This section uses a procedure proposed by Horvath and Watson (1993) to test for PPP. The Horvath-Watson (HW) procedure makes use of the same VAR as the Johansen procedure, which is repeated here for the reader's convenience

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \dots + \Gamma_{k-1} \Delta X_{t-(k-1)} + \Pi X_{t-k} + \mu + \eta D_t + \varepsilon_t \quad (4)$$

but it imposes the following restriction:

$$\Pi = \alpha \beta' = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} (1 \ 1 \ -1). \quad (5)$$

Horvath and Watson (1993) propose testing for cointegration with a known cointegrating vector--which is given here by the symmetry and proportionality restrictions--by using the Wald statistic for the hypothesis that  $\alpha_1, \alpha_2, \alpha_3$  are all equal to zero.<sup>13</sup> Alternatively, the HW procedure may be expressed using the real exchange rate,  $r = \beta'X = p+e-p^*$ .

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<sup>13</sup>The Wald statistic is computed from the covariance matrix of the coefficient estimates. The test statistics reported here employ a small-sample correction proposed by Whittle (1953) that shrinks the test statistic by the factor  $(T-p/q)/T$ , where T is the number of observations, p is the number of free parameters, and q is the number of equations.

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \dots + \Gamma_{k-1} \Delta X_{t-(k-1)} + \alpha r_{t-k} + \mu + \eta D_t + \varepsilon_t \quad (6)$$

To conduct the HW test, equation (6) is estimated via maximum likelihood with  $k = 4$  (the same lag length that was used in the Johansen procedure) for each pair of countries.<sup>14</sup>

Table 5 presents these results.

Table 5  
Watson Test Results, 1974:Q1 - 1992:Q4

Germany as Foreign Country													
	AU	BE	CA	FR	US	IT	JA	NE	SP	SD	SZ	TU	UK
$\alpha_1$	-0.006	-0.027	-0.008	-0.013	-0.014	-0.049	-0.007	-0.051	-0.030	-0.030	.010	-0.024	-0.034
$\alpha_2$	-0.052	-0.068	-0.099	-0.277	-0.082	.031	-.131	-.126	-.140	-.233	-.149	-.127	-.000
$\alpha_3$	-0.007	.021	-0.010	-0.009	-0.010	-0.001	-0.005	.014	.008	.009	-0.001	.006	-0.003
$\alpha_1 + \alpha_2 - \alpha_3$	-0.051	-.116	-.097	-.281	-.086	-.017	-.133	-.191	-.179	-.273	-.139	-.156	-.031
Watson Statistic (WS)	17.87 *	13.09 +	12.24 +	18.90 *	12.34 +	13.19 +	10.19	23.70 *	17.14 *	11.76 +	9.73	9.43	8.29

United States as Foreign Country													
	AU	BE	CA	FR	GE	IT	JA	NE	SP	SD	SZ	TU	UK
$\alpha_1$	-0.009	-0.001	-0.001	.005	.010	-0.007	-0.001	.002	-0.006	-0.001	.005	-0.008	-0.016
$\alpha_2$	-0.096	-0.071	-0.052	-0.152	-0.082	-.123	-.072	-.099	-.068	-.078	-.115	-.034	-.121
$\alpha_3$	-0.005	.010	.009	.011	.014	.003	.005	.012	.004	.007	.006	.008	-0.001
$\alpha_1 + \alpha_2 - \alpha_3$	-.100	-.083	-.062	-.158	-.086	-.134	-.078	-.109	-.079	-.086	-.116	-.050	-.135
Watson Statistic (WS)	17.30 *	7.61	4.13	11.32 +	12.34 +	7.92	4.68	8.84	6.16	6.04	5.51	5.03	8.00

Notes to table:

\* = significant at 5 percent level.

+ = significant at 10 percent level.

<sup>14</sup>To check the robustness of our results to different lag lengths, we also conducted the HW procedure after a sequential test down of all the individual elements of the  $\Gamma$  matrices from  $k = 13$  to 1--constraining the insignificant parameter estimates to zero--and obtained similar results.



Implementation of the HW test is once again complicated by our small sample size. Table 6 compares the HW statistic asymptotic critical values with our Monte Carlo critical values.<sup>15</sup> Use of the asymptotic critical values would result in more rejections of the hypothesis of non-cointegration than if our Monte Carlo critical values are used.

Size	Watson Test	
Size	Asymptotic	Monte Carlo
.01	15.41	19.24
.05	11.62	13.49
.10	9.72	11.11
.20	na	8.46

With Germany as the foreign country (Table 5, top panel), there is a significant cointegrating vector ( $HW > 13.49$ ) at the 5 percent level for four (Australia, France, Netherlands, and Spain) of the 13 countries. Note that for these same countries we also found one significant cointegrating vector using the Johansen trace test. In addition, nine of the 13 countries are significant at the 10 percent level. For all 13 countries the sum,  $\alpha_1 + \alpha_2 + \alpha_3$ , has the correct (negative) sign indicating short-run adjustment to PPP at rates ranging from 2 percent per quarter for Italy to 28 percent per quarter for France.

With the United States as the foreign country (Table 5, bottom panel) only one of the 13 countries (Australia) has a significant HW statistic at the 5 percent level. (Three of 13 countries are significant at the 10 percent level.) Thus, our results duplicate the finding in the

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<sup>15</sup>The HW asymptotic critical values come from Horvath and Watson (1993), using 1000 observations. The small sample critical values are based on 10,000 Monte Carlo replications of a sample of 76 observations. The DGP is identical to that used to calculate the size of the Johansen tests in the previous section. See the appendix for further details.

existing literature that evidence for PPP is weakest for the United States. However like the German case, the sum  $\alpha_1 + \alpha_2 - \alpha_3$  is always negative and of reasonable magnitude, implying a plausible speed of adjustment to PPP in every case.

In order to assess the power of the HW procedure to detect a stationary real exchange rate in a sample of this size, we ran 10,000 Monte Carlo replications of a sample of 76 observations generated by the average estimated parameters from the top panel of Table 6.<sup>16</sup>

Size	Watson
.01	.057
.05	.235
.10	.392
.20	.629

Our Monte Carlo trials indicate that the power of the HW test is 3 to 6 times the size.

Overall, using the HW test, there is significant evidence for PPP at the 5 percent level in five out of the 26 cases studied. At the 10 percent level, there is significant evidence for PPP in 12 of the 26 cases studied. Thus, the HW procedure provides moderately stronger evidence for PPP than the Johansen tests.

## V. Conclusion

The failure of most empirical studies to find evidence for PPP in the post-Bretton Woods era can be attributed largely to the low power of the tests employed. This paper documents the low power of the Johansen cointegration tests using Monte Carlo experiments based on post-Bretton Woods datasets. This paper also documents the power advantages of a

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<sup>16</sup>This is the same DGP that was used to measure the power of the Johansen trace and maximum eigenvalue tests.

testing procedure based on Horvath and Watson (1993). Using this procedure we find moderate evidence for PPP in post-Bretton Woods data.

A number of studies that use multivariate techniques claim to find significant evidence of cointegration between exchange rates and price levels, but the estimated cointegrating vectors usually reject the restrictions of symmetry and proportionality implied by PPP. This paper shows that these results are partly due to the use of inappropriate critical values in small samples. However, even with appropriate critical values we find several instances of significant cointegrating vectors that reject symmetry and proportionality. Unfortunately, there is no pattern in the coefficients of these cointegrating vectors, frustrating any attempt to provide an economic theory that might account for them.

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## Appendix Monte Carlo Experiments

### I. Trace, Maximum Eigenvalue, and Horvath-Watson Tests

Both the Johansen and Horvath-Watson (HW) procedures maintain the null hypothesis of no cointegration between the variables of interest. Two Monte Carlo experiments were conducted for each procedure. The first experiment determined the critical values for the trace, maximum eigenvalue, and HW test statistics to conduct tests of the desired size. The second experiment then judged the power of the tests to discriminate against a false null given the critical values calculated in the first experiment.

#### A. Size: $\Pr[\text{Reject } H_0 \mid H_0 \text{ is true}]$

There has been little previous work on small sample critical values for the trace, maximum eigenvalue, and HW tests.<sup>17</sup> To calculate small-sample critical values for our dataset, equation (3) from the text was estimated (with Germany as the foreign country using four lags) for each of the 13 countries, imposing the restriction that  $\Pi=0$ . Averaging the

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<sup>17</sup>Cheung and Lai (1993a) use response-surface Monte Carlo techniques to calculate adjustments to the asymptotic critical values of Johansen and Juselius (1990). To avoid undue specificity, their work maintains  $\Delta X_t = \epsilon_t$  as the Data Generating Process (DGP) under the null hypothesis. Given the richer dynamics in our dataset, this null hypothesis DGP did not seem appropriate, as will be shown below.

coefficients across the 13 countries yielded the following data generation process:

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \Gamma_3 \Delta X_{t-3} + \mu + \eta D_t + \varepsilon_t \quad (\text{A7})$$

$$\Gamma_1 = \begin{bmatrix} .286 & -.004 & .201 \\ -.038 & .138 & .724 \\ .025 & .016 & .243 \end{bmatrix} \quad \Gamma_2 = \begin{bmatrix} .199 & -.000 & .165 \\ -.088 & -.067 & -.377 \\ .053 & .008 & .085 \end{bmatrix} \quad \Gamma_3 = \begin{bmatrix} .170 & -.026 & -.144 \\ -.112 & .072 & .282 \\ .021 & .009 & .248 \end{bmatrix}$$

$$\mu = \begin{bmatrix} .012 \\ -.008 \\ .007 \end{bmatrix} \quad \eta = \begin{bmatrix} -.011 & -.008 & -.006 \\ .009 & .003 & -.012 \\ -.003 & -.011 & -.005 \end{bmatrix} \quad \sigma = \begin{bmatrix} .011 \\ .043 \\ .004 \end{bmatrix}$$

where  $\sigma$  is the vector of estimated standard deviations for  $\varepsilon$ . Random samples of 140 observations were then created for  $X_t$ , with  $\varepsilon$  normally distributed. The first 64 observations were used to initialize the process, leaving 76 observations for our analysis (the same number of observations used in estimation). The Monte Carlo experiment consisted of calculating the trace, maximum eigenvalue, and HW tests for 10,000 trials from the process in equation (A1). The test statistics were then ordered to determine the critical values for various size tests. The first six columns of Table A1 compare the trace and maximum eigenvalue small-sample critical values with the asymptotic critical values of Johansen and Juselius (1990) and the adjusted critical values of Cheung and Lai (1993a)<sup>18</sup>, while the last two columns compare the HW small-sample critical values with the asymptotic critical values of Horvath and Watson (1993).

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<sup>18</sup>The columns labelled Cheung/Lai are calculated by substituting our sample size (76) and lag length (4) into the response-surface equations of Cheung and Lai (1993a). These critical values are just slightly less (about 1.5 percent) than those calculated according to the degrees-of-freedom adjustment T/(T-nk) proposed by Reimers (1992).



Size	Trace Test			Maximum Eigenvalue Test			Horvath-Watson	
	Asymptotic	Cheung/Lai	Monte Carlo	Asymptotic	Cheung/Lai	Monte Carlo	Asymptotic	Monte Carlo
.01	37.29	na	44.76	26.15	na	31.69	15.41	19.24
.05	31.26	36.50	38.33	21.28	24.95	26.35	11.62	13.49
.10	28.44	33.18	35.08	18.96	22.18	23.68	9.72	11.11
.20	25.45	na	31.33	16.52	na	20.65	na	8.46

In order to ensure that the small-sample critical values were not sensitive to our averaging procedure, we calculated critical values for those countries that showed the largest disparity in terms of the exchange rate's adjustment to PPP. For the Johansen procedure, critical values were calculated for Japan, Switzerland, and the United Kingdom. The individual-country critical values were on either side of, and were quite close to, those calculated with the averaged coefficients. With a test size of 5 percent, the largest discrepancy between the averaged-coefficient critical values and the individual-country critical values was only 2.5 percent. For the HW test critical values were calculated for France, Italy, and the United Kingdom. This time, the individual-country critical values were slightly larger than the average-coefficient critical values, with the discrepancies at a size of 5 percent ranging from 0.6 to 8.8 percent. However, even the largest 5-percent critical value would not change any of our conclusions about the significance of estimated adjustments to PPP.

**B. Power:  $\Pr[\text{Reject } H_0 \mid H_0 \text{ is false}]$**

To determine the tests' ability to reject a false null, a DGP that implied long-run PPP was constructed. As before, equation (4) in the text was estimated for each of the 13 countries (with Germany as the foreign country), in this instance with the restriction that  $\beta' =$

[1,1,-1]. Averaging the coefficients across the 13 countries yielded the following data generation process:

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \Gamma_3 \Delta X_{t-3} + \Pi X_{t-4} + \mu + \eta D_t + \varepsilon_t \quad (A)$$

$$\Gamma_1 = \begin{bmatrix} .226 & -.017 & .219 \\ -.282 & .056 & .608 \\ .016 & .019 & .213 \end{bmatrix} \quad \Gamma_2 = \begin{bmatrix} .154 & -.012 & .205 \\ -.284 & -.134 & -.354 \\ .046 & .011 & .064 \end{bmatrix} \quad \Gamma_3 = \begin{bmatrix} .128 & -.035 & -.085 \\ -.276 & .011 & .392 \\ .016 & .011 & .234 \end{bmatrix}$$

$$\Pi = \alpha \beta' = \begin{bmatrix} -.022 \\ -.112 \\ .001 \end{bmatrix} [1 \ 1 \ -1] \quad \mu = \begin{bmatrix} .014 \\ .007 \\ .008 \end{bmatrix} \quad \eta = \begin{bmatrix} -.010 & -.008 & -.006 \\ .011 & .005 & -.011 \\ -.003 & -.011 & -.006 \end{bmatrix} \quad \sigma = \begin{bmatrix} .011 \\ .042 \\ .004 \end{bmatrix}$$

Once again, the Monte Carlo experiment involved 10,000 trials from the process in equation (A2). Table A2 shows the percent of the 10,000 trials for which non-cointegration was correctly rejected for the three tests.

Table A2 Small Sample Power Values			
Size	Trace Test	Maximum Eigenvalue Test	Horvath-Watson
.01	.017	.017	.057
.05	.068	.064	.235
.10	.129	.125	.392
.20	.242	.254	.629

For the Johansen tests, these results are much less optimistic than those reported in Cheung and Lai (1993b). However, the power calculations of Cheung and Lai (1993b) are

based on a simpler DGP and longer sample size (200 observations). In our notation, their DGP is given by

$$\Delta X_t = \Pi X_{t-1} + \varepsilon_t \quad (\text{A3})$$

$$\Pi = \alpha \beta' = \frac{(\rho - 1)}{9} \begin{bmatrix} 4 \\ 11 \\ 2 \end{bmatrix} [1 \ 1 \ -1] \quad \sigma = \begin{bmatrix} \frac{2}{3} \\ \frac{20}{9} \\ \frac{5}{9} \end{bmatrix}$$

The parameter  $\rho$  determines the speed of adjustment to deviations from PPP. When  $\rho=1$ , the system is not cointegrated. Cheung and Lai (1993b) consider three values for  $\rho$ : .9, .8, and .7. With  $\rho=.9$  the speeds of adjustment in (A2) and (A3) are quite close. In this instance, Cheung and Lai (1993b) with a test size of 5 percent find reasonable power (.239) for the trace test. This more optimistic finding is the result of a larger sample size. We recalculated critical values based on (A1) and conducted a power exercise with (A2) using a sample size of 200 in both calculations. The power of the trace test jumped from .068 to .421. The dramatic increase in power illustrates the advantage of datasets that span fifty instead of twenty years.

The small-sample power results are more encouraging for the HW test. Power is roughly four times size, quite an improvement over the Johansen tests where power was equal to size. Clearly, the imposition of the cointegrating vector, if it is known, increases the odds of rejecting a false null.

## II. Johansen Test of Restrictions on $\beta$

The test of the restrictions on  $\beta$ , for our application, maintains a composite null hypothesis of 1) a single cointegrating vector and 2) that the cointegrating vector is given by  $\beta=[1,1,-1]$ . Two Monte Carlo experiments were conducted. The first determined the appropriate critical values for our sample size, while the second judged the ability of the test to reject a false null.

### A. Size: $\Pr[\text{Reject } H_0 \mid H_0 \text{ is true}]$

To calculate small sample critical values, a DGP with  $\beta=[1,1,-1]$  was constructed. The potential conditionality of the test of the restrictions on  $\beta$  complicates the Monte Carlo analysis (i.e. the researcher may choose not to calculate the test unless the trace or maximum eigenvalue test is significant). Critical values for the test of restrictions could be calculated with the 10,000 trials from process (A2) used in the power calculations for the trace and maximum eigenvalue tests, since the DGP is formulated to satisfy the  $\{1,1,-1\}$  restriction. This approach would ignore the conditionality; the test of restrictions would be calculated regardless of the results of the trace or maximum eigenvalue test. Alternatively, sufficient trials on process (A2) could be conducted to generate 10,000 cases with a significant trace or maximum eigenvalue test, with the test on  $\beta$  conducted on these 10,000 trials. Given the low power of the trace and maximum eigenvalue tests, this second approach is computationally burdensome. For example, to obtain 10,000 trials with a significant trace statistic at the 5 percent level required 148,000 trials from process (A2). Unfortunately, critical values generated from the two methods are quite different, they are shown in columns 2 and 3 of Table A3. Those from the second method are well above those from the first, with both sets

much higher than the asymptotic critical values. To remain conservative with respect to the PPP hypothesis, which would be more readily accepted with the higher critical values, we used the critical values from the first method to make inferences.

Size	Asymptotic	Monte Carlo, small-sample			
	Chi-Square	Conditional	Unconditional		
		T=76	T=76	T=200	T=76, rapid
.01	9.21	30.04	20.71	14.45	12.10
.05	5.99	23.90	15.61	9.98	8.11
.10	4.61	21.12	13.04	7.81	6.21
.20	3.22	17.49	10.05	5.52	4.36

The difference between the asymptotic and the unconditional small-sample critical values is quite large. To investigate the discrepancy, two alternative Monte Carlos were conducted. The first experiment, column 5, increased the sample size to 200. With the larger sample size, the critical values move much closer to the asymptotic critical values. The second experiment (column 6) held the sample fixed at 76, but changed the vector  $\alpha$  in (A2) from [-.022,-.112,.001] to [-.1,-.1,.1], allowing for much more rapid adjustment to PPP. In this instance also, the small-sample critical values come much closer to the asymptotic critical values. In short samples with slow adjustment, there will be a substantial difference between asymptotic and small-sample critical values.

**B. Power: Pr[Reject  $H_0$  |  $H_0$  is false]**

To assess the power of the test of the restrictions on  $\beta$ , we considered a DGP with a cointegrating vector given by  $\beta=[1,2,-1]$ . Estimating the system with this cointegrating vector imposed yielded;

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \Gamma_3 \Delta X_{t-3} + \Pi X_{t-4} + \mu + \eta D_t + \varepsilon_t \quad (\text{A4})$$

$$\Gamma_1 = \begin{bmatrix} .269 & -.001 & .180 \\ -.082 & .064 & .608 \\ .016 & .017 & .207 \end{bmatrix} \quad \Gamma_2 = \begin{bmatrix} .184 & .004 & .144 \\ -.103 & -.139 & -.348 \\ .046 & .010 & .059 \end{bmatrix} \quad \Gamma_3 = \begin{bmatrix} .155 & -.021 & -.157 \\ -.111 & -.004 & .370 \\ .015 & .011 & .231 \end{bmatrix}$$

$$\Pi = \alpha \beta' = \begin{bmatrix} .003 \\ -.047 \\ .000 \end{bmatrix} [1 \ 2 \ -1] \quad \mu = \begin{bmatrix} .015 \\ -.020 \\ .008 \end{bmatrix} \quad \eta = \begin{bmatrix} -.010 & -.008 & -.006 \\ .010 & .003 & -.013 \\ -.003 & -.011 & -.005 \end{bmatrix} \quad \sigma = \begin{bmatrix} .011 \\ .042 \\ .004 \end{bmatrix}$$

Power calculations are shown in the table below, using the unconditional method discussed above. These results are encouraging and are similar to those for the HW test.

Table A4 Jonansen Small Sample Power Values	
Size	Restriction on $\beta$
.01	.068
.05	.230
.10	.368
.20	.578

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