REAL EXCHANGE RATE TARGETING AND MACROECONOMIC INSTABILITY

Martín Uribe

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ABSTRACT

This paper introduces a real exchange rate rule of the type analyzed by Dornbusch (1982) in an optimizing, two-sector, monetary model of a small open economy. By this rule the government increases the devaluation rate when the real exchange rate is below its long-run level and reduces it when the real exchange rate is above its long-run level. I show that the mere existence of such a rule can give room for extrinsic uncertainty to have real effects, that is, it can generate economic fluctuations due to self-fulfilling expectations. I also analyze the stabilizing role of these PPP rules when fluctuations are driven by shocks to fundamentals. I show that the volatility of real variables decreases with tighter rules when shocks to the supply of home goods or to the real rate of return are the main source of uncertainty, and increases when fluctuations are mainly due to shocks to the supply of traded goods. In all cases, PPP rules increase the volatility of nominal variables. Finally, PPP rules help stabilize both real and nominal variables when fluctuations originate from random but persistent deviations from the PPP rule itself.
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1 Introduction

It appears that policymakers frequently link the rate of devaluation to the level of the real exchange rate. Big devaluations usually take place when the relative price of traded goods in terms of home goods is unusually low. In a recent empirical study, Klein and Marion (1994) use a data set of sixty one spells of exchange rate pegs drawn from sixteen Latin American countries and Jamaica from the late 1950s to the early 1990s, to analyze the determinants of the duration of exchange rate pegs. They find strong evidence that a more appreciated real exchange rate is associated with a higher likelihood of a devaluation.

In this paper I analyze the macroeconomic effects of policy rules by which the government increases the devaluation rate when the real exchange rate is below its long-run level, and decreases it when the real exchange rate is above its long-run level. In particular, the attention will be focused on two questions. The first is whether the mere existence of such policies can give room for extrinsic uncertainty to affect real variables. The second is concerned with the stabilizing properties of this type of PPP rule when the economy is subject to shocks to fundamentals.

The last question is essentially the same that motivated Dornbusch’s (1982) paper, where he develops a reduced form, small open economy model with sticky wages due to overlapping contracts,

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in which aggregate demand is assumed to be an increasing function of the real exchange rate and aggregate supply a decreasing function of this relative price. In his framework, PPP rules may increase or decrease the volatility of output in response to supply shocks. In particular, if the supply channel dominates, a tighter PPP rule increases output volatility. Price volatility, on the other hand, always increases with tighter PPP rules.

Calvo, Reinhart and Végh (1994) study a related issue. They analyze, within an optimizing, flexible-price, cash-in-advance framework, the ability of governments to achieve a real exchange rate target. They show that the government can achieve this goal only temporarily and by generating an also temporary increase in the inflation rate. The reason is that in their model inflation enters as a tax on consumption, so the temporarily higher level of inflation induces agents to substitute future for current consumption. Given the real exchange rate, current demand for both, traded and non-traded goods goes down, and since the supply of nontraded goods is perfectly inelastic, equilibrium in the home good market requires that the relative price of the home good falls, that is, that the real exchange rate increases.

In this paper the analysis is carried out within a standard monetary model of a two-sector, small, open, endowment economy, in which the use of money is motivated through a transactions cost technology as in Kimbrough (1986). In this model, situations in which the public expect next period’s inflation to be above its long-run level, are associated with low current real exchange rates through basically the same channels as those in the Calvo et. al. model: the decrease in the demand for money induced by higher expected inflation, causes the marginal transactions cost to go up and thus a reduction in aggregate demand. Given the supply of home goods, its relative price has to go down in order for this market to clear, i.e., the real exchange rate has to increase.

In the presence of a sensitive enough exchange rate rule, the model described above can display endogenous fluctuations. To see why this might be the case, consider a negatively serially correlated sunspot variable and assume that economic agents associate high values of this variable with high levels of the current devaluation rate and low values with low current devaluations. Then a high value of the sunspot variable today, will induce people to believe that next period’s devaluation rate will be small, and so current real balances and aggregate demand will increase, and the real exchange rate will appreciate (i.e., it will go down). Given the exchange rate rule, this induces the government to devalue the domestic currency. If the PPP rule is sensitive enough, the current
deviation of the devaluation rate from its steady state will be larger than the absolute value of the one expected for next period, making the expectations of future devaluations self-fulfilling.

I also analyze the case of PPP rules that are not so sensitive as to generate indeterminacy of the rational expectations equilibrium. The sources of fluctuations I consider in this case, are exogenous shocks to the endowment of tradables and home goods and to the real rate of return on foreign assets, which can be regarded as supply shocks, and random deviations from the PPP rule, which can be interpreted as demand shocks. As in Dornbusch (1982), the volatility of nominal variables increases with the sensitivity of the PPP rule when the economy is hit by supply shocks. The use of PPP rules reduces the volatility of real variables when supply shocks affect mainly the real rate of return or the supply of home goods, and increases the variability of real variables when they affect mainly the supply of traded goods. When the economy is subject to persistent demand shocks of the type described above, tighter PPP rules help stabilize both real and nominal variables.

The welfare effects of PPP rules are mixed. In the case in which the rational expectations equilibrium is indeterminate due to a highly sensitive exchange rate rule, and in the absence of shocks to fundamentals, the endogenous fluctuations in consumption that result, are welfare reducing. PPP rules are also welfare reducing in the case in which the main source of fluctuations are (fundamental) shocks to the endowments of traded or home goods. If, on the other hand, aggregate fluctuations are mainly due to shocks to the real interest rate on foreign assets or to random deviations from the PPP rule itself, exchange rate rules are welfare increasing.

The rest of the paper is organized as follows. Section 2 presents the model and introduces the PPP rule. In the same section, the parametric restrictions imposed by the presence of exogenous long-run growth are described and the model is calibrated. Section 3 shows that when the PPP rule is sensitive enough, the model can display sunspot fluctuations, and proves that in the absence of such policies they can be ruled out. Section 4 analyzes the effects of PPP rules on the volatility and predictability of nominal and real variables and on the level of welfare of the representative agent, in the presence of shocks to fundamentals. Section 5 concludes.
2 The Model

In this section I embed a simple real exchange rate rule of the type analyzed in Dornbusch (1982) in a standard optimizing monetary model of a small open economy. By this rule, the government sets the devaluation rate above or below its average level depending on whether the level of the real exchange rate is below or above average. Specifically, let \( e_t \) denote the real exchange rate, i.e., the relative price of traded goods in terms of home goods, and \( \epsilon_t \) the devaluation rate in \( t \) (i.e., the growth rate of the nominal exchange rate between \( t - 1 \) and \( t \), where the nominal exchange rate is defined as the price of one unit of the foreign currency in terms of the domestic currency). Then the rule is assumed to be given by the following log-linear function

\[
\log \left( \frac{1 + \epsilon_t}{1 + \epsilon} \right) = -\alpha \log(e_t/e) + \hat{\mu}_t
\]

(1)

where \( \alpha > 0 \) and \( \epsilon \) and \( e \) denote the long-run values of the devaluation rate and of the real exchange rate, respectively. I allow for random deviations from the PPP rule by adding the term \( \hat{\mu}_t \) which is assumed to follow a stationary process. In each period \( t \geq 0 \) the government assures free convertibility of the domestic currency at the nominal exchange rate given by the nominal exchange rate in \( t - 1 \) times \( 1 + \epsilon_t \).

The economy is assumed to be populated by a large number of identical, infinitely-lived consumers with preferences defined over sequences of consumption, \( C_t \), and described by the utility function

\[
E_0 \sum_{t=0}^{\infty} \beta^t U(C_t)
\]

(2)

where \( \beta \in (0,1) \) denotes the subjective discount factor, \( U(\cdot) \) denotes the period utility function, assumed to be strictly increasing, strictly concave and twice continuously differentiable, and \( E_0 \) denotes the conditional expectations operator given information available in period 0. Consumption is a assumed to be a composite of traded and non-traded goods, \( C_t^T \) and \( C_t^N \); that is,

\[
C_t = A(C_t^T, C_t^N)
\]

(3)

where \( A(\cdot, \cdot) \) is an aggregator function assumed to be strictly increasing in both arguments, homo-
geneous of degree one, strictly quasi concave, and twice differentiable.¹

There are only two assets in non-zero aggregate net supply in this economy. An internationally traded bond denominated in foreign currency that pays the interest rate \( r_t \) in terms of traded goods between periods \( t \) and \( t+1 \), and money. As in Kimbrough (1986), money is assumed to reduce the transactions costs incurred while shopping for goods.² Let \( X_t \) denote the consumer's expenditure in \( t \) measured in terms of tradables, and \( M^d_t \) the demand for money in \( t \), also measured in terms of traded goods. Then the transactions cost, \( S_t \), measured in terms of the traded good, is given by

\[
S_t = v(X_t, M^d_t)
\]

(4)

where \( v(\cdot, \cdot) \) is assumed to be strictly increasing in its first argument, strictly decreasing in its second argument and strictly convex, with \( v_{xm} < 0 \) and \( \lim_{m \to -0} v_m(x, m) > 1 \) for any \( x > 0 \). Total expenditure, in turn, is given by

\[
X_t = C_t^T + \frac{C^N_t}{\epsilon_t}
\]

(5)

I will assume that the price of one unit of the traded good in terms of the foreign currency is always equal to one. This implies that the price of one unit of the traded good in terms of domestic currency is always equal to the nominal exchange rate.

Each period, the consumer starts with some financial assets carried over from the previous period and is endowed with exogenously given amounts of traded and home goods, \( Y_t^T \) and \( Y_t^N \). His budget constraint is then given by,

\[
D_t^c = (1 + r_{t-1})D_{t-1}^c - \frac{M^d_{t-1}}{(1 + \epsilon_t)} - Y_t^T - \frac{Y_t^N}{\epsilon_t} + M^d_t + X_t + S_t - T_t
\]

(6)

where \( D_t^c \) denotes the amount borrowed in period \( t \), and \( T_t \) is a lump-sum transfer received from the government. Both are measured in terms of the traded good.

The consumer's problem consists in choosing contingent plans for consumption and asset holdings so as to maximize his expected utility subject to (3)-(6) and to some borrowing constraint that

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¹These assumptions imply that \( A_{11} < 0, A_{22} < 0 \) and \( A_{12} > 0 \).

²Reinhart and Végh (1994a,b) also use this motivation to introduce money in a one-sector model of a small open economy.
prevents him from engaging in Ponzi-type games. The first order conditions of this problem are

\[ U'(C_t) = \beta(1 + r_t)E_t \left\{ U'(C_{t+1}) \frac{p_t}{p_{t+1}} \right\} \]  

(7)

\[ \epsilon_t = \frac{A_1(C_t^T, C_t^N)}{A_2(C_t^T, C_t^N)} \]  

(8)

\[ -v_m(X_t, M_t^d) = \frac{i_t}{1 + i_t} \]  

(9)

where \( p_t \) denotes the effective price of the consumption good in terms of the traded good, and is given by\(^3\)

\[ p_t = \frac{1 + v_r(X_t, M_t^d)}{A_1(C_t^T, C_t^N)} \]  

(10)

and \( i_t \) is the nominal interest rate (known in \( t \) and paid in \( t + 1 \)) on an asset denominated in domestic currency, and satisfies,

\[ U'(C_t) = \beta(1 + i_t)E_t \left\{ U'(C_{t+1}) \frac{p_t}{p_{t+1}} \frac{1}{1 + \epsilon_{t+1}} \right\} \]  

(11)

Note that given the real exchange rate, equations (9), (10) and (8) imply that the effective price of consumption is increasing in the nominal interest rate.

The government, on the other hand, is assumed to be able to hold debt in the form of the internationally traded bond, to issue money and to give lump-sum transfers to the public. Its budget constraint, expressed in terms of tradables per capita is given by,

\[ D_t^g = (1 + r_{t-1})D_{t-1}^g - M_t^s + \frac{M_{t-1}^s}{1 + \epsilon_t} + T_t \]  

(12)

where \( D_t^g \) denotes the stock of public debt and \( M_t^s \) the money supply in \( t \).

In equilibrium, both the money market and the home-good market have to clear,

\[ M_t^d = M_t^s \]

\(^3\)This notation was taken from Reinhart and Végh (1994a,b). In their model only traded goods are consumed, so the denominator in the right hand side of (10) is just one.
\[ C_t^N = Y_t^N \] (13)

I will ignore the wealth effects associated with inflation, by assuming that the transaction cost, \( S_t \), is returned to the household in a lump-sum fashion.\(^4\) This assumption, together with the two market clearing conditions and the budget constraints of the government and of the household (equations (12) and (6)), imply that the net foreign-asset holdings of the economy, \( D_t \equiv D_t^f + D_t^g \), evolve according to the following expression

\[ D_t = (1 + r_t-l)D_{t-1} - Y_t^T + C_t^T \] (14)

Following Senhadji (1994), I will assume that in equilibrium the external debt of this economy is positive, and that the real interest rate faced by the country in period \( t \), is an increasing function of the ratio of the stock of debt over the trend of income, that is

\[ 1 + r_t = \left[ 1 + r(D_t/(1 + \gamma)^t) \right] \kappa_t \] (15)

where \( \gamma \) denotes the long-run growth rate of output, \( \kappa_t \) is an exogenous and stationary random shock with mean equal to one, and \( r(\cdot) \) satisfies \( r'(\cdot) > 0, \ r(0) < \beta^{-1} - 1 \) and \( \lim_{d \to 0} r(d) = \infty \).

These assumptions assure the existence of a positive steady state level of debt. They also imply that this level is independent of the initial stock of debt.\(^5\)

**Dynamics around a balanced growth path**

I will assume that the endowments of traded and home goods are both trend-stationary variables. Specifically, their laws of motion are assumed to be given by

\[ Y_t^T = y_t^T (1 + \gamma)^t \] (16)

\(^4\)One way of rationalizing this is to think that \( S_t \) represents pure profits of financial institutions, which belong to the households.

\(^5\)It has become a standard practice in modeling small open economies, to impose some condition in order to induce independence of the steady state from initial conditions. The most frequently used are to assume that the subjective discount factor is decreasing in consumption, as proposed by Koopmans (1960), and to introduce finite lives, as in Blanchard (1985).
\[ Y_t^N = y_t^N (1 + \gamma)^t \] (17)

where \( y_t^T \) and \( y_t^N \) are stationary random variables denoting detrended income. As in King, Plosser and Rebelo (1988), I will analyze the dynamics of the model around a non-stochastic balanced growth path. I will require that along this path, the real exchange rate, the effective relative price of consumption and money velocity are constant. From (8) and given that the aggregator function is homogeneous of degree one, it follows that the real exchange rate will be constant in the steady state, only if \( C_t^T \) grows at the rate \( \gamma \). From the economy-wide budget constraint (14) and equation (15), it follows that the stock of debt also has to grow at the rate \( \gamma \). Then (15) itself implies that the real interest rate is constant in the steady state. Since both \( C_t^T \) and \( C_t^N \) grow at the rate \( \gamma \) in the steady state, and given that the aggregator function is linearly homogeneous, the composite good, \( C \), also grows at the rate \( \gamma \). The Euler equation (7) then implies that the marginal utility of consumption has to be homogeneous, i.e., that the period utility function, \( U(\cdot) \), has to be of the CRRA form,

\[
U(C) = \begin{cases} 
(C^{1-\sigma} - 1)/(1 - \sigma) & \text{for } \sigma \geq 0, \sigma \neq 1 \\
\log(C) & \text{for } \sigma = 1
\end{cases}
\]

From the fisherian equation, (11), it follows that the nominal interest rate is constant in the steady state. Finally, from the demand for money (9), and the expression for the effective price of consumption, (10), it follows that both \( v_x(\cdot, \cdot) \) and \( v_m(\cdot, \cdot) \) have to be homogeneous of degree zero functions. This implies a unit income elasticity of money demand.

The equilibrium conditions in terms of stationary variables, can be obtained by removing the log-linear trend from them (i.e., by dividing by \( (1 + \gamma)^t \)). The resulting set of equations is the following (detrended variables are in lower case letters)

\[
e_t = \frac{A_1(c_t^T, y_t^N)}{A_2(c_t^T, y_t^N)}
\] (18)

\[
1 = \tilde{\beta}(1 + r(d_t))\kappa_t E_t \left\{ \left( \frac{c_t}{c_{t+1}} \right)^\sigma \frac{p_t}{p_{t+1}} \right\}
\] (19)

\[
c_t = A(c_t^T, y_t^N)
\] (20)

\[
p_t = \frac{1 + v_x(x_t, m_t)}{A_1(c_t^T, y_t^N)}
\] (21)
\[-v_m(x_t, m_t) = \frac{i_t}{1 + i_t} \quad (22)\]

\[x_t = c_t^T + \frac{y^N_t}{e_t} \quad (23)\]

\[1 = \tilde{\beta} \frac{1 + i_t}{1 + \epsilon} E_t \left\{ \left( \frac{c_t}{c_{t+1}} \right)^{\alpha} \frac{p_t}{p_{t+1}} \left( \frac{e_{t+1}}{e_t} \right) \exp(-\tilde{\mu}_{t+1}) \right\} \quad (24)\]

\[d_t = \frac{1 + r(d_{t-1})}{1 + \gamma} \kappa_{t-1} d_{t-1} + c_t^T - y_t^T \quad (25)\]

where $\tilde{\beta} \equiv \beta(1 + \gamma)^{-\sigma}$.

Consider now the existence of the steady state. The assumptions made about the function $r(\cdot)$, guarantee the existence of a positive steady state level of debt, which can be obtained from the euler equation (19) as the solution to

\[\tilde{\beta}(1 + r(d)) = 1\]

Since in the steady state consumption grows at the rate $\gamma$, the level of utility given by (2) will be finite only if $(1 + \gamma)\tilde{\beta}$ is less than one, which together with the expression above implies that in the steady state

\[1 + r(d) > 1 + \gamma\]

A positive long-run value for the consumption of tradables will exist only if the following restriction, coming from the resource constraint (25) and involving the endowment of tradables, the subjective discount factor, the long-run growth rate and the function $r(\cdot)$, is imposed,

\[r^{-1}(\tilde{\beta}^{-1} - 1) \frac{1 + \gamma - \tilde{\beta}^{-1}}{1 + \gamma} + y^T > 0\]

The marginal condition (18) then gives the unique steady-state value of the real exchange rate, and (23) the steady level of domestic absorption. The Fisher equation (24) gives the steady state nominal interest rate $\beta^{-1}(1 + \epsilon) - 1$ and the money demand function (22) gives the unique level of real balances. Note that in this model neither the steady-state level of the real exchange rate nor the steady-state level of consumption of tradables depend on the long-run devaluation rate.

I will restrict my attention to the dynamics of the model around the deterministic growth path.
In order to do this, I will analyze a log-linear version of conditions (18) to (25). In the equations below, a hat on a variable denotes log-deviation from steady-state, except in two cases, \( \hat{\epsilon}_t \) and \( \hat{i}_t \) denote, the log deviations of \((1 + \epsilon_t)\) and \((1 + i_t)\) from their steady state, respectively,

\[
\hat{\epsilon}_t = \frac{\hat{c}_t^T - \hat{y}_t^T}{\rho_{TN}}
\]

(26)

\[
0 = \bar{\kappa}_t + \left( \frac{r}{1 + r} \right) \rho_d \hat{d}_t + \sigma(\hat{\epsilon}_t - E_t \hat{\epsilon}_{t+1}) + (\hat{p}_t - E_t \hat{p}_{t+1})
\]

(27)

\[
\hat{\epsilon}_t = \theta^T \hat{c}_t^T + (1 - \theta^T) \hat{y}_t^N
\]

(28)

\[
\hat{p}_t = \left( \frac{v_x}{1 + v_x} \right) \rho_{xz} (\hat{z}_t - \hat{m}_t) + (1 - \theta^T) \rho_{TN}^{-1} (\hat{c}_t^T - \hat{y}_t^N)
\]

(29)

\[
\hat{m}_t - \hat{z}_t = (i \rho_{mm})^{-1} \hat{i}_t
\]

(30)

\[
\hat{\epsilon}_t = \theta^T \hat{c}_t^T + (1 - \theta^T)(\hat{y}_t^N - \hat{\epsilon}_t)
\]

(31)

\[
0 = \dot{i}_t + \sigma(\hat{\epsilon}_t - E_t \hat{\epsilon}_{t+1}) + (\hat{p}_t - E_t \hat{p}_{t+1}) + \alpha E_t \hat{\epsilon}_{t+1} - E_t \bar{\mu}_{t+1}
\]

(32)

\[
\hat{d}_t = \left( \frac{1 + r}{1 + \gamma} \right) \left[ \left( 1 + \frac{r \rho_d}{1 + r} \right) \hat{d}_{t-1} + \bar{\kappa}_{t-1} \right] + \left( 1 + \frac{r}{1 + \gamma} - 1 \right) \left[ \frac{\theta^T}{\theta_{TB}} \hat{c}_t^T - \frac{\theta^T + \theta_{TB}}{\theta_{TB}} \hat{y}_t^T \right]
\]

(33)

where \( \rho_{TN} \) denotes the elasticity of substitution between traded and non-traded goods, \( \theta_T \) the share of consumption of tradables in gross national output, \( \theta_{TB} \) the share of the trade balance in gross national output, \( r \) the real interest rate, \( i \) the nominal interest rate, and \( \rho_{xz} \) and \( \rho_{mm} \) the elasticities of \( v_x \) with respect to \( x \) and of \( -v_m \) with respect to \( m \), respectively.

This linearized system can be reduced to a smaller one involving only three linear difference equations in the non-predetermined endogenous variables \( \hat{m}_t \) and \( \hat{x}_t \) and the pre-determined endogenous variable \( \hat{d}_{t-1} \).

\[
\begin{bmatrix}
E_t \hat{m}_{t+1} \\
E_t \hat{x}_{t+1} \\
\hat{d}_t
\end{bmatrix} = M 
\begin{bmatrix}
\hat{m}_t \\
\hat{x}_t \\
\hat{d}_{t-1}
\end{bmatrix} + R 
\begin{bmatrix}
E_t \hat{y}_t^T \\
E_t \hat{y}_t^N \\
E_t \bar{\mu}_{t+1}
\end{bmatrix} + Q 
\begin{bmatrix}
\hat{y}_t^T \\
\hat{y}_t^N \\
\bar{\mu}_t \\
\bar{\kappa}_{t-1}
\end{bmatrix}
\]

(34)

where the elements of the matrices \( M, R \) and \( Q \) are functions of the parameters of the linearized
system. The main concern of this paper is to investigate the type of dynamics that this model can generate as a function of the PPP rule’s parameter $\alpha$. In order to do this, I am first going to assign plausible values to all other parameters of the model.

**Calibration**

In order to be able to get numerical values for the matrices $M$, $R$ and $Q$, one needs to assign numerical values to the twelve parameters of the linearized model. I will base the calibration on data from Argentina for the period 1970-1990. The precise sources are described in Uribe (1994). The time unit is a month. The share of consumption of tradables in GNP, $\theta^T$, was set at 40%. and the share of the trade balance in GNP, $\theta^{TB}$, at 2%. The elasticity of substitution between traded and non-traded goods in producing the composite good, $\rho_{TN}$, was assumed to be one. The long-run devaluation (and inflation) rate, $\epsilon$, was set at 10% per month and the world real interest rate, $r$, at 6.5% per year. The growth rate of per capita output was set at 0.5% percent per year. Using moment conditions derived from a one sector version of the model described above, Reinhart and Végh (1994b), obtained GMM estimates of the intertemporal elasticity of substitution of around .2 for Argentina, that is, a value of $\sigma$ of around 5. This is the value used in this calibration exercise, which might seem a little high, but estimates using data from developing countries are in general lower than those obtained using data from developed countries. In the same paper, Reinhart and Végh find similar and even higher values of $\sigma$ for Brazil, Israel, Mexico and Uruguay. Giovannini (1985) also reports low values of the intertemporal elasticity of substitution using data from developing countries.

Consider now the marginal transactions cost, $v_x$, and the elasticities $\rho_{mm}$ and $\rho_{xx}$. From (30), it follows that $\rho_{mm}$ is the money demand elasticity with respect to its opportunity cost, $i_t/(1 + i_t)$. Reinhart and Végh (1994a) estimate this elasticity for Argentina and get a value of -0.10. Arrau et. al. (1990) estimate, for the same country, a semi-elasticity of -.44, which implies, at a monthly nominal interest rate of around 10%, an elasticity of -.12, which is comparable to the one obtained by Reinhart and Végh. In their paper, Reinhart and Végh (1994a) assume the following form for the transactions cost,

$$v(x, m) = Ax \left( \frac{x}{m} \right)^\eta$$
In this case the money demand elasticity is \( 1/(1 + \eta) \), that is \( \eta = 9 \). The elasticity of \( v_x \) with respect to \( x \), \( \rho_{xx} \), is then given by \( \eta = 9 \). Finally the marginal transactions cost, \( v_x \), is given by,

\[
v_x(x, m) = (1 + \eta) A \left( \frac{x}{m} \right)^\eta
= \frac{1 + \eta}{\eta} \frac{m}{x} (-v_m)
= \frac{1 + \eta}{\eta} \frac{m}{x} \frac{i_i}{1 + i_t}
\]

the last equality follows from the money demand equation (22). The ratio \( m/x \) is the inverse of the velocity of circulation of money, which in Argentina averaged 1.25 a month during the period 1970-1990. The implied value of \( v_x \) is then around .09.

There is not much evidence on the debt elasticity of the interest rate. My baseline parameterization will follow the assumptions made by Senhadji (1994) \( \rho_d \), which imply a value of 0.3 for \( \rho_d \).

\[
\rho_d
\]

Table 1 summarizes the calibration exercise.

3 Endogenous fluctuations

In this section I will analyze the possibility that real exchange rate rules of the type introduced above can give room for extrinsic uncertainty to affect real variables. In order to focus the attention on this issue, I will assume, only for this section, that the economy is not hit by shocks to fundamentals, that is

\[
\hat{y}^T_t = 0
\]
\[
\hat{y}^N_t = 0
\]
\[
\hat{\mu}_t = 0
\]
\[
\hat{\kappa}_{t-1} = 0
\]

\[\text{In his paper Senhadji (1994) cites evidence suggesting that developing countries pay an average interest premium of 1%, and assumes that the debt elasticity of the premium is 2. The implied elasticity of the interest rate, given the value of 6.5% assumed for the real interest rate, is } \rho_d = (.01/.065) \cdot 2 \approx 3.\]
for all $t \geq 0$. In this case the linear system (34) becomes

$$
\begin{bmatrix}
E_t \hat{m}_{t+1} \\
E_t \hat{x}_{t+1} \\
\hat{d}_t
\end{bmatrix}
= M
\begin{bmatrix}
\hat{m}_t \\
\hat{x}_t \\
\hat{d}_{t-1}
\end{bmatrix}
$$

(35)

This system will have a solution converging to its steady state, if and only if at least one of the three eigenvalues of $M$ lies inside the unit circle. The equilibrium will be unique if exactly one of the eigenvalues is inside the unit circle. Figure 1 shows the three eigenvalues of $M$ as functions of the sensitivity of the PPP rule, $\alpha$. For values of $\alpha$ below 3.9, two of the eigenvalues lie outside the unit circle and the other one inside, so the rational expectations equilibrium is determinate. For values of $\alpha$ bigger than that value, only one of the eigenvalues of $M$ is outside the unit circle, so the equilibrium becomes indeterminate. In this case, endogenous fluctuations due to self-fulfilling expectations can arise. I show in the appendix that the existence of a PPP rule is a necessary condition for the indeterminacy to occur. That is, I show that when $\alpha = 0$, the matrix $M$ has at most one eigenvalue inside the unit circle, independently of the particular values assumed for the other parameters of the model.

For self-fulfilling expectations to occur, agents have to base their expectations about future rates of devaluation on a negatively serially correlated sunspot variable. In this case, and assuming that, say, high levels of the sunspot variable are associated with high levels of inflation, a high level of the sunspot variable in period $t$, generates the expectation that in period $t + 1$ the devaluation rate will be below average, inducing people to increase their demand for money in $t$. This, in turn, decreases the marginal transactions cost and induces an increase in aggregate demand in $t$. The fixed supply of home goods implies that equilibrium in this market will occur only if the relative price of the traded good in terms of the home good (the real exchange rate), decreases. Given the PPP rule, this decrease in the real exchange rate induces the monetary authority to devalue the domestic currency. For sensitive enough PPP rules, the departure of the devaluation rate from its steady state level in $t$ can be bigger than the absolute value of the one expected to occur in $t + 1$, making the expectations of low devaluations self-fulfilling. In this case, solutions to (35) take the
form
\[
\begin{bmatrix}
\hat{x}_{t+1} \\
\hat{d}_{t+1}
\end{bmatrix} = \Pi \begin{bmatrix}
\hat{x}_t \\
\hat{d}_{t-1}
\end{bmatrix} + \begin{bmatrix}
\xi_{t+1} \\
0
\end{bmatrix}
\] (36)

where \( \xi_t \) is an i.i.d. innovation in domestic absorption, and \( \Pi \) is a 2x2 matrix with both eigenvalues inside the unit circle. The other variables of interest, such as consumption of tradables, the real exchange rate, real balances, and the devaluation rate, are functions of \( \hat{x}_t \) and \( \hat{d}_{t-1} \) and can be expressed in the following way,
\[
\begin{bmatrix}
m_t \\
\epsilon_t \\
\tilde{c}_t^T \\
\tilde{\epsilon}_t
\end{bmatrix} = \Gamma \begin{bmatrix}
\hat{x}_t \\
\hat{d}_{t-1}
\end{bmatrix}
\] (37)

Typically, when policymakers and economists make the case for the need of a "corrective" devaluation, they justify this necessity not only by observing that the real exchange rate is overvalued but also by pointing at the magnitude of the trade balance deficit or at the level of aggregate domestic absorption. In the absence of intrinsic uncertainty, the model presented above implies that targeting the real exchange rate is equivalent to targeting any of those two other variables. Specifically, it follows from the linearized equilibrium conditions (26) and (31) that the policy rule \( \hat{\xi}_t = -\alpha \hat{\epsilon}_t \), is equivalent to the rules \( \hat{\epsilon}_t = -\alpha_1 \hat{x}_t \) with \( \alpha_1 \equiv \alpha_1^l[(1 + \rho_{TN})\theta^T - 1] \) and \( \hat{\epsilon}_t = \alpha_2 \tilde{c}_t^T \) with \( \alpha_2 \equiv \alpha_2^B/(\rho_{TN}\theta^T) \), in the sense that they all deliver the same matrix \( M \).

The value of \( \alpha \) at which, given the baseline parameter values, endogenous fluctuations become a possibility, is rather high. A value of around 4 for this parameter means that for each percentage point of real exchange rate appreciation, the government increases the monthly devaluation rate by 4 percentage points. The low value assumed for the intertemporal elasticity of substitution (1/\( \sigma = .2 \)), has a lot to do with this outcome. For higher values of this elasticity, expectations of, say, higher future devaluations, would generate a bigger boom in aggregate demand because consumers would be more willing to substitute present for future consumption in response to the increase in the marginal transaction cost. Thus, the current appreciation of the real exchange rate would be more pronounced and given \( \alpha \), the current increase in the devaluation rate would also be

\[^7\]The last rule corresponds to targeting the trade balance. To see this let \( q_t \equiv y^T - c_t^T \) be the trade balance. Then \( \hat{q}_t = -\theta^T/\theta^{TB}c_t^T \) and from (26) \( \hat{c}_t^T = -\rho_{TN}\hat{\epsilon}_t \).
larger. Figure 2 shows the eigenvalues of $M$ as functions of $\alpha$ for two different values of $\sigma$: 1 and 5. When preferences are logarithmic, endogenous fluctuations can occur for values of $\alpha$ less than 2.

In the calibration exercise performed above, the only parameter which was fixed completely arbitrarily was the debt elasticity of the real interest rate, $\rho_d$. Figure 3 shows the three eigenvalues of $M$ as a function of $\rho_d$ for a value of $\alpha$ of 5, at which the baseline model (i.e., $\rho_d = .3$) displays indeterminacy. The figure shows that in this case the possibility of endogenous fluctuations does not depend upon the particular value assumed for $\rho_d$.

I interpret the cases of indeterminacy discussed above as showing that exchange rate policies that are too responsive to deviations from PPP, might generate results which are exactly the opposite to those they are designed to produce. Specifically, they might increase and not reduce the volatility of the real exchange rate and other key macroeconomic variables. This interpretation, however, should not be confused with an attempt to explain business cycle fluctuations in open economies as driven only by self-fulfilling expectations of the type described above.

### 3.1 The timing of the PPP rule

In generating indeterminacy of the competitive equilibrium, it is important that the current devaluation rate be highly sensitive to changes in the current level of the real exchange rate. To see this consider the following more general PPP rule

\[ \dot{z}_t = -\alpha \dot{z}_t \]

\[ \dot{z}_t = \lambda \dot{z}_{t-1} + (1 - \lambda)\dot{e}_t \]

with $\lambda \in [0, 1)$ and $\alpha > 0$. By this rule, the government links the devaluation rate to a weighted average of present and past values of the real exchange rate. The parameter $\lambda$ measures the weight assigned to past values of the real exchange rate. When $\lambda = 0$, the rule collapses to the one given by equation (1), by which the government cares only about the current level of the real exchange rate.

---

Note that as $\rho_d$ approaches zero, one of the eigenvalues of $M$ gets very close to one. In the limit, the frequently used assumption $\beta = 1/(1 + r)$ holds, and the solution to (35) displays random walks in consumption and the stock of debt.
Table 2 shows the minimum value of $\alpha$ for which the model displays indeterminacy, for different values of $\lambda$ and of the inverse of the intertemporal elasticity of substitution, $\sigma$. Given $\sigma$, the table shows that the heavier is the weight assigned to past values of the real exchange rate, the more sensitive the PPP rule has to be in order for the model to display indeterminacy. Each row of table 2 shows that the result obtained above that a higher value of the elasticity of substitution is associated with a smaller required value of $\alpha$ to generate indeterminacy, also holds with this more general rule.

Table 3 considers a rule by which the government measures deviations from PPP in $t$ using a weighted average of the real exchange rate in $t$ and $t + 1$. Formally,

$$\dot{\epsilon}_t = -\alpha \hat{\epsilon}_t$$

$$\hat{\epsilon}_t = \lambda \hat{\epsilon}_{t-1} + (1 - \lambda)\dot{\epsilon}_t$$

When the weight assigned to the past real exchange rate exceeds 50%, the model becomes determinate for any value of $\alpha$.

4 PPP rules and shocks to fundamentals

In this section I will set the parameter $\alpha$ in the PPP rule (1) at values for which endogenous fluctuations around the steady state cannot occur. Instead, I will assume that the economy is hit by shocks to fundamentals. In particular, I will assume the following autorregressive structure for the shocks to the endowments, to the devaluation rate, and to the real interest rate

$$\dot{y}_t^T = \lambda_T \dot{y}_t^{T-1} + \nu_t^T$$

$$\dot{y}_t^N = \lambda_N \dot{y}_t^{N-1} + \nu_t^N$$

$$\dot{\mu}_t = \lambda_\mu \dot{\mu}_{t-1} + \nu_t^\mu$$

$$\dot{\kappa}_t = \lambda_\kappa \dot{\kappa}_{t-1} + \nu_t^\kappa$$

where $\lambda_i \in [0,1)$ and $var(\nu^i) = 1 - \lambda_i^2$ for $i = T, N, \mu,$ and $\kappa$. 

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Figures 4 through 7 present the effects of varying the degree of sensitivity of the PPP rule, $\alpha$, on the volatility of two key macroeconomic variables, when the economy is hit by each of the shocks introduced above, separately. The upper left panel of each of the figures shows the standard deviation of the (log of) the real exchange rate, and the upper right panel the standard deviation of the devaluation rate as functions of $\alpha$. The lower panel of each figure presents the standard deviation of the one-period forecast error of the (log of) the real exchange rate (left panel) and of the devaluation rate (right panel). Solid lines show the case in which the corresponding shock is i.i.d., and broken lines the case in which the shock follows an AR(1) process with a half-life of 36 months.\footnote{This value was picked arbitrarily and implies that $\lambda_T = \lambda_N = \lambda_\mu = \lambda_\alpha = 0.5^{1/36} \approx .98.$}

Figure 4 shows the case in which the economy is hit only by shocks to the endowment of non-traded goods. When these shocks are serially correlated, the standard deviation of the log of the real exchange rate is decreasing in $\alpha$. The reduction in the volatility of the real exchange rate achieved through a more sensitive PPP rule, however, comes at a cost. The standard deviation of the devaluation rate rises from zero to 8 percentage points per month as $\alpha$ goes from zero to around 4. The lower panel of figure 4 shows that the standard deviations of the forecast errors of the log of the real exchange rate and of the devaluation rate follow patterns similar to those of their unconditional standard deviations. The predictability of the real exchange rate slightly increases with $\alpha$, while that of the devaluation rate sharply decreases as the PPP rule becomes tighter. Figure 8 provides some intuition for why in this case tighter PPP rules help curb the volatility and increase the predictability of the real exchange rate. The top panel shows the impulse response functions of the real exchange rate and of the devaluation rate following a positive and serially correlated shock to the endowment of home goods, for two values of $\alpha$: zero, presented using a solid line, and three, presented using a broken line. When the shock hits the economy, the increased supply of non-tradables drives its price down (or the real exchange rate up), and agents expect this situation to continue in the following periods, because of the persistent nature of the shock. In the presence of the PPP rule, the current and (more importantly) the expected future devaluation rates would be below average, inducing an increase in money holdings and, via a reduction in the marginal transaction cost, also in aggregate demand, preventing the real exchange rate from falling as much
as it would have fallen had the government kept the devaluation rate constant. The bottom panel of figure 8 presents an equivalent way of looking at the effect of varying $\alpha$ on the volatility of the real exchange rate. It shows the decomposition of the variance of the log of the real exchange rate across frequencies. The higher value of $\alpha$ (shown with a broken line) reduces the variance at high frequencies by much more than the amount by which it increases the variance at low frequencies. In the case in which the non-traded endowment shocks are i.i.d., the standard deviation of the real exchange is almost insensitive to changes in $\alpha$.\textsuperscript{10} The volatility of the devaluation rate, on the other hand, is significantly increasing in $\alpha$. It goes from zero to around 4 percentage points per month as $\alpha$ increases from zero to 4.

Figure 5 shows the case in which the economy is hit only by shocks to the endowment of tradables. When the shocks are serially correlated, the standard deviation of the real exchange rate is slightly increasing in $\alpha$, while the standard deviation of the forecast error is slightly decreasing in that parameter. Again, the slight effects on the volatility and predictability of the real exchange rate contrast with a pronounced increase in devaluation uncertainty as the PPP rule becomes more responsive. The intuition behind the effects on the volatility of the real exchange rate is the following. As shown in the top panel of figure 9, which presents impulse responses of the real exchange rate and of the devaluation rate for two values of $\alpha$, zero and three, when the economy is hit by a positive and persistent shock to the endowment of tradables, the PPP rule causes the endowment shocks to be spread over time by creating long periods of low inflation in response to positive realizations and vice versa. This smoothing effect implies that the consumption of tradables and the real exchange rate take longer to reach their steady state following an endowment shock, in the presence of a tighter PPP rule. So more sensitive PPP rules make the variance of these two variables higher at low frequencies and lower at high frequencies (see the spectrum shown in the bottom panel of figure 9).

Figure 6 shows the volatility and forecast error of the real exchange rate when the economy is hit only by shocks to the real interest rate. Again, when these shocks are serially correlated,

\textsuperscript{10}It is not completely independent of $\alpha$, though, because the consumption of tradables is affected by the change in relative prices caused by the temporary disturbances in the endowment of home goods, and thus so is future consumption of tradables. This implies that the current level of the real exchange rate conveys information about its future value. A PPP rule then makes the current devaluation rate itself convey information about its own future value, generating extra real effects.
the government can achieve some reduction in volatility and some increase in the predictability of the real exchange rate by following more sensitive PPP rules, but at the cost of very pronounced increases in the volatility of the devaluation rate. In the example shown in the figure, when the standard deviation of the exogenous part of the real interest rate is one percentage point per month, the government can reduce the volatility of the real exchange rate by a third by tightening the PPP rule, but at the expense of increasing the standard deviation of the devaluation rate by forty percentage points per months.

Finally, figure 7 shows the standard deviation of the real exchange rate and of the devaluation rate when fluctuations are originated in random deviations from the PPP rule. The more interesting case is when these deviations are serially correlated. In this case tighter PPP rules actually succeed in reducing the volatility of both nominal and real variables. The reason is that a persistent deviation from the PPP rule, say an increase in the devaluation rate, is associated with a long period of real exchange rate depreciation due to its depressing effect on aggregate demand, and will trigger, given the PPP rule, a persistent decreases in the devaluation rate itself, partially offsetting the effects of the exogenous deviation from the rule.

4.1 Welfare implications

In this section I analyze the question of whether using PPP rules to reduce the volatility of the real exchange rate is also welfare increasing. In order to do this, I will consider a second order approximation of the utility function (2) around the non-stochastic steady state. Let $V_0$ be this approximated utility function. Then $V_0$ is given by\(^{11}\)

$$V_0 = \frac{c^{1-\sigma}}{1 - \sigma} \sum_{t=0}^\infty \beta^t \left[ 1 + (1 - \sigma) \dot{c}_t - \frac{\sigma(1-\sigma)}{2} \sigma^2 \right]$$

where $\beta^* = \beta^{1-\sigma}$. The unconditional expectation of this expression is given by,

$$V = \frac{c^{1-\sigma}}{1 - \sigma} \frac{1}{1 - \beta^*} \left[ 1 - \frac{\sigma(1-\sigma)}{2} \sigma^2 \right]$$  \hspace{1cm} (38)

\(^{11}\)I am ignoring the constant term $[(1 - \beta)(\sigma - 1)]^{-1}$.  

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where \( \sigma^2_c \) stands for the variance of \( \hat{c}_t \). The expression above is decreasing in \( \sigma^2_c \). Since the steady state level of consumption, \( c \), is independent of the degree of sensitivity of the PPP rule, \( \alpha \), the unconditional expectation of \( V \) will be decreasing in \( \alpha \) only if the volatility of consumption increases this parameter.

It follows that if the economy is not hit by any fundamental shock, then setting \( \alpha \) at a high enough value that allows for sunspot fluctuations, is welfare reducing. Figure 10, on the other hand, shows the standard deviation of consumption as a function of \( \alpha \) when the economy is hit by shocks to fundamentals, and \( \alpha \) is not too high, so that the competitive equilibrium is always determinate. Reducing the volatility of the real exchange rate by increasing the sensitivity of the PPP rule is in conflict with increasing the level of welfare when the economy is hit by shocks to the endowment of home goods. In this case, more sensitive PPP rules help reduce the volatility of the real exchange rate, as shown in figure 4, but at the same time increase the volatility of consumption, and thus are welfare reducing. When the economy is hit by shocks to the endowment of tradables, more sensitive exchange-rate rules increase the volatility of the real exchange rate and reduce welfare. On the other hand, tighter PPP rules are welfare increasing and help reduce the volatility of real variables when the economy is hit by shocks to the real interest rate or to the PPP rule itself.

5 Conclusion

This paper suggests an answer to the question of whether PPP rules can help stabilize real and nominal macroeconomic variables. Within the context of the model developed in this paper, introducing tight PPP rules might generate macroeconomic instability via endogenous fluctuations due to self-fulfilling expectations. In this case the rule delivers results which are exactly the opposite from those it is designed to produce, namely, the reduction of the volatility of a key relative price such as the real exchange rate.

In the case in which endogenous fluctuations are not a possibility, which, in the context of this paper might be thought of as cases of mild PPP rules, the desirability of following real exchange rate rules is hardly evident. For instance, PPP rules stabilize the real exchanger rate only if shocks are serially correlated and do not originate in the traded good sector. More importantly, the extent to which real exchange rate volatility can be reduced when the economy is hit by shocks to the
endowment of home goods, the real interest rate or the PPP rule itself, is quantitatively small compared with the substantial increase in the variability of inflation brought about by this type of policies.

Finally, this paper shows that PPP rules might be welfare reducing. When the exchange rate rule is made so tight that endogenous fluctuations become a possibility, the volatility of consumption increases and thus the unconditional expected utility of the representative agent decreases. On the other hand, when the PPP rule is not too sensitive, so that the rational expectations equilibrium is determinate, exchange rate rules are welfare reducing when the main sources of fluctuations are shocks to the endowments of traded or home goods, and are welfare increasing when the economy is hit mainly by shocks to the real rate of return on foreign assets, or by random deviations from the PPP rule itself.
Appendix

In this appendix I prove that in the absence of a PPP rule, the model presented in the paper does not generate stationary sunspot equilibria. That is, that when \( \alpha \) equals zero in (1), the matrix \( M \) of the linear system (35) cannot have more than one eigenvalue inside the unit circle.

First, I will show that in this case (35) can be reduced to a 2x2 system in \( c_t^T \) and \( d_t \). From (19) and (24) it follows that when \( \alpha = 0 \) and in the absence of shocks to fundamentals, the nominal interest rate is given by

\[
1 + i_t = (1 + \epsilon)(1 + \tau(d_t))
\]

So \( i_t \) depends only on \( d_t \) and is strictly increasing in it. This and the fact that \( v_m \) is homogeneous of degree zero, imply that the money demand equation can be written as \( m_t = x_t h(d_t) \) where \( h' < 0 \). Taking into account that \( v_x \) is also homogeneous of degree zero, the effective price of consumption, \( p_t \), is then given by,

\[
p_t = \frac{1 + v_x(1, h(d_t))}{A_1(c_t^T, y^N)} \equiv P(c_t^T, d_t)
\]

where \( P_1 > 0 \) because \( A_{11} < 0 \), and \( P_2 > 0 \) because by assumption \( v_{xm} < 0 \). The Euler equation (19) can then be written in the following way,

\[
1 = \tilde{\beta}(1 + \tau(d_t)) E_t \left\{ \frac{G(c_t^T, d_t)}{G(c_{i+1}^T, d_{i+1})} \right\}
\]

where \( G(\cdot, \cdot) \) is given by \( G(c^T, d) \equiv A(c^T, y)^\sigma P(c^T, d) \); so \( G_1 \) and \( G_2 \) are both strictly positive.

The second equilibrium condition is the economy-wide budget constraint (25), which I reproduce below for convenience,

\[
d_t = \frac{1 + \tau(d_{t-1})}{1 + \gamma} d_{t-1} + c_t^T - y^T
\]

The log-linear version of these two equations is,

\[
0 = \eta_1(c_t^T - E_t \tilde{c}_{i+1}^T) + \eta_2(\hat{d}_t - E_t \hat{d}_{i+1}) + b \hat{d}_t
\]

\[
\hat{d}_t = \alpha_1 \hat{d}_{i-1} + \alpha_2 \tilde{c}_t^T
\]

where \( \alpha_1 \equiv \frac{1 + \tau}{1 + \gamma} (1 + \frac{\tau}{1 + \tau} \rho_d) > 1 \), \( \alpha_2 \equiv \frac{\tau + \gamma}{1 + \tau} \frac{\sigma}{\beta} > 0 \), \( b \equiv \frac{\tau}{1 + \tau} \rho_d > 0 \), and \( \eta_1 > 0 \) and \( \eta_2 > 0 \) denote

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the elasticities of $G(\cdot, \cdot)$ with respect to its first and second arguments, respectively. This system can be written as

$$
\begin{bmatrix}
E_{t}c_{t+1}^T \\
\hat{d}_{t}
\end{bmatrix} = M
\begin{bmatrix}
\hat{c}_{t}^T \\
\hat{d}_{t-1}
\end{bmatrix}
$$

(44)

where the matrix $M$ is given by,

$$
M = 
\begin{bmatrix}
w + \alpha_2 z & \alpha_1 z \\
\alpha_2 & \alpha_1
\end{bmatrix}
$$

(45)

and

$$
w \equiv \frac{\eta_1}{\eta_1 + \alpha_2 \eta_2} < 1
$$

$$
z \equiv \frac{\eta_2(\alpha_1 - 1) + b}{\eta_1 + \alpha_2 \eta_2}
$$

One of the necessary conditions for the matrix $M$ to have both of its eigenvalues inside the unit circle is that

$$
\text{trace}(M) < 1 + \text{det}(M)
$$

that is,

$$
w + \alpha_2 z + \alpha_1 < 1 + \alpha_1 w
$$

using the expressions for $w$ and $z$ and rearranging, one obtains

$$
\alpha_2 b < 2\alpha_2 \eta_2 (1 - \alpha_1)
$$

which is a contradiction since the left hand side is positive and the right hand side is negative.
References


Table 1:
Long-Run Data Relations and Calibrated Parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Description</th>
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<tbody>
<tr>
<td>$r$</td>
<td></td>
<td>6.5%</td>
<td>Steady-state real rate of return (per year)</td>
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<td>$\epsilon$</td>
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<td>10%</td>
<td>Devaluation rate (per month)</td>
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<td>$i$</td>
<td>$(1 + r)(1 + \epsilon) - 1$</td>
<td>10.6%</td>
<td>Nominal interest rate (per month)</td>
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<td>$x/m$</td>
<td></td>
<td>1.25</td>
<td>Money velocity (per month)</td>
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<td>$\gamma$</td>
<td></td>
<td>0.5%</td>
<td>per capita output growth rate (per year)</td>
</tr>
<tr>
<td>$\theta^T$</td>
<td>$\frac{\epsilon T}{c^T + y^N/c}$</td>
<td>40%</td>
<td>Share of tradables in GNP</td>
</tr>
<tr>
<td>$\theta^{TB}$</td>
<td>$\frac{\sqrt{y^T - \epsilon T}}{c^T + y^N/c}$</td>
<td>2%</td>
<td>Share of the trade balance in GNP</td>
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<td>$\rho_{TN}$</td>
<td>$\frac{A_{11}A_2}{A_{12}A}$</td>
<td>1</td>
<td>Elasticity of substitution between traded and home goods</td>
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<td>$\sigma^{-1}$</td>
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<td>0.2</td>
<td>Intertemporal elasticity of substitution</td>
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<td>$\nu_x$</td>
<td></td>
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<td>Marginal transactions cost</td>
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<td>$\rho_{mm}^{-1}$</td>
<td>$\frac{v_m}{(m/x)\cdot v_m}$</td>
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<td>Money demand elasticity</td>
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<td>$\rho_{xx}$</td>
<td>$\frac{(x/m)\cdot v_{xx}}{v_x}$</td>
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<td>Expenditure elasticity of the marginal transactions cost</td>
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<td>$\rho_d$</td>
<td>$\frac{r'(d)\cdot d}{r(d)}$</td>
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<td>$\alpha$</td>
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<td>Sensitivity of the PPP rule ((minus) the elasticity of the devaluation rate with respect to the real exchange rate)</td>
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</table>
\[ \hat{\epsilon}_t = -\alpha \hat{z}_t \]
\[ \hat{z}_t = \lambda \hat{z}_{t-1} + (1 - \lambda) \hat{\epsilon}_t \]

Table 2:
Minimum value of \( \alpha \) that generates indeterminacy as a function of \( \lambda \) and \( \sigma \), when the PPP rule is

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Table 3:
Minimum value of $\alpha$ that generates indeterminacy as a function of $\lambda$ and $\sigma$, when the PPP rule is

$$\dot{\epsilon}_t = -\alpha \dot{\epsilon}_t$$

$$\dot{z}_t = \lambda \dot{\epsilon}_{t-1} + (1 - \lambda) \dot{\epsilon}_t$$

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<td>15.0</td>
<td>21.6</td>
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<td>0.44</td>
<td>12.5</td>
<td>22.5</td>
<td>32.5</td>
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<tr>
<td>0.47</td>
<td>25.0</td>
<td>45.0</td>
<td>65.0</td>
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<tr>
<td>0.50</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
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Figure 1

The three eigenvalues of $M$
as functions of the sensitivity of the
PPP rule ($\alpha$)
The eigenvalues of the matrix $M$ as functions of $\alpha$, for two values of the intertemporal elasticity of substitution $(1/\sigma)$.
The eigenvalues of the matrix $M$ as functions of the debt elasticity of the real interest rate ($\rho_d$)
FIGURE 4
SHOCKS TO THE ENDOWMENT OF HOME GOODS
(Solid lines refer to iid shocks, broken lines to AR(1) shocks with half-lives of 36 months)

Standard Deviation as a Function of the Sensitivity of the PPP Rule ($\alpha$)

Standard Deviation of the One-Period Forecast Error as a Function of the Sensitivity of the PPP Rule ($\alpha$)
FIGURE 5
SHOCKS TO THE ENDOWMENT OF TRADABLES
(Solid lines refer to iid shocks, broken lines to AR(1) shocks with half-lives of 36 months)

Standard Deviation as a Function of the Sensitivity of the PPP Rule (\(\alpha\))

![Graphs showing real exchange rate and devaluation rate as functions of \(\alpha\)]

Standard Deviation of the One-Period Forecast Error as a Function of the Sensitivity of the PPP Rule (\(\alpha\))

![Graphs showing real exchange rate and devaluation rate of forecast error as functions of \(\alpha\)]
FIGURE 6
SHOCKS TO THE REAL INTEREST RATE
(Solid lines refer to iid shocks, broken lines to AR(1)
shocks with half-lives of 36 months)

Standard Deviation as a Function of the
Sensitivity of the PPP Rule ($\alpha$)

Standard Deviation of the One-Period Forecast Error as a Function of the
Sensitivity of the PPP Rule ($\alpha$)
FIGURE 7
RANDOM DEVIATIONS FROM THE PPP RULE
(Solid lines refer to iid shocks, broken lines to AR(1) shocks with half-lives of 36 months)

Standard Deviation as a Function of the Sensitivity of the PPP Rule ($\alpha$)

Standard Deviation of the One-Period Forecast Error as a Function of the Sensitivity of the PPP Rule ($\alpha$)
FIGURE 8
IMPULSE RESPONSES AND SPECTRUM WHEN THE ECONOMY IS HIT BY
SHOCKS TO THE ENDOWMENT OF HOME GOODS
(Solid lines refer to $\alpha=0$, broken lines to $\alpha=3$
The half life of the shock is always 36 months)

Impulse responses

real exchange rate

months after the shock

devaluation rate

months after the shock

Spectrum of the Real Exchange Rate

frequency
FIGURE 9
IMPULSE RESPONSES AND SPECTRUM WHEN THE ECONOMY IS HIT BY
SHOCKS TO THE ENDOWMENT OF TRADABLES
(Solid lines refer to $\alpha=0$, broken lines to $\alpha=3$
The half life of the shock is always 36 months)

Impulse responses

real exchange rate

-0.04
-0.06
-0.08
-0.1
-0.12
0 50 100 150 200
months after the shock

devaluation rate

0.3
0.2
0.1
0
-0.1
0 50 100 150 200
months after the shock

Spectrum of the Real Exchange Rate

1000
500
0
0 0.01 0.02 0.03 0.04 0.05 0.06 0.07
frequency
FIGURE 10
STANDARD DEVIATION OF CONSUMPTION AS A FUNCTION OF $\alpha$
WHEN THE ECONOMY IS HIT BY SHOCKS TO FUNDAMENTALS
(Solid lines refer to iid shocks, broken lines to AR(1)
shocks with half-lives of 36 months)

![Graphs showing standard deviation of consumption](image-url)
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