Board of Governors of the Federal Reserve System

International Finance Discussion Papers

Number 760

February 2003

The Present-Value Model of the Current Account Has Been Rejected: Round Up the Usual Suspects

James M. Nason

and

John H. Rogers

The Present-Value Model of the Current Account Has Been Rejected: Round Up the Usual Suspects

James M. Nason[†]
Rm. 997, Buchanan Tower
Department of Economics
University of British Columbia
1873 East Mall
Vancouver, British Columbia
CANADA V6T 1Z1
email : nason@econ.ubc.ca

John H. Rogers[†]
Mail Stop 22
International Finance Division
Board of Governors of the
Federal Reserve System
Washington, D.C.
U.S.A. 20551
email : John.H.Rogers@FRB.GOV

First Draft: June 26, 1999
Current Draft: February 5, 2003

Abstract

Tests of the present-value model of the current account are frequently rejected by the data. Standard explanations rely on the "usual suspects" of non-separable preferences, shocks to fiscal policy and the world real interest rate, and imperfect international capital mobility. We confirm these rejections on post-war Canadian data, then investigate their source by calibrating and simulating alternative versions of a small open economy, real business cycle model. Monte Carlo experiments reveal that, although each of the suspects matters in some way, a "canonical" RBC model moves closest to the data when it features exogenous world real interest rate shocks.

Key Words : current account; present-value model; world real interest rate; international capital mobility; Bayesian monte carlo.

JEL Classification Number : $F41$; $E32$.

[†] We wish to thank the co-editor, Enrique Mendoza, two anonymous referees, Paul Bergin, David Bowman, Michael Dooley, Allan Gregory, Mathias Hoffman, Beth Ingram, Pat Kehoe, Rody Manuelli, Tommaso Monacelli, Gregor Smith, Peter Tinsley, Shaun Vahey, conference participants at the 1999 Summer Meetings of the Econometric Society at the University of Wisconsin-Madison, the 2000 Midwest Macro Meetings at the University of Iowa, and the 2001 International Conference of the Society for Computational Economics at Yale University, and seminar participants at California-Davis, California-Santa Cruz, Cambridge, Georgetown, Québec à Montréal, Queen's, Simon Fraser, Southampton, Victoria, Virginia, the Bank of Canada, and the Federal Reserve Board for useful comments. The first author thanks the Research Department of the Federal Reserve Bank of Atlanta for its hospitality and support during his stay there. The views in this paper represent those of the authors and are not necessarily those of either the Board of Governors of the Federal Reserve System, the Federal Reserve Bank of Atlanta, or any member of these institutions. The appendix to the paper can be found at *http://eagle.econ.ubc.ca*.

1. Introduction

Current account fluctuations resist easy explanations. Large current account deficits have persisted in the U.S. through periods of large government budget deficits and surpluses, large and persistent real appreciations and depreciations of the dollar, and all phases of the business cycle. In Canada, the expansion of the 1980s coincided with large current account deficits, but an expansion in the 1990s witnessed current account surpluses.

Economists have increasingly used the intertemporal approach to study the current account. The intertemporal approach views the current account as a tool domestic residents use to smooth consumption by borrowing from or lending to the rest of the world. For example, if future income is expected to rise, say due to a technology shock, domestic agents attempt to smooth consumption by borrowing internationally prior to the high-income years, thereby running a current account deficit. As such, the intertemporal approach emphasizes permanent income fluctuations (driven by technology shocks) to explain current account movements. Compared to traditional Keynesian views, the intertemporal approach reduces emphasis on the economy's intratemporal competitiveness measured by the real exchange rate.

The intertemporal approach to the current account encompasses several classes of small open economy models. The most basic is the present-value model (PVM) of the current account. Sheffrin and Woo (1990), Otto (1992), Ghosh (1995), and Bergin and Sheffrin (2000) find that the testable implications of the PVM are routinely rejected by the data.

Despite rejections of the PVM's cross-equation restrictions, it is argued that abandonment of the underlying scheme is not warranted. Adherents point out that the (in-sample) current account forecast from the most unadorned PVM often tracks the actual current account fairly closely (e.g., Obstfeld and Rogoff (1996), pp. 92-94).[1] Thus, the PVM, although strictly rejected, is viewed in the literature as "useful" overall. This conclusion, while appropriate, is not fully satisfactory because it fails to say which elements of the intertemporal model are most responsible for the poor empirical performance of the PVM.

In this paper, we consider a set of "usual suspects" – i.e., factors that theory teaches us can matter for

[1] Other authors, such as Ahmed (1986) and more recently Glick and Rogoff (1995), İşcan (2000), and Nason and Rogers (2002), test a variety of implications of the intertemporal approach and present evidence that favors some aspects of it.

the current account – as potential sources of the empirical rejections of the PVM of the current account. These factors are non-separable preferences, country-specific fiscal shocks, a world real interest rate shock, and imperfect international capital mobility. We place the suspects in a "canonical" small open economy-real business cycle (RBC) model which nests the PVM and serves as our benchmark intertemporal model of the current account.

Our "testing" strategy compares moments of synthetic data produced by the RBC model to those of actual data. Rather than focus on the usual variances and covariances, the "moments" we study are the cross-equation restrictions of the PVM. The actual data we use is from post-war Canada, a proto-type small open economy for which rejections of the PVM are found uniformly in the literature.[2] We evaluate the empirical and theoretical distributions implied by the cross-equation restrictions using Bayesian Monte Carlo methods, which measure the fit of the RBC model to the actual data.

The canonical RBC model includes none of our suspects. This economy features a permanent, country-specific technology shock, but has no transitory shocks to fiscal policy or the world real interest rate. Capital is perfectly mobile internationally. Thus, the canonical model nests the PVM. Next, we add suspects to the canonical model one-at-a-time to create alternative specifications. With each alternative, we generate artificial time series, repeat tests of the cross-equation restrictions, and again compare outcomes to the actual data. These exercises give us evidence about the culpability of each suspect.

Our choice of suspects is guided by related work on the intertemporal approach. Non-separable preferences can matter for the current account in several ways. Ghosh and Ostroy (1997) find that incorporating precautionary saving into the PVM helps to explain current account volatility.[3] Gruber (2002) analyzes the role of habit persistence. Bergin and Sheffrin (2000) improve the fit of the basic PVM by allowing for non-separable utility (between tradable and non-tradable goods) and a stochastic (consumption-based) real world interest rate. The impact of fiscal and interest rate shocks on small open economies is well studied. Striking evidence of the importance of fiscal shocks, especially large ones, for intertemporal external borrowing decisions has been presented by Ahmed (1986) and Ahmed and Rogers (1995). Hercowitz (1986) and Blankenau, Kose, and Yi

[2]An example is Ghosh (1995), who obtains rejections for Canada, but ironically, finds the PVM holds better in U.S. data. He and Obstfeld and Rogoff (1996) report "excess smoothness" of the predicted Canadian current account, as we report below.

[3]Interestingly, Canadian data rejects the precautionary saving hypothesis according to Ghosh and Ostroy.

(2001) report that world real interest rate shocks help to explain aggregate fluctuations in small open economies. Cole and Obstfeld (1991) argue that small barriers to international capital mobility lead to a smaller current account on average because the benefits of consumption smoothing are negated. Barriers to international capital mobility have been modeled formally by Mendoza (1991b), Valderrama (2002), and Schmitt-Grohé and Uribe (2003), in different fashions. Mendoza adds capital controls to a small open economy-RBC model, Valderrama places an arbitrary bound on debt accumulation, while Schmitt-Grohé and Uribe introduce an endogenous risk premium into a similar model.

In the next section, we describe the testable predictions of the PVM of the current account and confirm rejections of those predictions on Canadian data. Section 3 presents our small open economy-RBC model, derives its optimality conditions, and explains the numerical methods we employ to solve our model. We report the results of the Monte Carlo experiments in section 4 and discuss our conclusions in section 5.

2. The Present-Value Model of the Current Account

Empirical studies of the PVM of the current account have adapted the permanent-income model of consumption to the small open economy to derive the tests of Campbell (1987) and Campbell and Shiller (1987). As Sheffrin and Woo (1990), Otto (1992), Ghosh (1995), and Bergin and Sheffrin (2000) demonstrate, the cross-equation restrictions implied by the PVM are almost always rejected by the data. In this section, we review these restrictions and confirm their rejections on a sample of post-war Canadian data. We then pursue the paper's main objective of uncovering the sources of these rejections.

2.1 Tests of the Present-Value Model of the Current Account

The PVM of the current account is derived from the permanent-income decision rule of a small open economy, $\bar{C}_t = rB_t + r(1+r)^{-1}\sum_{j=0}^{\infty}(1+r)^{-j}\mathbf{E}_t\{Y_{t+j} - I_{t+j} - G_{t+j}\}$, $0 < r$, where $\bar{C}_t$ is the permanent income-level of consumption, and r, Y_t, I_t, G_t, and B_t denote the non-stochastic world real interest rate, output, investment, government spending, and net domestic ownership of foreign assets, respectively. It is assumed that there is only one disturbance, a unit root country-specific technology shock that generates a permanent response in Y_t. Replacing C_t with $\bar{C}_t$ in the expenditure identity, $Y_t \equiv C_t + I_t + G_t + NX_t$, where net exports, NX_t, equals the current account minus income from net domestic ownership of foreign assets, $CA_t - rB_t$, it follows that

$$CA_t = -\sum_{j=1}^{\infty}\left(\frac{1}{1+r}\right)^j \mathbf{E}_t \Delta NY_{t+j}, \quad NY_t \equiv Y_t - I_t - G_t, \tag{1}$$

where Δ represents the difference operator. The current account-PVM relation (1) implies that when future net income is expected to be above trend, the current account is in deficit. This occurs because agents smooth consumption by borrowing in response to the (positive) shock to permanent income.

The cross-equation restrictions of the PVM rely on the assumption that the joint data generating process (DGP) of ΔNY and CA is an unrestricted pth-order bivariate autoregression, AR(p). Create a $2p-$dimensional vector AR(1), $\mathcal{W}_t = \mathcal{D}\mathcal{W}_{t-1} + \mathcal{V}_t$, from this AR($p$), where $\mathcal{D}$ is the companion matrix, $\mathcal{W}_t \equiv [\Delta NY_t \ldots \Delta NY_{t-p+1}\, CA_t \ldots CA_{t-p+1}]'$, and $\mathcal{V}_t \equiv [v_{\Delta NY,t}\, 0 \ldots 0\, v_{CA,t}\, 0 \ldots]'$ is the column vector of mean zero, homoskedastic unrestricted errors. The vector AR(1) yields an unrestricted forecast of $\mathcal{W}_{t+j}$, $\mathbf{E}_t\mathcal{W}_{t+j} = \mathcal{D}^j\mathcal{W}_t$. This and the row vector $\mathcal{F} \equiv [1\, \mathbf{0}_{p-1}]$ yield $\mathbf{E}_t\Delta NY_{t+j} = \mathcal{F}\mathcal{D}^j\mathcal{W}_t$. Substitute this into the present-value relation (1) to find $\mathcal{H} = -\mathcal{F}\mathcal{D}[\mathbf{I} - \mathcal{D}/(1+r)]^{-1}/(1+r)$, where $\mathcal{H}$ is a $2p$ row vector.

The cross-equation restrictions of the PVM are embodied in $\mathcal{H}$. Note that these restrictions imply the linear rational expectations forecast of the current account, $CA_{f,t} = \mathcal{H}\mathcal{W}_t$. The PVM predicts that $CA_{f,t}$ is identical to the actual current account because the contemporaneous current account is contained in the date t information set of the vector AR(1). Recalling that the $p+1$st element of $\mathcal{W}_t$ is CA_t, the null hypothesis $CA_{f,t} = CA_t$ holds when all elements of $\mathcal{H}$ are zero except its $p+1$st element, which equals one.

Another implication of the model is that the difference between the forecast and actual current account is unpredictable, given the history of $\mathcal{W}_t$. Campbell (1987) provides a way to test this prediction. When the PVM is assumed to be exact, only the country-specific technology shock drives the small open economy. In this case, $\mathcal{D}$ is singular because the difference between its $p+1$st row and first row equals its $p+2$nd row multiplied by $(1+r)$. We define the implied variable as $\mathcal{CA}_t$ (notice the script notation).[4] Given that $\mathcal{D}$ is singular, $\mathcal{CA}_t$ is orthogonal to lags of $\mathcal{W}_t$. However, these predictions fail to hold in the presence of a transitory demand shock. In

[4]The $p+1$st row of $\mathcal{D}$ contains the response of CA_t to $\mathcal{W}_{t-1}$, while the first and $p+2$nd rows of $\mathcal{D}$ are the responses of ΔNY_t and CA_{t-1}, respectively. Hence, $\mathcal{CA}_t \equiv CA_t - \Delta NY_t - (1+r)CA_{t-1}$.

this case, since $\mathcal{CA}_t$ and $\mathcal{W}_{t-1}$ are correlated, the PVM predicts that $\mathcal{W}_{t-1}$ has no power to forecast $\mathcal{CA}_{t+1}$ rather than $\mathcal{CA}_t$. This is a test of the extent to which current account fluctuations can be explained by shocks other than those that affect permanent income.

In sum, there are two tests of the cross-equation restrictions of the PVM of the current account: (i) all elements of $\mathcal{H}$ are zero except the $p+1$st element which equals one; and (ii) the forecast innovation is orthogonal to $\mathcal{W}_{t-1}$, so the coefficients of the regression of $\mathcal{CA}_t$ or $\mathcal{CA}_{t+1}$ on $\mathcal{W}_{t-1}$ are all zero.

2.2 The Data and Empirical Results

We employ data on the Canadian current account and GDP net of investment and government spending. Our data sources are Statistics Canada and Basic Economics. The entire sample covers $1961Q1$ through $1998Q1$. Our estimation sample period begins in $1963Q1$ and ends with $1997Q4$, for a total of $T = 140$ observations. The data is measured on a per capita basis in 1992 Canadian dollars and is seasonally adjusted at annual rates. We demean ΔNY_t and CA_t, and estimate a vector AR (VAR) conditional on $p = 4$. The calibration sets $r = 0.0091$ (or 3.70 percent on an annual basis) to compute $\mathcal{H}$.[5]

The cross-equation restrictions of the PVM are rejected on these data. The Wald statistic (computed as in Sheffrin and Woo (1990)) embodied in $\mathcal{H}$ is 16.12 with an asymptotic p-value of 0.04 given eight degrees-of-freedom. The LM test statistic, $T \times R^2$, of the orthogonality conditions also indicates a rejection of the PVM. When $\mathcal{CA}_{t+1}$ ($\mathcal{CA}_t$) is the dependent variable, the test statistic of 14.78 (19.55) has an asymptotic p-value of 0.06 (0.01) on eight degrees-of-freedom, suggesting that the regression coefficients are not all zero.

Rejections of the cross-equation restrictions of the PVM are confirmed by estimates of $\mathcal{H}$. These estimates, $[0.11 \ -0.03 \ -0.06 \ 0.06 \ 0.08 \ 0.06 \ -0.04 \ -0.10]'$, are all smaller than 0.12 in absolute value and none is significantly different from zero.[6] The key theoretical prediction that $\mathcal{H}_5 = 1.0$, implying that the PVM forecast moves one-for-one with the actual current account, is strongly rejected in the data.

[5]Sheffrin and Woo (1990), Otto (1992), and Ghosh (1995) also demean ΔNY_t and CA_t prior to estimating the VAR. To select p, we compute the Akaike information criterion (AIC) and general-to-specific likelihood ratio (LR) tests for VARs of ΔNY_t and CA_t. The AIC and the LR tests select $p = 4$. The Canadian sample average of r is calculated using Fisher's equation, the three-month Euro-dollar deposit rate, the Canadian dollar-U.S. dollar exchange rate, and the implicit GDP deflator of Canada.

[6]The associated asymptotic standard errors are $[0.20 \ 0.20 \ 0.14 \ 0.09 \ 0.41 \ 0.12 \ 0.12 \ 0.11]'$.

Finally, as derived in the previous section, $\widehat{\mathcal{H}}$ yields a forecast of the current account, $CA_{f,t}$. The left-side window of figure 1 plots this forecast (the dot-dash line) and compares it to the actual current account (the solid line). Clearly, $CA_{f,t}$ is less persistent and less volatile than the actual current account. This "excess smoothness", which is found uniformly for Canada in the literature, still exists when we account for estimation uncertainty in the parameters of the unrestricted VAR. This can be seen from the right-side window of figure 1, which contains the actual Canadian current account, the solid line, and the fifth and 95th percentiles of the piecewise probability bands of $CA_{f,t}$ (the dotted lines) which are generated using Bayesian Monte Carlo integration methods of Geweke (1999a).[7] For most of the sample, the actual current account falls outside of the 90 percent probability bands. This is particularly striking for the second half of the 1980s and the early 1990s, when the actual current account is below the lower probability band of the PVM current account forecast.

Thus we have confirmed rejections of the PVM of the current account that have been reported elsewhere. In the next section, we present a small open economy-RBC model that allows us to examine several possible sources of these empirical rejections.

3. A Small Open Economy-RBC Model

In this section, we construct a one-sector small open economy-RBC model. The model acts as a restricted DGP to explore the role of the usual suspects in explaining the empirical failures of the PVM of the current account. We solve our small open economy-RBC model numerically around its deterministic steady state and choose prior distributions of the model's parameters. We do this for our benchmark model containing only country-specific permanent technology shocks, which we label the "canonical RBC model", and for alternative specifications that feature our suspects.

3.1 Tastes, Technology, and the Impulse Structure

The small open economy-RBC model consists of a representative household, a constant returns to scale (CRS) technology, and an external sector. The household chooses uncertain streams of the single consumption

[7] We generate 5000 replications of $CA_{f,t}$ with software Geweke provides at *http://www.econ.umn.edu/∼bacc*. To account for the serial correlation in $CA_{f,t}$, the two largest principal components of the covariance matrix of the ensemble of synthetic $CA_{f,t}$ are computed to construct the 90 percent confidence bands.

good, C_t, and leisure, L_t, to maximize discounted expected lifetime utility

$$\mathbf{E}_t \left\{ \sum_{i=0}^{\infty} \beta^i U\left(C_{t+i},\ L_{t+i}\right) \right\}, \quad 0 \ < \ \beta \ < \ 1. \tag{2}$$

Leisure plus time supplied to the labor market, N_t, sums to the unit of time endowment the household receives each date t, $L_t = 1 - N_t$. Consumption and leisure enter the period utility function either separably

$$U\left(C_t,\ L_t\right) \quad = \quad \phi\, ln\left[C_t\right] \quad + \quad (1 - \phi)\, ln\left[1 \ - \ N_t\right], \tag{3}$$

or in non-separable form

$$U\left(C_t,\ L_t\right) \quad = \quad \frac{\left[C_t^{\phi}\left(1 \ - \ N_t\right)^{(1-\phi)}\right]^{(1-\psi)}}{1 \ - \ \psi}, \quad 0 \ < \ \phi \ < \ 1, \quad \psi \ \neq \ 1. \tag{4}$$

The restriction is $\psi = 1$ for the separable and log period utility function (3).

When period utility is non-separable in consumption and leisure, there is a link between demand for foreign assets (the current account) and current and expected future domestic labor market activity. Changes in time-worked alter the discount the small open economy applies to future expected returns, and hence the degree of consumption smoothing undertaken. Thus, under non-separable utility, there is an additional channel for shocks to produce current account fluctuations: through transitory movements in time-worked. A second way in which the assumption of non-separable utility can influence the current account is because the (non-log utility) risk aversion it implies yields "consumption-tilting" behavior. The greater the risk aversion, the more responsive is the current account to transitory shocks to consumption.

The household consumes, C_t, accumulates capital, K_{t+1}, through investment, I_t, or alters its stock of the internationally traded bond, B_{t+1}, through net exports, NX_t. The law of motion of capital is

$$K_{t+1} \quad = \quad (1 - \delta)K_t \quad + \quad \left(\frac{K_t}{I_t}\right)^{\alpha} I_t, \quad 0 \ < \ \alpha,\ \delta \ < \ 1. \tag{5}$$

This features installation costs in the flow of new capital as in Baxter and Crucini (1993).

An international bond is the only financial asset available to the household, which may purchase (sell) a unit of B_{t+1} from (to) the rest of the world at the end of date t. During date $t + 1$, the small open economy receives (sends) one unit of the consumption good plus a stochastic return, r_t, from (to) the rest of the world. The stochastic return is the world real interest rate. Thus, the law of motion of B_{t+1} is

$$B_{t+1} = (1 + r_t)B_t + NX_t, \tag{6}$$

where NX_t reflects the net flow of the good between the small open economy and the rest of the world.

There are two components to r_t, an exogenous and stochastic return q_t, which is common across the world, and a risk premium that is specific to the small open economy, and is assumed to be a linear function of the economy's bond-output ratio.[8] Thus,

$$r_t = q_t - \varphi\frac{B_t}{Y_t}, \quad 0 < \varphi. \tag{7}$$

In our model, movements in the world real interest rate are an additional source of consumption smoothing. Interest rate fluctuations can arise from exogenous shocks, q_t, or from endogenous fluctuations in the risk premium, $\varphi(B_t/Y_t)$, in response to other shocks. Exogenous shocks generate income and substitution effects in the small open economy and produce current account fluctuations by driving a wedge between expected returns to the international bond and domestic physical capital (which equals the marginal product of K_{t+1}). Movements in the risk premium have the same effect and imply, for example, that an economy that is a net debtor, $B_{t+1} < 0$, must pay a premium above q_t.[9]

We assume that the small open economy internalizes its risk premium, i.e., that its decision rules take account of fluctuations in the risk premium, $\varphi(B_t/Y_t)$.[10] From the perspective of the small open economy, imperfect capital mobility is equivalent to the (internalized) risk premium because either produces a wedge between q_t and r_t. We examine the extent to which the decisions of the small open economy affect current account fluctuations through the risk premium.

[8]This risk premium in the law of motion (6) negates a unit root in the *linearized* solution of our small open economy-RBC model. Schmitt-Grohé and Uribe (2003) study several devices that achieve a well-posed linearized solution for this class of RBC models, including an endogenous risk premium that is strictly increasing in the unit discount bond, and show that they all produce the same responses to one-time technology shocks.

[9]The risk premium a debtor pays is higher the more negative is its bond-output ratio. However, a debtor whose output grows faster than its foreign liabilities sees its risk premium fall. This allows our small open economy in principle to remain in debt permanently.

[10]Alternatively, one could assume that foreigners set the risk premium, in which case it is treated as given by the small open economy. This is discussed more fully in the appendix at *http://eagle.econ.ubc.ca*.

Our model is tied together by the resource constraint

$$Y_t \quad = \quad C_t \quad + \quad I_t \quad + \quad G_t \quad + \quad NX_t. \tag{8}$$

Output is produced with the CRS technology

$$Y_t \quad = \quad K_t^{\theta}[A_t\, N_t]^{(1-\theta)}, \quad 0 \;<\; \theta \;<\; 1, \tag{9}$$

where A_t denotes a country-specific technology shock. It evolves as a random walk with drift

$$A_{t+1} \quad = \quad A_t \exp\{\gamma \;+\; \varepsilon_{t+1}\}, \quad 0 \;<\; \gamma, \quad \varepsilon_{t+1} \;\sim\; \mathbf{N}\left(0,\, \sigma_{\varepsilon}^2\right). \tag{10}$$

Since A_t drives permanent movements in the model's quantity variables (except for N_t), it plays the role of the domestic disturbance to permanent income.

Shocks to the transitory component of government spending, $g_t = G_t/Y_t$, and the exogenous component of the world real interest rate, q_t are assumed to follow AR(1) processes

$$g_{t+1} \quad = \quad g^{*\,(1-\rho_g)}\, g_t^{\rho_g}\, exp\left\{\eta_{t+1}\right\}, \quad |\rho_g| \;<\; 1, \quad \eta_{t+1} \;\sim\; \mathbf{N}\left(0,\, \sigma_{\eta}^2\right), \tag{11}$$

and

$$1 \,+\, q_{t+1} \quad = \quad (1 \,+\, q^*)^{(1-\rho_q)}\, (1 \,+\, q_t)^{\rho_q}\, exp\left\{\xi_{t+1}\right\}, \quad |\rho_q| \;<\; 1, \quad \xi_{t+1} \;\sim\; \mathbf{N}\left(0,\, \sigma_{\xi}^2\right), \tag{12}$$

where g^* is the steady state or unconditional mean of g_t and q^* is the exogenous component of the world real interest rate. We assume that the innovations ε_t, η_t, and ξ_t are uncorrelated at all leads and lags.

3.2 Optimality and Equilibrium

We derive the following optimality conditions

$$\left(\frac{1 \,-\, \phi}{\phi}\right)\frac{C_t}{1 \,-\, N_t} \quad = \quad (1 \,-\, \theta)\frac{Y_t}{N_t}\left[1 \;+\; \varphi\left(\frac{B_t}{Y_t}\right)^2\right], \tag{13}$$

$$\left(\frac{I_t}{K_t}\right)^{\alpha} \quad = \quad \mathbf{E}_t\left\{\Gamma_{t+1}\left[\theta(1-\alpha)\frac{Y_{t+1}}{K_{t+1}}\left[1 + \varphi\left(\frac{B_{t+1}}{Y_{t+1}}\right)^2\right] \;+\; \left[1 - \delta \,+\, \alpha\left(\frac{I_{t+1}}{K_{t+1}}\right)^{1-\alpha}\right]\left(\frac{I_{t+1}}{K_{t+1}}\right)^{\alpha}\right]\right\}, \tag{14}$$

and

$$1 \quad = \quad \mathbf{E}_t\left\{\Gamma_{t+1}\left(1 \;+\; r_{t+1} \;-\; \varphi\frac{B_{t+1}}{Y_{t+1}}\right)\right\}, \tag{15}$$

from maximizing (2) subject to (5) – (12) and $K_{t+1},\; C_t,\; I_t$, and N_t non-negative. The stochastic discount factor, Γ_{t+1}, equals $\beta\,(C_{t+1}/C_t)^{\phi(1-\psi)-1}\,([1 \,-\, N_{t+1}]/[1 \,-\, N_t])^{(1-\phi)(1-\psi)}$ if period utility is given by (4) or

$\beta\,(C_{t+1}/C_t)^{-1}$ when period utility is (3).[11]

Equation (13) equates the marginal rate of substitution between leisure and consumption to the marginal product of labor gross of the response of the risk premium to a change in employment. Along an equilibrium path, employment is higher than under perfect international capital mobility because the risk premium produces a negative income effect. Hence, the international bond's risk premium ties demand for foreign assets to the marginal rate of substitution between leisure and consumption. This is independent of period utility being separable or non-separable in consumption and leisure.

Equation (14) sets the marginal cost of increasing date t investment equal to the expected discounted benefit of the extra unit of capital available at date $t+1$. The extra unit of capital contributes to greater production gross of the risk premium, higher depreciation, and smaller adjustment costs. This is the expected return to an additional unit of capital and is the right hand side of the Euler equation (14).

The Euler equation (15) describes optimality in the international bond market. It states that the unit of consumption foregone by holding one more bond is equal to the expected discounted benefit of holding that bond. The benefit includes the world real interest rate, r_{t+1}, net of the risk premium, $-\varphi(B_{t+1}/Y_{t+1})$, which moves endogenously with the bond-output ratio.

It is possible to price the risk premium. Excess returns on B_{t+1} are generated by

$$\mathbf{E}_t\left\{r_{t+1}\ -\ r_{F,t+1}\right\} \quad = \quad \varphi\left[\mathbf{E}_t\left\{\frac{B_{t+1}}{Y_{t+1}}\right\}\ +\ \mathbf{E}_t\left\{1+r_{F,t+1}\right\}Cov_t\left(\Gamma_{t+1},\ \frac{B_{t+1}}{Y_{t+1}}\right)\right]. \tag{16}$$

According to (16), large excess returns on the internationally traded bond occur for two reasons. First, at the margin a creditor small open economy demands a larger excess return to lend more to the rest of the world. Second, excess returns are large when the stochastic discount factor is high at the same time the economy's international borrowing is high. This occurs in states of the world in which the small open economy is a debtor and wants to bring utility forward in time through additional borrowing.[12]

The degree of international capital mobility is parameterized with φ. As seen in (16), a small open

[11]Notice that the optimality conditions $(13)-(15)$ reflect the social planner's problem. We compare and contrast the social planner's and the decentralized market economy's solutions of our small open economy model in the appendix. The appendix shows that for the cases that are most relevant the numerical solutions are very similar.

[12]This idea is responsible for much of the literature that tests equilibrium international asset pricing models (Mark (2001)).

economy that is a debtor (creditor) faces a higher (lower) world real interest rate, r_t, because its risk premium rises (falls) with φ. This drives the steady state bond-output ratio to zero as φ increases. The sense in which the risk premium is endogenous can be seen in that, for any φ greater than zero, it becomes increasingly expensive for a debtor economy to accumulate debt. Hence, greater imperfections in international capital mobility raise the cost of accumulating debt, and so limit the net benefits a small open economy derives from consumption smoothing.

Any candidate equilibrium path must satisfy the optimality conditions (13) – (15), the laws of motion of (5) – (7), and aggregate resource constraint (8) by necessity, given the production function (9) and exogenous shock processes (10) – (12). The transversality conditions $\lim_{j\to\infty} \beta^j \mathbf{E}_t \{\lambda_{K,t+j} K_{t+1+j}\} = 0$ and $\lim_{j\to\infty} \beta^j \mathbf{E}_t \{\lambda_{B,t+j} B_{t+1+j}\} = 0$, provide sufficient conditions for any candidate equilibrium, where $\lambda_{K,t+j}$ and $\lambda_{B,t+j}$ are shadow prices attached to the laws of motion of (5) and (6), respectively. Brock (1982) shows that a combination of the optimality, equilibrium, and transversality conditions establishes uniqueness of the equilibrium for a class of economies that covers our small open economy-RBC model.

3.3 The Numerical Solution and Priors

Our numerical solution proceeds by first, stochastically detrending the aggregate quantity variables (with the exception of N_t) using A_t. Second, we take a first-order Taylor expansion around the deterministic steady state of the stochastically detrended optimality and equilibrium conditions. Next, we arrange the results of the linearization into a two-sided linear vector stochastic difference equation for $\mathcal{M}_{t+1} = [\widetilde{K}_{t+1} \;\; \widetilde{B}_{t+1}]'$, where, for example, $\widetilde{K}_{t+1} = K_{t+1} A_t^{-1}/K^* - 1$ and K^* denotes the steady state value of capital. We conjecture the solution

$$\mathcal{M}_{t+1} \quad = \quad \mu_{\mathcal{M}}\mathcal{M}_t \quad + \quad \mu_{\mathcal{Z}}\mathcal{Z}_t \tag{17}$$

of the linearized system and follow Zadrozny (1998) to compute a solution.[13] The matrices $\mu_{\mathcal{M}}$ and $\mu_{\mathcal{Z}}$ of (17) are two-by-two and two-by-three matrices of coefficients to be determined by our numerical solution and $\mathcal{Z}_t = [\varepsilon_t \;\; \widetilde{g}_t \;\; \widetilde{q}_t]'$. The vector of state variables $\mathcal{S}_t = [\mathcal{M}_t' \;\; \mathcal{Z}_t']'$ drives the "flow" vector

$$\mathcal{C}_t \quad = \quad \pi_{\mathcal{S}}\mathcal{S}_t, \tag{18}$$

where $\mathcal{C}_t = [\widetilde{C}_t \;\; \widetilde{I}_t \;\; \widetilde{N}_t \;\; \widetilde{Y}_t]'$ and $\pi_{\mathcal{S}}$ is a four-by-five matrix.

[13]Zadrozny employs an eigenvalue method of undetermined coefficients – the elements of $\mu_{\mathcal{M}}$ – to solve the AR part of the linear vector stochastic difference equation of $\mathcal{M}_{t+1}$. Given this solution, the forward-looking moving average component of the difference equation imposes linear restrictions on the elements of the matrix $\mu_{\mathcal{Z}}$.

Our simulation exercises adapt the Bayesian methods of DeJong, Ingram, and Whiteman (1996). This requires us to calibrate prior distributions for the parameters of our small open economy-RBC model. Since the prior distributions induce probability distributions for the moments of interest (the tests of the PVM) implied by the theoretical model, we can measure the fit of the theoretical model to the actual data.

Priors for $\gamma,\ \sigma_\varepsilon,\ g^*,\ \rho_g\ ,\ \sigma_\eta,\ q^*,\ \rho_q$, and σ_ξ are based on sample observations. We impose a degenerate prior on the deterministic growth rate of technology, γ, calibrating it to 0.0024, the sample mean of $\Delta ln[A_t]$. The prior of its standard deviation, σ_ε, is assumed to be normal, centered on its sample mean of 0.0120 with a 95 percent coverage interval of [0.0115, 0.0125]. The sample mean of the government spending-output ratio, 0.2326, serves as the mean of the prior of g^*, while the means of the priors of the slope coefficient and standard deviation of the fiscal shock are calibrated from the relevant AR(1) estimates, $[\rho_g\ \sigma_\eta]' = [0.9923\ 0.0127]'$. This gives 95 percent coverage intervals for $g^*,\ \rho_g$, and σ_η of [0.2062, 0.2593], [0.9609, 0.9983], and [0.0121, 0.0133], respectively. These intervals are drawn from normal distributions, except for ρ_g whose prior is based on a log-normal distribution.

The priors of the parameters of the exogenous world real interest rate process are set in the same way. The mid-point of the prior of q^* is set at 0.0071 (or 2.87 percent on an annual basis), $[\rho_q\ \sigma_\xi]' = [0.9076\ 0.0040]'$ is calibrated from AR(1) estimates of q_t, and normal distributions produce the 95 percent coverage intervals for q^*, ρ_q, and σ_ξ of [0.0058, 0.0084], [0.8580, 0.9567], and [0.0035, 0.0045], respectively.

Priors for the technology and utility function parameters follow standard RBC calibration practice. We center the prior of capital's share of income, θ, on 0.35 and draw its 95 percent coverage interval, [0.3201, 0.3802], from a normal distribution. Likewise, the prior of the depreciation rate, δ, is given by a normal distribution with a median of 0.02, which yields a 95 percent coverage interval of [0.0149, 0.0250]. The prior of the utility function parameter ϕ is chosen to be consistent with the steady state of optimality condition (13). This centers the prior of ϕ at 0.3716. Along with a normal distribution, this gives a 95 percent coverage interval of [0.3498, 0.3941]. We choose a mid-point for the risk aversion parameter ψ of two to be consistent with Mendoza (1991a). The 95 percent coverage interval of ψ drawn from a normal distribution is [1.5025, 2.5100].

We choose the prior of the installation cost (on the flow) of new capital parameter, α, to be consistent with the international RBC literature (e.g., Mendoza (1991a), Baxter and Crucini (1993), Correia, Neves, and Rebelo (1995), and Mendoza and Tesar (1998)). The first moment of α, 0.0503, and the 95 percent coverage interval, [0.0301, 0.0703], are drawn from a normal distribution.

The literature provides little guidance for choosing a prior for the risk premium parameter φ. In our canonical RBC model, capital is perfectly mobile internationally, suggesting that $\varphi = 0.0$. However, in order to avoid the well-known problems associated with a unit root in the bond accumulation process (e.g., Mendoza (1991a), Rebelo, Correia, and Neves (1995), Senhadji (1997), and Schmitt-Grohé and Uribe (2003)) we cannot set $\varphi = 0$.[14] Thus, we assume that the risk premium is "small", just one basis point at an annual rate (which implies $\varphi = 0.000071$) at the steady state, in this case.

Under imperfect international capital mobility, we calibrate the prior for φ to the Canadian data. Estimates reported by Clinton (1998) and Fung, Mitnick, and Remolona (1999) are consistent with risk premia anywhere between 10 to 90 basis points at an annual rate on medium to long-term Canadian bonds in international financial markets. We fix φ in the middle of this range, implying a risk premium of 50 basis points. This yields $\varphi = 0.0035$ at the mean of the Canadian bond-output ratio (about -0.35). We also assume that the risk premium is about 75 basis points at the top end of the 95 percent coverage interval of φ and 25 basis points at the low end. Given a normal distribution, these imply a 95 percent coverage interval for φ of [0.0019, 0.0052].[15]

Under this calibration, the effects of the risk premium are similar to those of Backus, Kehoe, and Kydland (1992), as well as Bergin's (2001) estimate of the cost arising from current account flows. Our risk premia imply a loss of less than 0.03 percent of the net exports-output ratio at the sample mean of our Canadian data. Cole and Obstfeld (1991) argue that even small barriers to perfect capital mobility such as these may be sufficient to outweigh the benefits associated with consumption smoothing.

[14] This problem does not necessarily arise for non-linear solution methods when $r^* < (1-\beta)/\beta$, but does arise for linear approximate solution methods irrespective of the relationship of r^* and β.

[15] The calibration of φ ignores higher order moments of the data, which suggest a larger risk premium. Using the unconditional version of the excess return generating equation (16), implies a φ equal to 0.0104 and 0.0162 for the separable and log period utility function (3) and the non-separable power utility function (4), respectively. At the steady state, these values of φ imply risk premiums of 150 and 230 basis points, respectively, which is one half or more of the world real interest rate we calculate from the data. These seem much too large for Canada, so we opt for the range of values noted above.

4. Understanding Rejections of the Present-Value Model of the Current Account

Given the approximate linear equilibrium law of motion of the state vector (17), the associated flow relationship (18), and priors of our model, we construct distributions of the test statistics of the PVM from artificial time series. These series are generated by Bayesian Monte Carlo experiments.

4.1 Monte Carlo Strategy

We measure the fit of our small open economy-RBC model using PVM test statistics (described in section 2) as our "moments" of interest. Monte Carlo experiments generate $\mathcal{J} = 5000$ replications of the multivariate artificial time series $\{\mathcal{W}_{\mathcal{T},\mathcal{K}}\}_{\mathcal{K}=1}^{140}$, where $\mathcal{T}$ denotes an object conditional on the RBC model.[16] An unrestricted VAR(4) is estimated on the synthetic data to compute $\mathcal{H}_{\mathcal{T}}$, its Wald statistic, and $CA_{f,\mathcal{T},t}$.[17] In addition, a regression of $\mathcal{CA}_{\mathcal{T},t+1}$ on $\mathcal{W}_{\mathcal{T},t-1}$ is used to test the orthogonality condition. The empirical distributions, denoted $\mathcal{E}$, are posterior distributions of the actual data's PVM moments. We produce these $\mathcal{E}$ distributions using Bayesian simulation techniques described by Geweke (1999a). Geweke (1999b) argues that to measure the fit of a dynamic stochastic general equilibrium model to the data requires an explicit statistical model.[18] In our case, the statistical model is the unrestricted VAR or the orthogonality regression. The statistical models link population PVM moments implied by $\mathcal{T}$ distributions to the actual data's $\mathcal{E}$ distributions of the same moments.

An obvious requirement for our analysis is a metric of "closeness". Consider our comparison involving the individual elements of $\mathcal{H}$. The distance between $\mathcal{T}(\mathcal{H}_i)$ and $\mathcal{E}(\mathcal{H}_i)$ is measured by the difference in their ensemble averages, $\mathcal{H}_{\mathcal{T},i}$ and $\mathcal{H}_{\mathcal{E},i}$, normalized by the latter ensemble's standard deviation. DeJong, Ingram, and Whiteman (1996) refer to this as the standardized difference of means (SDM) statistic. Since the SDM statistic gauges how close the probability distribution of $\mathcal{T}(\mathcal{H}_i)$ is to $\mathcal{E}(\mathcal{H}_i)$ for each element of $\mathcal{H}$, a large value for this $t-$ratio-like statistic indicates our small open economy-RBC model fits the actual data poorly.

A second comparison of the $\mathcal{T}$ and $\mathcal{E}$ distributions examines the validity of the cross-equation

[16] We generate 315 synthetic observations, but drop the first 175 observations to remove dependency to initial conditions.

[17] The simulations use the mean of synthetic realizations of $\{r_{\mathcal{K}}\}_{\mathcal{K}=1}^{140}$ to calibrate the non-stochastic world real interest rate.

[18] Canova (1995) develops a Bayesian approach to model evaluation that treats the theoretical model in a similar fashion, as discussed by DeJong, Ingram, and Whiteman (1996) and Geweke.

restrictions, as measured by non-parametric estimates of the probability densities of (i) the Wald statistic of $\mathcal{H}_{\mathcal{T}}$ and (ii) the LM statistic, $T \times R^2$, of the orthogonality condition.[19] We display plots of the estimated $\mathcal{T}$ and $\mathcal{E}$ densities, which should be close and overlap if the RBC model is a good fit to the data. The confidence interval criterion (CIC) statistic developed by DeJong, Ingram, and Whiteman (1996) measures the intersection of these distributions. Given a $1-\omega$ percent confidence level, the CIC measures the fraction of $\mathcal{T}$ and $\mathcal{E}$ distributions that reside in an interval from the (lower) 0.5ω quantile $\mathcal{L}$ to the (upper) $1-0.5\omega$ quantile $\mathcal{U}$.[20] The larger the CIC the better the theoretical RBC model fits the actual data. In their study of a proto-type RBC model, DeJong, Ingram, and Whiteman regard CIC statistics of 0.3 or greater as indicating support for the theoretical model.

We focus on the orthogonality condition of the PVM that accounts for a transitory demand shock, rather than the orthogonality condition of the exact PVM, since the latter will always be strongly rejected by the small open economy-RBC model no matter its configuration. This is not surprising in light of the RBC model's structure and linearized solution. We discuss this point in the appendix.

4.2 The Canonical RBC Model

We begin with a "canonical" version of our small open economy-RBC model outlined in section 3 that embodies the restrictions of the PVM of section 2. The restrictions produce a prototype small open economy-RBC model with perfect international capital mobility, a constant world real interest rate, and a country-specific technology shock that is the dominant source of business cycle fluctuations. We show that the canonical model generates predictions that *are* consistent with the implications of the PVM of the current account, and hence could *not* be consistent with the actual data.

As noted above, by design, the canonical model rules out several of our "usual suspects", such as fiscal policy shocks. Seven parameter restrictions on the full model imply the canonical model. The household discount

[19]Non-parametric density estimation uses the normal kernel $\mathcal{N}(x) = \exp\{-0.5x^2\}/\sqrt{2\pi}$ where x is the distance between two points in the density. The density, $d(x) = (1/\mathcal{J})\sum_{i=1}^{\mathcal{J}}\mathcal{N}([\mathcal{X}-\mathcal{X}_i]/h)$, simply plugs in the kernel, where h is the bandwidth or smoothing parameter of the density and $\mathcal{X}_i$ is the ith Monte Carlo replication of either the Wald statistic of $\mathcal{H}_{\mathcal{T}}$ or the LM statistic, $T \times R^2$. We follow Silverman (1986) to compute $d(x)$ (see pp. $43-48$ in particular).

[20]This is $CIC_x = (1-\omega)^{-1}\int_{\mathcal{L}}^{\mathcal{U}}\mathcal{T}(x_j)dx_j$, where x is either the LM or Wald statistic. DeJong, Ingram, and Whiteman normalize the CIC by $1-\omega$ so that it equals $\int_{\mathcal{L}}^{\mathcal{U}}\mathcal{E}(x)dx_j$. We always set $\omega = 0.10$.

factor β is set equal to $1/(1+q^*)$, approximating the permanent income hypothesis (PIH) restriction. Unless otherwise noted, this restriction is always in effect. The restrictions $g^* = 0.0$, $\rho_g = 0.0$, and $\sigma_\eta = 0.0$ remove the fiscal shock from the model. Notice also that, because the PVM assumes the data generating process is a VAR in ΔNY_t and CA_t, our synthetic DGP must possess two fundamental disturbances. The first is naturally the country-specific unit root technology shock. We assume that the second shock in the canonical RBC model is ξ_t, the innovation to the exogenous component of the world real interest rate, q_t. However, to maintain the spirit of the basic intertemporal model, we make the innovation to q_t "small", setting the point mass prior of σ_ξ to 2.110×10^5 (its calibrated value is 0.004) and eliminate any persistence in q_t with $\rho_q = 0.0$. Also, as noted above, we cannot have international capital mobility being literally perfect. Therefore, we employ the degenerate prior of $\varphi = 0.000071$, which is consistent with a steady state risk premium of one basis point (at an annual rate).

The upper panels of figure 2 plot the distribution of the LM statistics from the $\mathcal{T}$ and $\mathcal{E}$ densities. There are two important elements to evaluating these densities, the comparison between: (1) the synthetic data and the theoretical orthogonality prediction (how far to the left of the sample test statistic is the $\mathcal{T}$ density?), and (2) the synthetic data and the actual data (how much do the $\mathcal{T}$ and $\mathcal{E}$ densities overlap?).

First, we see that the canonical RBC model has little problem satisfying the theoretical orthogonality prediction. Under either separable or non-separable utility, the model produces $\mathcal{T}$ densities of the LM statistic that are mostly to the left of the sample LM statistic. Thus, the canonical RBC model is consistent with the theoretical prediction that lags of ΔNY_t and CA_t have no power to forecast $\mathcal{CA}_{t+1}$. The match to this theoretical prediction is stronger for separable utility than for non-separable utility, as manifested in the CIC statistics of 0.33 for the model with separable utility and 0.84 for the non-separable utility model. This suggests that the assumption of separable utility is in part responsible for rejections of the PVM that we observe in the actual data.

The probability densities of the Wald statistic appear in the bottom panels of figure 2. The version with separable utility generates a distribution for $\mathcal{H}$ that resembles the PVM's theoretical cross-equations restrictions, as indicated by the fact that the density lies to the left of the sample Wald statistic. In this particular case, the overlap with the $\mathcal{E}$ distribution is substantial, as seen from the CIC statistic of 0.68. With non-separable utility, the $\mathcal{T}$ density of the Wald statistic is close to uniform, and hence matches neither the basic PVM theory nor the actual data well at all. Its CIC statistic is 0.03.

Table 1 displays the ensemble means of the elements of $\mathcal{H}_{\mathcal{T}}$ and their SDM statistics. As shown in column 1, the canonical RBC model with the separable and log period utility function (3) yields estimates of $\mathcal{H}_{\mathcal{T},i}$ that are all close to zero, except the fifth element which, at 0.97, is close to the theoretical value of unity. The SDM statistics indicate that the canonical RBC model does not appear to match the actual data very well. For example, $\mathcal{H}_{\mathcal{T},5}$ is nearly two standard deviations above its empirical counterpart.

The canonical RBC model with the non-separable period utility function (4) fails to replicate the theoretical predictions of the PVM and to resemble the actual data. In this case, there is excess sensitivity of the current account to contemporaneous and lagged changes in net output, as $\mathcal{H}_{\mathcal{T},1} = -1.66$ and $\mathcal{H}_{\mathcal{T},2} = -1.34$ are more than six standard deviations below $\mathcal{H}_{\mathcal{E},1}$ and $\mathcal{H}_{\mathcal{E},2}$. Also, the estimate of $\mathcal{H}_{\mathcal{T},5}$ $(= 1.02)$ indicates that allowing for non-separable utility does little to move the canonical RBC model closer to the data. Hence, in what follows we focus on versions of the RBC model with the separable and log period utility function (3).

The final piece of evidence about the fit of the canonical RBC model appears in the top row of figure 3. This displays the actual Canadian current account (the solid plot) and the fifth and 95th percentiles of the Bayesian Monte Carlo pointwise probability bands (the dot-dash plots) of the PVM forecast of the current account from the canonical RBC model with separable utility, the left-side window, and with non-separable utility, the right-side window.[21] The model with separable preferences matches the actual PVM forecasts because the 90 percent probability bands of $CA_{f,\mathcal{T}}$ contain zero except for a brief episode in the early 1980s and then during the late 1980s and early 1990s. With non-separable utility, the actual Canadian current account lies above the 90 percent probability bands of $CA_{f,\mathcal{T}}$ except, once again, during the late 1980s and early 1990s. Thus, the fit of the canonical model to the actual Canadian current account is poor irrespective of the restrictions on utility.

In sum, we have shown that in artificial data generated by the canonical RBC model, the familiar present-value restrictions on the current account and net output are typically not rejected. This finding ought not to be a surprise in retrospect given the restrictions on the canonical model. The canonical model does a relatively poor job explaining the actual current account data, as our results in section 2 would suggest. This indicates that

[21] The 90 percent probability bands use the two largest principal components of the covariance matrix of the $CA_{f,\mathcal{T}}$ ensemble.

the true DGP for the current account is more complex than the canonical RBC model. In the next section, we see if incorporating our "usual suspects" into the canonical RBC model improves its fit to the actual data.

4.3 The Usual Suspects

We investigate the impact of each of the usual suspects by introducing them one-by-one into the canonical RBC model with the separable utility function (3). The suspects are transitory shocks to fiscal policy and the world real interest rate, and imperfect international capital mobility. When we introduce a suspect the rest of the priors are the same as for the canonical model with separable, log period preferences.

The transitory shock to fiscal policy is specified by the univariate AR(1) process (11). These highlight the role of a country-specific demand shock for current account fluctuations. We set up the fiscal policy experiment with priors on the parameters of the AR(1) government spending-output ratio process, its steady state, g^*, the slope coefficient ρ_g, and the standard deviation σ_η. These priors reflect observation from Canadian data and are described in section 3.3. Movements in the identified transitory component of government spending exhibit a substantial amount of persistence and volatility. The prior on ρ_g is a near unit root and its 95 percent coverage interval implies that the half-life of an innovation to g_t ranges from 3.5 to more than 100 years. Transitory shocks to fiscal policy possess more volatility than either technology shocks or shocks to the exogenous component of the world real interest rate. Innovations to g_t, η_t, are more than 3 times as volatile as innovations to q_t.

In the interest rate experiment, shocks to q_t focus attention on the response of the current account to changes in the rate at which the rest of the world is willing to move consumption intertemporally. Although the exogenous component of the world real interest rate is persistent, it is not as persistent as shocks to fiscal policy. The 95 percent coverage interval of the prior of ρ_q implies that the half-life of a shock to q_t is between one year and four years. The prior on the standard deviation of the innovation of the exogenous component of the world real interest rate, σ_ξ, makes this shock the least volatile of the model.

Figure 4 depicts densities of the LM (upper panels) and Wald (lower panels) statistics of the fiscal policy and exogenous world real interest shock experiments. The fiscal policy experiment yields a $\mathcal{T}$ density of the LM statistic that has the most overlap and is closest to the $\mathcal{E}$ density of the LM statistic (top left), with a CIC statistic of 0.51. This statistic is 0.35 in the exogenous world real interest rate experiment (top right). The $\mathcal{T}$ and

$\mathcal{E}$ densities of the Wald statistic for the interest rate and fiscal policy experiments generate CIC statistics of 0.93 and 0.47, respectively (lower panels). This suggests that an exogenous shock to the world real interest rate is crucial to explain rejections of the PVM. This is consistent with the estimation work of Bergin and Sheffrin (2000), who report that adding a consumption-based real interest rate to the standard VAR used in present-value tests produces fewer rejections of the PVM's predictions. Our experiments distinguish between different potential sources of interest rate movements, which makes our results helpful to understand the underlying reasons for Bergin and Sheffrin's findings.[22]

The distributions of $\mathcal{H}_{\mathcal{T}}$ in the interest rate and fiscal policy experiments are qualitatively similar to one another, and similar to those of the canonical RBC model. This can be seen from columns 3 and 4 of table 1. The only substantive difference appears in the all-important fifth element of this vector. The fiscal policy experiment generates an $\mathcal{H}_{\mathcal{T},5}$ of 0.68 and $SDM = 1.33$. In the interest rate experiment, $\mathcal{H}_{\mathcal{T},5} = 1.24$, which is more than two standard deviations greater than its $\mathcal{H}_{\mathcal{E},5}$ counterpart. Thus, the fiscal policy experiment is closer to the actual data according to this test of the PVM.

The 90 percent probability bands that are associated with the $\mathcal{T}$ and $\mathcal{E}$ distributions of CA_f, appear in the bottom row of figure 3 along with the actual Canadian current account. When we consider these current account forecast plots, we see that the world real interest rate experiment generates $\mathcal{T}$ 90 percent probability bands (the dot-dash plot) that almost always contain the actual current account (the solid plot) and often do not cover zero or the $\mathcal{E}$ piecewise 90 percent probability bands (the dots plot). The $\mathcal{T}$ bands associated with the fiscal policy experiment, on the other hand, fail to contain the actual current account, except during the late 1980s and early 1990s (lower left window of figure 3).

As noted in section 3.3, imperfect international capital mobility is identified with the risk premium, $-\varphi B_t/Y_t$. Introducing this risk premium affects the current account of a debtor small open economy such as Canada for two reasons. First, the interest rate it faces, r_t, is higher than the common world real interest rate, q_t. This creates a flow of asset income out of the small open economy, generating current account fluctuations as households smooth consumption. Second, a larger risk premium is synonymous with a larger debt-output ratio

[22]Schmitt-Grohé (1998) finds that terms-of-trade and U.S. business cycle shocks help to explain aggregate fluctuations in Canada. The outcomes of our experiments are consistent with her results to the extent that our calibration of q_t captures the shocks she identifies.

(more negative B_t/Y_t) for the economy, which implies a smaller current account on average. Thus, the current account exhibits more sensitivity to any shock that affects the debt-output ratio.

As seen in the right-side windows of figure 5, the imperfect international capital mobility experiment produces $\mathcal{T}$ densities of the LM and Wald statistics with CICs of 0.32 and 0.66, respectively. These lie in the range of those of the world interest rate and fiscal policy experiments.

The ensemble averages of $\mathcal{H}_{\mathcal{T}}$ and the SDM statistics of the imperfect international capital mobility experiment appear in the final column of table 1. These statistics indicate that the elements of $\mathcal{H}_{\mathcal{T}}$ are much further away from their empirical counterparts than in the canonical RBC model, and in the fiscal policy or interest rate experiments. There is evidence of excess sensitivity of the current account to changes in net output, as seen from the fact that $\mathcal{H}_{\mathcal{T},1}$, $\mathcal{H}_{\mathcal{T},2}$, and $\mathcal{H}_{\mathcal{T},3}$ are negative (and more than 25 standard deviations below the same parameters of the $\mathcal{E}$ distribution). Also, this experiment generates a negative response of the PVM current account forecast to contemporaneous current account movements, as indicated by $\mathcal{H}_{\mathcal{T},5} = -7.70$. All the lagged revisions to this forecast are large, $[\mathcal{H}_{\mathcal{T},6} \;\; \mathcal{H}_{\mathcal{T},7} \;\; \mathcal{H}_{\mathcal{T},8}]' = [15.97 \;\; \text{-}4.24 \;\; \text{-}5.63]'$, suggesting substantial volatility in the 90 percent probability bands. The left-side window of figure 5 quite clearly reflects the additional variability in the current account induced by incorporating imperfect capital mobility.

Thus, there is mixed evidence on the importance of departures from perfect capital mobility in explaining rejections of the PVM. According to the LM and Wald tests, imperfect capital mobility is nearly as important a suspect as transitory shocks to fiscal policy and shocks to world interest rates. However, the forecasted current account from the model with imperfect capital mobility exhibits considerably more variability than is consistent with the data.

4.4 Changing Preferences: What if $r^* < (1-\beta)/\beta$?

Up to this point, we have imposed the PIH restriction, $q^* = (1-\beta)/\beta$. This restriction, or more generally $r^* \geq (1-\beta)/\beta$, is problematic when studying the limiting distributions of small open economies, if φ is identically zero. Under these conditions, domestic households are sufficiently patient that the small open economy eventually accumulates the entire universe of wealth. In this case, the joint limiting distribution of the endogenous variables is not defined.[23] This problem can be rendered moot if it is assumed that $r^* < (1 - \beta)/\beta$, even when $\varphi = 0$. In this case, the future discounted marginal utility of consumption converges in the limit, which produces a finite level of consumption and bond value. Therefore, it is potentially useful to study cases in which $r^* < (1 - \beta)/\beta$.

Table 2 contains the ensemble averages of the elements of $\mathcal{H}_{\mathcal{T}}$ and their SDM statistics for (i) our canonical RBC model and (ii) the alternative that allows for imperfect international capital mobility, given $r^* < (1 - \beta)/\beta$.[24] Densities of the LM and Wald statistics and plots of the 90 percent confidence bands for these experiments appear in figures 6 and 7.

The results in table 2 for the canonical RBC model given $r^* < (1 - \beta)/\beta$ show the model fails to match the actual data and the theoretical predictions of the PVM. For example, $\mathcal{H}_{\mathcal{T},1} = 0.48$ indicates excess sensitivity of the current account to movements in changes in net output, while $\mathcal{H}_{\mathcal{T},5} = 0.36$ is well below the theoretical restriction that this element of $\mathcal{H}$ equals one. This is reflected in the 90 percent confidence bands of the left-side window of figure 6. The bands are always below the actual current account. The restriction that the steady state world real interest rate is below the subjective rate of time preference gives the small open economy incentive to bring consumption forward, thereby leading to a lower current account balance. The left-side window of figure 6 suggests that such modifications to the canonical RBC model do not bring the current account forecast closer to the actual Canadian data.

The densities of the LM and Wald statistics provide further evidence on the effects of relaxing the PIH restriction. For the LM test, both specifications of the model yield CICs of about one-third. For the Wald test, relaxing the PIH restriction worsens the fit, as the CIC falls to only 0.03. Thus, the change in preferences afforded by $r^* < (1 - \beta)/\beta$ moves the model further away from the PVM predictions, but not in such a way as to move it closer to the actual data.

Finally, we conduct an experiment that assumes $r^* < (1 - \beta)/\beta$ and allows for imperfect international

[23] Chamberlain and Wilson (2000) discuss this problem in the context of a household saving problem. Aiyagari (1994) provides intuition and suggests a resolution, while chapter 14 of Ljungqvist and Sargent (2000) is a good introduction.

[24] We impose $\beta_j = 1/(1 + q_j^*) - (0.01 \times \breve{\beta}_j)$ where the prior of q^* is described in section 3.3 and $\breve{\beta}$ is normally distributed with mean 0.9940 and 95 percent coverage interval [0.9920, 0.9961].

capital mobility. Qualitatively, the results are little changed from those we presented in the previous section. The fit of the model in terms of the densities of the LM and Wald statistics is reasonable as shown in the right-side windows of figure 7. The problem is that the PVM current account forecast exhibits excess sensitivity to lags of net output changes and a negative response to contemporaneous and lagged current account movements. This evidence is found in the right-most column of table 2 and the left-side window of figure 7. Thus, our results show that even fairly small and conservatively calibrated imperfections in capital mobility induce too much volatility in the current account, and that this is true irrespective of the restrictions on r^* and β.

4.5 Caveats

As with any applied study of this topic, our results could be affected by plausible extensions of our framework. It may help to find such extensions, because our simulations fail to achieve *complete* success in matching the data. Our RBC model has no monetary sector. This allows us to avoid the well-known difficulties associated with modeling money demand, the behavior of price setters, and the instruments, targets, and objectives of monetary policy in the open economy. The omission of a monetary sector from our model is not likely to affect our results much, and is surely a reasonable starting point, given that we expect monetary shocks to have only a minor effect on real variables beyond the short-run, while PVM tests place weight on the medium and long-run as well as the short-run. It might also be argued that non-monetary foreign demand shocks ought to play more of a role in our small open economy model. In earlier versions of this paper, we appended to the law of motion of the international bond, equation (6), an additive term that we interpreted as a shock to foreign demand. We found that the effect of this shock was negligible, and hence dropped it from our list of suspects.[25] Adding a non-tradables sector to the small open economy model could also improve the fit to the data. However, there is some reason to suspect that this channel might have only minor effects too. When the Glick and Rogoff (1995) model is extended to include non-traded goods, the responses of both the current account and investment to relative price shocks (the terms of trade and nominal exchange rates) are insignificant (e.g., İşcan (2000)). Finally, one might consider incorporating physical trading costs of the type identified by Obstfeld and Rogoff

[25] Bergin's (2003) variance decomposition for Canada shows that at the 10 quarter horizon the sum of money demand and supply shocks account under 20 percent of current account variability while foreign demand shocks account for seven percent.

(2000) as going a long way to resolve several major puzzles on the real side of international macroeconomics. However, the results of the two-country complete markets model of Backus, Kehoe, and Kydland (1992) suggest that trading frictions may not be able to improve the fit of the intertemporal model to observed current account fluctuations. Continued progress on these fronts will undoubtedly shed more light on the questions that have been raised in this literature.

5. Conclusion

We study the importance of various explanations for the poor empirical performance of a basic intertemporal model of the current account, the present value model. First, we confirm the results of existing papers that reject the cross-equation restrictions and orthogonality conditions of the present-value model, in our case using a sample of post-war Canadian data. To understand these empirical results, we construct a small open economy-real business cycle model. We show that a “canonical” version of the model *is* consistent with the theoretical predictions of the present-value model, and hence is *not* consistent with the actual data.

The usual suspects we study, non-separable preferences, shocks to fiscal policy and the real interest rate, and imperfect international capital mobility, are portrayed in the literature as potential explanations of rejections of the present-value model. We conduct Bayesian Monte Carlo experiments to generate evidence about the culpability of our suspects. Although each is important to some extent, world real interest rate shocks appear to do the most to move the model closer to the data. Thus, although the attention paid to transitory movements in domestic fiscal policy to explain the current account has been appropriate, it may also have missed other important factors. Given, our finding that transitory shocks to the exogenous component of the world real interest matter more at business cycle horizons, future research should perhaps look for additional underlying macroeconomic factors that drive the current account. Finally, our results suggest that the sources of current account movements have more of a common, cross-country component than is perhaps usually suspected, at least in the case of Canada.

The intuition for our results rest with households hedging against country-specific permanent income shocks through the current account. Any transitory shock to consumption generates current account fluctuations independent of movements in permanent income and hence could produce rejections of the present-value model. Shocks to the world real interest rate, for example, produce these sorts of current account fluctuations, whether they come from an endogenous country-specific component, an exogenous world shock, or an endogenous common world component. This is especially important because ever since Cole and Obstfeld (1991) pointed out that even small imperfections in international capital markets can wipe out the (consumption smoothing) benefits of international portfolio diversification, the sources and causes of such imperfections have eluded researchers. We hope this paper invigorates this research agenda.

References

Ahmed, S., 1986. Temporary and permanent government spending in an open economy: Some evidence for the United Kingdom, Journal of Monetary Economics 17, 197 – 224.

Ahmed, S., Rogers J.H., 1995. Government budget deficits and trade deficits: Are present value constraints satisfied in long-term data?, Journal of Monetary Economics 36, 351 – 374.

Aiyagari, S.R., 1994. Uninsured idiosyncratic risk and aggregate saving, Quarterly Journal of Economics 103, 659 – 684.

Backus, D.K., Kehoe, P.J., Kydland, F.E., 1992. International real business cycles, Journal of Political Economy 100, 745 – 775.

Baxter, M., Crucini, M.J., 1993. Explaining saving-investment correlations, American Economic Review 83, 416 – 436.

Bergin, P.R., 2003. Putting the 'new open economy macroeconomics' to a test, Journal of International Economics, forthcoming.

Bergin, P.R., 2001. How well can we explain the current account in terms of optimizing behavior, manuscript, Department of Economics, University of California, Davis.

Bergin, P.R., Sheffrin, S.M., 2000. Interest rates, exchange rates, present value models of the current account, Economic Journal 110, 535 – 558.

Blankenau, W., Kose, M.A., Yi, K.M., 2001. Can world real interest rates explain business cycles in a small open economy?, Journal of Economic Dynamics and Control 25, 867 – 889.

Brock, W.A., 1982. Asset prices in a production economy, in: McCall, J.J. (Ed.). The Economics of Information and Uncertainty, University of Chicago Press, Chicago, pp. 1 – 43.

Campbell, J.Y., 1987. Does saving anticipate declining labor income: an alternative test of the permanent income hypothesis, Econometrica, 55, 1249 – 1273.

Campbell, J.Y., R. Shiller, 1987. Cointegration and tests of present value models, Journal of Political Economy 93, 1062 – 1088.

Canova, F., 1995. Sensitivity analysis and model evaluation in simulated dynamic general equilibrium economies, International Economic Review 36, 477 – 501.

Chamberlain, G., C.A. Wilson, 2000. Optimal intertemporal consumption under uncertainty, Review of Economic Dynamics 3, 365 – 395.

Clinton, K., 1998. Canada-U.S. long-term interest differentials in the 1990s, Bank of Canada Review (Spring), 17 − 38.

Cole, H., Obstfeld M., 1991. Commodity trade and international risk sharing: How much do financial markets matter?, Journal of Monetary Economics 28, 3 − 24.

Correia, I., Neves, J.C., Rebelo, S., 1995. Business cycles in a small open economy, European Economic Review 39, 1089 − 1113.

DeJong, D.N., Ingram, B.F., Whiteman, C.H., 1996. A Bayesian approach to calibration, Journal of Business and Economic Statistics 14, 1 − 9.

Fung, B., Mitnick, S., Remolona, E., 1999. Uncovering inflation expectations and risk premiums from internationally integrated financial markets, Working Paper 99 − 6, Bank of Canada, Ottawa, Canada.

Geweke, J., 1999*a*. Using simulation methods for Bayesian econometric models: Inference, development, communication, Econometric Reviews 18, 1 − 73.

Geweke, J., 1999*b*. Computational experiments and reality, manuscript, Department of Economics, University of Minnesota.

Ghosh, A.R., 1995. International capital mobility amongst the major industrialized countries: Too little or too much?, Economic Journal 105, 107 − 128.

Ghosh, A.R, Ostry, J.D., 1997, "Macroeconomic uncertainty, precautionary saving, the current account, Journal of Monetary Economics 40, 121-139.

Glick, R., Rogoff, K., 1995. Global versus country-specific productivity shocks and the current account, Journal of Monetary Economics 35, 159 − 192.

Gruber, J., 2002, "A Present Value Test of Habits and the Current Account", manuscript, Federal Reserve Board.

Hercowitz, Z., 1986. The real interest rate and aggregate supply, Journal of Monetary Economics 18, 121 − 145.

İşcan, T.B., 2000. The terms of trade, productivity growth, the current account, Journal of Monetary Economics 45, 587 − 611.

Ljungqvist, L., Sargent, T.J., 2000. Recursive Macroeconomic Theory, MIT Press, Cambridge, MA.

Mark, N.C., 2001. International Macroeconomics and Finance: Theory and Econometric Methods. Blackwell Publishers, New York, NY.

Mendoza, E.G., 1991*a*. Real business cycles in a small open economy, American Economic Review 81, 797 − 818.

Mendoza, E.G., 1991*b*. Capital controls and the gains from trade in a business cycle model of a small open economy, IMF Staff Papers 38, 480 − 504.

Mendoza, E.G., Tesar, L.L., 1998. The international ramification of tax reforms: Supply-side economics in the global economy, American Economic Review 88, 226 − 245.

Nason, J.M., J.H. Rogers, 2002. Investment and the current account in the short-run and long-run, Journal of Money, Credit, Banking, 34, 967 − 986.

Obstfeld, M., Rogoff, K. 1996, Foundations of International Macroeconomics, The MIT Press, Cambridge, MA.

Obstfeld, M., Rogoff, K., 2000. The six major puzzles in international macroeconomics: Is there a common cause?, in: Blanchard, O.J., Bernanke, B.S., (Eds.)., NBER Macroeconomics Annual, MIT Press, Cambridge, MA, pp. 339 − 389.

Otto, G., 1992. Testing a present value model of the current account: Evidence from the U.S. and Canadian time series, Journal of International Money and Finance 11, 414 − 430.

Schmitt-Grohé, S., 1998. The international transmission of economic fluctuations: Effects of U.S. business cycles on the Canadian economy, Journal of International Economics 44, 257 − 287.

Schmitt-Grohé, S., Uribe, M., 2003. Closing small open economy models, Journal of International Economics, forthcoming.

Senhadji, A., 1997. Sources of debt accumulation in a small open economy, Working Paper no. 97/146, International Monetary Fund, Washington, D.C.

Sheffrin, S.M., Woo, W.T., 1990. Present value tests of an intertemporal model of the current account, Journal of International Economics 29, 237 − 253.

Silverman, B.W., 1986. Density Estimation for Statistics and Data Analysis, Chapman and Hall, Ltd., London, England.

Valderrama, D., 2002. The impact of financial frictions on small open economies: when current account borrowing hits a limit, Working Paper 2002 − 15, Federal Reserve Bank of San Francisco.

Zadrozny, P.A., 1998. An eigenvalue method of undetermined coefficients for solving linear rational expectations models, Journal of Economic Dynamics and Control 22, 1353 − 1373.

Table 1: Tests of the Canonical and Alternative SOE-RBC Models

	Experiment				
	Separable Utility	Non-Separable Utility	Transitory Fiscal Shock	World Interest Rate Shock	Imperfect Capital Mobility
$\mathcal{H}_{\mathcal{T},1}$	-0.20	-1.66	-0.09	0.11	-4.98
	(-1.44)	(-8.64)	(-0.87)	(0.09)	(-25.02)
$\mathcal{H}_{\mathcal{T},2}$	-0.20	-1.34	-0.10	0.11	-15.11
	(-0.88)	(-6.84)	(-0.32)	(0.79)	(-79.16)
$\mathcal{H}_{\mathcal{T},3}$	-0.04	-0.24	-0.04	0.05	-5.73
	(0.21)	(-1.30)	(0.20)	(0.90)	(-42.52)
$\mathcal{H}_{\mathcal{T},4}$	0.03	-0.00	0.02	0.02	-0.04
	(-0.57)	(-0.86)	(-0.69)	(-0.57)	(-1.39)
$\mathcal{H}_{\mathcal{T},5}$	0.97	1.02	0.68	1.24	-7.70
	(1.87)	(-1.97)	(1.33)	(2.39)	(-14.69)
$\mathcal{H}_{\mathcal{T},6}$	0.14	0.12	0.02	-0.16	15.97
	(0.72)	(0.56)	(-0.18)	(-1.66)	(127.29)
$\mathcal{H}_{\mathcal{T},7}$	-0.13	-0.41	-0.06	0.15	-4.24
	(-0.81)	(-3.13)	(-0.25)	(1.59)	(35.57)
$\mathcal{H}_{\mathcal{T},8}$	-0.03	-0.16	-0.05	0.04	-5.63
	(0.77)	(-0.51)	(0.54)	(1.37)	(-52.26)

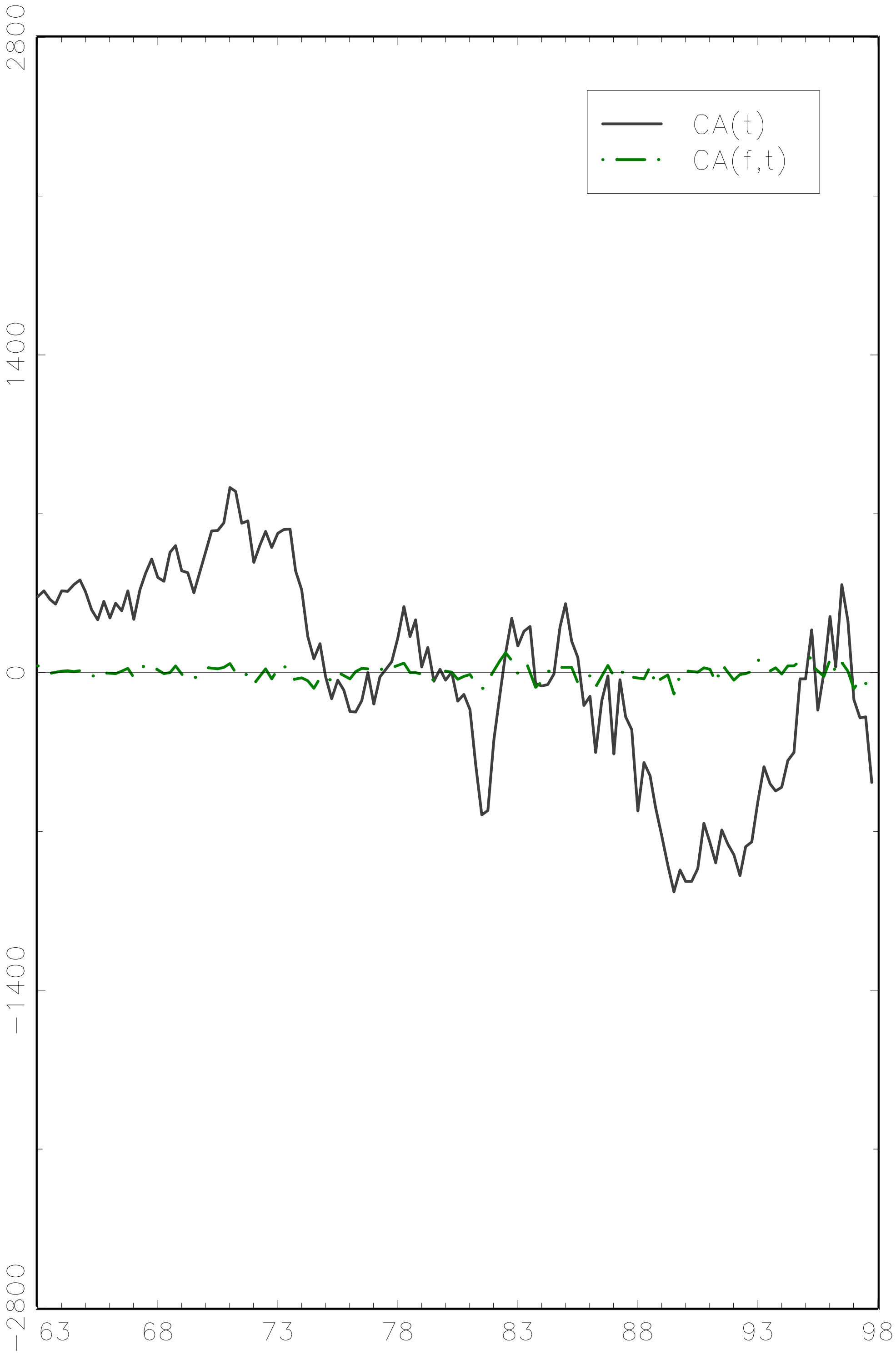
The canonical RBC specification employs log separable utility, $\beta_j = 1/(1+q_j^*)$, point mass priors of $g^* = \rho_g = \rho_q = \sigma_\eta = 0$, $\varphi = 0.000071$, $\sigma_\xi = 0.000021$. The non-separable utility specification employs a prior on the risk aversion parameter of $\psi \in [1.50,\ 2.51]$ centered on two. Imperfect international capital mobility is achieved with the prior $\varphi \in [0.0019,\ 0.0052]$ centered on $\varphi = 0.0035$. The prior on the transitory shock to the government spending-output ratio are $\rho_g \in [0.9609,\ 0.9983]$ and $\sigma_\eta \in [0.0121,\ 0.0133]$. The Monte Carlo experiment with a transitory exogenous world real interest rate is based on the priors $\rho_q \in [0.8580,\ 0.9567]$ and $\sigma_\xi \in [0.0035,\ 0.0045]$. Details about the priors of the model parameters are discussed in section 3.3 of the text. The simulation experiments rely on 5000 replications of 140 artificial observations of ΔNY_t and the CA_t generated by the linearized solution of our small open economy-RBC model over the priors of the model's parameters. The $SDM(\mathcal{H})$ statistics appear in parentheses and are computed as $[\mathcal{H}_{\mathcal{T},i} - \mathcal{H}_{\mathcal{E},i}]/\mathbf{STD}(\mathcal{H}_{\mathcal{E},i})$, $i = 1,\ldots,8$, where $\mathbf{STD}(\mathcal{H}_{\mathcal{E},i})$ is the standard deviation of $\mathcal{H}_{\mathcal{E},i}$.

Table 2: Tests of the Canonical RBC Specification with Smaller β to Force $r^* < (1-\beta)/\beta$

	Experiment	
	Canonical Model	Imperfect Capital Mobility
$\mathcal{H}_{\mathcal{T},1}$	0.48	-0.04
	(1.93)	(-0.66)
$\mathcal{H}_{\mathcal{T},2}$	0.19	-1.89
	(1.20)	(-9.74)
$\mathcal{H}_{\mathcal{T},3}$	0.05	-1.00
	(0.86)	(-7.04)
$\mathcal{H}_{\mathcal{T},4}$	-0.00	-0.01
	(-0.90)	(-0.94)
$\mathcal{H}_{\mathcal{T},5}$	0.36	-1.00
	(0.71)	(-1.88)
$\mathcal{H}_{\mathcal{T},6}$	0.19	2.41
	(1.18)	(18.87)
$\mathcal{H}_{\mathcal{T},7}$	0.09	-0.06
	(1.10)	(-0.18)
$\mathcal{H}_{\mathcal{T},8}$	0.03	-0.98
	(1.33)	(-8.29)

Details about the canonical and imperfect capital mobility RBC specifications are in the notes at the bottom of table 1. The restriction $r^* < (1-\beta)/\beta$ is satisfied at the jth replication by drawing from the prior of q^* to create $\beta_j = 1/(1+q^*_j) - (0.01 \times \breve{\beta}_j)$ where the prior of $\breve{\beta}$ is normally distributed with mean 0.9940 and 95 percent coverage interval [0.9920, 0.9961]. Otherwise, the priors of the model parameters are discussed in section 3.3 of the text.

Figure 1: Canadian CA and PVM Forecast



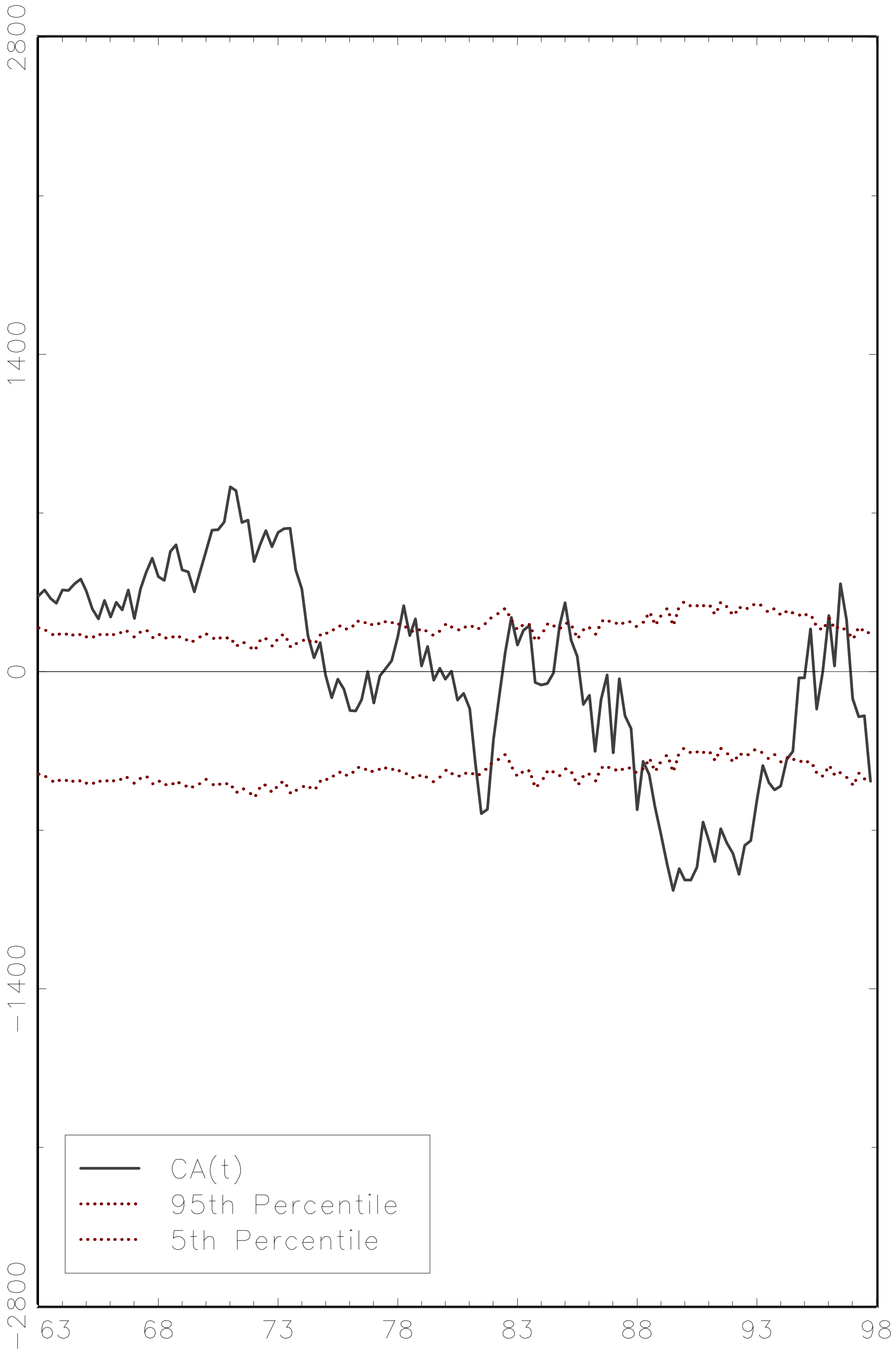


Figure 2: Fit of the Canonical RBC Model

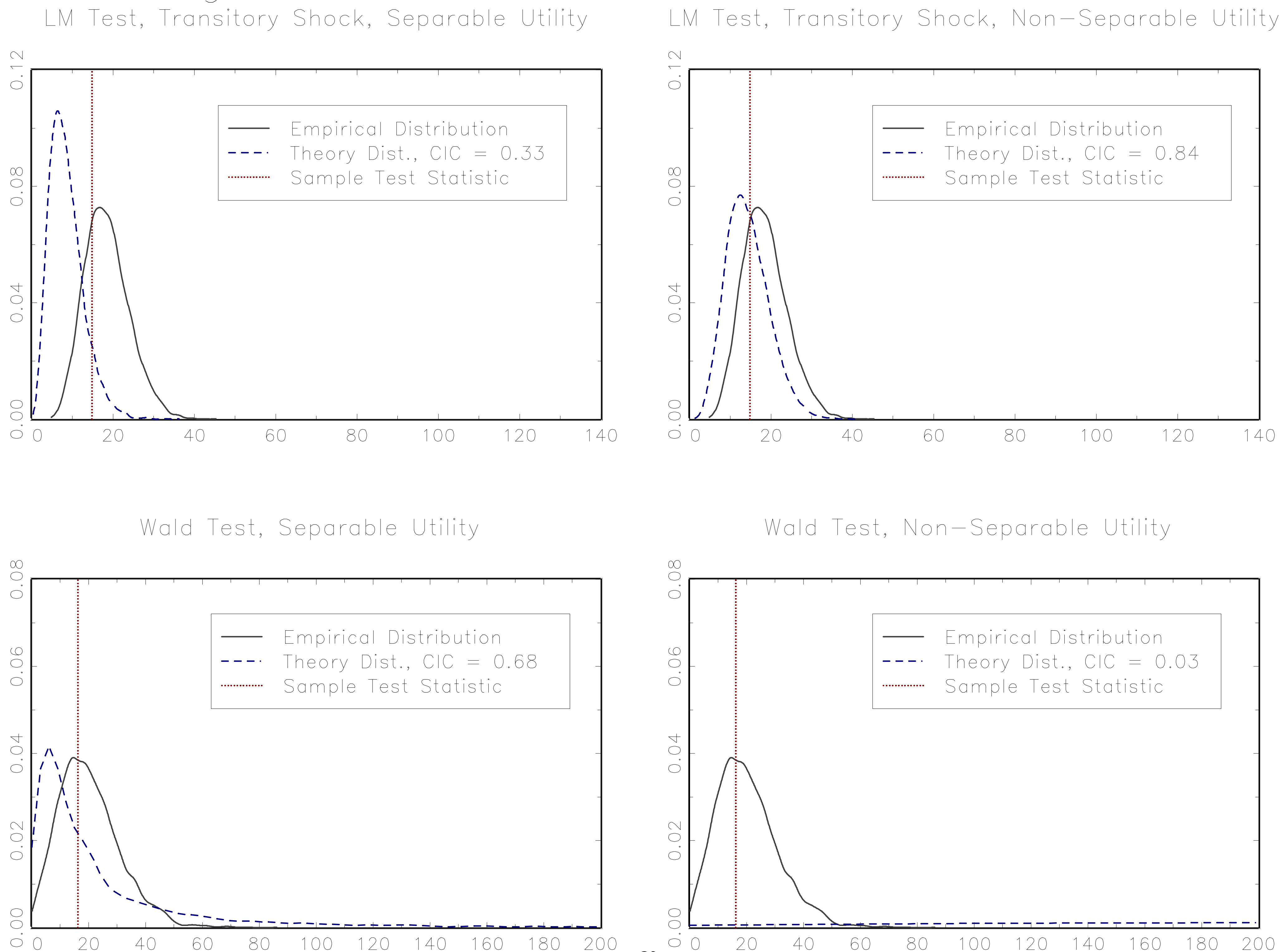


Figure 3: PVM Forecast and RBC Models

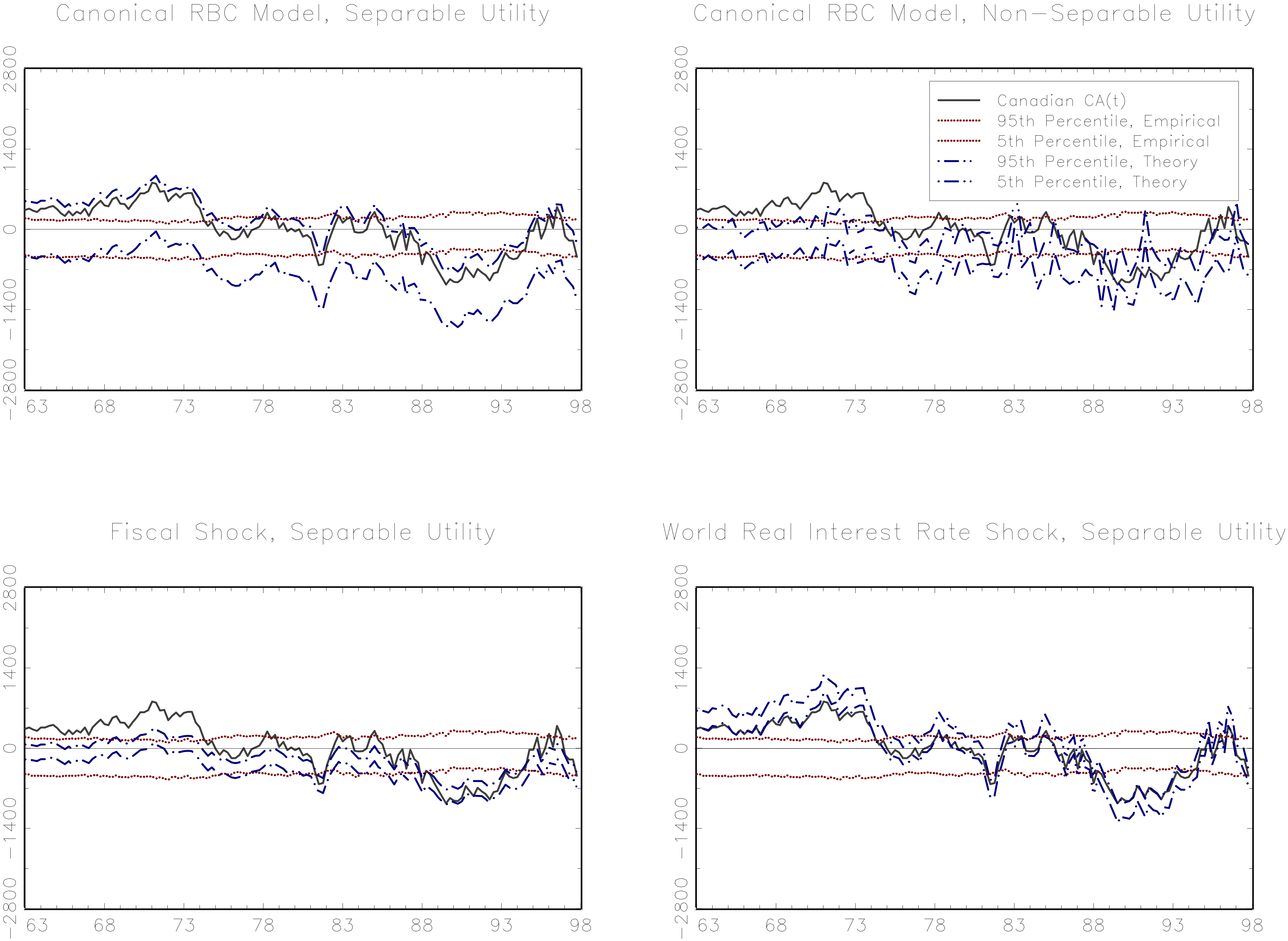


Figure 4: PVM Tests and the RBC Models

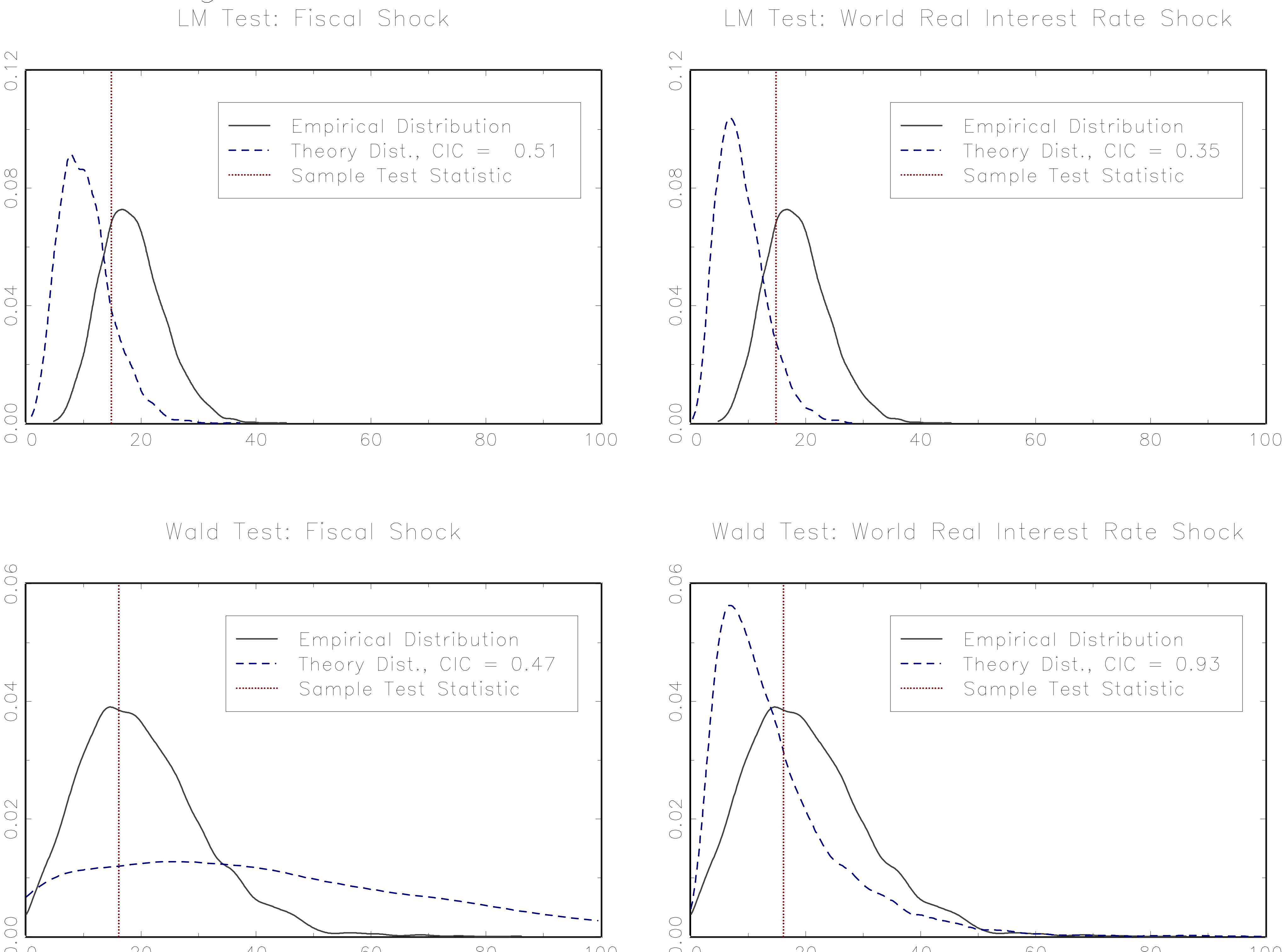


Figure 5: Imperfect Capital Mobility and RBC Model

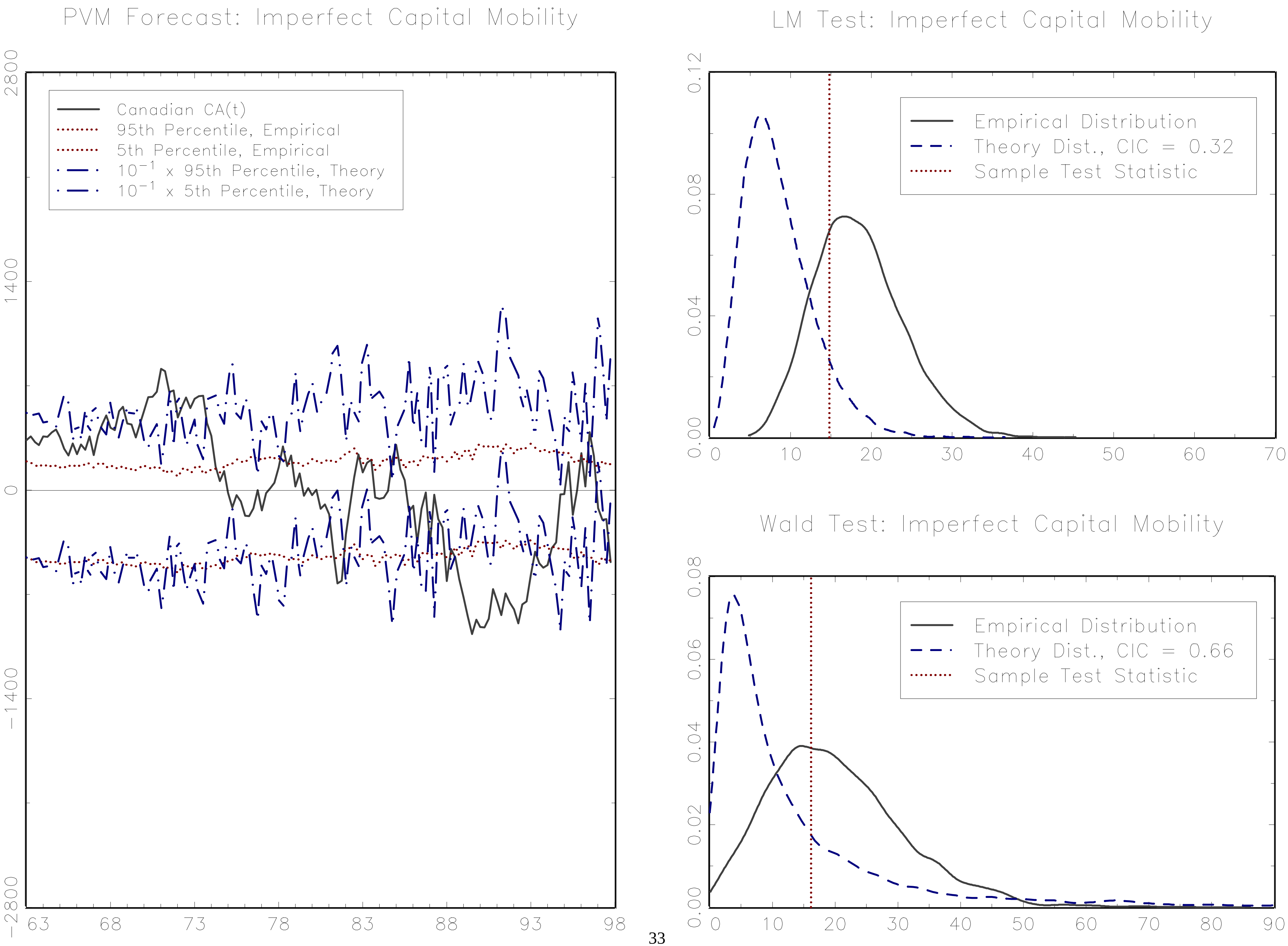


Figure 6: Lower β to Force $r^* < (1-\beta)/\beta$

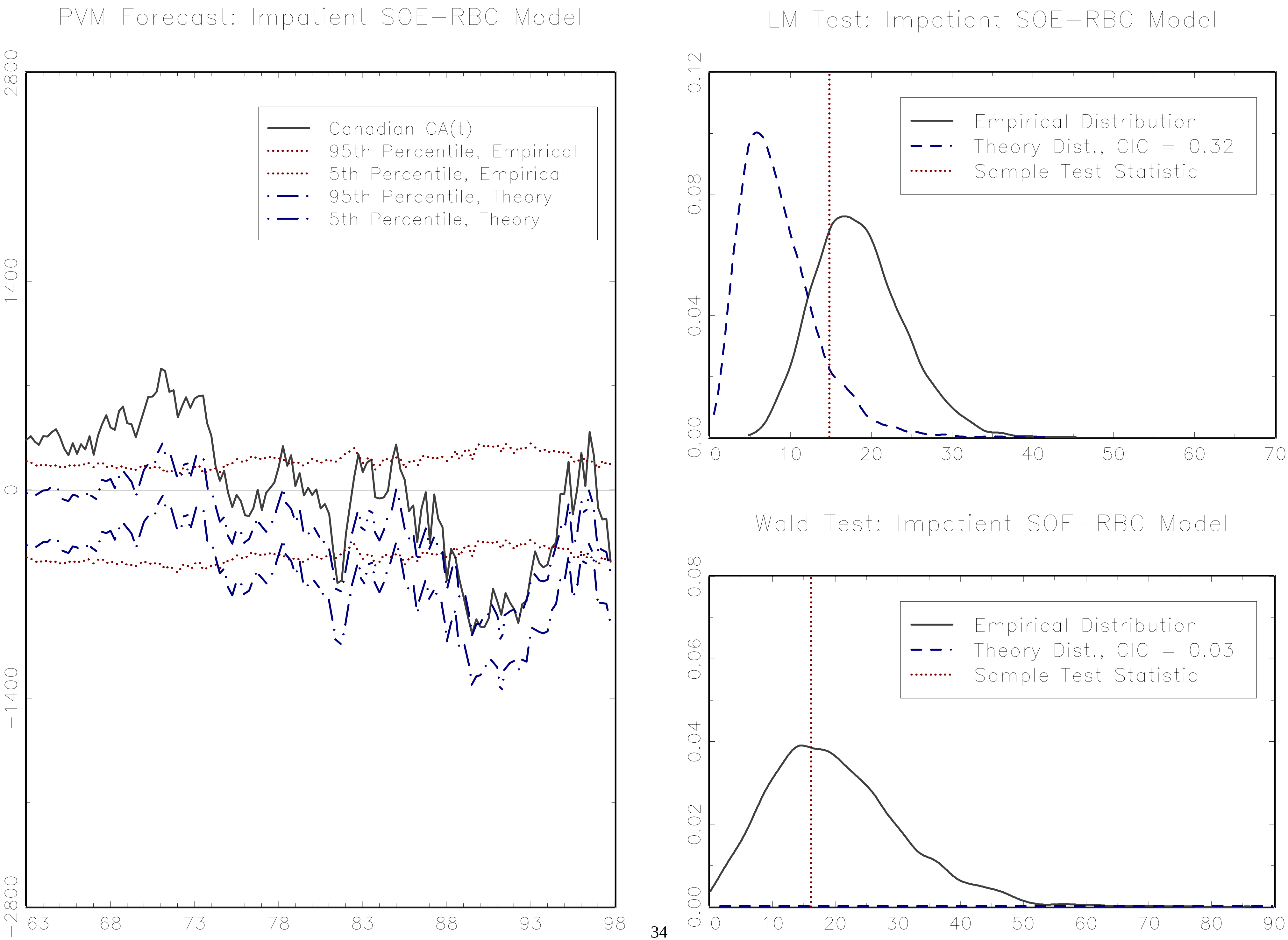


Figure 7: Lower β to Force $r^* < (1-\beta)/\beta$