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Sand in the Wheels of the Labor Market: The Effect of Firing Costs on Employment

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Abstract: This paper examines the effects of firing costs in a dynamic general equilibrium model where firms face stochastic demand. It derives analytically two simple closed-form equations, one for the supply of labor, the other for its demand. These equations determine the comparative static effects of changes in firing costs on the labor market. When negative shocks are more likely to occur than positive shocks, and when the frequency of these shocks is high, firing costs have a substantial negative impact on aggregate employment. In addition, product market integration, as it has occurred in the formation of the European Union, induces firms to be more wary of future possible downturns and therefore intensifies the negative consequences of firing costs.

Keywords: employment protection legislation, European labor markets.
JEL Classifications: E24, L16, J50.

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1 Introduction

Employment in Europe has been stagnant for the last thirty years; unemployment has risen and participation in the workforce has fallen. These dramatic events have occurred against a backdrop of legislative efforts to make it more difficult for European firms to layoff workers. These high and rising firing costs are among the leading suspects as the reason for the poor performance of labor markets in several European countries.[1]

There exists a large literature that analyzes the effects of job security regulations on the functioning of the labor market. Most available evidence indicates that firing costs have a negative impact on employment (Heckman and Pages, 2000). Yet, no consensus has emerged from the theoretical side of the debate. This disagreement is not surprising since the effect of firing costs on employment are deeply ambiguous. The first impact of job security provisions is to increase employment by discouraging layoffs when firms are hit by negative shocks. Conversely, the fear of high firing costs in the event of a future downturn acts as a hiring cost, effectively reducing the creation of new jobs when firms are hit by positive shocks. Which of the two channels dominates depends on the specification of the model and, in particular, on the nature of uncertainty.

This paper will develop a tractable general equilibrium model that delivers a clear and intuitive understanding of how the labor market is affected by job security regulations. Specifically, we will spell out precise conditions under which firing costs reduce aggregate employment and will illustrate these results with simple comparative statics.

We make three main assumptions. The first is that of monopolistic competition in the product market, which determines the size of the rent. The second is to represent uncertainty by letting demand for each product increase or decrease, according to a simple Markov process, in steps. The third is that of linear layoff costs, which yields partial but instantaneous adjustment.

This approach allows us to derive analytically simple expressions that characterize firms' hiring and firing policies. At each level of demand, there is an upper threshold of employment above which firms are firing workers; and a lower threshold, below which firms are hiring workers. We

[1] Much of the current job security regulations were introduced between the 1950s and the 1970s. The recession following the 1973 oil shock gave an additional impetus to goverments to adopt various protective measures. Since then, the broad evolution has been towards deregulation, but at an extremly slow pace (OECD, 1999).

can then solve for the steady-state probability of employment at all levels, and aggregate to derive an expression for the expected value of total labor demand. Finally, since demand for the firms' products depend upon the prices, we compute the optimal prices that the firms will charge for their goods, as well as the equilibrium wage rate that equates the supply of labor to the demand for labor.

The model derives explicit expressions for the supply of and the demand for labor, and thus sheds light on the ambiguity of the effects of firing costs on aggregate employment. It shows that when the economy is "depressed," these effects will be negative. Our notion of depressed economy refers to a situation in which the Markov process is such that the probability to move downward is greater than the probability to move upward, so firms are more likely to think that they will have to be firing workers than hiring them in the near future. On the other hand, firing costs will have a positive effect on employment when the economy is doing well, and firms are more likely to be hiring workers than firing them. Thus, the model yields the theory underlying the view that the poor performance of Europe's labor markets is the result of the interaction between "bad" labor market institutions and adverse shocks (Blanchard and Wolfers, 2000).

In addition, comparative static exercises indicate that these negative effects become stronger when the economy is more "turbulent". Our notion of turbulence pertains to the frequency of the demand shocks. Thus, the model also provides rigorous foundation for the long-standing argument that when demand is stable and growing, the hiring policy of firms is not affected by job security provisions; while, when demand turns flat and volatile, severance payments and rules become important obstacles to employment creation (Blanchard et al*., 1986,* Ljungqvist and Sargent, 2002).

The model provides the answer to another important question. Besides the increase in firing costs, another major change has been the creation of the European Union. This historical event, together with globalization and privatization of public companies, is changing the economic landscape from a collection of small national protected markets to a single large competitive market. We attribute to this ongoing process a general fall in the market power of existing firms. The model allows an assessment of how high firing costs interact with deregulation in the product market to determine aggregate employment.

The last part of the paper is devoted to the simulation of the model using plausible parameter values. As the aim of the paper is to explain the poor performance of Europe's labor market, we

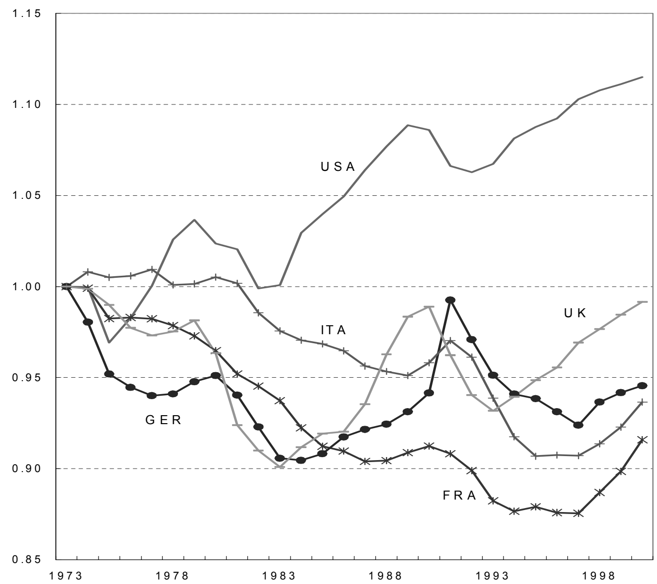

Figure 1: Employment-population ratio (base year 1973)

need to check how well the model can reproduce the experience of most European countries. Here, we need to keep in mind that employment and unemployment are not mirror images of each other. If workers' participation decisions are influenced by job protection policies (as shown by Lazear, 1990), a reduction in employment will be associated with a decline in participation rates. Thus, the unemployment rate is not the best indication of how institutional differences affect the functioning of the labor market. Figure 1 shows, instead, the ratio between employment and working age population for selected European countries and the US since 1973.[2] One fact strongly comes out: a significant downward trend in most European countries, which is even more striking if compared with the experience for the US.

[2]All employment figures in the paper are based on BLS data which put foreign countries on a similar basis as the US. See Capdeviellle and Sherwood (2002) for a detailed presentation of the BLS international data.

In Figure 1, we normalized the employment-population ratios across country using 1973 as a base year. This is because social norms have a substantial impact on labor partecipation, especially for women and young individuals. By indexing our data with a base year, we want to direct the reader's attention to the change in the employment-population ratio that occured over time.

Our model will deliver this downward trend. High layoff costs have a sizeable negative impact on employment when the economy is depressed and turbulent. In addition, we will show that a fall in the market power of firms, while it stimulates production and employment, also causes firms to be more fearful of possible future downturns and therefore increases the prospective costs of hiring workers. In other words, product market deregulations generate better results for employment when associated with low layoff costs.

The remainder of this paper is organized as follows. Section 2 discusses the related literature. Section 3 sets up a simple model of an economy with both product and labor market regulations. Section 4 solves for the steady-state general equilibrium, in which both employment and the wage are endogenously determined. Section 5 presents numerical simulations to examine how the various dimensions of regulation affect the functioning of the labor market. Finally, section 6 discusses possible extensions of the basic model and summarizes the conclusions.

2 Literature Review

The goal of this section is to motivate our contribution by discussing the existing literature on employment protection regulations. Both theoretical and applied work have been carried out on this topic.

On the theoretical side, we can identify at least three different approaches to the question of whether layoff costs have a significant impact on employment. Bentolila and Bertola (1989) analyze the case of a firm that faces uncertainty in the returns to labor in a dynamic partial equilibrium model. Assuming linear and asymmetric adjustment costs, they show that dismissal costs have a negligible effect on hiring decisions and, surprisingly, slightly increase average employment. These results are quite sensitive to different assumptions about the persistence of the shocks, the magnitude of the discount rate, and the cyclicality of voluntary quits. Thus, less persistent shocks and lower discount rates cause layoff costs to have larger negative effects on employment, because both factors reduce hiring relative to firing (Bentolila and Saint-Paul, 1994). In addition, allowing for a procyclical - rather than constant - quit rate increases the fear of dismissal costs as fewer workers leave their jobs voluntarily during downturns. In De Michelis (2003), we show that layoff costs can depress labor demand when quits are procyclical.

Hopenhayn and Rogerson (1993) develop a general equilibrium model that incorporates the structure presented by Bertola and Bentolila. Calibrating the stochastic process driving labor productivity to match US evidence on job creation and destruction, they find that layoff costs reduce the turnover rate and the overall efficiency of the economy, and have a sizable negative impact on aggregate employment. These results (however) depend greatly on the assumption of decreasing returns to scal: higher firing costs increase firms' size, and thus result into lower productivity, lower demand and lower employment.

The search and matching framework by Mortensen and Pissarides has been adapted to study how job protection provisions affect the functioning of the labor market. Blanchard (2000) and various coauthors show that costly layoffs reduce workers' flows to and from employment. This causes longer unemployment spells, while the impact on unemployment is ambiguous. Ljungqvist and Sargent (2002), on the other hand, calibrate a search model to show that the combination of high severance payments with increasing economic turbulence can generate a significant fall in the rate of employment.

On the empirical side, most available evidence shows a consistent, although not always statistically significant, negative impact of job security provisions on employment. This is true not only in the Western world (Lazear, 1990, Addison and Grosso, 1996) but in Latin America as well (Heckman and Pages, 2000). In contrast, the evidence regarding the impact on unemployment is ambiguous, but we suspect that there are conceptual reasons for such findings. Specifically, Bertola (1990), Blanchard (2000), and Nickell (1997) find no effect of job security regulations on unemployment, while Lazear (1990) and Scarpetta (1996) find positive effects. Yet, it should not be a surprise that a negative impact on employment is not always mirrored in a positive effect on unemployment. Lazear (1990) shows that job security policies affect workers' participation decision: thus, a reduction in employment will cause a decline in participation rates.

One point on which the literature has converged is the formalization of the adjustment cost function. A series of studies indicate that convexity à la Tobin's q is not the best way to proceed (Hamermesh 1993, 1995, and Hamermesh and Pfann, 1996). The last study, for example, concludes: "Adjustment costs are definitely not uniformly symmetric and convex." (p. 1281) Thus, we make the choice to follow the new standard assumption (since Nickell, 1986) of asymmetric linear adjustment costs. In particular, we formalize employment protection as a state-mandated cost that a firm has to pay if it wants to lay off an employee. We think of it as a cost to the firm-worker pair, rather than a transfer from the firm to the worker: the conventional wisdom is that, in most European countries, the legal and administrative costs associated with dismissals exceed the monetary value of severance payments (Blanchard, 2000).

Summarizing, the assertion that job security does not have a negative impact on employment is based on indirect evidence concerning unemployment, not employment. However, this finding is not supported by a rigorous theoretical argument. The ambition of this paper is to fill this gap. We will show that high layoff costs significantly reduce employment when the economy is in a phase of depression and high volatility, but not when it is booming and uncertainty is small. This result offers an explanation for why, in the early 1970s, European labor markets began to perform poorly. In addition, we will also explain how the interaction of layoff costs and the degree of competition among firms is important to assess the impact of job security regulations on aggregate employment.

It has been argued that product market constraints might significantly contribute to the poor performance of European labor markets. A recent and small literature attempts to formalize this idea in simple models[3]. In particular, Blanchard and Giavazzi (2001) develop a general equilibrium model to analyze how the interaction of product and labor market deregulations can give rise, in the short run, to lower real wages and higher unemployment and, in the long run, to a recovery of the labor share and a decrease of equilibrium unemployment.

We follow Blanchard and Giavazzi in modeling product market regulations as determining the degree of market power of firms. But, while they identify labor market regulations with the bargaining power of workers, we focus our attention on the impact of employment protection regulations. As high firing costs are likely to strengthen the hands of workers in bargaining, leading to higher wages, one might be tempted to argue that our paper has nothing new to add. However, costly layoffs also affect labor flows – layoffs directly, hirings indirectly – and not only the bargaining strength of workers. Thus, our contribution to this literature is to explain how the interaction

[3]To the best of our knowledge, Leonard and Van Audenrode (1993) were the first to argue this conjecture.

of product market regulations and firing restrictions affect aggregate employment in a stochastic environment.

Unfortunately, there is little direct evidence on the size and importance of product market constraints. A rare exception is a paper by Goldberg and Verboven (2001) in which the authors examine the European car market from 1980 to 1993: while they document a significant price dispersion across country, their findings also suggest that price discrimination plays a minor and diminishing role. Since labor demand is derived from the behavior of firms, it seems reasonable that regulations in the product market might inhibit the redeployment of workers and hence affect firms' hiring and firing policies.

Before we start spelling out the details of the model, we want to clarify what we mean by the term firing, in case the reader still had some doubt. In this paper, a fired worker is a laid off worker, not a worker fired with cause. This distinction is important because job security provisions can affect labor markets through two different channels. First, such regulations raise the costs that firms must bear in order to adjust their stock of employees. And this is what this paper is about. However, they also change the relation between employer and employee as it becomes harder to fire those workers who are not sufficiently productive. On this issue, see, among others, Kugler and Saint-Paul (2000). Anyway, for the remainder of the paper, we will use the terms "fire," "layoff" and "dismiss" interchangeably to indicate a decrease in the employment level of a given firm.

3 The Model

3.1 Setup

Demand side. This is a discrete-time model with infinite horizon. At time 0, the representative agent's preference are given by:

$$U_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+\delta}\right)^t \left[C_t^{\gamma} \left(\frac{M_t}{P_t}\right)^{1-\gamma} - \frac{\alpha}{\beta} N_t^{\beta} \right], \tag{1}$$

where δ is the rate of time preference, t denotes time, and $\mathbb{E}_0$ is the expectation operator conditional on information available in period 0. The first term inside the square brackets gives the effect on utility of a consumption index over differentiated good x_i. (See below for important details on C.)

The second term gives the effect of real money balances. Nominal money balances, M, are deflated by the nominal price index, P, associated with the composite good C. The parameter γ denotes the importance of C relative to M/P in utility; $\gamma \in [0, 1]$. The reason why we include money in the utility function is to avoid Say's law. It is a well-known fact in economic theory that the supply of products by the monopolistically competitive firms – as we choose to model the supply side in this model – automatically generates its own demand unless agents have the choice between these goods and something else (Hart, 1982). If there were only this type of good, the equilibrium in the labor market would be always indeterminate. Here, we follow Blanchard and Kiyotaki (1987), and assume that the choice is between buying goods and holding money. This is most simply and crudely achieved by having real money balances in the utility function.

The third term gives the disutility from work; N is the amount of work supplied by the household. The parameter α measures the importance of leisure in utility. The term $\beta - 1$ is the elasticity of marginal disutility of labor. We assume $\alpha > 0$ and $\beta > 1$. Since leisure enters utility as an additively separable term, we rule out any income effects on the supply of labor.

The budget constraint is:

$$\int_0^n p_{i,t} x_{i,t}^D di + M_t + V_{t+1} = w_t N_t + M_{t-1} + (1 + r_t) V_t + G_t, \tag{2}$$

where $p_{i,t}$ and $x_{i,t}^D$ denote the price and the demand for good x_i in period t, V_{t+1} denotes net assets (besides money) at the end of period t, w_t denotes the wage rate in period t, G_t denotes lump transfers in period t (see below for details). We assume that V_0 and M_0 are both positive and exogenously determined.

All of the above is quite standard. Now, we introduce our first new idea. We assume that there is a measure n of differentiated goods and demand varies across goods, that is consumers like different products differently. Specifically, we divide the x_i's goods into m taste groups and assume symmetric demand within each group. The taste parameter τ takes value in the set $\{\theta_j\}_{j=1}^m$, $\theta_j \geq 1$, and θ_j is increasing in j: thus, products located in higher taste groups (i.e. higher j) will be in higher demand. We use τ_i to denote the taste for good i. We can therefore define a "taste-adjusted" composite good:

$$C_t \equiv \left\{ n^{-1/\sigma} \int_0^n \left(\tau_{i,t} x_{i,t}\right)^{\frac{\sigma-1}{\sigma}} di \right\}^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1,$$

and a "taste-adjusted" price index (see appendix A.1 for details):

$$P_t \equiv \left\{ n^{-1} \int_0^n \tau_{i,t}^{\sigma-1} p_{i,t}^{1-\sigma} di \right\}^{\frac{1}{1-\sigma}}. \tag{3}$$

Thus, the composite good C is the usual index *à la* Dixit and Stiglitz except for the presence of the taste parameter, τ. The interpretation is straightforward: goods in higher demand, i.e. with higher τ, provide more consumption-utility. The term $n^{-1/\sigma}$ is added to neutralize the variety effect, thus an increase in the number of products does not increase utility directly. The parameter σ denotes the elasticity of substitution across all products. We make the usual assumption that $\sigma > 1$ to guarantee the existence of an equilibrium.

The price index is also standard but for τ. Again, the intuition is very simple. Recall that the price index measures the least expenditure that buys one unit of the composite good. Thus, equation (3) says that the prices of goods in higher demand must be discounted more as they provide more consumption-utility.

Uncertainty. Demand for each good x_i is uncertain. Specifically, we assume that the taste parameter τ is stochastic and follows an m-step Markov process where the upward transition probability is s and the downward transition probability is q. Note that s and q do not sum to 1 so that there is a nonzero probability that demand remains constant. If the demand shifter hits one of the extreme states θ_1 or θ_m, it stays there until demand reverts towards the center. Thus, for example, $\Pr\left\{\tau_{i,t} = \theta_1 \left| \tau_{i,t-1} = \theta_1 \right.\right\} = 1 - s$.

In section 4.2, we will show that this specification with taste shocks is perfectly equivalent to the standard assumption of productivity shocks. Thus, while our model attributes all uncertainty to shocks on the demand side, there is an alternative interpretation of the same structure in which the disturbances reflect supply shocks. Of course, the truth lies in the middle and both types of shock are important.

The justification for these idiosyncratic demand/productivity shocks is a series of studies by Leonard (1987) and Davis and Haltiwanger (1992). These authors provide evidence that gross rates of job creation and destruction are remarkably large. For the US manufacturing sector, they amount to roughly 10% in a typical year. In this paper, we suggest that idiosyncratic shocks of significant size are the source for this observed heterogeneity of employment changes across firms.

While these shocks occur at a micro-level, this setup provides a simple framework to analyze the effects of macro shocks as well. If $s = q$, firms are uniformly distributed over the line of measure n: there is the same number of firms in each taste group. Firms face an idiosyncratic risk but aggregate demand is stable. Furthermore, the closer to 1 is the sum of s and q, the higher is the uncertainty that firms must face. If $s \neq q$, the distribution of firms is no longer uniform. In particular, we can model a "depressed" economy setting $q > s$. As negative shocks are more likely to occur than positive shocks, the economy converges to a steady-state where the majority of firms are in states of low demand. Thus, we can formalize a recession - a macro shock - by raising q relative to s.

Regulations. As we discussed in the introduction, we (partially) follow Blanchard and Giavazzi (2001) in their modeling product market regulation and assume that governments can affect the elasticity of substitution. Specifically, we assume that the government sets σ.

We make the choice to identify labor market regulations with employment protection institutions which we formalize as state-mandated costs that a firm has to pay when it lays off an employee. We think of it as a cost to the firm-worker pair, rather than a transfer from the firm to the worker. This captures the fact that, in most European countries, firms consider legal and administrative costs associated with layoffs - due to notice periods, plant closing legislation, bureaucratic procedures - to vastly exceed the monetary value of the severance payments.[4]

In particular, we follow the recent literature and specify asymmetric linear adjustment costs. The firm bears a layoff cost, f, per dismissed worker while, for simplicity, we set hiring costs to

[4]Furthermore, the potential impact of severance payments could be undone by designing a wage contract that cancels out the effect of a transfer from firms to laid off workers. For example, as in the efficiency wage model of Shapiro and Stiglitz (1984), we could have workers post a bond of the value of the transfer which they would forfeit in case they are dismissed. Alternatively and more realistically, think of an employment package which pays rising wages over time. This is in fact equivalent to a constant wage, except that the firm keeps part of the early payments as a bond and returns it to the worker later if she is still employed.

zero:

$$F(Z_{i,t}) = \begin{cases} fZ_{i,t}, \text{ if } Z_{i,t} > 0 \\ 0, \text{ otherwise,} \end{cases} \tag{4}$$

where $Z_{i,t} \equiv L_{i,t-1} - L_{i,t}$, thus $Z_t > 0$ represents the number of layoffs in firm i.[5] Note that this implies that there are no voluntary quits. Recall the discussion of the literature in section 2 about how the presence of quits affects labor demand in this class of models. If the turnover rate is constant, there are fewer workers to lay off when firms are hit by a negative shock. Hence, labor demand is less negatively affected by dismissal costs (Bertola and Bentolila, 1989). However, this argument fails to realize that quits are procyclical (Akerlof et al., 1988, and Burda and Wyplosz, 1988). In this case, fewer workers will leave their jobs voluntarily during downturns, increasing the fear of high layoff costs and so inducing firms to be more reluctant to hire. In De Michelis (2003) , we show that firing costs can depress labor demand when quits are procyclical. Summarizing, we feel confident that allowing for voluntary labor turnover would not weaken the argument against employment protection laws. At the same time, algebra is much easier and we are able to derive closed form solutions.

While job security regulations impose a cost to the worker-firm pair, we assume that they are not a deadweight loss to the economy as a whole. We think that the related legal fees and administrative duties are eventually spent to purchase goods x_i's. Thus, for simplicity, we assume that they are rebated to the representative agent as a lump sum transfer; the expression for G_t in the budget constraint (2) is given by:

$$G_t = \int_0^n F(Z_{i,t})di.$$

This is a technical assumption to simplify algebra. Since total adjustment costs also enter the budget constraint through the expression for assets (the agent owns the firms), these two terms cancel each other out. Nonetheless, note that this simplification goes against the argument that costly layoffs negatively affect aggregate employment since we are ruling out any direct effect of

[5]The assumption of zero hiring cost does not affect the qualitative conclusions of the paper. It could be easily relaxed at the cost of longer and more cumbersome notation.

Furthermore, note that as labor is homogenoeus, net and gross labor flows coincide.

firing costs on total income.[6]

Firms and technology. There is a continuum of firms of measure n producing differentiated goods using homogeneous labor, L. Each product, x_i, is produced by a single firm but all firms use the same linear technology:

$$x_{i,t} = AL_{i,t} \ , \ i \in [0, n] \, . \tag{5}$$

Thus, labor productivity is always A. Of course, labor is supplied by the representative agent.

Firms are placed in a monopolistically competitive market. We make the standard assumption that firms take the behavior of other existing firms as given.

The firm chooses an employment and firing policy each period to maximize the present discounted value of expected net revenues over the infinite future:

$$v_{i,1} = \max_{\{x_{i,t},\, L_{i,t},\, Z_{i,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \left(\frac{1}{1+r_t} \right)^t \left[p_{i,t} x_{i,t} - w_t L_{i,t} - F(Z_{i,t}) \right] \right\} \tag{6}$$

subject to (1), (2), (4), (5),

and L_0 is given. Note that the information set in period t includes information on the value of τ_t. Finally, we assume that all firms are owned by the representative agents.

4 Solution of the Model

Definition of equilibrium. A competitive equilibrium for this economy is a collection of quantities $\left\{ x_{i,t}^D, V_{t+1}, M_t, N_t, x_{i,t}, Z_{i,t}, L_{i,t}, \ v_{i,t+1} \right\}_{t \in \mathbb{N},\, i \in \{0,\, n\}}$ and prices $\left\{ w_t, r_t, p_{i,t} \right\}_{t \in \mathbb{N},\, i \in \{0,\, n\}}$ such that:

- taking $\left\{ w_t, r_t, p_{i,t} \right\}$ as given, the representative agent chooses $x_{i,t}^D$, N_t, V_{t+1} and M_t to maximize (1) subject to the budget constraint (2) and $M_0 > 0$ and V_0 given;
- taking $\left\{ w_t, r_t, p_{i,t} \right\}$ and L_0 as given, each firm i chooses $x_{i,t}$, $L_{i,t}$ and $Z_{i,t}$ to solve (6);
- $\left\{ w_t, r_t, p_{i,t} \right\}$ are such that all markets clear:

[6]In fact, this is the argument exploited by Hopenhayn and Rogerson (1993): firing costs are equivalent to a less productive technology, and so reduce employment and welfare. Here, we want to show that firing costs can have a negative effect on employment through a different channel.

$x_{i,t}^D = x_{i,t}$ (market for good i)

$M_t = M_0$ (money market)

$\int_0^n \left(L_{i,t-1} + Z_{i,t}\right) di = N_t$ (labor market)

$\int_0^n v_{i,t+1} di = V_{t+1}$ (asset market).

We solve only for the steady-state equilibrium in which all prices and quantities are time invariant. Thus, in order to simplify notation, we choose, whenever possible, to omit the time index for the remainder of the paper.

We proceed by solving first for the partial equilibrium, in which prices are exogenous. Specifically, in section 4.1, we characterize the behavior of the representative agent, in section 4.2 the behavior of firms, and in sections 4.3 and 4.5, we aggregate the individual behavior in the steady-state. Finally, in section 4.5, we solve for the steady-state general equilibrium in which prices are endogenous and markets clear.

4.1 The representative agent's problem

As the economy is in a stationary state, the value of aggregate assets for the whole economy is not time dependent:

$$V_t = V_0,$$

for all t. Furthermore, for the same reason, consumption is stationary as well. Thus, the Euler condition implies that the interest rate is equal to the rate of time preference: $r_t = r = \delta$. Hence, the agent chooses not to save and each period, spends all of her capital income.

Since there are no intertemporal links, we can characterize the behavior of the representative agent with a relation between real money balances and aggregate demand, a demand function for each product and a labor supply equation (see appendix A.1 for details). Let $X \equiv \left(\int_0^n p_i x_i di\right)/P$ be an index for real aggregate consumption expenditures, which we call "aggregate demand" for short. Then, in equilibrium, we find that:

$$X = \frac{\gamma}{1-\gamma}\frac{M}{P}. \tag{7}$$

In equilibrium, desired real money balances are proportional to consumption expenditures.

Demand for good i s given by:

$$x_i^D = \frac{X}{n}\left(\frac{p_i}{P}\right)^{-\sigma} \tau_i^{\sigma-1}. \tag{8}$$

As in a standard monopolistically competitive model, the demand for each type of good relative to aggregate demand is a function of the ratio of its price to the price index, with elasticity $-\sigma$. Furthermore, a higher demand shifter, τ_i, causes demand to be higher.

Finally, aggregate labor supply is given by:

$$N = \left(\frac{\chi}{\alpha}\frac{w}{P}\right)^{\frac{1}{\beta-1}}, \tag{9}$$

where $\chi \equiv \gamma^{\gamma}(1-\gamma)^{1-\gamma}$. Thus, labor supply is increasing in the real wage.

4.2 The firm's problem

We characterize the behavior of firms using dynamic programming. All firms solve the same problem, thus, to simplify notation, we omit the goods' index, i.

Let the state variable be L_{-1} and the control variable L, where L_{-1} and L are yesterday's and today's levels of employment, respectively. Combine equations (5) and (8) to substitute for p and x into the firm's objective function. It is convenient to solve the firm's maximization problem using dynamic programming. Let $v(L_{-1}; \tau_{-1}, \tau)$ be the value function where L_{-1} is the state variable, τ_{-1} and τ the values of the demand shifter one period back and in the current period, respectively[7].

$$v(L_{-1}; \tau_{-1}, \tau) = \max_{L}\left\{\Omega\,(\tau L)^{\frac{\sigma-1}{\sigma}} - wL - F(Z) + \frac{1}{1+r}\mathbb{E}v(L; \tau, \tau_1)\right\}, \tag{10}$$

where $Z = L_{-1} - L$. Note that $\Omega \equiv P\,(X/n)^{\frac{1}{\sigma}}\,A^{\frac{\sigma-1}{\sigma}}$ is constant with respect to the choice variable, L. Furthermore, as A and τ have the same exponent, the above expression makes clear

[7]The value function depends on the lagged value of the demand shifter because today's adjustment costs depend on how a firm got to the current state, by firing or hiring workers.

that, while we model uncertainty coming from taste shocks, the same structure would arise if firm-level supply shocks were the source of risk. In other words, if τ denoted idiosyncratic productivity shocks and consumers had symmetric demands across all goods, firms would solve the same maximization problem as in equation (10).

The optimization problem defined in (10) is non-standard because the derivative of the objective function changes with the sign of Z. The first order condition with respect to L in fact yields:

$$
\begin{aligned}
\frac{\partial v(L_{-1};\tau_{-1},\tau)}{\partial L} &= \frac{\sigma-1}{\sigma}\Omega\tau^{\frac{\sigma-1}{\sigma}}L^{-\frac{1}{\sigma}} - w + \frac{1}{1+r}\frac{\partial \mathbb{E}v(L;\tau,\tau_1)}{\partial L} = \\
&= \frac{\partial F(Z)}{\partial L} = \begin{cases} -f, \text{ if } Z > 0 \\ \in \left[-f,\ 0\right], \text{ if } Z = 0 \\ 0,\ Z < 0. \end{cases}
\end{aligned}
\tag{11}
$$

The above system is analogous to the deterministic expressions derived by Nickell (1986). The difference between $\frac{\sigma-1}{\sigma}\Omega\tau^{\frac{\sigma-1}{\sigma}}L^{-\frac{1}{\sigma}}$ and w gives the net (of the wage) product the marginal worker while $\mathbb{E}v'(L;\tau,\tau_1)/(1+r)$ denotes the expected present discounted value of the marginal worker in the next period. Thus, the left hand side of the above equation can be interpreted as the net shadow value of the current marginal worker. If $Z > 0$, some workers are being laid off and equation (11) states that, at the optimum, the firm equates the expected discounted value of the future savings due to dismissing the marginal employee to the layoff cost today. Inaction, i.e. $Z = 0$, is optimal when the net expected cost of the marginal worker is smaller than the dismissal cost and greater than the zero cost of hiring. If $Z < 0$, the firm is hiring and equation (11) states that, at the optimum, the firm sets to zero the net shadow value of the marginal worker.

Recall that τ takes value in the set $\{\theta_j\}_{j=1}^m$. Thus, the first order condition in (11) implicitly defines the hiring and firing policy for each firm in state j. When a firm moves from a higher to a lower state, it will lay off some workers but it will set employment above the optimally desired level, at which the marginal product of labor is equal to the wage. The intuition behind this result is straightforward: as the firm faces adjustment costs, the adjustment will be partial. Furthermore, the linear specification of the adjustment cost implies that all the adjustment occurs immediately: there is no advantage to smooth layoffs over time. This result is in sharp contrast with the implication of convex adjustment costs à la Tobin's q. Thus, equation (11) identifies a firing threshold, L_j^F, at which employment will be set when a firm falls from state $j+1$ to state j. Similarly, when a firm moves from a higher to a lower state, it will hire additional workers but it will set employment level below the optimally desired level. The firm takes into account that some workers may have to be fired in the future if demand turns down, and this is costly. This prospective cost acts as a hiring cost, deterring the creation of new jobs in good states. Again, the linear specification of the adjustment cost implies that all the adjustment occurs immediately. Thus, for each j, equation (11) identifies a hiring threshold, L_j^H, at which employment will be set when a firm jumps from state $j-1$ to state j.

We set the values of structural parameters and exogenous variables to ensure that $L_{j-1}^F < L_j^H < L_j^F < L_{j+1}^H$ for all j's so that when a firm is hit by a negative shock it will lay off some workers and when it is hit by a positive shock it will hire some workers. In other words, we assume that the demand shocks, i.e. the difference between θ_j and θ_{j-1}, are large enough to induce the firm to change its employment level each time it is hit by a shock.[8]

In appendix B, we report the details of the derivation of the hiring and firing thresholds in the steady-state equilibrium, when the probability mass over the employment is not time dependent. Here, we just present and discuss the results. The firing threshold in state j is equal to:

$$L_j^F = \left(\frac{\sigma - 1}{\sigma} \frac{\Omega \theta_j^{\frac{\sigma-1}{\sigma}}}{w - \frac{r+s}{1+r} f} \right)^{\sigma}, \tag{12}$$

for $j = 1, ..., m-1$. As is intuitive[9], L_j^F varies positively with the demand shifter, θ_j, labor productivity, A, the dismissal cost, f, and the probability of a positive shock, s; negatively with the wage, w. Setting $f = 0$ in equation (12), we find the employment level at which the marginal product of labor is equal to the wage, L_j^*.[10] It is straightforward to check that $L_j^F > L_j^*$. In words, when a firm shifts from a higher to a lower state, the presence of dismissal costs deters firing and

[8] Alternatively, we could think that it takes two or more consecutive positive (or negative) shocks to make the firm want to hire (layoff). However, this alternative assumption would just complicate notation without adding any new insight.

[9] Recall that we have defined $\Omega = P \left(\frac{X}{n}\right)^{\frac{\mu}{1+\mu}} A^{\frac{1}{1+\mu}}$ and $\mu = \frac{1}{\sigma - 1}$.

[10] See the Appendix D for a complete analysis of the benchmark case of zero layoff costs.

thus increases labor demand. However, until a positive shock occurs in the future, these extra workers keep on being excessive.

The hiring threshold in state j is equal to:

$$L_j^H = \left(\frac{\sigma - 1}{\sigma} \frac{\Omega \theta_j^{\frac{\sigma-1}{\sigma}}}{w + \frac{q}{1+r} f} \right)^{\sigma}, \tag{13}$$

for $j = 2, ..., m$. As it is intuitive, L_j^H varies positively with the demand shifter, θ_j, and labor productivity, A; negatively with the wage, w, the dismissal cost, f, and the probability of a negative shock, q. It is straightforward to check that $L_j^H < L_j^*$. In words, when a firm shifts from a lower to a higher state, the presence of dismissal costs deters hiring and thus decreases labor demand. This fear of hiring, however, has a negligible impact unless the likelihood of a future negative shock is high enough.

Note that we have not discussed how L_j^F and L_j^H vary with σ. This is because Ω depends on P and both vary with σ. Thus, we postpone this discussion for when we find an expression for P in equilibrium.

4.3 The steady-state distribution of firms

In order to derive aggregate labor demand, we have first to determine the distribution of firms over the employment line; that is, for example, how many firms are employing L_{i1}^F workers. In the steady-state equilibrium, there is uncertainty at the firm-level but the aggregate economy is in a stationary state, for any given pair s and q.[11] A basic feature is that the number of new hires will be equal to the number of layoffs.

Recall that we set the values of structural parameters and exogenous variables to ensure that $L_{j-1}^F < L_j^H < L_j^F < L_{j+1}^H$ for all j's. Thus, even if a firm initially employs more than L_m^F, over

[11] Irrespective of the initial conditions, the stochastic process converges towards a steady-state where the probability mass at any level of employment is not time dependen. Formally, this result comes from the assumption that we are considering a discrete time finite-state Markov chain which is ergodic, i.e. irreducible with aperiodic, recurrent states. An ergodic Markov chain converges to a distribution where the probability of being in state j is independent of the initial state. For a proof of this result and a thorough analysis of the stationarity, the limit theorem and the ergodic theorem for Markov chains, see Grimmett and Stirzaker (1995), section 6.4, pp. 207-218, and section 9.5, pp. 367-380.

time, it will fall over time at or below $L^H_{i,m-1}$. Similarly, in steady-state, no firm will have L^H_1 workers. In other words, L^H_1 and L^F_m are the two extremes of the distribution.

We are now ready to derive the probability distribution of firms along the employment line. Let λ_j indicate the percentage of firms that employ L^H_j workers, and ω_j indicate the percentage of firms that employ L^F_j. Thus, for example, the probability that a firm is at L^H_j is equal to the probability that a firm was in the same state in the preceding period and did not move, $(1-s-q)\lambda_j$, plus the probability that the firm was in state $j-1$ and moved up to state j, $s\left(\omega_{j-1}+\lambda_{j-1}\right)$. Note that if a firm was in state $j+1$ and moved down, then it would set employment to L^F_j not to L^H_j: λ_j does not depend directly on the percentage of firms in state $j+1$. In steady state, these weights are time invariant and sum up to one, so that we need to solve the following system:

$$\begin{cases} \lambda_j = (1-s-q)\lambda_j + s\left(\omega_{j-1}+\lambda_{j-1}\right) \\ \omega_j = (1-s-q)\omega_j + q\left(\lambda_{j+1}+\omega_{j+1}\right) \\ \omega_1 = (1-s)\omega_1 + q\left(\lambda_2+\omega_2\right) \\ \lambda_m = (1-q)\lambda_m + s\left(\omega_{m-1}+\lambda_{m-1}\right) \\ \sum_{j=1}^{m}(\omega_j+\lambda_j) = 1 \\ \omega_m = \lambda_1 = 0. \end{cases}$$

The first two equations form a system of two second order linear difference equations in λ_j and ω_j. The other four equations give the boundary conditions. The solution of the above system yields:

$\lambda_j = \Psi\left(\frac{s}{q}\right)^{j-1}$ for $j = 2, 3, ..., m-1$,

$\omega_j = \Psi\left(\frac{s}{q}\right)^{j}$ for $j = 2, 3, ..., m-1$,

$\omega_1 = \Psi\left(\frac{s}{q}+1\right)$,

$\lambda_m = \Psi\left(\frac{s}{q}+1\right)\left(\frac{s}{q}\right)^{m-1}$,

where $\Psi \equiv \left\{\left(\frac{s}{q}\right)^m + 2\frac{s}{q}\frac{\left(\frac{s}{q}\right)^{m-1}-1}{\frac{s}{q}-1}+1\right\}^{-1}$. Note that Ψ is decreasing in $\frac{s}{q}$.

4.4 The partial equilibrium

As the measure of firm is n, there are $n\lambda_j$ firms employing L_j^H workers and $n\omega_j$ firms employing L_j^F workers. Thus, we can easily compute the steady-state aggregate labor demand as a weighted sum of L_j^H and L_j^F:

$$\begin{aligned} L_{PE} &= \sum_{j=1}^{m} \left(n\lambda_j L_j^H + n\omega_j L_j^F \right) \\ &= n\left(\frac{\sigma-1}{\sigma}\Omega\right)^{\sigma} \sum_{j=1}^{m} \left(\left(w + \frac{q}{1+r} f \right)^{-\sigma} \lambda_j \theta_j^{\sigma-1} + \left(w - \frac{r+s}{1+r} f \right)^{-\sigma} \omega_j \theta_j^{\sigma-1} \right), \end{aligned} \quad (14)$$

where L_{PE} denotes aggregate labor demand in Partial Equilibrium.

Unfortunately, the expression in (14) is not convenient to illustrate the qualitative effects of the key parameters, f and σ, on labor demand. The following approximation is extremely useful to understand the economic intuition behind the analytical result. As m increases, the total number of employees of firms in the extreme states 1 and m becomes very small since L_1^F and L_m^H become a small fraction of the total. Thus, for m large enough, we can ignore the impact of these two extremes on the aggregate labor demand. This methodology greatly simplifies algebra and notation and we can focus on where the driving forces are at work.[12] Since $\omega_j = \frac{s}{q}\lambda_j$ for $j = 2, 3, ...,$ $m-1$, one can show that L_{PE} is (approximately) proportional to:

$$L_{PE} \approx \Lambda \left[\left(w + \frac{q}{1+r} f \right)^{-\sigma} + \frac{s}{q} \left(w - \frac{r+s}{1+r} f \right)^{-\sigma} \right], \quad (15)$$

where $\Lambda \equiv n\left(\frac{\sigma-1}{\sigma}\Omega\right)^{\sigma} \sum_{j=2}^{m-1} \left(\lambda_j \theta_j^{\sigma-1} \right)$. The above expression reveals the source of the ambiguity about how job security provisions affect the labor market. On one hand, firing costs increase employment by discouraging layoffs when firms are hit by a negative shock: this is captured by the second term inside the square brackets of equation (15). On the other hand, the fear of firing costs in the event of a future downturn acts as hiring costs, effectively reducing the creation of new jobs when firms are hit by a positive shock: this is captured by the first term inside the square

[12] We check the accuracy of these approximation by carrying out simulations for both the exact case and for the approximate case.

brackets of equation (15). The latter channel dominates when the probability of a negative shock is greater than the probability of a positive shock, so firms are more likely to think that they will have to be laying off workers rather than hiring them. In such a case, the marginal hired worker will become excessive relatively soon. Thus, we find that layoff costs reduce labor demand when q is greater than s, and this effect becomes stronger when, for a given q/s, q gets large relative to r. We label such an economy "depressed" and "turbulent". By contrast, if we let the Markov process be symmetric, i.e. $q = s$, labor demand is slightly increasing in f – the main result of Bertola and Bentolila (1990). Summarizing, the interaction between institutions and the macroeconomic environment is crucial to understand the impact of firing costs on the labor market.

Finally, note that L_{PE} does not depend on the measure of firms, n. This can be easily checked substituting $P\,(X/n)^{\frac{1}{\sigma}}\,A^{\frac{\sigma-1}{\sigma}}$ for Ω in equation (14). This result comes from the specification of the utility function since aggregate demand turns out to be independent of n.

4.5 The general equilibrium

In partial equilibrium, each firm sets its price, p_i, freely according to the first order condition (11) and takes aggregate demand, X, as given. In general equilibrium, instead, prices and aggregate demand are endogenously determined.

In appendix C, we show that all firms at any firing thresholds charge the same price, p^F:

$$p^F = \frac{\sigma}{\sigma - 1}\frac{\left(w - \frac{r+s}{1+r}f\right)}{A} \tag{16}$$

Since $w - \frac{r+s}{1+r}f$ is the cost of the marginal worker in a firing threshold and A denotes labor productivity, $\left(w - \frac{r+s}{1+r}f\right)/A$ represents the cost of the marginal unit of output. Thus, equation (16) says that equilibrium price is a simple markup over marginal cost.[13] Similarly, all firms at any hiring thresholds charge the same price, p^H:

$$p^H = \frac{\sigma}{\sigma - 1}\frac{\left(w + \frac{q}{1+r}f\right)}{A} \tag{17}$$

[13] As usual, the markup rate, μ, is given demand's elasticty according to: $\mu \equiv 1/(\sigma - 1)$.

Since $w+\frac{q}{1+r}f$ is the cost of the marginal worker in a hiring threshold, we find once again the standard condition that equilibrium price is a markup over marginal cost. Substituting (16) and (17) for p^F and p^H into equation (3), we find an expression for the price index as a function of the structural parameters of the model:

$$P_{GE}=\frac{\sigma/(\sigma-1)}{A}\left\{\sum_{j=1}^{m}\theta_j^{\sigma-1}\left(\lambda_j\left(w+\frac{q}{1+r}f\right)^{1-\sigma}+\omega_j\left(w-\frac{r+s}{1+r}f\right)^{1-\sigma}\right)\right\}^{\frac{1}{1-\sigma}}. \quad (18)$$

Secondly, in general equilibrium, aggregate demand, X, is endogenously determined: in particular, it is proportional to real money balances. Hence, we need to express $\Omega=P\left(X/n\right)^{\frac{1}{\sigma}}A^{\frac{\sigma-1}{\sigma}}$ as a function of M and the structural parameters of the model, and then substitute this expression for Ω into equation (14). Simple algebra (again see appendix C for details) yields the following equation for aggregate labor demand in General Equilibrium:

$$L_{GE}=M\frac{\gamma}{1-\gamma}\frac{\sigma-1}{\sigma}\frac{\sum_{j=1}^{m}\theta_j^{\sigma-1}\left(\lambda_j\left(w+\frac{q}{1+r}f\right)^{-\sigma}+\omega_j\left(w-\frac{r+s}{1+r}f\right)^{-\sigma}\right)}{\sum_{j=1}^{m}\theta_j^{\sigma-1}\left(\lambda_j\left(w+\frac{q}{1+r}f\right)^{1-\sigma}+\omega_j\left(w-\frac{r+s}{1+r}f\right)^{1-\sigma}\right)}. \quad (19)$$

In order to compare partial and general equilibrium, it is again convenient to carry the same approximation as in section 4.5 which yield:

$$L_{GE}\approx M\frac{\gamma}{1-\gamma}\frac{\sigma-1}{\sigma}\frac{\left(w+\frac{q}{1+r}f\right)^{-\sigma}+\frac{s}{q}\left(w-\frac{r+s}{1+r}f\right)^{-\sigma}}{\left(w+\frac{q}{1+r}f\right)^{1-\sigma}+\frac{s}{q}\left(w-\frac{r+s}{1+r}f\right)^{1-\sigma}}, \quad (20)$$

and then put side by side expressions (15) and (20). If the economy is depressed and turbulent, that is if q is large relative to s and r, we still find that layoff costs have a negative effect on labor demand.[14] However, this effect is now weaker because of the presence of the term $\left(w+\frac{q}{1+r}f\right)^{1-\sigma}$

[14]Note that $\left(w+\frac{q}{1+r}f\right)^{\sigma}>\left(w+\frac{q}{1+r}f\right)^{\sigma-1}$ as $\sigma>1$ and that we still have $\left(w+\frac{q}{1+r}f\right)>\frac{s}{q}\left(w-\frac{r+s}{1+r}f\right)$. Thus, the dominating term is $\left(w+\frac{q}{1+r}f\right)^{-\sigma}$ which is decreasing in f.

in the denominator of the above expression. Intuitively, the presence of layoff raises the marginal cost of production, and thus the price, for all firms at any hiring threshold. Since these types of firms are dominating in a depressed and turbulent economy, the price index P rises with f. Ceteris paribus, a higher price level causes higher production and so higher employment. But, things are not equal, as we learned discussing L_{PE}, if q is large relative to s and r, the presence of layoff costs deters hiring more than firing. It turns out that this second effect dominates: L_{GE} is decreasing in f even if not as much as L_{PE}. Summarizing, layoff costs induce each firm at any hiring thresholds to reduce employment but the resulting fall in production raises the general level of prices, lessening the incentive to employ fewer workers.

Finally, equilibrium aggregate employment is given by the intersection of equation (19) with the labor supply schedule. Using the expression (18) to substitute in for P equation (9), it is straightforward to show that aggregate labor supply in general equilibrium is given by:

$$N_{GE} = \left(\frac{(\sigma - 1)\, A\chi w}{\sigma \alpha}\right)^{\frac{1}{\beta-1}} \left[\sum_{j=1}^{m} \theta_j^{\sigma-1} \left(\lambda_j \left(w + \frac{q}{1+r} f\right)^{1-\sigma} + \omega_j \left(w - \frac{r+s}{1+r} f\right)^{1-\sigma}\right)\right]^{\frac{1}{(\beta-1)(\sigma-1)}}, \tag{21}$$

and, with the usual approximation, we find:

$$N_{GE} \approx \left(\frac{(\sigma - 1)\, A\chi w}{\sigma \alpha}\right)^{\frac{1}{\beta-1}} \left\{\Lambda \left[\left(w + \frac{q}{1+r} f\right)^{1-\sigma} + \frac{s}{q} \left(w - \frac{r+s}{1+r} f\right)^{1-\sigma}\right]\right\}^{\frac{1}{(\beta-1)(\sigma-1)}}. \tag{22}$$

The above expression makes clear that, for a depressed and turbulent economy, aggregate labor supply is decreasing in f. Intuitively, this happens because the fall in production causes a rise in the price index, and thus a fall in the real wage. Note how this price effect counterbalances the price effect of labor demand, therefore intensifying the negative impact of firing costs on aggregate employment.

The intersection of equations (19) and (21) gives the equilibrium wage, w_{EQ}, and the equilibrium level of employment, L_{EQ}. As for a depressed and turbulent economy, firing costs shift to the left both labor demand and labor supply schedules, we find that firing costs hurt employment.

5 Numerical Simulations

In this section, we examine the effects of product and market regulations on the functioning of the labor market. We run numerical simulations using the exact expressions for labor demand and supply, equations (19) and (21), and plausible parameter values. The goal is to assess the quantitative effects of firing costs and the markup rate, and to see if we can replicate, at least partially, the downward trend in the employment-population ratio depicted in Figure 1.

First, we present and discuss the several assumptions we need to make about the parameter values.

The first two concern the size and the probability of shocks. The problem is that there are no estimates of q, s, and the θ_j's: we can only try to infer their sizes from firms' reaction to shocks. Davis and Haltiwanger (1992) measure establishment level employment changes in the US manufacturing sector. Three of the main findings of the paper are that: (i) "most of annual job creation and destruction reflects persistent establishment-level employment changes," (ii) "job destruction is highly concentrated," and (iii) the "rate of job reallocation is of impressive magnitude" (p. 821). Therefore, we suppose that firms are hit by large and infrequent shocks. In other words, the transition probabilities, q and s, seem to be small, but when these shocks occur the taste/productivity parameter, θ_j, seems to vary considerably.

Furthermore, most European countries experienced substantially higher productivity growth and substantially lower probability of negative demand shocks in the 1950s and 1960s than subsequently. In other words, the slowdown in the rate of productivity growth, the increase in real oil prices, and the increase in real interest rates and the fiscal cuts associated with the launch of the Euro profoundly altered the macroeconomic environment where firms operate. For this reason, we think we characterize as "depressed" most of the European economies in the last thirty years. In other words, we will simulate an economy where firms are hit more often by negative than by positive shocks, i.e. $q < s$.

Another important assumption must be made about the elasticity of labor supply with respect to the real wage. Equation (9) yields an elasticity equal to $\alpha/\chi(\beta - 1)$. Using $\gamma = 2/3$ which yields $\chi \approx 0.5$, so we can infer $\alpha/(\beta - 1)$ from estimates of the elasticity of the labor supply with respect to the real wage. Here, we set a value for the elasticity of 0.5, bigger than conventional micro estimates but smaller than those often assumed in aggregate models. Summarizing, we need $\alpha/(\beta - 1) = 2$, which is consistent with $\alpha = 2$ and $\beta = 2$.

Finally, our last key assumption pertains to the degree of competition among firms. The literature indicates that the markup rate varies considerably across industries (Roger, 1995). Unfortunately, our model does not allow for firm's heterogeneity besides the level of demand. Thus, with this caveat in mind, we set the markup rate $\mu \equiv \frac{1}{\sigma - 1} = 30\%$ (or alternatively $\sigma = 13/3 \approx 4$). We think this corresponds to a plausible situation where firms have significant market power.

The value of the other parameters are: $A = 1$, $M = 10$, $n = 10$, $m = 10$, $\delta = r = 4\%$.

- The benchmark case ($q = .08$, $s = .06$, $\mu = 30\%$)

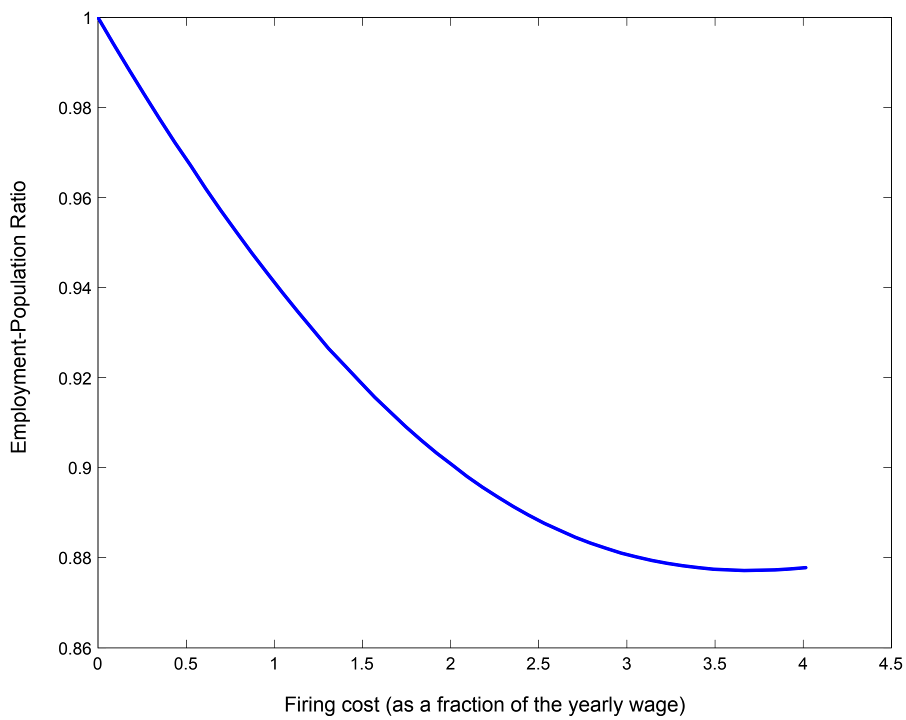

Figure 2: The benchmark case: a depressed and turbulent economy

For a given set of parameters, we first compute the employment and the wage when firing costs are zero. Then, we use this employment level as a measure of the population for our simulated economy, and this wage rate as a reference for the size of the firing cost.

Figure 2 shows the result of the simulation exercise. On the horizontal axes, we report the firing cost as a fraction of the reference wage level. On the vertical axis, we report the ratio between equilibrium employment and the measure for population. The message is clear: high layoff costs reduce employment. Even the magnitude of this change is consistent with Europe's experience in the past thirty years: an increase in the layoff costs from one to two times the reference annual wage causes a fall in the employment-population ratio of about 5%.

We also need to comment on the convex shape of the line depicted in Figure 2. Specifically, employment becomes slightly increasing when firing costs are extremely high. Not surprisingly, if job security provisions are very strict, firms stop firing workers: in our model, L_j^F moves very close to L_{j+1}^H. The other side of the coin is that adjustment costs paid by firms become very high, causing a sharp fall in profits. It seems reasonable that when profits are too low, some firms will leave the market, causing a fall in employment. Thus, future work should extend the model by allowing firms to exit (and enter) the market in order to account for the impact of firing costs on the extensive margin.

- Higher frequency ($q = .12$, $s = .09$, $\mu = 30\%$)

Ljungqvist and Sargent (2002) show how an increase in economic turbulence – which they measure as workers' income variability – in conjunction with high unemployment benefits and layoff costs can contribute to persistently high unemployment. In this paper, by contrast, we focus on the effect of job security provisions on the labor demand side. Still, we reach a similar conclusion. An increase in economic turbulence – here defined as the frequency of shocks to products' demand – is associated with a more negative impact of firing costs on employment. For example, in Figure 3, we raise the transition probabilities (while keeping their ratio constant), and we find that an increase in the layoff costs from one to two times the reference annual wage now causes a fall in the employment-population ratio of about 7%. The comparison with the benchmark case (the dotted line) makes evident that, in a depressed economy, an increase in the frequency of shocks intensifies the negative consequences of firing costs on employment.

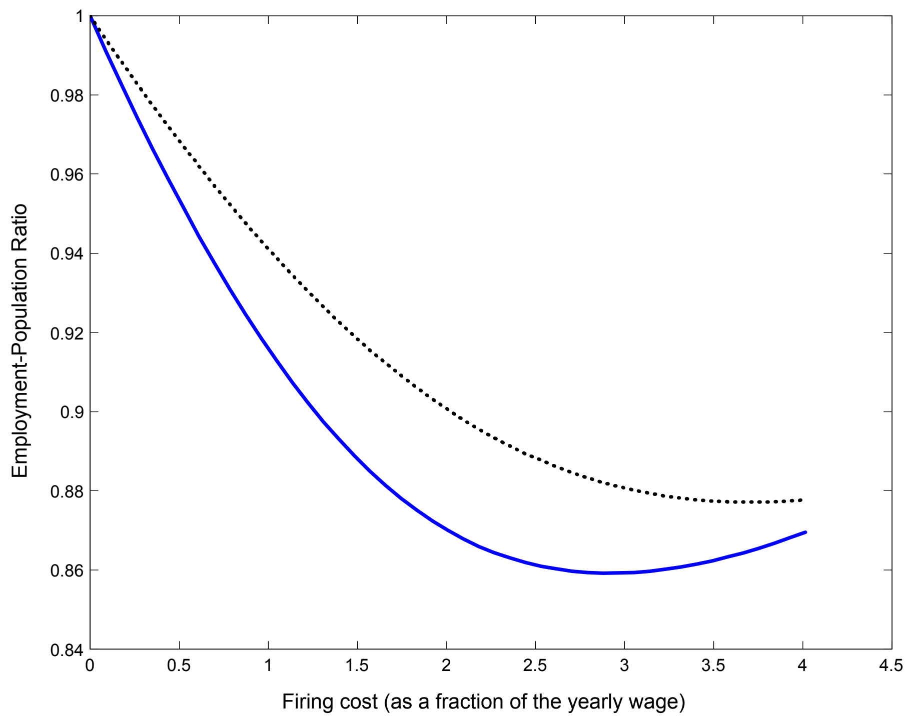

Figure 3: Higher frequency of shocks: a more turbulent economy

- Lower market power ($s = .06$, $q = .08$, $\mu = 15\%$)

  Martins *et al.* (1996) provide estimates of the markup rate over the period 1970-92, for 36 manufacturing industries in 14 OECD countries. Interestingly for us, they document a downward trend in all European countries they examine. Their results confirm the common belief that the ongoing product market deregulation – associated with the process of European integration – is reducing the market power of firms.

  In Figure 4, we reduce the markup rate from 30% to 15%, and the simulation exercise shows that lowering the market power intensifies the negative consequences of layoff costs. For example, an increase in the layoff costs from one to two times the reference annual wage causes a fall in the employment-population ratio of about 8%. Intuitively, the decrease in rents induces firms to be more worried about future possible adverse shocks when they are hiring new workers. The comparison with the benchmark case (the dotted line) clearly shows that, in a depressed economy, a fall in the markup rate increases the negative consequences

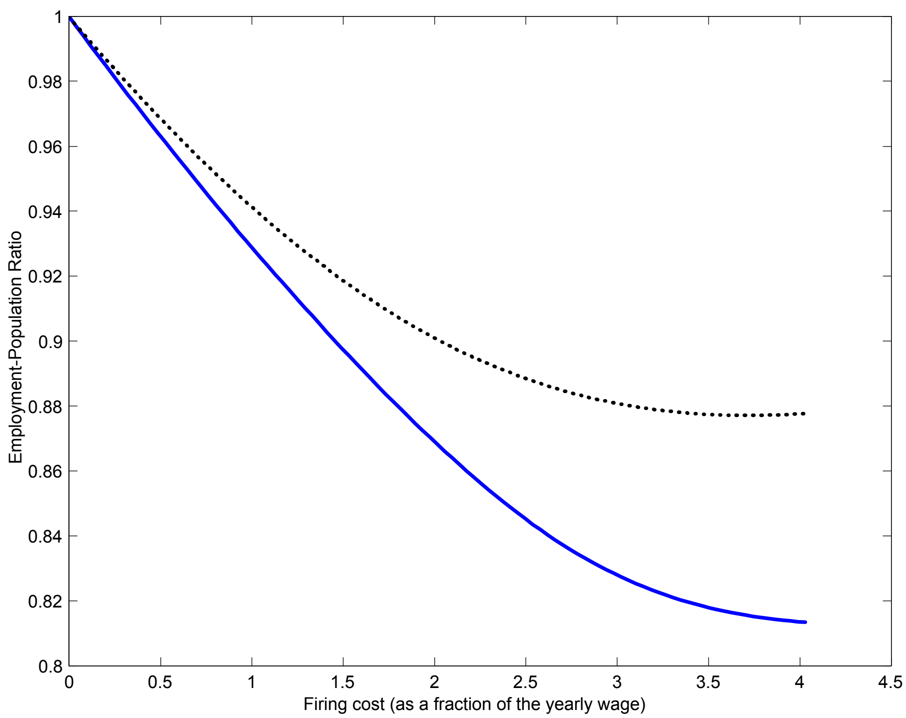

Figure 4: The effects of firing costs with low markup rate

of firing costs.

6 Conclusions

While most empirical evidence links high firing costs to low levels of employment, no agreement has yet emerged about the proper way to model this effect. We have argued that a suitable model must solve five problems simultaneously. The first of these is that the modeling of firing costs involves establishing, for different firms, an upper threshold of employment. Above this threshold, firms fire workers. Second, such a model must establish a lower threshold of employment. Below this threshold, firms hire workers. Third, because the effects of firing costs involves the hires and fires that are made, demand must be represented as subject to shocks; otherwise there will be neither hires nor fires. Fourth, it is necessary to solve for the steady-state probabilities of employment at all levels, and aggregate to compute total steady-state employment. Finally, since aggregate demand depends upon the prices that firms will charge for their goods, it is necessary to solve for the optimal prices, as it is also necessary to compute the wage rate that will clear the labor market.

We were able to accomplish all five of these steps using standard modeling techniques. We assumed that uncertainty is governed by a simple m-state Markov chain, which enabled us to derive analytical solutions for labor supply and labor demand. In contrast, a standard assumption in the previous related literature was to characterize uncertainty with a continuous-time stochastic process. This choice required that the models had to be solved numerically, so that the forces driving the results were hidden beneath the surface.

Our analytic solutions, instead, clearly show when firing costs have positive or negative effect on employment. The reasons for such effects are also clear. Job security legislation reduces aggregate employment when (i) negative shocks are more likely to occur than positive shocks, and when (ii) the frequency of shocks is high. In addition, we find that product market deregulation, with an associated fall in market power, induces firms to be more concerned about future possible downturns, intensifying the negative consequences of firing costs.

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A Appendix

A.1 The derivation of each individual demand functions and each individual labor supply

Let us begin with the definition of the price index, P. Formally, P is given by:

$$P = \min_{x_i} \int_0^n p_i x_i di \tag{23}$$

subject to $C \equiv \left\{ n^{-1/\sigma} \int_0^n (\tau_i x_i)^{\frac{\sigma-1}{\sigma}} di \right\}^{\frac{\sigma}{\sigma-1}} = 1$. Set up the Lagrangian:

$$L = \int_0^n p_i x_i di + \lambda_P \left\{ 1 - \left[n^{-1/\sigma} \int_0^n (\tau_i x_i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} \right\}.$$

Thus, the first order condition with respect to x_i yields:

$$x_i = \lambda_P^{\sigma} C n^{-1} \tau_i^{\sigma-1} p_i^{-\sigma}, \tag{24}$$

where λ_P is the Lagrangian multiplier. Combining terms in the above expression, and integrating over i gives:

$$\Rightarrow \lambda_P = \left(n^{-1} \int_0^n p_i^{1-\sigma} \tau_i^{\sigma-1} di \right)^{\frac{1}{1-\sigma}}. \tag{25}$$

Plugging (24) and (25) into (23), we get:

$$\begin{aligned} \min_{x_i} \int_0^n p_i x_i di &= \int_0^n p_i^{1-\sigma} \lambda_P^{\sigma} C n^{-1} \tau_i^{\frac{\sigma-1}{\sigma}} \tau_i^{\sigma-1} di = \\ &= \ldots = \left(n^{-1} \int_0^n p_i^{1-\sigma} \tau_i^{\sigma-1} di \right)^{\frac{1}{1-\sigma}} = P, \end{aligned}$$

which is equation (3).

Recall that we solve the model for the stationary equilibrium. Thus, consumption, money holding and labor supply is constant over time. The Euler condition then yields that $r - \delta$, and so no savings: $V_t = V_0$. This implies that the representative agent consumes the annuity value of its wealth, which is equal to total firms' profits: $rV = \int_0^n p_i x_i di - wL - \int_0^n F(Z_i)di$.

We are now ready to solve the representative agent's utility maximization problem within each period. We do this into steps. First, each worker chooses the optimal composition of consumption and money holding for a given level of income. Setup the Lagrangian:

$$L = C^{\gamma}\left(\frac{M}{P}\right)^{1-\gamma} + \lambda\left\{I + M_0 - \int_0^n p_i x_i^D di - M\right\},$$

where $I \equiv wN + rV + G$ is total period's income. Thus, the first order conditions with respect to M and x_i^D are, respectively:

$$(1-\gamma)C^{\gamma}\left(\frac{M}{P}\right)^{-\gamma}\frac{1}{P} = \lambda \tag{26}$$

$$\gamma C_l^{\gamma-1}\left(\frac{M}{P}\right)^{1-\gamma} C^{\frac{1}{\sigma}} n^{-\frac{1}{\sigma}} \tau_i^{\frac{\sigma-1}{\sigma}} \left(x_i^D\right)^{\frac{-1}{\sigma}} = \lambda p_i^{1-\sigma}, \tag{27}$$

Since (27) holds for all goods i:

$$\frac{x_i^D}{x_v^D} = \left(\frac{p_v}{p_i}\right)^{\sigma}\left(\frac{\tau_i}{\tau_v}\right)^{\sigma-1},$$

where v is a dummy index for the goods. Integrating over v and using the definition of P, we get:

$$x_i^D = \frac{(I + M_0 - M)/P}{n}\left(\frac{p_i^{1-\sigma}}{P}\right)^{-\sigma}\tau_i^{\sigma-1}. \tag{28}$$

Let $X \equiv \left(\int_0^n p_i x_i^D di\right)/P$ be "aggregate demand," so taking the ratio of (27) over (26) yields:

$$\frac{M}{P} = \frac{1-\gamma}{\gamma}X,$$

which is the same as equation (7). Note that we have used the fact that, in equilibrium, all markets must clear: $x_i^D = x_i$, $M = M_0$, and $L = N$; and that all adjustment costs are rebated to the agent as lump-sum transfers: $G = \int_0^n F(Z_i)di$.

Plugging the above expression into (28) gives the demand function for good i, equation (8):

$$x_i^D = \frac{X}{n}\left(\frac{p_i}{P}\right)^{-\sigma} \tau_i^{\sigma-1}.$$

Hence, demand of the composite good for a given level of income is:

$$C = \gamma \ (I + M_0) / P. \tag{29}$$

The second and final step is to determine labor supply, given the demand functions for money and the composite good:

$$\begin{aligned} N &= \arg\max_N U = C^\gamma \left(\frac{M}{P}\right)^{1-\gamma} - \frac{\alpha}{\beta} N^\beta \\ &= \gamma^\gamma (1-\gamma)^{1-\gamma} \frac{wN + rV + G + M_0}{P} - \frac{\alpha}{\beta} N_l^\beta. \end{aligned}$$

The first order condition with respect to N_l gives:

$$N = \left(\frac{\gamma^\gamma (1-\gamma)^{1-\gamma}}{\alpha} \frac{w}{P}\right)^{1/(\beta-1)},$$

which is equation (9). As the utility function is additively separable in consumption and real balances, on the one hand, and leisure, on the other, there are no income effects on the labor supply.

B The derivation of the hiring and the firing thresholds

We set the values of structural parameters and exogenous variables to ensure that $L_{j-1}^F < L_j^H < L_j^F < L_{j+1}^H$ for all j's so that when a firm is hit by a negative shock it will lay off some workers and when it is hit by a positive shock it will hire some workers.

Recall the Bellman equation (10) and the first order condition (11). The main issue is to compute $\partial \mathbb{E} v(L; \tau, \tau_1)/\partial L$ inside the first order condition. The key to the solution of this problem is to understand that since future employment decisions will be taken optimally, the envelope theorem ensures us that they can be taken as given (in probability distribution) when setting the current policy. In other words, the marginal worker at period t is perceived as marginal for the infinite future.

Let L_τ be the optimal value of the employment level in the current period as defined by equation (11) and τ_1 is next period's value of the demand shifter. Taking the derivative of next period value function with respect to today's control variable yields:

$$\begin{aligned}\frac{\partial V(L_\tau;\tau,\tau_1)}{\partial L_\tau} &= \left\{\frac{\sigma-1}{\sigma}\Omega\tau_1^{\frac{\sigma-1}{\sigma}}L_1^{-\frac{1}{\sigma}} - w - \frac{\partial F(Z_1)}{\partial L_1}\right.\\ &\quad \left.+\frac{1}{1+r}\frac{\mathbb{E}_1\partial V(L_1;\tau_1,\tau_2)}{\partial L_1}\right\} - \frac{\partial F(Z_1)}{\partial L_\tau} \qquad (30)\end{aligned}$$

where $Z_1 = L_\tau - L_1$. Note that τ_2 denotes the value of the demand shifter two period ahead and $\mathbb{E}_1$ the expectation operator conditional to the information available in the next period. Since the function $F(Z)$ is not continuously differentiable at zero, we have to be careful when we expand the expression in (30). Let consider first the case when the firm will lay off some workers in the next period, i.e. $Z_1 > 0$. Then, next period's adjustment costs are equal to:

$$F(Z_1) = fZ_1.$$

Using this result in equation (30), we find that the one period ahead shadow value of the current marginal employee is equal to:

$$\frac{\partial V(L_\tau;\tau,\tau_1)}{\partial L_\tau} = \frac{\sigma-1}{\sigma}\Omega\tau_1^{\frac{\sigma-1}{\sigma}}L_1^{-\frac{1}{\sigma}} - w + f + \frac{1}{1+r}\frac{\mathbb{E}_1\partial V(L_1;\tau_1,\tau_2)}{\partial L_1} - f = -f. \qquad (31)$$

We used equation (11) to substitute in 0 for $\frac{\sigma-1}{\sigma}\Omega\tau_1^{\frac{\sigma-1}{\sigma}}L_1^{-\frac{1}{\sigma}} - w + f + \frac{1}{1+r}\frac{\mathbb{E}_1\partial V(L_1;\tau_1,\tau_2)}{\partial L_1}$. Intuitively, if the firm will be laying off workers in period $t+1$, the shadow value of the marginal employee in period t will be equal to the dismissal cost, f. Let now consider the case when the firm will not lay off any worker in the next period, $Z_1 \leq 0$. Then:

$$F(Z_1) = 0.$$

Plugging the above expression into equation (30), we find:

$$\frac{\partial V(L_\tau;\tau,\tau_1)}{\partial L_\tau}=\frac{\sigma-1}{\sigma}\Omega\tau_1^{\frac{\sigma-1}{\sigma}}L_1^{-\frac{1}{\sigma}}-w+\frac{1}{1+r}\frac{\mathbb{E}_1\partial V(L_1;\tau_1,\tau_2)}{\partial L_1}. \tag{32}$$

Using equations (31) and (32) to expand the first order condition, we find:

$$\frac{\partial F(Z)}{\partial L}=\frac{\sigma-1}{\sigma}\Omega\theta_j^{\frac{\sigma-1}{\sigma}}L^{-\frac{1}{\sigma}}-w+\frac{1}{1+r}\left[0+(1-s-q)\frac{\partial V(L;\theta_j,\theta_j)}{\partial L}-qf\right], \tag{33}$$

for $j=2,\ ...,\ m-1$. We are interested in solving for the steady-state equilibrium, when the probability mass at any level of employment is not time dependent. In steady-state, all firms in state j have a number of employees equal to either L_j^H, the hiring threshold, or to L_j^F, the firing threshold. Thus, using once again equations (31) and (32), we find:

$$\frac{\partial V(L;\theta_j,\theta_j)}{\partial L}=\begin{cases}-f, \text{ if } L=L_j^F\\ 0, \text{ if } L=L_j^H.\end{cases} \tag{34}$$

We are now ready to find the hiring and the firing thresholds. Suppose that, in the last period, the firm i was in state $j+1$ and this period is in state j. Combing equations (33) and (34), we find an expression for the firing threshold in state j:

$$\frac{\sigma-1}{\sigma}\Omega\theta_j^{\frac{\sigma-1}{\sigma}}\left(L_j^F\right)^{-\frac{1}{\sigma}}-w+\frac{1}{1+r}\left[0-(1-s-q)f-qf\right]=-f$$

$$\Rightarrow L_j^F=\left(\frac{\sigma-1}{\sigma}\frac{\Omega\theta_j^{\frac{\sigma-1}{\sigma}}}{w-\frac{r+s}{1+r}f}\right)^{\sigma} \text{ for } j=1,...,m-1,$$

which is the same as equation (12)[15]. Suppose now that, in the last period, the firm i was in state $j-1$ and this period is in state j. Combing equations (33) and (34), we find an expression for the

[15]Note that in state 1, firms will remain in the same state next period with probability $(1-s)$ and will move to state 2 with probablity s. Thus, the first order condition is:
$\frac{\sigma-1}{\sigma}\Omega\theta_1^{\frac{\sigma-1}{\sigma}}\left(L_1^F\right)^{-\frac{1}{\sigma}}-w+\frac{1}{1+r}\left[s0-(1-s)f\right]=0$. This expression still implies equation (12).

hiring threshold in state j:

$$\frac{\sigma-1}{\sigma}\Omega\theta_j^{\frac{\sigma-1}{\sigma}}\left(L_j^H\right)^{-\frac{1}{\sigma}}-w+\frac{1}{1+r}\left[0+0-qf\right]=0$$

$$\Rightarrow L_j^H=\left(\frac{\sigma-1}{\sigma}\frac{\Omega\theta_j^{\frac{\sigma-1}{\sigma}}}{w+\frac{q}{1+r}f}\right)^{\sigma}\quad\text{for } j=2,...,m,$$

which is the same as equation (13)[16]. Note that no firm is ever hiring in the lowest state, 1, or is firing in the highest state, m. Thus, we do not find expressions for L_1^H and L_m^F.

C The characterization of the general equilibrium

We start by finding the equilibrium prices. Recall from the first order condition of the consumer maximization problem:

$$\frac{p_\upsilon}{p_i}=\left(\frac{x_i^D}{x_\upsilon^D}\right)^{\frac{1}{\sigma}}\left(\frac{\tau_\upsilon}{\tau_i}\right)^{\frac{\sigma-1}{\sigma}},\tag{35}$$

where υ is (again) a dummy index for goods. The above expression implies:

$$\frac{p_\upsilon}{p_i}=\left(\frac{\theta_j}{\theta_z}\right)^{\frac{\sigma}{\sigma-1}}\left(\frac{\theta_z}{\theta_j}\right)^{\frac{\sigma-1}{\sigma}}=1.$$

Thus, $p_i=p_i^F=p^F$. Similarly, one can check that $p_i=p_i^H=p^H$. Finally, equation (35) implies that the ratio between p^H and p^F is:

$$p^H=\left(\frac{w+\frac{q}{1+r}f}{w-\frac{r+s}{1+r}f}\right)p^F.$$

Using these results into the expression for the price index P, we get:

[16]Note that in state m, firms will remain in the same state next period with probability $(1-q)$ and will move to state $m-1$ with probablity q. Thus, the first order condition is:
$\frac{\sigma-1}{\sigma}\Omega\theta_m^{\frac{\sigma-1}{\sigma}}\left(L_m^H\right)^{-\frac{1}{\sigma}}-w+\frac{1}{1+r}\left[q0-(1-q)f\right]=0$. This expression still implies equation (13).

$$P=\left\{\left(\frac{w-\frac{r+s}{1+r}f}{p^F}\right)^{\sigma-1}\sum_{j=1}^{m}\theta_j^{\sigma-1}\left(\lambda_j\left(w+\frac{q}{1+r}f\right)^{1-\sigma}+\omega_j\left(w-\frac{r+s}{1+r}f\right)^{1-\sigma}\right)\right\}^{-\frac{1}{\sigma-1}}. \tag{36}$$

Using the above expression and recalling that aggregate demand is proportional to real money balances, we can express Ω as a function of p^F:

$$\begin{aligned}\Omega &= P\left(\frac{X}{n}\right)^{\frac{1}{\sigma}}A^{\frac{\sigma-1}{\sigma}}=\ldots=\left(\frac{Ap^F}{w-\frac{r+s}{1+r}f}\right)^{\frac{\sigma-1}{\sigma}}\left(\frac{\gamma}{1-\gamma}\frac{M'}{n}\right)^{\frac{1}{\sigma}}\\ &\quad\cdot\left\{\sum_{j=1}^{m}\theta_j^{\sigma-1}\left(\lambda_j\left(w+\frac{q}{1+r}f\right)^{1-\sigma}+\omega_j\left(w-\frac{r+s}{1+r}f\right)^{1-\sigma}\right)\right\}^{-\frac{1}{\sigma}} \end{aligned}\tag{37}$$

Therefore, in general equilibrium, the aggregate supply of goods x_i's is:

$$\begin{aligned}\int_0^n p_i x_i di &= nA\sum_{j=1}^{m}\left(\lambda_j p^H L_j^H+\omega_j p^F L_j^F\right)=\ldots=nAp^F\left(\frac{\sigma-1}{\sigma}\Omega\right)^{\sigma}\left(w-\frac{r+s}{1+r}f\right)^{-1}\\ &\quad\cdot\sum_{j=1}^{m}\theta_j^{\sigma-1}\left(\lambda_j\frac{w+\frac{q}{1+z}f}{w-\frac{r+s}{1+r}f}\left(w+\frac{q}{1+r}f\right)^{1-\sigma}+\omega_j\left(w-\frac{r+s}{1+r}f\right)^{1-\sigma}\right).\end{aligned}$$

Using equation (37) to substitute in for Ω in the above expression yields:

$$\int_0^n p_i x_i di=\ldots=\left[\frac{Ap^F}{\frac{\sigma}{\sigma-1}\left(w-\frac{r+s}{1+r}f\right)}\right]^{\sigma}\frac{\gamma}{1-\gamma}M.$$

Recall $\int_0^n p_i x_i di = PX$ and $X=\frac{\gamma}{1-\gamma}M/P$, thus, the equilibrium price charged by all firms at any firing threshold is given by:

$$\left[\frac{Ap^F}{\frac{\sigma}{\sigma-1}\left(w-\frac{r+s}{1+r}f\right)}\right]^{\sigma}=1\Rightarrow p^F=\frac{\sigma}{\sigma-1}\frac{\left(w-\frac{r+s}{1+r}f\right)}{A},$$

which is equation (16). Similarly, we can find that $p^H=\frac{\sigma}{\sigma-1}\left(w+\frac{q}{1+r}f\right)/A$, which is equation (17). Combining the above expression with equation (36), we can solve for the price index as a function of the structural parameters of the model, equation (18):

$$P_{GE} = \left\{ \left(\frac{\sigma-1}{\sigma} A \right)^{\sigma-1} \sum_{j=1}^{m} \theta_j^{\sigma-1} \left(\lambda_j \left(w + \frac{q}{1+r} f \right)^{1-\sigma} + \omega_j \left(w - \frac{r+s}{1+r} f \right)^{1-\sigma} \right) \right\}^{-\frac{1}{\sigma-1}},$$

We now have all the elements to write down an expression for aggregate labor demand in general equilibrium, L_{GE}:

$$\begin{aligned} L_{GE} &\equiv \sum_{j=1}^{m} \left(n\lambda_j L_j^H + n\omega_j L_j^F \right) = \ldots = \\ &= \frac{\gamma M}{1-\gamma} \frac{\sigma-1}{\sigma} \sum_{j=1}^{m} \theta_j^{\sigma-1} \left(\lambda_j \left(w + \frac{q}{1+r} f \right)^{-\sigma} + \omega_j \left(w - \frac{r+s}{1+r} f \right)^{-\sigma} \right) \\ &\quad \cdot \left\{ \sum_{j=1}^{m} \theta_j^{\sigma-1} \left(\lambda_j \left(w + \frac{q}{1+r} f \right)^{1-\sigma} + \omega_j \left(w - \frac{r+s}{1+r} f \right)^{1-\sigma} \right) \right\}^{-1}, \end{aligned}$$

which the same as equation (19).

The final step is to solve for labor supply in general equilibrium, equation (21):

$$\begin{aligned} N_{GE} &= \left(\frac{\chi}{\alpha} \frac{w}{P_{GE}} \right)^{\frac{1}{\beta-1}} \\ &= \left\{ Aw \frac{\chi}{\alpha} \frac{\sigma-1}{\sigma} \left[\sum_{j=1}^{m} \theta_j^{\sigma-1} \left(\lambda_j \left(w + \frac{q}{1+r} f \right)^{1-\sigma} + \omega_j \left(w - \frac{r+s}{1+r} f \right)^{1-\sigma} \right) \right]^{\frac{1}{\sigma-1}} \right\}^{\frac{1}{\beta-1}}. \end{aligned}$$

D The benchmark case of no adjustment costs

When layoff costs are equal to zero, the firm i sets marginal the product of labor is equal to the wage in all periods. Therefore, we have:

$$L_j^* = \left(\frac{\sigma-1}{\sigma} \frac{\Omega^* \theta_j^{\frac{\sigma-1}{\sigma}}}{w} \right)^{\sigma}. \tag{38}$$

It is easy to check that the case of zero layoff costs is yields the same steady-state distribution of firms along the employment line. Define with η_j the steady-state probability that a firm is in state j, then $\eta_j = \omega_j + \lambda_j$. In fact, it is straightforward to show that the solution of $\eta_j = s\eta_{j-1} + (1-s-q)\eta_j + q\eta_{j+1}$ with boundary conditions $\eta_1 = (1-s)\eta_1 + q\eta_2$ and $\eta_m = (1-q)\eta_m + s\eta_{m-1}$ is given by $\eta_j = \frac{\frac{s}{q}-1}{\left(\frac{s}{q}\right)^m - 1}\left(\frac{s}{q}\right)^{j-1}$ for $j = 1, 2, ..., m$.

Given this, it is immediate to find an expression for equilibrium aggregate employment without layoff costs. Let us begin with deriving aggregate labor demand in partial equilibrium, L_{PE}. Recall that equation (38) defines the employment level, L_i^*, at which the marginal product of labor is equal to the wage. Thus, we find:

$$L_{PE}^* = n\left(\frac{(\sigma - 1)\,\Omega^*}{\sigma w}\right)^{\sigma} \sum_{j=1}^{m} \eta_j \theta_j^{\sigma-1}.$$

In general equilibrium, we need to consider that all firms set the same price $p^* = \left(\frac{\sigma}{\sigma-1}\right) w/A$ so that:

$$P_{GE}^* = \frac{\left(\frac{\sigma}{\sigma-1}\right) w}{A}\left(\sum_{j=1}^{m} \eta_j \theta_j^{\sigma-1}\right)^{-\frac{1}{\sigma-1}}.$$

Similarly, we need to adjust the labor supply equation. Thus, aggregate labor demand and aggregate labor supply without layoff costs are given by:

$$\begin{aligned} L_{GE}^* &= n\left(\frac{(\sigma - 1)\,\Omega^*}{\sigma w}\right)^{\sigma} \sum_{j=1}^{m} \eta_j \theta_j^{\sigma-1} \\ &= \frac{\gamma}{1-\gamma}\frac{(\sigma - 1)\,M'}{\sigma w}. \end{aligned}$$

For the numerical simulations, it is also useful to compute L_j^* in general equilibrium:

$$L_j^* = \frac{\gamma}{1-\gamma}\frac{(\sigma - 1)\,M'}{\sigma w}\theta_j^{\sigma-1}.$$

Finally, aggregate labor supply in general equilibrium is given by:

$$N_{GE}^* = \left(\frac{\sigma - 1}{\sigma}\frac{\chi A}{\alpha}\left(\sum_{j=1}^{m} \eta_j \theta_j^{\sigma-1}\right)^{\frac{1}{\sigma-1}}\right)^{\frac{1}{\beta-1}}.$$

Summarizing, both L^*_{GE} and N^*_{GE} are decreasing in μ, thus both schedules shift out following a fall in μ resulting into higher employment and higher wage.