Appendix: Derivation of Equation (24)
Growth-Led Exports: Is Variety the Spice of Trade?

Take the logarithm of equation (23) and totally differentiate. Make the following notational simplifications: \( P^x/R = PX, \ P^x* = PE*, \ P^d* = PD*. \)

\[
\begin{align*}
(1) & \quad \frac{d\log(NX)}{d\log(PX/PE*)} + \frac{d\log(E*/PE*)}{d\log(Y/(Y+Y*))} \\
(2) & \quad + (\sigma-1) \frac{d\log(A*)}{d\log((Y+Y*)/Y*)} + \frac{d\log(Z/Z*)}{d\log(Y*+Y)}/(Y*+Y)} \end{align*}
\]

Terms (1)-(3) above are the same as in equation (24) except that “d” is replaced by “Δ”. Making use of dlog(X)=dX/X, term (8) can be written:

**Term 8**

\[-\{Z \ Y \ (1-\sigma) \ PX \ [PD^* \ A^* \ dpX - PX \ (PD^* \ dA^* + A^* \ dPD^*)]/[Z^* \ Y^* \ (PD^* \ A^*)^3] + [Z^* \ Y^* \ (Y \ dZ + Z \ dY) - Z \ Y \ (Y^* \ dZ^* + Z^* \ dY^*)] / (Z^* \ Y^*)^2 \}/(1 + Z \ Y \ [PX/(PD^* \ A^*)]^{1-\sigma}/(Z^* \ Y^*)) \]

Use of initial conditions -- A*=1, PX=PD*, Z=Z* -- allows simplification to

\[-\{Y \ (1-\sigma) \ (dPX - PD^* \ dA^* - dPD^*)/(Y* \ PD^*) + [Y* \ Y \ (dZ - dZ^*) + Z(Y* \ dY - Y \ dY^*)]/(Z \ Y^*)^2 \}/(1 + Y/Y*) \]

dividing both numerator and denominator by (1+Y/Y*)

\[
\begin{align*}
[Y/(Y*+Y)](1-\sigma)dA^* & \quad - [Y/(Y*+Y)](1-\sigma)(dPX - dPD^*)/PD^* \\
& \quad - [Y/(Y*+Y)](dZ - dZ^*)/Z - (Y* \ dY - Y \ dY^*)/[(Y*+Y)Y*] \end{align*}
\]
Term 4

\[(\sigma-1)\frac{dA^*}{A^*} \quad (A^*=1)\]

Combine with first term of simplified term 8 to yield

\[(\sigma-1)[\frac{Y^*}{(Y+Y^*)}]dA^*/A^* \text{ which is the fourth term in equation (24).}\]

Term 5

\[\frac{[Y^*/(Y+Y^*)][Y^*(dY+dY^*)-(Y+Y^*)dY^*]}{Y^*^2} = \frac{[Y^*/(Y+Y^*)][Y^* dY - Y dY^*]}{Y^*^2} = \frac{[Y^* dY - Y dY^*]}{[(Y+Y^*)Y^*]}\]

which cancels out the fourth term of simplified term 8.

Term 6

\[(\frac{Z^*}{Z})(\frac{Z^* dZ - Z dZ^*}{Z^*^2}) \quad (Z=Z^*)\]

\[= \frac{dZ - dZ^*}{Z}\]

Combine with third term of simplified term 8 to yield

\[\frac{[Y^*/(Y+Y^*)](dZ - dZ^*)}{Z} \text{ which is the fifth term in equation (24).}\]

Term 7

Use the definition of PE* in equation (22), defining

\[w = PD^* D^*/E^* \text{ and } (1-w) = PX X/E^* .\]

\[(1-\sigma) \log\{[w PD^* + (1-w) PX]/PD^*\} = (1-\sigma)[PD^*(w dPD^* + PD^* dW + dPX - w dPX - PX dW) - w PD^* dPD^*
- PX dPD^* + w PX dPD^*]/PD^*^2\]

Substituting the initial condition: PX = PD*.

\[(1-\sigma)(dPX - w dPX - dPD^* + w dPD^*)/PD^*\]
\[(1-\sigma)(1-w)(dPX - dPD*)/PD*\]

Under the initial condition of no home bias (A*=1) the share of imports in expenditures (1-w) equals exporter’s share of world output \([Y/(Y+Y*)]\).

\[(1-\sigma)[Y/(Y+Y*)](dPX - dPD*)/PD*\]

which cancels out the second term of simplified term 8.

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