SIGMA: A New Open Economy Model for Policy Analysis*
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Abstract

In this paper, we describe a new multi-country open economy SDGE model named “SIGMA” that we have developed as a quantitative tool for policy analysis. We compare SIGMA’s implications to those of an estimated large-scale econometric policy model (the FRB/Global model) for an array of shocks that are often examined in open-economy policy simulations. We show that SIGMA’s implications for the near-term (2-3 year) responses of key variables are generally similar to those of FRB/Global. Two features of our modeling framework, including rational expectations with learning, and the inclusion of some non-Ricardian agents, play an important role in giving SIGMA more flexibility to generate responses akin to the econometric policy model; nevertheless, some quantitative disparities between the two models remain due to certain restrictive aspects of SIGMA’s optimization-based framework. We conclude by using long-term simulations to illustrate some areas of comparative advantage of our SDGE modeling framework. These include linking model responses to underlying structural features of the economy, and fully articulating the endogenous channels through which “imbalances” arising from various shocks are alleviated.

Keywords: SDGE model, open-economy macroeconomics

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1 Introduction

In the wake of the Lucas critique and the early RBC literature, a wide gap emerged between the models examined in the academic literature and those utilized in policy analysis by most central banks. While central bank policy models retained a heavy emphasis on fitting short-run properties of the data, academic models increasingly heeded the methodological imperative of the RBC literature that required optimizing-agent foundations and model consistent expectations. But the focus of the latter on coherent theoretical underpinnings came at the expense of empirical realism.

In recent years, there has been a surge of interest in developing optimization-based models that are more suited to fitting the data. Consistent with this more empirical orientation, “state-of-the-art” stochastic dynamic general equilibrium (SDGE) models have evolved to include a large array of nominal and real rigidities. Important work by Christiano, Eichenbaum, and Evans (2005) shows that their optimization-based model, which includes both sticky nominal wages and various types of adjustment costs in the expenditure components, is quite successful in accounting for the estimated effects of a monetary policy shock. Smets and Wouters (2003) demonstrate that the forecasting ability of a similar model appears comparable to that of an unconstrained Bayesian VAR.

A salient motivation of this recent research has been to enhance the latitude of the SDGE models to contribute to policy analysis. In this vein, a number of central banks and other institutions such as the IMF are in the process of developing micro-founded models that can provide quantitative input into the policy process.1 But while recent empirical work validating certain features of the micro-founded models is encouraging, it remains an open question whether these models can yield plausible

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1The IMF’s new SDGE model “GEM” is described in a recent paper by Laxton and Pesenti (2003). The Bank of England’s new model “BEQM” is described by Harrison, Nikolov, Quinn, Ramsay, Scott, and Thomas (2005).
implications across the rather broad spectrum of shocks considered routinely in policy work.

In this paper, we address this question using a new multi-country open economy SDGE model (SIGMA) that we have developed for quantitative policy analysis. Our new model has its antecedents in the seminal open-economy modeling framework of Obstfeld and Rogoff (1995). However, it includes many of the nominal and real frictions that have been identified as empirically important in the closed economy models of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003), such as habit persistence in consumption and adjustment costs in investment. Moreover, it incorporates analogous frictions relevant in an open economy framework, including both costs of adjusting trade flows and local currency pricing (e.g., Betts and Devereux (1996) or Devereux and Engel (2002)).

Our approach consists in comparing the implications of SIGMA with those of the FRB/Global model, a large-scale econometric model used extensively in policy analysis at the Federal Reserve Board. Our comparisons involve examining the impulses response functions to a number of shocks often considered in policy work. These include domestic monetary and fiscal shocks, a taste shock to home (U.S.) consumption demand, a shock to the risk premium in the uncovered interest parity equation, and alternative shocks to foreign demand. The large-scale models form an important benchmark for evaluating the new SDGE models both because they pro-

\[\text{While the FRB/Global model essentially has neo-classical properties in the long-run, the behavioral equations are formulated to allow for considerable flexibility in accounting for the short-run properties of the data. This model assumes that expectations are formed adaptively, i.e., expectations are derived from small-scale VARs. In our analysis, we focus on the implications for the U.S. block of the model, which consists of about 80 estimated behavioral equations, and 300 identities; the foreign sector consists of 29 country blocks and roughly 4000 equations. See Brayton, Levin, Tryon, and Williams (1997) for an overview. The U.S. block of the FRB/Global model, augmented with a small external sector, can also be run as an independent model (FRB/US) under either adaptive or rational expectations. See Brayton and Tinsley (1996) for an overview; Kiley (2001) details the specification and estimation of the business investment sector; and Elmendorf and Reifschneider (2002) provide an application to fiscal policy.}\]
vide a reasonably accurate reduced-form characterization of the data, and because they embed the priors of policymakers who have applied these models to policy questions for several decades. While we do not exclude an eventual departure from the responses of the large-scale econometric models if the data provide sufficient grounds for doing so, it seems crucial that the new SDGE models not a priori rule out such responses due to arbitrary structural restrictions in the theoretical framework.

Our model comparisons indicate that the short-run responses of SIGMA are qualitatively similar to FRB/Global for a large array of macroeconomic variables, including output, prices, interest rates, real exchange rates, and the trade balance. We highlight two features of our framework that are instrumental in giving SIGMA greater flexibility to generate responses that are more closely aligned with FRB/Global quantitatively, including more persistent responses of both real and nominal variables. These features include “information frictions,” which posit agents as having incomplete information about the persistence of shocks, and non-Ricardian households.

With information frictions, agents use the Kalman filter to estimate the nature of shocks affecting the economy, and to make projections. This simple learning mechanism typically implies gradual responses to underlying shocks that are similar to adaptive expectations models, while retaining the appealing property of model-consistent expectations. As emphasized by Erceg and Levin (2003), it implies that model dynamics may depend crucially on the credibility and transparency of policy changes.

Our model breaks Ricardian Equivalence by assuming that some fraction of households simply consume their current after-tax disposable income. Accordingly, the short-run fiscal multiplier associated with temporary increases in government spending exceeds unity and the response of private consumption is positive. The inclusion of both non-Ricardian agents and of information frictions in our model can

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3Our approach is similar to that of Galí, López-Salido, and Vallés (2003) and Mankiw (2000).
account for a highly persistent fiscal multiplier similar to that in FRB/Global.\footnote{The highly persistent fiscal multiplier and positive consumption response are in line with the empirical findings of Fatas and Mihov (2001) and of Blanchard and Perotti (2002).}

But our comparisons also reveal some noticeable quantitative disparities between the SIGMA and FRB/Global models. First, import prices adjust much more quickly and completely to exchange rate changes in SIGMA, with full exchange rate passthrough after a few quarters. Second, SIGMA tends to imply substantially greater volatility in the expenditure components of GDP than FRB/Global. Finally, SIGMA tends to imply smaller and less persistent spillover effects from foreign shocks to the domestic economy (though the disparity is small if the increase in foreign demand is investment-driven).

We argue that these differences are particularly significant insofar as they stem from features of our SDGE framework that appear robust to reasonable departures from our benchmark calibration; moreover, they reflect certain restrictive theoretical constraints that are likely to hold in a broad class of current open economy SDGE models. Thus, high exchange rate passthrough after a few quarters is likely to characterize any model in which the desired price markup is either fixed (as in SIGMA) or exhibits only modest variation (as in other models that imply nonconstant demand elasticities, e.g., models with distribution costs). In turn, the large volatility in expenditure components is partly attributable to high passthrough, and to a marked sensitivity of private absorption to very persistent changes in real interest rates. Finally, insofar as SIGMA relies on trade linkages to account for spillover effects from foreign demand disturbances – a feature common to most open economy SDGE models – it may understate the importance of spillovers arising through other channels, including financial linkages.

Notwithstanding some limitations, the SDGE framework possesses some key advantages over that of existing econometric policy models. The last section of our paper focuses on some of these advantages in the context of longer-term simula-
tions of SIGMA, including simulations of cuts in distortionary tax rates, and of a productivity acceleration. One advantage of the SDGE framework is that it facilitates assessing how structural features of the economy affect its responses to shocks. For example, it is often of interest to consider how the effects of a tax cut depend on the elasticity of household labor supply, or on extent to which households are Ricardian in their consumption behavior. Such experiments are more difficult to design in typical large-scale econometric models in which there may exist no clear linkage between structural features and reduced form parameters. Another major advantage of the SDGE framework for policy questions is that it fully articulates the channels through which the economy returns to its balanced growth path following initial “imbalances.” For instance, in the case of the productivity acceleration, we illustrate the economic forces that induce the trade balance to eventually move into surplus following an initial trade deficit, and discuss how different initial perceptions about the underlying shock influence adjustment dynamics.

The remainder of this paper is organized as follows. Section 2 presents our basic open economy model. The calibration is discussed in Section 3. Section 4 compares short-run impulse response functions in SIGMA and FRB/Global for an array of shocks frequently considered in policy analysis. Section 5 examines long-run simulations in SIGMA under alternative calibrations. Section 6 concludes and provides a discussion of directions for future research.

2 The Model

Our model consists of two countries that may differ in size, but are otherwise isomorphic.\(^5\) Hence, our exposition below focuses on the “home” country. Each country

\(^5\)For expositional purposes, we focus on a two country variant of SIGMA in this paper. However, in actual policy analysis we have used a variant with seven country blocks that includes the United States, the euro area, Japan, Canada, Mexico, Developing Asia, and a rest-of-the-world sector. While the expanded model is useful for assessing how shocks may have disparate effects on different countries, the effects of the shocks considered
in effect produces a single domestic output good, although we adopt a standard monopolistically competitive framework to rationalize stickiness in the aggregate price level. While household utility depends on consumption of both the domestic output good and imported goods, it is convenient to assume that a competitive distribution sector purchases both inputs, and simply resells a final consumption good to households. Similarly, we assume that competitive distributors combine the domestic output good with imports to produce a final investment good.

To introduce non-Ricardian consumption behavior, we assume that there are two types of households. One type of households maximize welfare subject to an intertemporal budget constraint. These households own the entire capital stock, accumulate capital subject to adjustment costs, and exhibit habit persistence in their consumption decisions. They are also regarded as monopolistic competitors in the labor market to account for aggregate wage stickiness. The other set of households (“hand-to-mouth” households) simply consume their entire after-tax disposable income.

2.1 Firms and Price Setting

Production of Domestic Intermediate Goods. There is a continuum of differentiated intermediate goods (indexed by $i \in [0, 1]$) in the home country, each of which is produced by a single monopolistically competitive firm. As in Betts and Devereux (1996), intermediate goods firms charge different prices at home and abroad (i.e., they practice local currency pricing). In the home market, firm $i$ faces a demand function that varies inversely with its output price $P_{Di}(i)$ and directly with aggregate demand at home $Y_{Dt}$:

$$
Y_{Di}(i) = \left[ \frac{P_{Di}(i)}{P_{Dt}} \right]^{-(1+\delta_p)} \sigma_p Y_{Dt},
$$

(1)

in our analysis on the home country (i.e., the United States) are similar across model variants.
where $\theta > 0$, and $P_{Dt}$ is an aggregate price index defined below. Similarly, in the foreign market, firm $i$ faces the demand function:

$$X_t(i) = \left[ \frac{P_{Mt}(i)}{P_{Mt}^*} \right]^{-(1+\theta_p)} M_t^*,$$

(2)

where $X_t(i)$ denotes the foreign quantity demanded of home good $i$, $P_{Mt}(i)$ denotes the price that firm $i$ sets in the foreign market (denominated in foreign currency), $P_{Mt}^*$ is the foreign import price index, and $M_t^*$ is aggregate foreign imports (we use an asterisk to denote foreign variables).

Each producer utilizes capital services $K_t(i)$ and a labor index $L_t(i)$ (defined below) to produce its respective output good. The production function is assumed to have a constant-elasticity of substitution (CES) form:

$$Y_t(i) = \left( \omega_K^{\frac{1}{\rho}} K_t(i)^{\frac{1}{1+\rho}} + \omega_L^{\frac{1}{\rho}} (Z_t L_t(i))^{\frac{1}{1+\rho}} \right)^{1+\rho}.$$

(3)

The production function exhibits constant-returns-to-scale in both inputs, and technological progress $Z_t$ is given by:

$$Z_t = \exp(g_z t + z_t),$$

(4)

where $z_t$ is a country-specific shock to the level of technology and $g_z$, the deterministic rate of technological growth, is assumed to be the same in both countries. Firms face perfectly competitive factor markets for hiring capital and labor. Thus, each firm chooses $K_t(i)$ and $L_t(i)$, taking as given both the rental price of capital $R_{Kt}$ and the aggregate wage index $W_t$ (defined below). Firms can costlessly adjust either factor of production. Thus, the standard static first-order conditions for cost minimization imply that all firms have identical marginal cost per unit of output, $MC_t$.

We assume that the home and foreign prices of the intermediate goods are determined by Calvo-style staggered contracts (see Calvo (1983)). In each period, a firm faces a constant probability, $1 - \xi_p$, of being able to reoptimize its price at
home \( (P_{Dt}(i)) \) and \( 1 - \xi_{p,x} \) probability of being able to reoptimize its price abroad \( (P^*_{Mt}(i)) \). These probabilities are assumed to be independent across firms, time, and countries. If a firm is not allowed to optimize its prices, we follow Christiano, Eichenbaum, and Evans (2001) and assume the firm must reset its home price based on lagged aggregate inflation. Prices are updated according to \( P_{Dt}(i) = \pi_{t-1} P_{Dt-1}(i) \)

\[ \text{6} \]

where \( \pi_t = P_{Dt}/P_{Dt-1} \). Similarly, in foreign markets, if a firm cannot reoptimize its price, the price is changed according to the rule, \( P^*_{Mt}(i) = \pi^*_{Mt-1} P^*_{Mt-1}(i) \) where \( \pi^*_{Mt} = P^*_M/P^*_{M-1} \). This form of lagged indexation is a mechanism for introducing inflation inertia into the key price-setting equations.

When firm \( i \) is allowed to reoptimize its price in the domestic market in period \( t \), the firm maximizes

\[ \tilde{E}_t \sum_{j=0}^{\infty} \xi^j_p \psi_{t,t+j} [V_{Dt+j} P_{Dt} (i) Y_{Dt+j} (i) - MC_{t+j} Y_{Dt+j} (i)]. \] (5)

The operator \( \tilde{E}_t \) represents the conditional expectation based the information available to agents at period \( t \). The firm discounts profits received at date \( t + j \) by the state-contingent discount factor \( \psi_{t,t+j} \); for notational simplicity, we have suppressed all of the state indices.\(^7\) Also, \( V_{Dt+j} \) is defined by

\[ V_{Dt+j} = \prod_{h=1}^{j} \pi_{t+h-1}. \] (6)

The first-order condition for setting the contract price of good \( i \) in the home market is

\[ \tilde{E}_t \sum_{j=0}^{\infty} \psi_{t,t+j} \xi^j_p \left( \frac{V_{Dt+j} P_{Dt} (i)}{(1 + \theta_p)} - MC_{t+j} \right) Y_{Dt+j} (i) = 0. \] (7)

\(^6\)In alternative calibrations of SIGMA, we also consider the specification used by Yun (1996) and Erceg, Henderson, and Levin (2000) where \( P_{Dt}(i) = \pi P_{Dt-1}(i) \) so that \( V_{Dt+j} = \pi^j \) in the profit functional defined below. For this alternative calibration, prices are updated according to \( P_{Mt}(i) = \pi^* P_{Mt-1}(i) \) in foreign markets.

\(^7\)We define \( \xi_{t,t+j} \) to be the price in period \( t \) of a claim that pays one dollar if the specified state occurs in period \( t + j \) (see the household problem below); then the corresponding element of \( \psi_{t,t+j} \) equals \( \xi_{t,t+j} \) divided by the probability that the specified state will occur.
Defining a similar profit functional to equation (5) for a firm’s optimal choice of its contract price in the foreign market at date t, the associated first-order condition is:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \psi_{t,t+j}^x \left( \frac{e_{t+j} V_{Mt+j+1}^x P_{Mt}^x(i)}{(1 + \theta_p)} - MC_{t+j} \right) X_{t+j}(i) = 0. \quad (8)$$

In equation (8), $e_t$ is the price of a unit of foreign currency in terms of the home currency (so that a rise in $e_t$ corresponds to a depreciation of the home currency), and $V_{Mt+j}^x$ is defined as:

$$V_{Mt+j}^x = \prod_{h=1}^{j} \pi_{Mt+h-1}^x. \quad (9)$$

**Production of the Domestic Output Index.** Because households have identical Dixit-Stiglitz preferences, it is convenient to assume that a representative aggregator combines the differentiated intermediate products into a composite home-produced good $Y_{Dt}$:

$$Y_{Dt} = \left[ \int_0^1 Y_{Dt}(i) \frac{1}{1+\theta_p} \, di \right]^{1+\theta_p}. \quad (10)$$

The aggregator chooses the bundle of goods that minimizes the cost of producing $Y_{Dt}$, taking the price $P_{Dt}^x(i)$ of each intermediate good $Y_{Dt}(i)$ as given. The aggregator sells units of each sectoral output index at its unit cost $P_{Dt}^x$:

$$P_{Dt}^x = \left[ \int_0^1 P_{Dt}^x(i) \frac{1}{\theta_p} \, di \right]^{-\theta_p}. \quad (11)$$

We also assume a representative aggregator in the foreign economy who combines the differentiated home products $X_t(i)$ into a single index for foreign imports:

$$M_t^* = \left[ \int_0^1 X_t(i) \frac{1}{1+\theta_p} \, di \right]^{1+\theta_p}, \quad (12)$$

and sells $M_t^*$ at price $P_{Mt}^*$:

$$P_{Mt}^* = \left[ \int_0^1 P_{Mt}^*(i) \frac{1}{\theta_p} \, di \right]^{-\theta_p}. \quad (13)$$
Production of Consumption and Investment Goods. Final consumption goods are produced by a representative “consumption good distributor.” This firm combines purchases of domestically-produced goods with imported goods to produce a final consumption good \( (C_t) \) according to a constant-returns-to-scale CES production function:

\[
C_t = \left( \frac{\varphi}{\varphi + C} C_t^{\frac{1}{\varphi + C}} + (1 - \varphi) (\varphi M_t) \right)^{1 + \varphi_C}, \tag{14}
\]

where \( C_{Dt} \) denotes the consumption good distributor’s demand for the index of domestically-produced goods, \( M_Ct \) denotes the distributor’s demand for the index of foreign-produced goods, and \( \varphi_Ct \) reflects costs of adjusting consumption imports. The form of the production function mirrors the preferences of households over consumption of domestically-produced goods and imports. Accordingly, the quasi-share parameter \( \omega_C \) may be interpreted as determining household preferences for home relative to foreign goods, or equivalently, the degree of home bias in household consumption expenditure. Finally, the adjustment cost term \( \varphi_Ct \) is assumed to take the quadratic form:

\[
\varphi_Ct = \left[ 1 - \frac{\varphi_M}{2} \left( \frac{M_Ct}{C_{Dt}} - 1 \right)^2 \right]. \tag{15}
\]

This specification implies that it is costly to change the share of the imported good in total consumption. It has the attractive features that the import share in consumption is relatively unresponsive in the short-run to changes in the relative price of imported goods, even while allowing the level of imports to jump costlessly in response to changes in overall consumption demand.

Given the presence of adjustment costs, the representative consumption goods distributor chooses (a contingency plan for) \( C_{Dt} \) and \( M_Ct \) to minimize its discounted
expected costs of producing the aggregate consumption good:

\[
\min_{C_{Dt}, M_{Ct}} \sum_{k=0}^{\infty} \psi_{t,t+k} \left( P_{Dt+k} C_{Dt+k} + P_{Mt+k} M_{Ct+k} \right)
+ P_{Ct} \left[ C_t - \left( \omega_C \frac{\psi_{Ct}}{\psi_{Ct}} C_{Dt}^{\frac{1}{1+\rho_C}} + (1 - \omega_C) \frac{\psi_{Ct}}{\psi_{Ct}} (\varphi_{Ct} M_{Ct})^{\frac{1}{1+\rho_C}} \right)^{1+\rho_C} \right].
\]  

(16)

The distributor sells the final consumption good to households at a price \( P_{Ct} \), which may be interpreted as the consumption price index (or equivalently, as the shadow cost of producing an additional unit of the consumption good).

We model the production of final investment goods in an analogous manner. Thus, the representative “investment goods distributor” produces a final investment good by combining its purchases of domestically-produced goods with purchases of foreign-produced goods according to a constant-returns-to-scale CES production function:

\[
I_t = \left( \omega_I \frac{\varphi_{It}}{\psi_{It}} I_{Dt}^{\frac{1}{1+\rho_I}} + (1 - \omega_I) \frac{\varphi_{It}}{\psi_{It}} (\varphi_{It} M_{It})^{\frac{1}{1+\rho_I}} \right)^{1+\rho_I},
\]  

(17)

where \( I_{Dt} \) denotes the investment goods distributor’s demand for the index of domestically-produced goods, \( M_{It} \) denotes the distributor’s demand for the index of foreign goods, and \( \varphi_{It} \) reflects costs of adjusting imports of investment goods. As in case of consumption goods, the quasi-share parameter \( \omega_I \) may be interpreted as the degree of home bias in the production of final investment goods. The adjustment cost \( \varphi_{It} \) takes a form that is analogous to the adjustment cost specification for imported consumption goods, so that:

\[
\varphi_{It} = \left[ 1 - \varphi \frac{M_{It}}{2} \left( \frac{M_{It}}{I_{Dt-1}} - 1 \right)^2 \right].
\]  

(18)

Investment goods distributors solve an intertemporal cost minimization problem isomorphic to that of consumption goods distributors. The distributor sells the final investment good to households at a price \( P_{It} \), which may be interpreted as the
investment price index. This price may differ from the price index of the consumption good $P_{Ct}$, even in the absence of the adjustment costs for consumption and investment imports because of differences in import composition.

2.2 Households and Wage Setting

We assume a continuum of monopolistically competitive households (indexed on the unit interval), each of which supplies a differentiated labor service to the intermediate goods-producing sector (the only producers demanding labor services in our framework). It is convenient to assume that a representative labor aggregator (or “employment agency”) combines households’ labor hours in the same proportions as firms would choose. Thus, the aggregator’s demand for each household’s labor is equal to the sum of firms’ demands. The aggregate labor index $L_t$ has the Dixit-Stiglitz form:

$$L_t = \left[ \int_0^1 (\zeta_t N_t(h))^{1+\psi_w} dh \right]^{1+\theta_w},$$

where $\theta_w > 0$ and $N_t(h)$ is hours worked by a typical member of household $h$. Also, $\zeta_t$ is the size of a household of type $h$ and evolves according to $\zeta_t = g_n \zeta_{t-1}$ (effectively, $\zeta_t$ and $g_n$ determine the size and growth rate of the population). The aggregator minimizes the cost of producing a given amount of the aggregate labor index, taking each household’s wage rate $W_t(h)$ as given, and then sells units of the labor index to the production sector at their unit cost $W_t$:

$$W_t = \left[ \int_0^1 W_t(h)^{-1/\psi_w} dh \right]^{-\theta_w}. \quad (20)$$

It is natural to interpret $W_t$ as the aggregate wage index. The aggregator’s demand for the labor services of a typical member of household $h$ is given by

$$N_t(h) = \left[ \frac{W_t(h)}{W_t} \right]^{-1+\theta_w/\psi_w} L_t/\zeta_t. \quad (21)$$
The utility functional of a representative member of household \( h \) is

\[
\tilde{E}_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{1}{1-\sigma} \left( C_{t+j} (h) - \frac{C_{t+j-1}}{\zeta_{t+j-1}} - \nu_{ct} \right)^{1-\sigma} + \frac{\chi^{0.7} \zeta_{t+j}^{1-\sigma}}{1-\chi} (1 - N_{t+j} (h))^{1-\chi} + \frac{\mu_0}{1-\mu} \left( \frac{MB_{t+j+1} (h)}{P_{Ct+j}} \right)^{1-\mu} \right\},
\]

where the discount factor \( \beta \) satisfies \( 0 < \beta < 1 \). As in Smets and Wouters (2003), we allow for the possibility of external habits, where each household member cares about its consumption relative to lagged aggregate consumption per capita. The period utility function depends on each member’s current leisure \( 1 - N_t (h) \), his end-of-period real money balances, \( \frac{MB_{t+j+1} (h)}{P_{Ct+j}} \), and a preference shock, \( \nu_{ct} \). We allow for preferences over leisure to shift with the level of technology so that the model is consistent with balanced growth, even if the subutility function over consumption is not logarithmic.\(^8\)

We assume that there are two types of households: households that make intertemporal consumption, labor supply, and capital accumulation decisions in a forward-looking manner by maximizing utility subject to an intertemporal budget constraint (FL households, for “forward-looking”); and the remainder that simply consume their after-tax disposable income (HM households, for “hand-to-mouth” households). The latter type receive no capital rental income or profits, and choose to set their wage to be the average wage of optimizing households. Given that households of each type grow at the same rate, the share of each type of household in the population is fixed. We denote the share of FL households by \( \zeta \) and the share of HM households by \( 1 - \zeta \).

We consider first the problem faced by FL households. Household \( h \) faces a flow budget constraint in period \( t \) which states that its combined expenditure on goods

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\(^8\)This statement is only strictly true in the absence of permanent country-specific technology shocks. In this case, a permanent increase in technology in the home country that does not occur abroad will be associated with a permanent deterioration in the home country’s terms of trade that moves the home economy off its balanced growth path.
and on the net accumulation of financial assets must equal its disposable income:

\[ P_{Ct}C_t(h) + P_{It}I_t(h) + MB_{t+1}(h) - g_n^{-1}MB_t(h) + \int_s \xi_{t,t+1}B_{Dt+1}(h) \]

\[ -g_n^{-1}B_{Dt}(h) + P_{Bt}B_{Gt+1} - g_n^{-1}B_{Gt} + \frac{\epsilon_t P_{Bt}B_{Ft+1}(h)}{\phi_{Bt}} - g_n^{-1}e_tB_{Ft}(h) \]

\[ = (1 - \tau_{Nt})W_t(h) N_t(h) + \Gamma_t(h) + TR_t(h) - T_t(h) + (1 - \tau_{Kt})g_n^{-1}R_{Kt}K_t(h) + \]

\[ P_{It}\tau_{Kt}\delta g_n^{-1}K_t(h) - P_{Dt}\phi_{Kt}(h) - P_{ Dt} \phi_{It}(h). \]

(23)

The presence of the population growth parameter \( g_n \) in the household’s budget constraint reflects that equation (23) is expressed in per capita terms as well as the assumption that new household members are born without any initial holdings of bonds, capital, or money. Final consumption goods are purchased at a price \( P_{Ct} \), and final investment goods at a price \( P_{It} \). Investment in physical capital augments the per capita capital stock \( K_{t+1}(h) \) according to a linear transition law of the form:

\[ K_{t+1}(h) = (1 - \delta)g_n^{-1}K_t(h) + I_t(h), \]

(24)

where \( \delta \) is the depreciation rate of capital.

Financial asset accumulation of a typical member of FL household \( h \) consists of increases in nominal money holdings \((MB_{t+1}(h) - g_n^{-1}MB_t(h))\) and the net acquisition of bonds. We assume that agents within a country can engage in frictionless trading of a complete set of contingent claims, while trade in international assets is restricted to a non-state contingent nominal bond. The term \( P_{Bt}B_{Gt+1} - g_n^{-1}B_{Gt} \) represents each household member’s net purchases of domestic government bonds, while \( \int_s \xi_{t,t+1}B_{Dt+1}(h) - g_n^{-1}B_{Dt}(h) \) are net purchases of state-contingent domestic bonds. We denote \( \xi_{t,t+1} \) as the price of an asset that will pay one unit of domestic currency in a particular state of nature at date \( t + 1 \), while \( B_{Dt+1}(h) \) represents the quantity of such claims purchased by a typical member of household \( h \) at time \( t \).
Thus, the gross outlay on new state-contingent domestic claims is given by integrating over all states at time $t + 1$, while $B_{Dt}(h)$ indicates the value of the household’s existing claims (on a per capita basis) given the realized state of nature.

In equation (23), $B_{Ft+1}(h)$ represents the quantity of a non-state contingent bond purchased by a typical member of household $h$ at time $t$ that pays one unit of foreign currency in the subsequent period, $P_{Bt}^*$ is the foreign currency price of the bond, and $e_t$ is the exchange rate expressed in units of home currency per unit of foreign currency. We follow Turnovsky (1985) and assume there is an intermediation cost $\phi_{bt}$ paid by households in the home country for purchases of foreign bonds, which ensures that net foreign assets are stationary in the model. More specifically, the intermediation costs depend on the ratio of economy-wide holdings of net foreign assets to nominal output and are given by:

$$\phi_{bt} = \exp \left( -\phi_b \left( \frac{e_t B_{Ft+1}}{P_{Dt} Y_t} \right) + \nu_{bt} \right).$$

In the above, $\nu_{bt}$ is a mean-zero stochastic process, which we interpret as a risk-premium shock or shock to the uncovered interest-rate parity condition. Abstracting from this shock, if the home economy has an overall net lender position internationally, then a household will earn a lower return on any holdings of foreign bonds. By contrast, if the economy has a net debtor position, a household will pay a higher return on any foreign debt.

Each member of FL household $h$ earns after-tax labor income, $(1 - \tau_{Nt}) W_t(h) N_t(h)$, where $\tau_{Nt}$ is a stochastic tax on labor income. The household leases capital to firms at the after-tax rental rate $(1 - \tau_{Kt}) R_{Kt}$, where $\tau_{Kt}$ is a stochastic tax on capital income. The household receives a depreciation writeoff of $P_{lt} \tau_{Kt} \delta$ per unit of capital. Each member also receives an aliquot share $\Gamma_t(h)$ of the profits of all firms and a lump-sum government transfer, $TR_t(h)$ and pays a lump-sum tax $T_t(h)$.

---

9 This intermediation cost is asymmetric, as foreign households do not face these costs. Rather, they collect profits on the monopoly rents associated with these intermediation costs.
We allow for two types of costs associated with adjusting the capital stock. First, there is a cost associated with changing the net stock of physical capital, as in the standard q-theory literature; specifically, these costs (in per capita terms) are given by
\[ \phi_{Kt}(h) = \frac{1}{2} \phi_K g_n^{-1} K_t(h) \left( \frac{I_t(h)}{K_t(h)} - \delta g_z g_n \right)^2, \]  
and penalize the adjustment of the deviation of the investment-to-capital ratio from its steady state level, \( \delta g_z g_n \), where \( g_z \) is the steady state growth rate of technology. Second, as in Christiano, Eichenbaum, and Evans (2005), it is also costly to change the level of gross investment from the previous period, so that the acceleration in the capital stock is penalized:
\[ \phi_{It}(h) = \frac{1}{2} \phi_t \left( \frac{I_t(h) - g_z g_n I_{t-1}(h)}{I_{t-1}(h)} \right)^2. \]

In every period \( t \), each member of FL household \( h \) maximizes the utility functional (22) with respect to its consumption, investment, (end-of-period) capital stock, money balances, holdings of contingent claims, and holdings of foreign bonds, subject to its labor demand function (21), budget constraint (23), and transition equation for capital (24). In doing so, a household takes as given prices, taxes and transfers, and aggregate quantities such as lagged aggregate consumption and the aggregate net foreign asset position.

Forward-looking (FL) households set nominal wages in staggered contracts that are analogous to the price contracts described above. In particular, with probability \( 1 - \xi_w \), each member of a household is allowed to reoptimize its wage contract. If a household is not allowed to optimize its wage rate, we assume each household member resets its wage according to:
\[ W_t(h) = \omega_{t-1} W_{t-1}(h), \]
where \( \omega_t = W_t/W_{t-1} \) and in steady state \( \omega = \pi g_z \).\(^{10}\) Each member of household \( h \) chooses the value of \( W_t(h) \) to maximize its utility functional (22), yielding the

\(^{10}\)In alternative specifications, we also consider \( W_t(h) = \omega W_{t-1}(h) \).
following first-order condition:

\[
\tilde{\mathbb{E}}_t \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t \left\{ \frac{1-\tau_{t+j}}{(1+\theta_{t+j})} \Lambda_{t+j} V_{wt+j} W_t(h) - \right. \\
\left. \chi_{0t+j} Z_{t+j}^{1-\sigma} (1 - N_{t+j}(h))^{-\chi} \right\} N_{t+j}(h) = 0,
\]

(29)

where \( \Lambda_t \) is the marginal value of a unit of consumption, and \( V_{wt+j} \) is defined as:

\[
V_{wt+j} = \prod_{h=1}^{j} \omega_{t+h-1}.
\]

(30)

Roughly speaking, equation (29) says that the household chooses its contract wage to equate the present discounted value of working an additional unit of time to the discounted marginal cost.

Finally, we consider the determination of consumption and labor supply of the hand-to-mouth (HM) households. A typical member of a HM household simply equates his nominal consumption spending to his current after-tax disposable income, which consists of labor income plus net lump-sum transfers from the government:

\[
P_{C_t} C_t(h) = (1 - \tau_{N_t}) W_t(h) N_t(h) + T\tau_t(h) - T_t(h).
\]

(31)

The HM households set their wage to be the average of the forward-looking households. Since HM households face the same labor demand schedule as the forward-looking households, each HM household works the same number of hours as the average for forward-looking households.

2.3 Monetary Policy

We assume that the central bank follows an interest rate reaction function similar in form to the historical rule estimated by Orphanides and Wieland (1998) over the Volcker-Greenspan period. Thus, the short-term nominal interest rate is adjusted so that the \textit{ex post} real interest rate rises when inflation exceeds its constant target value, or when output \textit{growth} rises above some target value. With some allowance
for interest rate smoothing, monetary policy is described by the following interest rate reaction function:

\[ i_t = \gamma_i i_{t-1} + \tau + \pi_t + \gamma_\pi (\pi_t^{(4)} - \tilde{\pi}_t) + \gamma_y (y_t - y_{t-4} - g_y) + \epsilon_{it}. \] (32)

In the above, \( i_t \) is the annualized nominal interest rate, \( \pi_t^{(4)} \) is the four-quarter inflation rate of the GDP deflator (i.e., \( \pi_t^{(4)} = \sum_{j=0}^{3} \pi_{t-j} \)), \( \tau \) and \( \tilde{\pi} \) are the steady-state real interest rate and the central bank’s constant inflation target (both expressed at annual rate). Also, \( y_t - y_{t-4} \) is the four-quarter growth rate of output, and \( g_y \) is its corresponding steady state value.

### 2.4 Fiscal Policy

Some of the domestically-produced good is purchased by the government. Government purchases (\( G_t \)) are assumed to have no direct effect on the utility of a household.\(^{11}\) We also assume that government purchases as a fraction of output, \( g_t = G_t / Y_t \), follow an exogenous stochastic process.

The government can issue debt \( B_{Gt+1} \) to finance a deficit so that its budget constraint is given by:

\[ P_{Dt} B_{Gt+1} - B_{Gt} = P_{Dt} G_t + TR_t - T_t - \tau_N W_t L_t - (\tau_K R_K - \delta P_{It}) K_t \]

\[ -(MB_{t+1} - MB_t). \] (33)

In equation (33), we have aggregated the capital stock, money and bond holdings, and transfers and taxes over all households so that, for example, \( T_t = \zeta_t \int_0^T T_t(h)dh \).

As noted above, labor and capital taxes are determined exogenously, while we assume that real transfers as a fraction of domestic output, \( TR_t = \frac{TR_t}{P_{Dt}} \), evolve according to a exogenous stochastic process. Given that the central bank uses the nominal

\(^{11}\) We could have assumed instead that government purchases enter separably in the utility function. This would not alter the model’s dynamics but would have different welfare consequences.
interest rate as its policy instrument, the level of seignorage revenues are determined by nominal money demand.

Lump-sum taxes are adjusted in a manner that the government satisfies an intertemporal solvency constraint, requiring that the present discounted value of the government debt stock tends toward zero in the long run. In particular, we assume that the real lump-sum tax rate, \( \tau_t = \frac{T_t}{P_t Y_t} \), is determined according to the following reaction function:

\[
\tau_t = \nu_0 \tau_{t-1} + \nu_1 (b_{Gl+1} - b_G) + \nu_2 (b_{Gl+1} - b_G),
\]

where \( b_{Gl+1} = \frac{B_{Gl+1}}{P_{DtM_t}} \) and \( b_G \) is the government’s target value for the ratio of government debt to nominal output.

### 2.5 Resource Constraint and Net Foreign Assets

The home economy’s aggregate resource constraint can be written as:

\[
Y_t = C_{Dt} + I_{Dt} + G_t + M_t^* + \phi_{Kt} + \phi_{It},
\]

where \( M_t^* = M_{Ct}^* + M_{It}^* \) and \( \phi_{Kt} \) and \( \phi_{It} \) are the adjustment costs on capital and investment aggregated across all households, and \( G_t \) is government consumption.

The evolution of net foreign assets can be expressed as:

\[
\frac{e_t P_{Bt} B_{Ft+1}}{\phi_{mb}} = e_t B_{Ft} + e_t P_{Mt} M_t^* - P_{Mt} M_t.
\]

This expression can be derived from the budget constraint of the FL households after imposing the government budget constraint, the consumption rule of the HM households, the definition of firm profits, and the condition that domestic bonds \((B_{Dt+1})\) are in zero net supply.\(^{12}\)

Finally, we assume that the structure of the foreign economy (the “rest of the world”) is isomorphic to that of the home country.

\(^{12}\)The derivation of the evolution of net foreign assets also requires that \( P_{Cl} C_t = P_{Dt} C_{Dt} + P_{Mt} M_{Cl} \) and \( P_{It} I_t = P_{Dt} I_{Dt} + P_{Mt} M_{It} \). It is possible to show that these conditions are satisfied even in the presence of the adjustment costs on imported goods (i.e., equations (15) and (18)).
2.6 Description of Shocks and the Optimal Filtering Problem

As discussed above, our model includes shocks to the level of home productivity ($Z_t$), real government spending ($g_t$), real transfers ($tr_t$), labor tax rate ($\tau_{Nt}$), capital tax rate ($\tau_{Kt}$), preferences ($\nu_c$), and similar shocks to the foreign country (which may be regarded as alternative sources of foreign demand shocks from the perspective of the home country). In addition, it includes a shock to the financial intermediation technology ($\nu_{ht}$) or a risk premium shock.

We assume that each shock in the model has two underlying components with different levels of persistence. Agents observe the shock, but they cannot observe the separate components. It is helpful to illustrate the nature of the forecasting problem with reference to a particular shock, since the treatment of the other shocks is basically isomorphic. Thus, focusing on the case of the productivity growth shock examined in Section 5, we begin by noting that equation (4) implies that productivity growth can be written as $\Delta \log(Z_t) = g_z + \Delta z_t$ where movements in $z_t$ are attributable to two separate components:

$$\Delta z_t = \Delta z_{Pt} + \Delta z_{Tt}.$$  

The first component, $\Delta z_{Pt}$, is a highly persistent shock that shifts the “trend” level of productivity growth, and the second, $\Delta z_{Tt}$, is a transient shock to productivity growth. The bivariate process determining the evolution of each component may be represented as:

$$
\begin{bmatrix}
\Delta z_{Pt} \\
\Delta z_{Tt}
\end{bmatrix} =
\begin{bmatrix}
\rho_p & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta z_{Pt-1} \\
\Delta z_{Tt-1}
\end{bmatrix} +
\begin{bmatrix}
\varepsilon_{Pt} \\
\varepsilon_{Tt}
\end{bmatrix},
$$

where the persistence parameter $\rho_p$ is assumed to be slightly below unity, and the transient shock is assumed to be $i.i.d.$ in the case of this particular shock (for other shocks, e.g., to government spending, we allow the transient component to be
somewhat persistent, though much less persistent than the permanent shock). Thus, an innovation to the growth rate has a permanent effect on the level of productivity, but no effect on the future growth rate. Moreover, the innovations $\varepsilon_{Pt}$ and $\varepsilon_{Tt}$ are mutually uncorrelated with variances $v_P$ and $v_T$, respectively.

Agents observe the current level of productivity in the economy, and hence observe $\Delta z_t$, but cannot observe the growth rate of the underlying components $\Delta z_{Pt}$ and $\Delta z_{Tt}$. Thus, agents solve a signal extraction problem to forecast the future level of productivity. Given that agents know the underlying parameters of the bivariate process for productivity growth, the Kalman filter can be used to obtain an optimal solution.

The expected level of productivity at a future date $k$ periods ahead depends only on the current level of productivity, and on the expected growth rate of the permanent component:

$$
\mathbb{E}_t \log(Z_{t+K}) = Kg_z + \log(Z_t) + \mathbb{E}_t(z_{Pt+k} - z_{Pt}) =
Kg_z + \log(Z_t) + \sum_{J=1}^{K} \mathbb{E}_t \Delta z_{Pt+J}.
$$

The Kalman filtering algorithm implies that the expected growth rate of the permanent component is updated according to

$$
\mathbb{E}_t \Delta z_{Pt} = \rho_p \mathbb{E}_{t-1} \Delta z_{Pt-1} + k_g \rho_p (\Delta z_t - \rho_p \mathbb{E}_{t-1} \Delta z_{Pt-1}).
$$

Thus, agents update their assessment of the persistent component of the productivity growth process by the product of the forecast error innovation and a constant coefficient. This coefficient, which is proportional to the scalar Kalman gain parameter $k_g$, is an increasing function of the signal-to-noise ratio $v_P/v_T$ (the ratio of the variances of the persistent and transitory components of the productivity growth process).
3 Solution Method and Calibration

Because the levels of technology and the population are non-stationary, real variables (including output and the expenditure components of GDP) are also non-stationary. Accordingly, prior to solving the model, we scale real variables in the home country by the level of home technology $Z_t$ and the population size, $\zeta_t$, and the real variables in the foreign country by $exp(g_z t)$ and $\zeta^*_t$. Nominal variables are scaled to account both for growth in the corresponding real variable, and for the steady state inflation rate. By construction, the model is stationary in the transformed variables provided that home and foreign economies have the same steady-state population and technological growth rates.

We solve the model by log-linearizing the equations (specified in terms of the transformed variables) around the steady state associated with common growth rates of technology and population in the two countries. To obtain the reduced-form solution of the model, we use the numerical algorithm of Anderson and Moore (1985), which provides an efficient implementation of the method proposed by Blanchard and Kahn (1980) (see also Anderson (1997)).

3.1 Calibration of Parameters

The model is calibrated at a quarterly frequency. Structural parameters are set at identical values for each of the two countries, except for the parameters determining population size (as discussed below). We assume that the discount factor $\beta = .997$ and the rate of technological growth $g_z = 1.0037$. These values are consistent with a steady-state annualized real interest rate $r$ of about 3 percent.

\footnote{The steady state around which we linearize depends on the relative level of technology in each country, which we initialize to unity (so that per capita income in each country is identical in the steady state, though GDP may differ across countries due to population differences). We evaluated the robustness of our solution procedure by using a nonlinear Newton-Raphson algorithm that does not rely on linearization around an initial steady state, and found that the results were nearly identical to those reported.}
The utility functional parameter $\sigma$ is set equal to 2, while the parameter determining the degree of habit persistence in consumption $\tau = 0.8$. We set $\chi = 10$, implying a Frisch elasticity of labor supply of $1/5$, which is considerably lower than if preferences were logarithmic in leisure, but well within the range of most empirical estimates. The utility parameter $\chi_0$ is set so that employment comprises one-third of the household’s time endowment, while the parameter $\mu_0$ on the subutility function for real balances is set an arbitrarily low value (so that variation in real balances has a negligible impact on other variables). We choose $\varsigma = 0.5$ so that 50 percent of households are Ricardian FL agents and the rest are HM agents.

The depreciation rate of capital $\delta = .025$ (consistent with an annual depreciation rate of 10 percent). The price and wage markup parameters $\theta_p = \theta_w = 0.20$, similar to the estimated values obtained by Rotemberg and Woodford (1997) and Amato and Laubach (1999).\textsuperscript{14} We set $\xi_p$ and $\xi_w$ to be consistent with four-quarter contracts (subject to full indexation). The parameter $\xi_{p,x}$ is chosen to be consistent with two-quarter contracts. We set the steady state inflation rate $\pi$ to yield an annual inflation rate of four percent.

The parameter $\rho$ in the CES production function of the intermediate goods producers is set to -2, implying an elasticity of substitution between capital and labor of $1/2$. Thus, capital and labor are less substitutable than the unitary elasticity case implied by the Cobb-Douglas specification. The quasi-capital share parameter $\omega_K$ is chosen to imply a steady state investment to output ratio of 20 percent. The private consumption to output ratio is 70 percent, while government consumption is 10 percent of steady state output. We set the cost of adjusting investment parameter $\phi_I = 3$, slightly below the value used by Christiano, Eichenbaum, and Evans (2001) ($\phi_K = 0$, so that there are only adjustment costs for investment in our baseline

\textsuperscript{14}Rotemberg and Woodford (1997) found $\theta_p = 0.15$, while Amato and Laubach (1999) obtained $\theta_p = 0.19$ and $\theta_w = 0.13$. Given our assumption that there is perfect capital mobility across firms within a country, the parameter $\theta_p$ only affects the steady state capital-output ratio, and does not otherwise appear in the dynamic equations of the log-linearized model.
The parameter $\omega_C$ is chosen to match the estimated average share of imports in total U.S. consumption of about 7 percent (over the 1983-2000 period); while the parameter $\omega_I$ is chosen to match the average share of imports in total U.S. investment of about 30 percent. Given that trade is balanced in steady state, this parameterization implies an import or export to GDP ratio for the home country (the United States) of about 13 percent. We choose the initial population levels $\zeta_0$ and $\zeta_0^*$ so that the home country constitutes about 25 percent of world output. This implied an import (or export) share of output of the foreign country of about 3 percent.

We assume that $\rho_C = \rho_I = 2$, consistent with a long-run price elasticity of demand for imported consumption and investment goods of 1.5. While this is higher than most empirical estimates using macro data, we emphasize that the presence of adjustment costs translates into a much lower relative price sensitivity in the short to medium-term. In particular, we set the adjustment cost parameters $\varphi_{MC} = \varphi_{MI} = 10$, implying a price-elasticity near unity after four quarters. We choose a small value (0.001) for the financial intermediation cost $\phi_b$, which is necessary to ensure the model has a unique steady state.

We estimated the parameters of the monetary policy rule using U.S. data from 1983:1-2003:4.\footnote{We estimated the rule using instrumental variables with lags of inflation and output growth as instruments.} Our estimates implied $\gamma_\pi = 0.6$, $\gamma_g = 0.28$, and $\gamma_i = 0.8$. For the tax rate reaction function, we choose $\nu_0 = 1$, $\nu_1 = 0.1$, $\nu_2 = 0.001$, and $b_C = 0.6$. We set the steady state capital and labor tax rates equal to 0.3 and 0.2, respectively.

## 4 Comparisons of SIGMA and FRB/Global

We now turn to assessing the short-run properties of our SIGMA model, comparing the implications of SIGMA with those of FRB/Global for an array of shocks often
considered in policy analysis. To facilitate comparison across models, we specify the monetary policy rule in FRB/Global to be the same as in the benchmark version of SIGMA.

4.1 Loosening of Monetary Policy

We begin by examining the effects of a transient innovation to the monetary policy rule in SIGMA, i.e., a rise in $\epsilon_{it}$ in equation (32). The shock is scaled to induce an initial decline in the short-term nominal interest rate of about 75 basis points. As shown in Figure 1, this policy shock raises output by slightly less than 0.2 percent after 2-3 quarters. The response of investment is roughly three times larger than the response of consumption, reflecting the higher interest-sensitivity of the former. The fall in domestic real interest rates drives a modest depreciation of the real exchange rate (a rise in the figure), which pushes up import prices (not shown). The combination of a positive output gap and higher import prices causes consumer price inflation to rise.

While the figure shows that the qualitative effects of the shock are similar in FRB/Global, the peak effects on real GDP and the expenditure components are noticeably larger in FRB/Global than in SIGMA. This primarily reflects that long-term real interest rates (which are determined by a small-scale VAR in FRB/Global) drop much more sharply and persistently than in SIGMA (in which long-term real rates are effectively determined by the expectations hypothesis). Given this disparity in the long-term real interest rate responses, it is useful to control for differences in the simulation results that are attributable to alternative term structure equations. Accordingly, the green dash-dotted line in the figure shows an alternative calibration of the shock in SIGMA that implies a response of the 5 year real interest rate that comes very close to matching that in FRB/Global (this calibration assumes that the persistence of the monetary policy error $\epsilon_{it}$ in the interest rate reaction function is 0.6 rather than zero as in our benchmark). In this case, the peak responses of
GDP and the real expenditure components are very close across the two models; the notable difference is in the inflation response, which is much larger in SIGMA.

These results indicate that the responses of the two models to a monetary policy shock that has similar effects on long-term real interest rates is quite similar across the two models (at least for real variables). However, we caution that the similarity in responses appears sensitive to the persistence of the underlying shock. Thus, the results should not be interpreted more broadly as indicating that the interest-sensitivity of aggregate demand is similar across the two models, irrespective of persistence of the shock’s effect on real interest rates. As we show below, SIGMA appears to exhibit a somewhat greater interest-elasticity to shocks that exert highly persistent effects on long-term real interest rates.\(^{16}\)

### 4.2 Rise in Government Spending

Figure 2 shows the effects of an exogenous rise in the U.S. government spending share of GDP of one percentage point relative to baseline. The shock is highly persistent \((\rho_g = 0.975)\), so that the government spending share remains about 0.6 percentage point above baseline after five years.

The rise in government spending induces an immediate expansion of output. The government spending multiplier exceeds unity in the impact period due to the sharp rise in consumption of the HM households (as shown by the dotted light blue line, the impact multiplier would be below unity if all households were optimizers, reflecting a sharp immediate fall in aggregate consumption). However, rising real interest rates

\(^{16}\)The results from our SIGMA model also appear broadly consistent with the implications of the empirical VAR literature that estimates responses to a monetary policy innovation. For example, the results of Christiano, Eichenbaum, and Evans (2005) indicate that a monetary policy innovation that reduces the nominal interest rate by about 75 basis points induces output to rise about 0.4 percent, which is very close to our model for the calibration in which the monetary policy error follows an AR(1) with a persistence parameter of 0.6. However, direct comparisons of the quantitative effects are somewhat difficult because model results are fairly sensitive to the form of the monetary policy rule, which differs from that imposed in the VAR.
quickly crowd out private investment and the consumption of the interest-sensitive optimizing households, which is depressed further due to the negative wealth effect of higher government spending. In fact, the fall in consumption for the optimizing households is exacerbated by the presence of the HM households, as real interest rates rise even more than would occur if all households were optimizers. Thus, overall private consumption falls below baseline after only a few quarters, and most of the output expansion is reversed. The small but more persistent component of the output increase reflects a rise in labor supply that is induced by the negative wealth effect.

Figure 2 also shows impulse responses derived from the FRB/Global model for a similar-sized rise in government spending. Clearly, the output response is noticeably more persistent in FRB/Global, as output remains nearly 0.4 percent above baseline three years after the shock occurs. The greater persistence of the output response reflects that the crowding out of private investment and consumption spending occurs much more gradually. This partly reflects that negative wealth effects play a less prominent role in depressing consumption in FRB/Global. In addition, private absorption (especially investment) is considerably less sensitive to persistent changes in the long-term real interest rate in FRB/Global, especially in the short-run. Thus, while the responses in SIGMA and FRB/Global are similar qualitatively, the responses in the latter tend to exhibit noticeably greater persistence. The empirical VAR evidence of Blanchard and Perotti (2002) also suggests that government spending shocks exert highly persistent effects on output and on the expenditure components that appears more consistent with the FRB/Global responses.

A plausible channel for inducing greater persistence in the impulse responses of the SIGMA model is to allow for imperfect information, as also shown in the figure
Because agents initially perceive that the increase in government spending is temporary under imperfect information, there is a larger increase in GDP in this case than in the benchmark. Furthermore, the rise in GDP is more persistent, as agents only slowly update their beliefs about the persistence of the shock and are continually surprised by the higher-than-expected levels of government spending. Given both a less pronounced rise in real longer-term interest rates and a smaller negative wealth effect (since agents think the government spending rise will be temporary), there is much less crowding out of consumption and investment spending – with consumption even remaining above baseline for a sustained duration. Thus, the inclusion of both non-Ricardian agents and information frictions would seem to provide a tractable channel for increasing the flexibility of SDGE models to account for responses that are similar to large-scale policy models, and to encompass the range of responses derived from empirical VARs.

4.3 Rise in Home Consumption Demand

Figure 3 assesses the effects of a taste shock $\nu_{ct}$ to equation (22) that raises the marginal utility of consumption. This shock may regarded as tantamount to the “autonomous shift in consumption demand” that is often considered in policy simulations. The shock is scaled so that it induces private consumption to rise by one percent above baseline at peak impact, and has a persistence of 0.975.

The taste shock exerts a highly persistent positive effect on real interest rates, accounting for the immediate rise in the 5 year real interest rate shown in the figure. As a result, the stimulative effects of the rise in consumption demand on output are partly offset by a contraction in investment demand, and by a reduction in real net exports. The latter occurs because higher real interest rates generate an appreciation

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17The temporary shock follows an AR(1) with a persistence parameter of 0.5. The innovations variances imply a Kalman gain parameter on the permanent component of .07. We use a similar calibration for the home consumption taste shock and foreign investment demand shocks considered below.
of the real exchange rate. The exchange rate appreciation causes consumer price inflation to fall in the near-term, although higher demand pressures eventually push up domestic goods prices by enough to cause consumer prices to rise.

These effects are qualitatively similar in the case of an autonomous shock to consumption demand in FRB/Global, i.e., a shock to the statistical residual in the consumption equation that is scaled to have the same effect on consumption. But from a quantitative perspective, the output effects are considerably larger in FRB/Global, because there is less crowding out of investment and a smaller decline in real net exports in that model (not shown). The smaller investment decline in FRB/Global reflects that investment is less sensitive to the long-term real interest rate (noting that long rates rise by a roughly commensurate amount in each model). The smaller decline in real net exports in FRB/Global reflects several factors, including less passthrough of the exchange rate to import prices, modestly lower trade price elasticities, and somewhat smaller real appreciation.

As in the case of the government spending rise, allowing for imperfect information about the persistence of the taste shock allows SIGMA to imply a larger and more persistent output response that is very similar to that in FRB/Global. The more gradual rise in long-term interest rates reduces the magnitude of real exchange rate appreciation, leading to a smaller export decline.

4.4 Fall in Home Currency Risk Premium

Figure 4 shows the effects on the home country of a decline in the risk-premium ($\nu_b$) on home currency-denominated assets. As in McCallum and Nelson (1999) and in Kollman (2001), in this simulation we shock the exogenous component of the risk premium in the uncovered interest parity condition implied by the log-linearized model. The shock is scaled so that it induces an initial real appreciation of 10 percent, and the persistence of the shock is $\rho_b = 0.95$.

This shock reduces the required real return on all home-currency denominated
assets relative to the return on foreign assets. The lower required real return on home-currency assets occurs through a combination of persistently lower real interest rates, and through expected real currency depreciation. Thus, longer-term real interest rates fall (not shown), and given that the shock has no long-run effect on the real exchange rate, the exchange rate is required to appreciate sharply in the impact period.

The appreciation of the real exchange rate depresses real exports, and raises imports – and thus exerts a contractionary effect on real GDP. However, private domestic absorption is stimulated by lower real interest rates and lower import prices. As a result, real GDP shows only a modest contraction in the near-term, and actually rises above baseline after a few years as higher investment spending leads to progressive capital deepening. The combination of stronger domestic demand and real exchange rate appreciation induce a significant deterioration of the nominal trade balance exceeding 1-1/2 percentage points of GDP. Finally, PCE inflation shows a sizeable but transient drop due to declining import prices.

The qualitative effects in FRB/Global are very similar, with the exchange rate appreciation leading to an initial output decline, lower real interest rates, some decline in inflation, and a trade balance deterioration. But while the response of GDP is fairly similar across models, there is considerable quantitative disparity in the responses of the expenditure components: private absorption rises much more in SIGMA than in FRB/Global, and real net exports correspondingly exhibit a larger contraction (not shown, though suggested by the larger trade balance response). A key factor accounting for these differences is that the passthrough of exchange rate changes to import prices is effectively 100 percent after a couple of quarters in SIGMA, whereas it is only about 30 percent in FRB/Global. The higher passthrough directly accounts for the larger effects on real imports and exports in the former model, and contributes significantly to driving the sharper swings in PCE price inflation and private absorption. As noted in the discussion of the government
spending shock, the substantial response of private absorption in SIGMA also reflects a highly elastic response to the persistent decline in long-term real interest rates (not shown).

SIGMA’s implication of very high exchange rate passthrough to import prices seems at odds with empirical evidence for the United States that estimates long-run passthrough in the range of 20-30 percent.\textsuperscript{18} Importantly, we emphasize that any model in which the desired markup is fixed (as in SIGMA) would appear unable to match the empirical pattern of import price response incorporated into FRB/Global, in which import prices react fairly quickly to exchange rate changes, but then adjust little subsequently. For example, the dash-dotted green lines marked “sluggish import prices” use a calibration of SIGMA in which the mean duration of export prices is eight quarters (rather than two quarters as in the baseline). Even in this case of long-lived local currency pricing, exchange rate passthrough is nearly complete after two years, and there is much larger trade adjustment than in FRB/Global.

Given the central role of exchange rate passthrough in affecting the transmission of open economy shocks, we believe that it is of crucial importance to develop a theoretical framework that has the flexibility to account for the empirical features of passthrough evident in U.S. data. While some microfounded models can account for long-run passthrough that is below unity, including models with a distribution sector for retail goods as in Corsetti and Dedola (2003), and Corsetti, Dedola, and Leduc (2004), such models do not allow enough variation in desired markups to come close to matching the low level of passthrough in U.S. data.\textsuperscript{19} Until such frameworks are developed, the implausibly large expenditure-switching effects implied by models such as SIGMA may limit their usefulness in addressing some important policy questions. Moreover, such restrictions may lead to large biases in the estimates of

\textsuperscript{18}The low long-run passthrough in FRB/Global is consistent with recent empirical estimates for the United States; see Marazzi, Sheets, and Vigfusson (2005).

\textsuperscript{19}For example, Corsetti, Dedola, and Leduc (2004) find that their inclusion of a distribution sector can account for a reduction in long-run passthrough from unity to slightly under 0.9.
4.5 Alternative Foreign Demand Shocks

Figure 5 shows the effect on the home country of a rise in foreign investment demand. Specifically, investment in the foreign country increases due to a highly persistent decline in its capital income tax rate; but the shock may be regarded somewhat more broadly as any change that boosts the marginal productivity of capital in the foreign country. The shock is scaled so that foreign output eventually rises by 1 percent relative to baseline.

The foreign demand shocks operates through similar channels both in SIGMA and in FRB/Global, and has qualitatively similar effects in each. Thus, the rise in foreign demand stimulates home real net exports, both directly because of the rise in foreign absorption, and indirectly through a depreciation of the home country’s real exchange rate (not shown). This raises home output, though the stimulative effect on net exports is partly offset by declining private absorption as domestic interest rates rise. The nominal trade balance improves despite rising import prices, while higher import prices and stronger activity put upward pressure on consumer prices (not shown).

From a quantitative perspective, the “spillover effects” of the foreign demand increase on the home country are broadly similar for the first two years following the shock across the two models. This suggests that SDGE models may account for substantial spillover effects through trade channels in response to certain types of shocks, although we caution that the fact that the shock affects foreign investment spending – which is heavily import-intensive – plays an important role in accounting for the relatively large effects. In addition, the spillover effects would decline if SIGMA incorporated lower exchange rate passthrough, since this would diminish the magnitude of the home country’s export improvement. It is also evident that SIGMA implies much greater volatility in the expenditure components.
than FRB/Global. The trade balance exhibits a much larger improvement, and the components of private absorption fall much more sharply than in FRB/Global. As might be expected, the inclusion of imperfect information can markedly damp the volatility of the expenditure components in SIGMA by generating a smaller and more gradual response of long-term real interest rates, and by damping the impact on the real exchange rate.

Figure 6 illustrates that spillover effects on the home economy would be much smaller if the foreign output expansion were instead attributable to higher consumption spending. In particular, this alternative simulation assumes that foreign consumption demand rises due to a taste shock, with the shock again scaled so that foreign output peaks at 1 percent above baseline. Given that that foreign investment is crowded out by the shock, the shock has much smaller effects on the home country’s real exports, and induces only a small and transient rise in output in our benchmark calibration of SIGMA. While it is interesting economically that SIGMA implies that the source of the foreign demand shock may be quite important in determining its effects on the domestic economy (whereas the domestic output increase in FRB/Global is broadly similar across the foreign demand shocks), it is plausible that certain features of SIGMA may unduly restrict the ability of the model to generate substantial spillover effects. These include both the high interest sensitivity of private absorption to the long-term real interest rate, and the omission of other potentially important financial channels. Recent work by Gilchrist, Hairault, and Kempf (2002) suggests that a financial accelerator may serve to enhance the ability of open economy SDGE models to account for larger spillover effects.

5 Long-run simulations in SIGMA

We now examine our model’s implications for several supply-side shocks. Each shock exerts a highly persistent effect on output, in part because capital accumulation is
very gradual. In addition to illustrating the model’s long-run implications for the path of adjustment to each of these shocks, our analysis highlights the endogenous channels through which this adjustment occurs.

5.1 Persistent Rise in Productivity Growth

Figure 7 shows the responses of key variables to a productivity growth rate shock under alternative assumptions about the information structure. The shock raises technology growth by 1 percentage point per year over the first five years of the simulation horizon, and then decays slowly following an AR1 process with an autocorrelation parameter of $\rho_p = 0.975$. The magnitude of the technology growth shock is similar to that experienced in the United States between 1996 and 2000; given the decay rate, it is consistent with agents projecting growth 5 years-ahead to rise immediately by about 3/4 percentage point above baseline.

We begin by analyzing the effects of the shock under the assumption that agents have full information, and hence correctly ascertain that the shock will have highly persistent effects on the future growth rate of productivity (see the solid blue lines in the figure). Households project a much sharper rise in their future income than in the pre-shock baseline. This immediately stimulates consumption demand, and depresses the saving rate. The increase in the expected marginal product of capital induces investment to rise (after a short delay). The expansion of domestic demand leads to higher real interest rates, putting upward pressure on the real exchange rate (not shown), and inducing a prolonged deterioration of the trade balance.

A hallmark feature of microfounded models such as SIGMA is that they completely articulate the longer-term economic forces that operate to correct any “imbalances,” including in trade, its components, and the real exchange rate. The imposition of intertemporal budget constraints (and a debt-elastic risk premium) play a key role in generating these endogenous adjustments. In the case of the productivity acceleration, several aspects of the adjustment process account for the
eventual movement of the trade balance into surplus. First, there is a long-run de-
preciation of the real exchange rate due to an increase in the supply of U.S. goods,
which stimulates exports and reduces imports. Second, while the saving rate declines
initially, it eventually increases as current income converges towards permanent in-
come. Finally, after peaking about 10 years after the initial shock, the investment
rate declines as capital approaches its new long-run level.

While these simulation results are instructive in understanding the forces that
bring about long-run adjustment, the assumption that agents immediately recognize
a productivity acceleration as permanent may be somewhat implausible. For exam-
ple, in the U.S. experience of the late 1990s, forecasts of long-term output growth
(i.e., 5-10 years ahead) did not change noticeably until several years after the initial
rise in productivity growth. This provides some empirical motivation for consider-
ing a "gradual learning" case in which agents do not immediately recognize that the
productivity growth shock is highly persistent: given our calibration of the steady
state Kalman gain parameter, they initially believe the shock is mainly transitory.20
In this case, agents project a much less pronounced rise in their income profile, so
that consumption jumps much less than in the full information case; correspond-
ingly, the figure shows that the consumption share of output remains nearly flat.
Given reduced aggregate demand pressure relative to the full information case, real
interest rates rise much less abruptly, which accounts for the somewhat larger rise
in the investment share. While the trade deficit expands, the deterioration is also
somewhat muted relative to the case of full information. This gradual adjustment
of both domestic demand and external variables in response to the shock is more in
line with the U.S. experience in the late 1990s, including with the relative constancy
of the saving rate during that period.

20The temporary shock in this case is iid. The Kalman gain parameter on the permanent component is .10,
implying that it takes roughly 3 years for agents to essentially recognize the shock as permanent.
5.2 Reduction in Labor Tax Rate

As another illustration of the forces that ensure long-run adjustment in the SIGMA model, Figure 8 shows the effects of a cut in the labor tax rate. The tax cut is scaled so that government’s labor tax revenue would fall by one percentage point of GDP if pre-tax labor income and output were unaffected. The shock is assumed to be highly persistent, with the autoregressive parameter $\rho_N$ set to 0.975. This labor tax cut induces the fiscal deficit to rise initially by about 1 percent of GDP, and to decay slowly thereafter.\(^\text{21}\)

The cut in labor taxes induces a sharp initial rise in output. The shock exerts a strong stimulative effect on aggregate demand in the short-run, as the HM households immediately expand their consumption in response to the increase in their after-tax income. The high level of persistence of aggregate consumption reflects that the consumption of the HM households remains high for an extended duration (given that the cut to labor taxes is very persistent, and lump-sum taxes adjust slowly). Output declines from its initial peak as higher real interest rates crowd out investment spending and the consumption of optimizing households. However, output remains persistently above its pre-shock even in the longer-term because the shock exerts substantial supply-side effects: lower tax rates induce households to work more by raising the cost of leisure, and the rise in labor supply in turn encourages capital accumulation.

A clear advantage of our framework is that it is well-suited to explore sensitivity to various structural characteristics determining consumption and labor supply behavior. In the context of assessing the effects of tax cuts, it is often of interest to policymakers to ascertain how results depend on the extent to which households are “Ricardian,” or on their labor supply elasticity. We explore each of these alternatives in the figures. The dashed green lines show the case in which all households are optimizers (rather than assuming 50 percent are optimizers as in the baseline).

\(^{21}\)We provide a more detailed discussion of the effects on the trade deficit in Erceg, Guerrieri, and Gust (2005).
Because all households internalize the future tax increases necessary to satisfy the government intertemporal budget constraint, consumption shows a much smaller initial increase. In the longer-term, the responses are similar to the benchmark, though there is somewhat greater capital deepening in this case. The red dash-dotted line shows the case of a much higher Frisch labor supply elasticity (equal to unity, rather than 0.2 in our baseline). The larger labor supply response induces a much higher level of capital accumulation; accordingly, the longer-run output and consumption rise is greatly accentuated relative to the benchmark calibration.

The implications for the real exchange rate and trade balance under the alternative calibrations are shown in the lower panels. In the benchmark, the sharp rise in the real interest rate induces an initial appreciation in the real exchange rate, which drives the trade balance to deteriorate by around 0.1 percentage point of GDP. In the longer-term, the supply-side effects dominate. Thus, the real exchange rate depreciates due to the higher supply of U.S. goods, while the trade balance shifts into surplus for the same reasons as in the case of the productivity acceleration. The trade balance exhibits a slightly larger deterioration in the calibration with a high labor supply elasticity (given the large stimulative effect on investment), while real exchange rates and trade show little reaction in the case in which all agents optimize.

5.3 Reduction in the Capital Tax Rate

Figure 9 shows the effects of a cut in the capital income tax rate that is scaled so that capital tax revenue would fall by one percentage point of GDP if pre-tax labor income and output were unaffected. The shock is assumed to be highly persistent, with the autoregressive parameter set to 0.95. The capital tax cut induces the fiscal deficit to rise initially by about 1 percent of GDP, and to decay slowly thereafter.

Given that the HM agents do not pay capital taxes, the capital tax rate reduction does not impart the same sort of short-run aggregate demand stimulus evident in the case of the labor tax cut. Accordingly, output increases slowly in line with
the gradual rise in the capital stock. Higher real interest rates encourage somewhat greater saving by optimizing agents, even though the aggregate saving rate responds somewhat less due to the presence of the HM agents. The sharp rise in investment and high import-content of investment goods encourages a substantial rise in imports, and this pressure on the trade balance is reinforced by a short-run real exchange rate appreciation: as a result, the trade balance experiences a peak deterioration of roughly 1/4 percentage point of GDP. In the long-run, real exchange rate depreciation, a fall in the investment rate, and some rise in the saving rate (in part due to higher taxes on HM agents) induce the trade balance to move into persistent surplus.

Figure 9 also shows that output and investment would exhibit larger responses to a capital tax cut if all agents were optimizers (green dotted lines). This reflects that the larger response of domestic saving in the latter case reduces pressure on real interest rates. The output response would be markedly accentuated if all agents were optimizers and had a much higher Frisch elasticity of labor supply of unity (five times higher than in the baseline case; see the red dash-dotted lines), since the larger labor supply response would encourage higher capital accumulation.

6 Conclusion

The recent surge in interest in developing SDGE models for policy analysis seems quite warranted. The SDGE framework possesses some key advantages over that of existing large-scale econometric models by providing a clear linkage between structural features of the economy and its responses to shocks. Moreover, it offers a theoretically consistent framework for analyzing both short- and long-run responses that is helpful in assessing how the economy returns to its balanced growth path following disturbances.

While estimation remains an important future research objective, we have ar-
guessed that an essential prerequisite involves identifying theoretical constraints of the particular SDGE framework that may preclude fitting the data on key dimensions. In this vein, even though it is encouraging that SIGMA implies responses to policy shocks that are generally similar to FRB/Global, there are at least two salient differences that seem attributable to restrictive aspects of our framework: namely, SIGMA implies much larger responses of both trade flows and the domestic expenditure components in response to shocks, and smaller and more transient spillover effects in response to foreign disturbances. Importantly, SIGMA’s implications along these dimensions are likely to characterize a broad class of current SDGE models that imply fixed (or nearly fixed) desired markups, and that adopt a fairly standard framework for modeling investment and consumption behavior.

Given the importance of the magnitude of expenditure-switching effects for a wide range of open economy questions, we believe that it will be important in future work to develop mechanisms that can account for much lower long-term passthrough than implied by our model. Moreover, it will also be desirable to incorporate features that can potentially account for larger responses to foreign disturbances, at least under some conditions. It seems plausible that allowing for sectoral attachments of factors of production or the inclusion of a financial accelerator may allow SDGE models greater flexibility on this dimension.

Finally, while comparisons with FRB/Global are useful in evaluating the flexibility of SIGMA to fit responses similar to that of a data-oriented econometric model, we intend to adopt an estimation strategy that would allow significant departures from the responses of FRB/Global if the data provide a strong enough rationale. In particular, FRB/Global responses could be helpful in setting priors over certain estimated parameters in SIGMA in the context of a Bayesian maximum likelihood approach, though sizeable differences could emerge between the responses of the two models if the posterior distribution over the parameters diverged significantly from the prior.
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Figure 1: Monetary Policy Loosening

- **Real GDP**
  - Benchmark
  - Match 5-year real rate
  - FRB/Global

- **Real Interest Rate (5 year)**

- **Private Consumption**

- **Investment**

- **Real Exchange Rate**

- **Consumer Price Inflation**
Figure 2: Rise in Government Spending

- **Real GDP**
- **Real Interest Rate (5 yr)**
- **Private Consumption**
- **Investment**
- **Real Exchange Rate**
- **Consumer Price Inflation**
Figure 3: Rise in Home Consumption Demand

- **Real GDP**
- **Real Interest Rate (5 yr)**
- **Private Consumption**
- **Investment**
- **Real Exchange Rate**
- **Consumer Price Inflation**
Figure 4: Fall in Home Currency Risk Premium

- **Real GDP**
- **Import Prices**
- **Private Absorption**
- **Consumer Price Inflation**
- **Real Exchange Rate**
- **Trade Balance/GDP**

The graphs show the trends in various economic indicators over the years 2004 to 2008. The indicators include Real GDP, Import Prices, Private Absorption, Consumer Price Inflation, Real Exchange Rate, and Trade Balance/GDP. The graphs highlight the impact on benchmark, sluggish import prices, and FRB/Global scenarios.
Figure 5: Rise in Foreign Investment Demand

- Real GDP
- Real Interest Rate (5 yr)
- Private Consumption
- Investment
- Foreign Real GDP
- Trade Balance/GDP

Legend:
- Benchmark (full information)
- Imperfect Information
- Frb/Global
Figure 6: Rise in Foreign Consumption Demand

- Real GDP
- Real Interest Rate (5 year)
- Private Consumption
- Investment
- Foreign GDP
- Trade Balance/GDP

Legend:
- Benchmark
- Lower Interest Sensitivity
- FRB/Global

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Figure 7: Rise in Home Productivity Growth Rate

Real GDP Growth (yr/yr)

Real Interest Rate (5 year)

Private Consumption/GDP

Investment/GDP

Consumer Price Inflation (yr/yr, AR)

Trade Balance/GDP
Figure 8: Fall in Home Labor Tax Rate

- **Real GDP**
- **Real Interest Rate (5 year)**
  - Benchmark
  - All Households Optimize
  - High Labor Supply Elasticity
- **Private Absorption**
- **Government Deficit/GDP**
- **Real Exchange Rate**
- **Trade Balance/GDP**
Figure 9: Fall In Home Capital Tax Rate

- **Real GDP**
- **Real Interest Rate (5 year)**
- **Private Consumption**
- **Investment**
- **Real Exchange Rate**
- **Trade Balance/GDP**