Fully Modified Estimation with Nearly Integrated Regressors

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Erik Hjalmarsson*

Division of International Finance

Federal Reserve Board, Mail Stop 20, Washington, DC 20551, USA

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Abstract

I show that the test procedure derived by Campbell and Yogo (2005, Journal of Financial Economics, forthcoming) for regressions with nearly integrated variables can be interpreted as the natural t-test resulting from a fully modified estimation with near-unit-root regressors. This clearly establishes the methods of Campbell and Yogo as an extension of previous unit-root results.

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Keywords: Fully modified estimation; Near-unit-roots; Predictive regressions.

*Tel.: +1-202-452-2436; fax: +1-202-263-4850; email: erik.hjalmarsson@frb.gov. The views presented in this paper are solely those of the author and do not represent those of the Federal Reserve Board or its staff.

1 Introduction

In the recent past, there has been much effort spent on the econometric analysis of forecasting regressions with nearly persistent regressors. Using Monte Carlo simulations, Mankiw and Shapiro (1986) showed in an influential paper that when the regressor variables in a predictive regression are almost persistent and endogenous, test statistics will no longer have standard distributions. Since then, a much better understanding of this phenomenon has been established and several alternative methods have been proposed; e.g. Cavanagh et al. (1995), Stambaugh (1999), Jansson and Moreira (2004), Lewellen (2004), and Campbell and Yogo (2005).

The issues encountered when performing inference in forecasting regressions with near-persistent variables are, of course, similar to those in the cointegration literature where the regressors are assumed to follow unit-root processes. Indeed, the case with nearly persistent regressors can be seen as a generalization of the standard unit-root setup.

In this note, I show that the efficient test for inference in predictive regressions derived by Campbell and Yogo (2005) can also be seen as the natural test resulting from a generalization of fully modified estimation (Phillips and Hansen, 1990, and Phillips, 1995) to the case of near-unit-root regressors. In addition, the optimality properties of the Campbell and Yogo (2005) test-statistic can be seen as a direct analogue of the optimal inference results derived by Phillips (1991) for cointegrated unit-root systems. These results firmly establish the link between current work on predictive regressions and earlier work on cointegration between unit-root variables.

2 Model and assumptions

Let the dependent variable be denoted y_t, and the corresponding vector of regressors, x_t, where x_t is an $m \times 1$ vector and $t = 1, ..., T$. The behavior of y_t and x_t are assumed to satisfy,

$$y_t = \alpha + \beta x_{t-1} + u_t, \tag{1}$$

$$x_t = A x_{t-1} + v_t, \tag{2}$$

where $A = I + C/T$ is an $m \times m$ matrix.

Assumption 1 *Let $w_t = (u_t, \epsilon_t)'$ and $\mathcal{F}_t = \{w_s | s \leq t\}$ be the filtration generated by w_t. Then*

1. $v_t = D(L)\epsilon_t = \sum_{j=0}^{\infty} D_j \epsilon_{t-j}$, *and* $\sum_{j=0}^{\infty} j\,||D_j|| < \infty$.

2. $E[w_t|\mathcal{F}_{t-1}] = 0$, $E[u_t^4] < \infty$, *and* $E\left[||\epsilon_t||^4\right] < \infty$.

3. $E[w_t w_t'|\mathcal{F}_{t-1}] = \Sigma = [(\sigma_{11}, \sigma_{12}), (\sigma_{21}, I)]$.

The model described by equations (1) and (2) and Assumption 1 captures the essential features of a predictive regression with nearly persistent regressors. It states the usual martingale difference (mds) assumption for the errors in the dependent variables but allows for a linear time-series structure in the errors of the predictors. The error terms u_t and v_t are also often highly correlated. The auto-regressive roots of the regressors are parametrized as being local-to-unity, which captures the near-unit-root behavior of many predictor variables, but is less restrictive than a pure unit-root assumption.

The local-to-unity parameter C is generally unknown and not consistently estimable. Following Campbell and Yogo (2005), I derive the results under the assumption that C *is* known. Bonferroni type methods can then be used to form feasible tests, as in Cavanagh et al. (1995) and Campbell and Yogo (2005); such methods are extensively explored in these papers and will not be further discussed here.

Let $E_t = (u_t, v_t)'$ be the joint innovations process. Under Assumption 1, by standard arguments, $\frac{1}{\sqrt{T}} \sum_{t=1}^{[Tr]} E_t \Rightarrow B(r) = BM(\Omega)(r)$, where $\Omega = [(\omega_{11}, \omega_{12}), (\omega_{21}, \Omega_{22})]$, $\omega_{11} = \sigma_{11}$, $\omega_{21} = D(1)\sigma_{12}$, $\omega_{12} = \omega_{21}'$, $\Omega_{22} = D(1)D(1)'$, and $B(\cdot) = (B_1(\cdot), B_2(\cdot))'$ denotes an $1+m-$dimensional Brownian motion. Also, let $\Lambda_{22} = \sum_{k=1}^{\infty} E(v_k v_0')$ be the one-sided long-run variance of v_t. The following lemma sums up the key asymptotic results for the nearly integrated model in this paper (Phillips 1987, 1988).

Lemma 1 *Under Assumption 1, as* $T \to \infty$, *(a)* $T^{-1/2} x_{i,[Tr]} \Rightarrow J_C(r)$, *(b)* $T^{-3/2} \sum_{t=1}^{T} x_t \Rightarrow \int_0^1 J_C(r)\,dr$, *(c)* $T^{-2} \sum_{t=1}^{T} x_t x_t' \Rightarrow \int_0^1 J_C(r) J_C(r)'\,dr$, *(d)* $T^{-1} \sum_{t=1}^{T} u_t x_{t-1}' \Rightarrow \int_0^1 dB_1(r) J_C(r)'$, *and (e)* $T^{-1} \sum_{t=1}^{T} v_t x_{t-1}' \Rightarrow \int_0^1 dB_2(r) J_C(r)' + \Lambda_{22}$, *where* $J_C(r) = \int_0^r e^{(r-s)C} dB_2(s)$.

Analogous results hold for the demeaned variables $\underline{x}_t = x_t - T^{-1} \sum_{t=1}^{n} x_t$, with the limiting process J_C replaced by $\underline{J}_C = J_C - \int_0^1 J_C$.

3 Fully modified estimation

Let $\hat{\beta}$ denote the standard OLS estimate of β in equation (1). By Lemma 1 and the continuous mapping theorem (CMT), it follows that

$$T\left(\hat{\beta}-\beta\right) \Rightarrow \left(\int_0^1 dB_1 \underline{J}_C'\right)\left(\int_0^1 \underline{J}_C \underline{J}_C'\right)^{-1}, \tag{3}$$

as $T \to \infty$. Analogous to the case with pure unit-root regressors, the OLS estimator does not have an asymptotically mixed normal distribution due to the correlation between B_1 and B_2, which causes B_1 and J_C to be correlated. Therefore, standard test procedures cannot be used.

In the pure unit-root case, one popular inferential approach is to "fully modify" the OLS estimator as suggested by Phillips and Hansen (1990) and Phillips (1995). In the near-unit-root case, a similar method can be considered. Define the quasi-differencing operator

$$\Delta_C x_t = x_t - x_{t-1} - \frac{C}{T} x_{t-1} = v_t, \tag{4}$$

and let $y_t^+ = y_t - \hat{\omega}_{12}\hat{\Omega}_{22}^{-1}\Delta_C x_t$ and $\hat{\Lambda}_{12}^+ = -\hat{\omega}_{12}\hat{\Omega}_{22}^{-1}\hat{\Lambda}_{22}$, where $\hat{\omega}_{12}, \hat{\Omega}_{22}^{-1}$, and $\hat{\Lambda}_{22}$ are consistent estimates of the respective parameters.[1] The fully modified OLS estimator is now given by

$$\hat{\beta}^+ = \left(\sum_{t=1}^T \underline{y}_t^+ \underline{x}_{t-1}' - T\hat{\Lambda}_{12}^+\right)\left(\sum_{t=1}^T \underline{x}_{t-1}\underline{x}_{t-1}'\right)^{-1}, \tag{5}$$

where $\underline{y}_t^+ = \underline{y}_t - \hat{\omega}_{12}\hat{\Omega}_{22}^{-1}\Delta_C x_t$ and $\underline{y}_t = y_t - T^{-1}\sum_{t=1}^t y_t$. The only difference in the definition of (5), to the FM-OLS estimator for the pure unit-root case, is the use of the quasi-differencing operator, as opposed to the standard differencing operator. Once the innovations v_t are obtained from quasi-differencing, the modification proceeds in exactly the same manner as in the unit-root case.

Define $\omega_{11\cdot 2} = \omega_{11} - \omega_{12}\Omega_{22}^{-1}\omega_{21}$ and the Brownian motion $B_{1\cdot 2} = B_1 - \omega_{12}\Omega_{22}^{-1}B_2 = BM\left(\omega_{11\cdot 2}\right)$. The process $B_{1\cdot 2}$ is now orthogonal to B_2 and J_C. Using the same arguments as Phillips (1995), it follows that, as $T \to \infty$,

$$T\left(\hat{\beta}^+ - \beta\right) \Rightarrow \left(\int_0^1 dB_{1\cdot 2}\underline{J}_C{}'\right)\left(\int_0^1 \underline{J}_C\underline{J}_C'\right)^{-1} \equiv MN\left(0, \omega_{11\cdot 2}\left(\int_0^1 \underline{J}_C\underline{J}_C'\right)^{-1}\right). \tag{6}$$

[1] The definition of $\hat{\Lambda}_{12}^+$ is slightly different from the one found in Phillips (1995). This is due to the predictive nature of the regression equation (1), and the martingale difference sequence assumption on u_t.

The corresponding test-statistics will now have standard distributions asymptotically. For instance, the $t-$test of the null hypothesis $\beta_k = \beta_k^0$ satisfies

$$t^+ = \frac{\hat{\beta}_k^+ - \beta_k^0}{\sqrt{\hat{\omega}_{11\cdot 2} a' \left(\sum_{t=1}^T \underline{x}_{t-1} \underline{x}_{t-1}' \right)^{-1} a}} \Rightarrow N(0,1) \tag{7}$$

under the null, as $T \to \infty$. Here a is an $m \times 1$ vector with the k'th component equal to one and zero elsewhere.

The t^+-statistic is identical to the $Q-$statistic of Campbell and Yogo (2005). Whereas Campbell and Yogo (2005) attack the problem from a test point-of-view, the derivation in this paper starts with the estimation problem and delivers the test-statistic as an immediate consequence. However, presenting the derivation in this manner makes clear that this approach is a generalization of fully modified estimation.

In addition, if Assumption 1 is replaced by the stronger condition that both u_t and v_t are martingale difference sequences, it is easy to show that OLS estimation of the augmented regression

$$y_t = \alpha + \beta x_{t-1} + \gamma \Delta_C x_t + u_{t \cdot v} \tag{8}$$

yields an estimator of β with an asymptotic distribution identical to that of $\hat{\beta}^+$. This is, of course, a straightforward extension of the results in Phillips (1991) for unit-root regressors. Moreover, in the unit-root case, Phillips (1991) shows that the OLS estimator of β in equation (8) is identical to the gaussian full system maximum likelihood estimator of β. The optimality properties of Campbell and Yogo's (2005) $Q-$test is thus a direct extension of the optimality results developed in Phillips (1991).

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