Board of Governors of the Federal Reserve System

International Finance Discussion Papers

Number 884

November 2006

International Asset Markets And Real Exchange Rate Volatility

Martin Bodenstein

International Asset Markets And Real Exchange Rate Volatility*

Martin Bodenstein†

November 2006

Abstract

The real exchange rate is very volatile relative to major macroeconomic aggregates and its correlation with the ratio of domestic over foreign consumption is negative (Backus-Smith puzzle). These two observations constitute a puzzle to standard international macroeconomic theory. This paper develops a two country model with complete asset markets and limited enforcement for international financial contracts that provides a possible explanation of these two puzzles. The model performs poorly with respect to asset pricing. However, with limited enforcement for both domestic and international financial contracts, the model's asset pricing implications are brought into line with the empirical evidence, albeit at the expense of raising real exchange rate volatility.

Keywords: limited enforcement, Backus-Smith puzzle, asset prices

JEL classifications: F 31, G 12

*I benefited from discussions with Roc Armenter, Larry Christiano, Marty Eichenbaum, Etienne Gagnon and Sergio Rebelo. I am also thankful to seminar participants at Northwestern University, SAIS Johns Hopkins, University of Rochester (Finance), the Federal Reserve Board, and the European Central Bank. All remaining errors are mine. Financial support from the Center for International Economics and Development at Northwestern University is gratefully acknowledged. The views expressed in this paper are solely the responsibility of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or any other person associated with the Federal Reserve System.

†Board of Governors of the Federal Reserve System, 20th Street and Constitution Avenue NW, Washington, D.C. 20551. Martin.R.Bodenstein@frb.gov.

1 Introduction

This paper analyses the interplay of three classic puzzles about the real exchange rate and asset prices:

1. the high volatility of the real exchange rate relative to the volatility of consumption (real exchange rate volatility puzzle),

2. the negative correlation of the real exchange rate with the ratio of domestic over foreign consumption (Backus-Smith puzzle),

3. the volatility of asset prices and the associated volatility of the intertemporal marginal rate of substitution (equity premium puzzle).

In their simplest form the first two puzzles can be stated as follows.[1] If preferences over consumption are given by the power utility function and all financial markets are complete, the real exchange rate between two countries is driven by the ratio of domestic and foreign consumption. Since there are no wealth effects under complete markets, consumption is highly correlated across countries. Therefore, the real exchange rate hardly fluctuates. Furthermore, the correlation between the real exchange rate and relative consumption equals unity, as the real exchange rate is solely a function of relative consumption.

Given this apparent contradiction with the data, most international macroeconomists have concluded that international financial risk sharing is not complete. Although it is nowadays standard to assume that international financial markets are limited to one non-state-contingent bond, there has been only little progress in explaining the first two puzzles. Two notable exceptions are Corsetti, Dedola and Leduc (2005) and Benigno and Thoenissen (2006).[2]

Brandt, Cochrane and Santa-Clara (2006) have recently challenged the view that international consumption risk sharing is very limited. Their analysis draws on the high volatility of asset prices and the implied high volatility of the intertemporal marginal rate of substitution. Real exchange rates between industrialized economies fluctuate by as much as 10% per annum. However, the intertemporal marginal rate of substitution estimated using asset returns varies by 40%. As the real exchange rate depreciates by the difference between the domestic and foreign intertemporal marginal rates of substitution, these estimated volatilities imply that the intertemporal marginal rates of substitution are highly correlated between countries.

[1] See Obstfeld and Rogoff (1996, 2000) for the volatility puzzle and Backus and Smith (1993) for the consumption-real exchange rate correlation puzzle.

[2] Lewis (1996) performs an econometric test on complete risk sharing, but finds little support for it.

Brandt et al interpret their findings as clear evidence, that international risk sharing is very good.

This paper attempts to clarify these contradictory conclusions about international risk sharing. I first follow Kehoe and Levine (1993) in assuming that international financial markets are complete but enforcement of international financial contracts is limited. Contracts are sustainable only to the extent that they can be enforced by the threat of permanent exclusion from trade in international financial markets if an agent reneges on her obligations.[3]

The production/trade side of the economy is modelled as in Corsetti at al (2005). The distinguishing feature of their model is that the implied elasticity of substitution between traded goods is low since non-traded goods have to be used for the distribution of traded goods. This feature implies that if there were no international financial markets at all the real exchange rate is very volatile and the correlation between the real exchange rate and relative consumption is negative.

The key finding of my paper is that the model with complete asset markets and enforcement constraints can resolve the real exchange rate volatility puzzle and the Backus-Smith puzzle provided that agents are sufficiently impatient. If agents are impatient, only limited risk sharing can be sustained and the model behaves close to a model without international financial markets. If agents are very patient, contract enforcement works well and agents can share risk efficiently across countries. In this case consumption is highly correlated across countries. The real exchange rate is very smooth and the correlation between the real exchange rate and relative consumption is close to unity.

Because I follow the international finance literature in assuming complete and frictionless domestic asset markets and standard preferences, the model inherits all the puzzles of domestic asset pricing. In particular all asset prices are very smooth and the equity premium is too low.[4] One potential resolution of the equity premium puzzles in a closed economy is offered by Alvarez and Jermann (2001). In line with empirical findings, these authors assume that agents' idiosyncratic incomes are volatile relative to aggregate income. Also, asset markets are assumed to be complete but enforcement of financial contracts is limited.

Following the ideas in Alvarez and Jermann (2001), I subsequently enrich my model by assuming that contract enforcement is also limited for domestic financial contracts. My main findings are: first, as in the data, the intertemporal marginal rates of substitution are volatile and so are asset prices. The standard deviation of the marginal rate of substitution is about 40%. Second, the model can still explain the Backus-Smith puzzle. Third, in sharp contrast to the original model, the real exchange rate is too volatile. The standard deviation of the

[3] Kehoe and Perri (2002) analyze a two country model with limited contract enforcement. However, since there is only one good in their model all trade is intertemporal and the real exchange rate is constant and equal to 1.

[4] See in particular Mehra and Prescott (1985) and Hansen and Jagannathan (1991).

real exchange jumps from 7% to 60%.

What explains this drastic increase? Note that the change in the real exchange rate equals the difference between the log of the foreign and the domestic intertemporal marginal rates of substitution. In the original model, the volatility of the marginal rates of substitution is determined by the low volatility of aggregate consumption. However, the marginal rates of substitution are volatile enough to imply exchange rates that are roughly as volatile as in the data.

In the extended model, the high volatility of the marginal rates of substitution stems from high idiosyncratic income risk that cannot be insured efficiently due to limited enforcement in domestic asset markets. Highly volatile marginal rates of substitution with a standard deviation of roughly 40% can only be reconciled with an exchange rate volatility of around 7% if the correlation between the foreign and the domestic marginal rates of substitution is larger than 0.9. However, a correlation of 0.9 cannot arise in the model with limited enforcement. This class of models implies volatile marginal rates of substitution only if risk is not shared efficiently both domestically and internationally. Consequently, the correlation of the marginal rates of substitution implied by the model is 0.16.

This paper is closely related to the works of Corsetti et al (2005) and Brandt et al (2006). Corsetti et al address the exchange rate volatility puzzle and the Backus-Smith puzzle in a model similar to mine. However, they assume that international financial markets are exogenously incomplete: the only asset that is traded internationally is one non-state-contingent bond. This assumption, although widely used, is very strong from an empirical perspective. I show in this paper how their results extend to an environment with a larger set of available assets.

Based on the Backus-Smith puzzle, Corsetti et al (2005) conclude like many others before that international risk sharing is very limited.[5] This conclusion stands in sharp contrast to Brandt et al (2006) who argue the opposite based on asset return data. This contradiction arises since each group of authors considers only two out of the three puzzles mentioned above. In line with the international finance literature Corsetti et al are silent with respect to the volatility of asset prices (3. puzzle). Brandt et al do not relate their findings to the Backus-Smith puzzle (2. puzzle).

Colacito and Croce (2006) and Verdelhan (2006) also provide insight into the work of Brandt et al. They suggest modelling frameworks that are consistent with the observed volatility of the real exchange rate and the volatility of asset returns (1. and 3. puzzle). Unfortunately, neither approach provides a satisfying answer to the Backus-Smith puzzle. The correlation between the real exchange rate and relative consumption is close to or equal to unity in both papers.

[5] See Lewis (1999) for a summary of the literature on international risk sharing.

The remainder of the paper is organized as follows. Section 2 provides a deeper introduction to the puzzles that are analysed in this paper. In Section 3, I present a two country model with complete international financial markets and enforcement constraints. Section 4 presents and discusses the qualitative and quantitative implications of the benchmark model. In order to address the evidence provided in Brandt et al (2006), Section 5 extends the benchmark model to a two country model with heterogenous agents. Section 6 concludes.

2 Real exchange rate puzzles

Under complete markets, the real exchange rate between two countries is given by the ratio of marginal utilities

$$q_t = \kappa \frac{MU_{2,t}}{MU_{1,t}}, \text{ with } \kappa = q_0 \frac{MU_{2,0}}{MU_{1,0}}, \tag{1}$$

or equivalently the change of the real exchange rate equals the difference of marginal utility growth between two countries

$$\log \frac{q_{t+1}}{q_t} = \log \frac{MU_{2,t+1}}{MU_{2,t}} - \log \frac{MU_{1,t+1}}{MU_{1,t}}, \tag{2}$$

where

q_t	is the real exchange rate defined as the price level in country 2 over the price level in country 1 $(P_{2,t}/P_{1,t})$,
$MU_{i,t}$	is the marginal utility in country i.

Backus, Foresi and Telmer (1996) and Appendix A provide a derivation of these expressions using the law of one price and the no-arbitrage condition.

2.1 The correlation puzzle

Assume that agents have preferences described by the power utility function, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, where c is consumption and γ is the coefficient of relative risk aversion. The predicted correlation between the real exchange rate and relative consumption for a given country pair equals unity, i.e.,

$$\rho_{q,\frac{c_1}{c_2}} = corr\left(-\gamma \log \frac{c_{1,t}}{c_{2,t}}, \log \frac{c_{1,t}}{c_{2,t}}\right) = 1.$$

However, Backus and Smith (1993) find that the actual correlation between exchange rates and relative consumption is low and often negative. Chari, Kehoe and McGrattan (2002), Corsetti, Dedola and Leduc (2004) confirm these findings. The latter report an average estimate of -0.25.

2.2 The volatility of the real exchange rate

Testing the implications of equations (1) and (2) for the volatility of the real exchange rate requires data on marginal utility. Unfortunately, marginal utility cannot be observed directly in the data. One way to get around this problem is by assuming a particular utility function and measure marginal utility as a function of consumption.

When equation (1) is embedded into a general equilibrium model of the international business cycle, the predicted volatility of the real exchange rate (σ_q) relative to consumption (σ_{c_1}) is too low for reasonable levels of risk aversion (γ). In the data, the real exchange rate is roughly four times as volatile as consumption. However, models with complete international financial markets typically predict a very high correlation of consumption across countries $\left(\rho_{c_1c_2}\right)$. With $\rho_{c_1c_2}$ close to 1,

$$\frac{\sigma_q}{\sigma_{c_1}} = \gamma\sqrt{2\left(1-\rho_{c_1c_2}\right)},$$

one can match a volatility ratio of $\frac{\sigma_q}{\sigma_{c_1}} \approx 4$ only if γ is implausibly large. Of course, the model's implication of $\rho_{c_1c_2}$ being close to unity is already unrealistic, as the data suggests a value of 0.33.

2.2.1 The volatility of asset prices

Another way to evaluate equation (2) is through estimating the intertemporal marginal rate of substitution, $\frac{MU_{i,t+1}}{MU_{i,t}}$, which can be done using asset prices. As shown in Appendix A, the $IMRS$ can be estimated directly from the data using only asset prices. Although my estimates for the standard deviation of the $IMRS$ are lower than in the literature, the annualized standard deviation of the $IMRS$ is still about 40% and therefore much higher than the roughly 6% of the real exchange rate.[6] From equation (2), this implies that the $IMRS$ for the U.S. and the aggregate of the remaining G7 countries must be very highly correlated with a correlation coefficient of more than 0.98!

Using equity and bond returns for the G7, the correlation between the $IMRS_{US}$ and the $IMRS_{G7\backslash US}$ varies between 0.9908 and 0.9916 depending on the aggregation method. For more details on the data and the aggregation, the interested reader is referred to Appendix A. To see that this high correlation is not simply an artefact of aggregation, I also report correlations of the $IMRS$ for each country pair

[6] The general consensus is that the MRS varies by at least 50% for US stock market data. In my case the standard deviations are lower since I have to use a more volatile proxy for the risk free rate to calculate excess returns for equity for the reason of data availability.

	CAN	FRA	GER	ITA	JAP	UK	USA
CAN	1.0000	0.9730	0.9843	0.9580	0.9384	0.9665	0.9961
FRA		1.0000	0.9995	0.9929	0.9842	0.9800	0.9828
GER			1.0000	0.9971	0.9399	0.9942	0.9841
ITA				1.0000	0.9643	0.9781	0.9766
JAP					1.0000	0.9635	0.9651
UK						1.0000	0.9763
USA							1.0000

Table 1

These results are comparable to Brandt, Cochrane and Santa-Clara (2006), who compare the behavior of the $IMRS$ for the U.S. with the UK, Germany and Japan respectively. Brandt et al interpret the high correlation as an indication of substantial risk sharing between countries.

3 Setup

3.1 Financial Markets

Each period t the economy experiences one of finitely many events $s \in S$. Let the transition probability from state s to s' follow a Markov chain denoted by $\pi(s'|s)$. $s^t = (s_0, s_1, ...s_t)$ denotes the history of events up through and including period t. The probability, as of period 0, of any history s^t is $\pi(s^t)$. With the initial realization s_0, the Markov transition probabilities induce the probability distribution

$$\pi\left(s^t\right) = \pi\left(s_t|s_{t-1}\right)\pi\left(s_{t-1}|s_{t-2}\right)...\pi\left(s_1|s_0\right). \tag{3}$$

There are two countries, $i = 1, 2$, each of which is populated by a large number of identical, infinitely lived households. At the beginning of each period, households are endowed with $y_i^T(s_t)$ units of a tradable good and $y_i^N(s_t)$ units of a non-tradable good. The domestic and foreign tradable good are imperfect substitutes. Let $y(s_t) = \left(y_1^T, y_2^T, y_1^N, y_2^N\right)$ be the endowment vector in state s_t. The endowment vector depends solely on the current realization s_t. Final consumption in country i in history s^t, $c_i(s^t)$, is a function of the consumption of the two tradables and the non-tradable good. A more explicit structure of the goods market is introduced in section 3.5. For now, all that is assumed, is that the endowment vector at time t can be mapped into an aggregate international resource constraint

$$\tilde{F}(c_1, c_2, y) \leq 0. \tag{4}$$

The set of feasible consumption allocations $\left\{(c_1, c_2) \,|\tilde{F}(c_1, c_2, y) \le 0\right\}$ is non-empty, bounded, and strictly convex for each realization of the endowment vector. The latter is an immediate implication of the imperfect substitutability of the domestic and the foreign tradable good. The function $F(\cdot)$, defined as $F(c_1, c_2, y) = 0$, is differentiable with respect to its first two arguments. Since the real exchange rate is defined as the price of the consumption basket in country 2 relative to country 1, the real exchange rate is linked to $F(\cdot)$ through $q = \frac{F_2(c_1,c_2,y)}{F_1(c_1,c_2,y)} = -\frac{dc_1}{dc_2}$. F_i is the derivative of F with respect to its ith argument.

Households in country i rank consumption streams $\{c_i(s^t)\}_{t=0}^{\infty}$ according to

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) u\left[c_i\left(s^t\right)\right]. \tag{5}$$

$u(c)$ is strictly increasing, strictly concave and continuously differentiable. The budget constraint of the agent is given by

$$\begin{aligned} & P_i\left(s^t\right) c_i\left(s^t\right) + \sum_{s^{t+1}|s^t} Q_i\left(s^{t+1}|s^t\right) a_i\left(s^{t+1}|s^t\right) \\ \le\ & a_i\left(s^t|s^{t-1}\right) + \bar{P}_{ii}^T\left(s^t\right) y_i^T\left(s^t\right) + P_i^N\left(s^t\right) y_i^N\left(s^t\right), \end{aligned} \tag{6}$$

where P_i is the (currency) price of one unit of the final consumption good in country i, $\bar{P}_{ij}^T$ is the price of tradable good i in country j, and P_i^N is the price of the non-tradable good in country i.

In this economy, financial markets are complete, i.e., agents have access to a complete set of one-period state-contingent claims. The holdings of such claims by the representative agent in country i are denoted by $a_i(s^{t+1}|s^t)$. Each claim pays one unit of country $i's$ currency in period $t+1$ if the particular state s_{t+1} occurs and 0 otherwise. $Q_i(s^{t+1}|s^t)$ is the price of such a claim in country i's currency.

Building on the seminal work of Kehoe and Levine (1993) and Kocherlakota (1996) international loans are assumed to be sustainable to the extent that they can be enforced by the threat of exclusion from future trade in asset markets.[7] The enforcement constraint is therefore given by

$$\sum_{r=t}^{\infty} \sum_{s^r|s^t} \beta^{r-t} \pi\left(s^r|s^t\right) u\left[c_i\left(s^r\right)\right] \ge V_i\left(s^t\right), \tag{7}$$

where $V_i(s^t)$ is the value for agent i in financial autarchy from s^t onwards. Notice that like in the original paper by Kehoe and Levine (1993) but unlike in the one-good models of

[7] Fitzgerald (2006) reports empirical evidence that is in line with the assumption of limited contract enforceability at the international level.

Alvarez and Jermann (2000) or Kehoe and Perri (2002), I assume that agents can still trade in the international goods markets after default. Exclusion applies to financial markets only.

In Kehoe and Perri (2002), the decision to default is made by the government. In this case the value of financial autarchy, $V_i\left(s^t\right)$, is given by the discounted present value at the prices that actually occur in autarchy. If the default decision is made by the individual agent, however, each agent assumes that her decision to default will not affect prices in the goods market. The agent does not take into account that other agents might default, as well.

In any case, the value of financial autarchy is determined from

$$V_i\left(s^t\right) = \max_{\{c_i(s^r)\}} \sum_{r=t}^{\infty} \sum_{s^r|s^t} \beta^{r-t} \pi\left(s^r|s^t\right) u\left[c_i\left(s^r\right)\right] \tag{8}$$

$s.t.$

$$P_i\left(s^t\right) c_i\left(s^t\right) \leq \bar{P}_{ii}^T\left(s^t\right) y_i^T\left(s^t\right) + P_i^N\left(s^t\right) y_i^N\left(s^t\right), \tag{9}$$

where the perceived prices $P_i\left(s^t\right)$, $\bar{P}_{ii}^T\left(s^t\right)$, $P_i^N\left(s^t\right)$ depend on who decides whether to default.

The maximization problem of each agent can now be stated as

$$\max_{\{c_i(s^t)\}} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) u\left[c_i\left(s^t\right)\right]$$

subject to (6) and (7).

3.2 Equilibrium

Definition 1 (Competitive Equilibrium) *An equilibrium in the economy with enforcement constraints is a collection of allocations $c_i\left(s^t\right)$, $a_i\left(s^t\right)$, $i = 1,2$ and prices $P_i\left(s^t\right)$, $\bar{P}_{ij}^T\left(s^t\right)$, $P_i^N\left(s^t\right)$, $Q_i\left(s^{t+1}|s^t\right)$, $i = 1,2$ and $j = 1,2$ such that* (1) *the consumer allocations solve the consumers' problem in both countries, and in particular the enforcement constraints are satisfied;* (2) *the resource constraint holds for all s^t, $F\left(c_1\left(s^t\right), c_2\left(s^t\right), y\left(s_t\right)\right) = 0$; and* (3) *asset markets clear, $a_1\left(s^{t+1}|s^t\right) + a_2\left(s^{t+1}|s^t\right) = 0$.*

Since I consider a real economy, the nominal exchange rate is fixed at 1. Furthermore, the price of the final consumption good in each country is normalized to 1 and the real exchange rate is defined to be $q\left(s^t\right) = P_2\left(s^t\right)/P_1\left(s^t\right)$.

3.3 Solution

Let $\beta^t \pi(s^t)\mu_i(s^t)$ denote the Lagrangian multipliers on the enforcement constraints in the optimization problem of the representative agent in country i. Using the "partial summation formula of Abel" this problem can be written as

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)\left[u\left[c_i\left(s^t\right)\right]+\mu_i\left(s^t\right)\left(\sum_{r=t}^{\infty}\sum_{s^r|s^t}\beta^{r-t}\pi\left(s^r|s^t\right)u\left[c_i\left(s^r\right)\right]-V_i\left(s^t\right)\right)\right]$$

$$=\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)\left[M_i\left(s^{t-1}\right)u\left[c_i\left(s^t\right)\right]+\mu_i\left(s^t\right)\left\{u\left[c_i\left(s^t\right)\right]-V_i\left(s^t\right)\right\}\right],$$

where

$$M_i\left(s^t\right)=M_i\left(s^{t-1}\right)+\mu_i\left(s^t\right), \tag{10}$$

and $M_i(s_0)=1$. $\mu_i(s^t)>0$ if the enforcement constraint (7) is binding for country i and zero otherwise. Note that at each point in time, at most one country can be constrained.[8]

The first order conditions of the representative agent in country i are summarized by

$$\begin{aligned}\lambda_i\left(s^t\right)P_i\left(s^t\right) &= \left[M_i\left(s^{t-1}\right)+\mu_i\left(s^t\right)\right]u_c\left[c_i\left(s^t\right)\right], &(11)\\ Q_i\left(s^{t+1}|s^t\right) &= \beta\pi\left(s_{t+1}|s_t\right)\frac{\lambda_i\left(s^{t+1}\right)}{\lambda_i\left(s^t\right)}, &(12)\end{aligned}$$

where $\lambda_i(s^t)$ is the Lagrangian multiplier on the budget constraint of an agent located in country i. Define the following variables

$$\begin{aligned}z\left(s^t\right) &= \frac{M_2\left(s^t\right)}{M_1\left(s^t\right)},\ z\left(s_0\right)=1,\\ v_i\left(s^t\right) &= \frac{\mu_i\left(s^t\right)}{M_i\left(s^t\right)},\ i=1,2.\end{aligned}$$

Equation (10) implies a law of motion for $z(\cdot)$

$$z\left(s^t\right)=\frac{1-v_1\left(s^t\right)}{1-v_2\left(s^t\right)}z\left(s^{t-1}\right). \tag{13}$$

[8] Messner and Pavano (2004) have recently hinted to some pitfalls of this approach. However, for an endowment economy their criticism does not apply.

Absent arbitrage opportunities, $Q_1\left(s^{t+1}|s^t\right)=Q_2\left(s^{t+1}|s^t\right)$, and I obtain an explicit expression for the real exchange rate

$$\frac{q\left(s^{t+1}\right)}{q\left(s^t\right)}=\frac{z\left(s^{t+1}\right)}{z\left(s^t\right)}\frac{u_c\left[c_2\left(s^{t+1}\right)\right]}{u_c\left[c_2\left(s^t\right)\right]}\frac{u_c\left[c_1\left(s^t\right)\right]}{u_c\left[c_1\left(s^{t+1}\right)\right]}. \tag{14}$$

Iterating on this expression, delivers

$$q\left(s^t\right)=z\left(s^t\right)\kappa\frac{u_c\left[c_2\left(s^t\right)\right]}{u_c\left[c_1\left(s^t\right)\right]}, \tag{15}$$

where $\kappa=\frac{q(s_0)}{z(s_0)}\frac{u_c[c_1(s_0)]}{u_c[c_2(s_0)]}$.

3.4 Interpretation

Computing equilibria in economies with limited enforcement involves finding the correct relative weights z. For a given sequence of Pareto weights $\{z\left(s^t\right)\}_{t=0}^{\infty}$, the problem of the planner can be thought of as

$$\begin{aligned}&\max_{c_1,c_2} u\left(c_1\left(s^t\right)\right)+z\left(s^t\right)u\left(c_2\left(s^t\right)\right)\\ &s.t.\\ &F\left(c_1\left(s^t\right),c_2\left(s^t\right),y\left(s_t\right)\right)=0.\end{aligned} \tag{16}$$

For given z, the planner's problem at time t resembles the static optimal allocation problem.

3.4.1 Partial risk sharing

Why is the relative weight time-varying? In the economy with enforcement constraints, full risk sharing is achieved only if $z\left(s^t\right)=1$ for all s^t. However, full risk sharing cannot be implemented if agents are sufficiently impatient.

To understand the forces that operate in the economy with enforcement constraints, it is helpful to compare the allocations under full risk sharing with the allocations in financial autarchy. Due to the concavity of $u\left(\cdot\right)$, consumption in country i varies less across states of the world under complete markets than in financial autarchy. Consequently, there is at least one realization $\tilde{s}\in S$, such that in this particular state the agent in country i receives higher consumption in financial autarchy than under full risk sharing. Obviously, full risk sharing cannot be implemented if the discount factor β is close to zero. If $\tilde{s}$ is realized, the utility loss from giving up the ability to share risk efficiently in the future is lower than the utility gain due to higher current consumption.

However, partial risk sharing might still be feasible. For simplicity, assume that at time $t-1$ the realized relative weight is $z\left(s^{t-1}\right)=1$. Now, suppose that the enforcement constraint binds for country 1 at s^t. To obtain partial risk sharing, the consumption of agent 1 has to be less than under financial autarchy but higher than under full risk sharing. To compensate agent 1 for lower contemporary consumption relative to financial autarchy, her future consumption must increase relative to full risk sharing. From equations (13) and (16) this means that the weight on country 1 has to increase, i.e., $z\left(s^{t-1}\right)<1$ which implies an appreciation of the real exchange rate in the decentralized economy.

3.4.2 Consumption-real exchange rate correlation

Equation (15) reveals, how the model with enforcement constraints breaks the tight link between the real exchange rate and relative consumption that arises under frictionless and complete markets.

Let $\sigma(x)$ be the standard deviation of variable x and let $\rho\left(x, \frac{c_1}{c_2}\right)$ denote the correlation between variable x and the relative consumption $\frac{c_1}{c_2}$ with $u(c)=\frac{c^{1-\gamma}}{1-\gamma}$. The correlation between the real exchange rate and relative consumption can be expressed as

$$\rho\left(q, \frac{c_1}{c_2}\right)=\frac{\left(\frac{\sigma(z)}{\sigma\left(\frac{c_1}{c_2}\right)}\rho\left(z, \frac{c_1}{c_2}\right)+\gamma\right)}{\left[\left(\frac{\sigma(z)}{\sigma\left(\frac{c_1}{c_2}\right)}\rho\left(z, \frac{c_1}{c_2}\right)+\gamma\right)^2+\left(\frac{\sigma(z)}{\sigma\left(\frac{c_1}{c_2}\right)}\right)^2\left(1-\rho\left(z, \frac{c_1}{c_2}\right)^2\right)\right]^{\frac{1}{2}}}. \tag{17}$$

In the standard complete market framework without enforcement constraint z is constant and $\sigma(z)=0$. Hence, $\rho\left(q, \frac{c_1}{c_2}\right)=1$. In the economy with enforcement constraints z is not constant and the correlation between z and relative consumption $\frac{c_1}{c_2}$ is negative. If the enforcement constraint binds for country 1, the planner increases the weight on country 1 and increases current consumption in country 1 relative to country 2 as described previously. Equation (17) then implies that $\rho\left(q, \frac{c_1}{c_2}\right)<1$.

3.5 Goods markets

The aggregate resource constraint of the global economy, $\tilde{F}\left(c_1, c_2, y\left(s_t\right)\right) \leq 0$, is derived from the underlying endowments with traded and non-traded goods. One possible specification that allows me to address the real exchange rate volatility puzzle and the consumption real exchange rate puzzle has been proposed by Corsetti et al (2005). There are four key features: imperfect substitutability between the domestic and the foreign tradable good, non-traded goods, distribution costs, and purchasing power parity for tradable goods at the producer level.[9]

3.5.1 Deriving the international resource constraint

The final consumption good c_i is an aggregate of tradable and non-tradable goods:

$$c_i = \left[\left(\alpha_i^T\right)^{1-\phi}\left(c_i^T\right)^{\phi} + \left(\alpha_i^N\right)^{1-\phi}\left(c_i^N\right)^{\phi}\right]^{\frac{1}{\phi}},\ \phi < 1,\ \alpha_i^T + \alpha_i^N = 1, \tag{18}$$

where c_i^T is the consumption of an aggregate of the tradable goods and c_i^N is the consumption of the non-traded good in country i.

The consumption index c_i^T is determined by

$$c_i^T = \left[\alpha_{i1}^{1-\rho}\left(c_{i1}^T\right)^{\rho} + \alpha_{i2}^{1-\rho}\left(c_{i2}^T\right)^{\rho}\right]^{\frac{1}{\rho}},\ \rho < 1,\quad \alpha_{i1} + \alpha_{i2} = 1, \tag{19}$$

where c_{ij}^T denotes country i's consumption of the tradable good that originates in country j. For $\rho < 1$, the domestic and the foreign tradable goods are imperfect substitutes. If $\alpha_{ii} > \frac{1}{2}$, there is home-bias in consumption.

Following Erceg and Levin (1996), Burstein, Neves and Rebelo (2003) I assume that brining one unit of any traded good to consumers in country i requires η units of country i's non-traded good. Any allocation of tradable and non-tradable goods therefore has to satisfy

$$c_{1i}^T + c_{2i}^T \leq y_i^T,\ i = 1, 2 \tag{20}$$

and

$$c_i^N + \eta c_{i1}^T + \eta c_{i2}^T \leq y_i^N,\ i = 1, 2. \tag{21}$$

Let P_{ij}^T denote the consumer price of the tradable good that originates in country j and is consumed in country i. $\bar{P}_{ij}^T$ denotes this price at the producer level. If the distribution sector is assumed to be perfectly competitive, the consumer price and the producer price are related by

$$P_{ij}^T = \bar{P}_{ij}^T + \eta P_i^H,$$

where P_i^H is the price of one unit of the non-traded good in country i. Under the assumption that purchasing power parity holds for nominal prices at the producer level, $\bar{P}_{1j}^T$ is equal to $\bar{P}_{2j}^T$. Given prices and the total income of an agent in country i, NI_i, the consumption

[9] See Betts and Kehoe (2001) and Burstein, Eichenbaum and Rebelo (2002) for supportive evidence.

choices, c_{i1}^T, c_{i2}^T, c_i^N, can be determined from a standard static utility maximization program:

$$\begin{aligned}
&\max_{c_{i1}^T, c_{i2}^T, c_i^N} \left[\left(\alpha_i^T\right)^{1-\phi}\left(c_i^T\right)^{\phi} + \left(\alpha_i^N\right)^{1-\phi}\left(c_i^N\right)^{\phi}\right]^{\frac{1}{\phi}} \\
&s.t. \\
&NI_i = P_{i1}^T c_{i1}^T + P_{i2}^T c_{i2}^T + P_i^N c_i^N
\end{aligned} \tag{22}$$

and c_i^T is defined by equation (19).

For the purpose of this paper it is convenient to summarize the allocations of the final good in terms of an international resource constraint $\left\{(c_1, c_2) \,|\, \tilde{F}(c_1, c_2, y) \leq 0\right\}$. The efficient frontier $F(c_1, c_2, y) = 0$ is obtained by

$$\begin{aligned}
&\max_{\substack{c_{11}^T, c_{12}^T, \\ c_{21}^T, c_{22}^T}} c_1 = \left[\left(\alpha_1^T\right)^{1-\phi}\left(c_1^T\right)^{\phi} + \left(\alpha_1^N\right)^{1-\phi}\left(c_1^N\right)^{\phi}\right]^{\frac{1}{\phi}} \\
&s.t. \\
&c_2 = \left[\left(\alpha_2^T\right)^{1-\phi}\left(c_2^T\right)^{\phi} + \left(\alpha_2^N\right)^{1-\phi}\left(c_2^N\right)^{\phi}\right]^{\frac{1}{\phi}}
\end{aligned}$$

and equations (19)-(21). Given an allocation (c_1, c_2) that satisfies $F(c_1, c_2, y) = 0$, the remaining consumption allocations $\left(c_{i1}^T, c_{i2}^T, c_i^T, c_i^N\right)$, $i = 1, 2$ are uniquely determined. The prices for tradables and non-tradables that support such an allocation can be found from the first order conditions of (22).[10] Furthermore, the real exchange rate is given by $q = \frac{F_2(c_1, c_2, y)}{F_1(c_1, c_2, y)}$.

3.5.2 Discussion

Figure 1 shows how the shape of the international resource constraint changes with the introduction of non-traded goods and distribution costs for a given endowment vector y. The elasticity of substitution is set equal to 4. The solid line characterizes the allocations for a state with $y_1^T = y_2^T$ and $y_1^N = y_2^N$. In the economy with only traded goods, the boundary of the consumption set is almost linear. Adding non-traded goods to the model increases the curvature and introducing distribution costs increases the curvature even more.

The curvature of the consumption set is key to understanding the volatility of the real exchange rate. Consider an increase in y_1^T. For reasonable parameterizations of the model and the shock, the international resource constraint hardly changes. In Figure 1, the dotted

[10] One important assumption in the derivation of the function $F(\cdot)$, is that agents have access to free disposal. Since consumption of traded goods requires η units of the non-traded good, there is an interior optimum for the consumption of the traded goods for a given endowment with the non-traded good.

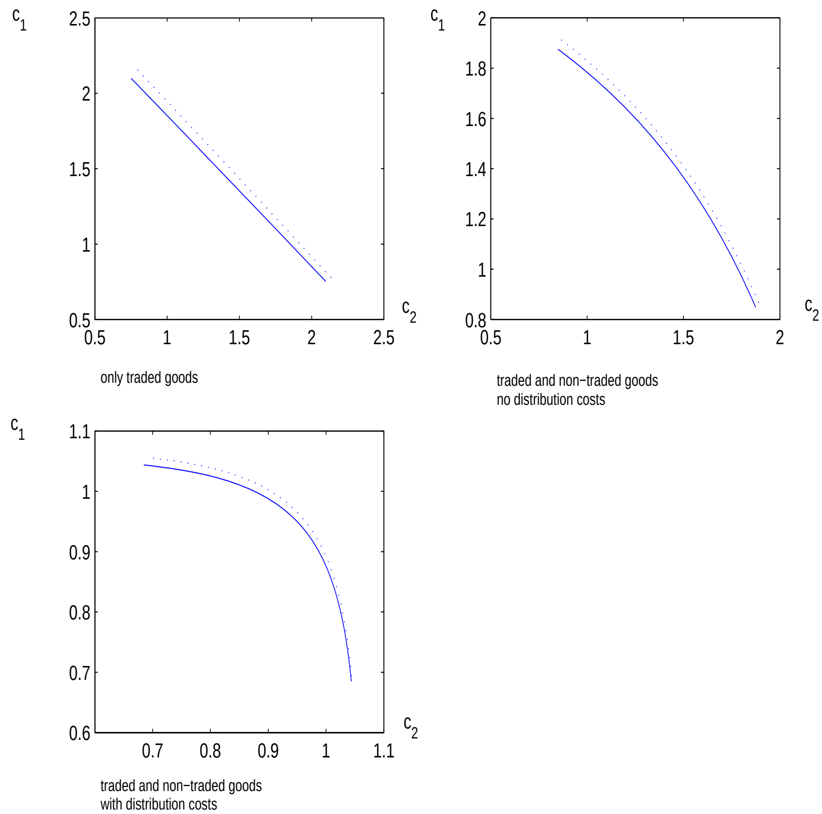

Figure 1: Set of Feasible Consumption Allocations

line $\left(y_1^T > y_2^T\right)$ hardly differs from the solid line $\left(y_1^T = y_2^T\right)$. Remember that $q = \frac{F_2(c_1,c_2,y)}{F_1(c_1,c_2,y)}$. Due to the low curvature of the resource constraint in the economy with only traded goods, large swings in $\frac{c_1}{c_2}$ are needed across states to generate substantial real exchange rate volatility.[11] Although adding non-traded goods increases the curvature of the resource constraint, the increase is not large enough quantitatively. Only with distribution costs small variations in c_1/c_2 cause large swings in the real exchange rate. Put differently, in accord with the stylized facts, large movements in the real exchange rate have little impact on the actual

[11] Heathcote and Perri (2002) examine such a model for the case of complete markets, exogenously incomplete markets with one non-state-contingent bond and financial autarchy. They find little real exchange rate volatility since consumption turns out to be highly correlated across countries. Chari, Kehoe and McGrattan (2002) show that with nominal rigidities the simple model with only traded goods can generate substantial real exchange rate volatility.

allocations. The real exchange rate is disconnected from macroeconomic fundamentals.

4 Calibration and results

4.1 Calibration

The values of the benchmark parameters and the endowment process are listed in tables (2) and (3).

Parameter values

risk aversion	$\gamma = 2$
discount factor	$\beta = 0.95$
elasticity of substitution:	
– domestic and foreign tradables	$\frac{1}{1-\rho} = 4.00$
– tradables and non-tradables	$\frac{1}{1-\phi} = 0.74$
distribution costs	$\eta = 1.09$
share of domestic tradables	$\alpha_{11} = \alpha_{22} = 0.72$
share of non-tradables	$\alpha_1^N = 0.45$

Table 2

Preferences are represented by the power utility function, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. In the benchmark calibration the coefficient of relative risk aversion γ is set equal to 2. This value lies well within the range of other studies where γ is usually assumed to be between 1 and 6. Two comments are in place to explain the choice of β. First, the model is calibrated to annual data. Second and more important, partial risk sharing as an equilibrium phenomenon only arises if agents are sufficiently impatient. Otherwise, the equilibrium outcome is close to or identical to the full risk sharing scenario. In terms of the economics it is the value of the risk free rate that matters, which turns out to be around 2%.[12]

The remaining parameter values are taken from Corsetti et al (2005) except for the elasticity of substitution between the domestic and the foreign tradables, $(1-\rho)^{-1}$, which I choose to set equal to 4. The quantitative literature has proposed a variety of values for the elasticity of substitution between traded goods. For instance, Backus, Kydland and Kehoe (1995) set it equal to 1.5, whereas Heathcote and Perri (2002) estimate its value to be 0.9. Using disaggregate data Broda and Weinstein (2006) find a mean estimate for the elasticity of substitution of 6.

[12] See Alvarez and Jermann (2001) for a discussion about the time discount factor β in models with enforcement constraints.

Mendoza (1991) estimates the value of ϕ to be 0.74 in a sample of industrialized countries. According to the evidence presented in Burstein, Neves and Rebelo (2003), the share of the retail price of traded goods accounted for by local distribution services ranges from 40% to 50% for the U.S. The value of $\eta = 1.09$ implies a share of roughly 50% in my setup.

The weights of the domestic and foreign tradables, α_{ii} and α_{ij}, have been chosen to be 0.72 and 0.28 respectively. Depending on the exact choices for the remaining parameters, these values imply imports of $5\% - 9\%$ of total income. The average ratio of U.S. imports from Europe, Canada and Japan to U.S. GDP between 1960-2002 is 5%. However, due to the enormous growth in international trade since 1960, this value is substantially larger than 5% towards the end of the sample. Stockman and Tesar (1995) suggest that the share of tradables in the consumption basket of the seven largest OECD countries is roughly 50%. This motivates the choice of $\alpha_i^T = 0.55$ and $\alpha_i^N = 0.45$.

Endowment process

data (annualized)	
standard deviations	
$\sigma\left(y_{US}^T\right) = 0.0285$	$\sigma\left(y_{G7\backslash US}^T\right) = 0.0161$
$\sigma\left(y_{US}^N\right) = 0.0082$	$\sigma\left(y_{G7\backslash US}^H\right) = 0.0043$
$\sigma\left(GDP_{US}\right) = 0.0121$	$\sigma\left(GDP_{G7\backslash US}\right) = 0.0070$
correlations	
$\rho\left(y_{US}^T, y_{G7\backslash US}^T\right) = 0.5166$	$\rho\left(y_{US}^T, y_{G7\backslash US}^N\right) = 0.4049$
$\rho\left(y_{US}^N, y_{G7\backslash US}^N\right) = 0.6488$	$\rho\left(y_{US}^N, y_{G7\backslash US}^T\right) = 0.6818$
$\rho\left(GDP_{US}, GDP_{G7\backslash US}\right) = 0.3741$	
calibration	
endowment vector	
$y_i^T(h) = 1.0257$	$y_i^T(l) = 0.9743$
$y_i^N(h) = 2.4684$	$y_i^N(l) = 2.4316$
properties of VAR	
$\sigma\left(y_1^T\right) = 0.0254$	$\sigma\left(y_2^T\right) = 0.0254$
$\sigma\left(y_1^N\right) = 0.0074$	$\sigma\left(y_2^N\right) = 0.0074$
$\rho\left(y_1^T, y_2^T\right) = 0.4500$	$\rho\left(y_1^T, y_2^N\right) = 0.6400$
$\rho\left(y_1^N, y_2^N\right) = 0.7600$	$\rho\left(y_1^N, y_2^T\right) = 0.6400$

Table 3

The endowment process for tradable and non-tradable goods is calibrated as follows. Consistent with the literature and the evidence provided in Betts and Kehoe (2001), non-tradables are identified in the data as service output and tradables as manufacturing output. Using annual data for manufacturing and services from the OECD STAN database for the G7 countries, I obtain an estimate for the relative size of the two sectors. The estimates for the ratio of sectorial GDP, $\bar{P}_{ii}^{T} y_i^T / P_i^N y_i^N$, range from 0.2 to 0.45. In the baseline calibration I target a value of 0.36. Given the benchmark calibration this value translates into y_i^T / y_i^N $= 1/2.45$.

The endowment vector $y = \left(y_1^T, y_2^T, y_1^N, y_2^N\right)$ is assumed to follow a Markov chain with transition matrix Π. Each element of the endowment vector can take on two values. Hence, there are 16 exogenous states of the aggregate economy. To calibrate the transition matrix, I generate artificial data from a VAR with time series properties similar to the data (see Table 3). The transition probabilities are then estimated from the artificial data using sample averages. Table 3 also shows properties of the actual data. The U.S. time series are more volatile than the series for the aggregate of the remaining G7 countries.[13] This is partly due to aggregation. Also, manufacturing output is more volatile than service output and the volatility of total output lies in between the two. Given the symmetric nature of the model, the endowment process is calibrated to match closer the behavior of the U.S. data than of the remaining G7.

4.2 Results and interpretation

4.2.1 Benchmark calibration

Appendix B provides the details on the computational procedure that is used in order to find the policy functions. The economy is simulated 200 times over 500 periods. Unless mentioned otherwise the artificial data is HP-filtered and the relevant statistics are computed for each simulation. The reported numbers are the averages over the 200 simulations. Table 4 reports data from the U.S. and the remaining G7 countries along with the results for the benchmark calibration for three different arrangements of the international financial markets: complete markets with enforcement constraints (LC), complete markets without enforcement constraints (CM) and financial autarchy (FA). In this section, it is assumed that the government is responsible for the default decision. As it is shown in section 5.2.2

[13] The aggregation method follows Chari, Kehoe and McGrattan (2002). Countries are weighted by GDP in U.S. dollars. Purchasing power parities for a given baseline year are used in order to convert national currencies into U.S. dollars.

the qualitative results do not depend on this choice.[14]

The poor performance of the model with complete markets (CM) restates the exchange rate disconnect puzzle and the Backus-Smith puzzle: the real exchange rate is barely more volatile than consumption and its correlation with relative consumption equals 1. These two failures have their common cause in the very high correlation of cross country consumption under complete markets. The model with enforcement constraints (LC) does reasonably well in comparison with the data both qualitatively and quantitatively. The real exchange rate is considerably more volatile than consumption and it is negatively correlated with relative consumption. In addition, consumption across countries is positively correlated, but far from perfect. Comparing the economy with enforcement constraint to the financial autarchy model (FA) reveals why the model is so successful in replicating the data. Although the quantitative effects are somewhat too strong under financial autarchy, the qualitative behavior is in line with the data: real exchange rates are volatile and negatively correlated with relative consumption. Depending on the impatience of the agents, risk sharing in the economy with enforcement constraints can be very limited and the economy behaves qualitatively like under financial autarchy.[15]

Business cycle statistics: benchmark calibration with $\beta = 0.95$, $\rho = 0.75$

		$\gamma = 2$			$\gamma = 4$		
	Data	LC	CM	FA	LC	CM	FA
HP-filtered statistics							
$\sigma(C_1)$	0.0150	0.0100	0.0096	0.0100	0.0100	0.0095	0.0100
$\sigma(q)$	0.0504	0.0700	0.0117	0.0728	0.0687	0.0205	0.0728
$\rho(C_1, C_2)$	0.4300	0.6808	0.8166	0.6737	0.6873	0.8563	0.6737
$\rho(q, C_1/C_2)$	-0.3500	-0.5503	1.0000	-0.5672	-0.5370	1.0000	-0.5672
Non-filtered variables							
$E\left(\bar{M}_1\right)$	0.9919	0.9872	0.9503	0.9503	0.9908	0.9513	0.9514
$\sigma\left(\log \bar{M}_1\right)$	0.4509	0.0641	0.0263	0.0273	0.0798	0.0519	0.0545
$\rho\left(\log \bar{M}_1, \log \bar{M}_2\right)$	0.9920	-0.0010	0.0117	0.0097	0.0045	0.0118	0.0097

Table 4

[14] Changes in the default decision change the value of financial autarchy. By adjusting the discount factor β the behavior of the model can be brought in line with the data.

[15] As shown in Bodenstein (2005) the differences between the model of financial autarchy and the model with enforcement constraints become more pronounced in a production economy with labor.

4.2.2 Interpretation

How does the model generate the negative correlation between the real exchange rate and relative consumption? Consider the two extreme cases of complete markets and financial autarchy. In both cases the allocations do not depend on the time discount factor β. However, these two economies are the limits of the model with limited enforcement as the time discount factor varies: if β approaches 1, agents are patient and full risk sharing becomes feasible. In contrast, if β is sufficiently small, agents have a strong incentive to default. As a result, risk sharing is severely limited and the economy behaves like under financial autarchy.

For ease of exposition, denote the two countries U.S. and Europe. Each period the U.S. receives an endowment of meat and Europe receives an endowment of vegetables. Meat and vegetables are the two (imperfectly substitutable) tradable goods. In order to consume a meal (a combination of meat and vegetables), cooking services are needed. Each period the two countries also receive an endowment of these non-tradable cooking services.

Consider first an increase of the meat endowment in the U.S. under financial autarchy. As meat becomes relatively abundant, the price of meat relative to vegetables declines. If there is home bias in consumption, this effect acts towards a decline of the U.S. price level relative to the European price level, i.e., a depreciation of the real exchange rate. However, because of the wealth effect demand for cooking services rises in the U.S. and drives up its price in the U.S. This second effect acts towards an increase of the U.S. price level relative to the European price level, i.e., an appreciation of the real exchange rate. If this second effect is strong enough to overcome the first effect, the model can account for the observation of Backus and Smith (1993): the real exchange rate appreciates while U.S. consumption of meals increases relative to European consumption.[16]

Under complete markets, however, there is no wealth effect. The extra endowment of meat is shared more equally between the two countries.[17] Hence, the price of cooking services increases in both countries and the aforementioned second effect on the real exchange rate is weak. In contrast with the data, the real exchange rate now depreciates while U.S. consumption of meals increases relative to European consumption.

It is crucial to note, that the explanation of the Backus-Smith puzzle depends on the presence of shocks in the tradable goods sector. Shocks to the non-tradable goods sector induce a positive correlation between the real exchange rate and relative consumption irrespective

[16] The simple endowment economy in this paper implies that the terms of trades and the real exchange rate move in opposite directions. Empirical evidence suggests, however, that these two variables move in the same direction over the business cycle. As shown in Bodenstein (2006) and Corsetti et al (2005) this problem is overcome in a production economy. Furthermore, shocks to non-traded goods and consumption taste shocks (not considered here) induce comovement of the terms of trade and the real exchange rate.

[17] Due to home bias in consumption US consumption of meals still increases relative to European consumption of meals.

of the financial market structure.

4.2.3 Sensitivity analysis

Table 4 also shows the simulation results for $\gamma = 4$. Higher risk aversion means, that agents have a stronger taste for smooth consumption. Therefore, more risk sharing is feasible at higher values of γ for a given value of β. The quantitative effects of an increase in γ are, however, small. The results are almost identical for the two different values of γ.

Changes in β have a stronger impact on the results. Table 5 summarizes the simulation results for several values of β. For $\beta \geq \beta_{CM}$, full risk sharing is feasible in the economy with enforcement constraints. As under frictionless complete markets, the real exchange rate is very smooth and the correlation between the real exchange rate and relative consumption is 1. Lowering β, brings the model in line with the data. For values as high as $\beta = 0.975$, the correlation of the real exchange rate with relative consumption is significantly below 1 and the real exchange rate volatility is higher than the volatility of consumption. Also, if agents become very impatient, $\beta \leq \beta_{FM}$, the economy with enforcement constraints behaves identical to the economy without international financial markets.

Business cycle statistics: sensitivity wrt β with $\gamma = 2$, $\rho = 0.75$

β	$\geq \beta_{CM}$	0.9750	0.9600	0.9500	0.9400	$\leq \beta_{FM}$
HP-filtered statistics						
$\sigma(C_1)$	0.0096	0.0097	0.0100	0.0100	0.0100	0.0100
$\sigma(q)$	0.0117	0.0312	0.0646	0.0700	0.0713	0.0728
$\rho(C_1, C_2)$	0.8166	0.8002	0.6937	0.6808	0.6776	0.6737
$\rho(q, C_1/C_2)$	1.0000	0.0706	-0.5165	-0.5503	-0.5582	-0.5672
Non-filtered variables						
$E\left(\bar{M}_1\right)$	0.9603	0.9862	0.9934	0.9872	0.9779	0.9604
$\sigma\left(\log \bar{M}_1\right)$	0.0263	0.0340	0.0595	0.0641	0.0652	0.0273
$\rho\left(\log \bar{M}_1, \log \bar{M}_2\right)$	0.0117	0.0096	-0.0001	-0.0010	-0.0012	0.0097

Table 5

5 A closer look at asset prices

The benchmark model with limited contract enforcement can account both for the volatility of the real exchange rate and the observed low or even negative correlation between the real exchange rate and relative consumption (Backus-Smith puzzle).

Brandt, Cochrane and Santa-Clara (2006) emphasize, that real exchange rate volatility is tightly linked to the volatility of asset prices. As shown in equation (2) the growth rate of the real exchange rate equals the difference in the intertemporal marginal rates of substitution ($IMRS$) between the two countries when markets are complete.

Both the benchmark model with enforcement constraints and the model of Corsetti et al (2005) imply that asset prices (other than the real exchange rate) are smooth and the equity premium is too low. In Tables 4 and 5 the volatility of the $IMRS$, $\sigma\left(\log \bar{M}_1\right)$, is at least 5 times smaller in the model than in the data. Under the benchmark calibration, the real exchange rate is also more volatile than the $IMRS$. This finding is hardly surprising as I have merely extended the equity premium puzzle to its international dimension. As shown by Mehra and Prescott (1985) for a closed economy, standard preferences and complete frictionless domestic financial markets imply little volatility of the $IMRS$ since aggregate endowment shocks are small. In the benchmark model domestic financial markets are complete and frictionless and the calibrated endowment shocks – which can even be smoothed to some extent in international financial markets – are relatively small.

One potential resolution to the equity premium puzzle in a closed economy is offered by Alvarez and Jermann (2001). In line with empirical findings, these authors assume that agents' idiosyncratic incomes are volatile relative to aggregate income. In addition, they assume that asset markets are complete, but enforcement of financial contracts is limited.

In this section, I extend the simple two country model along the lines of Alvarez and Jermann in order to simultaneously address the three puzzles mentioned in the introduction: the volatility of the real exchange rate, the consumption real exchange rate puzzle and the volatility of (other) asset prices. From now on I assume that both domestic and international financial contracts can only be enforced by the threat of permanent exclusion from all financial markets.

5.1 The extended model

There are two groups of agents in country 1 which are denoted by 1 and 2. The agents in country 2 are labeled agents 3 and 4. Each agent i in country j faces a maximization problem similar to the one of the representative agents in section 3.1:

$$\begin{aligned}
& \max_{\{c_i(s^t)\}} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) \left[M_i\left(s^{t-1}\right) u\left[c_i\left(s^t\right)\right] + \mu_i\left(s^t\right) \left\{u_i\left[c\left(s^t\right)\right] - V_i\left(s^t\right)\right\}\right] \\
& s.t. \\
& P_j\left(s^t\right) c_i\left(s^t\right) + \sum_{s^{t+1}|s^t} Q_j\left(s^{t+1}|s^t\right) a_i\left(s^{t+1}|s^t\right) \\
\leq\ & a_i\left(s^t|s^{t-1}\right) + \xi_i\left(s^t\right)\left[\bar{P}_{jj}^T\left(s^t\right) y_j^T\left(s^t\right) + P_j^N\left(s^t\right) y_j^N\left(s^t\right)\right]
\end{aligned}$$

and

$$M_i\left(s^t\right) = M_i\left(s^{t-1}\right) + \mu_i\left(s^t\right).$$

$\xi_i\left(s^t\right)$ is the share of agent i in the aggregate income of her home country. The outside option, $V_i\left(s^t\right)$, is defined by

$$\begin{aligned}
& V_i\left(s^t\right) = \max_{\{c_i(s^r)\}} \sum_{r=t}^{\infty} \sum_{s^r|s^t} \beta^{r-t} \pi\left(s^r|s^t\right) u\left[c_i\left(s^r\right)\right] \\
& s.t. \\
& P_j\left(s^t\right) c_i\left(s^t\right) \leq \xi_i\left(s^t\right)\left[\bar{P}_{jj}^T\left(s^t\right) y_j^T\left(s^t\right) + P_j^N\left(s^t\right) y_j^N\left(s^t\right)\right].
\end{aligned}$$

In this version of the model, I assume that the default decision is made by each agent individually. Therefore she ignores the effect of her behavior on goods market prices. An agent who defaults on any contract is banned from all financial markets, but she can still trade in the spot markets for goods.

The solution of the model is fully characterized by the first order conditions

$$\begin{aligned}
\eta_1\left(s^t\right) &\equiv \frac{M_2\left(s^t\right)}{M_1\left(s^t\right)} = \frac{M_2\left(s^{t-1}\right) + \mu_2\left(s^t\right)}{M_1\left(s^{t-1}\right) + \mu_1\left(s^t\right)} = \frac{u_c\left(c_1\left(s^t\right)\right)}{u_c\left(c_2\left(s^t\right)\right)}, \\
\eta_2\left(s^t\right) &\equiv \frac{M_4\left(s^t\right)}{M_3\left(s^t\right)} = \frac{M_4\left(s^{t-1}\right) + \mu_4\left(s^t\right)}{M_3\left(s^{t-1}\right) + \mu_3\left(s^t\right)} = \frac{u_c\left(c_3\left(s^t\right)\right)}{u_c\left(c_4\left(s^t\right)\right)}, \\
z\left(s^t\right) &\equiv \frac{M_3\left(s^t\right)}{M_1\left(s^t\right)} = \frac{M_3\left(s^{t-1}\right) + \mu_3\left(s^t\right)}{M_1\left(s^{t-1}\right) + \mu_1\left(s^t\right)} = \frac{u_c\left(c_1\left(s^t\right)\right)}{u_c\left(c_3\left(s^t\right)\right)} q\left(s^t\right), \\
\mu_i\left(s^t\right) &\geq 0,
\end{aligned}$$

the resource constraints

$$\begin{aligned}
C_1\left(s^t\right) &= c_1\left(s^t\right) + c_2\left(s^t\right), \\
C_2\left(s^t\right) &= c_3\left(s^t\right) + c_4\left(s^t\right), \\
0 &= F\left(C_1\left(s^t\right), C_2\left(s^t\right), y\left(s_t\right)\right),
\end{aligned}$$

$q\left(s^t\right)=\frac{F_2\left(C_1\left(s^t\right),C_2\left(s^t\right),y(s_t)\right)}{F_1\left(C_1\left(s^t\right),C_2\left(s^t\right),y(s_t)\right)}$, and the enforcement constraints. C_j denotes aggregate consumption in country j. The solution of this model is found by following the same steps as in the benchmark model with the additional complication that the system now contains three endogenous state variables (η_1, η_2 and z). The relevant asset pricing kernels are given by

$$m_i\left(s^{t+1}\right)=\frac{\beta}{1-\varphi_i\left(s^{t+1}\right)}\frac{u_c\left[c_i\left(s^{t+1}\right)\right]}{u_c\left[c_i\left(s^t\right)\right]},$$

with $\varphi_i\left(s^{t+1}\right)=\frac{\mu_i\left(s^{t+1}\right)}{M_i\left(s^{t+1}\right)}$. The price of a contingent claim is given by the $IMRS$ of the unconstrained agents. Within a country the pricing kernels satisfy $m_1\left(s^{t+1}\right)=m_2\left(s^{t+1}\right)$ and similarly for agents 3 and 4. The $IMRS$ across countries are related by

$$\frac{q\left(s^{t+1}\right)}{q\left(s^t\right)}=\frac{m_3\left(s^{t+1}\right)}{m_1\left(s^{t+1}\right)}=\frac{\bar{M}_2\left(s^{t+1}\right)}{\bar{M}_1\left(s^{t+1}\right)}, \tag{23}$$

where $\bar{M}_i$ is the marginal rate of substitution for country i.

5.2 A numerical example

5.2.1 Calibration

As in the benchmark model, the endowment with traded goods can be either high or low in each country. However, the endowment with non-traded goods is assumed to be constant in this part of the analysis in order to keep the state space manageable.[18] I calibrate the Markov process for the agents' income in each country following Heaton and Lucas (1996):[19]

$$\xi_i^L=0.3772 \text{ and } \xi_i^H=0.6228,\ i=1,2,3,4$$

where $\xi_i\left(s^t\right)$ is the share of agent i in the aggregate income of her home country. Obviously,

$$\begin{aligned}\xi_1\left(s^t\right)+\xi_2\left(s^t\right) &= 1,\\ \xi_3\left(s^t\right)+\xi_4\left(s^t\right) &= 1,\end{aligned}$$

[18] The 4 state endowment process is calibrated to match the business cycle statistics of the manufacturing sectors in the U.S. and the remaining G7 countries reported in table 3.

[19] Based on a large sample from the PSID, Heaton and Lucas (1996) find that the log of an agent's income, relative to the aggregate is stationary with a first order serial correlation of 0.5 and a standard deviation of 0.29 for annual data. Alvarez and Jermann (2001) and Lustig (2004) also calibrate their models based on the estimates in Heaton and Lucas (1996).

for all s^t. The transition matrix for the income distribution in country 1 is given by

$$\begin{array}{ccc} & (\xi_1^L, \xi_2^H) & (\xi_1^H, \xi_2^L) \\ (\xi_1^L, \xi_2^H) & 0.7423 & 0.2577 \\ (\xi_1^H, \xi_2^L) & 0.2577 & 0.7423 \end{array}$$

and similarly for country 2. These income processes for the agents are assumed to be independent across countries. The remaining parameters are taken from Table 2 unless explicitly noted otherwise in Table 6.

5.2.2 Results and interpretation

The model is simulated 200 times over 500 periods. The artificial data is HP-filtered and the relevant business cycle statistics are computed. The moments for the $IMRS$ are calculated from non-filtered data. Table 2.4 summarizes the results for the for $\beta = 0.70$ and $\beta = 0.95$. I will refer to these to scenarios as low and high risk sharing, respectively. The model generates volatile $IMRS$ only in the low risk sharing scenario. Since individual income is very volatile, the gains from risk sharing are potentially very high. Hence, agents need to be fairly impatient ($\beta = 0.70$) for enforcement constraints to matter.

For $\gamma = 2$, the model predicts that the $IMRS$ in the two countries $\bar{M}_i$ are volatile and reasonably close to the data (40% in the model compared to my estimates of 45%) in the low risk sharing scenario. In addition, the implied risk-free rate is 2%. Also, the model predicts a negative correlation between the relative consumption and the real exchange rate. However, the real exchange rate moves too much now: its volatility is about 53 times the volatility of consumption for the HP-filtered time series, whereas this ratio is less than 4 in the data. Similarly, the growth rate of q fluctuates too much.

In the high risk sharing scenario, income heterogeneity within a country does not matter. Agents make efficient use of the domestic financial markets and individual consumption behaves similar to aggregate consumption. While the model correctly predicts the real exchange volatility and the negative correlation between the real exchange rate and relative consumption, it fails to generate volatile asset prices. The $IMRS$ varies about only 7%.

Business cycle statistics: extended vs benchmark model with $\rho = 0.75$

	Extension $\beta = 0.70$			Benchmark $\beta = 0.95$		
γ	2	3	4	2	3	4
HP-filtered statistics						
$\sigma(C_1)$	0.0117	0.0083	0.0056	0.0063	0.0063	0.0063
$\sigma(q)$	0.6203	0.2604	0.0103	0.0703	0.0699	0.0690
$\rho(C_1, C_2)$	-0.4625	-0.0023	0.9876	0.6667	0.6701	0.6756
$\rho(q, C_1/C_2)$	-0.3345	-0.6125	-0.2216	-1.0000	-1.0000	-1.0000
Non-filtered variables						
$E\left(\bar{M}_1\right)$	0.9795	0.8035	0.7007	0.9884	0.9907	0.9929
$\sigma\left(\log\left(\bar{M}_1\right)\right)$	0.3949	0.1283	0.0236	0.0698	0.0730	0.0769
$\rho\left(\log\left(\bar{M}_1\right), \log\left(\bar{M}_2\right)\right)$	0.1673	0.2579	0.9413	-0.0001	0.0013	0.0027
$\sigma\left(\log\left(q_{t+1}/q_t\right)\right)$	0.5033	0.1593	0.0091	0.0106	0.0100	0.0092

Table 6

Remember that the real exchange rate depreciates by the difference between the log of the foreign and the domestic $IMRS$:

$$\log \frac{q_{t+1}}{q_t} = \log \bar{M}_{2,t+1} - \log \bar{M}_{1,t+1}.$$

For $\beta = 0.70$, risk sharing between agents within each country and across countries is severely limited and the correlation between the stochastic discount factors for the two countries is low (0.1673).[20] Given the volatility of the stochastic discount factors, the real exchange rate fluctuates too much.

By assuming higher values of the coefficient of relative risk aversion, more risk sharing becomes sustainable in equilibrium. For $\gamma = 3$, the volatility of aggregate consumption declines and cross-country consumption correlations increase. The real exchange rate is smoother, although it is still 31 times more volatile than aggregate consumption. The stochastic discount factors become smoother and more correlated. The extended model falls short of explaining asset pricing behavior for $\gamma > 2$ given $\beta = 0.70$.

Although the correlation between the $IMRS$ is even lower for $\beta = 0.95$, this parameterization of the model does not imply too much volatility in the real exchange rate. With low volatility of the $IMRS$, the low correlation does not pose any problems for the real exchange

[20] The low degree of international risk sharing is also reflected in the negative correlation of cross country consumption.

rate. Hence, the model of limited enforcement presented in this paper cannot simultaneously account for the observed volatility in the real exchange rate, asset prices and the Backus-Smith puzzle. It either fails with respect to the volatility of the real exchange rate or of the asset prices.

6 Conclusions

Most international macroeconomists believe that international risk sharing is limited by financial market frictions and that these frictions are key to understanding the international business cycle. This paper examines the extent to which models with endogenous incomplete markets can resolve the exchange rate volatility puzzle and the real exchange rate correlation puzzle (Backus-Smith puzzle). A model with complete markets and enforcement constraints for international financial contracts but frictionless domestic asset markets provides a candidate explanation of these two puzzles if agents are not too patient. For sufficiently impatient agents, international risk sharing is very limited. As a result the correlation between cross country consumption levels is low and real exchange rates are volatile and negatively correlated with relative consumption across countries.

However, since asset markets are complete within each country and aggregate income fluctuations are low, the model inherits all the standard asset pricing puzzles. In particular, it implies stochastic discount factors that are too smooth vis-à-vis the data. Once I extend the benchmark model by introducing enforcement constraints also into each country's local financial markets, the model delivers more volatile asset prices. However, it now fails to deliver the right amount of real exchange rate volatility. As risk sharing is low both within and across countries, the marginal rates of substitution in the two countries are not very correlated and the real exchange rate is too volatile in comparison to the data. It seems that models that severely restrict the amount of international risk sharing for all agents will be subject to this failure, once it has been enriched to deliver realistic asset pricing behavior.

References

[1] Alvarez, F., Jermann, U., 2001. Quantitative Asset Pricing Implications Of Endogenous Solvency Constraints. Review of Financial Studies 14, 1117-1151.

[2] Backus, D., Foresi, S., Telmer, C., 1996. Affine Models Of Currency Pricing. NBER Working Paper 5623.

[3] Backus, D., Kehoe, P., Kydland, F., 1995. International Business Cycles: Theory And Evidence. In Cooley, T. (Ed.), Frontiers of Business Cycle Research. Princeton University Press, Princeton, pp. 331-356.

[4] Backus, D., Smith, G., 1993. Consumption And Real Exchange Rates In Dynamic Economies With Non-Traded Goods. Journal of International Economics 35, 297-316.

[5] Benigno, G., Thoenissen, C., 2006. Consumption and Real Exchange Rates with Incomplete Markets and Non-Traded Goods. CEPR Discussion Papers 5580.

[6] Betts, C., Kehoe, T., 2001. Tradability Of Goods And Real Exchange Rate Fluctuations. Federal Reserve Bank of Minneapolis, Research Department Staff Report.

[7] Bodenstein, M., 2005. On Real Exchange Rate Volatility And Financial Markets. PhD Dissertation Northwestern University.

[8] Brandt, M., Cochrane, J., Santa-Clara, P., 2003. International Risk Sharing Is Better Than You Think, Or Exchange Rates Are Too Smooth. Journal of Monetary Economics 53, 671-698.

[9] Broda, C., Weinstein, D., 2006. Globalization and the gains from variety. Quarterly Journal of Economics 121, 541-585.

[10] Burstein, A., Eichenbaum, M., Rebelo, S., 2002. Large Devaluations and the Real Exchange Rate. Journal of Political Economy 113, 742-784.

[11] Burstein, A., Neves, J., Rebelo, S., 2003. Distribution Costs And Real Exchange Rate Dynamics During Exchange-Rate-Based Stabilizations. Journal of Monetary Economics 50, 1189-1214.

[12] Chari, V.V., Kehoe, P., McGrattan, E., 2002. Can Sticky Prices Generate Volatile And Persistent Real Exchange Rates. Review of Economic Studies 69, 633-663.

[13] Cochrane, J., 2001. Asset Pricing, Princeton University Press, Princeton.

[14] Colacito, R., Croce, M., 2006. Risk for the Long Run and the Real Exchange Rate. New York University.

[15] Cole, H., Obstfeld, M., 1991. Commodity Trade And International Risk Sharing: How Much Do Financial Markets Matter? Journal of Monetary Economics 28, 3-24.

[16] Corsetti, G., Dedola, L., Leduc, S., 2005. International Risk-Sharing And The Transmission of Productivity Shocks. International Finance Discussion Paper 826, Board of Governors of the Federal Reserve System.

[17] Erceg, C., Levin, A., 1996. Structures And The Dynamic Behavior Of The Real Exchange Rate, manuscript, Board of Governors of the Federal Reserve.

[18] Fitzgerald, D., 2006. Trade Costs, Asset Market Frictions and Risk Sharing: A Joint Test, manuscript, Stanford University.

[19] Hansen, L., Jagannathan, R., 1991. Implications Of Security Market Data For Models Of Dynamic Economies. Journal of Political Economy 99 225-262.

[20] Heathcote, J., Perri, F., 2002. Financial Autarky And International Business Cycles. Journal of Monetary Economics 49, 601-627.

[21] Heaton, J., Lucas, D., 1996. Evaluating The Effects Of Incomplete Markets On Risk-Sharing And Asset Pricing. Journal of Political Economy 104, 443-487.

[22] Kehoe, T., Levine, D., 1993. Debt-Constrained Asset Markets. Review of Economic Studies 60, 865-888.

[23] Kehoe, P., Perri, F., 2002. International Business Cycles With Endogenous Incomplete Markets. Econometrica 70, 907-928.

[24] Kocherlakota, N., 1996. Implications Of Efficient Risk Sharing Without Commitment. Review of Economic Studies 63, 595-609.

[25] Lewis, K., 1996. What Can Explain the Apparent Lack of International Consumption Risk Sharing? Journal of Political Economy 104, 267-297.

[26] Lewis, K., 1999. Trying to Explain the Home Bias in Equity and Consumption. Journal of Economic Literature 37, 571-608.

[27] Lustig, H., 2004. The Market Price of Aggregate Risk and the Wealth Distribution, manuscript, UCLA.

[28] Marcet, A., Marimon, R., 1999. Recursive Contracts, manuscript, Universitat Pompeu Fabra.

[29] Mehra, R., and E. Prescott 1985. The Equity Premium: A puzzle. Journal of Monetary Economics 15, 145-161.

[30] Messner, M., and N. Pavoni 2004. A Note on the Recursive Saddle Point Method, manuscript, University College London.

[31] Obstfeld, M., and K. Rogoff 1996. Foundations Of International Macroeconomics. MIT Press, Cambridge.

[32] Obstfeld, M., and K. Rogoff 2000. The Six Major Puzzles in International Macroeconomics: Is there a Common Cause? In: Bernanke, B., Rogoff, K. (Eds.), NBER Macroeconomics Annual 2000. MIT Press, Cambridge, pp. 339-390.

[33] Verdelhan, A., 2006. A Habit-Based Explanation of the Exchange Rate Risk Premium, manuscript, Boston University.

A International stochastic discount factors

In this appendix, I show how to estimate stochastic discount factors using only asset prices and real exchange rate data. This section builds heavily on Cochrane (2001) and Brandt et al (2006).

A.1 Basic relations

There is a strictly positive discount factor m_t such that $p_t = E_t\left(m_{t+1}x_{t+1}\right)$ if and only if there are no arbitrage opportunities, i.e., a positive payoff has a positive price. In complete markets, no arbitrage implies that there exists a unique $m_t > 0$ such that $p_t = E_t\left(m_{t+1}x_{t+1}\right)$. This fundamental equation of asset pricing can also be written as

$$E_t\left(m_{t+1}R_{t+1}\right) = 1 \tag{24}$$

for any return series. Suppose that there is free trade in international assets. Consider an agent in country 1 who buys a foreign asset at price $p_{2,t}$. Equation (24) implies that

$$\begin{aligned} p_{2,t}q_t &= E_t\left(m_{1,t+1}q_{t+1}x_{2,t+1}\right) \\ p_{2,t} &= E_t\left(m_{1,t+1}\frac{q_{t+1}}{q_t}x_{2,t+1}\right), \end{aligned}$$

where q_t is the real exchange rate defined as $e_tP_{2,t}/P_{1,t}$ where $P_{i,t}$ is the price level in country i and e_t is the nominal exchange rate. For an agent in country 2 who buys the same asset (24) implies,

$$p_{2,t} = E_t\left(m_{2,t+1}x_{2,t+1}\right).$$

Hence,

$$E_t\left(m_{1,t+1}\frac{q_{t+1}}{q_t}R_{2,t+1}\right) = E_t\left(m_{2,t+1}R_{2,t+1}\right).$$

If markets are complete Backus et al (1996) show that

$$\log\frac{q_{t+1}}{q_t} = \log m_{2,t+1} - \log m_{1,t+1}. \tag{25}$$

If markets are not complete, the (positive) stochastic discount factor is not unique. If m_t is positive and satisfies (24) so does $m_t + \varepsilon_t$ with $E\left(\varepsilon_tR_t\right) = 0$. However, if m_t satisfies (25), $m_t + \varepsilon_t$ will usually not satisfy this relation.

A.2 Calculating discount factors

Following Hansen and Jagannathan (1991) and Cochrane (2001), the minimum variance stochastic discount factor can be calculated using only asset returns:

$$\begin{aligned} m_{1,t+1} &= E(m_1) - \left[E(m_1) E\left(\tilde{R}_1\right) - 1\right]' \Sigma_1^{-1} \left[\tilde{R}_{1,t+1} - E\left(\tilde{R}_1\right)\right] \\ \Sigma_1 &= E\left(\left[\tilde{R}_1 - E\left(\tilde{R}_1\right)\right]\left[\tilde{R}_1 - E\left(\tilde{R}_1\right)\right]'\right), \end{aligned}$$

where $E(\cdot)$ denotes the unconditional expectation. From the perspective of the agent in country 1 the return series are $\tilde{R}_{1,t} = \left[R_{1,t}, \frac{q_{t+1}}{q_t} R_{2,t}\right]$. Σ_1 and $E\left(\tilde{R}_1\right)$ can be estimated directly from the data. Note that the above formula holds for any $E(m_1)$. The risk free rate satisfies $E(m_i) = 1/R_{i,f}$. The minimum variance stochastic discount factors satisfy the fundamental equation (25), whether markets are complete or not

$$\log \frac{q_{t+1}}{q_t} = \log m_{2,t+1} - \log m_{1,t+1}.$$

To see this,

$$\begin{aligned} m_{2,t+1} &= 1/R_{2,f} - 1/R_{2,f} \left[E\left(\tilde{R}\right) - R_{2,f}\right]' \Sigma_2^{-1} \left[\tilde{R}_{2,t+1} - E\left(\tilde{R}_2\right)\right] \\ &= 1/R_{2,f} - 1/R_{2,f} \left[E\left(\tilde{R}_2\right) - R_{2,f}\right]' \frac{q_{t+1}}{q_t} \\ &\quad \times \left(\Sigma_2 \frac{q_{t+1}}{q_t} \frac{q_{t+1}}{q_t}\right)^{-1} \left[\tilde{R}_{2,t+1} - E\left(\tilde{R}_2\right)\right] \frac{q_{t+1}}{q_t} \end{aligned}$$

$$\begin{aligned} & \frac{q_t}{q_{t+1}} m_{2,t+1} \\ = \; & \frac{1}{R_{1,f}} - \frac{1}{R_{1,f}} \left[E\left(\tilde{R}_1\right) - R_{1,f}\right]' \\ & \times \left(\left[\left(\tilde{R}_{1,t} - E\left(\tilde{R}_1\right)\right)\left(\tilde{R}_{1,t} - E\left(\tilde{R}_1\right)\right)'\right]\right)^{-1} \left[\tilde{R}_{1,t+1} - E\left(\tilde{R}_1\right)\right] \\ = \; & m_{1,t+1} \end{aligned}$$

Brandt et al (2006) have an explicit formal and quantitative analysis for the case of market incompleteness.

A.3 Estimation results

In order to estimate stochastic discount factors, I use country stock market indices, interest rates, nominal exchange rates and inflation rates for the G7 countries. All the results reported in this appendix are based on quarterly data for the period Q4 1978-Q4 2003. However, the results do not depend on the frequency of the data or the exact time window. The stock indices are total market returns from Datastream and the interest rates are for one-month Eurocurrency deposits. Nominal exchange rates are taken from the OECD database and CPI data comes from the International Monetary Fund's IFS database.

When I construct an aggregate of the G7 countries excluding the U.S., I use market capitalization from Datastream, real output data and trade shares from the OECD to construct the country weights in the index.

A.3.1 Summary statistics

Table A1 summarizes the estimates for the equity premium, $R^e - R^f$, in the G7 countries.

	CAN	FRA	GER	ITA	JAP	UK	U.S.
mean	4.47	7.66	4.64	6.83	3.78	6.06	6.88
std	16.98	23.60	21.41	29.81	21.55	17.50	16.54

Table A1

As in Brandt et al (2003), who only consider the U.S., the UK, Germany and Japan, equity premia are high and volatile. Also, excess stock market returns are strongly correlated within the G7. These correlations range from 0.39 for Italy and Japan to 0.89 for Canada and the US. For comparison, the correlation of U.S. GDP with the aggregate of the remaining G7 countries is about 0.65.

	CAN	*FRA*	*GER*	*ITA*	*JAP*	*UK*	*USA*
CAN	1.00	0.64	0.62	0.48	0.48	0.69	0.84
FRA		1.00	0.76	0.62	0.42	0.63	0.69
GER			1.00	0.59	0.42	0.65	0.69
ITA				1.00	0.39	0.54	0.52
JAP					1.00	0.55	0.50
UK						1.00	0.77
USA							1.00

Table A2

To obtain an empirical analogue for the two country model, I constructed an aggregate index for stock market returns and interest rates for the G7 countries excluding the U.S. Results for the following indices are reported:

1. value weighted stock market index (market capitalization) and the same weights for interest rates,
2. equally weighted stock market index and the same weights for interest rates,
3. real GDP weighted stock market index (1990 real GDP) and the same weights for interest rates,
4. real GDP weighted stock market index (2000 real GDP) and the same weights for interest rates,
5. value weighted stock market index (market capitalization) and equally weighted interest rates,
6. value weighted stock market index (market capitalization) and real GDP weighted interest rates (1990 real GDP),
7. value weighted stock market index (market capitalization) and real GDP weighted interest rates (2000 real GDP).

The correlation of each country's excess returns with any of the seven aggregate indices is very high:

	CAN	FRA	GER	ITA	JAP	UK
Index 1	0.68	0.64	0.65	0.58	0.92	0.77
Index 2	0.78	0.86	0.83	0.80	0.66	0.82
Index 3	0.73	0.83	0.82	0.79	0.73	0.80
Index 4	0.74	0.82	0.82	0.75	0.77	0.80
Index 5	0.68	0.62	0.61	0.59	0.91	0.74
Index 6	0.68	0.63	0.62	0.59	0.91	0.75
Index 7	0.68	0.64	0.62	0.59	0.91	0.76

Table A3

Weighting excess return series by GDP (Index 3 and 4) results in uniformly high correlations. For value weighted indices (1, 5, 6 and 7), Japan's excess returns are clearly more correlated with the index than the returns of the remaining countries. This is simply an artefact of the relatively high market capitalization of the Japanese stock market.

The correlation of the U.S. return series with any of the indices is very similar to the correlation of U.S. returns with any individual country:

Correlation of U.S. excess returns with aggregate indices for G7\U.S.

Index 1	Index 2	Index 3	Index 4	Index 5	Index 6	Index 7
0.66	0.75	0.72	0.72	0.66	0.66	0.66

Table A4

A.3.2 Estimates

Table A5 reports the (annualized) standard deviations of the minimum variance stochastic discount factors for country pairs. The first row denotes the "home" country and the first column denotes the "foreign" country in the estimation.

	CAN	FRA	GER	ITA	JAP	UK	USA
CAN		0.3833	0.4163	0.3192	0.3362	0.3656	0.4513
FRA	0.4150		1.1516	0.3581	0.4893	0.4029	0.5136
GER	0.3386	1.1699		0.7280	0.2586	0.5386	0.4595
ITA	0.3472	0.3459	0.7845		0.3964	0.3563	0.4792
JAP	0.2734	0.4223	0.3046	0.3173		0.3753	0.4222
UK	0.3841	0.3903	0.6037	0.3547	0.4390		0.4656
USA	0.4402	0.4823	0.5065	0.4516	0.4552	0.4402	

Table A5

Similar to stochastic discount factors, that are calculated from only domestic data, the international stochastic discount factors are very volatile, too. In addition, they are highly correlated.

	CAN	FRA	GER	ITA	JAP	UK	USA
CAN	1.0000	0.9730	0.9843	0.9580	0.9384	0.9665	0.9961
FRA		1.0000	0.9995	0.9929	0.9842	0.9800	0.9828
GER			1.0000	0.9971	0.9399	0.9942	0.9841
ITA				1.0000	0.9643	0.9781	0.9766
JAP					1.0000	0.9635	0.9651
UK						1.0000	0.9763
USA							1.0000

Table A6

The correlations in Table A6 are much higher than the correlation of stock market returns in Table A3. These results confirm the findings of Brandt et al (2006) who estimate stochastic discount factors for the three country pairs U.S.-Japan, U.S.-UK and U.S.-Germany. Using any index for the G7 excluding the U.S. reveals that these properties also carry over in the aggregate.

Correlation of U.S. sdf with G7\U.S. sdf using aggregate indices

Index 1	Index 2	Index 3	Index 4	Index 5	Index 6	Index 7
0.9913	0.9911	0.9908	0.9908	0.9921	0.9916	0.9915

Table A7

B Computations

The computations of the solution of the model has two parts. First, given the endowment process I solve for the set of feasible allocations of the aggregate consumption good. Given the consumption set, the optimal allocations for the economy with enforcement constraints are then determined using a policy function iteration algorithm.

B.1 The consumption set

The set of feasible allocations is found by solving the following problem:

$$\begin{aligned}
&\max_{\substack{c_{11}^T, c_{12}^T, c_1^N \\ c_{21}^T, c_{22}^T, c_2^N}} c_1 = \left[\left(\alpha_1^T\right)^{1-\phi} \left(c_1^T\right)^{\phi} + \left(\alpha_1^N\right)^{1-\phi} \left(c_1^N\right)^{\phi} \right]^{\frac{1}{\phi}} \\
&s.t. \\
&c_2 = \left[\left(\alpha_2^T\right)^{1-\phi} \left(c_2^T\right)^{\phi} + \left(\alpha_2^N\right)^{1-\phi} \left(c_2^N\right)^{\phi} \right]^{\frac{1}{\phi}} \\
&y_1^T \geq c_{11}^T + c_{21}^T \\
&y_2^T \geq c_{12}^T + c_{22}^T \\
&y_1^N \geq c_1^N + \eta c_{11}^T + \eta c_{12}^T \\
&y_2^N \geq c_2^N + \eta c_{21}^T + \eta c_{22}^T
\end{aligned}$$

where

$$c_i^T = \left[\alpha_{i1}^{1-\rho} \left(c_{i1}^T\right)^{\rho} + \alpha_{i2}^{1-\rho} \left(c_{i2}^T\right)^{\rho} \right]^{\frac{1}{\rho}}, \ \rho < 1, \ \alpha_{i1} + \alpha_{i2} = 1, \ i = 1, 2.$$

Define a grid $\bar{C}_2$ on the space of country 2 consumption. For each $c_2 \in \bar{C}_2$ the above program is solved to find the corresponding value of c_1. This provides an approximation of $F(c_1, c_2, y) = 0$ on the grid $\bar{C}_2$. Using cubic splines, I obtain a continuous approximation of the boundary of the consumption set.

Continuous approximations for goods prices and the real exchange rate as functions of (c_1, c_2) are easily found by solving (22) in section 3.5 and using cubic spline interpolation.

B.2 Policy functions

The computational procedure used to solve for the policy functions in an economy with enforcement constraints is based on the work of Kehoe and Perri (2002) and Marcet and Marimon (1999). If the default decision is made at the aggregate level, $V_i(\cdot)$ can be directly calculated from the allocations that arise absent international financial markets. I discuss the additional complications that arise when each household makes her own decision about quitting the risk sharing arrangement at the end of this appendix.

Let $x = (z, s)$ be the state of the economy. Each $s \in S$ corresponds to one particular realization of the endowment vector $y = \left(y_1^T, y_2^T, y_1^N, y_2^N\right)$. The goal is to find policy functions for current consumption, $c_i(x)$, the real exchange rate, $q(x)$, the multipliers on the enforcement constraints, $v_i(x)$, and the future relative weight $z'(x)$. For convenience, I also define the functions $W_i(x)$ that satisfy

$$W_i(x) = u[c_i(x)] + \beta \sum_{s'|s} \pi(s'|s) W_i(x'). \tag{26}$$

A solution has to satisfy the following set of equations

$$q(x) = z'(x) \kappa \frac{u_c(c_2(x))}{u_c(c_1(x))} = \frac{F_2(c_1(x), c_2(x), s)}{F_1(c_1(x), c_2(x), s)}, \tag{27}$$

$$F(c_1(x), c_2(x), s) = 0, \tag{28}$$

$$z'(x) = \frac{1 - v_1(x)}{1 - v_2(x)} z, \tag{29}$$

$$u[c_i(x)] + \beta \sum_{s'|s} \pi(s'|s) W_i(x') \geq V_i(x), \tag{30}$$

and $v_i(x) \geq 0$ with $v_i(x) = 0$ whenever (30) holds as a strict inequality. In practice, I define a grid X on the state space and restrict the search to within the class of functions that take arbitrary values for every $x \in X$ and are equal to the cubic spline interpolation of those values for every $x \notin X$. These functions are completely characterized over the state space by specifying their value for every $x \in X$.

The initial guess is taken to be the policy and value functions under full risk sharing. Denote this initial guess by $(c_i^0(x), v_i^0(x), W_i^0(x), q^0(x))$ for every $x \in X$. Given the above equations a new set of values is found as follows. Under the assumption that neither enforcement constraint is binding, $c_1(x)$, $c_2(x)$ and $q(x)$ can easily be calculated from (27) and (28), since $v_1(x) = v_2(x) = 0$ and $z'(x) = z$. Remains to check whether the enforcement constraints are indeed not binding, i.e. whether (30) is satisfied. If (30) is satisfied, full risk sharing is possible in state x.

Now, suppose that the enforcement constraint is violated for agent 1. Find $c_1(x)$ such that the enforcement constraint is just binding given the current guess of the value functions $W_1(\cdot)$. Equations (26)-(28) deliver the new values for $c_2(x)$, $q(x)$ and $z'(x)$. $v_1(x)$ is found from equation (29). The new values of the value function at x are $W_1(x) = V_1(x)$ for country 1 and $W_2(x) = u[c_2(x)] + \beta \sum_{s'|s} \pi(s'|s) W_2(x')$ for country 2. Similarly, if the the enforcement constraint for country 2 is binding. Clearly, it cannot be that both countries are constrained simultaneously. This procedure is repeated for every $x \in X$ until convergence is reached.

If the default decision is made by each agent individually, the computation $V_i(\cdot)$ depends on the current guess of the policy and value functions. In her decision to quit the risk sharing agreement, the agent does not take into account that the remaining agents in her country face the same decision problem. Therefore, she will assume that prices remain unchanged after her default. In this case the values for $(c_i(x), v_i(x), W_i(x), q(x), V_i(x))$ for each $x \in X$ are computed simultaneously. Given the current guess for $(c_i(x), v_i(x), W_i(x), q(x))$ for all $x \in X$, $V_i(\cdot)$ is found by solving the dynamic programming problem of an agent in financial autarchy given the policy function for prices which can be derived from the policy functions for $c_i(x)$ and the solution to (22).