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Abstract

The real exchange rate is very volatile relative to major macroeconomic aggregates and its correlation with the ratio of domestic over foreign consumption is negative (Backus-Smith puzzle). These two observations constitute a puzzle to standard international macroeconomic theory. This paper develops a two country model with complete asset markets and limited enforcement for international financial contracts that provides a possible explanation of these two puzzles. The model performs better than a standard incomplete markets model with a single non-contingent bond unless very tight borrowing constraints are imposed in the latter. With limited enforcement for both domestic and international financial contracts, the model's asset pricing implications are brought into line with the empirical evidence, albeit at the expense of raising real exchange rate volatility.

Keywords: risk-sharing, limited enforcement, real exchange rate, Backus-Smith puzzle, asset prices

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1 Introduction

This paper analyses the interplay of three classic puzzles about the real exchange rate and asset prices:

1. the high volatility of the real exchange rate relative to the volatility of consumption (real exchange rate volatility puzzle),
2. the negative correlation of the real exchange rate with the ratio of domestic over foreign consumption (Backus-Smith puzzle),
3. the low correlation of consumption across countries.

I develop a two country model with complete asset markets and limited enforcement of international financial contracts that provides a possible explanation of the these puzzles.

In their simplest form these three puzzles can be stated as follows.[1] If preferences over consumption are given by the power utility function and all financial markets are complete, the real exchange rate between two countries is determined by the ratio of domestic and foreign consumption. This immediately implies that the correlation between the real exchange rate and relative consumption equals unity. Since there are no wealth effects under complete markets, consumption is highly correlated across countries. Therefore, the real exchange rate hardly fluctuates.

Given this apparent contradiction with the data, most international macroeconomists have concluded that international financial risk sharing is not complete. Lewis (1996) provides also direct empirical evidence that international risk sharing is incomplete. Although it is nowadays standard to assume that there are frictions in international financial markets, there has been little progress in explaining the first two puzzles. Two notable exceptions are Corsetti, Dedola and Leduc (2007) and Benigno and Thoenissen (2007).

[1] See Obstfeld and Rogoff (1996) for the volatility puzzle and Backus and Smith (1993) for the consumption-real exchange rate correlation puzzle.

Brandt, Cochrane and Santa-Clara (2006) have recently challenged the view that international consumption risk sharing is very limited. Their analysis draws on the high volatility of asset prices and the implied high volatility of the intertemporal marginal rate of substitution. Real exchange rates between industrialized economies fluctuate by as much as 10% per annum. However, the intertemporal marginal rate of substitution estimated using asset returns varies by 40%. As the real exchange rate depreciates by the difference between the domestic and foreign intertemporal marginal rates of substitution, these estimated volatilities imply that the intertemporal marginal rates of substitution are highly correlated between countries. Brandt et al interpret this finding as evidence, that international risk sharing is very good.[2]

This paper attempts to clarify these contradictory conclusions about international risk sharing. I first follow Kehoe and Levine (1993) in assuming that international financial markets are complete but enforcement of international financial contracts is limited.[3] Contracts are sustainable only to the extent that they can be enforced by the threat of permanent exclusion from trade in international financial markets if an agent reneges on her obligations.[4]

The production/trade side of the economy is modelled as in Corsetti et al (2007). The distinguishing feature of their model is that the implied elasticity of substitution

[2]While Brandt et al (2006) stronlgy advocate this interpretation of their findings, the following interpretation puts their results more in line with the general consensus in the literature. With complete markets and additive power utility over consumption the high correlation of the intertemporal marginal rates of substitution that Brandt et al document implies highly correlated consumption growth. However, consumption and consumption growth are not very correlated across countries at business cycle frequencies. I am thankful to an editor of this journal for suggesting this interpretation of Brandt et al. (2003).

[3]Models with limited contract enforceability have been applied to a number of questions: Krueger and Perri (2006) address the question of consumption inequality in the U.S., Kehoe and Perri (2002) analyze the implications for the international business cycle, Lustig (2004) and Alvarez and Jermann (2001) investigate asset pricing implications.

[4]Kehoe and Perri (2002) analyze a two country model with limited contract enforcement. However, since there is only one good in their model all trade is intertemporal and the real exchange rate is constant and equal to 1.

between traded goods is low since non-traded goods are used in the distribution of traded goods. This feature implies that absent international financial markets the real exchange rate is very volatile and the correlation between the real exchange rate and relative consumption is negative.

The key finding of my paper is that the model with complete asset markets and enforcement constraints can resolve the real exchange rate volatility puzzle and the Backus-Smith puzzle provided that agents are sufficiently impatient. If agents are impatient, only limited risk sharing can be sustained and the model behaves close to a model without international financial markets. If agents are very patient, contract enforcement works well and agents can share risk efficiently across countries. In this case consumption is highly correlated across countries, the real exchange rate is very smooth, and the correlation between the real exchange rate and relative consumption is close to unity. I also compare the model with limited contract enforcement to a model with a single non-contingent bond. The latter model fails to deliver substantial exchange rate volatility and a negative correlation between the real exchange rate and relative consumption unless tight constraints on international borrowing are imposed.[5]

Because I follow the international finance literature in assuming complete and frictionless domestic asset markets and standard preferences, the model inherits all the puzzles of domestic asset pricing. In particular all asset prices are very smooth and the equity premium is too low.[6] One potential resolution of the equity premium puzzles in a closed economy is offered by Alvarez and Jermann (2001).[7]

[5]This finding is at odds with the results in Corsetti et al (2007) and Benigno et al (2007). However, there are several differences between their works and mine: I assume (i) an endowment economy in contrast to their production economies, (ii) a somewhat higher elasticity of substitution between traded goods and (iii) I solve the model using non-linear methods as opposed to a methond that is based log-linearization of the first order equations around the deterministic steady state.

[6]See in particular Mehra and Prescott (1985) and Hansen and Jagannathan (1991).

[7]In line with the empirical findings presented in Heaton and Lucas (1996), these authors assume that agents' idiosyncratic incomes are volatile relative to aggregate income. Also, asset markets are assumed to be complete

Following these ideas, I subsequently enrich my model by assuming that contract enforcement is also limited for domestic financial contracts and that agents face high personal income risk. My main findings are: first, as in the data, the intertemporal marginal rates of substitution are volatile and so are asset prices. The standard deviation of the marginal rate of substitution is about 40%. Second, the model can still replicate the findings of Backus and Smith (1993). Third, in sharp contrast to the original model, the real exchange rate is too volatile as its standard deviation rises from 7% to 60%.

What explains this drastic increase? In the model changes in the real exchange rate are equal to the difference between the log of the foreign and the domestic intertemporal marginal rates of substitution. If domestic contracts are fully enforceable, the volatility of the marginal rates of substitution is determined by the low volatility of aggregate consumption and the real exchange rate is roughly as volatile as in the data.

In the extended model, the high volatility of the marginal rates of substitution stems from high idiosyncratic income risk that cannot be insured efficiently due to limited enforcement in domestic asset markets. Highly volatile marginal rates of substitution with a standard deviation of roughly 40% can only be reconciled with an exchange rate volatility of around 7% if the correlation between the foreign and the domestic marginal rates of substitution is larger than 0.9. However, a correlation of 0.9 cannot arise in the model with limited enforcement as this class of models implies volatile marginal rates of substitution only if risk is not shared efficiently both domestically and internationally. The correlation of the marginal rates of substitution implied by the model, however, is 0.16.

This paper is closely related to the works of Corsetti et al (2007) and Brandt et al (2006). Corsetti et al address the exchange rate volatility puzzle and the Backus-Smith puzzle in a model similar to mine. However, they assume that international

but enforcement of financial contracts is limited.

financial markets are exogenously incomplete: the only asset that is traded internationally is one non-state-contingent bond. I show in this paper how their results extend to an environment with a larger set of available assets.

Based on the Backus-Smith puzzle, Corsetti et al (2007) conclude like many others before that international risk sharing is very limited.[8] This conclusion stands in sharp contrast to Brandt et al (2006) who argue the opposite based on asset return data. This contradiction arises since each group of authors considers only two of the three puzzles mentioned above. Corsetti et al are silent on the volatility of asset prices; Brandt et al do not relate their findings to the Backus-Smith puzzle.

Scholl (2005) shows that limited enforceability substantially alters cross country-correlations and the dynamics of net exports. While this finding is in line with the results reported in this paper, her work does not address the Backus-Smith puzzle or the volatility of the real exchange rate.

Colacito and Croce (2006) and Verdelhan (2006) provide useful insight into the work of Brandt et al (2006) from a different perspective. They suggest modelling frameworks that are consistent with the observed volatility of the real exchange rate and the volatility of asset returns. Unfortunately, neither approach provides a satisfying answer to the Backus-Smith puzzle. The correlation between the real exchange rate and relative consumption is close to or equal to unity in both papers.

The remainder of the paper is organized as follows. Section 2 provides a deeper introduction to the puzzles that are analyzed in this paper. In Section 3, I present a two country model with complete international financial markets and enforcement constraints. Section 4 presents and discusses the qualitative and quantitative implications of the benchmark model. In order to address the evidence provided in Brandt et al (2006), Section 5 extends the benchmark model to a two country model with heterogenous agents. Section 6 concludes.

[8] See Lewis (1999) for a summary of the literature on international risk sharing.

2 Real exchange rate puzzles

2.1 The correlation puzzle

Assume international market completeness and that agents have preferences described by $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, where c is consumption and γ is the coefficient of relative risk aversion. The predicted correlation between the real exchange rate and relative consumption for a given country pair equals unity, i.e.,

$$\rho_{q,\frac{c_1}{c_2}} = corr\left(-\gamma \log \frac{c_{1,t}}{c_{2,t}}, \log \frac{c_{1,t}}{c_{2,t}}\right) = 1.$$

However, Backus and Smith (1993) find that the actual correlation between the real exchange rates and relative consumption is low and often negative. Corsetti, Dedola and Leduc (2007) report pairwise estimates for the US with major industrialized economies. Their median estimate for HP-filtered data is -0.42. The estimate for the US with an aggregate of industrialized economies is -0.71.

2.2 The volatility of the real exchange rate

When embedding the assumption of complete markets and power utility into a general equilibrium model of the international business cycle, the predicted volatility of the real exchange rate σ_q relative to consumption σ_{c_1} is too low for reasonable levels of risk aversion γ. In the data, the real exchange rate is roughly four times as volatile as consumption. However, models with complete international financial markets typically predict a very high correlation of consumption across countries $\rho_{c_1c_2}$. With $\rho_{c_1c_2}$ close to 1,

$$\frac{\sigma_q}{\sigma_{c_1}} = \gamma\sqrt{2\left(1-\rho_{c_1c_2}\right)},$$

one can match a volatility ratio of $\frac{\sigma_q}{\sigma_{c_1}} \approx 4$ only if γ is implausibly large. Obviously, the model's implication of $\rho_{c_1c_2}$ being close to unity is already unrealistic, as the data suggests values around 0.4.

3 Setup

3.1 Financial Markets

Each period t the economy experiences one of finitely many events $s \in S$. Let the transition probability from state s to s' follow a Markov chain denoted by $\pi(s'|s)$. $s^t = (s_0, s_1, ...s_t)$ denotes the history of events up through and including period t. The probability, as of period 0, of any history s^t is $\pi(s^t)$. With the initial realization s_0, the Markov transition probabilities induce the probability distribution $\pi(s^t) = \pi(s_t|s_{t-1})\pi(s_{t-1}|s_{t-2})...\pi(s_1|s_0)$.

There are two countries, $i = 1, 2$, each of which is populated by a large number of identical, infinitely lived households. At the beginning of each period, households are endowed with $y_i^T(s_t)$ units of a tradable good and $y_i^N(s_t)$ units of a non-tradable good. The domestic and foreign tradable good are imperfect substitutes. Let $y(s_t) = \left(y_1^T, y_2^T, y_1^N, y_2^N\right)$ be the endowment vector in state s_t. The endowment vector depends solely on the current realization s_t. Final consumption in country i in history s^t, $c_i(s^t)$, is a function of the consumption of the two tradables and the non-tradable good. A more explicit structure of the goods market is introduced in section 3.5. For now, all that is assumed, is that the endowment vector at time t can be mapped into an aggregate international resource constraint of the form $\left\{(c_1, c_2)|\tilde{F}(c_1, c_2, y) \leq 0\right\}$. I assume that this set is non-empty, bounded, and strictly convex for each realization of the endowment vector. The latter is an immediate implication of the imperfect substitutability of the domestic and the foreign tradable good. The function $F(\cdot)$, defined as $F(c_1, c_2, y) = 0$, is differentiable with respect to its first two arguments. Since the real exchange rate is the price of the consumption basket in country 2 relative to country 1, the real exchange rate is linked to $F(\cdot)$ through $q = \frac{F_2(c_1,c_2,y)}{F_1(c_1,c_2,y)} = -\frac{dc_1}{dc_2}$. F_i is the derivative of F with respect to its ith argument.

Households in country i rank consumption streams $\{c_i(s^t)\}_{t=0}^{\infty}$ according to

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)u\left[c_i\left(s^t\right)\right]. \tag{1}$$

$u(c)$ is strictly increasing, strictly concave and continuously differentiable. The budget constraint of the agent is given by

$$\begin{aligned} & P_i\left(s^t\right)c_i\left(s^t\right)+\sum_{s^{t+1}|s^t}Q_i\left(s^{t+1}|s^t\right)a_i\left(s^{t+1}|s^t\right) \\ \leq\ & a_i\left(s^t|s^{t-1}\right)+\bar{P}_{ii}^T\left(s^t\right)y_i^T\left(s^t\right)+P_i^N\left(s^t\right)y_i^N\left(s^t\right), \end{aligned} \tag{2}$$

where P_i is the (currency) price of one unit of the final consumption good in country i, $\bar{P}_{ij}^T$ is the price of tradable good i in country j, and P_i^N is the price of the non-tradable good in country i.

In this economy, financial markets are complete, i.e., agents have access to a complete set of one-period state-contingent claims. The holdings of such claims by the representative agent in country i are denoted by $a_i(s^{t+1}|s^t)$. Each claim pays one unit of country $i's$ currency in period $t+1$ if the particular state s_{t+1} occurs and 0 otherwise. $Q_i(s^{t+1}|s^t)$ is the price of such a claim in country i's currency.

Building on the seminal work of Kehoe and Levine (1993) and Kocherlakota (1996) international loans are assumed to be sustainable to the extent that they can be enforced by the threat of exclusion from future trade in asset markets.[9] The enforcement constraint is therefore given by

$$\sum_{r=t}^{\infty}\sum_{s^r|s^t}\beta^{r-t}\pi\left(s^r|s^t\right)u\left[c_i\left(s^r\right)\right]\geq V_i\left(s^t\right), \tag{3}$$

where $V_i(s^t)$ is the value for agent i in financial autarchy from s^t onwards. Notice that like in Kehoe and Levine (1993) but unlike in the one-good models of Alvarez and Jermann (2000) or Kehoe and Perri (2002), I assume that agents can still trade in the international goods markets after default.

[9]Fitzgerald (2006) reports empirical evidence that is in line with the assumption of limited contract enforceability at the international level.

In Kehoe and Perri (2002), the decision to default is made by the government. In this case the value of financial autarchy, $V_i(s^t)$, is given by the discounted present value at the prices that actually occur in autarchy. If the default decision is made by the individual agent, however, each agent assumes that her decision to default will not affect prices in the goods market. The agent does not take into account that other agents might default, as well. In either case, the value of financial autarchy is determined from

$$V_i\left(s^t\right) = \max_{\{c_i(s^r)\}} \sum_{r=t}^{\infty} \sum_{s^r|s^t} \beta^{r-t} \pi\left(s^r|s^t\right) u\left[c_i\left(s^r\right)\right] \tag{4}$$

$s.t.$

$$P_i\left(s^t\right) c_i\left(s^t\right) \leq \bar{P}_{ii}^T\left(s^t\right) y_i^T\left(s^t\right) + P_i^N\left(s^t\right) y_i^N\left(s^t\right), \tag{5}$$

where the perceived prices $P_i(s^t)$, $\bar{P}_{ii}^T(s^t)$, $P_i^N(s^t)$ depend on who decides whether to default.

The maximization problem of each agent can now be stated as

$$\max_{\{c_i(s^t)\}} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) u\left[c_i\left(s^t\right)\right]$$

subject to (2) and (3).[10]

3.2 Equilibrium

Definition 1 (Competitive Equilibrium) *An equilibrium in the economy with enforcement constraints is a collection of allocations* $c_i(s^t)$, $a_i(s^t)$, $i = 1, 2$ *and prices* $P_i(s^t)$, $\bar{P}_{ij}^T(s^t)$, $P_i^N(s^t)$, $Q_i(s^{t+1}|s^t)$, $i = 1, 2$ *and* $j = 1, 2$ *such that* (1) *the consumer allocations solve the consumers' problem in both countries, and in particular the enforcement constraints are satisfied;* (2) *the resource constraint holds*

[10] Jeske (2006) compares a centralized arrangement for international debt where only governments borrow and lend internationally with a decentralized arrangement where individual borrowers have access to international capital markets. He shows that more risk sharing can be sustained in equilibrium in a centralized setup.

for all s^t, $F\left(c_1\left(s^t\right), c_2\left(s^t\right), y\left(s_t\right)\right) = 0$*; and* (3) *asset markets clear,* $a_1\left(s^{t+1}|s^t\right) + a_2\left(s^{t+1}|s^t\right) = 0$.

Since I consider a real economy, the nominal exchange rate is fixed at 1. Furthermore, the price of the final consumption good in each country is normalized to 1 and the real exchange rate is defined to be $q\left(s^t\right) = P_2\left(s^t\right)/P_1\left(s^t\right)$.

3.3 Solution

The approach follows Marcet and Marimon (1999) and Kehoe and Perri (2002).[11] Let $\beta^t \pi\left(s^t\right)\mu_i\left(s^t\right)$ denote the Lagrangian multipliers on the enforcement constraints in the optimization problem of the representative agent in country i. Using the "partial summation formula of Abel" this problem can be written as

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)\left[u\left[c_i\left(s^t\right)\right] + \mu_i\left(s^t\right)\left(\sum_{r=t}^{\infty}\sum_{s^r|s^t}\beta^{r-t}\pi\left(s^r|s^t\right)u\left[c_i\left(s^r\right)\right] - V_i\left(s^t\right)\right)\right]$$

$$= \sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi\left(s^t\right)\left[M_i\left(s^{t-1}\right)u\left[c_i\left(s^t\right)\right] + \mu_i\left(s^t\right)\left\{u\left[c_i\left(s^t\right)\right] - V_i\left(s^t\right)\right\}\right],$$

where

$$M_i\left(s^t\right) = M_i\left(s^{t-1}\right) + \mu_i\left(s^t\right), \tag{6}$$

and $M_i\left(s_0\right) = 1$. $\mu_i\left(s^t\right) > 0$ if the enforcement constraint (3) is binding for country i and zero otherwise. Note that at each point in time, at most one country can be constrained.

The first order conditions of the representative agent in country i are summarized by

$$\lambda_i\left(s^t\right)P_i\left(s^t\right) = \left[M_i\left(s^{t-1}\right) + \mu_i\left(s^t\right)\right]u_c\left[c_i\left(s^t\right)\right], \tag{7}$$

$$Q_i\left(s^{t+1}|s^t\right) = \beta\pi\left(s_{t+1}|s_t\right)\frac{\lambda_i\left(s^{t+1}\right)}{\lambda_i\left(s^t\right)}, \tag{8}$$

[11] Messner and Pavano (2004) have recently hinted to some pitfalls of this approach. However, for an endowment economy their criticism does not apply.

where $\lambda_i(s^t)$ is the Lagrangian multiplier on the budget constraint of an agent located in country i. Define the following variables

$$\begin{aligned} z\left(s^t\right) &= \frac{M_2\left(s^t\right)}{M_1\left(s^t\right)},\ z\left(s_0\right)=1, \\ v_i\left(s^t\right) &= \frac{\mu_i\left(s^t\right)}{M_i\left(s^t\right)},\ i=1,2. \end{aligned}$$

Equation (6) implies a law of motion for $z(\cdot)$

$$z\left(s^t\right)=\frac{1-v_1\left(s^t\right)}{1-v_2\left(s^t\right)}z\left(s^{t-1}\right). \tag{9}$$

Absent arbitrage opportunities, $Q_1(s^{t+1}|s^t)=Q_2(s^{t+1}|s^t)$, and I obtain an explicit expression for the real exchange rate

$$\frac{q\left(s^{t+1}\right)}{q\left(s^t\right)}=\frac{z\left(s^{t+1}\right)}{z\left(s^t\right)}\frac{u_c\left[c_2\left(s^{t+1}\right)\right]}{u_c\left[c_2\left(s^t\right)\right]}\frac{u_c\left[c_1\left(s^t\right)\right]}{u_c\left[c_1\left(s^{t+1}\right)\right]}. \tag{10}$$

Iterating on this expression, delivers

$$q\left(s^t\right)=z\left(s^t\right)\kappa\frac{u_c\left[c_2\left(s^t\right)\right]}{u_c\left[c_1\left(s^t\right)\right]}, \tag{11}$$

where $\kappa=\frac{q(s_0)}{z(s_0)}\frac{u_c[c_1(s_0)]}{u_c[c_2(s_0)]}$.

3.4 Interpretation

Computing equilibria in economies with limited enforcement involves finding the correct relative weights z. For a given sequence of Pareto weights $\{z(s^t)\}_{t=0}^{\infty}$, the problem of the planner can be thought of as

$$\begin{aligned} &\max_{c_1,c_2} u\left(c_1\left(s^t\right)\right)+z\left(s^t\right)u\left(c_2\left(s^t\right)\right) \\ &s.t. \\ &F\left(c_1\left(s^t\right),c_2\left(s^t\right),y\left(s_t\right)\right)=0. \end{aligned} \tag{12}$$

For given z, the planner's problem at time t resembles the static optimal allocation problem.

3.4.1 Partial risk sharing

To understand the forces that operate in the economy with enforcement constraints, I compare the allocations under full risk sharing with the allocations in financial autarchy. Due to the concavity of $u(\cdot)$, consumption in country i varies less across states of the world under complete markets than in financial autarchy. Consequently, there is at least one realization $\tilde{s} \in S$, such that in this particular state the agent in country i receives higher consumption in financial autarchy than under full risk sharing. Obviously, full risk sharing ($z(s^t) = 1$ for all s^t) cannot be implemented if the discount factor β is close to zero. If $\tilde{s}$ is realized, the utility loss from giving up the ability to share risk efficiently in the future is lower than the utility gain due to higher current consumption.

However, partial risk sharing might still be feasible. For simplicity, assume that at time $t-1$ the realized relative weight is $z(s^{t-1}) = 1$. Suppose that at s^t country 1 receives a positive shock to its endowment. Rather than sharing the additional wealth with country 2, agents in country 1 prefer consuming this wealth by themselves, i.e., the enforcement constraint binds for country 1. To provide an incentive for country 1 to hand some of her wealth to country 2 in the current period t, the planner promises to raise her average consumption in the future. Under partial risk sharing, consumption in country 1 rises relative to consumption in country 2 both in period t and in future periods compared to the allocations under full risk sharing. From equations (9) and (12) this means that the weight on country 1 has to increase, i.e., $z(s^t) < 1$ which implies an appreciation of the real exchange rate in the decentralized economy.

3.4.2 Consumption-real exchange rate correlation

Equation (11) reveals, how the model with enforcement constraints breaks the tight link between the real exchange rate and relative consumption that arises under frictionless and complete markets.

Let $\sigma(x)$ be the standard deviation of variable x and let $\rho\left(x, \frac{c_1}{c_2}\right)$ denote the correlation between variable x and the relative consumption $\frac{c_1}{c_2}$ with $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. The correlation between the real exchange rate and relative consumption can be expressed as

$$\rho\left(q, \frac{c_1}{c_2}\right) = \frac{\left(\frac{\sigma(z)}{\sigma\left(\frac{c_1}{c_2}\right)}\rho\left(z, \frac{c_1}{c_2}\right) + \gamma\right)}{\left[\left(\frac{\sigma(z)}{\sigma\left(\frac{c_1}{c_2}\right)}\rho\left(z, \frac{c_1}{c_2}\right) + \gamma\right)^2 + \left(\frac{\sigma(z)}{\sigma\left(\frac{c_1}{c_2}\right)}\right)^2\left(1 - \rho\left(z, \frac{c_1}{c_2}\right)^2\right)\right]^{\frac{1}{2}}}. \quad (13)$$

In the standard complete market framework without enforcement constraint z is constant and $\sigma(z) = 0$. Hence, $\rho\left(q, \frac{c_1}{c_2}\right) = 1$. In the economy with enforcement constraints z is not constant and the correlation between z and relative consumption $\frac{c_1}{c_2}$ is negative. If the enforcement constraint binds for country 1, the planner increases the weight on country 1 and increases current consumption in country 1 relative to country 2 as described previously. Equation (13) then implies that $\rho\left(q, \frac{c_1}{c_2}\right) < 1$.

3.5 Goods markets

The aggregate resource constraint of the global economy, $\tilde{F}(c_1, c_2, y(s_t)) \leq 0$, is derived from the underlying endowments with traded and non-traded goods. One possible specification that allows me to address the real exchange rate volatility puzzle and the consumption real exchange rate puzzle has been proposed by Corsetti et al (2007). There are four key features: imperfect substitutability between the domestic and the foreign tradable good, non-traded goods, distribution costs, and purchasing power parity for tradable goods at the producer level.[12]

[12]See Betts and Kehoe (2001) and Burstein, Eichenbaum and Rebelo (2002) for supportive evidence.

3.5.1 Deriving the international resource constraint

The final consumption good c_i is an aggregate of tradable and non-tradable goods:

$$c_i = \left[\left(\alpha_i^T\right)^{1-\phi} \left(c_i^T\right)^{\phi} + \left(\alpha_i^N\right)^{1-\phi} \left(c_i^N\right)^{\phi} \right]^{\frac{1}{\phi}}, \ \phi < 1, \ \alpha_i^T + \alpha_i^N = 1, \tag{14}$$

where c_i^T is the consumption of an aggregate of the tradable goods and c_i^N is the consumption of the non-traded good in country i.

The consumption index c_i^T is determined by

$$c_i^T = \left[\alpha_{i1}^{1-\rho} \left(c_{i1}^T\right)^{\rho} + \alpha_{i2}^{1-\rho} \left(c_{i2}^T\right)^{\rho} \right]^{\frac{1}{\rho}}, \ \rho < 1, \quad \alpha_{i1} + \alpha_{i2} = 1, \tag{15}$$

where c_{ij}^T denotes country i's consumption of the tradable good that originates in country j. For $\rho < 1$, the domestic and the foreign tradable goods are imperfect substitutes. If $\alpha_{ii} > \frac{1}{2}$, there is home-bias in consumption.

As in Burstein, Neves and Rebelo (2003) brining one unit of any traded good to consumers in country i requires η units of country i's non-traded good. Any allocation of tradable and non-tradable goods therefore has to satisfy

$$c_{1i}^T + c_{2i}^T \leq y_i^T, \ i = 1, 2 \tag{16}$$

$$c_i^N + \eta c_{i1}^T + \eta c_{i2}^T \leq y_i^N, \ i = 1, 2. \tag{17}$$

Let P_{ij}^T denote the consumer price of the tradable good that originates in country j and is consumed in country i. $\bar{P}_{ij}^T$ denotes this price at the producer level. If the distribution sector is assumed to be perfectly competitive, the consumer price of tradables is $P_{ij}^T = \bar{P}_{ij}^T + \eta P_i^H$, where P_i^H is the price of one unit of the non-traded good in country i. Under the assumption that purchasing power parity holds for nominal prices at the producer level, $\bar{P}_{1j}^T$ is equal to $\bar{P}_{2j}^T$. Given prices and the total income of an agent in country i, NI_i, the consumption choices, c_{i1}^T, c_{i2}^T, c_i^N, can be determined from a standard static utility maximization program:

$$\max_{c_{i1}^T, c_{i2}^T, c_i^N} \left[\left(\alpha_i^T\right)^{1-\phi} \left(c_i^T\right)^{\phi} + \left(\alpha_i^N\right)^{1-\phi} \left(c_i^N\right)^{\phi} \right]^{\frac{1}{\phi}} \tag{18}$$

$s.t.$

$$NI_i = P_{i1}^T c_{i1}^T + P_{i2}^T c_{i2}^T + P_i^N c_i^N$$

and c_i^T is defined by equation (15).

For the purpose of this paper it is convenient to summarize the allocations of the final good in terms of an international resource constraint $\left\{(c_1, c_2) \,|\tilde{F}\,(c_1, c_2, y) \leq 0\right\}$. The efficient frontier $F\,(c_1, c_2, y) = 0$ is obtained by

$$\max_{\substack{c_{11}^T, c_{12}^T, \\ c_{21}^T, c_{22}^T}} c_1 = \left[\left(\alpha_1^T\right)^{1-\phi} \left(c_1^T\right)^{\phi} + \left(\alpha_1^N\right)^{1-\phi} \left(c_1^N\right)^{\phi}\right]^{\frac{1}{\phi}}$$
$$s.t.$$
$$c_2 = \left[\left(\alpha_2^T\right)^{1-\phi} \left(c_2^T\right)^{\phi} + \left(\alpha_2^N\right)^{1-\phi} \left(c_2^N\right)^{\phi}\right]^{\frac{1}{\phi}}$$

and equations (15)-(17). Given an allocation (c_1, c_2) that satisfies $F\,(c_1, c_2, y) = 0$, the remaining consumption allocations $\left(c_{i1}^T, c_{i2}^T, c_i^T, c_i^N\right)$, $i = 1, 2$ are uniquely determined. The prices for tradables and non-tradables that support such an allocation can be found from the first order conditions of (18).[13] Furthermore, the real exchange rate is given by $q = \frac{F_2(c_1,c_2,y)}{F_1(c_1,c_2,y)}$.

3.5.2 Discussion

Figure 1 shows how the shape of the international resource constraint changes with the introduction of non-traded goods and distribution costs for a given endowment vector y. The elasticity of substitution between traded goods is set equal to 4. The solid line characterizes the allocations for a state with $y_1^T = y_2^T$ and $y_1^N = y_2^N$. In the economy with only traded goods, the boundary of the consumption set is almost linear. Adding non-traded goods to the model increases the curvature and introducing distribution costs increases the curvature even more.

The curvature of the consumption set is key to understanding the volatility of the real exchange rate. Consider an increase in y_1^T. For reasonable parameterizations of the model and the shock, the international resource constraint hardly changes. In

[13]One important assumption in the derivation of the function $F\,(\cdot)$, is that agents have access to free disposal. Since consumption of traded goods requires η units of the non-traded good, there is an interior optimum for the consumption of the traded goods for a given endowment with the non-traded good.

Figure 1, the dotted line $\left(y_1^T > y_2^T\right)$ is close to the solid line $\left(y_1^T = y_2^T\right)$. Remember that $q = \frac{F_2(c_1,c_2,y)}{F_1(c_1,c_2,y)}$. Due to the low curvature of the resource constraint in the economy with only traded goods, large swings in $\frac{c_1}{c_2}$ are needed across states to generate substantial real exchange rate volatility.[14] Although adding non-traded goods increases the curvature of the resource constraint, the increase is not large enough quantitatively. Only with distribution costs small variations in c_1/c_2 cause large swings in the real exchange rate. Put differently, in accord with the stylized facts, large movements in the real exchange rate have little impact on the actual allocations.

4 Calibration and results

4.1 Calibration

The values of the benchmark parameters and the endowment process are listed in Tables 1 and 2. Preferences are represented by the power utility function, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. In the benchmark calibration the coefficient of relative risk aversion γ is set equal to 2. Two comments are in place to explain the choice of β. First, the model is calibrated to annual data due to data availability. Second and more important, partial risk sharing as an equilibrium phenomenon only arises if agents are sufficiently impatient. Otherwise, the equilibrium outcome is close to or identical to the full risk sharing scenario. In terms of the economics it is the value of the risk free rate that matters, which turns out to be around 1.5%.[15]

[14] Heathcote and Perri (2002) examine such a model for the case of complete markets, exogenously incomplete markets with one non-state-contingent bond and financial autarchy. They find little real exchange rate volatility since consumption turns out to be highly correlated across countries. Chari, Kehoe and McGrattan (2002) show that with nominal rigidities the simple model with only traded goods can generate substantial real exchange rate volatility.

[15] See Alvarez and Jermann (2001) for a discussion about the time discount factor β in models with enforcement constraints.

The remaining parameter values are taken from Corsetti et al (2007) except for the elasticity of substitution between the domestic and the foreign tradables, $(1-\rho)^{-1}$, which I choose to set equal to 4. The quantitative literature has proposed a variety of values for the elasticity of substitution between traded goods. For instance, Backus, Kydland and Kehoe (1995) set it equal to 1.5, whereas Heathcote and Perri (2002) estimate its value to be 0.9. Using disaggregate data Broda and Weinstein (2006) find a mean estimate for the elasticity of substitution of 6.

Mendoza (1991) estimates the value of the elasticity between traded and non-traded goods to be 0.74 in a sample of industrialized countries. According to the evidence presented in Burstein, Neves and Rebelo (2003), the share of the retail price of traded goods accounted for by local distribution services ranges from 40% to 50% for the U.S. The value of $\eta = 1.09$ implies a share of roughly 50% in my setup.

The weights of the domestic and foreign tradables, α_{ii} and α_{ij}, have been chosen to be 0.72 and 0.28 respectively. Depending on the exact choices for the remaining parameters, these values imply imports of $5\% - 9\%$ of total income. The average ratio of U.S. imports from Europe, Canada and Japan to U.S. GDP between 1960-2002 is 5%. However, due to the enormous growth in international trade since 1960, this value is substantially larger than 5% towards the end of the sample. Stockman and Tesar (1995) suggest that the share of tradables in the consumption basket of the seven largest OECD countries is roughly 50%. This motivates the choice of $\alpha_i^T = 0.55$ and $\alpha_i^N = 0.45$.

The endowment process for tradable and non-tradable goods is calibrated as follows. Consistent with the literature and the evidence provided in Betts and Kehoe (2001), non-tradables are identified in the data as service output and tradables as manufacturing output. Using annual data for manufacturing and services from the OECD STAN database for the G7 countries, I obtain an estimate for the relative size of the two sectors.[16] The estimates for the ratio of sectorial GDP, $\bar{P}_{ii}^T y_i^T / P_i^N y_i^N$, range from 0.2 to 0.45. I target a value of 0.36 which translates into $y_i^T / y_i^N = 1/2.45$ in the steady state.

The endowment vector $y = \left(y_1^T, y_2^T, y_1^N, y_2^N\right)$ is assumed to follow a Markov chain with transition matrix Π. Each element of the endowment vector can take on two values. Hence, there are 16 exogenous states of the aggregate economy. To calibrate the transition matrix, I generate artificial data from a VAR with time series properties similar to the data (see Table 2). The transition probabilities are then estimated from the artificial data using sample averages. Table 2 also shows properties of the actual data. The U.S. time series are more volatile than the series for the aggregate of the remaining G7 countries.[17] This is partly due to aggregation. Also, manufacturing output is more volatile than service output and the volatility of total output lies in between the two.

4.2 Results and interpretation

4.2.1 Benchmark calibration

The economy is simulated 200 times over 500 periods.[18] Unless mentioned otherwise the artificial data is HP-filtered and the relevant statistics are computed for each simulation. The reported numbers are the averages over the 200 simulations. Table 3 reports data from the U.S. and the remaining G7 countries along with the results for the benchmark calibration for three different arrangements of the international financial markets: complete markets with enforcement constraints, complete markets without enforcement constraints and financial autarchy. In this section, it is

[16] While the OECD STAN database provides quarterly data for manufacturing and services for some countries, it does not do so for all the G7 countries.

[17] The aggregation method follows Chari, Kehoe and McGrattan (2002). Countries are weighted by GDP in U.S. dollars. Purchasing power parities for a given baseline year are used in order to convert national currencies into U.S. dollars.

[18] See Kehoe and Perri (2002) and Bodenstein (2005) for a description of the solution algorithm.

assumed that the government is responsible for the default decision. As shown in section 5.2.2 the qualitative results do not depend on this choice.[19]

The poor performance of the model with complete markets restates the exchange rate disconnect puzzle and the Backus-Smith puzzle: the real exchange rate is barely more volatile than consumption and its correlation with relative consumption equals 1. These two failures have their common cause in the high correlation of cross country consumption under complete markets. The model with enforcement constraints does reasonably well in comparison with the data, both qualitatively and quantitatively. The interest rate implied by the model about 1.5% since the expected value of the intertemporal marginal rate of substitution is 0.987. The real exchange rate is considerably more volatile than consumption and it is negatively correlated with relative consumption. In addition, consumption across countries is positively correlated, but far from perfect. Comparing the economy with enforcement constraint to the financial autarchy model reveals why the model is so successful in replicating the data. Although the quantitative effects are somewhat too strong under financial autarchy, the qualitative behavior is in line with the data: real exchange rates are volatile and negatively correlated with relative consumption. Depending on the impatience of the agents, risk sharing in the economy with enforcement constraints can be very limited and the economy behaves qualitatively like under financial autarchy.[20]

The last column in table 3 shows the results for an incomplete markets economy with one non-contingent and borrowing limits that never bind in the simulation. As is apparent form the table, the economy with one bond is much closer to the economy with complete markets than to the data: the real exchange rate is about as volatile as consumption and the correlation between the real exchange rate and

[19]Changes in the default decision change the value of financial autarchy. By adjusting the discount factor β the behavior of the model can be brought in line with the data.

[20]As shown in Bodenstein (2005) the differences between the model of financial autarchy and the model with enforcement constraints become more pronounced in a production economy with labor.

relative consumption is positive. This result is in stark contrast to the findings presented in Corsetti, Dedola and Leduc (2007) and Benigno and Thoenissen (2007). However, there are several differences between their works and mine: I assume (i) an endowment economy in contrast to their production economies, (ii) a somewhat higher elasticity of substitution between traded goods and (iii) I solve the model using non-linear methods as opposed to a method that is based log-linearization of the first order equations around the deterministic steady state. The last candidate that possibly accounts for the different finding is the way stationarity of bond holdings is induced across different models. Corsetti, Dedola and Leduc (2007) assume that agents' time discount factor is endogenous and agents grow less patient if their utility realization has been high. This mechanism acts against borrowing and lending. Similarly, if one imposes borrowing constraints in my model that are tight enough, then the predictions of the one bond economy can be brought in line with the data.

4.2.2 Interpretation

How does the model generate the negative correlation between the real exchange rate and relative consumption? Consider the two extreme cases of complete markets and financial autarchy. In both cases the allocations do not depend on the time discount factor β. However, these two economies are the limits of the model with limited enforcement as the time discount factor varies: if β approaches 1, agents are patient and full risk sharing becomes feasible. In contrast, if β is sufficiently small, agents have a strong incentive to default. As a result, risk sharing is severely limited and the economy behaves like under financial autarchy.

For ease of exposition, denote the two countries U.S. and Europe. Each period the U.S. receives an endowment of meat and Europe receives an endowment of vegetables. Meat and vegetables are the two (imperfectly substitutable) tradable goods. In order to consume a meal (a combination of meat and vegetables), cooking services are needed. Each period the two countries also receive an endowment of these non-tradable cooking services.

Consider first an increase of the meat endowment in the U.S. under financial autarchy. As meat becomes relatively abundant, the price of meat relative to vegetables declines. If there is home bias in consumption, this effect acts towards a decline of the U.S. price level relative to the European price level, i.e., a depreciation of the real exchange rate. However, because of the wealth effect demand for cooking services rises in the U.S. and drives up its price in the U.S. This second effect acts towards an increase of the U.S. price level relative to the European price level, i.e., an appreciation of the real exchange rate. If this second effect is strong enough to overcome the first effect, the model can account for the observation of Backus and Smith (1993): the real exchange rate appreciates while U.S. consumption of meals increases relative to European consumption.[21]

Under complete markets, however, there is no wealth effect. The extra endowment of meat is shared more equally between the two countries. Hence, the price of cooking services increases in both countries and the aforementioned second effect on the real exchange rate is weak. In contrast with the data, the real exchange rate now depreciates while U.S. consumption of meals increases relative to European consumption.

It is crucial to note, that the explanation of the Backus-Smith puzzle depends on the presence of shocks in the tradable goods sector. Shocks to the non-tradable goods sector induce a positive correlation between the real exchange rate and relative consumption irrespective of the financial market structure.

[21]The simple endowment economy in this paper implies that the terms of trades and the real exchange rate move in opposite directions. Empirical evidence suggests, however, that these two variables move in the same direction over the business cycle. As shown in Bodenstein (2006) and Corsetti et al (2007) this problem is overcome in a production economy. Furthermore, shocks to non-traded goods and consumption taste shocks (not considered here) induce comovement of the terms of trade and the real exchange rate.

4.2.3 Sensitivity analysis

Since the ability to share risk depends on how patient agents are, changes in the time discount factor β have a strong impact on the results. Table 4 summarizes the simulation results for several values of β. For $\beta \geq \beta_{CM}$, full risk sharing is feasible in the economy with enforcement constraints. As under frictionless complete markets, the real exchange rate is smooth and the correlation between the real exchange rate and relative consumption is 1. Lowering β, brings the model in line with the data. For values as high as $\beta = 0.975$, the correlation of the real exchange rate with relative consumption is significantly below 1 in this annual model and the real exchange rate volatility is higher than the volatility of consumption. Also, if agents become very impatient, $\beta \leq \beta_{FM}$, the economy with enforcement constraints behaves identical to the economy without international financial markets.

Table 5 offers sensitivity results for the value of risk aversion (first column), γ, the elasticity of substitution between traded goods (second and third column), $\frac{1}{1-\rho}$, the elasticity of substitution between traded and non-traded goods (fourth and fifth column), $\frac{1}{1-\phi}$, and the case without distribution costs (last column).

Higher risk aversion means, that agents have a stronger taste to smooth consumption. Therefore, more risk sharing is feasible at higher values of γ ($\gamma = 4$) for a given value of β. The quantitative effects of an increase in γ are, however, small. The results are almost identical for the two different values of γ.

Interpreting changes in the parameters ρ or ϕ is less straightforward than changes in β and γ. The time discount factor and the coefficient of risk aversion are the two parameters that directly control agents' willingness to share risk. Any changes in these parameters only affect the utility function of the agents but leave the set of feasible consumption allocations unchanged. However, the substitution elasticities between traded goods or traded and non-traded goods determine the set of feasible consumption allocations. Therefore, in response to changes of these parameters the business cycle moments change for any financial market arrangement. For higher trade elasticity, $\rho = 0.83$, the correlation between the real exchange rate and relative consumption becomes more negative for both the economy with limited contract enforcement (-0.56) and financial autarchy (-0.58). For $\rho = 0$, i.e., a trade elasticity of 1, the respective values of the correlation are 0.23 and -0.90. Despite the absence of a universally accepted measure of risk sharing, it seems safe to claim that there is more risk sharing under the low trade elasticity.

The model with a lower elasticity of substitution between traded and non-traded goods, $\phi = -1$, displays higher real exchange rate and consumption volatility, but lower consumption correlation and a lower correlation between the real exchange rate and relative consumption than the model with $\phi = 0$, -0.75 vs -0.16. Both economies behave very closely to their respective outcomes under financial autarchy: the correlation between the real exchange rate and relative consumption under financial autarchy are -0.75 for $\phi = -1$ and -0.17 for $\phi = 0$.

It remains to be mentioned, that a model without distribution costs delivers a negative correlation between the real exchange rate and relative consumption. However, the volatility of the real exchange rate is much lower than in the data both in absolute value and relative to consumption.[22]

5 A closer look at asset prices

5.1 The volatility of asset prices

The benchmark model with limited contract enforcement can account both for the volatility of the real exchange rate and the observed low or even negative correlation between the real exchange rate and relative consumption (Backus-Smith puzzle).

Brandt, Cochrane and Santa-Clara (2006) emphasize, that real exchange rate

[22]The range of additional sensitivity analysis is huge. Reducing home bias in the economy, α_{11} closer to 0.5, leads to a dcrease in the volatility of the real exchange rate, and to a higher correlation between the real exchange rates and relative consumption compared to the baseline calibration.

volatility is tightly linked to the volatility of asset prices. As shown in their work the growth rate of the real exchange rate equals the difference in the intertemporal marginal rates of substitution ($IMRS$) between the two countries when markets are complete. Brandt et al (2006) and Bodenstein (2005) show how the $IMRS$ is estimated using only data on asset returns. The annualized standard deviation of the $IMRS$ is about 40% to 50% depending on the data used for the estimation and therefore much higher than the roughly 6% of the real exchange rate.[23] This implies that the $IMRS$ for the U.S. and the aggregate of the remaining G7 countries must be highly correlated with a correlation coefficient of more than 0.98 to be consistent with a real exchange rate volatility of 6%.

Using equity and bond returns for the G7, the correlation between the $IMRS$ for the US and the $IMRS$ for the G7 excluding the US varies between 0.9908 and 0.9916 depending on the aggregation method. Brandt et al (2006) find similar numbers and interpret the high correlation as an indication of substantial risk sharing between countries

Both the benchmark model with enforcement constraints and the model of Corsetti et al (2007) imply that asset prices (other than the real exchange rate) are smooth and the equity premium is too low. The volatility of the $IMRS$ in the model is exceeded by a factor of 5 in the data. Under the benchmark calibration, the real exchange rate is also more volatile than the $IMRS$. This finding is hardly surprising as I have merely extended the equity premium puzzle to its international dimension. As shown by Mehra and Prescott (1985) for a closed economy, standard preferences and complete frictionless domestic financial markets imply little volatility of the $IMRS$ since aggregate endowment shocks are small. In the benchmark model domestic financial markets are complete and frictionless and the calibrated

[23] The general consensus is that the $IMRS$ varies by at least 50% for US stock market data. In Bodenstein (2005) the standard deviations are lower since I use a more volatile proxy for the risk free rate to calculate excess returns for equity for the reason of data availability.

endowment shocks – which can even be smoothed to some extent in international financial markets – are relatively small.

One potential resolution to the equity premium puzzle in a closed economy is offered by Alvarez and Jermann (2001). In line with empirical findings, these authors assume that agents' idiosyncratic incomes are volatile relative to aggregate income. In addition, they assume that asset markets are complete, but enforcement of financial contracts is limited.

In this section, I extend the simple two country model along the lines of Alvarez and Jermann in order to simultaneously address the three puzzles mentioned in the introduction: the volatility of the real exchange rate, the consumption real exchange rate puzzle and the volatility of (other) asset prices. From now on I assume that both domestic and international financial contracts can only be enforced by the threat of permanent exclusion from all financial markets.

5.2 The extended model

There are two groups of agents in country 1 which are denoted by 1 and 2. The agents in country 2 are labeled agents 3 and 4. Each agent i in country j faces a maximization problem similar to the one of the representative agents in section 3.1:

$$\max_{\{c_i(s^t)\}} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi\left(s^t\right) \left[M_i\left(s^{t-1}\right) u\left[c_i\left(s^t\right)\right] + \mu_i\left(s^t\right)\left\{u_i\left[c\left(s^t\right)\right] - V_i\left(s^t\right)\right\}\right]$$

$s.t.$

$$\begin{aligned} & P_j\left(s^t\right) c_i\left(s^t\right) + \sum_{s^{t+1}|s^t} Q_j\left(s^{t+1}|s^t\right) a_i\left(s^{t+1}|s^t\right) \\ \leq\ & a_i\left(s^t|s^{t-1}\right) + \xi_i\left(s^t\right)\left[\bar{P}_{jj}^T\left(s^t\right) y_j^T\left(s^t\right) + P_j^N\left(s^t\right) y_j^N\left(s^t\right)\right] \end{aligned}$$

and $M_i(s^t) = M_i(s^{t-1}) + \mu_i(s^t)$. $\xi_i(s^t)$ is the share of agent i in the aggregate income of her home country. The outside option, $V_i(s^t)$, is defined by

$$V_i\left(s^t\right) = \max_{\{c_i(s^r)\}} \sum_{r=t}^{\infty} \sum_{s^r|s^t} \beta^{r-t} \pi\left(s^r|s^t\right) u\left[c_i\left(s^r\right)\right]$$

$$s.t.$$

$$P_j\left(s^t\right) c_i\left(s^t\right) \leq \xi_i\left(s^t\right) \left[\bar{P}_{jj}^T\left(s^t\right) y_j^T\left(s^t\right) + P_j^N\left(s^t\right) y_j^N\left(s^t\right)\right].$$

In this version of the model, I assume that the default decision is made by each agent individually. Therefore she ignores the effect of her behavior on goods market prices. An agent who defaults on any contract is banned from all financial markets, but she can still trade in the spot markets for goods.

The solution of the model is fully characterized by the first order conditions

$$\begin{aligned}
\eta_1\left(s^t\right) &\equiv \frac{M_2\left(s^t\right)}{M_1\left(s^t\right)} = \frac{M_2\left(s^{t-1}\right) + \mu_2\left(s^t\right)}{M_1\left(s^{t-1}\right) + \mu_1\left(s^t\right)} = \frac{u_c\left(c_1\left(s^t\right)\right)}{u_c\left(c_2\left(s^t\right)\right)}, \\
\eta_2\left(s^t\right) &\equiv \frac{M_4\left(s^t\right)}{M_3\left(s^t\right)} = \frac{M_4\left(s^{t-1}\right) + \mu_4\left(s^t\right)}{M_3\left(s^{t-1}\right) + \mu_3\left(s^t\right)} = \frac{u_c\left(c_3\left(s^t\right)\right)}{u_c\left(c_4\left(s^t\right)\right)}, \\
z\left(s^t\right) &\equiv \frac{M_3\left(s^t\right)}{M_1\left(s^t\right)} = \frac{M_3\left(s^{t-1}\right) + \mu_3\left(s^t\right)}{M_1\left(s^{t-1}\right) + \mu_1\left(s^t\right)} = \frac{u_c\left(c_1\left(s^t\right)\right)}{u_c\left(c_3\left(s^t\right)\right)} q\left(s^t\right), \\
\mu_i\left(s^t\right) &\geq 0,
\end{aligned}$$

and the national and international resource constraints, the real exchange rate $q(s^t) = \frac{F_2\left(C_1\left(s^t\right), C_2\left(s^t\right), y(s_t)\right)}{F_1\left(C_1(s^t), C_2(s^t), y(s_t)\right)}$, and the enforcement constraints. C_j denotes aggregate consumption in country j. The solution of this model is found by following the same steps as in the benchmark model with the additional complication that the system now contains three endogenous state variables (η_1, η_2 and z). The relevant asset pricing kernels are given by

$$m_i\left(s^{t+1}\right) = \frac{\beta}{1 - \varphi_i\left(s^{t+1}\right)} \frac{u_c\left[c_i\left(s^{t+1}\right)\right]}{u_c\left[c_i\left(s^t\right)\right]},$$

with $\varphi_i(s^{t+1}) = \frac{\mu_i\left(s^{t+1}\right)}{M_i(s^{t+1})}$. The price of a contingent claim is given by the $IMRS$ of the unconstrained agents. Within a country the pricing kernels satisfy $m_1(s^{t+1}) = m_2(s^{t+1})$ and similarly for agents 3 and 4. The $IMRS$ across countries are related by

$$\frac{q\left(s^{t+1}\right)}{q\left(s^{t}\right)}=\frac{m_{3}\left(s^{t+1}\right)}{m_{1}\left(s^{t+1}\right)}=\frac{\bar{M}_{2}\left(s^{t+1}\right)}{\bar{M}_{1}\left(s^{t+1}\right)}, \tag{19}$$

where $\bar{M}_i$ is the marginal rate of substitution for country i.

5.3 A numerical example

5.3.1 Calibration

As in the benchmark model, the endowment with traded goods can be either high or low in each country. However, the endowment with non-traded goods is assumed to be constant in this part of the analysis in order to keep the state space manageable.[24] I calibrate the Markov process for the agents' income in each country following Heaton and Lucas (1996): $\xi_i^L = 0.3772$ and $\xi_i^H = 0.6228$, $i = 1, 2, 3, 4$ where $\xi_i\left(s^t\right)$ is the share of agent i in the aggregate income of her home country.[25] The transition matrix for the income distribution in country 1 is given by

	$\left(\xi_1^L, \xi_2^H\right)$	$\left(\xi_1^H, \xi_2^L\right)$
$\left(\xi_1^L, \xi_2^H\right)$	0.7423	0.2577
$\left(\xi_1^H, \xi_2^L\right)$	0.2577	0.7423

and similarly for country 2. These income processes for the agents are assumed to be independent across countries. The remaining parameters are taken from Table 2 unless explicitly noted otherwise in Table 6.

[24]The 4 state endowment process is calibrated to match the business cycle statistics of the manufacturing sectors in the U.S. and the remaining G7 countries reported in table 3.

[25]Based on a large sample from the PSID, Heaton and Lucas (1996) find that the log of an agent's income, relative to the aggregate is stationary with a first order serial correlation of 0.5 and a standard deviation of 0.29 for annual data. Alvarez and Jermann (2001) and Lustig (2004) also calibrate their models based on the estimates in Heaton and Lucas (1996).

5.3.2 Results and interpretation

The model is simulated 200 times over 500 periods. The artificial data is HP-filtered and the relevant business cycle statistics are computed. The moments for the $IMRS$ are calculated from non-filtered data. Table 2.4 summarizes the results for the for $\beta = 0.70$ and $\beta = 0.95$. I will refer to these to scenarios as low and high risk sharing, respectively. The model generates volatile $IMRS$ only in the low risk sharing scenario. Since individual income is very volatile, the gains from risk sharing are potentially very high. Hence, agents need to be fairly impatient ($\beta = 0.70$) for enforcement constraints to matter.

For $\gamma = 2$, the model predicts that the $IMRS$ in the two countries $\bar{M}_i$ are volatile and reasonably close to the data (40% in the model compared to my estimates of 45%) in the low risk sharing scenario. In addition, the implied risk-free rate is 2%. Also, the model predicts a negative correlation between the relative consumption and the real exchange rate. However, the real exchange rate moves too much now: its volatility is about 53 times the volatility of consumption for the HP-filtered time series, whereas this ratio is less than 4 in the data. Similarly, the growth rate of q fluctuates too much.

In the high risk sharing scenario, income heterogeneity within a country does not matter. Agents make efficient use of the domestic financial markets and individual consumption behaves similar to aggregate consumption. While the model correctly predicts the real exchange volatility and the negative correlation between the real exchange rate and relative consumption, it fails to generate volatile asset prices. The $IMRS$ varies about only 7%.Remember that the real exchange rate depreciates by the difference between the log of the foreign and the domestic $IMRS$:

$$\log \frac{q_{t+1}}{q_t} = \log \bar{M}_{2,t+1} - \log \bar{M}_{1,t+1}.$$

For $\beta = 0.70$, risk sharing between agents within each country and across countries is severely limited and the correlation between the stochastic discount factors for the two countries is low (0.1673). Given the volatility of the stochastic discount factors, the real exchange rate fluctuates too much.

By assuming higher values of the coefficient of relative risk aversion, more risk sharing becomes sustainable in equilibrium. For $\gamma = 3$, the volatility of aggregate consumption declines and cross-country consumption correlations increase. The real exchange rate is smoother, although it is still 31 times more volatile than aggregate consumption. The stochastic discount factors become smoother and more correlated. The extended model falls short of explaining asset pricing behavior for $\gamma > 2$ given $\beta = 0.70$.

Although the correlation between the $IMRS$ is even lower for $\beta = 0.95$, this parameterization of the model does not imply too much volatility in the real exchange rate. With low volatility of the $IMRS$, the low correlation does not pose any problems for the real exchange rate. Hence, the model of limited enforcement presented in this paper cannot simultaneously account for the observed volatility in the real exchange rate, asset prices and the Backus-Smith puzzle. It either fails with respect to the volatility of the real exchange rate or of the asset prices.

6 Conclusions

Most international macroeconomists believe that international risk sharing is limited by financial market frictions and that these frictions are key to understanding the international business cycle. This paper examines the extent to which models with endogenous incomplete markets can resolve the exchange rate volatility puzzle and the Backus-Smith puzzle. A model with complete markets and enforcement constraints for international financial contracts but frictionless domestic asset markets provides a candidate explanation of these two puzzles if agents are not too patient. For sufficiently impatient agents, international risk sharing is very limited. As a result cross country consumption levels are lowly correlated and real exchange rates are volatile and negatively correlated with relative consumption across countries.

However, since asset markets are complete within each country and aggregate income fluctuations are low, the model inherits all the standard asset pricing puzzles. In particular, it implies stochastic discount factors that are too smooth vis-à-vis the data. Once I extend the benchmark model by introducing enforcement constraints also into each country's local financial markets, the model delivers more volatile asset prices. However, it now fails to deliver the right amount of real exchange rate volatility. As risk sharing is low both within and across countries, the marginal rates of substitution in the two countries are not very correlated and the real exchange rate is too volatile in comparison to the data. It seems that models that severely restrict the amount of international risk sharing for all agents will be subject to this failure, once it has been enriched to deliver realistic asset pricing behavior.

References

[1] Alvarez, F., Jermann, U., 2001. Quantitative Asset Pricing Implications Of Endogenous Solvency Constraints. Review of Financial Studies 14, 1117-1151.

[2] Backus, D., Kehoe, P., Kydland, F., 1995. International Business Cycles: Theory And Evidence. In Cooley, T. (Ed.), Frontiers of Business Cycle Research. Princeton University Press, Princeton, pp. 331-356.

[3] Backus, D., Smith, G., 1993. Consumption And Real Exchange Rates In Dynamic Economies With Non-Traded Goods. Journal of International Economics 35, 297-316.

[4] Benigno, G., Thoenissen, C., 2007. Consumption and Real Exchange Rates with Incomplete Markets and Non-Traded Goods. Journal of International Money and Finance (forthcoming).

[5] Betts, C., Kehoe, T., 2001. Tradability Of Goods And Real Exchange Rate Fluctuations. Federal Reserve Bank of Minneapolis, Research Department Staff Report.

[6] Bodenstein, M., 2005. On Real Exchange Rate Volatility And Financial Markets. PhD Dissertation Northwestern University.

[7] Brandt, M., Cochrane, J., Santa-Clara, P., 2003. International Risk Sharing Is Better Than You Think, Or Exchange Rates Are Too Smooth. Journal of Monetary Economics 53, 671-698.

[8] Broda, C., Weinstein, D., 2006. Globalization and the gains from variety. Quarterly Journal of Economics 121, 541-585.

[9] Burstein, A., Eichenbaum, M., Rebelo, S., 2002. Large Devaluations and the Real Exchange Rate. Journal of Political Economy 113, 742-784.

[10] Burstein, A., Neves, J., Rebelo, S., 2003. Distribution Costs And Real Exchange Rate Dynamics During Exchange-Rate-Based Stabilizations. Journal of Monetary Economics 50, 1189-1214.

[11] Chari, V.V., Kehoe, P., McGrattan, E., 2002. Can Sticky Prices Generate Volatile And Persistent Real Exchange Rates. Review of Economic Studies 69, 633-663.

[12] Colacito, R., Croce, M., 2006. Risk for the Long Run and the Real Exchange Rate. New York University.

[13] Corsetti, G., Dedola, L., Leduc, S., 2007. International Risk-Sharing And The Transmission of Productivity Shocks. Review of Economic Studies (forthcoming).

[14] Fitzgerald, D., 2006. Trade Costs, Asset Market Frictions and Risk Sharing: A Joint Test, manuscript, Stanford University.

[15] Hansen, L., Jagannathan, R., 1991. Implications Of Security Market Data For Models Of Dynamic Economies. Journal of Political Economy 99 225-262.

[16] Heathcote, J., Perri, F., 2002. Financial Autarky And International Business Cycles. Journal of Monetary Economics 49, 601-627.

[17] Heaton, J., Lucas, D., 1996. Evaluating The Effects Of Incomplete Markets On Risk-Sharing And Asset Pricing. Journal of Political Economy 104, 443-487.

[18] Jeske, K., 2006. Private International Debt with Risk of Repudiation. Journal of Political Economy 114, 576-593.

[19] Kehoe, T., Levine, D., 1993. Debt-Constrained Asset Markets. Review of Economic Studies 60, 865-888.

[20] Kehoe, P., Perri, F., 2002. International Business Cycles With Endogenous Incomplete Markets. Econometrica 70, 907-928.

[21] Kocherlakota, N., 1996. Implications Of Efficient Risk Sharing Without Commitment. Review of Economic Studies 63, 595-609.

[22] Krueger, D., Perri, F., 2006. Does Income Inequality Lead to Consumption Inequality? Evidence and Theory. Review of Economic Studies 73, 163-193.

[23] Lewis, K., 1996. What Can Explain the Apparent Lack of International Consumption Risk Sharing? Journal of Political Economy 104, 267-297.

[24] Lewis, K., 1999. Trying to Explain the Home Bias in Equity and Consumption. Journal of Economic Literature 37, 571-608.

[25] Lustig, H., 2004. The Market Price of Aggregate Risk and the Wealth Distribution, manuscript, UCLA.

[26] Marcet, A., Marimon, R., 1999. Recursive Contracts, manuscript, Universitat Pompeu Fabra.

[27] Mehra, R., and E. Prescott, 1985. The Equity Premium: A puzzle. Journal of Monetary Economics 15, 145-161.

[28] Mendoza, E., 1991. Real Business Cycles in a Small Open Economy. American Economic Review 81, 797-818.

[29] Messner, M., and N. Pavoni, 2004. A Note on the Recursive Saddle Point Method, manuscript, University College London.

[30] Obstfeld, M., and K. Rogoff, 1996. Foundations Of International Macroeconomics. MIT Press, Cambridge.

[31] Scholl, A., 2005. Limited Enforceable International Loans, International Risk Sharing and Trade, manuscript, University of Frankfurt.

[32] Stockman, A., and L. Tesar, 1995. Taste and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovement. American Economic Review 83, 473-486.

[33] Verdelhan, A., 2007. A Habit-Based Explanation of the Exchange Rate Risk Premium, manuscript, Boston University.

Parameter values

risk aversion	$\gamma = 2$
discount factor	$\beta = 0.95$
elasticity of substitution:	
– domestic and foreign tradables	$\frac{1}{1-\rho} = 4.00$
– tradables and non-tradables	$\frac{1}{1-\phi} = 0.74$
distribution costs	$\eta = 1.09$
share of domestic tradables	$\alpha_{11} = \alpha_{22} = 0.72$
share of non-tradables	$\alpha_1^N = 0.45$

Table 1

Endowment process

data (annualized)	
standard deviations	
$\sigma\left(y_{US}^{T}\right)=0.0285$	$\sigma\left(y_{G7\backslash US}^{T}\right)=0.0161$
$\sigma\left(y_{US}^{N}\right)=0.0082$	$\sigma\left(y_{G7\backslash US}^{H}\right)=0.0043$
$\sigma\left(GDP_{US}\right)=0.0121$	$\sigma\left(GDP_{G7\backslash US}\right)=0.0070$
correlations	
$\rho\left(y_{US}^{T},y_{G7\backslash US}^{T}\right)=0.5166$	$\rho\left(y_{US}^{T},y_{G7\backslash US}^{N}\right)=0.4049$
$\rho\left(y_{US}^{N},y_{G7\backslash US}^{N}\right)=0.6488$	$\rho\left(y_{US}^{N},y_{G7\backslash US}^{T}\right)=0.6818$
$\rho\left(GDP_{US},GDP_{G7\backslash US}\right)=0.3741$	
calibration	
endowment vector	
$y_i^T(h)=1.0257$	$y_i^T(l)=0.9743$
$y_i^N(h)=2.4684$	$y_i^N(l)=2.4316$
properties of VAR	
$\sigma\left(y_1^T\right)=0.0254$	$\sigma\left(y_2^T\right)=0.0254$
$\sigma\left(y_1^N\right)=0.0074$	$\sigma\left(y_2^N\right)=0.0074$
$\rho\left(y_1^T,y_2^T\right)=0.4500$	$\rho\left(y_1^T,y_2^N\right)=0.6400$
$\rho\left(y_1^N,y_2^N\right)=0.7600$	$\rho\left(y_1^N,y_2^T\right)=0.6400$

y_i^T, y_i^N, GDP_i, $i=US$, $G7\backslash US$ denote output of tradables, non-tradables, and GDP in the US and the $G7$ excluding the US, respectively. $\sigma(\cdot)$ denotes the standard deviation of a variable, $\rho(\cdot,\cdot)$ measures the correlation between two variables.

Table 2

Benchmark calibration and different market arrangements

	data	enforcement constraints	complete markets	financial autarchy	bond economy
HP-filtered statistics					
$\sigma(C_1)$	0.0150	0.0096	0.0094	0.0097	0.0094
$\sigma(q)$	0.0504	0.0804	0.0120	0.0825	0.0140
$\rho(C_1, C_2)$	0.4300	0.6871	0.7934	0.6831	0.7921
$\rho(q, C_1/C_2)$	-0.4200	-0.4902	1.0000	-0.5019	0.8278
Non-filtered variables					
$E\left(\bar{M}_1\right)$	0.9919	0.9872	0.9500	0.9500	0.9500

$\sigma(\cdot)$ denotes the standard deviation of a variable, $\rho(\cdot,\cdot)$ measures the correlation between two variables. $E\left(\tilde{M}_1\right)$ is the expected intertemporal marginal rate of substitution.

Table 3

Sensitivity analysis economy with enforcement constraints

wrt to time discount factor β

β	$\geq \beta_{CM}$	0.9750	0.9600	0.9500	0.9400	$\leq \beta_{FA}$
$\sigma(C_1)$	0.0094	0.0094	0.0096	0.0096	0.0096	0.0097
$\sigma(q)$	0.0120	0.0325	0.0712	0.0804	0.0819	0.0825
$\rho(C_1, C_2)$	0.7934	0.4870	0.7027	0.6871	0.6830	0.6831
$\rho(q, C_1/C_2)$	1.0000	0.1752	-0.4250	-0.4902	-0.4937	-0.5019

$\sigma(\cdot)$ denotes the standard deviation of a variable, $\rho(\cdot,\cdot)$ measures the correlation between two variables.

Table 4

Sensitivity analysis economy with enforcement constraints wrt other parameters

	higher risk aversion	higher trade elasticity	lower trade elasticity	higher elast. traded vs non-traded	lower elast traded vs non-traded	no distrib. costs
	$\gamma = 4$	$\rho = 0.83$	$\rho = 0$	$\phi = 0$	$\phi = -1$	$\eta = 0$
$\sigma(C_1)$	0.0096	0.0097	0.0094	0.0086	0.0118	0.0164
$\sigma(q)$	0.0794	0.0890	0.0359	0.0738	0.0794	0.0104
$\rho(C_1, C_2)$	0.6889	0.6529	0.7871	0.7050	0.6285	0.4865
$\rho(q, C_1/C_2)$	-0.4768	-0.5582	0.2279	-0.1618	-0.7492	-0.8894

$\sigma(\cdot)$ denotes the standard deviation of a variable, $\rho(\cdot,\cdot)$ measures the correlation between two variables.

Table 5

Business cycle statistics economy with enforcement constraints and heterogenous agents extended vs benchmark model

	Extension $\beta = 0.70$			Benchmark $\beta = 0.95$		
γ	2	3	4	2	3	4
HP-filtered statistics						
$\sigma(C_1)$	0.0117	0.0083	0.0056	0.0063	0.0063	0.0063
$\sigma(q)$	0.6203	0.2604	0.0103	0.0703	0.0699	0.0690
$\rho(C_1, C_2)$	-0.4625	-0.0023	0.9876	0.6667	0.6701	0.6756
$\rho(q, C_1/C_2)$	-0.3345	-0.6125	-0.2216	-1.0000	-1.0000	-1.0000
Non-filtered variables						
$E\left(\bar{M}_1\right)$	0.9795	0.8035	0.7007	0.9884	0.9907	0.9929
$\sigma\left(\log\left(\bar{M}_1\right)\right)$	0.3949	0.1283	0.0236	0.0698	0.0730	0.0769
$\rho\left(\log\left(\bar{M}_1\right), \log\left(\bar{M}_2\right)\right)$	0.1673	0.2579	0.9413	-0.0001	0.0013	0.0027
$\sigma(\log(q_{t+1}/q_t))$	0.5033	0.1593	0.0091	0.0106	0.0100	0.0092

$\sigma(\cdot)$ denotes the standard deviation of a variable, $\rho(\cdot,\cdot)$ measures the correlation between two variables.

Table 6

Figure 1: **Set of Feasible Consumption Allocations**