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# Optimal Fiscal and Monetary Policy with Costly Wage Bargaining\*

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## Abstract

Costly nominal wage adjustment has received renewed attention in the design of optimal policy. In this paper, we embed costly nominal wage adjustment into the modern theory of frictional labor markets to study optimal fiscal and monetary policy. Our main result is that the optimal rate of price inflation is highly volatile over time despite the presence of sticky nominal wages. This finding contrasts with results obtained using standard sticky-wage models, which employ Walrasian labor markets at their core. The presence of shared rents associated with the formation of long-term employment relationships sets our model apart from previous work on this topic. The existence of rents implies that the optimal policy is willing to tolerate large fluctuations in real wages that would otherwise not be tolerated in a standard model with Walrasian labor markets; as a result, any concern for stabilizing nominal wages does not translate into a concern for stabilizing nominal prices. Our model also predicts that smoothing of labor tax rates over time is a much less quantitatively-important goal of policy than standard models predict. Our results demonstrate that the level at which nominal wage rigidity is modeled — whether simply laid on top of a Walrasian market or articulated in the context of an explicit relationship between workers and firms — can matter a great deal for policy recommendations.

**Keywords:** inflation stability, real wage, Ramsey model, Friedman Rule, labor search

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# 1 Introduction

Studying optimal monetary policy in the presence of nominally-rigid wages has enjoyed a resurgence of late. The typical story behind models featuring nominal wage rigidities is that wage negotiations are costly or time-consuming, which leads to infrequent adjustments. However, it is somewhat difficult to understand the idea of wage negotiations, costly or not, when the underlying model of the labor market is Walrasian, which is true of existing sticky-wage models that study optimal policy. In Walrasian markets, there are no negotiations. Instead, models that feature explicit bilateral relationships between firms and workers seem to be called for in order to study the consequences of costly wage negotiations. In this paper, we embed costly nominal wage negotiations into the modern theory of frictional labor markets to study optimal fiscal and monetary policy. Our central result is that the optimal inflation rate is quite volatile over time despite the presence of the nominal friction. This result is robust to several different specifications of our underlying environment and stands in contrast to that obtained in environments with fundamentally Walrasian labor markets. Thus, the level at which nominal wage rigidity is modeled — whether simply lain on top of a Walrasian market or articulated in the context of an explicit relationship between workers and firms — can matter a great deal for policy recommendations.

The reason behind optimal inflation volatility in basic Ramsey monetary models is well-understood. In a coordinated program of fiscal and monetary policy, the Ramsey planner prefers surprise movements in the price level to changes in proportional taxes in response to shocks to the government budget. This result was first quantitatively demonstrated by Chari, Christiano, and Kehoe (1991) in a model with fully-flexible nominal prices and nominal wages. The Ramsey literature has recently re-examined this issue in models featuring nominally rigid prices and wages. Schmitt-Grohe and Uribe (2004b) and Siu (2004) showed that with even a small degree of nominal rigidity in prices, optimal inflation volatility is quite small. Chugh (2006a) showed that stickiness in nominal wages by itself also makes Ramsey-optimal inflation very stable over time, but in the latter the wage rigidity is introduced in an otherwise Walrasian labor market.

The contrast between our results here and those in Chugh (2006a) stems from the importance the planner attaches to delivering a stable path of realized *real* wages for the economy. The key to understanding the result in Chugh (2006a) is that if real wage growth is determined essentially by technological features of the economy (such as productivity) that do not fluctuate too much, then any desire to stabilize nominal wages shows up as a concern for stabilizing nominal prices. If real wages are not tied so tightly to an economy's production possibilities but instead are free to adjust without much welfare consequence, as is the case in our model here, then such an effect need not occur. In our model, which builds on the basic labor search and matching framework, wages are determined after a worker and a firm meet. In general, there is a continuum of real wages that

is acceptable for both parties to agree to consummate the match and begin production. In this sense, the real wage is (within certain boundaries) not allocational in our model. Thus, any desire to stabilize nominal wages does not immediately translate into a desire to stabilize nominal prices because the planner takes into account the fact that real wages do not critically affect allocations.

We articulate these ideas by incorporating two new elements into a standard model of labor search and wage bargaining. First, we assume that workers and firms negotiate over nominal wages, rather than real wages as is typically assumed in this class of models. We think it seems empirically descriptive of actual wage negotiations that bargaining occurs in terms of a nominal unit of account, but we do not claim to have any novel explanation for why this occurs. By itself, this assumption is innocuous because, as we show, bargaining in either nominal or real units has no consequence for the basic labor search model. Instead, we assume it in order to have a well-defined notion of resource costs of changing nominal wages. Once again, we do not claim we have an explanation any deeper than existing ones for why there are costs of changing nominal wages; such costs may be administrative costs of recording, reporting, and implementing a new nominal wage for an employee, for example. By pushing the notion of costly nominal wage contracting down to a more clearly-defined concept of a worker-firm pair, though, we show that monetary policy should be conducted in a very different way than predicted by sticky-nominal-wage models as typically formulated.

The idea that real wages may play a very different role than predicted by neoclassical models of course has a rich history in policy discussions. We cannot do justice to this entire line of thought. Instead, we find it useful to relate our findings to Goodfriend and King's (2001) discussion, which cogently distills much of the previous thinking regarding this issue, of the consequences sticky nominal wages may or may not have for the conduct of monetary policy. Goodfriend and King (2001, p. 48-51) conjecture that costs of adjusting nominal wages ought not to have much consequence for the dynamics of optimal inflation because firms and workers engaged in long-term relationships have incentives to arrange rent payments among themselves to neutralize any allocative distortions. The labor search and bargaining framework provides a modern structure with which to think about such issues. Indeed, our results show that costly nominal wage adjustment does not affect the basic Ramsey prescription of price volatility.

The lack of a neoclassical labor mechanism via which the time- $t$  real wage influences time- $t$  allocations leads us to explore the robustness of our results to a decision margin that does resemble a standard model, an intensive (hours) margin of labor supply that may depend on the contemporaneous real wage. When we add to our basic model an hours margin, we find that some protocols by which hours are determined (bargaining between firms and workers) do not change our basic results while some protocols by which hours are determined (firms choosing their employees' hours)

do. Thus, the operation, or lack thereof, of a neoclassical labor margin is important in determining the optimal degree of inflation stability in the presence of nominally-rigid wages.

In addition to our central result regarding the volatility of inflation despite the presence of a nominal friction, a few other novel short-run and long-run properties of optimal policy emerge from our model. Dynamic tax-smoothing incentives are not nearly as strong in (both flexible-wage and sticky-wage versions of) our model as in basic Ramsey models; we find optimal labor tax rates are an order of magnitude more volatile than benchmark results in the literature (e.g., Chari, Christiano, and Kehoe (1991)). As we discuss, crucial to thinking about this result seems to be a *dynamic bargaining power effect* in our model in which cyclical variations in tax rates and inflation affect the relative bargaining power of workers and firms, which have consequences for splits of match surpluses but not efficient formation of matches. With regard to the steady state, the optimal inflation rate trades off three forces. Two forces are standard in monetary models: inefficient money holdings due to a deviation from the Friedman Rule versus resource losses stemming from nominal adjustment due to non-zero inflation. The third force influencing steady-state inflation in our model is inefficiencies in job creation, which positive inflation in some cases can offset. This latter policy channel is one about which Ramsey models based on Walrasian labor markets are silent.

There is a large recent literature focused on the dynamic properties of real wages in the basic labor search model. After Shimer (2005) and Hall (2005) pointed out that the workhorse Pissarides (2000) search model falls short in explaining the dynamics of some of its key endogenous variables, it has been understood that a model that does better would require the real wage to be less volatile than the one that emerges from simple Nash bargaining, which is the typical wage determination mechanism used in the literature. In our model, we stick with Nash bargaining because it is still the benchmark wage mechanism for these models. Our results show that the costlier is adjustment of nominal wages, the more volatile is the real wage under the optimal policy, a result seemingly at odds with recent modeling efforts to reduce real wage volatility. We do not view this as problematic because our immediate concern here is not explaining the data; rather, our focus here is on the policy implications that emerge from such environments, and we think it makes sense to begin with the most well-understood framework.

This paper is also a building block in a larger research program aimed at studying optimal policy in models with deep-rooted non-Walrasian features in key markets. Aruoba and Chugh (2006) study optimal fiscal and monetary policy in a model in which monetary exchange expands the set of feasible trades; they find results in sharp contrast to the standard Ramsey monetary literature, suggesting that the way in which money is modeled may matter a lot for policy recommendations. Arseneau and Chugh (2006) study possible implications of labor matching frictions in concert with

ex-post welfare heterogeneity between employed and unemployed individuals for optimal capital taxation; our work in this paper adds a monetary dimension to their model. Our research horizon is characterizing optimal fiscal and monetary policy in a model featuring deep descriptions of both monetary exchange and labor market frictions. Money markets and labor markets have long been thought to be important in understanding business cycles. Given recent advances in both monetary theory and labor market theory, the time seems ripe for exploring standard macro questions in these new, richer environments.

The rest of our paper is organized as follows. Section 2 builds our basic model. Section 3 presents the Ramsey problem, and Section 4 presents our main results. In Section 5, we allow for an intensive margin to demonstrate how the presence or absence of a neoclassical mechanism alters our results. Section 6 offers concluding thoughts and possible avenues for continued research.

## 2 The Basic Model

As many other recent studies have done, our model embeds the Pissarides (2000) textbook search model into a general equilibrium framework. There is full consumption insurance between employed and unemployed individuals. Bargaining occurs between individual workers and the representative firm. We present in turn the composition of the representative household, the representative firm, how wages are determined, the actions of the government, and the definition of equilibrium.

### 2.1 Households

There is a continuum of identical households in the economy. The representative household consists of a continuum of measure one of family members. Each member of the household either works during a given time period or is unemployed and searching for a job. At time  $t$ , a measure  $n_t$  of individuals in the household are employed and a measure  $1 - n_t$  are unemployed. We assume that total household income is divided evenly amongst all individuals, so each individual has the same consumption.<sup>1</sup>

The household's discounted lifetime utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(c_{1t}, c_{2t}) - \int_0^{n_t} A^i \bar{h} di + \int_{n_t}^1 v^i di \right], \quad (1)$$

where  $u(c_1, c_2)$  is each family member's utility from consumption of cash goods ( $c_1$ ) and credit goods ( $c_2$ ),  $\bar{h}$  is a fixed number of hours that an employed individual works,  $A^i$  is the disutility per unit time an employed individual  $i$  suffers, and  $v^i$  is the utility experienced by individual  $i$

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<sup>1</sup>Thus, we follow Merz (1995), Andolfatto (1996), and much of the subsequent literature in this regard by assuming full consumption insurance between employed and unemployed individuals.

from non-work. The function  $u$  satisfies  $u_j > 0$  and  $u_{jj} < 0$ ,  $j = 1, 2$ . We assume symmetry in the disutility of work amongst the employed, so that  $A^i = A$ , as well as symmetry in the utility of non-work amongst the unemployed, so that  $v^i = v$ . Thus, household lifetime utility can be expressed as

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_{1t}, c_{2t}) - n_t A \bar{h} + (1 - n_t)v]. \quad (2)$$

The household does not choose how many family members work. As described below, the number of people who work is determined by a labor matching process. We also assume that each employed individual works a fixed number of hours  $\bar{h} < 1$ ; as described below, we calibrate  $\bar{h}$  to make the quantitative results of the model readily comparable to our richer model in Section 5 in which we allow for adjustment at the intensive labor margin.

The household chooses sequences of consumption of each good, nominal money holdings, and nominal bond holdings  $\{c_{1t}, c_{2t}, M_t, B_t\}$ , to maximize lifetime utility subject to an infinite sequence of flow budget constraints

$$M_t - M_{t-1} + B_t + R_{t-1}B_{t-1} = (1 - \tau_{t-1}^n)W_{t-1}n_{t-1}\bar{h} - P_{t-1}c_{1t-1} - P_{t-1}c_{2t-1} + P_{t-1}d_{t-1} \quad (3)$$

and cash-in-advance constraints

$$P_t c_{1t} \leq M_t. \quad (4)$$

$M_{t-1}$  is the nominal money the household brings into period  $t$ ,  $B_{t-1}$  is nominal bonds brought into  $t$ ,  $W_t$  is the nominal wage,  $P_t$  is the price level,  $R_t$  is the gross nominally risk-free interest rate on government bonds held between  $t$  and  $t + 1$ ,  $\tau_t^n$  is the tax rate on labor income, and  $d_t$  is profit income of firms received by households lump-sum. The timing of the budget and cash-in-advance constraints conforms to the timing described by Chari, Christiano, and Kehoe (1991) and used by Siu (2004) and Chugh (2006a, 2006b).

Associate the Lagrange multipliers  $\phi_t/P_{t-1}$  with the sequence of budget constraints and  $\lambda_t/P_t$  with the sequence of cash-in-advance constraints. The household's first-order conditions with respect to cash good consumption, credit good consumption, money holdings, and bond holdings are thus

$$u_{1t} - \lambda_t - \beta E_t \phi_{t+1} = 0, \quad (5)$$

$$u_{2t} - \beta E_t \phi_{t+1} = 0, \quad (6)$$

$$-\frac{\phi_t}{P_{t-1}} + \frac{\lambda_t}{P_t} + \beta E_t \left( \frac{\phi_{t+1}}{P_t} \right) = 0, \quad (7)$$

$$-\frac{\phi_t}{P_{t-1}} + \beta R_t E_t \left( \frac{\phi_{t+1}}{P_t} \right) = 0, \quad (8)$$

respectively, where the notation  $u_{1t}$  denotes the value of marginal utility of cash goods in period  $t$ , and similarly for  $u_{2t}$ .

From (8), we get a usual Fisher relation,

$$1 = R_t E_t \left[ \frac{\beta \phi_{t+1}}{\phi_t} \frac{1}{\pi_t} \right], \quad (9)$$

where  $\pi_t \equiv P_t/P_{t-1}$  is the gross rate of price inflation between period  $t - 1$  and period  $t$ . The stochastic discount factor  $E_t [(\beta \phi_{t+1}/\phi_t)(1/\pi_t)]$  prices a nominally risk-free one-period asset. Combining (5) and (7), we get

$$\phi_t = \frac{u_{1t}}{\pi_t}. \quad (10)$$

Substituting this expression into the previous one gives us the pricing formula for a one-period nominally risk-free bond,

$$1 = R_t E_t \left[ \frac{\beta u_{1t+1}}{u_{1t}} \frac{1}{\pi_{t+1}} \right]. \quad (11)$$

As is standard in this type of cash/credit setup, the household first-order conditions also imply that the gross nominal interest rate equals the marginal rate of substitution between cash and credit goods,

$$R_t = \frac{u_{1t}}{u_{2t}}. \quad (12)$$

In a monetary equilibrium,  $R_t \geq 1$ , otherwise consumers could earn unbounded profits by buying money and selling bonds.

## 2.2 Production

The production side of the economy features a representative firm that must open vacancies, which entail costs, in order to hire workers and produce. The representative firm is “large” in the sense that it operates many jobs and consequently has many individual workers attached to it through those jobs.

To be more specific, the firm requires only labor to produce its output. The firm must engage in costly search for a worker to fill each of its job openings. In each job  $k$  that will produce output, the worker and firm bargain over the pre-tax nominal wage  $W_{kt}$  paid in that position. Output of job  $k$  is given by  $y_{kt} = z_t f(\bar{h})$ , which is subject to a common technology realization  $z_t$ . We allow for curvature in  $f(\cdot)$  to enhance comparability with our model in Section 5; of course, in the model in this section, the curvature does not matter because  $\bar{h}$  is fixed anyway.

Any two jobs  $k_a$  and  $k_b$  at the firm are identical, so from here on we suppress the second subscript and denote by  $W_t$  the nominal wage in any job, and so on. Total output of the firm thus depends on the production technology and the measure of matches  $n_t$  that produce,

$$y_t = n_t z_t f(\bar{h}). \quad (13)$$

The total nominal wage paid by the firm in any given job is  $W_t \bar{h}$ , and the total nominal wage bill of the firm is the sum of wages paid at all of its positions,  $n_t W_t \bar{h}$ .



The firm begins period  $t$  with employment stock  $n_t$ . Its future employment stock depends on its current choices as well as the random matching process. With probability  $k^f(\theta)$ , taken as given by the firm, a vacancy will be filled by a worker. Labor-market tightness is  $\theta \equiv v/u$ , and matching probabilities depend only on tightness given the Cobb-Douglas matching function we will assume.

The firm also faces a cost of adjusting nominal wages. For each of its workers, the real cost of changing nominal wages between period  $t - 1$  and  $t$  is

$$\frac{\psi}{v} \left( \frac{W_t}{W_{t-1}\pi_t^\chi} - 1 \right)^v, \quad (14)$$

where  $\chi \in [0, 1]$  measures the degree to which nominal wage adjustment is indexed to contemporaneous price inflation. If  $\chi = 0$ , there is no indexation; if  $\chi = 1$ , there is full indexation; and if  $\chi \in (0, 1)$ , there is partial indexation. There are two reasons we allow for indexation. First, there seems to be a good deal of empirical support for wage indexation. Second, comparing our steady-state results under full indexation and no indexation allows us to disentangle some aspects of optimal policy in our model.

If we use  $v = 2$ , which we do in the results we report, then the cost function is of the Rotemberg quadratic variety. We also explored the sensitivity of our results with respect to values of  $v$  around two and found little difference. If  $\psi = 0$ , clearly there is no cost of wage adjustment. This Rotemberg type of nominal adjustment cost specification is a fairly common convention in typical sticky wage or sticky price models. At the expense of a heavier computational burden, an alternative specification one may want to pursue is a Calvo structure, in which wages in only a fraction of jobs can be re-set every period. As we mentioned earlier, though, our goal is not to provide a compelling micro-foundation for sticky nominal wages; adopting a fairly-conventional reduced-form specification is just a tractable way to get at our ultimate objective. Part of our reason in choosing the Rotemberg approach is that it makes solving our model computationally a bit easier because it avoids the introduction of further leads and lags in dynamic equations associated with a Calvo structure. Moreover, this specification enhances comparability with the results in Chugh (2006a), who uses a Rotemberg wage adjustment cost function.

Regardless of whether or not nominal wages are costly to adjust, wages are determined through bargaining, which we describe below. In the firm's profit maximization problem, the wage-setting protocol is taken as given. The firm thus chooses vacancies to post  $v_t$  and future employment stock  $n_{t+1}$  to maximize discounted nominal profits starting at date  $t$ ,

$$E_t \sum_{s=0}^{\infty} \beta^s \left\{ \left( \frac{\beta \phi_{t+1+s}}{P_{t+s}} \right) \left[ P_{t+s} n_{t+s} z_{t+s} f(\bar{h}) - W_{t+s} n_{t+s} \bar{h} - \gamma P_{t+s} v_{t+s} - \frac{\psi}{v} \left( \frac{W_{t+s}}{W_{t+s-1} \pi_{t+s}^\chi} - 1 \right)^v n_{t+s} P_{t+s} \right] \right\}. \quad (15)$$

The representative firm discounts period- $t$  profits using  $\beta \phi_{t+1}/P_t$  because this is the value to the

household of receiving a unit of nominal profit.<sup>2</sup> In period  $t$ , the firm's problem is thus to choose  $v_t$  and  $n_{t+1}$  to maximize (15) subject to the law of motion for employment

$$n_{t+1} = (1 - \rho^x)(n_t + v_t k^f(\theta_t)). \quad (16)$$

Firms incur the real cost  $\gamma$  for each vacancy created, and job separation occurs with exogenous fixed probability  $\rho^x$ .

Associate the multiplier  $\mu_t$  with the employment constraint. The first-order conditions with respect to  $n_{t+1}$  and  $v_t$  are, respectively,

$$\frac{\mu_t \beta \phi_{t+1}}{P_t} = E_t \left[ \beta \left( \frac{\beta \phi_{t+2}}{P_{t+1}} \right) \left( z_{t+1} f(\bar{h}) - W_{t+1} \bar{h} - \frac{\psi}{v} \left( \frac{W_{t+1}}{W_t \pi_t^x} - 1 \right)^v P_{t+1} + (1 - \rho^x) \mu_{t+1} \right) \right], \quad (17)$$

$$E_t \left\{ \frac{\beta \phi_{t+1}}{P_t} \left[ -\gamma P_t + (1 - \rho^x) \mu_t k^f(\theta_t) \right] \right\} = 0. \quad (18)$$

Combining the optimality conditions (17) and (18) yields the job-creation condition

$$\frac{\gamma}{k^f(\theta_t)} = E_t \left[ \beta \left( \frac{\beta \phi_{t+2}}{\beta \phi_{t+1}} \right) (1 - \rho^x) \left( z_{t+1} f(\bar{h}) - w_{t+1} \bar{h} - \frac{\psi}{v} \left( \frac{\pi_{t+1}^w}{\pi_t^x} - 1 \right)^v + \frac{\gamma}{k^f(\theta_{t+1})} \right) \right], \quad (19)$$

where we have defined  $\pi_{t+1}^w \equiv W_{t+1}/W_t$  as the gross nominal wage inflation rate and  $w_{t+1} \equiv W_{t+1}/P_{t+1}$  is the real wage rate. The job-creation condition states that at the optimal choice, the vacancy-creation cost incurred by the firm is equated to the discounted expected value of profits from the match. Profits from a match take into account the wage cost of that match, including future nominal wage adjustment costs, as well as future marginal revenue product from the match. This condition is a free-entry condition in the creation of vacancies and is one of the critical equilibrium conditions of the model. In equilibrium,  $(\beta \phi_{t+2})/(\beta \phi_{t+1}) = u_{2t+1}/u_{2t}$ , which can be seen from the household's optimality condition with respect to credit good consumption, condition (6).

### 2.3 Government

The government's flow budget constraint is

$$M_t + B_t + \tau_{t-1}^n W_{t-1} n_{t-1} \bar{h} = M_{t-1} + R_{t-1} B_{t-1} + P_{t-1} g_{t-1}. \quad (20)$$

Thus, the government finances its spending through labor income taxation, issuance of nominal debt, and money creation. Note that government consumption is a credit good, following Chari,

<sup>2</sup>To understand this, note from the household budget constraint that period- $t$  profits are received, in keeping with the usual timing of income receipts in a cash/credit model, in period  $t+1$ . The multiplier associated with the period- $t$  household flow budget constraint is  $\phi_t/P_{t-1}$ . Hence, the derivative of the Lagrangian of the household problem with respect to  $d_t$  is  $\beta \phi_{t+1}/P_t$ .

Christiano, and Kehoe (1991), because  $g_{t-1}$  is not paid for until period  $t$ . In equilibrium, the government budget constraint can be expressed in real terms as

$$c_{1t}\pi_t + b_t\pi_t + \tau_{t-1}^n w_{t-1} n_{t-1} \bar{h} = c_{1t-1} + \frac{u_{1t-1}}{u_{2t-1}} b_{t-1} + g_{t-1}, \quad (21)$$

where  $\pi_t \equiv P_t/P_{t-1}$  is the gross rate of price inflation.

## 2.4 Nash Wage Bargaining

As is standard in the literature, we assume that the wage paid in any given job is determined in a Nash bargain between a matched worker and firm. Thus, the wage payment divides the match surplus. Our departure from the standard Nash bargaining convention used in the literature is that we assume bargaining occurs over the nominal wage payment rather than the real wage payment. With zero costs of wage adjustment, the real wage that emerges is identical to the one that emerges from bargaining directly over the real wage. The reason that nominal bargaining and real bargaining are identical if wage adjustment is costless is straightforward. A firm and worker in negotiations take the price level  $P$  as given. Bargaining over  $W$  thus pins down  $w$ ; alternatively, bargaining over  $w$  pins down  $W$ . With no impediment to adjusting wages, there is no problem adjusting either  $w$  or  $W$  to achieve some desired split of the surplus, and the optimal split itself is independent of whether a real unit of account or a nominal unit of account is used in bargaining.

In addition to bargaining over nominal wages, though, we assume that nominal wage adjustment may entail a resource cost of the Rotemberg-type described in Section 2.2. Details of the solution of the Nash bargain with costly wage adjustment are given in Appendix A. Here we present only the outcome of the Nash bargain. Bargaining over the nominal wage payment yields

$$\begin{aligned} \frac{\omega_t}{1 - \omega_t} \left[ z_t f(\bar{h}) - w_t \bar{h} - \frac{\psi}{v} \left( \frac{\pi_t^w}{\pi_t^x} - 1 \right)^v + \frac{\gamma}{k^f(\theta_t)} \right] = & \quad (22) \\ (1 - \tau_t^n) w_t \bar{h} - \frac{A \bar{h}}{u_{2t}} - \frac{v}{u_{2t}} & \\ + (1 - \theta_t k^f(\theta_t)) \beta E_t \left[ \left( \frac{\omega_{t+1}}{1 - \omega_{t+1}} \right) \left( \frac{u_{2t+1}}{u_{2t}} \right) (1 - \rho^x) \left[ z_{t+1} f(\bar{h}) - w_{t+1} \bar{h} - \frac{\psi}{v} \left( \frac{\pi_{t+1}^w}{\pi_{t+1}^x} - 1 \right)^v + \frac{\gamma}{k^f(\theta_{t+1})} \right] \right], & \end{aligned}$$

which characterizes the real wage  $w_t$  agreed upon in period  $t$ . In (22),  $\omega_t$  is the effective bargaining power of the worker and  $1 - \omega_t$  is the effective bargaining power of the firm. Specifically,

$$\omega_t \equiv \frac{\eta}{\eta + (1 - \eta) \Delta_t^F / \Delta_t^W}, \quad (23)$$

where  $\Delta_t^F$  and  $\Delta_t^W$  measure marginal changes in the value of a filled job and the value of being employed, respectively, and  $\eta$  is the weight given to the worker's individual surplus in Nash bargaining.<sup>3</sup>

<sup>3</sup>Our notation surrounding the time-varying bargaining weights is adapted from Gertler and Trigari (2006).

As we say, we provide the details behind (22) and (23) in Appendix A, but there are three points worth mentioning here. First, effective bargaining power  $\omega_t$  is related to the Nash weight  $\eta$ . With flexible nominal wages and no labor taxation, it is straightforward to show that  $\omega_t = \eta \forall t$  (because in that case  $\Delta_t^F/\Delta_t^W = 1$ ). The presence of proportional taxes and sticky wages drives a time-varying wedge between  $\eta$  and  $\omega$ . Second, the expected future cost of adjusting the nominal wage affects the time- $t$  wage payment. Third, the labor tax rate appears in (22) both directly as well as through effective bargaining power. The weight  $\omega_t$  depends on  $\tau_t^n$ ; thus the weight  $\omega_{t+1}$ , which affects the time- $t$  split of the surplus, depends on  $\tau_{t+1}^n$ . Indeed, as can also be seen in our Appendix A, if wages are not at all sticky ( $\psi = 0$ ), the bargaining weight varies only because of variations in the tax rate,

$$\omega_t = \frac{\eta}{\eta + (1 - \eta) \frac{1}{1 - \tau_t^n}}. \quad (24)$$

The fact that current and future tax rates affect wage-setting may be important in understanding some of the results we present in Section 4.

## 2.5 Matching Technology

Matches between unemployed individuals searching for jobs and firms searching to fill vacancies are formed according to a matching technology,  $m(u_t, v_t)$ , where  $u_t$  is the number of searching individuals and  $v_t$  is the number of posted vacancies. A match formed in period  $t$  will produce in period  $t + 1$  provided it survives exogenous separation at the beginning of period  $t + 1$ . The evolution of total employment is thus given by

$$n_{t+1} = (1 - \rho^x)(n_t + m(u_t, v_t)). \quad (25)$$

## 2.6 Private-Sector Equilibrium

The equilibrium conditions of the model are the Fisher equation (11) describing the household's optimal intertemporal choices; the household intratemporal optimality condition (12), which is standard in cash/credit models; the restriction  $R_t \geq 1$ , which states that the net nominal interest rate cannot be less than zero, a requirement for a monetary equilibrium; the job-creation condition describing firm profit-maximization

$$\frac{\gamma}{kf(\theta_t)} = E_t \left[ \left( \frac{\beta u_{2t+1}}{u_{2t}} \right) (1 - \rho^x) \left( z_{t+1} f(\bar{h}) - w_{t+1} \bar{h} - \frac{\psi}{v} \left( \frac{\pi_{t+1}^w}{\pi_{t+1}^x} - 1 \right)^v + \frac{\gamma}{kf(\theta_{t+1})} \right) \right], \quad (26)$$

in which the household discount factor for credit resources,  $\beta u_{2t+1}/u_{2t}$ , appears; the flow government budget constraint, expressed in real terms, (21) (in which we have substituted  $R_{t-1} = u_{1t-1}/u_{2t-1}$  from (12) as well as the cash-in-advance constraint (4) holding with equality); the

Nash wage characterized by (22); the law of motion for employment (25); the identity

$$n_t + u_t = 1 \tag{27}$$

restricting the size of the labor force to one; a condition relating the rate of real wage growth to nominal price inflation and nominal wage inflation

$$\frac{\pi_t^w}{\pi_t} = \frac{w_t}{w_{t-1}}; \tag{28}$$

and the resource constraint

$$c_{1t} + c_{2t} + g_t + \gamma u_t \theta_t + \frac{\psi}{v} \left( \frac{\pi_t^w}{\pi_t^x} - 1 \right)^v = n_t z_t f(\bar{h}). \tag{29}$$

Condition (28) is typically thought of as an identity, but is one that does not hold trivially in a model with nominally-rigid wages and thus must be included as part of the description of equilibrium; see Chugh (2006a, p. 692) for an intuitive explanation. In (29), total costs of posting vacancies  $\gamma u_t \theta_t$  are a resource cost for the economy, as are wage adjustment costs; in the resource constraint, we have made the substitution  $v_t = u_t \theta_t$ , eliminating  $v_t$  from the set of endogenous processes of the model. The private-sector equilibrium processes are thus  $\{c_{1t}, c_{2t}, n_{t+1}, u_t, \theta_t, w_t, \pi_t, \pi_t^w, b_t\}$ , for given processes  $\{z_t, g_t, \tau_t^n, R_t\}$ .

### 3 Ramsey Problem in Basic Model

The problem of the Ramsey planner is to raise exogenous revenue for the government through labor income taxes and money creation in such a way that maximizes the welfare of the representative household, subject to the equilibrium conditions of the economy. In period zero, the Ramsey planner commits to a policy rule. Because of the complexity of the model, we cast the Ramsey problem as one of choosing both allocation and policy variables rather than in the pure primal form often used in the literature, in which it is just allocations that are chosen directly by the Ramsey planner. The Ramsey problem is to choose  $\{c_{1t}, c_{2t}, n_{t+1}, u_t, \theta_t, w_t, \pi_t, \pi_t^w, b_t, \tau_t^n\}$  to maximize (2) subject to (11), (21), (22), (25), (26), (27), (28), and (29) and taking as given exogenous processes  $\{z_t, g_t\}$ . In principle, we must also impose the inequality condition

$$u_1(c_{1t}, c_{2t}) - u_2(c_{1t}, c_{2t}) \geq 1 \tag{30}$$

as a constraint on the Ramsey problem. This inequality constraint ensures (in terms of allocations — refer to condition (12)) that the zero-lower-bound on the nominal interest rate is not violated. We thus refer to constraint (30) as the ZLB constraint. The ZLB constraint in general is an occasionally-binding constraint. Because our model likely is too complex, given current technology,

to solve using global approximation methods that would be able to properly handle occasionally-binding constraints, for our dynamic results we drop the ZLB constraint and then check whether the ZLB constraint is ever violated. As we discuss when we present our parameterization in Section 4.1, using this approach raises an issue for one aspect of our model calibration.

Throughout, we assume that the first-order conditions of the Ramsey problem are necessary and sufficient and that all allocations are interior.

## 4 Optimal Policy in Basic Model

We characterize the Ramsey steady-state of our model numerically. Before turning to our results, we describe how we parameterize the model. Because a number of our steady-state results have a close analog in the optimal capital taxation results of Arseneau and Chugh (2006), we adopt, where possible, their calibration to enhance comparability.

### 4.1 Model Parameterization

We assume that the instantaneous utility function over cash and credit goods is

$$u(c_{1t}, c_{2t}) = \frac{\left\{ \left[ (1 - \kappa)c_{1t}^\phi + \kappa c_{2t}^\phi \right]^{1/\phi} \right\}^{1-\sigma} - 1}{1 - \sigma}, \quad (31)$$

with, as is typical in cash/credit models, a CES aggregator over cash and credit goods. For the aggregator, we adopt the calibration used by Siu (2004) and Chugh (2006a, 2006b) and set  $\kappa = 0.62$  and  $\phi = 0.79$ . The time unit of the model is meant to be a quarter, so we set the subjective discount factor to  $\beta = 0.99$ , yielding an annual real interest rate of about four percent. We set the curvature parameter with respect to consumption to  $\sigma = 1$ , consistent with many macro models.

Our timing assumptions are such that production in a period occurs after the realization of separations. Following the convention in the literature, we suppose that the unemployment rate is measured *before* the realization of separations. We set the quarterly probability of separation at  $\rho^x = 0.10$ , consistent with Shimer (2005). Thus, letting  $n$  denote the steady-state level of employment,  $n(1 - \rho^x)^{-1}$  is the employment rate, and  $1 - n(1 - \rho^x)^{-1}$  is the steady-state unemployment rate.

The match-level production function in general displays diminishing returns in labor,

$$f(\bar{h}) = \bar{h}^\alpha, \quad (32)$$

and we set the fixed number of hours a given individual works to  $\bar{h} = 0.35$ , making our baseline model comparable to our richer model in Section 5. In the richer model, we allow intensive labor

adjustment and calibrate utility parameters so that steady-state hours are  $h = 0.35$ . Thus, we set  $\bar{h} = 0.35$  here. Regarding curvature, we choose  $\alpha = 0.70$ , a conventional value in DGE models.<sup>4</sup>

As in much of the literature, the matching technology is Cobb-Douglas,

$$m(u_t, v_t) = \psi^m u_t^{\xi_u} v_t^{1-\xi_u}, \quad (33)$$

with the elasticity of matches with respect to the number of unemployed set to  $\xi_u = 0.40$ , following Blanchard and Diamond (1989), and  $\psi^m$  a calibrating parameter that can be interpreted as a measure of matching efficiency.

We normalize the utility of non-work to  $v = 0$ . With this normalization, there are two natural cases to consider regarding the calibration of  $A$ , the disutility per unit time of working. The first case is  $A = v = 0$ , so that there is no difference at all in the realized welfare of employed versus unemployed individuals. Although this calibration may not be an accurate description of the relative welfare between unemployed and employed individuals, it serves as a very useful benchmark for our main results, as it did in Arseneau and Chugh (2006).

In the second case, we introduce ex-post heterogeneity between employed and unemployed individuals by allowing  $A\bar{h}$  to differ from  $v$ . As in Arseneau and Chugh (2006), our choice of a specific value of  $A$  is guided by Shimer (2005), who calibrates his model so that unemployed individuals receive, in the form of unemployment benefits, about 40 percent of the wages of employed individuals. With his linear utility assumption, unemployed individuals are therefore 40 percent as well off as employed persons. Our model differs from Shimer's (2005) primarily in that we assume full consumption insurance, but also in that we have curvature in utility. Thus, when we allow for welfare heterogeneity, we interpret Shimer's (2005) calibration to mean that unemployed individuals must receive 2.5 times more consumption of both cash goods and credit goods (in steady-state) than employed individuals in order for the total utility of the two types of individuals to be equalized. That is, we set  $A$  such that in steady-state

$$u(2.5\bar{c}_1, 2.5\bar{c}_2) + v = u(\bar{c}_1, \bar{c}_2) - A\bar{h}, \quad (34)$$

where  $\bar{c}_j$  denotes steady-state consumption,  $j = 1, 2$ . The resulting value is  $A = -2.6$ , but we point out that our qualitative results do not depend on the exact value of  $A$ . As we discuss in Section 4, all that is important is that  $A\bar{h} < v$ .

We choose steady-state government purchases  $\bar{g}$  so that they constitute about 18 percent of total output. The same value of  $\bar{g}$  ( $\bar{g} = 0.07$ ) delivers a government share of output very close to 18 percent in both models (as well as the models in Section 5). Finally, the steady-state value of

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<sup>4</sup>Note, however, that because part of output is absorbed by vacancy posting costs,  $\alpha$  does not correspond to labor's share of output; nonetheless, we choose  $\alpha = 0.70$  to remain as close as possible to conventional models.

government debt is to  $b = 0.14$ , making government debt about 40 percent of total output in steady state, in line with the long-run average for the U.S. economy.

Regarding the Nash bargaining weight  $\eta$ , we face a bit of a tension, driven purely by practical concerns about solving our model. We would like to focus on the case  $\eta = \xi_u = 0.40$  so that the usual Hosios (1990) parameterization is satisfied. The Nash bargaining weight being a relatively esoteric parameter, it is hard to say whether such a parameterization is empirically-justified. Nonetheless, it is a parameterization of interest because many results in the quantitative labor search literature are obtained assuming it. We thus present our primary steady-state and dynamic results using  $\eta = 0.40$ . However, when we turn to dynamics, we run into a problem because with costless wage adjustment and  $\eta = 0.40$ , the zero-lower-bound on the nominal interest rate is violated during simulations.<sup>5</sup> For the cases with costly wage adjustment, using  $\eta = 0.40$  does not pose a problem. Rather than fiddle with the calibration in an ad-hoc way to make the flexible-wage version also satisfy the ZLB constraint, though, we simply present the results as they are. We have reason to think that the main ideas that emerge from our model are unaffected by this issue, but we defer further discussion until our presentation of dynamic results.

Finally, regarding the cost-adjustment parameter  $\psi$ , we adopt Chugh’s (2006a) calibration strategy and consider four different values for our main results:  $\psi = 0$  (flexible wages),  $\psi = 1.98$  (nominal wages sticky for two quarters on average),  $\psi = 5.88$  (nominal wages sticky for three quarters on average), and  $\psi = 9.61$  (nominal wages sticky for four quarters on average). We recognize that Chugh’s (2006a) mapping of duration of wage-stickiness to the cost-adjustment parameter may need to be modified because we have a fundamentally different model, but we think it is a useful starting point and allows us to demonstrate our main points. We leave an empirical investigation of a “wage Phillips curve” in the presence of labor search frictions to future work.

## 4.2 Ramsey Steady State

We begin by analyzing how costly nominal wage bargaining influences allocations and policy variables in the Ramsey steady state. We first discuss the case of no wage adjustment costs ( $\psi = 0$ ) and no wage indexation ( $\chi = 0$ ) under our two alternative assumptions regarding the value of  $A$ . These two sets of results serve as useful benchmarks that will help in understanding how things change when we introduce wage adjustment costs.

We provide a thorough analysis of how Ramsey policy operates in the long run because ours is one of the first studies of optimal policy in these types of models; as such, we think it worthwhile to spend some effort understanding the forces at work, knowing that future work will reveal some

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<sup>5</sup>As we pointed out in Section 3, our dynamic solution method drops the ZLB constraint (30) and then checks whether the ZLB constraint is violated during simulated runs.



of these mechanisms to be more important than others. Readers primarily interested in understanding the dynamic policy implications of our model, however, may safely skip to Section 4.3 with the following summary of the steady-state results in mind. First, as is the case in the existing Ramsey literature, a tension between minimizing the monetary distortion (which in isolation calls for implementing the Friedman deflation) and minimizing the cost of nominal rigidities (which in isolation calls for implementing zero inflation) is present. In line with the existing literature, the tension is resolved overwhelmingly in favor of minimizing the distortions arising from nominal rigidities for even very small costs of wage adjustment. Second, and this is the most novel aspect of our steady-state results, the optimal inflation rate may actually be *above* zero, reflecting the consequences of a third force not present in standard models. This third force is that positive inflation can be used as an indirect way of addressing inefficiently-high job creation, an example of the inflation tax proxying for a missing tax instrument. In all cases, however, the steady-state inflation rate is never very far from zero.

#### 4.2.1 No Adjustment Costs, No Indexation ( $\psi = 0, \chi = 0$ )

Table 1 presents steady-state allocations and policy variables under the Ramsey plan assuming costless wage adjustment. The left panels of the table present results for both  $g = 0$  and our benchmark  $g > 0$  under the assumption that  $A\bar{h} = v$ . With no utility heterogeneity between unemployed and employed individuals, the Hosios condition delivers efficient job creation, so the only concern of the Ramsey planner with regard to monetary policy is minimizing the monetary distortion. It does so by implementing the Friedman deflation, thereby driving the nominal interest rate to zero and eliminating any wedge between cash good consumption and credit good consumption. The government budget constraint binds even though  $g = 0$  because, with lump-sum instruments ruled out, a sustained deflation must be financed by a small positive labor tax. This can be seen in the first column of Table 1, which shows that the labor tax is non-zero despite the fact that  $g = 0$ .

With positive government spending and no costs of wage adjustment, the Ramsey planner maintains the Friedman Rule. Doing so merely crowds out private consumption and, despite the fact that workers' effective bargaining power  $\omega$  falls, does not affect steady-state labor market allocations. In other words, the proportional labor tax acts as a lump-sum instrument in the special case of no wage adjustment costs and no welfare heterogeneity. Cast in this light, the optimal financing problem becomes quite transparent: the Ramsey planner chooses to finance all government spending with the non-distortionary proportional labor tax. This idea was developed in Arseneau and Chugh (2006) in a non-monetary economy.

Next, we introduce welfare heterogeneity, so that  $A\bar{h} < v$ ; results for this case are presented in the right panels of Table 1. As can be seen by comparing the  $g = 0$  columns in the table,

heterogeneity by itself lowers the bargained wage. The reason for this, also developed in Arseneau and Chugh (2006), is that individuals value the state of employment more highly and are thus willing to accept a lower wage to move out of unemployment. As the wage falls, the increased incentive for firms to post vacancies results in inefficiently-high job creation. On balance, the incentive to remove the monetary distortion remains; doing so requires, as above, a positive labor tax to finance the Friedman deflation. In the presence of heterogeneity, however, the labor tax is distortionary. As the labor tax rises it erodes the bargaining power of workers, thereby putting additional downward pressure on the wage. This further fuels job creation, which is already inefficiently high due to the presence of heterogeneity. Thus, the optimal policy equates the marginal benefit of reducing the monetary distortion to the marginal cost of further distorting the labor market in order to finance the required deflation. With flexible wages and  $g = 0$ , the optimal policy calls for a rate of inflation that is slightly above that implied by the Friedman Rule, but the departure from the Friedman deflation is obviously quantitatively very small.

In the presence of heterogeneity, any incentive to inflate away from the Friedman Rule in order to lower the labor tax rate is completely overwhelmed if  $g > 0$ . With flexible wages, the Ramsey planner implements the Friedman Rule and finances all government expenditures through the labor tax. Thus, the costs of reintroducing the monetary distortion are high relative to the marginal improvement in the labor market that comes from easing off on the rate of deflation and allowing the labor tax to fall by a bit.

#### 4.2.2 Adjustment Costs, No Indexation ( $\psi > 0, \chi = 0$ )

We now analyze how the presence of costly nominal wage adjustment influences the benchmark results presented above. Figure 1 plots the key Ramsey steady-state allocation and policy variables as a function of the cost of adjustment parameter,  $\psi$ , when  $A\bar{h} = v$ . Varying  $\psi \in (0, 10)$  varies the average length of nominal wage-stickiness between zero and four quarters. As the first two panels in the upper row of Figure 1 show, when nominal wage adjustment is costly,  $\psi > 0$ , the Friedman Rule ceases to be optimal. As  $\psi$  rises, the optimal rate of price inflation approaches zero. The reason behind this result is well-understood: minimizing the resource cost of nominal adjustment – be it nominal price adjustment or nominal wage adjustment – is a quantitatively much more important goal of optimal policy than is removing the monetary friction. This aspect of our steady-state results echoes that of Schmitt-Grohe and Uribe (2004b), Siu (2004), and Chugh (2006a).

The second row of Figure 1 shows a few key labor market allocations. As  $\psi$  increases, the real wage rises and total employment and labor market tightness each fall, although the effects are quantitatively small. This labor market response stems from the fact that the resource costs associated with nominal wage adjustment effectively shifts bargaining power away from firms and

towards workers. To understand this point, consider how a change in  $\psi$  affects workers' effective bargaining power  $\omega$  as well as the marginal change in the value of a filled job  $\Delta^F$ .<sup>6</sup> As can be deduced from the derivation of the Nash bargaining solution presented in Appendix A, in steady-state these partials are

$$\frac{\partial \omega}{\partial \psi} = -\eta \left( \frac{1}{\eta + (1-\eta) \Delta^F / \Delta^W} \right)^2 (1-\eta) \frac{1}{\Delta^W} \frac{\partial \Delta^F}{\partial \psi} \quad (35)$$

and

$$\frac{\partial \Delta^F}{\partial \psi} = - \left( \pi^{1-\chi} - 1 \right)^{v-1} \frac{\pi^{1-\chi}}{w} (1 - (1 - \rho^x) \beta), \quad (36)$$

where we have used the fact that  $\pi^w = \pi$  in steady state and have left the indexation parameter  $\chi$  in place. The important thing to note here is that the sign of  $\partial \Delta^F / \partial \psi$  and hence the sign of  $\partial \omega / \partial \psi$  may depend on whether or not there is inflation or deflation in the steady state. With no indexation ( $\chi = 0$ ), if the steady state features  $\pi < 1$ , then  $\partial \Delta^F / \partial \psi > 0$ , implying  $\partial \omega / \partial \psi > 0$ .<sup>7</sup> Thus, with deflation in the Ramsey steady state, which is indeed the case in the absence of heterogeneity, a higher cost of wage adjustment effectively transfers bargaining power away from firms and to workers, resulting in higher real wages. The reason this transfer of bargaining power occurs is because in a deflationary environment with costly wage adjustment, raising the nominal wage by an additional dollar pushes the absolute level of wage inflation closer to zero, marginally reducing both current and expected future costs of nominal wage adjustment. All else equal, the firm benefits from this and is thus willing to cede a bit of bargaining power in order to realize the cost savings. This effect gets stronger as the costs of wage adjustment rise. Increased bargaining power on the part of workers drives up the bargained wage, meaning that an individual job is less profitable to a firm. Vacancy postings fall and, as a consequence, both labor market tightness and the total number of people working in the economy fall.

Next, turn to the case of welfare heterogeneity, so that  $A\bar{h} < v$ . Figure 2 shows that, as was the case in the absence of welfare heterogeneity, costly nominal wage adjustment introduces an incentive to move toward zero inflation because doing so minimizes the resource costs associated with nominal wage adjustment. A notable difference, however, is that with heterogeneity, as  $\psi$  rises, the Ramsey inflation rate actually moves above zero because anticipated inflation is used to indirectly stifle job creation. The intuition behind this result lies in a complicated interaction between anticipated inflation, the costs of nominal wage adjustment, and effective bargaining power. The Ramsey planner exploits this interaction by using inflation to dampen the incentive for firms to post vacancies. Doing so, however, involves incurring greater resource costs associated with nominal wage adjustment. The optimal inflation tax balances the welfare gain from mitigating

<sup>6</sup>Both  $\omega$  and  $\Delta^F$  were introduced in Section 2.4.

<sup>7</sup>Because, as shown in Appendix A,  $\Delta^W < 0$ .

inefficiently-high job creation against the costs of nominal wage adjustment that arise from doing so.

The precise economic mechanisms at work here seem to be quite complex, but we can numerically verify our intuition by modifying our model to allow the Ramsey planner to have access to a vacancy tax. Following Domeij (2005) and Arseneau and Chugh (2006), we replace  $\gamma$  with  $\gamma(1 + \tau_t^s)$  in the firm's profit function and the resulting job-creation condition, and we introduce  $\tau_t^s \gamma v_t$  as a revenue item in the government budget constraint, where  $\tau_t^s$  is a proportional vacancy tax rate. If  $\tau_t^s > 0$ , the firm must pay a tax for each vacancy it created, while if  $\tau_t^s < 0$ , the firm receives a subsidy for each vacancy. Note that the total vacancy tax adds to government revenues and is now part of the optimal financing problem.

The vacancy tax offers the Ramsey planner a more efficient instrument with which to correct the labor market distortion. Thus, if our intuition about why inflation is above zero for high enough  $\psi$  without a vacancy tax is correct, the optimal policy mix in the presence of a vacancy tax should involve slight deflation (reflecting the usual tradeoff between the monetary distortion and the resource costs of wage adjustment) and a positive vacancy tax (reflecting the fundamental labor market distortion). As shown in the bottom right panel of Table 1, numerical results support our conjecture; in the cases in which long-run inflation was positive with no vacancy tax available, inflation is now between zero and the Friedman Rule and there is a tax on vacancy creation. Having demonstrated that positive inflation rates act as a proxy for a vacancy tax, we now continue our analysis by again omitting the direct vacancy instrument. Some justification for this might be that, given how much attention is usually paid to *promoting* job creation, an explicit vacancy tax may be politically infeasible.

### 4.2.3 Full Wage Indexation ( $\chi = 1$ )

The results so far have all been under the assumption of no indexation ( $\chi = 0$ ) and show that the optimal rate of steady-state inflation is highly sensitive to the costs of nominal wage adjustment. This raises the question of whether or not these results are robust to wage indexation. Expression (36) shows that if  $\chi = 1$ ,  $\partial \Delta^F / \partial \psi = 0$ , implying  $\partial \omega / \partial \psi = 0$ . With full indexation, effective bargaining power is invariant to the costs of nominal wage adjustment.

Figure 3 summarizes how steady-state inflation depends on  $\psi$  when the Ramsey planner must finance positive government expenditures. If nominal wages are unindexed to price inflation, the optimal rate of inflation rises away from the Friedman Rule as the costs of wage adjustment rise. In contrast, with full wage indexation, the Friedman Rule is always optimal regardless of assumptions regarding heterogeneity. Partial indexation, meaning  $0 < \chi < 1$ , would lead to an optimal rate of inflation that is simply a convex combination of these two cases, putting steady state inflation

somewhere between the Friedman deflation and the solid or dotted lines shown in Figure 3 for a given level of  $\psi$ . We proceed from here on assuming no indexation ( $\chi = 0$ ) because it simplifies obtaining dynamic results.

### 4.3 Ramsey Dynamics

To study dynamics, we approximate our model by linearizing in levels the Ramsey first-order conditions for time  $t > 0$  around the non-stochastic steady-state of these conditions. Our numerical method is our own implementation of the perturbation algorithm described by Schmitt-Grohe and Uribe (2004c). As in Khan, King, and Wolman (2003) and others, we assume that the initial state of the economy is the asymptotic Ramsey steady state. Throughout, we assume, as is common in the literature, that the first-order conditions of the Ramsey problem are necessary and sufficient and that all allocations are interior. We also point out that because we assume full commitment on the part of the Ramsey planner, the use of state-contingent inflation is not a manifestation of time-inconsistent policy. The “surprise” in surprise inflation is due solely to the unpredictable components of government spending and technology and not due to a retreat on past promises.

We conduct 5000 simulations, each 100 periods long. To make the comparisons meaningful as we vary  $\psi$ , the same realizations for government spending shocks and productivity shocks are used across versions of our model. We limit the length of each repetition because, as the results below show, there is a near-unit root in real government debt and thus we must prevent the model from wandering too far from initial conditions. For each simulation, we then compute first and second moments and report the medians of these moments across the 5000 simulations. By averaging over so many short-length simulations, we are likely obtaining a fairly accurate description of model dynamics even if a handful of simulations drift far away from the steady state.

Table 2 presents simulation-based moments for the key policy and allocation variables of our model for various degrees of nominal wage rigidity. We divide the discussion of results into two parts: first, we analyze the dynamics of policy variables, and then we discuss the dynamics of labor market variables. The important link between the dynamics of policy variables and the dynamics of labor market variables is the dynamics of the real wage. As we pointed out earlier, we face an issue regarding the zero-lower-bound in our flexible-wage model; we discuss this issue below.

#### 4.3.1 Policy Dynamics

The top panel of Table 2 shows that if nominal wages are costless to adjust, the average level of inflation is near the Friedman deflation and price inflation volatility is quite high. The basic reason for inflation volatility with flexible wages and prices, as mentioned in the introduction and as is well-known in the Ramsey literature, is that the Ramsey planner finds price-level variations a

relatively costless way of financing innovations to the government budget. Ex-post inflation renders nominally risk-free debt payments state-contingent in real terms. In the basic Ramsey literature, generating this ex-post variation in debt returns via unanticipated inflation allows the planner to finance a large share of innovations to the government budget without changing the labor tax rate very much. A tax-smoothing incentive is thus the source of inflation volatility in a basic Ramsey model. However, as Table 2 shows, with  $\psi = 0$  our model displays a fair amount of tax rate variability, an order of magnitude larger than in a basic Ramsey model.<sup>8</sup> We discuss this point further below. With  $\psi = 0$ , nominal wage inflation is also quite volatile; coupled with volatile price inflation, the path for the real wage turns out to be relatively stable, with a standard deviation of about 1 percent, much less than the volatility of output, which has a standard deviation of about 1.8 percent.<sup>9</sup>

If nominal wages are instead costly to adjust, wage inflation is near zero with very low variability. The reason behind low and stable nominal wage inflation is that the Ramsey planner largely eliminates the direct resource cost changes in nominal wages entail. However, the underlying incentive for the planner to generate movements in the price level is of course still present. The tradeoff thus facing the planner is the welfare loss due to any induced volatility in the real wage versus the welfare gain due to the shock absorption afforded by state-contingent inflation. In our model, relatively large fluctuations in the real wage apparently do *not* affect welfare very much, so price inflation continues to be quite volatile. This result is directly opposite that in Chugh (2006a), who finds that even two quarters of nominal wage rigidity lowers price inflation volatility by an order of magnitude. With two or three quarters of nominally-rigid wages on average, we find that price inflation volatility is still around five percent, little changed from the fully-flexible case. With four quarters of wage stickiness, inflation volatility is actually higher than in the fully-flexible case, and wage inflation volatility rises a bit as well compared to the two-quarter and three-quarter cases. Comparing our results with those of Chugh (2006a), it clearly matters for prescriptions regarding optimal inflation in what type of underlying environment — a Walrasian labor market or a labor market with fundamental frictions — nominal wage rigidity is modeled. This is the central result of our study.

There are other novel aspects of the dynamic Ramsey policy in this model, as well. The labor tax rate fluctuates around its steady-state value of about 23 percent. However, as noted above, its volatility is different than in basic Ramsey models. With flexible wages, the standard deviation of the tax rate is over one percent, an order of magnitude higher than benchmark tax-smoothing results

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<sup>8</sup>Tax rate volatility does not arise because our overall model is excessively volatile: the coefficient of variation of total output is about 1.8 percent, in line with the data and with basic Ramsey models.

<sup>9</sup>These standard deviations in percentage terms are simply equal to the raw standard deviations presented in Table 2 divided by the means. We could have equivalently computed the standard deviation of the logged variables.

in the Ramsey literature. In basic Ramsey models, the planner refrains from tax rate variability because doing so would disrupt equilibrium marginal rates of substitution between consumption and leisure, which harms welfare; shocks to the government budget are instead financed by varying realized returns on nominal government debt via state-contingent variations in the price level. In contrast, our results show that variability of proportional labor taxes are not as undesirable in a bargaining framework because there is no margin that is *directly* affected by the tax rate. The tax rate affects the bargained wage, which in turn influences the evolution of *expected future wages*, which in turn affects vacancy-postings by firms. This mechanism by which variations in taxes affect allocations apparently has very different quantitative welfare consequences than in models with standard Walrasian labor markets. Furthermore, the fact that our flexible-wage model predicts a more volatile tax rate than do basic Ramsey models may be related to the result that inflation is a bit *less* variable than in basic Ramsey models. For comparison, in standard flexible-price/flexible-wage Ramsey models without capital, Chari, Christiano, and Kehoe (1991) report inflation volatility of about 20 percent, while Schmitt-Grohe and Uribe (2004b), Siu (2004), and Chugh (2006a) all report inflation volatility of about 7 percent. The dynamic tradeoff in our model thus falls a little more on the side of tax variability and a little less on the side of inflation variability; nonetheless, we would call a standard deviation of inflation of nearly 5 percent quite volatile.<sup>10</sup> The tax rate becomes more volatile as the costs of nominal adjustment rise; this finding is line with Schmitt-Grohe and Uribe (2004b), Siu (2004), and Chugh (2006a).

To try to shed a little more light on why tax rate variability is not so undesirable in our model, consider the time- $t$  wage payment described by (22). This sharing rule reveals that the way in which variations in the labor tax rate influence allocations in our model is very different than in standard models. In a standard model, a labor tax creates only a static wedge. With our bargaining specification, both  $\tau_t^n$  and  $\tau_{t+1}^n$  affect the time- $t$  split of the match surplus through their influences on period  $t$  and  $t + 1$  effective bargaining weights. In addition, if  $\psi > 0$ , price inflation and nominal wage inflation also influence the split. The last row in each panel of Table 2 shows the dynamics of the worker's effective bargaining power  $\omega_t$  (defined in expression (23)). The mean and persistence of worker effective bargaining power are essentially invariant to the stickiness of nominal wages. In line with how the variability of the tax rate changes, though,  $\omega$  becomes more volatile as  $\psi$  rises. Figure 4 presents dynamic realizations of  $\omega_t$  and  $\tau_t^n$  from one simulation for various degrees of nominal wage rigidity. With  $\psi = 0$ , there is a negative linear relation between the two, as expression (24) confirms there should be. With  $\psi > 0$ , it is harder to analytically see the relation between  $\tau_t^n$  and  $\omega_t$  because variations in  $\pi_t$  affect the relationship as well. However, Figure 5 shows there is virtually no dynamic relationship between realized inflation and  $\omega$  no matter the degree

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<sup>10</sup>And believe that most central bankers would call it volatile as well.

of wage rigidity. In conjunction with Figure 5, Figure 4 then suggests that the dynamics of  $\omega$  can still be thought of as being driven primarily by the dynamics of  $\tau^n$ , evidenced by the continued negative relationship between the two even as wages become more costly to adjust. Time-variation in  $\omega_t$  affects how the surplus is divided between a worker and a firm; splits of the surplus have little to do with economic efficiency, and it is economic efficiency that is a primitive concern of the Ramsey planner, not the split of the surplus. Loosely speaking, our interpretation of our results is thus that tax rate variability is tolerable because all it does is affect the split of the surplus, not allocations.

Real government debt obligations, defined as  $b_t \equiv B_t/P_{t-1}$ , display a near-unit root no matter the degree of nominal rigidity. At first glance, this result does not seem readily reconcilable with existing results in the Ramsey literature. Aiyagari et al (2002) showed that incomplete (real) government bond markets render government debt highly persistent.<sup>11</sup> Schmitt-Grohe and Uribe (2004b) and Siu (2004) subsequently demonstrated that with sticky nominal prices and nominally-riskless government debt, real debt exhibits the same property. Chugh (2006a) shows that nominally-rigid wages, even with fully-flexible prices, also impart this feature to optimal policy. This set of results leave the impression that it is something about the presence of nominal rigidities *per se* (or, more precisely, the inability or undesirability of making real debt payments state-contingent) that renders real debt highly persistent. In our model, debt is highly persistent even with no nominal friction. A common theme running through the results in the existing literature is that a near-unit root in government debt is associated with relatively high variability in labor tax rates. Our model, including, notably, our flexible-wage model, exhibits this association. Indeed, high persistence of government debt goes hand-in-hand with high measured variability of the tax rate. Figure 6 shows the labor tax rate and real government debt obligations from a representative simulation (the same simulation underlying Figures 4 and 5) of our flexible-wage model. The tax rate is quite stable around 23 percent until about period 55, when it enters a large cyclical downswing, at the same time real debt enters a persistent upswing; high government debt allows for low taxes. As we just discussed, however, swings in tax rates do not seem to be as welfare-diminishing in our model as in basic models.

The dynamics of taxes and real government debt thus seem to be driven by a dynamic bargaining power effect – that is, by the (Ramsey-optimal) fluctuations in  $\omega_t$ . We do not claim we yet fully understand the nature of this dynamic bargaining power effect, but we think studying how bargaining and policy-setting interact seems an interesting future avenue of research.

The first panel of Table 2 indicates that the zero-lower-bound is violated during simulations

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<sup>11</sup>The basic intuition is that with incomplete markets, a one-time shock has (near-) permanent effects, hence the Ramsey government tries to insure the representative agent by maintaining at all times a very high asset position.



of the flexible-wage version of our model. Recall from our analysis of the steady state that the Friedman Rule is optimal with flexible wages. Without handling the occasionally-binding ZLB constraint (30), which our dynamic numerical implementation does not, the nominal interest rate is free to go below zero. The fluctuations are not very large, but, because technically the simulations violate the conditions of a monetary equilibrium, one may be inclined to not place too much faith in the results from the flexible-wage simulations. We think the results are still interpretable, however, because if we raise the Nash weight  $\eta$  on the worker's surplus and simulate the flexible-wage version of our model, we find a steady-state deviation from the Friedman Rule and no dynamic violations of the zero lower bound.<sup>12</sup> For  $\eta = 0.44$ , not too far above our preferred Hosios parameterization, we find that the zero-lower-bound is never violated dynamically and the cyclical properties of policy and quantity variables are very close to those presented in Table 2. Thus, the general picture emerging from our model does not hinge on properly handling the ZLB constraint. To avoid yet another fundamental distortion in our model, though, we have chosen to focus on just the  $\eta = 0.40$  case.<sup>13</sup>

The evolution of real wage payments is critical in understanding why the Ramsey planner continues to implement highly variable inflation as  $\psi$  rises. With volatile price inflation and quite stable nominal wage inflation, real wage volatility rises as  $\psi$  rises: moving from fully-flexible wages to four quarters of wage stickiness, the volatility of the real wage increases three-fold. However, the consequences of this increase in real wage variability on labor market outcomes is negligible, as we discuss in the next section, and this is the reason that optimal inflation volatility remains high no matter the degree of nominal wage rigidity.

### 4.3.2 Labor Market Dynamics

The volatility of unemployment, vacancies, and labor market tightness predicted by labor search models has received much attention lately. Since Shimer (2005) and Hall (2005) pointed out that the basic labor search model with Nash bargaining predicts far too little volatility in these labor market measures compared to empirical evidence, many studies have tried to address this issue. The main line of attack on this question has been to investigate the consequences of alternative (real) wage-setting mechanisms because, as Shimer (2005) pointed out, wage dynamics emerging

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<sup>12</sup>The reason for this is that with a static departure from the Hosios parameterization, the underlying model is already distorted; with  $\eta$  too high, the real wage is too *low*, and the Ramsey planner can indirectly tip the bargaining balance back towards firm by taxing consumers indirectly via inflation.

<sup>13</sup>A very similar issue arises in Cooley and Quadrini (2004). When studying the dynamics of their model, in order to ensure that the simulations are bounded away from the zero lower bound, they alter the Nash bargaining weight from the Hosios condition as well as introduce a second source of inefficiency. They report (p. 188), however, that the introduction of these features just induce a level shift of variables without altering the basic cyclical properties of the model; the same is true in our model.

from Nash bargaining are the root cause of the problem.<sup>14</sup> Here, we assess what happens to the volatility of these measures under the optimal policy as the degree of nominal wage rigidity varies.

Table 2 shows that the standard deviations of unemployment (which can be inferred from the volatility of employment,  $n$ ), vacancies, and labor-market tightness ( $\theta$ ) are all smallest when there are no costs of nominal wage adjustment. In terms of relative volatilities (relative to the volatility of output), unemployment is about 80 percent as volatile as output, vacancies are about 30 percent as volatile as output, and tightness is about 80 percent as volatile as output.<sup>15</sup> Compared to the empirical evidence reported by Gertler and Trigari (2006), the volatilities of each of these variables in the flexible-wage version of our model is substantially lower than in the data; this, in fact, is simply the Shimer (2005) critique.<sup>16</sup>

With costs of nominal wage adjustment, the volatilities of all of these variables rise, but only slightly. Indeed, once nominal wages are at all sticky, the volatilities of these variables do not change much with  $\psi$ . At the same time, the real wage becomes quite a bit more volatile as  $\psi$  rises: its volatility is 50 percent higher with three quarters of nominal wage stickiness than with flexible wages and about three times more volatile with four quarters of wage stickiness. This result goes in the opposite direction of the recent thrust in the literature that explores mechanisms to make real wages *less* volatile and quantity variables *more* volatile. As we mentioned in the introduction, we do not view this as a problematic prediction of our model because our primary concern is not explaining the data. We are interested in understanding optimal policy, and the reason that real wages become more volatile as  $\psi$  rises — namely, because the Ramsey planner keeps nominal price inflation volatility high even while muting nominal wage inflation volatility — is clear. We think it is useful to know that inducing more volatile real wages in the face of rigid nominal wages is in fact the optimal thing to do if one believes this class of models is useful for studying policy, regardless of what the data say. Indeed, in this sense, our results echo those of Erceg, Henderson, and Levin (2000).

Perhaps the most interesting aspect of this result is that highly volatile real wages seem to harm welfare so little in this model of labor markets. The reason for this is that the *actual* real wage simply divides the existing surplus between a matched worker and firm and has little to do with the *formation* of the match in the first place.<sup>17</sup> As Hall (2005) stressed, given that a match exists,

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<sup>14</sup>An alternative approach to this issue has been to retain the Nash bargaining assumption but employ key parameter values that are quite different from those conventionally used in this class of models. A prominent example of this type of approach is Hagedorn and Manovskii (2006).

<sup>15</sup>These relative volatilities are computed as a ratio of standard deviation percentages.

<sup>16</sup>For the U.S. economy between 1964 and 2005, Gertler and Trigari (2006, p. 20) report relative standard deviations of unemployment of over 5, vacancies of over 6, and tightness of over 11.

<sup>17</sup>Note that in the job-creation condition (19), it is the expected future real wage, not the contemporaneous real wage, that appears.

there is a continuum of real wages, bounded by the threat points (outside values) of the parties, that are acceptable to both the worker and the firm. In implementing optimal policy, the Ramsey planner exploits this feature of equilibrium in generating state-contingent inflation, which in turn leads to state-contingent movements in the realized real wage and thus state-contingent splits of the surplus. Efficiency and the efficient tax mix of course have nothing to say about how rents are divided amongst parties. This is somewhat of a restatement of our earlier point that movements in the labor tax rate are not terribly disruptive in our model because they primarily just lead to movements in effective bargaining power, which also lead only to movements in how rents are divided amongst parties.

Finally, because the labor search model is so well-suited to thinking about issues regarding unemployment, one may wonder whether a Phillips Curve arises in our model. Figure 7 shows a negative relationship between cyclical inflation rates and cyclical unemployment rates does arise if wages are flexible. However, this Phillips relation is not a feature of optimal policy with sticky nominal wages.

In summary, our results so far suggest that if the realized real wage did affect allocations more directly than they do in the model we have developed so far, then the optimal degree of price inflation volatility may fall as the cost of nominal wage adjustment rises. In Section 5 we pursue this idea by introducing an intensive margin of labor adjustment that potentially is affected by the realized real wage.

## 5 Intensive Margin

The way in which we introduce an hours margin follows closely that of Arseneau and Chugh (2006), who in turn build on Trigari (2006). The basic reason why we explore the consequences of allowing an intensive margin is that it potentially re-introduces to the model a neoclassical mechanism regarding a component — the hours choice — of labor supply. To the extent that the real wage *does* affect labor supply along the neoclassical intensive margin, we might expect to find that inflation volatility and/or tax-rate variability is welfare-diminishing, in contrast to the main findings in our basic model.

We consider three protocols by which hours are determined: simultaneous Nash bargaining between the firm and the worker over both hours and the wage payment; a right-to-manage (RTM) system in which the firm unilaterally sets its workers' hours, taking as given the bargained wage; and a right-to-work (RTW) convention in which each individual worker unilaterally chooses how many hours he works, taking as given the bargained wage. Of these three conventions, the latter two are the most likely candidates to make real wage fluctuations costly (precisely because in each case some party chooses hours *taking as given the contemporaneous real wage*) and thus dampen

inflation volatility. Our results show that inflation volatility is indeed dampened in these two cases, but still remains quite high.

## 5.1 Modifications to the Model

We briefly describe the main modifications to the basic model of Section 2 and relegate to Appendix B the implementation details behind each hours-determination arrangement. As we mention below and show more fully in Appendix B, the differences between the three models of hours-determination essentially lie in differences in effective bargaining weights of workers and firms.

### 5.1.1 Households

Suppose an individual family member who works  $h^i$  hours experiences disutility of effort  $e(h^i)$  that varies with hours. Thus, total household utility is

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(c_{1t}, c_{2t}) - \int_0^{n_t} e(h_t^i) di + \int_{n_t}^1 v^i di \right]. \quad (37)$$

In the household's budget constraint (3), we replace  $\bar{h}$  by  $h_t$ . Once again, we assume symmetry amongst all employed individuals and amongst all unemployed individuals, so household utility can be expressed as

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_{1t}, c_{2t}) - n_t e(h_t) + (1 - n_t)v]. \quad (38)$$

### 5.1.2 Firms

Each filled job now produces output  $y_{jt} = z_t f(h_{jt})$ ; with symmetry across all jobs, total output of the firm is  $y_t = n_t z_t f(h_t)$ .

### 5.1.3 Bargaining Over Hours

The first hours-determination scheme we consider is that the firm and worker (Nash) bargain simultaneously over the wage payment and hours. This setup is quite common in labor search models with both extensive and intensive margins. It is straightforward to show that the solution for the wage is still given by (22), with appropriate replacement of  $\bar{h}$  by  $h_t$ . The bargaining solution for hours takes a similar form as (22) except the (time-varying) bargaining weights are different, reflecting how changes in hours worked affect the marginal values of working, not working, and having a filled job. Further details are provided in Appendix B.

#### 5.1.4 Right To Manage

The second hours-determination scheme we consider is one in which firms unilaterally choose their workers' hours. Following Trigari (2006), suppose the firm is able to unilaterally set hours after bargaining over the wage. As in Trigari (2006), we call this system right-to-manage (RTM), which emphasizes the idea that firms retain the power to decide their employees' activities, including their hours. The firm chooses  $h_t$  to maximize the value of a filled job. As we show in Appendix B, this optimization yields  $w_t = z_t f'(h_t)$ , a standard condition from a neoclassical labor market. We can invert this function to express hours as the function  $h\left(\frac{W_t}{P_t}\right)$ , which shows that hours worked depend on the realized real wage. Both the worker and the firm take this function as given when bargaining over the nominal wage. The wage payment is then given by an expression again of the form (22) except the bargaining weights again differ (and differ also from the Nash hours-bargaining case).

#### 5.1.5 Right To Work

Finally, a plausible alternative to firms unilaterally choosing hours after wages have been negotiated is that workers unilaterally choose hours. We dub this protocol right-to-work (RTW).<sup>18</sup> Specifically, suppose a worker chooses  $h_t$  to maximize his individual value from working. As shown in Appendix B, this yields  $\frac{e'(h_t)}{u_{2t}} = (1 - \tau_t^n)w_t$ , also a standard condition from a neoclassical labor market. Inverting this function gives  $\tilde{h}\left(\frac{W_t}{P_t}\right)$ , which shows that hours worked depend on the realized real wage, as in the RTM case. The function  $\tilde{h}(\cdot)$  is distinct from the function  $h(\cdot)$  in the RTM protocol. As in the RTM protocol, though, both the worker and the firm take as given this function when bargaining over the wage. The wage payment is then once again given by an expression of the form (22) except the bargaining weights again differ (and differ also from the Nash hours-bargaining case and the RTM case).

#### 5.1.6 Equilibrium

The equilibrium variables and conditions are the same as in Section 2.6 (with appropriate replacement of  $\bar{h}$  by  $h_t$ ), with the addition of  $h_t$  as an endogenous stochastic process and, depending on which model we are studying, one of the three conditions presented in Appendix B that pin down hours worked.

### 5.2 Ramsey Problem

We extend the formulation of the Ramsey problem in the obvious way, adding  $\{h_t\}$  to the Ramsey choice variables described in Section 2.6 and adding as a constraint, depending on the hours-

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<sup>18</sup>Another appropriate term may be right-to-shirk.

determination scheme under consideration, the appropriate expression that pins down the intensive margin, as well as appropriately replacing  $\bar{h}$  by  $h_t$  in the constraints that are unchanged from Section 2.6. The same issue regarding the zero-lower-bound on the nominal interest rate arises here as arose in the model without the hours margin; our treatment of this issue is as above, namely, when studying dynamics, we drop the ZLB constraint and check whether or not it is satisfied.

## 5.3 Optimal Policy

### 5.3.1 Parameterization

The functional forms we use are the same as in the basic model, except now we must also specify a disutility function for hours. We choose a fairly standard specification,

$$e(h) = \frac{Ah^{1+\nu}}{1+\nu}, \quad (39)$$

and we set  $\nu = 5$ , which implies hours elasticity of  $1/5$ , in line with micro evidence about labor supply elasticity. We then calibrate  $A$  so that steady-state hours worked is  $h = 0.35$  in the Nash bargaining model and hold  $A$  at that value when we move to the RTM and RTW models. We continue using the diminishing-returns match-level production function  $f(h) = h^\alpha$ .

In this model, we consider only the case in which employed individuals have higher total utility than unemployed individuals. This requires us to calibrate the utility parameter  $v$ , rather than normalizing it to zero as we did in the basic model. Using the same calibration strategy as earlier (namely, requiring unemployed individuals to have 2.5 times as much consumption as employed individuals in order to be just as well off), our calibrated value of  $v$  is  $v = -1.1$ .

### 5.3.2 Ramsey Steady State

We again must numerically compute the Ramsey steady-state. Figures 8, 9, and 10 show the key Ramsey policy and allocation variables under the bargaining, RTM, and RTW protocols, respectively. The main result that emerges from all three models is that introducing endogenous hours reduces the degree to which costly wage adjustment matters for the job creation decision. This is evident by looking at the top left panel of all three figures. Regardless of which bargaining protocol is used, for low levels of  $\psi$  the ability to indirectly stifle job creation by inflating away from the Friedman rule is diminished to the point at which the costs of reintroducing the monetary distortion ensure the optimality of the Friedman rule. For  $\psi$  high enough, reintroducing the monetary distortion is tolerable in order to promote job creation, but clearly the tradeoff is less favorable; optimal inflation rises much more slowly with the costs of wage adjustment in comparison to the model with fixed hours. Again, this is true regardless of which bargaining protocol is used. The

main story of how the introduction of hours influences our top line results, however, comes through its effect on the dynamically optimal policy, which we turn to next.

### 5.3.3 Ramsey Dynamics

Table 3 presents simulation-based moments for the key policy and allocation variables in the bargaining model, Table 4 presents results for the RTM model, and Table 5 presents results for the RTW model. We highlight just the main ideas that emerge here.

There is a clear difference in how inflation volatility depends on nominal wage rigidity across the three different specifications. With bargaining over hours (Table 3), price inflation volatility is around 10 percent no matter how costly is nominal wage adjustment. Inflation is thus even more volatile than in the model without the intensive margin (Table 2), but the more important message is that volatility is relatively invariant to costly wage adjustment. In line with what we found in the basic model, nominal wage inflation volatility *does* decline, not surprisingly, as  $\psi$  rises. As a consequence, realized real wages become more volatile; its volatility rises by about 50 percent moving from flexible nominal wages to four quarters of nominal wage rigidity. Thus, with bargaining over hours, real wage volatility is tolerable from the Ramsey planner's perspective, for the same reasons as in the basic model.

Under the RTM protocol, Table 4 shows that price inflation volatility falls four-fold moving from costless wage adjustment to four-quarter nominal wage rigidity. In line with this noticeable decline in inflation variability and the fall in nominal wage inflation variability, the volatility of the real wage is essentially invariant to how costly is nominal wage adjustment. Thus, real wage volatility *is* welfare-diminishing in the RTM specification because part of the allocation — the firm's choice of hours, characterized by  $w_t = z_t f'(h_t)$  — depends directly on the *realized* (not the expected future) real wage. The reason that volatility in hours worked is undesirable — and notice that hours actually become a bit more stable as  $\psi$  rises in Table 4 — is that it leads to variation in the equilibrium marginal rate of substitution between an individual's consumption and hours worked. Thus, the RTM mechanism leads to reduced inflation volatility in the presence of costly nominal wage adjustment, similar to the neoclassical channel operating in Chugh (2006a).

Our conjecture was that the RTW mechanism in principle should operate in a similar neoclassical fashion, but Table 5 shows results that are not quite as clear quantitatively. Inflation volatility dips a bit moving from flexible nominal wages to two quarters of wage rigidity, but then rises quite a bit moving to three quarters of wage rigidity. With four quarters of wage rigidity, inflation volatility became unreasonably large, over 70 percent per year, so we do not even report results for the four-quarter case. The basic reason why the RTW mechanism does not seem to dampen inflation volatility is that in our calibration, hours supply is quite inelastic. As we stated when

we discussed our calibration, our parameterization  $\nu = 5$  is in line with micro estimates of low labor supply elasticity. With low labor supply elasticity, variations in real wages do not affect the consumption-hours margin much. Hence, inflation volatility and real wage volatility (and notice the real wage becomes very volatile with even just three quarters of nominal wage rigidity) do not affect allocations very much and are thus tolerable.

Another notable result in these three models is that tax rate variability is again an order of magnitude larger than in basic Ramsey models based on Walrasian markets. Indeed, the standard deviation of the labor tax rate, generally ranging between four and eight percent, is even larger than we found in our basic model. Finally, we also point out that in a few instances the zero-lower-bound is violated, as it was in the flexible-wage version of our basic model. As was the case there, we can shift the model away from the zero lower bound by increasing workers' bargaining power; doing so, we found that the qualitative results were unchanged, so we once again have reason to think that the ZLB issue is not one that blurs the basic ideas our models articulate.

## 6 Conclusion

The goal of our work here was to explore the implications of nominally-rigid wages, articulated in a model with an explicit notion of jobs, on optimal policy. The results turn out to be quite different than in models with nominal rigidities in wages modeled in otherwise-Walrasian labor markets. In our model, realized real wages are not critical for efficiency as they are in a labor market with neoclassical underpinnings. Thus, although unanticipated fluctuations in inflation cause unanticipated fluctuations in real wages, job formation and production are largely unaffected. Our results give quantitative voice to the Goodfriend and King (2001) conjecture that sticky nominal wages ought not to have much consequence for optimal monetary policy because firms and workers engaged in ongoing relationships have the proper incentives to neutralize any allocative effects.

To connect our results to the main thread of the monetary policy literature, a feature that many may think is a natural one to investigate in our model is nominally rigid goods prices. It seems clear to us, based on existing results in the Ramsey literature, that introducing sticky prices would render optimal inflation stable. Thus, we did not pursue this idea in this paper. Others — such as Blanchard and Gali (2006b), Walsh (2005), Trigari (2006), Christoffel and Linzert (2005), and Krause and Lubik (2005), to name just a few — have begun exploring the consequences of sticky prices in labor search and matching environments. We view our work as complementary to these efforts.

Primarily for tractability, we used a Rotemberg-type specification for wage-stickiness, in which all worker-firm pairs are able to reset nominal wages every period, albeit at a cost. A natural alternative to explore would be a Calvo specification, in which nominal wages could only be reset



in a fraction of jobs. In a Calvo world, an interesting question is whether or not potential wage and thus employment dispersion amongst different households has an important quantitative impact on optimal policy. Related work by Schmitt-Grohe and Uribe (2006) suggests the answer to this question (at least to a first-order approximation) is “no” in more standard labor markets. It is not clear how these findings would extend to a model with labor search and matching frictions.

As we mentioned at the outset, this paper is also part of a larger project studying the policy implications of deep-rooted, non-Walrasian frictions in money markets and labor markets. A central focus of this larger project has been to think about what sorts of departures from typical Walrasian frameworks make consumer price inflation stability an important goal of policy, but along the way we have uncovered other aspects of policy not evident in standard models. In this paper, we characterized optimal policy when labor markets are non-Walrasian but money markets are standard. Aruoba and Chugh (2006) characterized optimal policy when money markets are non-Walrasian but labor markets are standard. We now turn to studying optimal policy when labor markets and money markets both feature fundamental frictions.

## A Nash Bargaining Over Wages

Here we derive the Nash-bargaining solution between an individual worker and the firm in the model without an intensive margin. For notational simplicity, we omit the conditional expectations operator  $E_t$  where it is understood. Individuals' and firms' asset values are defined in nominal terms. The marginal (nominal) value to the household of an individual who works is

$$\mathbf{W}_t = (1 - \tau_t^n)W_t \bar{h} - \frac{P_t A \bar{h}}{u_{2t}} + \beta E_t \left[ \left( \frac{u_{2t+1}}{u_{2t}} \right) \left( \frac{P_t}{P_{t+1}} \right) ((1 - \rho^x) \mathbf{W}_{t+1} + \rho^x \mathbf{U}_{t+1}) \right]. \quad (40)$$

The marginal (nominal) value to the household of an individual who is unemployed and searching is

$$\mathbf{U}_t = \frac{P_t v}{u_{2t}} + \beta E_t \left[ \left( \frac{u_{2t+1}}{u_{2t}} \right) \left( \frac{P_t}{P_{t+1}} \right) (\theta_t k^f(\theta_t)(1 - \rho^x) \mathbf{W}_{t+1} + (1 - \theta_t k^f(\theta_t)(1 - \rho^x)) \mathbf{U}_{t+1}) \right]. \quad (41)$$

Note that because these asset values are defined as nominal, the nominal discount factor, which involves  $P_t/P_{t+1}$ , appears. The value to an intermediate goods producer of a filled job is

$$\mathbf{J}_t = P_t z_t f(\bar{h}) - W_t h_t - \frac{\psi}{v} \left( \frac{W_t}{W_{t-1} \pi_t^\chi} - 1 \right)^v P_t + \beta E_t \left[ \left( \frac{u_{2t+1}}{u_{2t}} \right) \left( \frac{P_t}{P_{t+1}} \right) (1 - \rho^x) \mathbf{J}_{t+1} \right], \quad (42)$$

where  $\frac{\psi}{v} \left( \frac{W_t}{W_{t-1}} - 1 \right)^v$  is a Rotemberg-type resource cost of nominal wage adjustment. This is the way in which we model nominal rigidity in the wage bargaining process. The typical Rotemberg quadratic specification sets  $v = 2$ .

Bargaining occurs every period over  $W_t$ . The firm and worker maximize the Nash product

$$(\mathbf{W}_t - \mathbf{U}_t)^\eta \mathbf{J}_t^{1-\eta}, \quad (43)$$

where  $\eta \in (0, 1)$  is the fixed weight given to the worker's individual surplus. The first-order condition of the Nash product with respect to  $W_t$  is

$$\eta (\mathbf{W}_t - \mathbf{U}_t)^{\eta-1} \left( \frac{\partial \mathbf{W}_t}{\partial W_t} - \frac{\partial \mathbf{U}_t}{\partial W_t} \right) \mathbf{J}_t^{1-\eta} + (1 - \eta) (\mathbf{W}_t - \mathbf{U}_t)^\eta \mathbf{J}_t^{-\eta} \frac{\partial \mathbf{J}_t}{\partial W_t} = 0. \quad (44)$$

We have  $\frac{\partial \mathbf{W}_t}{\partial W_t} = (1 - \tau_t^n) \bar{h}$ ,  $\frac{\partial \mathbf{U}_t}{\partial W_t} = 0$ , and

$$\frac{\partial \mathbf{J}_t}{\partial W_t} = -\bar{h} - \psi \left( \frac{\pi_t^w}{\pi_t^\chi} - 1 \right)^{v-1} \frac{\pi_t^{1-\chi}}{w_{t-1}} + \psi (1 - \rho^x) \beta E_t \left[ \frac{u_{2t+1}}{u_{2t}} \frac{P_t}{P_{t+1}} \left( \frac{\pi_{t+1}^w}{\pi_{t+1}^\chi} - 1 \right)^{v-1} \pi_{t+1}^w \frac{\pi_{t+1}^{1-\chi}}{w_t} \right]. \quad (45)$$

In computing the latter, we defined  $\pi_t^w \equiv W_t/W_{t-1}$  as the gross rate of nominal wage inflation between  $t$  and  $t+1$  and of course had to take into account that  $W_t$  affects  $\mathbf{J}_{t+1}$  through the adjustment cost function. A wage Phillips Curve is essentially subsumed inside  $\partial \mathbf{J}_t / \partial W_t$ .

Define

$$\Delta_t^W \equiv - \left( \frac{\partial \mathbf{W}_t}{\partial W_t} - \frac{\partial \mathbf{U}_t}{\partial W_t} \right), \quad (46)$$

Variable	No heterogeneity					Heterogeneity				
	Flex Wage	2-qtr rigid	3-qtr rigid	4-qtr rigid		Flex Wage	2-qtr rigid	3-qtr rigid	4-qtr rigid	
$g$	0	0.07	0.07	0.07	0.07	0	0.07	0.07	0.07	0.07
$R - 1$	0	0	3.8718	4.0169	4.0463	0.0002	0	4.0649	4.6049	4.7203
$\pi - 1$	-3.9404	-3.9404	-0.2212	-0.0818	-0.0535	-3.9402	-3.9404	-0.0357	0.4831	0.5939
$\tau^n$	0.0009	0.2293	0.2287	0.2287	0.2287	0.0009	0.2350	0.2343	0.2343	0.2343
$n$	0.8305	0.8305	0.8305	0.8304	0.8304	0.8743	0.8748	0.8748	0.8749	0.8751
$\theta$	0.7287	0.7287	0.7285	0.7284	0.7284	1.3074	1.3167	1.3166	1.3195	1.3224
$w$	1.0744	1.0744	1.0745	1.0745	1.0745	0.9965	0.9955	0.9955	0.9951	0.9948
$k^f(\theta)$	0.7440	0.7470	0.7471	0.7471	0.7471	0.5912	0.5896	0.5896	0.5891	0.5886
$\theta k^f(\theta)$	0.5443	0.5443	0.5442	0.5442	0.5442	0.7730	0.7763	0.7763	0.7773	0.7783
$c_1$	0.0284	0.0222	0.0213	0.0212	0.0212	0.0279	0.0217	0.0208	0.0207	0.0207
$c_2$	0.2918	0.2280	0.2289	0.2290	0.2290	0.2875	0.2236	0.2245	0.2246	0.2246
$gdp$	0.3983	0.3983	0.3983	0.3983	0.3983	0.4193	0.4195	0.4195	0.4196	0.4197
$\gamma v/gdp$	0.1960	0.1960	0.1960	0.1960	0.1960	0.2477	0.2484	0.2484	0.2486	0.2488
$v$	0.1235	0.1235	0.1235	0.1235	0.1235	0.1643	0.1649	0.1649	0.1650	0.1652
adj. cost	0	0	0.0000	0.0000	0.0000	0	0	0.0000	0.0000	0.0000
profit	0.0079	0.0079	0.0079	0.0079	0.0079	0.0105	0.0105	0.0105	0.0105	0.0105
$\omega$	0.3998	0.3394	0.3397	0.3397	0.3397	0.3998	0.3378	0.3380	0.3375	0.3370

	With vacancy tax				
$g$	0	0.07	0.07	0.07	0.07
$R - 1$	0	0	3.8898	4.0286	4.0570
$\pi - 1$	-3.9404	-3.9404	-0.2039	-0.0706	-0.0432
$\tau^n$	0.0008	0.2119	0.2113	0.2113	0.2113
$n$	0.8743	0.8674	0.8674	0.8674	0.8674
$\theta$	1.3069	1.1791	1.1792	1.1792	1.1792
$w$	0.9965	0.9807	0.9808	0.9808	0.9808
$k^f(\theta)$	0.5913	0.6162	0.6162	0.6162	0.6162
$\theta k^f(\theta)$	0.7728	0.7265	0.7266	0.7266	0.7266
$c_1$	0.0279	0.0219	0.0210	0.0210	0.0210
$c_2$	0.2875	0.2252	0.2261	0.2261	0.2261
$gdp$	0.4193	0.4160	0.4160	0.4160	0.4160
$\gamma v/gdp$	0.2476	0.2377	0.2377	0.2377	0.2377
$v$	0.1643	0.1564	0.1564	0.1564	0.1564
adj. cost	0	0	0.0000	0.0000	0.0000
profit	0.0105	0.0108	0.0108	0.0108	0.0108
$\omega$	0.3998	0.3444	0.3447	0.3447	0.3447
$\tau^s$	0.0003	0.0862	0.0859	0.0859	0.0859

Table 1: Steady-state Ramsey allocations and policies.

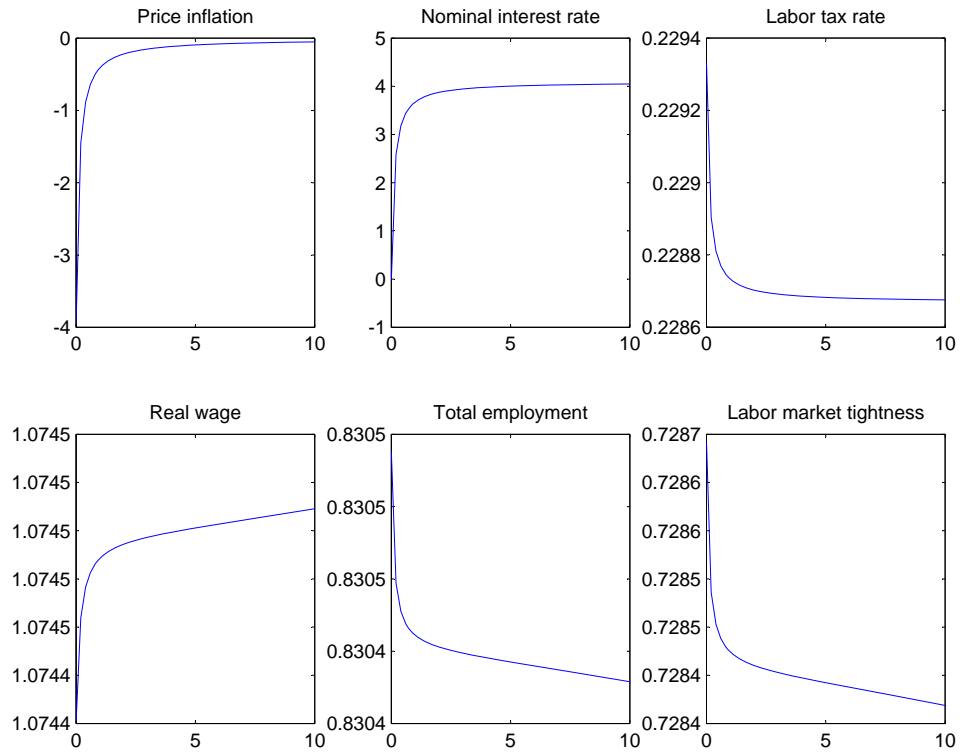


Figure 1: In model with no welfare heterogeneity, key steady-state Ramsey allocation and policy variables as a function of nominal wage adjustment cost parameter  $\psi$ .  $\pi$  and  $R$  expressed in annualized percentage points.

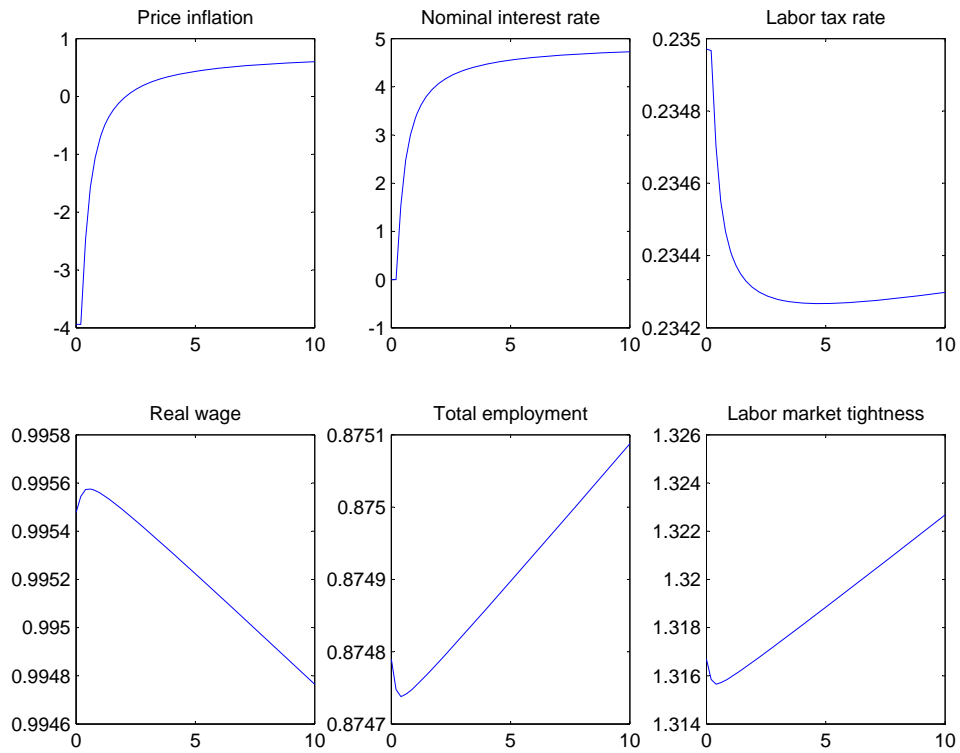


Figure 2: In model with welfare heterogeneity, key steady-state Ramsey allocation and policy variables as a function of nominal wage adjustment cost parameter  $\psi$ .  $\pi$  and  $R$  expressed in annualized percentage points.

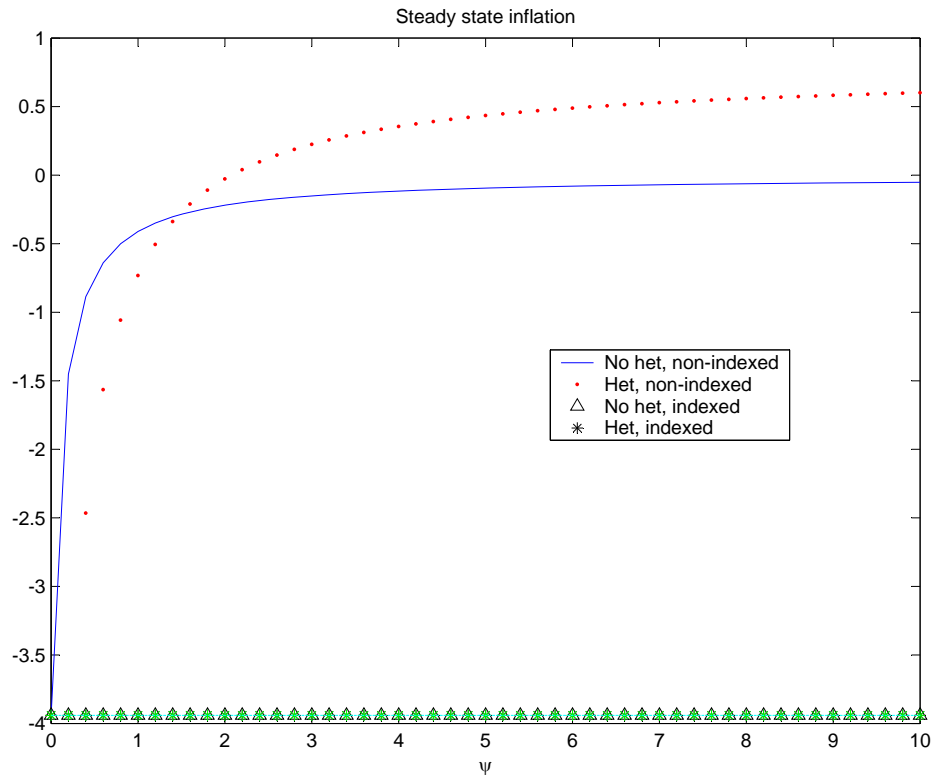


Figure 3: Steady-state inflation rate as a function of nominal wage adjustment cost parameter  $\psi$  in four different models: no heterogeneity without wage indexation, heterogeneity without wage indexation, no heterogeneity with wage indexation, heterogeneity with wage indexation.

Variable	Mean	Std. Dev.	Auto corr.	Corr( $x, gdp$ )	Corr( $x, z$ )	Corr( $x, g$ )
<u>Flexible wages</u>						
$\tau^n$	0.2346	0.0120	0.9793	0.1490	0.1172	0.0677
$\pi - 1$	-3.9173	4.5430	0.6556	0.1590	0.1155	-0.0431
$\pi^w - 1$	-3.9084	4.6534	0.8749	0.8521	0.8353	-0.0149
* $R - 1$	0.0189	0.1182	0.9932	0.3543	0.3324	-0.1652
$gdp$	0.4194	0.0078	0.9135	1.0000	0.9969	-0.0421
$w$	0.9954	0.0096	0.9473	0.9722	0.9727	0.0200
$n$	0.8747	0.0019	0.9560	0.7916	0.7421	-0.2650
$v$	0.1648	0.0008	0.4668	0.1420	0.1890	-0.0483
$\theta$	1.3161	0.0196	0.9430	0.8744	0.8412	-0.2921
$b$	0.1399	0.0446	0.9966	-0.1427	-0.1099	0.0412
$\omega$	0.3379	0.0049	0.9793	-0.1490	-0.1172	-0.0677
<u>Two quarters of nominal wage stickiness</u>						
$\tau^n$	0.2341	0.0225	0.9588	0.4916	0.3983	-0.0007
$\pi - 1$	0.0002	4.3283	0.1789	-0.5752	-0.6307	0.0168
$\pi^w - 1$	-0.0787	0.5914	0.9023	-0.1199	-0.1605	0.1259
$R - 1$	4.1206	0.2291	0.9604	-0.7690	-0.7474	0.0813
$gdp$	0.4194	0.0082	0.9201	1.0000	0.9930	-0.0423
$w$	0.9954	0.0133	0.6583	0.8578	0.9032	-0.0026
$n$	0.8747	0.0030	0.9774	0.7933	0.7177	-0.1844
$v$	0.1647	0.0009	0.5753	0.2585	0.3448	-0.0844
$\theta$	1.3161	0.0303	0.9713	0.8942	0.8387	-0.2117
$b$	0.1620	0.2929	0.9992	0.1815	0.2089	-0.0339
$\omega$	0.3383	0.0105	0.9426	-0.3946	-0.3018	-0.0304
<u>Three quarters of nominal wage stickiness</u>						
$\tau^n$	0.2337	0.0221	0.9840	0.4649	0.3737	-0.0051
$\pi - 1$	0.5735	5.5750	0.0166	-0.4523	-0.5104	0.0185
$\pi^w - 1$	0.4544	0.5261	0.7514	-0.1762	-0.2374	0.1857
$R - 1$	4.6827	0.2367	0.9538	-0.7464	-0.7180	0.0830
$gdp$	0.4195	0.0082	0.9193	1.0000	0.9926	-0.0427
$w$	0.9951	0.0155	0.5203	0.7422	0.8005	-0.0061
$n$	0.8749	0.0029	0.9786	0.7756	0.6952	-0.1881
$v$	0.1648	0.0009	0.6321	0.2738	0.3615	-0.0900
$\theta$	1.3188	0.0300	0.9736	0.8798	0.8191	-0.2173
$b$	0.1687	0.3689	0.9994	0.1895	0.2221	-0.0218
$\omega$	0.3381	0.0135	0.9555	-0.2138	-0.1213	-0.0908
<u>Four quarters of nominal wage stickiness</u>						
$\tau^n$	0.2337	0.0308	0.4773	0.3202	0.2734	-0.0109
$\pi - 1$	1.1630	12.9058	-0.1151	-0.1881	-0.2417	0.0280
$\pi^w - 1$	0.5670	0.8526	0.4385	-0.1215	-0.2031	0.1746
$R - 1$	4.8418	0.3111	0.9158	-0.5682	-0.5171	0.0497
$gdp$	0.4195	0.0081	0.9182	1.0000	0.9906	-0.0405
$w$	0.9948	0.0308	0.2897	0.3613	0.4398	-0.0344
$n$	0.8750	0.0030	0.9810	0.7256	0.6278	-0.1688
$v$	0.1650	0.0009	0.7655	0.2844	0.3733	-0.0932
$\theta$	1.3216	0.0310	0.9780	0.8269	0.7471	-0.1969
$b$	0.1696	0.3736	0.9977	0.1566	0.1926	-0.0072
$\omega$	0.3379	0.0206	0.9197	-0.0642	0.0407	-0.1497

Table 2: Simulation-based moments in model with heterogeneity between workers and non-workers. Nash bargaining weight is  $\eta = 0.4$ .  $\pi$ ,  $\pi^w$ , and  $R$  reported in annualized percentage points. Asterisk denotes zero-lower-bound is violated during simulations.

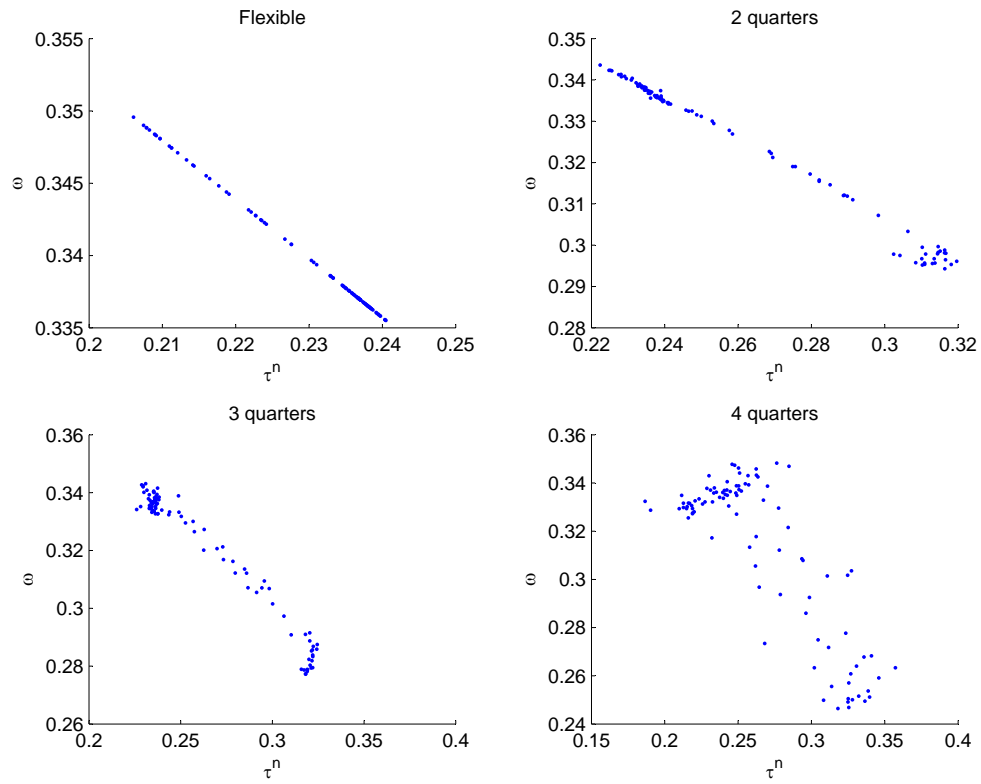


Figure 4: Dynamic relationship between worker's effective bargaining power ( $\omega$ ) and labor tax rate under the Ramsey policy for various degrees of nominal wage rigidity.



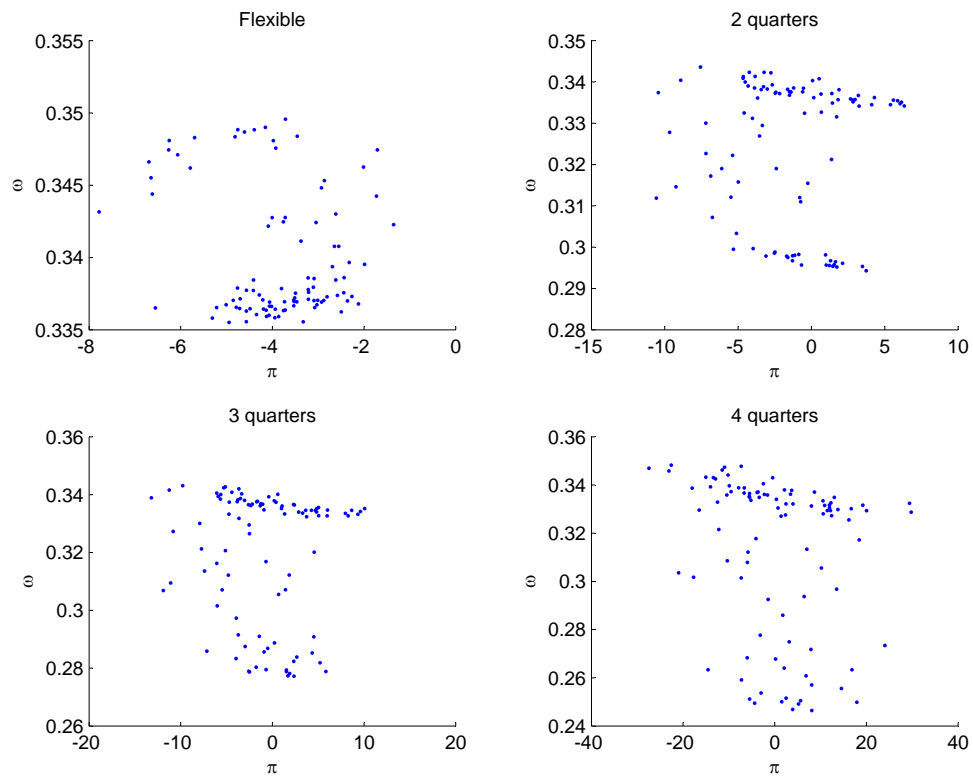


Figure 5: Dynamic relationship between worker's effective bargaining power ( $\omega$ ) and ex-post inflation rate under the Ramsey policy for various degrees of nominal wage rigidity.

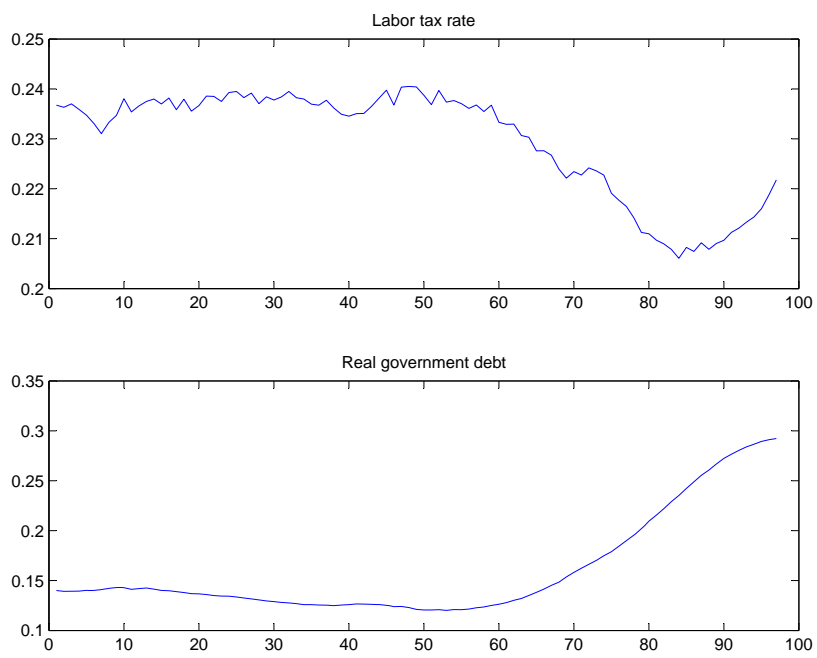


Figure 6: Simulation of the labor tax rate and real government debt obligations in flexible-wage model.

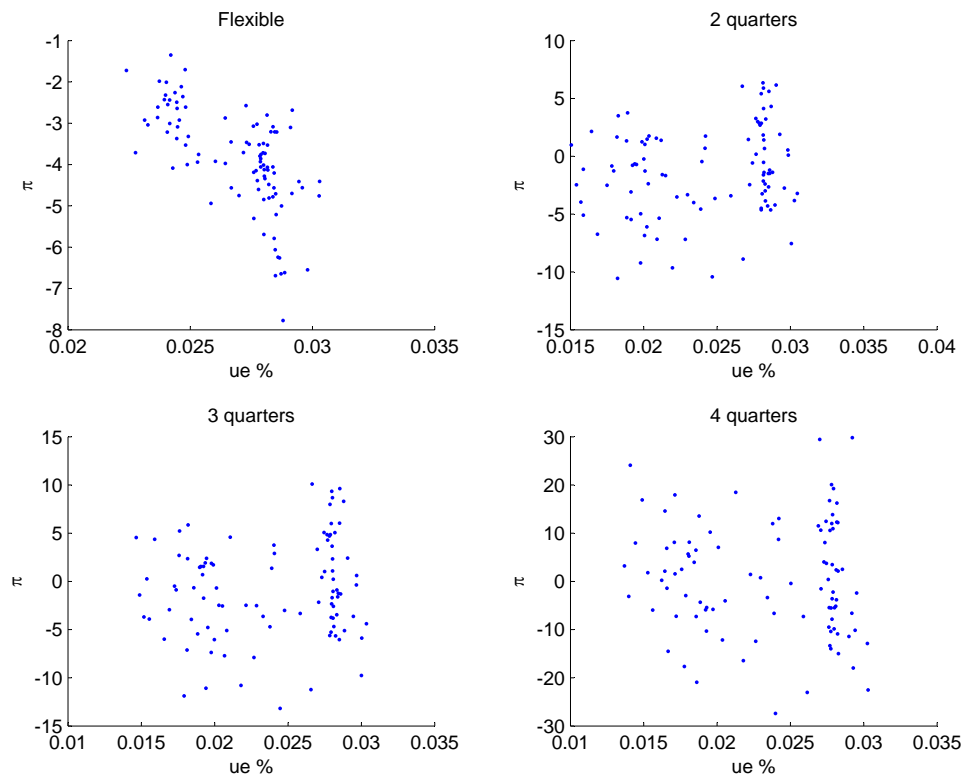


Figure 7: Realizations of inflation and unemployment rate under the Ramsey policy for various degrees of nominal wage rigidity.

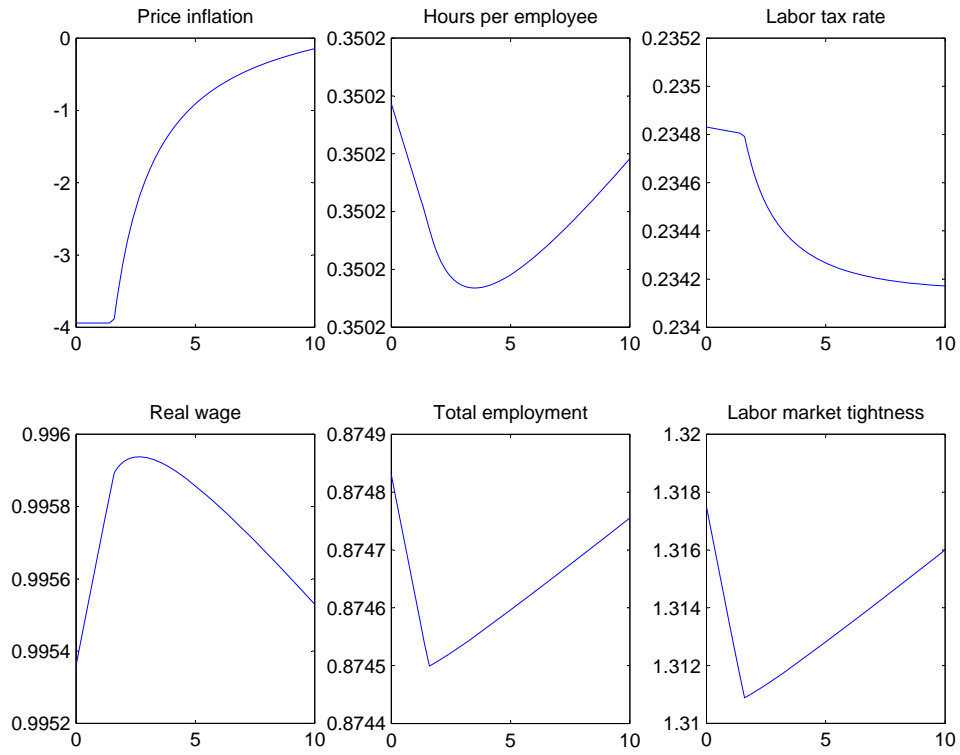


Figure 8: In model with bargaining over hours, key steady-state Ramsey allocation and policy variables as a function of nominal wage adjustment cost parameter  $\psi$ .  $\pi$  and  $R$  expressed in annualized percentage points.

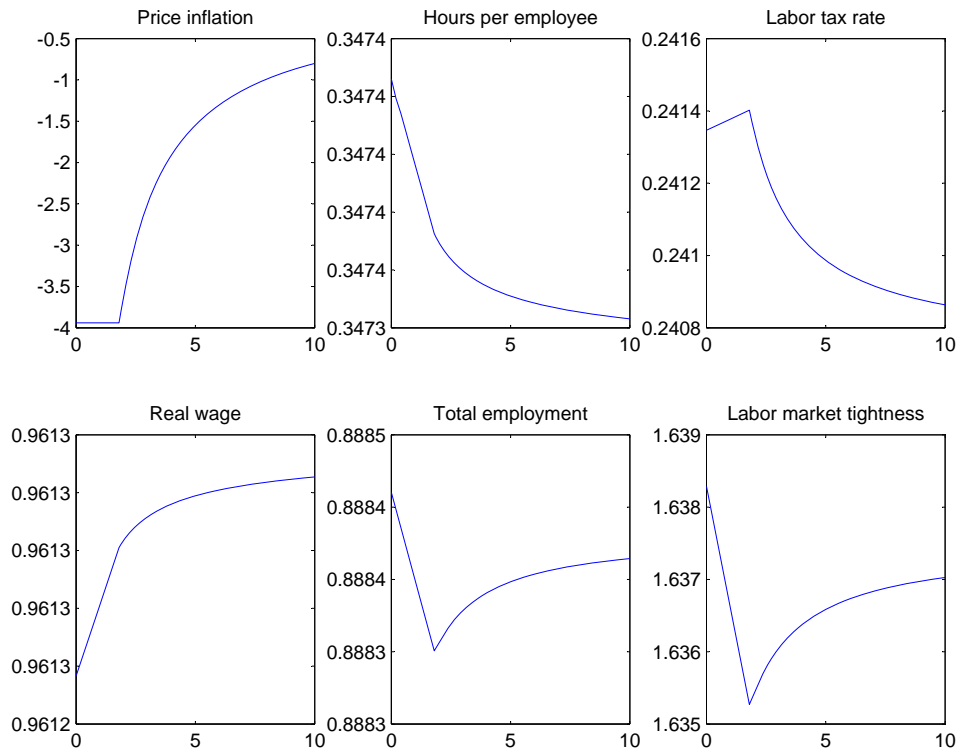


Figure 9: In RTM model, key steady-state Ramsey allocation and policy variables as a function of nominal wage adjustment cost parameter  $\psi$ .  $\pi$  and  $R$  expressed in annualized percentage points.

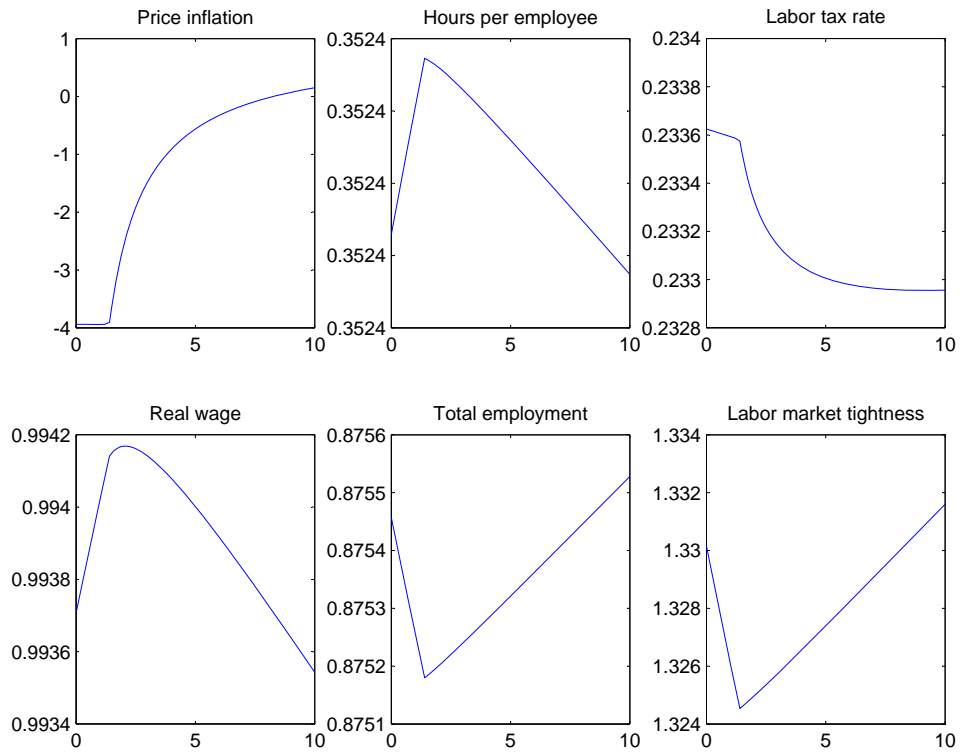


Figure 10: In RTW model, key steady-state Ramsey allocation and policy variables as a function of nominal wage adjustment cost parameter  $\psi$ .  $\pi$  and  $R$  expressed in annualized percentage points.

Variable	Mean	Std. Dev.	Auto corr.	Corr( $x, gdp$ )	Corr( $x, z$ )	Corr( $x, g$ )
<u>Flexible wages</u>						
$\tau^n$	0.2359	0.0842	0.9909	-0.6654	-0.0718	0.2613
$\pi - 1$	-3.1063	13.8521	0.9802	-0.0041	-0.0043	-0.1088
$\pi^w - 1$	-3.3516	11.3765	0.9806	0.2866	0.2139	-0.1353
* $R - 1$	0.3726	0.1945	0.9879	-0.0073	0.0123	-0.1295
$gdp$	0.4196	0.0108	0.9484	1.0000	0.7299	-0.0970
$w$	0.9951	0.0208	0.9729	0.6661	0.5077	0.0233
$h$	0.3502	0.0056	0.9886	0.6331	0.0218	-0.1539
$n$	0.8749	0.0051	0.9828	-0.3742	0.2369	0.1470
$v$	0.1646	0.0015	0.7707	0.2606	0.1245	0.0445
$\theta$	1.3178	0.0526	0.9802	-0.3210	0.2743	0.1595
$b$	0.1378	0.2007	0.9906	0.5226	0.0819	-0.1113
$\omega$	0.3372	0.0493	0.9909	0.6654	0.0718	-0.2613
<u>Two quarters of nominal wage stickiness</u>						
$\tau^n$	0.2362	0.0548	0.9968	-0.3653	0.1634	0.2396
$\pi - 1$	-2.7976	6.2892	0.2819	-0.5021	-0.4350	0.0946
$\pi^w - 1$	-3.0089	2.6568	0.9583	-0.0397	0.0883	-0.1960
* $R - 1$	1.2392	0.5276	0.9201	-0.1949	-0.1605	-0.4909
$gdp$	0.4194	0.0080	0.9149	1.0000	0.7957	-0.1071
$w$	0.9956	0.0176	0.6627	0.8254	0.7973	-0.2941
$h$	0.3501	0.0037	0.9933	0.3052	-0.2309	-0.0773
$n$	0.8746	0.0037	0.9805	0.0654	0.5593	0.0210
$v$	0.1643	0.0010	0.5798	0.2783	0.2645	0.0459
$\theta$	1.3116	0.0373	0.9756	0.1401	0.6419	0.0294
$b$	0.1362	0.2228	0.9974	0.1566	-0.0995	0.1557
$\omega$	0.3380	0.0320	0.9774	0.3829	-0.1368	-0.2618
<u>Three quarters of nominal wage stickiness</u>						
$\tau^n$	0.2359	0.0536	0.9972	-0.3968	0.1245	0.2748
$\pi - 1$	-0.2907	8.6085	0.0746	-0.4541	-0.4373	0.2061
$\pi^w - 1$	-0.6727	1.7862	0.9387	-0.1458	-0.0654	-0.2067
$R - 1$	3.6750	0.5879	0.8707	-0.0899	-0.1396	-0.6049
$gdp$	0.4194	0.0082	0.9255	1.0000	0.7999	-0.1290
$w$	0.9954	0.0245	0.5586	0.7111	0.6830	-0.3922
$h$	0.3501	0.0036	0.9922	0.3315	-0.1972	-0.1029
$n$	0.8747	0.0033	0.9806	0.0999	0.5655	0.0000
$v$	0.1645	0.0009	0.6757	0.2939	0.2799	0.0474
$\theta$	1.3139	0.0338	0.9771	0.1701	0.6519	0.0084
$b$	0.1430	0.3022	0.9980	0.1308	0.0253	0.2324
$\omega$	0.3378	0.0321	0.9440	0.4409	-0.0496	-0.3183
<u>Four quarters of nominal wage stickiness</u>						
$\tau^n$	0.2358	0.0571	0.9970	-0.4675	0.0811	0.3028
$\pi - 1$	0.3880	10.6319	0.0252	-0.4009	-0.3980	0.2239
$\pi^w - 1$	-0.1625	1.7287	0.9441	-0.2274	-0.1193	-0.1847
$R - 1$	4.2816	0.6724	0.8514	-0.0249	-0.0874	-0.6216
$gdp$	0.4195	0.0086	0.9352	1.0000	0.7783	-0.1560
$w$	0.9951	0.0296	0.5097	0.6389	0.6244	-0.4067
$h$	0.3501	0.0038	0.9907	0.4031	-0.1535	-0.1403
$n$	0.8748	0.0032	0.9804	0.0662	0.5425	-0.0098
$v$	0.1646	0.0009	0.7399	0.2544	0.2740	0.0472
$\theta$	1.3162	0.0324	0.9778	0.1265	0.6268	-0.0021
$b$	0.1449	0.3579	0.9982	0.1261	0.0543	0.2490
$\omega$	0.3374	0.0351	0.9249	0.5156	0.0199	-0.3363

Table 3: Simulation-based moments in the model with Nash bargaining over hours and heterogeneity between workers and non-workers; driving processes are  $g_t$  and  $z_t$ .  $\pi$ ,  $\pi^w$ , and  $R$  reported in annualized percentage points. Asterisk denotes zero-lower-bound is violated during simulations.

Variable	Mean	Std. Dev.	Auto corr.	Corr( $x, gdp$ )	Corr( $x, z$ )	Corr( $x, g$ )
<u>Flexible wages</u>						
$\tau^n$	0.2422	0.0678	0.9899	-0.6929	-0.0425	0.2944
$\pi - 1$	-3.5022	9.6131	0.9771	0.0437	-0.0073	-0.1298
$\pi^w - 1$	-3.5096	9.6361	0.9748	0.1576	0.2063	-0.1229
* $R - 1$	1.4390	1.1351	0.9883	0.0502	0.0145	-0.1633
$gdp$	0.4237	0.0117	0.9546	1.0000	0.6903	-0.1350
$w$	0.9612	0.0089	0.9061	0.3933	0.9202	0.0458
$h$	0.3474	0.0039	0.9846	0.6263	-0.0457	-0.1625
$n$	0.8884	0.0043	0.9790	0.9341	0.5305	-0.2421
$v$	0.1824	0.0015	0.6131	-0.0249	0.1318	-0.0654
$\theta$	1.6373	0.0598	0.9769	0.9788	0.5941	-0.2693
$b$	0.1387	0.1465	0.9900	0.4291	0.0529	-0.0960
$\omega$	0.2866	0.0117	0.9823	0.3739	-0.2991	-0.3548
<u>Two quarters of nominal wage stickiness</u>						
$\tau^n$	0.2440	0.0620	0.9968	-0.5056	0.2801	-0.0381
$\pi - 1$	-3.6109	3.4865	0.8307	-0.4485	-0.5538	-0.1654
$\pi^w - 1$	-3.6053	2.8654	0.9975	-0.2081	0.0493	-0.2575
* $R - 1$	0.4503	0.3678	0.9787	-0.3443	-0.4322	-0.2699
$gdp$	0.4234	0.0091	0.9361	1.0000	0.5863	0.0814
$w$	0.9613	0.0099	0.9192	0.3465	0.9496	-0.0531
$h$	0.3472	0.0040	0.9918	0.4374	-0.3548	0.1741
$n$	0.8882	0.0029	0.9631	0.9009	0.5630	-0.0656
$v$	0.1824	0.0013	0.4311	0.0828	0.1210	-0.0613
$\theta$	1.6336	0.0399	0.9583	0.9729	0.6330	-0.0853
$b$	0.1376	0.2992	0.9980	0.3757	-0.0488	0.2126
$\omega$	0.2865	0.0147	0.9868	0.2106	-0.5627	-0.0281
<u>Three quarters of nominal wage stickiness</u>						
$\tau^n$	0.2412	0.0446	0.9960	-0.3394	0.2686	-0.1081
$\pi - 1$	-1.3764	2.7970	0.7703	-0.6099	-0.8176	-0.0179
$\pi^w - 1$	-1.4067	1.4120	0.9984	-0.2283	-0.1435	-0.1412
$R - 1$	2.7258	0.2818	0.9665	-0.4980	-0.7025	-0.1449
$gdp$	0.4237	0.0082	0.9234	1.0000	0.7510	0.1398
$w$	0.9612	0.0094	0.9127	0.5825	0.9680	-0.0742
$h$	0.3473	0.0030	0.9877	0.2551	-0.3571	0.2844
$n$	0.8884	0.0026	0.9553	0.8707	0.7342	-0.0509
$v$	0.1825	0.0013	0.4137	0.1120	0.0914	-0.0257
$\theta$	1.6362	0.0365	0.9496	0.9604	0.8112	-0.0634
$b$	0.1682	0.4185	0.9990	0.3403	0.1754	0.1242
$\omega$	0.2870	0.0115	0.9813	-0.0498	-0.6206	0.0189
<u>Four quarters of nominal wage stickiness</u>						
$\tau^n$	0.2396	0.0408	0.9956	-0.3500	0.2001	-0.1094
$\pi - 1$	-0.8514	2.5151	0.7361	-0.7239	-0.8777	0.0171
$\pi^w - 1$	-0.9008	1.0222	0.9985	-0.3316	-0.1598	-0.1137
$R - 1$	3.2653	0.2474	0.9595	-0.6179	-0.7644	-0.1241
$gdp$	0.4239	0.0084	0.9280	1.0000	0.7973	0.1336
$w$	0.9612	0.0091	0.9087	0.6420	0.9697	-0.0775
$h$	0.3474	0.0027	0.9867	0.2507	-0.3057	0.3035
$n$	0.8884	0.0027	0.9577	0.8758	0.7699	-0.0585
$v$	0.1825	0.0013	0.4148	0.0979	0.0844	-0.0173
$\theta$	1.6372	0.0377	0.9523	0.9614	0.8469	-0.0689
$b$	0.1831	0.5530	0.9993	0.4373	0.2154	0.0811
$\omega$	0.2874	0.0108	0.9805	-0.0558	-0.5705	0.0169

Table 4: Simulation-based moments in the right-to-manage model with heterogeneity between workers and non-workers; driving processes are  $g_t$  and  $z_t$ .  $\pi$ ,  $\pi^w$ , and  $R$  reported in annualized percentage points. Asterisk denotes zero-lower-bound is violated during simulations.



Variable	Mean	Std. Dev.	Auto corr.	Corr( $x, gdp$ )	Corr( $x, z$ )	Corr( $x, g$ )
<u>Flexible wages</u>						
$\tau^n$	0.2348	0.0742	0.9928	-0.6559	-0.0845	0.2429
$\pi - 1$	-3.4824	13.3103	0.9818	-0.1041	0.0216	-0.0917
$\pi^w - 1$	-3.4406	11.3883	0.9830	0.1537	0.2447	-0.1073
* $R - 1$	0.4124	0.2392	0.9872	-0.0919	0.0433	-0.1091
$gdp$	0.4217	0.0113	0.9501	1.0000	0.7174	-0.0581
$w$	0.9936	0.0170	0.9644	0.7727	0.5972	0.0242
$h$	0.3523	0.0056	0.9907	0.6559	0.0463	-0.1147
$n$	0.8755	0.0044	0.9828	-0.2951	0.2648	0.1192
$v$	0.1654	0.0012	0.7193	0.3475	0.1308	0.0385
$\theta$	1.3306	0.0465	0.9798	-0.2232	0.3064	0.1290
$b$	0.1388	0.2086	0.9927	0.5955	0.0712	-0.1001
$\omega$	0.3358	0.0425	0.9928	0.6488	0.0825	-0.2470
<u>Two quarters of nominal wage stickiness</u>						
$\tau^n$	0.2349	0.0441	0.9885	-0.2549	0.2366	0.1353
$\pi - 1$	-2.4350	5.1105	0.3241	-0.5324	-0.4708	0.0151
$\pi^w - 1$	-2.5646	2.4221	0.9282	-0.0571	0.0753	-0.2243
* $R - 1$	1.6443	0.4616	0.9250	-0.3023	-0.2639	-0.4851
$gdp$	0.4215	0.0080	0.9079	1.0000	0.8240	-0.0568
$w$	0.9939	0.0140	0.7003	0.8639	0.8760	-0.2104
$h$	0.3523	0.0033	0.9915	0.2768	-0.2186	0.0070
$n$	0.8752	0.0033	0.9750	0.2165	0.6460	-0.0556
$v$	0.1652	0.0010	0.4874	0.3156	0.2496	0.0212
$\theta$	1.3254	0.0335	0.9677	0.3063	0.7389	-0.0548
$b$	0.1383	0.2039	0.9975	0.1430	-0.1026	0.2124
$\omega$	0.3366	0.0251	0.9935	0.2668	-0.2219	-0.1651
<u>Three quarters of nominal wage stickiness</u>						
$\tau^n$	0.2347	0.0814	0.8599	-0.5712	0.0392	0.1725
$\pi - 1$	0.2225	18.9486	-0.0839	-0.2659	-0.2276	0.1763
$\pi^w - 1$	-0.2737	3.1364	0.7279	-0.1806	-0.0704	-0.1779
$R - 1$	4.9116	1.5836	0.8636	0.0888	-0.0062	-0.5927
$gdp$	0.4215	0.0119	0.9413	1.0000	0.6837	-0.2241
$w$	0.9935	0.0457	0.3539	0.4677	0.4088	-0.3858
$h$	0.3523	0.0060	0.9929	0.6821	0.0481	-0.2261
$n$	0.8754	0.0039	0.9712	-0.1723	0.4284	-0.0005
$v$	0.1653	0.0013	0.7043	0.1801	0.1898	0.0592
$\theta$	1.3286	0.0400	0.9669	-0.1281	0.4966	0.0117
$b$	0.1420	0.4470	0.9973	0.1279	0.0178	0.2905
$\omega$	0.3360	0.0455	0.9572	0.6779	0.0587	-0.2975

Table 5: Simulation-based moments in the right-to-work model with heterogeneity between workers and non-workers; driving processes are  $g_t$  and  $z_t$ .  $\pi$ ,  $\pi^w$ , and  $R$  reported in annualized percentage points. Asterisk denotes zero-lower-bound is violated during simulations.

$$\Delta_t^F \equiv \frac{\partial \mathbf{J}_t}{\partial W_t}, \quad (47)$$

and

$$\omega_t \equiv \frac{\eta}{\eta + (1 - \eta)\Delta_t^F/\Delta_t^W}. \quad (48)$$

The latter means

$$1 - \omega_t = \frac{(1 - \eta)\Delta_t^F/\Delta_t^W}{\eta + (1 - \eta)\Delta_t^F/\Delta_t^W}. \quad (49)$$

With these definitions, we can write the Nash sharing rule as

$$(1 - \omega_t)(\mathbf{W}_t - \mathbf{U}_t) = \omega_t \mathbf{J}_t, \quad (50)$$

which is a generalization of the usual Nash sharing rule.

Using the Bellman equation for the value of a match along with the job-creation condition,  $\mathbf{J}_t = P_t f(\bar{h}) - W_t \bar{h} + \frac{P_t \gamma}{k^f(\theta_t)} - \frac{\psi}{v}(\pi^w - 1)^v$ . Using this as well as the values  $\mathbf{W}_t$  and  $\mathbf{U}_t$ , we can, after some tedious algebra, express the outcome of the Nash bargain as

$$\begin{aligned} \frac{\omega_t}{1 - \omega_t} \left[ P_t z_t f(\bar{h}) - W_t \bar{h} - \frac{\psi}{v} \left( \frac{\pi_t^w}{\pi_t^\chi} - 1 \right)^v P_t + \frac{P_t \gamma}{k^f(\theta_t)} \right] = \\ (1 - \tau_t^n) W_t \bar{h} - \frac{P_t A \bar{h}}{u_{2t}} - \frac{P_t v}{u_{2t}} \\ + (1 - \theta_t k^f(\theta_t)) \beta E_t \left[ \left( \frac{\omega_{t+1}}{1 - \omega_{t+1}} \right) \left( \frac{u_{2t+1}}{u_{2t}} \right) \left( \frac{P_t}{P_{t+1}} \right) (1 - \rho^x) \left[ P_{t+1} z_{t+1} f(\bar{h}) - W_{t+1} \bar{h} - \frac{\psi}{v} \left( \frac{\pi_{t+1}^w}{\pi_{t+1}^\chi} - 1 \right)^v P_{t+1} + \right. \right. \end{aligned}$$

Next, divide through by  $P_t$  and define the real wage as  $w_t \equiv W_t/P_t$  to write the outcome of bargaining as

$$\begin{aligned} \frac{\omega_t}{1 - \omega_t} \left[ z_t f(\bar{h}) - w_t \bar{h} - \frac{\psi}{v} \left( \frac{\pi_t^w}{\pi_t^\chi} - 1 \right)^v + \frac{\gamma}{k^f(\theta_t)} \right] = \\ (1 - \tau_t^n) w_t \bar{h} - \frac{A \bar{h}}{u_{2t}} - \frac{v}{u_{2t}} \\ + (1 - \theta_t k^f(\theta_t)) \beta E_t \left[ \left( \frac{\omega_{t+1}}{1 - \omega_{t+1}} \right) \left( \frac{u_{2t+1}}{u_{2t}} \right) (1 - \rho^x) \left[ z_{t+1} f(\bar{h}) - w_{t+1} \bar{h} - \frac{\psi}{v} \left( \frac{\pi_{t+1}^w}{\pi_{t+1}^\chi} - 1 \right)^v + \frac{\gamma}{k^f(\theta_{t+1})} \right] \right], \end{aligned}$$

which is expression (22) in the text.

Note that if  $\psi = 0$ , then  $\partial \mathbf{J}_t / \partial W_t = -\bar{h}$ , hence

$$\omega_t = \frac{\eta}{\eta + (1 - \eta) \frac{1}{1 - \tau_t^n}}, \quad (51)$$

so that it is only fluctuations in the labor tax rate that drive fluctuations between  $\omega_t$  and  $\eta$ .

## B Hours Determination

Here, we describe the details behind the three alternative protocols we consider for hours determination. The Bellman equations describing the value of a given state for an individual worker, an unemployed individual, and a filled job are, respectively:

$$\mathbf{W}_t = (1 - \tau_t^n)W_t h_t - \frac{P_t e(h_t)}{u_{2t}} + \beta E_t \left[ \left( \frac{u_{2t+1}}{u_{2t}} \right) \left( \frac{P_t}{P_{t+1}} \right) ((1 - \rho^x)\mathbf{W}_{t+1} + \rho^x \mathbf{U}_{t+1}) \right], \quad (52)$$

$$\mathbf{U}_t = \frac{P_t v}{u_{2t}} + \beta E_t \left[ \left( \frac{u_{2t+1}}{u_{2t}} \right) \left( \frac{P_t}{P_{t+1}} \right) (\theta_t k^f(\theta_t)(1 - \rho^x)\mathbf{W}_{t+1} + (1 - \theta_t k^f(\theta_t)(1 - \rho^x))\mathbf{U}_{t+1}) \right], \quad (53)$$

$$\mathbf{J}_t = P_t z_t f(h_t) - W_t h_t - \frac{\psi}{v} \left( \frac{W_t}{W_{t-1} \pi_t^x} - 1 \right)^v + \beta E_t \left[ \left( \frac{u_{2t+1}}{u_{2t}} \right) \left( \frac{P_t}{P_{t+1}} \right) (1 - \rho^x)\mathbf{J}_{t+1} \right]. \quad (54)$$

The difference between the protocols comes down to differences in the effective bargaining powers wielded by workers and firms: in each case, hours are determined by

$$\frac{\varrho_t}{1 - \varrho_t} \mathbf{J}_t = (1 - \tau_t^n)w_t h_t - \frac{e(h_t)}{u_{2t}} - \frac{v}{u_{2t}} + (1 - \theta_t k^f(\theta_t))\beta E_t \left[ \left( \frac{\varrho_{t+1}}{1 - \varrho_{t+1}} \right) \left( \frac{u_{2t+1}}{u_{2t}} \right) (1 - \rho^x)\mathbf{J}_{t+1} \right]. \quad (55)$$

The difference between the models amounts to differences in the specification of the weight  $\varrho_t$

### B.1 Nash Bargaining Over Hours

If the parties bargain simultaneously over the nominal wage and hours worked — i.e., they maximize the Nash product  $(\mathbf{W}_t - \mathbf{U}_t)^\eta \mathbf{J}_t^{1-\eta}$  with respect to both  $W_t$  and  $h_t$  — the wage payment is still described by (22) (with appropriate replacement of  $\bar{h}$  by  $h_t$ ). Defining

$$\delta_t^F = \frac{\partial \mathbf{J}_t / \partial h_t}{P_t} = z_t f'(h_t) - w_t h_t, \quad (56)$$

$$\delta_t^W = -\frac{\partial \mathbf{W}_t / \partial h_t}{P_t} = -(1 - \tau_t^n)w_t + \frac{e'(h_t)}{u_{2t}}, \quad (57)$$

and

$$\varphi_t = \frac{\eta}{\eta + (1 - \eta)\delta_t^F / \delta_t^W}, \quad (58)$$

the solution for hours is given by

$$\begin{aligned} \frac{\varphi_t}{1 - \varphi_t} \left[ z_t f(h_t) - w_t h_t - \frac{\psi}{v} \left( \frac{\pi_t^w}{\pi_t^x} - 1 \right)^v + \frac{\gamma}{k^f(\theta_t)} \right] = \\ (1 - \tau_t^n)w_t h_t - \frac{e(h_t)}{u_{2t}} - \frac{v}{u_{2t}} \\ + (1 - \theta_t k^f(\theta_t))\beta E_t \left[ \left( \frac{\varphi_{t+1}}{1 - \varphi_{t+1}} \right) \left( \frac{u_{2t+1}}{u_{2t}} \right) (1 - \rho^x) \left[ z_{t+1} f(h_{t+1}) - w_{t+1} h_{t+1} - \frac{\psi}{v} \left( \frac{\pi_{t+1}^w}{\pi_{t+1}^x} - 1 \right)^v + \frac{\gamma}{k^f(\theta_{t+1})} \right] \right], \end{aligned}$$

in which we have substituted the equilibrium value of  $\mathbf{J}_t$  using the job-creation condition:  $\mathbf{J}_t = z_t f(h_t) - w_t h_t - \frac{\psi}{v} (\pi_t^w - 1)^v + \frac{\gamma}{k^f(\theta_t)}$ . This sharing rule is different from the wage-setting rule only in that the worker's effective bargaining power in hours determination is  $\varphi_t$  rather than  $\omega_t$ .

## B.2 Right To Manage

In the right-to-manage (RTM) specification, the firm chooses  $h_t$  to maximize  $\mathbf{J}_t$  after wage negotiations are over. This second-stage optimization yields  $w_t = z_t f'(h_t)$ , a standard condition from a neoclassical labor market. Invert this expression to express hours as a function of the nominal wage,  $h\left(\frac{W_t}{P_t}\right)$ . Both the worker and the firm take as given this function when bargaining over the wage, hence we modify the Bellman equations:

$$\mathbf{W}_t = (1 - \tau_t^n)W_t h\left(\frac{W_t}{P_t}\right) - \frac{P_t e\left(h\left(\frac{W_t}{P_t}\right)\right)}{u_{2t}} + \beta E_t \left[ \left(\frac{u_{2t+1}}{u_{2t}}\right) \left(\frac{P_t}{P_{t+1}}\right) \left( (1 - \rho^x)\mathbf{W}_{t+1} + \rho^x \mathbf{U}_{t+1} \right) \right], \quad (59)$$

$$\mathbf{J}_t = P_t z_t f\left(h\left(\frac{W_t}{P_t}\right)\right) - W_t h\left(\frac{W_t}{P_t}\right) - \frac{\psi}{v} \left(\frac{W_t}{W_{t-1} \pi_t^\chi} - 1\right)^v + \beta E_t \left[ \left(\frac{u_{2t+1}}{u_{2t}}\right) \left(\frac{P_t}{P_{t+1}}\right) (1 - \rho^x) \mathbf{J}_{t+1} \right]. \quad (60)$$

Taking into account the dependence of  $h$  on  $W/P$ , we can express the marginal values with respect to the nominal wage  $W_t$  as

$$\frac{\partial \mathbf{W}_t}{\partial W_t} = (1 - \tau_t^n)h_t + (1 - \tau_t^n)w_t h'(w_t) - \frac{e'(h_t)h'(w_t)}{u_{2t}} \quad (61)$$

and

$$\frac{\partial \mathbf{J}_t}{\partial W_t} = z_t f'(h_t)h'(w_t) - [h_t + w_t h'(w_t)] + \Xi_t = -h_t + \Xi_t, \quad (62)$$

where

$$\Xi_t \equiv -\psi \left(\frac{\pi_t^w}{\pi_t^\chi} - 1\right)^{v-1} \frac{\pi_t^{1-\chi}}{w_{t-1}} + \psi(1 - \rho^x)\beta E_t \left[ \frac{u_{2t+1}}{u_{2t}} \frac{P_t}{P_{t+1}} \left(\frac{\pi_{t+1}^w}{\pi_{t+1}^\chi} - 1\right)^{v-1} \pi_{t+1}^w \frac{\pi_{t+1}^{1-\chi}}{w_t} \right]. \quad (63)$$

The second equality above uses the fact that under RTM,  $w_t = z_t f'(h_t)$ . Defining  $\zeta_t^F \equiv \partial \mathbf{J}_t / \partial W_t$ ,  $\zeta_t^W \equiv -\partial \mathbf{W}_t / \partial W_t$ , and

$$\xi_t \equiv \frac{\eta}{\eta + (1 - \eta)\zeta_t^F / \zeta_t^W}, \quad (64)$$

the bargained wage is determined according to

$$\begin{aligned} \frac{\xi_t}{1 - \xi_t} \left[ z_t f(h_t) - w_t h_t - \frac{\psi}{v} \left(\frac{\pi_t^w}{\pi_t^\chi} - 1\right)^v + \frac{\gamma}{k^f(\theta_t)} \right] = \\ (1 - \tau_t^n)w_t \bar{h} - \frac{e(h_t)}{u_{2t}} - \frac{v}{u_{2t}} \\ + (1 - \theta_t k^f(\theta_t))\beta E_t \left[ \left(\frac{\xi_{t+1}}{1 - \xi_{t+1}}\right) \left(\frac{u_{2t+1}}{u_{2t}}\right) (1 - \rho^x) \left[ z_{t+1} f(h_{t+1}) - w_{t+1} h_{t+1} - \frac{\psi}{v} \left(\frac{\pi_{t+1}^w}{\pi_{t+1}^\chi} - 1\right)^v + \frac{\gamma}{k^f(\theta_{t+1})} \right] \right], \end{aligned}$$

in which  $\zeta_t$  is the relevant effective worker bargaining power.

### B.3 Right To Work

An alternative to firms unilaterally choosing how many hours their employees work is that workers choose their hours themselves. In this right-to-work (RTW) specification, we assume that it is the worker that chooses  $h_t$  to maximize  $\mathbf{W}_t$  after wage negotiations are over. This second-stage optimization yields  $w_t = \frac{e'(h_t)}{u_{2t}}$ , also a standard condition from a neoclassical labor market. Inverting this expression to express hours as a function of the nominal wage gives  $\tilde{h}\left(\frac{W_t}{P_t}\right)$ . The function  $\tilde{h}$  is distinct from the function  $h$  from the RTM protocol. Both the worker and the firm take as given  $\tilde{h}$  when bargaining over the nominal wage. The Bellman equations under the RTW protocol are thus

$$\mathbf{W}_t = (1 - \tau_t^n)W_t \tilde{h}\left(\frac{W_t}{P_t}\right) - \frac{P_t e\left(\tilde{h}\left(\frac{W_t}{P_t}\right)\right)}{u_{2t}} + \beta E_t \left[ \left(\frac{u_{2t+1}}{u_{2t}}\right) \left(\frac{P_t}{P_{t+1}}\right) ((1 - \rho^x)\mathbf{W}_{t+1} + \rho^x \mathbf{U}_{t+1}) \right], \quad (65)$$

$$\mathbf{J}_t = P_t z_t f\left(\tilde{h}\left(\frac{W_t}{P_t}\right)\right) - W_t \tilde{h}\left(\frac{W_t}{P_t}\right) - \frac{\psi}{v} \left(\frac{W_t}{W_{t-1} \pi_t^\chi} - 1\right)^v + \beta E_t \left[ \left(\frac{u_{2t+1}}{u_{2t}}\right) \left(\frac{P_t}{P_{t+1}}\right) (1 - \rho^x) \mathbf{J}_{t+1} \right]. \quad (66)$$

Taking into account the dependence of  $\tilde{h}$  on  $W/P$ , we can express the marginal values with respect to the nominal wage  $W_t$  as

$$\frac{\partial \mathbf{W}_t}{\partial W_t} = (1 - \tau_t^n) h_t + (1 - \tau_t^n) w_t \tilde{h}'(w_t) - \frac{e'(\tilde{h}_t) \tilde{h}'(w_t)}{u_{2t}} \quad (67)$$

and

$$\frac{\partial \mathbf{J}_t}{\partial W_t} = z_t f'(h_t) \tilde{h}'(w_t) - [h_t + w_t \tilde{h}'(w_t)] + \Xi_t, \quad (68)$$

where, just as in the RTM model,

$$\Xi_t \equiv -\psi \left(\frac{\pi_t^w}{\pi_t^\chi} - 1\right)^{v-1} \frac{\pi_t^{1-\chi}}{w_{t-1}} + \psi(1 - \rho^x) \beta E_t \left[ \frac{u_{2t+1}}{u_{2t}} \frac{P_t}{P_{t+1}} \left(\frac{\pi_{t+1}^w}{\pi_{t+1}^\chi} - 1\right)^{v-1} \pi_{t+1}^w \frac{\pi_{t+1}^{1-\chi}}{w_t} \right]. \quad (69)$$

The difference between the RTM and RTW models is that here we cannot make the substitution  $w_t = z_t f'(h_t)$  whereas under the RTM protocol we could. The definitions of  $\zeta_t^F$  and  $\zeta_t^W$  are thus appropriately modified compared to the RTM version, and the bargained wage payment is determined according to this modified weight.

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