Board of Governors of the Federal Reserve System

International Finance Discussion Papers

Number 907

October 2007

A Residual-Based Cointegration Test for Near Unit Root Variables

Erik Hjalmarsson and Pär Österholm

A Residual-Based Cointegration Test for Near Unit Root Variables*

Erik Hjalmarsson† Pär Österholm‡

October 9, 2007

Abstract

Methods of inference based on a unit root assumption in the data are typically not robust to even small deviations from this assumption. In this paper, we propose robust procedures for a residual-based test of cointegration when the data are generated by a near unit root process. A Bonferroni method is used to address the uncertainty regarding the exact degree of persistence in the process. We thus provide a method for valid inference in multivariate near unit root processes where standard cointegration tests may be subject to substantial size distortions and standard OLS inference may lead to spurious results. Empirical illustrations are given by: (i) a re-examination of the Fisher hypothesis, and (ii) a test of the validity of the cointegrating relationship between aggregate consumption, asset holdings, and labor income, which has attracted a great deal of attention in the recent finance literature.

JEL classification: C12, C22.

Keywords: Bonferroni test; Cointegration; Near unit roots.

*We have benefitted from comments by Meredith Beechey, David Bowman, Mike McCracken, Chris Erceg, Dale Henderson, Lennart Hjalmarsson, Randi Hjalmarsson, George Korniotos, Rolf Larsson, Andy Levin, Johan Lyhagen, John Rogers, Jonathan Wright, and seminar participants at the Federal Reserve Board. The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. Österholm gratefully acknowledges financial support from Jan Wallander's and Tom Hedelius' Foundation.

†Division of International Finance, Federal Reserve Board, Mail Stop 20, Washington, DC 20551, USA. email: erik.hjalmarsson@frb.gov Phone: +1 202 452 2426

‡Department of Economics, Uppsala University, Box 513, 751 20 Uppsala, Sweden. email: par.osterholm@nek.uu.se Phone: +1 202 378 4135

1 Introduction

Cointegration tests have been among the most important and influential tools in empirical economics since their introduction over two decades ago. In essence, cointegration tests attempt to identify common driving factors in stochastically trending data, thus identifying long-run equilibrium relationships between economic variables. The most common cointegration tests are based on the assumption that the individual variables are unit root processes. The unit root assumption, however, is often hard to fully justify for actual economic data. In finite samples, many economic variables appear highly, but not totally, persistent; that is, the largest autoregressive root is close to, but not necessarily equal to, unity. Unfortunately, inferencial procedures designed for unit root data tend not to be robust to even small deviations from the unit root assumption. For instance, Elliot (1998) shows that large size distortions can occur when performing inference on the cointegrating vector in a system where the individual variables follow near unit root processes rather than pure unit root processes.

Unit root tests go some way toward alleviating the uncertainty regarding the persistence in a given time series but do not provide a definitive answer. Since unit root tests have low power against local alternatives, a failure to reject the null hypothesis of a unit root does not rule out the possibility of a root slightly different from unity. On the other hand, rejecting the null of a unit root does not rule out that the process is still fairly persistent and leaves open the possibility of spurious regressions. It is thus far from obvious how to deal with a multivariate near unit root process: Standard cointegration tests will not be valid under deviations from the pure unit root assumption and the possibility of spurious regressions invalidates standard OLS inference.[1]

The aim of this paper is to design a test of cointegration that is robust to deviations from the pure unit root assumption. In particular, we extend the standard framework to the case where the original data possess autoregressive roots that are local-to-unity, rather than identically equal to unity. The methods developed here are useful from two different perspectives. First, they provide a robustness check to standard cointegration tests in the typical situation where it is not known with certainty that there is an exact unit root in the data. Second, and just as importantly, the test procedures in this paper allow for valid inference in the case when the data is likely not a pure unit root process, but still highly persistent.

[1] In most cointegration studies, the regressors are endogenous, in which case OLS inference would be further complicated and invalid even in the strictly stationary case. Stock (1997) provides a detailed discussion on many of the issues that arise in inference with near unit root variables.

While there is a large literature on cointegrating regressions with near unit root regressors, the focus has been on inference on the slope parameter in these regressions, rather than actual tests of cointegration; see, for example, Cavanagh *et al.* (1995), Elliot (1998), Campbell and Yogo (2006) and Jansson and Moreira (2006). Typically, the models in this literature have been specified such that under the null hypothesis of a zero slope coefficient, the dependent variable is a stationary process. Tests on the slope coefficient therefore become joint tests of cointegration as well, and the issue of spurious regressions never occurs. Although this is a useful specification, for instance, in tests of stock-return predictability which motivated much of this literature, it is less convenient in most typical economic applications where both dependent and independent variables are near-integrated. The closest related literature to the current paper is the work on stationarity tests (Leybourne and McCabe, 1993, and Shin, 1994) and the work by Wright (2000). In particular, Wright (2000) develops a joint test of a specific hypothesis regarding the cointegrating vector and a test of the null hypothesis of cointegration that is robust to deviations from the pure unit root framework.

We focus on a residual-based test of cointegration. Following the work of Phillips and Ouliaris (1990), we extend the asymptotic results for a residual-based test to the case of near-integrated processes. Unlike the pure unit root case, the asymptotic distribution of the test statistic now depends on an unknown nuisance parameter; the local-to-unity root. Since this parameter is not consistently estimable, feasible tests cannot be directly constructed from the asymptotic distribution. Instead, we propose to replace the unknown parameter value for the local-to-unity root with a conservative estimate.

In order to understand the intuition behind our procedure, it is useful to consider the potential errors when applying a standard, pure unit root case, cointegration test to a set of near unit root variables. A residual-based cointegration test evaluates whether the residuals from the empirical regression contain a unit root. Now, if the original data are in fact near-integrated, with a root less than unity, the test will over-reject since the residuals will not contain a unit root even if there is no cointegration. But, by instead using critical values based on a conservative estimate of the local-to-unity root in the original data, a valid test is obtained. Intuitively, if one views a residual-based test of cointegration as a test of whether there is less persistence in the residuals than in the original data, then this test is only valid if the persistence of the original data is not overstated.[2] In a spirit similar to the Bonferroni

[2] Although, perhaps, less obvious, the same also holds true for non-residual-based tests, such as those of Johansen (1988,1991); see Hjalmarsson and Österholm (2007).

methods proposed by Cavanagh *et al.* (1995), we show how an appropriately conservative estimate of the local-to-unity root is obtained.

The rest of the paper is organized as follows. Section 2 outlines the modelling assumptions and the theoretical results. Section 3 describes the Bonferroni methods. In Section 4, the proposed procedure is evaluated using Monte Carlo simulations. We show that once the conservative estimate for the local-to-unity parameter is chosen appropriately, the resulting test has both good size and power properties. This is in contrast to standard cointegration tests, based on the unit root assumption, which are shown to severely over-reject as the data generating process deviates from a pure unit root setup. As an illustration of the method, two empirical applications are considered in Section 5. First, we re-examine the Fisher hypothesis and show that using the robust methods proposed in this paper, one can no longer find significant support for a long-run equilibrium relationship between nominal interest rates and inflation; using standard unit root based cointegration tests on the other hand, the null hypothesis of no cointegration is rejected. In a second illustration, we consider the robustness of the long-run relationship between aggregate consumption, asset holdings, and labor income, which was initially studied by Lettau and Ludvigson (2001) and has since received a great deal of attention in the finance literature. We find that after controlling for the unknown persistence in the variables, there is still strong evidence of cointegration between the three variables. Section 6 concludes and the Appendix contains tables of critical values for the test statistic.

2 Theoretical framework

2.1 Model and assumptions

Let $\{z_t\}_0^\infty$ be an $m-$vector of nearly integrated processes, such that the data generating process satisfies

$$z_t = Az_{t-1} + u_t \tag{1}$$

where $A = I + C/T$ is an $m \times m$ matrix with $A = diag\,(a_1, ..., a_m)$ and $C = diag\,(c_1, .., c_m)$, and T is the sample size. That is, each component process in z_t is generated as a near unit root process with individual local-to-unity parameters c_i, $i = 1, ..., m$. The initial conditions are set at $t = 0$ and z_0 is assumed randomly distributed with finite variance. Although none of the formal results depend upon it, we will work under the assumption that $c_i \leq 0$ for all i, which rules out explosive processes. The innovations u_t satisfy a general linear process.

Assumption 1 *1.* $u_t = D(L)\epsilon_t = \sum_{j=0}^{\infty} D_j \epsilon_{t-j}$, $\sum_0^{\infty} j\,||D_j|| < \infty$, $|D(1)| \neq 0$.

2. ϵ_t *is iid with mean zero, variance matrix* Σ_ϵ*, and finite fourth-order moment.*

By standard results, e.g. Phillips and Solo (1992), $T^{-1/2}\sum_{t=1}^{[Tr]} u_t \Rightarrow B(r) \equiv BM(\Omega)$, where $B(r)$ is a Brownian motion with covariance matrix $\Omega = D(1)\Sigma_\epsilon D(1)'$. Partition $z_t = (y_t, x_t')'$ such that y_t is a scalar and x_t is an $n-$vector $(n = m-1)$. Let $B(r) = \left(B_1(r), B_2(r)'\right)'$, $\Omega = [(\omega_{11}, \omega_{21}'), (\omega_{21}, \Omega_{22})]$, and $C = [(c_1, 0), (0, C_2)]$ be conformable partitions of $B(r)$, Ω, and C, respectively. We assume that $\Omega_{22} > 0$ and write $\Omega = L'L$. Denote an $m-$vector standard Brownian motion as $W(r)$, and it follows that $B(r) = L'W(r)$. Further, as $T \to \infty$, $z_t/\sqrt{T} \Rightarrow J_C(r) = \int_0^r e^{(r-s)C} dB(s)$. Partition J_C conformably with B and let $J_C^W(r) = \int_0^r e^{(r-s)C} dW(s)$.

We consider residual-based tests of the null of no cointegration using the regression residuals, $\hat{v}_t$, from the following empirical regression:

$$y_t = \beta' x_t + v_t. \tag{2}$$

2.2 The test statistic

We focus on the traditional Augmented Engle-Granger $t-$test (Engle and Granger, 1987) of the null of no cointegration, which is probably the most commonly used residual-based test of cointegration. Our analysis could easily be extended to cover the Z_α and Z_t cointegration tests proposed by Phillips and Ouliaris (1990), but for brevity we restrict ourselves to the Augmented Engle-Granger test (henceforth denoted AEG test).

The AEG test is defined as the $t-$statistic for $\hat{\alpha}_*$ from the regression $\Delta\hat{v}_t = \alpha_* \hat{v}_{t-1} + \sum_{i=1}^{p} \varphi_i \Delta\hat{v}_{t-i} + w_t$. The below result follows from the results in Phillips and Ouliaris (1990) and the results for near-integrated processes in Phillips (1987,1988).

Theorem 1 *Let the data generating process satisfy equation (1) for some given* $C = diag(c_1, ..., c_m)$*, and let Assumption 1 hold. Suppose that the autoregressive order in the AEG regression satisfies*

$p \to \infty$ as $T \to \infty$ such that $p = o\left(T^{1/3}\right)$. Then, under the null of no cointegration, as $T \to \infty$,

$$AEG \Rightarrow c_1 \left(\int_0^1 \left(J_{1\cdot 2,C}^W\right)^2 \right)^{1/2} + \frac{\int_0^1 J_{1\cdot 2,C}^W dW_{1\cdot 2}}{\left(\int_0^1 \left(J_{1\cdot 2,C}^W\right)^2 \right)^{1/2}} \tag{3}$$

where

$$J_{1\cdot 2,C}^W(r) = J_{1,c_1}^W(r) - \left(\int_0^1 J_{1,c_1}^W J_{2,C_2}^{W\prime} \right) \left(\int_0^1 J_{2,C_2}^W J_{2,C_2}^{W\prime} \right)^{-1} J_{2,C_2}^W(r) \tag{4}$$

and

$$W_{1\cdot 2}(r) = W_1(r) - \left(\int_0^1 W_1 W_2' \right) \left(\int_0^1 W_2 W_2' \right)^{-1} W_2(r), \tag{5}$$

are the $\mathcal{L}_2-$projection residuals of J_{1,c_1}^W and W_1 on the spaces spanned by J_{2,C_2}^W and W_2 respectively.

Remark 1.1 The limiting distribution of the AEG statistic depends on the unknown parameter C, but is otherwise free of nuisance parameters. For a given C, the asymptotic distribution can thus easily be tabulated. The next section describes a feasible implementation of the test when C is unknown.

Remark 1.2 Effectively, the AEG test evaluates whether the persistence in the residuals is less than that predicted under the null hypothesis of no cointegration. However, since the original data is not necessarily a unit root process, the critical values reflect this fact. In the special case of $C = 0$, the limiting distribution reduces to the usual one for pure unit root variables.

Remark 1.3 In empirical work, a constant or a constant and a linear trend are typically included in the empirical regression (2). As in standard cointegration analysis, this will affect the limiting distribution in a straightforward manner (e.g. Phillips and Ouliaris, 1990) and thus the critical values used, but will otherwise not alter the analysis.

3 Feasible implementation

For a known C, the above test is trivial to use once critical values for the asymptotic distribution are obtained. Unfortunately, C is typically not known. We therefore consider a Bonferroni test approach, which is similar to that used by Cavanagh *et al.* (1995) and Campbell and Yogo (2006) in their pursuit of inference in predictive regression with near-integrated variables.

Consider confidence intervals for c_i, $i = 1, ..., m$, of the shape $\{[\underline{c}_i, \overline{c}_i]\}_{i=1}^m$ with an overall coverage rate equal to $100 \times (1 - \alpha_1)$ percent. Let $\{\widetilde{c}_i \in [\underline{c}_i, \overline{c}_i]\}_{i=1}^m$ be the set of parameter values in this confidence region for which the critical value of the asymptotic distribution of the AEG statistic is most conservative, for some given α_2 percent level (e.g. five percent). If the AEG statistic is evaluated using this conservative critical value, calculated at the α_2 percent level, the size of the resulting cointegration test will be less than or equal to $\alpha = \alpha_1 + \alpha_2$, by Bonferroni's inequality.

However, relative to the Cavanagh *et al.* (1995) and Campbell and Yogo (2006) studies, there is an additional complication in the current setup. In those papers, there is only one local-to-unity process, whereas here there are at least two in the simplest case with just one regressor. In the univariate case, confidence intervals of the local-to-unity parameter can be obtained by inverting a unit root test statistic (Stock, 1991). In the $m-$dimensional case, a confidence region for C could be obtained by inverting individual unit root test statistics in order to obtain confidence intervals $[\underline{c}_i, \overline{c}_i]$, $i = 1, ..., m$, each with coverage rate $1 - \alpha_1/m$. The overall confidence level of $\{[\underline{c}_i, \overline{c}_i]\}_{i=1}^m$ is at least $100 \times (1 - \alpha_1)$ percent, again by Bonferroni's inequality. Although theoretically sound, such an approach suffers from the practical disadvantage that it would be virtually impossible to tabulate the critical values for the asymptotic distribution beyond the simple two-dimensional case. We therefore propose a simpler approach that allows for tabulation of critical values and seems to give up little in robustness.

Intuitively, the AEG test evaluates whether the persistence, or autoregressive root, in the regression residuals, v_t, is less than in the original data, y_t. As seen in equations (3) and (4), the critical values of the test depend on both the persistence in the 'dependent' variable, y_t, and the regressors, x_t, denoted c_1 and C_2 respectively. However, it seems reasonable to conjecture that the main determinant of the asymptotic distribution will be c_1, rather than C_2. Thus, using $\hat{C} = \hat{C}_1 = diag\,(\hat{c}_1, ..., \hat{c}_1)$ for some $\hat{c}_1$, rather than $\tilde{C} = diag\,(\tilde{c}_1, \tilde{c}_2, ..., \tilde{c}_m)$, to form critical values might not cause a large size distortion in the test. Although this conjecture is difficult to evaluate analytically, extensive simulation evidence supports it. For instance, Figure 1 shows the critical values for the AEG test in the two-dimensional case with an intercept in the empirical regression. As is evident, the primary changes come from changing c_1, whereas the critical values are almost constant across C_2. Additional evidence supporting this conclusion is provided by simulations in the following section.

Furthermore, if $C_1 = diag\,(c_1, ..., c_1)$ is used to calculate the critical values for the asymptotic distribution in Theorem 1, the AEG cointegration test will be more conservative as the value of c_1

decreases; that is, as c_1 becomes more negative, so do the corresponding critical values, as shown in Table A3. Only the lower bound on c_1, say $\underline{c}_1$, is therefore of interest in constructing a conservative test; for a given confidence level, such a lower bound can be obtained from a one-sided confidence interval for c_1, $[\underline{c}_1, +\infty)$.

By restricting the attention to the parameter c_1, and calculating critical values based on $\underline{C}_1 = diag\left(\underline{c}_1, ..., \underline{c}_1\right)$, it now becomes easy to implement the Bonferroni method. The lower confidence bound for c_1, $\underline{c}_1$, is obtained by inverting a unit root test statistic for the variable y_t. Based on this lower bound of c_1, the test is evaluated using the corresponding critical value for $\underline{C}_1 = diag\left(\underline{c}_1, ..., \underline{c}_1\right)$. If the lower bound $\underline{c}_1$ has confidence level $1 - \alpha_1$ and the AEG test is evaluated at the α_2 level, the resulting test will have a size no larger than $\alpha = \alpha_1 + \alpha_2$.[3]

In general, Bonferroni's inequality is strict, and the size of the test will be less than α. To obtain a correctly sized test of size $\tilde{\alpha}$, which is distinct from $\alpha = \alpha_1 + \alpha_2$, we first fix α_2 at some level and then find α_1 such that the resulting test has size $\tilde{\alpha}$. Finding α_1 is effectively a trial and error exercise. In the simulations below, we let $\tilde{\alpha} = \alpha_2 = 0.05$ and show that setting α_1 equal to 50 percent will approximately result in an overall five percent test. Thus, by effectively using a median unbiased estimate of c_1, an approximately correctly-sized test is obtained. These results are discussed more extensively in conjunction with the Monte Carlo simulations in the next section.

In terms of practical implementation, we follow Campbell and Yogo (2006) and invert Elliot *et al.*'s (1996) DF-GLS unit root test statistic to obtain a lower bound for c_1. Table A1 provides the lower 95th, 75th, 50th, 25th, and 5th percent confidence bounds of c_1, given a value of the DF-GLS test statistic.[4] For instance, the lower confidence bound that corresponds to $\alpha_1 = 0.05$ is given in the $100 \times (1 - \alpha_1)\,\% = 95\%$ column. Table A2 provides the corresponding bounds when a trend is allowed for in the DF-GLS regression. Table A3 tabulates the five percent critical values for the AEG statistic, for $c_1 = 0$ to $c_1 = -60$, assuming that $c_1 = c_2 = ... = c_m$; values for one to five regressors are provided for the cases of no intercept, intercept, and intercept and a linear trend in the empirical regression.

Henceforth, we will refer to the cointegration test constructed in the manner above as the Bonferroni AEG test, with the additional specification of the value of α_1 when necessary. Unless otherwise noted, we let $\alpha_2 = 0.05$.

[3] Since C_2 is assumed not to play an important role in the distribution of the test-statistic, the only uncertainty regarding the persistence of the data comes from uncertainty regarding c_1. The confidence level of the lower bound $\underline{C}_1$ is therefore $1 - \alpha_1$ rather than $1 - m \times \alpha_1$, as discussed above.

[4] Note that, for instance, the two lower confidence bounds at the 5 percent and 95 percent level provide a two-sided confidence interval with confidence level 90 percent.

4 Finite-sample properties

4.1 Size properties

We analyze the finite-sample properties of the proposed test procedure through a series of Monte Carlo simulations. Starting with the size properties, it is assumed that the data generating process (DGP) is given by equation (1), with the innovations u_t drawn from a multivariate normal distribution such that $E[u_t] = 0$ and $E[u_t u_t'] = I$. The sample size is set to either $T = 100$ or 500 and the number of regressors, $n = m - 1$, is equal to either one or three. The regression

$$y_t = \alpha + \beta' x_t + \upsilon_t \tag{6}$$

is estimated, which is a spurious regression given the above DGP, and the cointegration tests are applied to the fitted residuals, $\hat{\upsilon}_t$. Each simulated $m-$dimensional time-series z_t is thus partitioned as $z_t = (y_t, x_t')'$, as described previously. When all components in z_t are ex-ante identical, i.e. have the same persistence c, the first component series is set to y_t and the remainder to x_t. When c_i varies between each series, we describe explicitly which series are set as y_t and x_t. All tests are performed at the five percent significance level and are evaluated using the critical values given in Table A3. The results are based on $10,000$ repetitions.

In the first round of simulations, we let the local-to-unity matrix for z_t be given by $C = diag(c, ..., c)$, so that all the series have identical persistence. The local-to-unity parameter c varies from 0 to -30.

Figure 2 shows the size properties for the traditional AEG cointegration test, which by definition is evaluated at $c = 0$, as a function of the local-to-unity parameter c. The nominal size of the test is five percent, and for c close to zero, the actual rejection rate is also close to five percent. However, as c decreases in value, the test starts over-rejecting and the rejection rates already approach ten percent for $c = -5$. The rejection rates become even larger and approach one as c becomes even smaller. It should be stressed that this is not a small-sample bias, but a reflection of the inconsistency of the test when $c < 0$. Since the autoregressive root of the residual in equation (6) is less than one for $c < 0$, the AEG test, evaluated under the assumption of $c = 0$, will reject the null of a unit root in the residuals more frequently than its nominal size. For time series that do not necessarily have a unit root, standard cointegration tests can thus be highly misleading. This raises questions regarding previous studies that have relied on cointegrating methods, despite having found evidence of stationarity of the included variables; see, for example, Crowder and Hoffman (1996).

We next consider the size properties of the Bonferroni AEG test using a conservative estimate of C. As discussed in the previous section, we use $\underline{C}_1 = diag\left(\underline{c}_1, ..., \underline{c}_1\right)$ where $\underline{c}_1$ is the lower bound on the persistence in y_t. A direct application of the Bonferroni method suggests choosing $\underline{c}_1$ such that the one-sided confidence interval $[\underline{c}_1, +\infty)$ has confidence level $100 \times (1 - \alpha_1)$ percent, and then evaluating the AEG test-statistic at the α_2 percent level for a total size of $\alpha = \alpha_1 + \alpha_2$ percent. In practice, however, such an approach will deliver extremely conservative tests. For instance, if $\alpha_1 = \alpha_2 = 0.05$, the rejection rate for the resulting test is virtually identical to zero in the simulations considered here. Instead, we follow the approach outlined above and fix $\alpha_2 = 0.05$ and choose α_1 such that the size of the overall test is close to five percent. In particular, we consider setting $\alpha_1 = 0.25, 0.50$ and 0.75. That is, $\underline{c}_1$ is chosen as the lower bound in one-sided confidence intervals with confidence levels equal to 75, 50, and 25 percent, respectively. To obtain these values for $\underline{c}_1$, the DF-GLS unit root test-statistic is inverted, using the values in Table A1.[5]

Figure 3 shows the results for the Bonferroni AEG test using these different estimates of $\underline{C}_1$. It is immediately apparent that for small values of c, the test tends to over-reject when $\alpha_1 = 0.75$, and under-reject when $\alpha_1 = 0.25$. For $\alpha_1 = 0.50$, the test still tends to under-reject somewhat, except for small values of c in the case of $T = 500$ and $n = 1$, where there is instead a slight over-rejection. Overall, however, for $\alpha_1 = 0.50$, the rejection rate is typically between two and five percent. One could achieve rejection rates that are somewhat closer to the nominal size by letting α_1 vary with $\underline{c}_1$ in some manner, but at the cost of a substantially more cumbersome procedure. Using a fixed value of $\alpha_1 = 0.50$, for all values of $\underline{c}_1$, yields a very simple test to implement. The procedure would simply be given as:

(i) Obtain the value of the AEG test statistic from a standard implementation of the Engle and Granger test.

(ii) Calculate the DF-GLS unit root statistic for the y_t variable and obtain the corresponding value of $\underline{c}_1$ from Table A1 or A2.

(iii) Compare the AEG test statistic to the critical value corresponding to $\underline{c}_1$ in Table A3.

[5] The number of lags included in the DF-GLS test is chosen using the Schwarz (1978) information critierion, with a maximum number of two allowed in order to keep the simulation times managable. The same number of lags is also included in the AEG regression; that is, in $\Delta \hat{v}_t = \alpha_* \hat{v}_{t-1} + \sum_{i=1}^{p} \varphi_i \Delta \hat{v}_{t-i} + w_t$.

It may seem surprising that using, for instance, a lower bound with only a 25 percent confidence level, does not result in a larger size distortion. Figure 4 helps shed some light on this puzzle. The results in the figure are based on $10,000$ simulations of a univariate local-to-unity process, with local-to-unity parameter c, *iid* normal innovations and sample size $T = 500$. It shows estimates of the lower bounds of c, with confidence levels of 25, 50, and 75 percent, using the inversion of the DF-GLS statistic in Table A1. The panels in Figure 4 show the densities for the lower-bounds estimates for $c = -5, -10, -20$, and -30. As expected, the bounds estimates at the 75 percent confidence level are furthest to the left. However, the densities are far from symmetric, especially for c close to zero; the density for the 25 percent confidence bound is also less symmetric than the density for the 75 percent bound. Thus, although the density is shifted further to the right as the confidence level decreases, which leads to estimates of c closer to zero, the shift is not symmetric and the risk of vastly over-estimating c is not increased dramatically. This explains, to some extent, why the rejection rates in the cointegration test only increase slowly as the confidence level of the lower bound is decreased.

In the last set of size simulations, shown in Figure 5, we analyze the properties of the Bonferroni AEG test when the local-to-unity parameters c_i, $i = 1, ..., m$ are not identical; i.e. when the processes in z_t do not have the same persistence. Two different cases are considered. In the first case, there are two regressors with persistence parameters equal to -10 and -20. In the second setup, there are three regressors with persistence parameters $0, -10$, and -20. In both cases, it is assumed that the persistence in y_t, c_1, varies between 0 and -30. Thus, in the first case, $C = diag\,(c_1, -10, -20)$, and in the second case $C = diag\,(c_1, 0, -10, -20)$. The same methods as in the case with identical $c_i s$ are used and the results for $\alpha_1 = 0.25, 0.50$, and 0.75, are shown. Overall, the results in Figure 5 are very similar to those in Figure 3. Using $\alpha_1 = 0.50$ and a nominal size of five percent results in actual rejection rates around three percent. Given the results shown previously in Figure 1, it is not surprising that the test also performs well when the $c_i s$ are not identical.

In summary, the proposed procedure for tests of cointegration in data with an unknown C appears to work well in finite samples, once the confidence level of the lower bound is chosen appropriately. Additional fine tuning of this confidence level could be done to bring the actual size even closer to the nominal size, but at the cost of adding some complexity.

4.2 Power properties

We next perform a second Monte Carlo simulation to evaluate the finite-sample rejection rates under the alternative of cointegration. The 'independent' variable x_t is still generated according to equation (1) using *iid* standard normal innovations. However, the 'dependent' variable y_t, is now generated as

$$y_t = \beta' x_t + v_t, \tag{7}$$

where v_t is an $AR(1)$ process with an auto-regressive root ρ; the innovations to this AR process are *iid* standard normal. β is set to an n-vector of ones. The same empirical regression, including the constant, as in the size simulations is estimated, and the Bonferroni AEG test with $\alpha_1 = 0.50$ is applied to the estimated residuals $\hat{v}_t$. The critical values that are used are thus for the case with a constant in the regression. Two different sample sizes, $T = 100$ and 500, and $n = 1$ and 3 regressors, are considered. In the case of one regressor, the persistence in x_t is set equal to either $C_2 = -2, -10$, or -20. In the case of three regressors, it is assumed that $C_2 = diag\,(0, -10, -20)$.

Figure 6 shows the results in four sub-plots corresponding to the different combinations of sample size and number of regressors. The vertical axes of the graphs show the power of the Bonferroni AEG test plotted against the persistence ρ in the error term v_t. In the case of $T = 100$, results for $\rho \in [0.5, 1]$ are shown and for the $T = 500$, results for $\rho \in [0.8, 1]$ are shown. As is to be expected, power is a monotone and declining function of the persistence, ρ. It should be noted that for very large values of ρ, we expect the test to have low power; for example, in the bivariate case, a residual that is less persistent than y_t cannot be generated by regressing y_t on x_t when $\rho > 1 + C_2/T$. For most values of ρ, however, the test appears to exhibit good power properties and appears sufficiently powerful that it would be a useful tool in many empirical applications, including those with relatively small sample sizes.

5 Empirical illustrations

To illustrate the empirical use of the Bonferroni AEG test, we next consider two applications where the variables in question are all fairly persistent, but not necessarily pure unit root processes. As a comparison to the robust methodology proposed in this paper, we will also conduct the traditional AEG test.

5.1 The Fisher hypothesis

It is well known that both nominal interest rates and inflation are fairly persistent in most countries. Accordingly, cointegration techniques have been a popular approach to test the Fisher hypothesis in more recent years; see, for example, Mishkin (1992), Wallace and Warner (1993), Evans and Lewis (1995), and Crowder and Hoffman (1996). However, the assumption made in most of these studies of exact unit roots in both nominal interest rates and inflation can be questioned on both theoretical and empirical grounds.[6] It is therefore worth re-interesting this issue using the Bonferroni *AEG* test.

A common formulation of the Fisher hypothesis is that the m-period nominal interest rate (i_t^m) is related to the real interest rate (r_t^m) and inflation (π_t^m) according to

$$i_t^m = E_t\left(r_t^m\right) + E_t\left(\pi_t^m\right). \tag{8}$$

Relying on the commonly made assumption of a constant or mean-reverting real interest rate, an empirical version of the Fisher hypothesis can be written as

$$i_t^m = \alpha + \beta\pi_t^m + v_t, \tag{9}$$

where the constant α has the interpretation of the (constant) equilibrium real interest rate, the error term v_t is assumed to be a stationary ARMA process and β, in the most traditional interpretation, should be equal to unity.[7]

Monthly data on the short nominal interest rate − given by the three month treasury bill − and CPI inflation from January 1955 to October 2006 in the United States were provided by the Board of Governors of the Federal Reserve System. Table 1 shows the results from the DF-GLS unit root test and the KPSS stationarity test, as well as the median unbiased estimate of c, denoted $\hat{c}$, and a 90 percent confidence interval for c; the estimates and confidence intervals of c are derived using the values in Table A1 and linear interpolation.[8] As can be seen, the evidence for a unit root in the interest rate appears reasonably strong; the DF-GLS test fails to reject the null of a unit root whereas the KPSS test rejects the null of stationarity. For inflation, on the other hand, the evidence is more

[6] See, for example, Wu and Zhang (1996), Culver and Papell (1997), and Wu and Chen (2001).

[7] Note that in the estimations below, time t inflation is given as future inflation between t and $t+m$. This can be motivated by assuming rational expectations; see, for example, Mishkin (1992).

[8] Regarding the specification of deterministic terms in the unit root tests, it should be noted that we test for mean reversion around a constant level.

mixed since the DF-GLS test rejects a unit root but the KPSS test rejects stationarity.[9]

Table 1: Unit root tests.

	i_t	π_t
DF-GLS	-1.40	-2.54*
KPSS	0.53*	0.52*
$\hat{c}$	-3.40	-12.91
90% CI for c	[-9.06, 2.00]	[-21.37, -3.46]

Notes: * indicates significance at the five percent level.

The cointegration tests are conducted using a significance level of five percent. For the Bonferroni AEG test, based on the simulation results in the previous section, we set $\alpha_1 = 0.5$; thus $\hat{c} = -3.40$, the median unbiased estimate for the nominal interest rate, is used to establish the critical value in the Bonferroni AEG test. The results from the cointegration tests based on the specification in equation (9) are given in Table 2.[10] Asymptotic critical values are used for both the standard Engle-Granger test (denoted AEG) and the Bonferroni AEG test (denoted AEG^C) and are provided in Table 2; the AEG^C critical value is obtained from Table A3 and linear interpolation.

Table 2: Cointegration tests.

Test statistic	-3.43
Critical value AEG^C	-3.47
Critical value AEG	-3.34

Notes: Nominal size is 0.05.

As can be seen, the null hypothesis of no cointegration is rejected if the standard method is used, as the test statistic is smaller than the critical value for the traditional AEG test. However, when the Bonferroni AEG test is used, the null hypothesis is not rejected. Thus, performing inference using robust methods, there is no strong evidence of cointegration, or co-movement, between the nominal interest rate and inflation in U.S. data. This raises doubts about the validity of the Fisher hypothesis, and also illustrates the importance of controlling for the unknown degree of persistence in the data; assuming unit roots in the data, the cointegration test would have resulted in evidence favorable of

[9] Lag length in the DF-GLS test was determined using the Schwarz (1978) information criterion. For the KPSS test, a Newey-West estimator was employed to correct for serial correlation.

[10] As in the DF-GLS test, lag length in the test equation is determined using the Schwarz (1978) criterion.

the Fisher hypothesis. Having looked at a traditional application from the macroeconomic literature, we next turn to a recent issue from financial economics.

5.2 Consumption, aggregate wealth and stock returns

Many studies argue that financial valuation ratios such as the dividend- and earnings-price ratios may have predictive power for excess stock returns over the risk-free rate. In a novel attempt to tie macroeconomic variables more closely to financial markets, Lettau and Ludvigson (2001) argue that consumption is a function of aggregate wealth. Based on this claim, they suggest that aggregate consumption (k_t), asset holdings (a_t) and labour income (y_t) are cointegrated and that the deviation from equilibrium is useful in terms of predicting both excess stock returns and real stock returns. The empirical specification used by Lettau and Ludvigson accordingly takes its starting point in a cointegrating relationship of the type

$$k_t = \mu + \theta a_t + \lambda y_t + \chi_t, \tag{10}$$

where the error term χ_t is assumed to be a stationary ARMA process which has predictive power for future returns.

However, there is no strong *a priori* reason to assume that the above variables contain pure unit roots.[11] We therefore investigate the sensitivity of Lettau and Ludvigson's results when the uncertainty regarding the persistence in the data is taken into account. Quarterly data on US consumption, asset holdings and labour income ranging from the first quarter 1952 to the fourth quarter 2006 were obtained from Professor Ludvigson's web page;[12] all variables are given by the natural logarithm of real, per capita data.

Table 3 shows the results from unit root tests and stationarity tests for all variables and also provides the median unbiased estimates of c, $\hat{c}$, as well as 90 percent confidence intervals.[13] The

[11] As was shown above, the persistence of the dependent variable is of special importance when using the *AEG* test. The assumption of a unit root in consumption is thus of particular interest. Although this conjecture finds some support – see, for example, Hall (1978) and Gali (1993) – the opinion in the litterature is far from unanimous. For instance, the vast literatue that uses linear trends to detrend consumption – see, for example, Cooper and Ejarque (2000) and Casares (2007) – implicitly or explicitly assumes that consumption is trend stationary rather than generated by a unit root process. Furthermore, it has been argued that consumption and output should be integrated of the same order. Thus, if output is trend stationary (e.g. Flavin, 1981 and Diebold and Senhadji, 1996) then consumption should be as well.

[12] http://www.econ.nyu.edu/user/ludvigsons/

[13] Note that in this application, the unit root tests have both constant and trend included in the specification. Thus, the estimates and confidence intervals of c are derived using the values in Table A2; again, linear interpolation is used.

evidence for unit roots in consumption and labour income seems strong, whereas it is mixed for asset holdings.

Table 3: Unit root tests.

	k_t	a_t	y_t
DF-GLS	-1.95	-2.54	-0.78
KPSS	0.36*	0.20*	0.38*
$\hat{c}$	-4.06	-9.98	2.32
90% CI for c	[-12.28, 3.35]	[-19.63, 2.32]	[-2.18, 4.44]

Notes: * indicates significance at the five percent level.

As in the previous application, we choose a significance level of five percent for the cointegration tests and set $\alpha_1 = 0.5$. The results from the *AEG* and Bonferroni *AEG* cointegration tests are shown in Table 4. The null hypothesis of no cointegration is rejected regardless of which test is used. The robust cointegration methods developed here thus support the conclusion of Lettau and Ludvigson that US consumption, asset holdings and labour income are cointegrated.

Table 4: Cointegration tests.

Test statistic	-4.03
Critical value AEG^C	-3.86
Critical value AEG	-3.77

Notes: Nominal size is 0.05.

6 Conclusion

For many economic time series, it is difficult to justify theoretically that they are generated by unit root processes. This is problematic from an empirical point of view since cointegration tests may be misleading when the data follow near-integrated, rather than pure unit root, processes. The size distortions of cointegration tests relying on the unit root assumption – combined with the fact that standard OLS inference could lead to spurious results – makes it unclear how to analyze a multivariate time series of near-integrated variables.

In this paper, we have extended a standard residual-based cointegration test to allow for an unknown local deviation from the unit root assumption. This more robust test is easy to implement and Monte Carlo simulations show that it works well in finite samples. Unlike standard cointegration tests, the methods developed in this paper thus provide a means of performing valid inference on a multivariate near unit root process. The framework suggested in this paper therefore takes another step towards addressing the problems associated with inference when variables are near-integrated. The methods presented here take their starting point in the work of Engle and Granger (1987). In future research it would also be of interest to see Johansen's (1988,1991) VAR-based framework extended to a setting with near-integrated variables.

References

[1] Campbell, J.Y. and M. Yogo, 2006. Efficient Tests of Stock Return Predictability, *Journal of Financial Economics* 81, 27-60.

[2] Casares, M., 2007. The New Keynesian Model and the Euro Area Business Cycle, *Oxford Bulletin of Economics and Statistics* 69, 209-244.

[3] Cavanagh, C., G. Elliot, and J. Stock, 1995. Inference in models with nearly integrated regressors, *Econometric Theory* 11, 1131-1147.

[4] Cooper, R. and J. Ejarque, 2000. Financial Intermediation and Aggregate Fluctuations: A Quantitative Analysis, *Macroeconomic Dynamics* 4, 423-447.

[5] Crowder, W.J., and D.L. Hoffman, 1996. The Long-Run Relationship between Nominal Interest Rates and Inflation: The Fisher Equation Revisited, *Journal of Money, Credit and Banking* 28, 102-118.

[6] Culver, S.E. and D.H. Papell, 1997. Is There a Unit Root in the Inflation Rate? Evidence from Sequential Break and Panel Data Models, *Journal of Applied Econometric*s 12, 435-444.

[7] Diebold, F.X. and A.S. Senhadji, 1996. The Uncertain Unit Root in Real GNP: Comment, *American Economic Review* 86, 1291-1298.

[8] Elliot, G., 1998. On the Robustness of Cointegration Methods When Regressors Almost Have Unit Roots, *Econometrica* 66, 149-158.

[9] Elliot G., T.J. Rothenberg, and J.H. Stock, 1996. Efficient Tests for an Autoregressive Unit Root, *Econometrica* 64, 813-836.

[10] Engle, R.F. and C.W.J. Granger, 1987. Co-Integration and Error Correction: Representation, Estimation, and Testing, *Econometrica* 55, 251-276.

[11] Evans, M. and K. Lewis, 1995. Do Expected Shifts in Inflation Affect Estimates of the Long-Run Fisher Relation?, *Journal of Finance* 50, 225-253.

[12] Flavin, M., 1981. The Adjustment of Consumption to Changing Expectations about Future Income, *Journal of Political Economy* 89, 974-1009.

[13] Gali, J., 1993. Variability of Durable and Nondurable Consumption: Evidence for Six O.E.C.D. Countries, *Review of Economics and Statistics* 75, 418-428.

[14] Hall, R.E., 1978. Stochastic Implications of the Life-Cycle-Permanent-Income Hypothesis: Theory and Evidence, *Journal of Political Economy* 86, 971-987.

[15] Hjalmarsson, E., and P. Österholm, 2007. Testing for Cointegration Using the Johansen Methodology when Variables are Near-Integrated, IMF Working Paper 07/141, International Monetary Fund.

[16] Jansson, M., and M.J. Moreira, 2006. Optimal Inference in Regression Models with Nearly Integrated Regressors, *Econometrica* 74, 681-714.

[17] Johansen, S., 1988. Statistical Analysis of Cointegration Vectors, *Journal of Economic Dynamics and Control* 12, 231-254.

[18] Johansen, S., 1991. Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models, *Econometrica* 59, 1551-1580.

[19] Lettau, M. and S. Ludvigson, 2001. Consumption, Aggregate Wealth and Expected Stock Returns, *Journal of Finance* 56, 815-849.

[20] Leybourne, S.J. and B.P.M. McCabe, 1993. A Simple Test for Cointegration, *Oxford Bulletin of Economics and Statistics* 55, 97-103.

[21] Mishkin, F.S., 1992. Is the Fisher effect for real?, *Journal of Monetary Economics* 30, 195-215.

[22] Phillips, P.C.B., 1987. Towards a Unified Asymptotic Theory of Autoregression, *Biometrika* 74, 535-547.

[23] Phillips, P.C.B., 1988. Regression Theory for Near-Integrated Time Series, *Econometrica* 56, 1021-1043.

[24] Phillips, P.C.B., ans S. Ouliaris, 1990. Asymptotic Properties of Residual Based Tests for Cointegration, *Econometrica* 58, 165-193.

[25] Phillips, P.C.B., and V. Solo, 1992. Asymptotics for Linear Processes, *Annals of Statistics*, 20, 971-1001.

[26] Schwarz, G., 1978. Estimating the Dimension of a Model, *Annals of Statistics* 6, 461–464.

[27] Shin, Y., 1994. A Residual-Based Test of the Null of Cointegration Against the Alternative of No Cointegration, *Econometric Theory* 10, 91-115.

[28] Stock, J.H., 1991. Confidence intervals for the largest autoregressive root in U.S. economic time-series. *Journal of Monetary Economics* 28, 435-460.

[29] Stock, J.H., 1997. Cointegration, Long-Run Comovements, and Long-Horizon Forecasting, in D. Kreps and K.F. Wallis (eds), *Advances in Econometrics: Proceedings of the Seventh World Congress of the Econometric Society*, vol. III. Cambridge: Cambridge University Press, 34-60.

[30] Wallace, M. and J. Warner, 1993. The Fisher Effect and the Term Structure of Interest Rates: Test of Cointegration. *Review of Economics and Statistics* 75, 320-324.

[31] Wright J.H., 2000. Confidence Sets for Cointegrating Coefficients Based on Stationarity Tests, *Journal of Business and Economic Statistics* 18, 211-222.

[32] Wu, J.-L. and S.-L. Chen, 2001. Mean Reversion of Interest Rates in the Eurocurrency Market, *Oxford Bulletin of Economics and Statistics* 63, 459-474.

[33] Wu, Y. and H. Zhang, (1996). Mean Reversion in Interest Rates: New Evidence from a Panel of OECD Countries, *Journal of Money, Credit and Banking* 28, 604-621.

Table A1: Lower confidence bounds for c based on the DF-GLS statistic. For a given value of the DF-GLS statistic, without a time trend included, the following columns give lower confidence bounds of the local-to-unity parameter c with confidence levels of $95, 75, 50, 25$, and 5 percent, respectively.

DF-GLS	95%	75%	50%	25%	5%	DF-GLS	95%	75%	50%	25%	5%
1.0	-0.29	0.72	1.47	2.39	4.23	-2.0	-14.90	-10.41	-7.44	-4.51	-0.31
0.9	-0.40	0.65	1.41	2.34	4.19	-2.1	-15.95	-11.37	-8.25	-5.21	-0.88
0.8	-0.50	0.57	1.35	2.29	4.15	-2.2	-17.14	-12.35	-9.09	-5.94	-1.36
0.7	-0.63	0.49	1.29	2.24	4.12	-2.3	-18.34	-13.38	-9.97	-6.74	-1.98
0.6	-0.76	0.40	1.23	2.19	4.08	-2.4	-19.57	-14.42	-10.92	-7.48	-2.55
0.5	-0.91	0.30	1.15	2.13	4.04	-2.5	-20.84	-15.48	-11.89	-8.32	-3.28
0.4	-1.07	0.20	1.07	2.07	4.00	-2.6	-22.15	-16.61	-12.91	-9.19	-3.95
0.3	-1.25	0.09	0.99	2.02	3.95	-2.7	-23.53	-17.78	-13.95	-10.06	-4.69
0.2	-1.46	-0.03	0.90	1.94	3.90	-2.8	-24.93	-18.98	-15.03	-11.06	-5.52
0.1	-1.66	-0.17	0.80	1.87	3.85	-2.9	-26.34	-20.20	-16.13	-12.06	-6.28
0.0	-1.89	-0.31	0.70	1.79	3.80	-3.0	-27.71	-21.49	-17.29	-13.08	-7.14
-0.1	-2.14	-0.46	0.59	1.71	3.75	-3.1	-29.27	-22.81	-18.45	-14.12	-7.97
-0.2	-2.41	-0.63	0.48	1.62	3.69	-3.2	-30.86	-24.17	-19.62	-15.21	-8.88
-0.3	-2.72	-0.82	0.34	1.53	3.63	-3.3	-32.44	-25.53	-20.87	-16.33	-9.84
-0.4	-3.05	-1.03	0.18	1.42	3.57	-3.4	-34.06	-26.94	-22.15	-17.52	-10.83
-0.5	-3.45	-1.29	-0.02	1.30	3.51	-3.5	-35.78	-28.39	-23.49	-18.70	-11.80
-0.6	-3.84	-1.60	-0.23	1.15	3.40	-3.6	-37.43	-29.87	-24.85	-19.87	-12.87
-0.7	-4.31	-1.94	-0.47	0.98	3.28	-3.7	-39.09	-31.44	-26.24	-21.16	-13.96
-0.8	-4.87	-2.32	-0.75	0.78	3.17	-3.8	-40.85	-32.98	-27.65	-22.48	-15.09
-0.9	-5.44	-2.78	-1.10	0.54	3.06	-3.9	-42.69	-34.55	-29.11	-23.82	-16.25
-1.0	-6.04	-3.27	-1.47	0.28	2.91	-4.0	-44.52	-36.22	-30.62	-25.18	-17.52
-1.1	-6.73	-3.79	-1.90	-0.06	2.71	-4.1	-46.35	-37.87	-32.17	-26.55	-18.71
-1.2	-7.45	-4.37	-2.35	-0.39	2.49	-4.2	-48.24	-39.50	-33.70	-27.93	-19.87
-1.3	-8.19	-4.97	-2.88	-0.76	2.29	-4.3	-50.14	-41.27	-35.31	-29.44	-21.22
-1.4	-9.04	-5.66	-3.40	-1.18	2.01	-4.4	-52.14	-43.07	-36.94	-30.94	-22.57
-1.5	-9.90	-6.33	-3.97	-1.65	1.74	-4.5	-53.96	-44.86	-38.58	-32.45	-23.89
-1.6	-10.82	-7.05	-4.60	-2.15	1.38	-4.6	-56.08	-46.68	-40.23	-34.00	-25.21
-1.7	-11.75	-7.85	-5.23	-2.72	1.03	-4.7	-58.20	-48.54	-41.95	-35.67	-26.59
-1.8	-12.78	-8.65	-5.94	-3.27	0.59	-4.8	-60.27	-50.39	-43.70	-37.29	-28.05
-1.9	-13.84	-9.51	-6.69	-3.86	0.22	-4.9	-62.38	-52.31	-45.50	-38.90	-29.53

Table A2: Lower confidence bounds for c based on the DF-GLS statistic with a linear time trend included. For a given value of the DF-GLS statistic, allowing for a linear time trend, the following columns give lower confidence bounds of the local-to-unity parameter c with confidence levels of $95, 75, 50, 25$, and 5 percent, respectively.

DF-GLS	95%	75%	50%	25%	5%	DF-GLS	95%	75%	50%	25%	5%
1.0	2.20	2.63	3.07	3.72	5.24	-2.0	-12.90	-8.15	-4.56	1.69	3.27
0.9	2.16	2.60	3.04	3.69	5.20	-2.1	-14.04	-9.15	-5.54	1.28	3.12
0.8	2.12	2.57	3.01	3.65	5.16	-2.2	-15.26	-10.17	-6.48	-2.10	2.95
0.7	2.09	2.53	2.97	3.62	5.13	-2.3	-16.52	-11.28	-7.47	-3.22	2.76
0.6	2.05	2.50	2.93	3.58	5.09	-2.4	-17.85	-12.40	-8.49	-4.26	2.58
0.5	2.02	2.46	2.90	3.55	5.05	-2.5	-19.14	-13.55	-9.59	-5.30	2.39
0.4	1.97	2.42	2.86	3.51	5.01	-2.6	-20.49	-14.77	-10.67	-6.33	2.19
0.3	1.93	2.38	2.82	3.47	4.97	-2.7	-21.97	-16.04	-11.80	-7.41	1.96
0.2	1.88	2.34	2.78	3.42	4.93	-2.8	-23.44	-17.35	-12.98	-8.47	1.61
0.1	1.83	2.30	2.74	3.38	4.88	-2.9	-24.97	-18.67	-14.20	-9.62	-1.55
0.0	1.78	2.26	2.70	3.33	4.84	-3.0	-26.55	-20.02	-15.47	-10.75	-3.10
-0.1	1.72	2.22	2.65	3.29	4.79	-3.1	-28.14	-21.48	-16.78	-11.91	-4.27
-0.2	1.64	2.17	2.61	3.24	4.75	-3.2	-29.86	-22.97	-18.10	-13.19	-5.55
-0.3	1.56	2.12	2.56	3.20	4.70	-3.3	-31.64	-24.49	-19.51	-14.48	-6.68
-0.4	1.47	2.07	2.52	3.15	4.64	-3.4	-33.42	-26.05	-20.96	-15.80	-7.91
-0.5	1.32	2.02	2.47	3.10	4.59	-3.5	-35.21	-27.67	-22.45	-17.15	-9.12
-0.6	-0.81	1.95	2.42	3.05	4.54	-3.6	-37.09	-29.37	-23.95	-18.53	-10.30
-0.7	-1.58	1.89	2.36	3.01	4.49	-3.7	-38.99	-31.09	-25.56	-19.93	-11.62
-0.8	-2.29	1.82	2.31	2.95	4.43	-3.8	-40.97	-32.85	-27.19	-21.42	-12.96
-0.9	-2.95	1.75	2.26	2.89	4.37	-3.9	-43.06	-34.64	-28.85	-22.97	-14.34
-1.0	-3.70	1.61	2.18	2.82	4.31	-4.0	-45.18	-36.50	-30.56	-24.57	-15.79
-1.1	-4.43	1.45	2.10	2.76	4.25	-4.1	-47.18	-38.45	-32.34	-26.18	-17.31
-1.2	-5.15	-0.72	2.03	2.69	4.17	-4.2	-49.36	-40.35	-34.13	-27.89	-18.77
-1.3	-6.01	-1.85	1.92	2.60	4.09	-4.3	-51.66	-42.37	-36.01	-29.56	-20.19
-1.4	-6.83	-2.75	1.80	2.52	4.01	-4.4	-53.91	-44.46	-37.90	-31.31	-21.83
-1.5	-7.74	-3.62	1.63	2.42	3.91	-4.5	-56.27	-46.60	-39.83	-33.15	-23.44
-1.6	-8.69	-4.46	1.36	2.31	3.81	-4.6	-58.74	-48.74	-41.89	-35.04	-25.00
-1.7	-9.67	-5.33	-1.56	2.19	3.69	-4.7	-61.20	-50.98	-43.94	-36.99	-26.66
-1.8	-10.65	-6.22	-2.69	2.06	3.56	-4.8	-63.78	-53.32	-46.07	-38.96	-28.52
-1.9	-11.76	-7.17	-3.64	1.89	3.42	-4.9	-66.25	-55.64	-48.29	-40.95	-30.28

Table A3: Five percent critical values for the AEG statistic. This table gives the critical values for the AEG statistic at the five percent level, for different values of c under the assumption that $c_1 = ... = c_m = c$, and for one to five regressors. The first set of values provide the critical values when no intercept is included in the cointegrating regression. The second set provides the values when an intercept, but no time trend is included and the third set of values represent the case with both an intercept and a linear time trend. The values are based on $100,000$ repetitions with $T = 1,000$.

	No constant					Constant					Constant and trend				
c	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
0	-2.77	-3.30	-3.73	-4.09	-4.41	-3.34	-3.77	-4.10	-4.42	-4.72	-3.79	-4.14	-4.44	-4.73	-5.00
-1	-2.80	-3.32	-3.74	-4.09	-4.42	-3.37	-3.76	-4.12	-4.43	-4.73	-3.79	-4.14	-4.46	-4.72	-5.01
-2	-2.88	-3.35	-3.75	-4.10	-4.42	-3.40	-3.78	-4.12	-4.44	-4.73	-3.82	-4.16	-4.45	-4.74	-5.01
-3	-2.96	-3.39	-3.78	-4.12	-4.44	-3.45	-3.82	-4.15	-4.46	-4.75	-3.86	-4.18	-4.48	-4.76	-5.03
-4	-3.05	-3.45	-3.82	-4.15	-4.45	-3.50	-3.86	-4.17	-4.47	-4.76	-3.89	-4.21	-4.50	-4.78	-5.03
-5	-3.15	-3.51	-3.86	-4.17	-4.46	-3.56	-3.89	-4.21	-4.49	-4.77	-3.94	-4.24	-4.53	-4.79	-5.05
-6	-3.23	-3.58	-3.91	-4.21	-4.51	-3.62	-3.94	-4.24	-4.53	-4.80	-3.98	-4.27	-4.56	-4.81	-5.07
-7	-3.32	-3.64	-3.96	-4.26	-4.53	-3.68	-4.00	-4.28	-4.56	-4.81	-4.03	-4.32	-4.58	-4.85	-5.09
-8	-3.41	-3.72	-4.01	-4.30	-4.57	-3.75	-4.05	-4.33	-4.60	-4.86	-4.08	-4.36	-4.63	-4.89	-5.11
-9	-3.50	-3.79	-4.07	-4.34	-4.61	-3.82	-4.11	-4.37	-4.64	-4.89	-4.14	-4.41	-4.66	-4.91	-5.16
-10	-3.58	-3.86	-4.13	-4.40	-4.65	-3.89	-4.16	-4.43	-4.68	-4.92	-4.19	-4.44	-4.71	-4.93	-5.18
-11	-3.68	-3.93	-4.19	-4.45	-4.69	-3.97	-4.22	-4.47	-4.72	-4.95	-4.26	-4.51	-4.74	-4.98	-5.21
-12	-3.75	-4.01	-4.26	-4.50	-4.73	-4.03	-4.29	-4.52	-4.76	-4.99	-4.32	-4.56	-4.79	-5.02	-5.23
-13	-3.84	-4.08	-4.33	-4.56	-4.78	-4.10	-4.34	-4.58	-4.81	-5.03	-4.37	-4.60	-4.84	-5.06	-5.28
-14	-3.92	-4.16	-4.38	-4.60	-4.83	-4.18	-4.41	-4.64	-4.85	-5.07	-4.44	-4.66	-4.88	-5.09	-5.30
-15	-4.01	-4.23	-4.44	-4.66	-4.88	-4.25	-4.47	-4.69	-4.90	-5.11	-4.50	-4.71	-4.93	-5.14	-5.34
-16	-4.06	-4.30	-4.50	-4.72	-4.94	-4.30	-4.54	-4.74	-4.96	-5.16	-4.55	-4.78	-4.97	-5.19	-5.39
-17	-4.15	-4.37	-4.57	-4.78	-4.98	-4.38	-4.60	-4.80	-5.00	-5.20	-4.61	-4.82	-5.02	-5.23	-5.42
-18	-4.22	-4.43	-4.63	-4.83	-5.03	-4.44	-4.66	-4.85	-5.05	-5.25	-4.67	-4.88	-5.07	-5.27	-5.46
-19	-4.29	-4.50	-4.69	-4.89	-5.07	-4.50	-4.72	-4.90	-5.11	-5.29	-4.73	-4.94	-5.12	-5.32	-5.50
-20	-4.37	-4.57	-4.76	-4.94	-5.13	-4.58	-4.77	-4.97	-5.16	-5.34	-4.79	-4.99	-5.18	-5.37	-5.55
-21	-4.44	-4.62	-4.81	-5.00	-5.19	-4.65	-4.83	-5.01	-5.21	-5.39	-4.85	-5.04	-5.21	-5.41	-5.59
-22	-4.51	-4.69	-4.87	-5.04	-5.23	-4.70	-4.89	-5.07	-5.25	-5.42	-4.91	-5.08	-5.27	-5.45	-5.62
-23	-4.57	-4.76	-4.93	-5.11	-5.28	-4.77	-4.96	-5.13	-5.30	-5.47	-4.97	-5.15	-5.33	-5.49	-5.67
-24	-4.65	-4.81	-5.00	-5.16	-5.34	-4.83	-5.00	-5.19	-5.35	-5.52	-5.03	-5.19	-5.38	-5.54	-5.72
-25	-4.71	-4.88	-5.04	-5.22	-5.39	-4.89	-5.06	-5.23	-5.41	-5.57	-5.08	-5.25	-5.42	-5.59	-5.76
-26	-4.77	-4.93	-5.11	-5.27	-5.44	-4.95	-5.11	-5.29	-5.45	-5.62	-5.14	-5.29	-5.47	-5.64	-5.81
-27	-4.84	-5.00	-5.16	-5.32	-5.49	-5.01	-5.18	-5.34	-5.50	-5.67	-5.19	-5.36	-5.53	-5.69	-5.85
-28	-4.90	-5.06	-5.21	-5.38	-5.53	-5.08	-5.23	-5.39	-5.55	-5.70	-5.25	-5.41	-5.56	-5.72	-5.88
-29	-4.96	-5.12	-5.28	-5.43	-5.59	-5.13	-5.29	-5.45	-5.60	-5.76	-5.30	-5.47	-5.63	-5.77	-5.93
-30	-5.02	-5.18	-5.32	-5.47	-5.64	-5.19	-5.34	-5.49	-5.64	-5.81	-5.36	-5.52	-5.66	-5.82	-5.98

Table A3: Critical values for the AEG statistic (continued).

	No constant					Constant					Constant and trend				
c	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
-31	-5.09	-5.23	-5.39	-5.53	-5.69	-5.25	-5.39	-5.55	-5.70	-5.85	-5.41	-5.57	-5.72	-5.86	-6.02
-32	-5.14	-5.30	-5.44	-5.59	-5.74	-5.30	-5.46	-5.61	-5.75	-5.90	-5.47	-5.62	-5.76	-5.92	-6.07
-33	-5.20	-5.35	-5.49	-5.63	-5.79	-5.36	-5.51	-5.66	-5.80	-5.94	-5.52	-5.67	-5.82	-5.95	-6.11
-34	-5.26	-5.41	-5.54	-5.69	-5.83	-5.42	-5.56	-5.70	-5.85	-5.99	-5.57	-5.72	-5.86	-6.01	-6.15
-35	-5.32	-5.46	-5.60	-5.75	-5.89	-5.47	-5.61	-5.76	-5.90	-6.04	-5.62	-5.76	-5.91	-6.05	-6.20
-36	-5.38	-5.51	-5.65	-5.80	-5.94	-5.53	-5.66	-5.81	-5.95	-6.09	-5.67	-5.82	-5.96	-6.10	-6.24
-37	-5.44	-5.56	-5.71	-5.84	-5.98	-5.58	-5.71	-5.86	-5.99	-6.13	-5.72	-5.87	-6.01	-6.14	-6.28
-38	-5.50	-5.63	-5.76	-5.90	-6.03	-5.64	-5.77	-5.91	-6.04	-6.18	-5.79	-5.92	-6.06	-6.19	-6.33
-39	-5.55	-5.68	-5.81	-5.94	-6.07	-5.69	-5.82	-5.95	-6.09	-6.22	-5.84	-5.97	-6.10	-6.24	-6.36
-40	-5.60	-5.73	-5.86	-5.99	-6.12	-5.74	-5.87	-6.00	-6.13	-6.27	-5.88	-6.01	-6.15	-6.28	-6.41
-41	-5.65	-5.78	-5.91	-6.04	-6.17	-5.79	-5.92	-6.05	-6.18	-6.31	-5.93	-6.06	-6.20	-6.32	-6.46
-42	-5.71	-5.84	-5.96	-6.09	-6.22	-5.84	-5.97	-6.10	-6.22	-6.35	-5.99	-6.11	-6.24	-6.37	-6.49
-43	-5.77	-5.89	-6.01	-6.13	-6.26	-5.90	-6.02	-6.15	-6.27	-6.40	-6.04	-6.16	-6.28	-6.41	-6.54
-44	-5.81	-5.93	-6.06	-6.18	-6.30	-5.94	-6.06	-6.19	-6.32	-6.44	-6.08	-6.20	-6.33	-6.45	-6.58
-45	-5.87	-5.98	-6.11	-6.23	-6.36	-6.00	-6.11	-6.24	-6.36	-6.49	-6.13	-6.25	-6.38	-6.49	-6.63
-46	-5.92	-6.04	-6.16	-6.28	-6.41	-6.04	-6.17	-6.29	-6.41	-6.54	-6.18	-6.30	-6.42	-6.55	-6.68
-47	-5.96	-6.09	-6.20	-6.33	-6.44	-6.09	-6.22	-6.33	-6.45	-6.58	-6.22	-6.35	-6.46	-6.59	-6.71
-48	-6.01	-6.14	-6.26	-6.37	-6.49	-6.14	-6.26	-6.38	-6.49	-6.61	-6.27	-6.40	-6.51	-6.63	-6.75
-49	-6.07	-6.19	-6.30	-6.43	-6.54	-6.19	-6.32	-6.43	-6.55	-6.67	-6.32	-6.44	-6.56	-6.68	-6.80
-50	-6.13	-6.23	-6.36	-6.47	-6.59	-6.25	-6.36	-6.48	-6.59	-6.72	-6.37	-6.48	-6.60	-6.72	-6.84
-51	-6.17	-6.29	-6.40	-6.51	-6.63	-6.29	-6.41	-6.53	-6.63	-6.76	-6.42	-6.54	-6.65	-6.76	-6.89
-52	-6.22	-6.33	-6.44	-6.56	-6.67	-6.35	-6.45	-6.56	-6.68	-6.80	-6.47	-6.58	-6.69	-6.81	-6.92
-53	-6.27	-6.39	-6.49	-6.60	-6.71	-6.39	-6.50	-6.62	-6.72	-6.83	-6.51	-6.63	-6.74	-6.85	-6.95
-54	-6.31	-6.43	-6.54	-6.65	-6.76	-6.43	-6.55	-6.66	-6.76	-6.88	-6.55	-6.67	-6.78	-6.89	-7.00
-55	-6.38	-6.47	-6.58	-6.69	-6.80	-6.49	-6.59	-6.70	-6.81	-6.92	-6.60	-6.71	-6.82	-6.93	-7.04
-56	-6.41	-6.52	-6.62	-6.75	-6.85	-6.52	-6.64	-6.74	-6.86	-6.97	-6.64	-6.76	-6.86	-6.98	-7.09
-57	-6.46	-6.57	-6.68	-6.78	-6.89	-6.58	-6.68	-6.80	-6.89	-7.01	-6.69	-6.80	-6.91	-7.01	-7.13
-58	-6.51	-6.62	-6.72	-6.83	-6.93	-6.62	-6.73	-6.83	-6.94	-7.05	-6.74	-6.85	-6.95	-7.06	-7.16
-59	-6.56	-6.66	-6.77	-6.87	-6.98	-6.67	-6.77	-6.89	-6.98	-7.09	-6.78	-6.88	-7.00	-7.10	-7.21
-60	-6.60	-6.71	-6.82	-6.92	-7.02	-6.71	-6.82	-6.93	-7.03	-7.13	-6.82	-6.93	-7.04	-7.15	-7.25

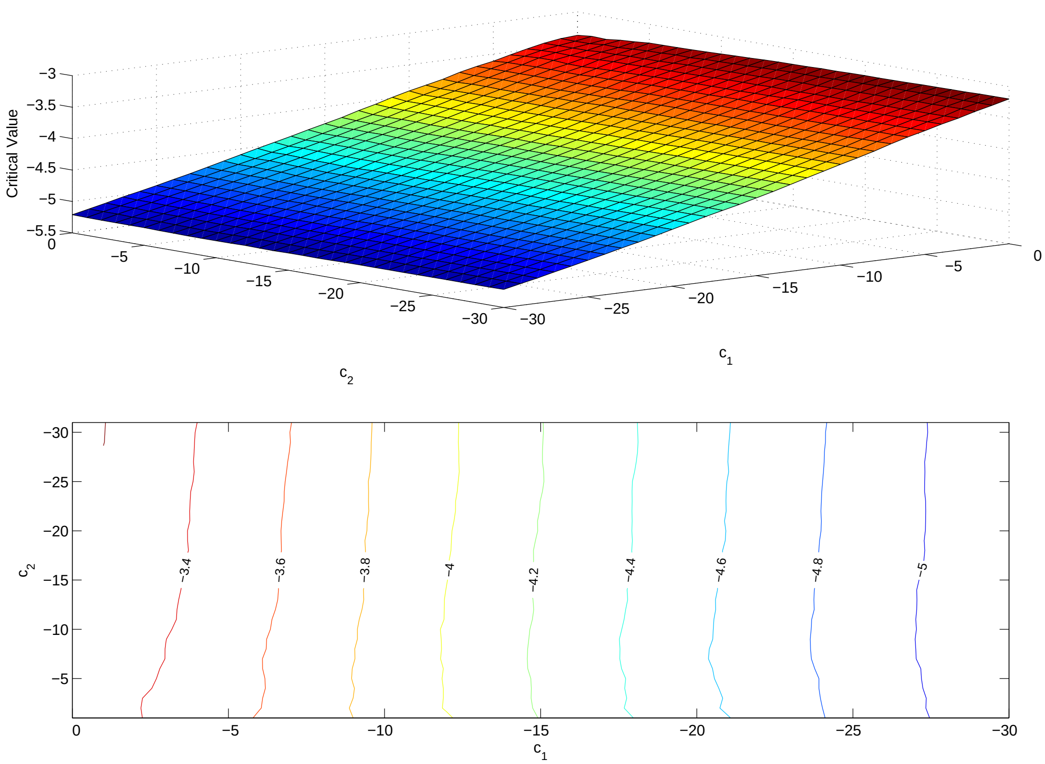

Figure 1: Critical values at the five percent level for the AEG test as a function of c_1 and c_2. The top panel shows the surface describing the five percent critical values of the AEG test, in the case of an intercept and one regressor, when c_1 and c_2 are non-identical. The bottom panel shows the corresponding contour plot. The values are based on $10,000$ repetitions with $T = 1,000$.

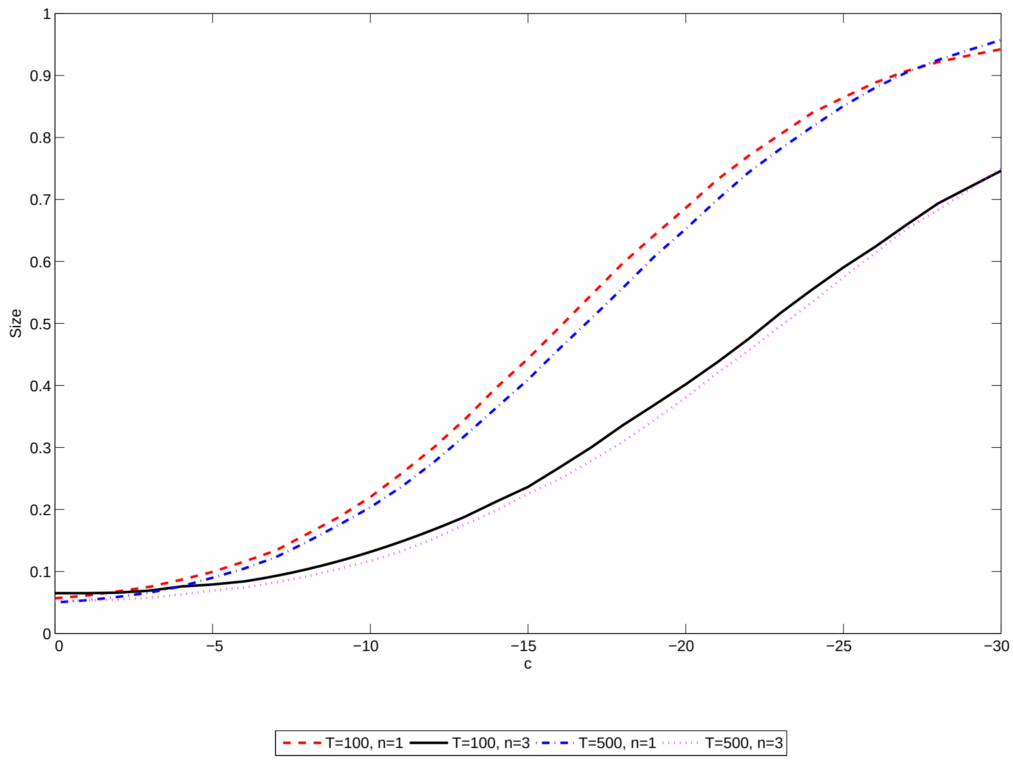

Figure 2: Size properties of the Engle and Granger (1987) test of cointegration, as a function of the local-to-unity parameter c. The graph shows the average rejection rates under the null hypothesis of no cointegration for the Engle and Granger test of cointegration, i.e. the standard AEG test evaluated under the assumption that $c = 0$, for different true values of c. The sample size is equal to either $T = 100$ or 500, and the number of regressors equal to either $n = 1$ or 3. The true persistence in the data is equal to $C = diag\,(c, ..., c)$, where c varies between 0 and -30. The results are based on $10,000$ repetitions.

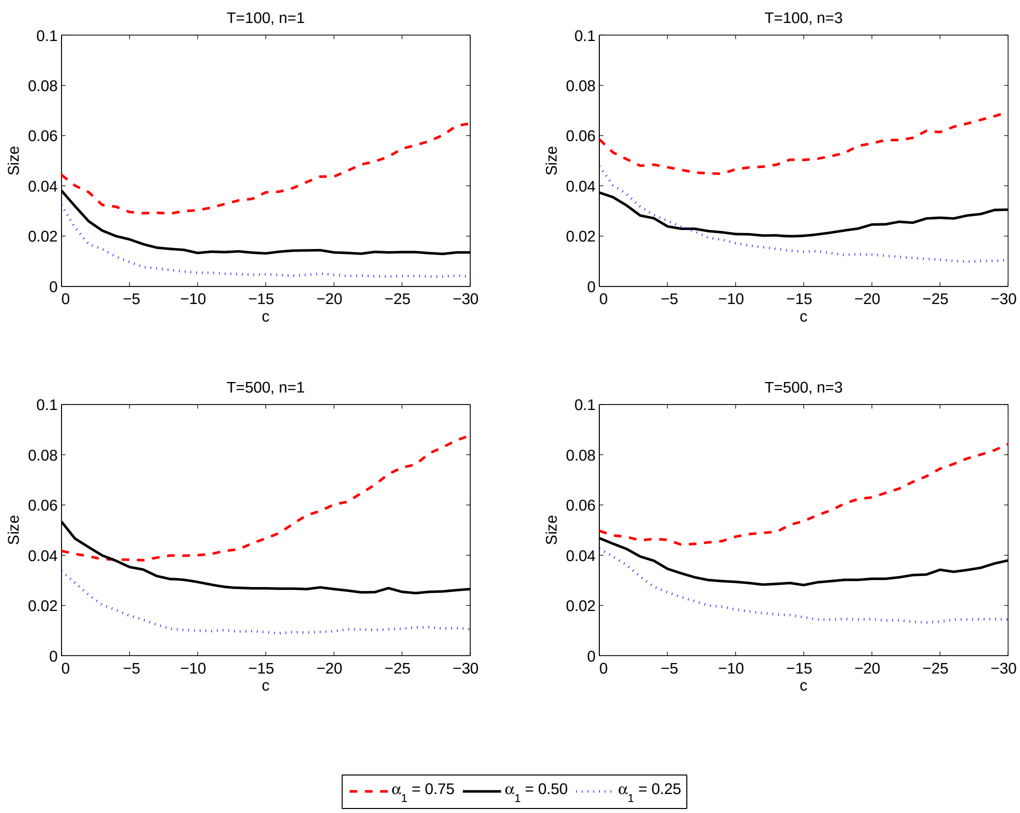

Figure 3: Size properties of the Bonferroni AEG test when the variables all have equal persistence. The graphs show the average rejection rates for the Bonferroni AEG test, under the null hypothesis of no cointegration, for $\alpha_1 = 0.75, 0.50$, and 0.25. The sample size is equal to either $T = 100$ or 500, and the number of regressors is equal to either $n = 1$ or 3. The true persistence in the data is equal to $C = diag\,(c, ..., c)$, where c varies between 0 and -30. The results are based on $10,000$ repetitions.

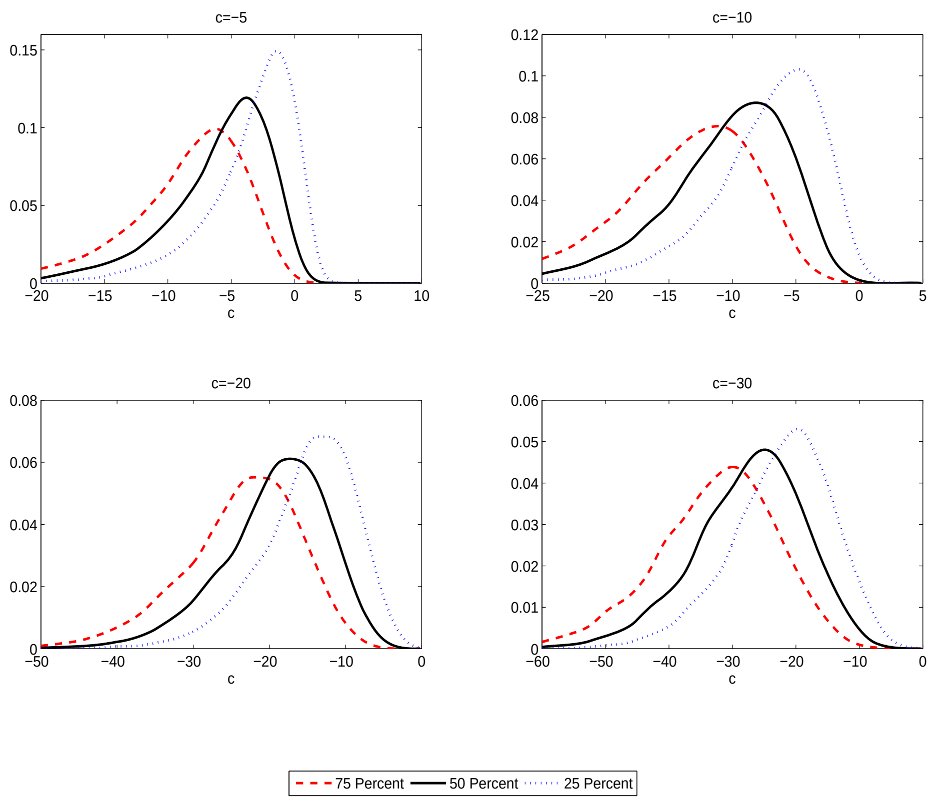

Figure 4: Estimates of the lower bounds of c. The graphs show the density of the estimates of the lower bounds of c, with confidence levels of 75, 50, and 25 percent, based on inversion of the DF-GLS statistic. The results are obtained from $10,000$ simulations of a univariate local-to-unity process, with local-to-unity parameter c, *iid* normal innovations and sample size $T = 500$.

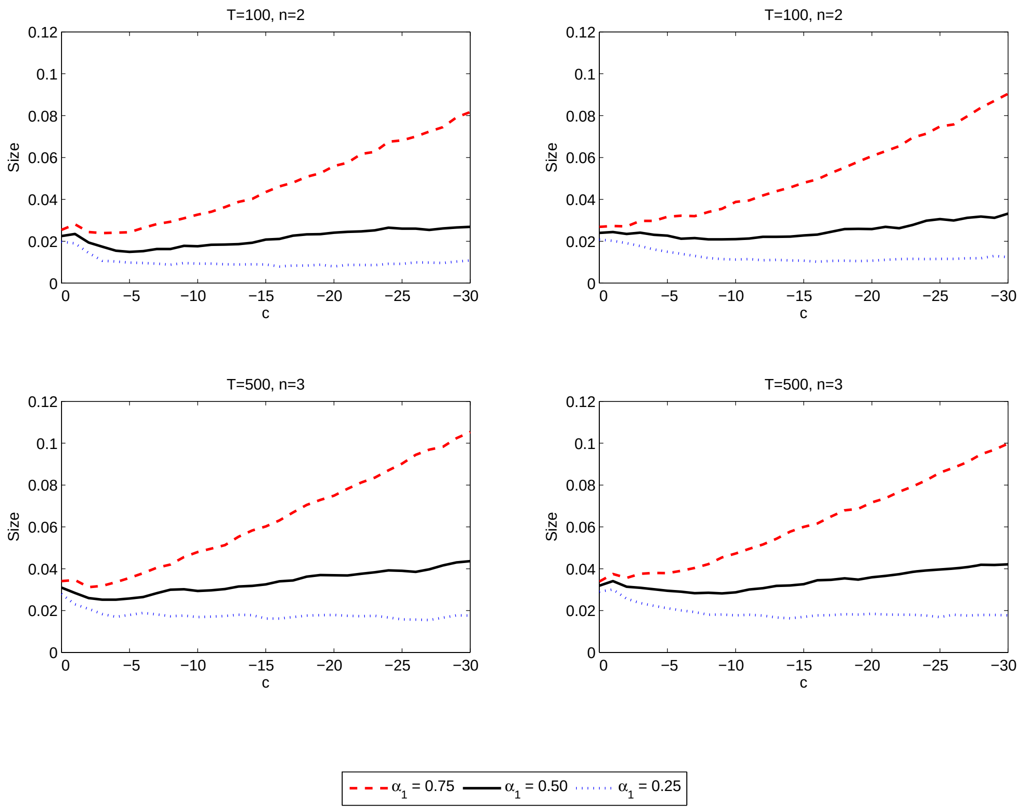

Figure 5: Size properties of the Bonferroni AEG test when c_i is not identical for all i. The graphs show the average rejection rates for the Bonferroni AEG test, under the null hypothesis of no cointegration, for $\alpha_1 = 0.75, 0.50$, and 0.25. The sample size is equal to either $T = 100$ or 500, and the number of regressors is equal to either $n = 2$ or 3. For $n = 2$, the true persistence in the data is equal to $C = diag\,(c_1, -10, -20)$, and for $n = 3$, $C = diag\,(c_1, 0, -10, -20)$, where c_1 varies between 0 and -30. The results are based on $10,000$ repetitions.

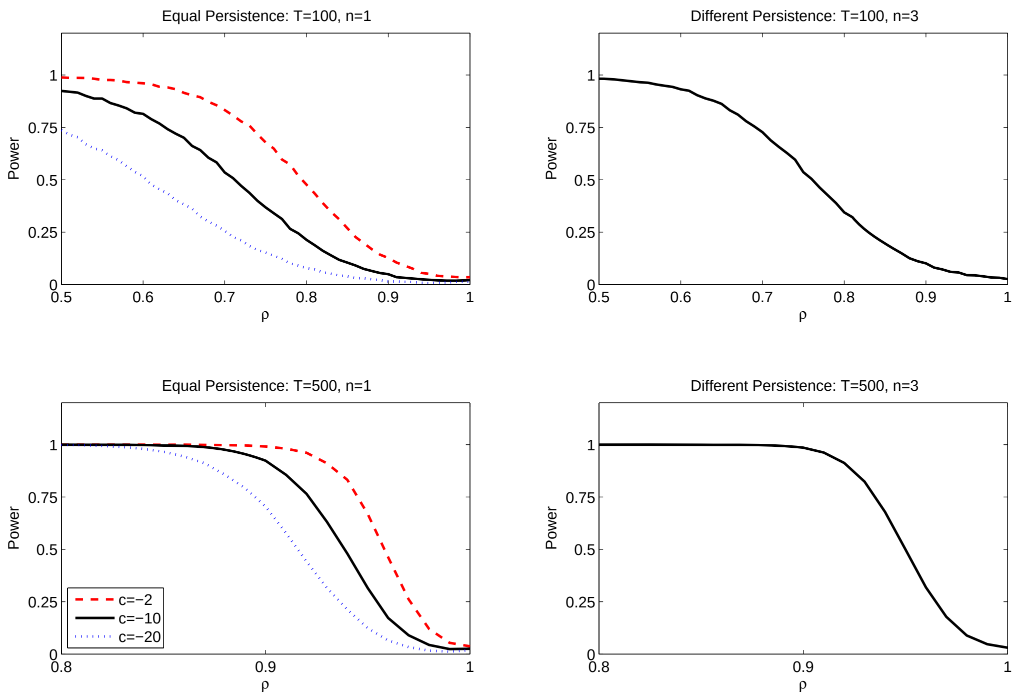

Figure 6: Power properties of the Bonferroni AEG test. The graphs show the average rejection rates of the Bonferroni AEG test, for $\alpha_1 = 0.50$, under the alternative of cointegration. The power is plotted as a function of ρ, the $AR(1)$ persistence parameter in the cointegrating residuals. The sample size is set equal to either $T = 100$ or 500. The left column gives results for the case of one regressor with persistence $C_2 = -2, -10$, or -20. The right column gives the results for the case with three regressors and $C_2 = diag(0, -10, -20)$. The results are based on $10,000$ repetitions.