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Abstract

Central banks, whether independent or not, may occasionally be subject to external pressures to change policy objectives. We analyze the optimal response of central banks to such pressures and the resulting macroeconomic consequences. We consider several alternative scenarios regarding policy objectives, the degree of commitment and the timing of external pressures. The possibility to adopt "more liberal" objectives in the future increases current inflation through an accommodation effect. Simultaneously, the central bank tries to anchor inflation by promising to be even "more conservative" in the future. The immediate effect is an output contraction, the opposite of what the pressures to adopt "more liberal" objectives may be aiming. We also discuss the opposite case, where objectives may become "more conservative" in the future, which may be the relevant case for countries considering the adoption of inflation targeting.

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1 Introduction

In the last decades, there has been a big effort to build monetary institutions that are independent from political forces and other types of external pressures. In most cases, this process has implied the definition of specific goals to be assigned to central banks, typically in terms of inflation and output stabilization. However, it has often been the case that central banks are subject to some pressures to change their policy goals. For instance, the European Central Bank has been recently criticized for adopting policies that do not properly take into account output growth. In other cases, as it is currently happening in the United States, the adoption of inflation targeting or other institutional reforms are being discussed. Even if there is not an immediate reaction to such pressures, the central bank and the public may attach some probability that monetary policy may change its objectives in the future. Changes in policy objectives may also simply be due to the the appointment of a new central bank chairman or staff, who may have different views from the predecessors.

The aim of this paper is to study how the possibility of changes in future policy goals affects current economic outcomes. It is widely accepted in the modern macroeconomic literature that firms and households, when setting current prices and wages, take into account their expectations about future inflation. In this context, when a policy that generates a higher inflation bias (henceforth, a "liberal policy") is expected in the future, inflation expectations increase and will be reflected in current prices and wages. As a consequence, given the policy chosen by the central bank, the possibility of a future change in policy objectives affects the economic variables already in the current period. In this paper, we analyze the optimal response of a central bank in such situation and its macroeconomic effects.

Optimal monetary policy is typically characterized under the assumptions that a central bank is in charge for an infinite horizon, it can credibly commit to future policy plans, and its policy goals do not change. In this paper, we relax these assumptions. First, we need to consider that current and future policy objectives may not necessarily coincide. Second, when considering the possibility of changes in policy objectives, the assumption about the credibility of the central bank needs to be rethought. For instance, if the central bank has received some external pressures to change its policy goals, it will adopt the best possible policy to fulfill its new objectives, and the plans made when priorities were different will become irrelevant.

We consider several alternative scenarios regarding policy objectives, the degree of commitment and the timing of external pressures. The main part of our analysis refers to the case where an independent central bank has been appointed, possibly one with more conservative objectives than society.[1] The central bank is not completely insulated from the opinions prevailing in the economic environment, and may occasionally be subject to some external pressures to adopt more liberal objectives in the future. This typically happens when countries face adverse economic conditions, like recessions or financial crisis. In all the settings taken into consideration, the possibility that liberal policies may be implemented implies an increase in current inflation through an accommodation effect. At the same time, the central bank tries to anchor inflation expectations by promising to be even more conservative in the future. The immediate effect is an output contraction, which is the opposite of what the pressures to implement liberal policies may be aiming for. These effects are stronger the higher is the probability that objectives change. In this respect, adopting inflation targeting may be better than appointing a conservative central banker. In the former case, changing objectives usually requires costly institutional reform. In the latter, it just requires that the central bank chairman and advisors, at the end of their term of office, are substituted by ones with different policy objectives.

We also analyze the opposite case, where objectives may become more conservative in the future. This is the relevant scenario for countries discussing the adoption of more stringent and explicit inflation objectives, which is, arguably, the case of the United States. In this context, the possibility of more conservative policy in the future creates a positive externality.

Our paper is related to the literature about political economy in monetary policy, following the seminal contribution of Alesina (1987). We want to emphasize that our goal is not to provide a partisan analysis of monetary policy, where one attributes economic cycles to political parties. According to the empirical literature, it seems difficult to match timely and systematically certain parties with effective changes in monetary policy.[2] It seems implausible that all governments have always successfully

[1] As shown by Rogoff (1985), appointing a central banker with higher aversion towards inflation than the overall society reduces the inefficiencies associated with time-inconsistency.

[2] Alesina et al. (1997) point out several empirical successes of the Alesina (1987) model, while Sheffrin (1989) and Drazen (2000) point out some empirical failures. Chappell et al. (1993) conclude that political influences on the Federal Reserve are not clearly connected with a party tenure, since they occur indirectly through the appointments of FOMC members and thus different ideologies overlap in the committee. Faust and Irons (1999) conclude that partisan effects in US macroeconomic data are fragile, and that there is little evidence that the partisan effects on the

influenced central bank's objectives, and that they did so as soon as they were elected. However, there have been frequent cases where central banks have been subject to external pressures. Our goal is to understand what are the effects of certain types of pressures, and what is the optimal response of a central bank in such situation. Also, our analysis can provide a rationale for the changes in interest rate rules, as analyzed by Davig and Leeper (2007).

We carry out our analysis in the framework commonly used in the literature on monetary policy design. As it has become standard, we do not assume a New Classical Phillips curve and consider instead its New Keynesian counterpart. The features of modern science of monetary policy considered here have important consequences, and induce different mechanisms than the ones modeled in Alesina (1987) and other "partisan models" of monetary policy. In the New Keynesian framework and unlike its New Classical counterpart, the possibility of a future change in policy objectives implies a different level of inflation and output in earlier periods. This is because firms and households, when setting current prices and wages, take into account future economic conditions. With respect to the partisan theory of economic fluctuations, our analysis suggests that it is difficult to find a link between economic outcomes and the objectives of certain policymakers.

We also study the importance of central bank credibility when policy objectives can change. To do so, we relax the discretion assumption, typical of the political economy literature. Instead, and following the recent contributions of Schaumburg and Tambalotti (2007) and Debortoli and Nunes (2006a), we model the central bank as possessing a loose commitment technology. In this context, in order to lower inflation expectations created by the possibility to adopt liberal policies, a conservative central bank optimally promises to be even more conservative in the future. If the conservative central bank does not possess a commitment technology, as in Alesina (1987) or Rogoff (1985), such policy is not possible and the effects of future liberal objectives are stronger. Even for a Rogoff central banker, our analysis suggests that, besides other considerations, credibility or commitment is important to counteract the negative externalities generated by the pressures for liberal policies.

We first consider a simplified model where changes in policy objectives can occur in any period. This simplification allows us to obtain several analytical results. However, in reality, policy objectives can not always be changed immediately because of institutional features and policy implementation lags. We then consider a richer model where the central bank has complete insulation from external pressures in the short-run, but understands that objectives may change in the future. For example,

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economy operate through changes in monetary policy.

this can be due to the fact that the current chairman, at the end of his tenure can then be replaced by one with different views about the conduct of monetary policy. This second model also identifies more clearly the key effects arising from possible changes in future objectives.

The paper is organized as follows. Section 2, examines a simple model with an analytical solution. In that framework, we discuss the importance of commitment and inflation targeting. In section 3, we solve a model where external pressures do not materialize immediately. Section 4 examines several alternative scenarios where the basic mechanisms identified are still present. It considers different types of changes in the policy objectives, the strategic use of lagged inflation in a hybrid Phillips curve, and a case with full-commitment and no disagreement. Section 5 concludes.

2 The model

We base our analysis on a simple monetary model. Inflation dynamics are described by a New Keynesian Phillips curve (NKPC). As it is well known, the NKPC is a reduced form approximation of the relationship between inflation and output in an economy with monopolistic competition and staggered price setting.[3] The NKPC takes the form:

$$\pi_t = \kappa y_t + \beta E_t \pi_{t+1} \tag{1}$$

where π_t denotes price inflation and y_t measures the output-gap, i.e. the difference between current output and the output level that would prevail under flexible prices.

As it is standard in the optimal monetary policy literature, we assume that the central bank controls inflation directly.[4] The objectives of the monetary authority are characterized by a period quadratic loss function taking the following form:

$$U = \frac{1}{2}\left[\pi_t^2 + w(y_t - \widetilde{y}_t)^2\right] \tag{2}$$

The monetary policy authority aims at minimizing a weighted average of deviations of inflation and output-gap from their respective targets. The parameter w

[3] The theoretical framework underlying such relationship is described in Woodford (2003) and Galí (2008). This specification of the NKPC holds in a neighborhood of a zero inflation steady state. Throughout our analysis, we abstract from the changes that may derive from having a different steady state level of inflation.

[4] The interest rate i_t required to implement the desired inflation level can be obtained from the demand side of the economy, not modeled here.

measures the relative importance of output vs. inflation stabilization. The inflation target is normalized to zero, while $\widetilde{y}_t \geq 0$ represents the (exogenously given) output target. The latter variable indicates that monetary policy aims at correcting some inefficiencies in the economy, like the presence of monopolistic power, distortionary taxes, frictions in the labor market, etc. The parameter $\widetilde{y}_t \geq 0$ can therefore be interpreted as the difference between the efficient level of output and the output that would prevail under flexible prices. This difference generates a trade-off between output and inflation stabilization around their respective targets.

In what follows, we consider that the objectives of the central bank, depending on the values assigned to $\widetilde{y}$ and w, can be either "liberal" (ℓ) or "conservative" (c). For convenience, we use the term "liberal central bank" to refer to a case where the output target and the relative weight to output stabilization are higher than a "conservative central bank", that is $\widetilde{y}^{\ell} > \widetilde{y}^{c}$, $w^{\ell} > w^{c}$, or both.

2.1 Modeling policy changes

To keep the problem as simple as possible, we start by considering a case where a conservative central bank is in charge. At any point in time, the monetary authority is subject to external pressures and may adopt more liberal objectives. In this simplified model, we can derive analytical results that illustrate the main points of our analysis. In section 3, we consider the case where the central bank faces some insulation from external pressures, and hence policy objectives can change in the future but not immediately.

Consider that the objectives of a conservative central bank are characterized by a relative weight for output w^{c}, and a target for the output gap $\widetilde{y}^{c} > 0$.[5] At any point in time, with probability $0 < q < 1$ the objectives of the central bank will remain unchanged, while with probability $1 - q$ the central bank will be subject to external pressures and more liberal objectives will be adopted, i.e. setting $\widetilde{y}^{\ell} > \widetilde{y}^{c}$, $w^{\ell} > w^{c}$ or both.[6]

Moreover, we assume that the central bank can only make credible commitments about future policy while objectives remain unchanged. If objectives do change, the more liberal central bank will set a new policy, and previous commitments will be disregarded. This assumption can be justified on the grounds that if the central bank has received some external pressures to change its policy goals, it will adopt

[5]The simple case where $\widetilde{y}^{c} = 0$ is discussed in section 2.3.

[6]Our analysis can be easily extended to analyze the symmetric case where a liberal central bank faces the possibility of adopting more conservative objectives. We discuss this case in section 4.4.

the best possible policy to fulfill its new objectives, and thus disregards the plans made when priorities were different.[7]

In this context, it can be shown that the central bank makes its policy plans to solve the following problem:

$$V^{c,c} = \max_{\{\pi_t, y_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta q)^t \{-\frac{1}{2}\left[\pi_t^2 + w^c(y_t - \widetilde{y}^c)^2\right] + \beta(1-q)V^{c,\ell}\} \quad (3)$$

$$s.t. \quad \pi_t = \kappa y_t + \beta q \pi_{t+1} + \beta(1-q)\pi_0^\ell \qquad \forall t = 0, 1, ... \quad (4)$$

The objective function is given by an infinite sum, discounted at the rate βq. This reflects the positive probability that objectives will remain unchanged over the infinite future. Each term in the summation is composed by two parts. The first part, in the square brackets, is the period loss function. The second part is the value function $V^{c,\ell}$, summarizing the utility the central bank obtains if next period policy objectives change.[8]

The central bank faces a sequence of constraints represented by the NKPC, where for any period t inflation expectations are an average between two terms. The first term, with weight q, is the inflation that would prevail under the current (conservative) central bank (π_{t+1}) and upon which there is credible commitment. The second term, with weight $(1-q)$, is the inflation that would be implemented under liberal objectives (π_0^ℓ), which is taken as given by the current central bank. In equilibrium, such level of inflation is determined by solving a symmetric problem to the one described above, using as preferences those of a more liberal central bank.

This framework nests many possible alternative settings, as summarized in table 1. For example, if we set $\widetilde{y}^c = \widetilde{y}^\ell$ and $w^\ell = w^c$ we have a typical problem of a central bank, without changes in policy goals. In addition, setting $q = 1$ we have the standard full-commitment case, while by setting $q = 0$, we have the problem of a central bank operating under discretion.

2.2 Effects of changes in the output target $\widetilde{y}$

We start analyzing the case where a change in policy objectives only implies a change in the output target, while keeping unchanged the relative weight of output

[7]In section 4.3, we analyze the case where the central bank can make credible commitments contingent on its policy objectives.

[8]In the present framework, the function $V^{c,\ell}$ is just a constant, and is irrelevant for the maximization problem. The current policymaker does not have a state variable to influence the choices when policy objectives change. This analysis changes in a model with endogenous state variables, like the one considered in section 4.2.

Table 1: Possible settings

	Same objectives ($w^\ell = w^c$ and $\widetilde{y}^c = \widetilde{y}^\ell$)	Different objectives ($w^\ell > w^c$ or $\widetilde{y}^c > \widetilde{y}^\ell$)
$q = 1$	Full Commitment (Objectives never change)	
$0 < q < 1$	Loose Commitment	Probabilistic changes Commitment only if no changes
$q = 0$	Discretion	Objectives change every period Commitment is not possible

stabilization ($w^\ell = w^c = w$). It can be shown that while the current conservative objective function prevails, the output gap and inflation evolve according to

$$\pi_t = \gamma_2^{-t}\pi_0 \tag{5}$$

$$y_t - \widetilde{y}^c = -\frac{\kappa}{w}\frac{1-\gamma_2^{-(t+1)}}{1-\gamma_2^{-1}}\pi_0, \tag{6}$$

where

$$\pi_0 = \frac{1}{\gamma_2(1-\gamma_1)}\left(\kappa\widetilde{y}^c + \beta(1-q)\pi_0^\ell\right) \tag{7}$$

and $0 < \gamma_1 < 1 < \gamma_2$ are coefficients satisfying $\gamma_1\gamma_2 = \beta q$ and $\gamma_1 + \gamma_2 = (1 + \beta q + \frac{\kappa^2}{w})$, which are thus independent of the policy objectives.[9] Such coefficients can also be shown to be strictly increasing in q.[10]

Equation (6) implies that the output gap is always below its target and that such difference increases over time. Inflation is always positive, declines over time and converges asymptotically to zero. In the initial period, the central bank can

[9]In the definition of γ_1 and γ_2 (see equations (A-5) and (A-6) in the appendix A), the only parameter related to the policy objectives is the relative weight of output stability w, which is assumed to be constant. This assumption is relaxed in section 4.1.

[10]See appendix A for the derivation of these results.

use inflation to expand output towards its efficient level. To do so, it also needs to keep inflation expectations at a low level by promising to reduce inflation in the subsequent periods. As time passes by, inflation is then reduced and output converges to a permanently and inefficiently low level. The policy plan prescribed by (5) and (6) is in general time-inconsistent. Whenever the central bank is allowed to review its policies, it has an incentive to surprise the economy and to implement a higher inflation than expected.

From these equations we can observe that, at any point in time, the distance between output gap and inflation from their respective targets can be expressed as increasing functions of the initial level of inflation π_0. The latter variable, as we can see in (7), depends on two factors. First, on the output target of the current policymaker $\widetilde{y}^c$, which is a measure of the amplitude of the distortions in the economy. Second, it depends on the inflation that is implemented if liberal objectives are adopted, π_0^ℓ. This reflects the externality that the possibility of changing the policy objectives generates on the current central bank. In this respect, it is worth noticing the presence of an inflation bias even when $\widetilde{y}^c = 0$, that is in the absence, according to the current objectives, of any trade-off between output and inflation stabilization. Section 2.3 discusses this particular case in detail. Moreover, we must emphasize that equation (5) holds independently of the specific institutional setting (in terms of the central bank credibility, duration of the tenure, etc.) prevailing once the liberal objectives are adopted. The only factor that matters is the initial level of inflation π_0^ℓ implemented in such circumstance. For convenience, we assume that once the change in objectives occurs, the central bank faces a symmetric problem to the one described above.[11]

We first study the effects of limited commitment alone, assuming that policy goals cannot be changed, but that the monetary authority is not fully credible. In particular, we consider that at any given point in time a reoptimization may occur with probability $0 < (1-q) < 1$, but the objectives of the central bank will be the same, i.e. $\widetilde{y}^c = \widetilde{y}^\ell$. As shown by Schaumburg and Tambalotti (2007) the optimal path of inflation under limited commitment is given by:[12]

$$\pi_t^{LC} = \left(\frac{\bar{\gamma}_2}{\gamma_2}\right)^t \frac{\bar{\gamma}_2 - \beta}{\gamma_2 - \beta}\bar{\pi}_t, \tag{8}$$

where $\bar{\pi}_t$ is the level of inflation prevailing under the assumption of full commitment and no uncertainty about policy objectives, as derived in appendix A.1. Since

[11] The previous assumption can be easily relaxed without affecting qualitatively our results.

[12] See appendix A.2 for the derivation of these results.

$\gamma_2 < \bar{\gamma}_2$ we have that in any period, $\pi_t^{LC} \geq \bar{\pi}_t$. This means that limited commitment introduces an inflation bias. Since $\frac{\partial \gamma_2}{\partial q} > 0$, such bias is decreasing in the probability of commitment q.

We can now turn to the main part of our analysis, namely the effects of the possibility that policy goals may change. In this context, at any point in time and with probability $1 - q$, the central bank may adopt more liberal objectives, thus setting $\widetilde{y}^\ell > \widetilde{y}^c$. It can be shown that in that case

$$\pi_t = \underbrace{\left(\frac{\bar{\gamma}_2}{\gamma_2}\right)^t \frac{\bar{\gamma}_2 - \beta}{\gamma_2 - \beta}}_{\substack{\text{Limited Commitment} \\ > 1}} \quad \underbrace{\frac{\widetilde{y}^c + \Phi \widetilde{y}^\ell}{\widetilde{y}^c (1 + \Phi)}}_{\substack{\text{Liberal Objectives} \\ > 1}} \quad \bar{\pi}_t \tag{9}$$

where $\Phi \equiv \frac{\beta(1-q)}{\gamma_2(1-\gamma_1)} = \frac{\beta - \beta q}{\gamma_2 - \beta q} < 1$ and $\frac{\partial \Phi}{\partial q} < 0$.

When more liberal objectives may be adopted in the future, two different forces affect current inflation, independently of whether such changes ultimately occur or not. The first is related to the reduction in credibility associated with the possibility of a change in policy objectives. This is represented by the first fraction on the left-hand side of equation (9), which is the same as in (8). The second force is related to the adoption of more liberal objectives, as represented by the second fraction on the left-hand side. We can thus conclude that the possibility that policy objectives become more liberal introduces an additional inflation bias with respect to the limited commitment case. Such additional bias can be shown to be increasing in the difference between $\widetilde{y}^c$ and $\widetilde{y}^\ell$ and in the probability of policy changes $(1-q)$.[13] In other words, the more liberal the alternative objectives are and the more likely the change, the higher is current inflation. Moreover, higher inflation is associated with a reduction in output. This can be clearly seen in equation (6), a higher initial inflation (π_0) lowers output in all periods.

The main message of our analysis is twofold. First, consider an economy experiencing a low productivity level or a recession. In that case, discussing the adoption of more liberal monetary policy objectives may seem appropriate but is in fact detrimental for the economy. It generates inflationary pressures and it exacerbates the recession. Second, when there is the possibility that more liberal objectives are adopted, even the most conservative central bank should accommodate the inflationary pressures to avoid a stronger recession. In other words, the fact that inflation is higher in a given period should not be necessarily interpreted as a signal that a more

[13] For the derivation of these results see appendix A.3.

liberal monetary policy is being implemented. It can be just the optimal reaction of a conservative central bank to the externalities generated by the possibility that liberal policies prevail in the future.

In order to give a quantification of the effects related to the uncertainty about policy objectives, in figure 1 we plot the optimal path of inflation and output gap while conservative objectives prevail. We assume that the central bank has an output target $\widetilde{y}^c = 0.01$ and that with probability $1 - q = 10\%$ it will adopt more liberal objectives, i.e. setting $\widetilde{y}^\ell = .1$. For the remaining parameters, we used a standard calibration of the model, with $\beta = .99$, $\kappa = .1$, and $w = .048$.[14] For comparison, we also plot the allocations prevailing under full-commitment and under limited commitment. When there is the possibility that more liberal objectives are adopted (solid line), in the first period there is a 0.25% increase in (annualized) inflation and a .5% reduction in output with respect to the standard case with full-commitment and no possibility of policy changes (dashed line). More importantly, the reduction in output is even bigger at a longer horizon, reaching a level of about 1.5%. From the picture, it is also clear that most of the effects are due to the changes in objectives, rather than to the associated loss of credibility, measured by the difference between the full-commitment (dashed line) and the limited commitment case (dashed-dotted line).

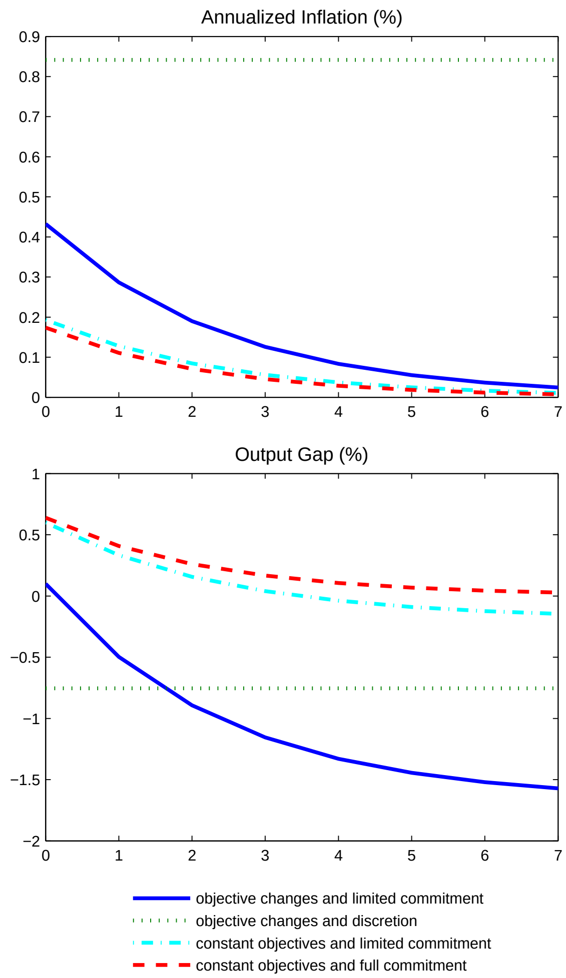
We can conclude that the mere possibility that more liberal policies can be adopted in the future, even if this ultimately never occurs, induces an increase in inflation in the short-run and a significant and permanent reduction in output. The intuition for this result goes as follows. Other things equal, the possibility of having more liberal policies in the future increases inflation expectations. As a consequence, on the one hand the central bank increases current inflation, thus accommodating the higher inflation expectations in the effort to limit the reduction in output. On the other hand, the central bank promises to reduce inflation in the future, for the case the conservative objectives still prevail.[15] In this way, it reduces the inflation expectations, limiting the negative externality generated by the possibility that policy objectives become more liberal. This means that as time passes by, inflation will not be used to foster production, and thus output will be lower.

Finally, we want to understand how the credibility of the central bank matters

[14]This calibration is used for instance in Woodford (2003) and Schaumburg and Tambalotti (2007).

[15]These two effects, namely the accommodation effect and the anchoring expectations effect, can be better disentangled in a framework where policy objectives can change only occasionally, say every T periods, as the one considered in section 3.1.

Figure 1: Adoption of liberal objectives: Optimal Inflation and Output under a conservative central bank



Note: The figure plots inflation and the output gap that the conservative central bank implements under several scenarios. The figure plots the case of objective changes and limited commitment (continuous line), objective changes and discretion (dotted line), no objective changes and limited commitment (dashed-dotted line), and no objective changes and full commitment (dashed line).

when policy objectives can change. To do so, we compare two alternative settings. The first is the one described above, where the central bank can make credible promises regarding the policies that will be implemented if the objectives remain unchanged. The second is a setting where, by assumption, the central bank is not credible at all, no matter whether the policy objectives may change or not. In that case, we have that inflation (π^{NC}) and output (y^{NC}) are constant over time and given by

$$\pi^{NC} = \frac{1}{1+\frac{\kappa^2}{w}-\beta q}\left[\kappa \widetilde{y}^c + \beta\left(1-q\right)\pi_0^{\ell}\right] \tag{10}$$

$$y^{NC} - \widetilde{y}^c = -\frac{\kappa}{w}\pi^{NC} \tag{11}$$

In order to compare the two alternative settings, we combine (5) and (7) and divide the resulting expression by (10), considering that the inflation implemented with liberal objectives (π^{ℓ}) is the same in the two cases. We then obtain that

$$\frac{\pi_t}{\pi^{NC}} = \gamma_2^{-t}\frac{1+\frac{\kappa^2}{w}-\beta q}{\gamma_2-\beta q} < 1 \tag{12}$$

where the last inequality follows from the fact that $\gamma_2 > 1+\frac{\kappa^2}{w}$. Moreover, dividing (6) by (11) we have

$$\frac{y_t - \widetilde{y}^c}{y^{NC}-\widetilde{y}^c} = \frac{1-\gamma_2^{-(t+1)}}{1-\gamma_2^{-1}}\frac{\pi_t}{\pi^{NC}}, \tag{13}$$

which, at least for $t = 0$, is smaller than 1, implying $y_0 > y^{NC}$.[16]

The difference between the two alternative settings is also shown in figure 1. In a context where policy goals can be changed, when the central bank can commit (solid line) inflation in the first period ($t = 0$) is approximately half of the value prevailing when commitment is ruled out (dotted line).

Our analysis suggests that when policy objectives can change, having credibility is important. It allows to implement a lower inflation and, at least in the short-run, a higher output. This is because a credible central bank can keep inflation expectations relatively low by promising to lower inflation in case liberal objectives are not adopted. In other words, a credible institution can counteract more efficiently, the inflationary pressures arising from the possibility of adopting more liberal objectives.

To summarize, the possibility of adopting more liberal objectives generates the following effects on the conservative central bank:

[16]To see this result notice that, from (11), $y^{NC} - \widetilde{y}^c < 0$.

- In any period, it generates an inflation bias, with respect to the case with full-commitment and constant objectives.
- In any period, it generates a lower output, with respect to the case with full-commitment and constant objectives. This is the opposite effect of what liberal pressures are aiming at.
- The previous effects are due in part to the loss of credibility, and in part to the change in objectives, as described in (9). The latter seems quantitatively more important.
- The more liberal are the policies (i.e. the higher is $\widetilde{y}^{\ell}$) and the more likely is the change in objectives (lower q), the higher is the inflation bias.
- The higher is the inflation bias, the bigger is the associated reduction in output, as described by (6).
- When there is the possibility policy goals can be changed, credibility of the central bank allows to keep inflation and output gap closer to their targets, as described in equation (12) and (13).

2.3 Discussion: commitment, the conservative central banker and inflation targeting

The optimal monetary policy literature has proposed many ways to limit the time-inconsistency problem. In a remarkable contribution, Rogoff (1985) suggested to appoint a conservative central bank that is more averse than society towards inflation.[17] Rogoff shows that appointing a conservative central banker, even operating under discretion, can significantly reduce the time-inconsistency problem, and in some cases can implement the same policy of a benevolent planner with full-commitment, i.e. the best possible policy. As a result, it may be concluded that the credibility of a central bank may be of little importance, as long as its degree of inflation aversion is high enough.

We have shown previously, that when there is the possibility that objectives may change, if the conservative central bank does have some commitment then it can achieve more favorable allocations. We can further illustrate this point in the following example. Consider an (extreme) case where the conservative central bank

[17]Walsh (1995) and Svensson (1997) also suggested alternative ways to solve the time-inconsistency problem.

has a target for the output gap $\widetilde{y}^c = 0$.[18] In this case, the central bank perceives that there are no distortions in the economy to be corrected through the use of inflation. As a consequence, if policy objectives cannot be changed, the central bank implements the allocation $\pi_t = 0$ and $y_t = 0$ in all periods, no matter what is the degree of commitment. In this context, appointing such a conservative central bank is equivalent to the adoption of a strict inflation targeting policy. The interesting question is to understand what happens if such a conservative central bank faces the possibility that future objectives can change to $\widetilde{y}^\ell > 0$.

As suggested from our discussion in the previous section, and as can be seen in figure 2, the conservative central bank, when facing possible changes in objectives, implements a positive inflation and a negative output gap.[19] The figure also plots the policy where the conservative can make credible commitments (solid line), and the policy where credible commitments are ruled out (dotted line). The difference between the two lines indicates that introducing credible commitments helps significantly in keeping inflation and output close to their targets. Note that we have identified this effect even when the Rogoff conservative central banker implements the benevolent planner policy.

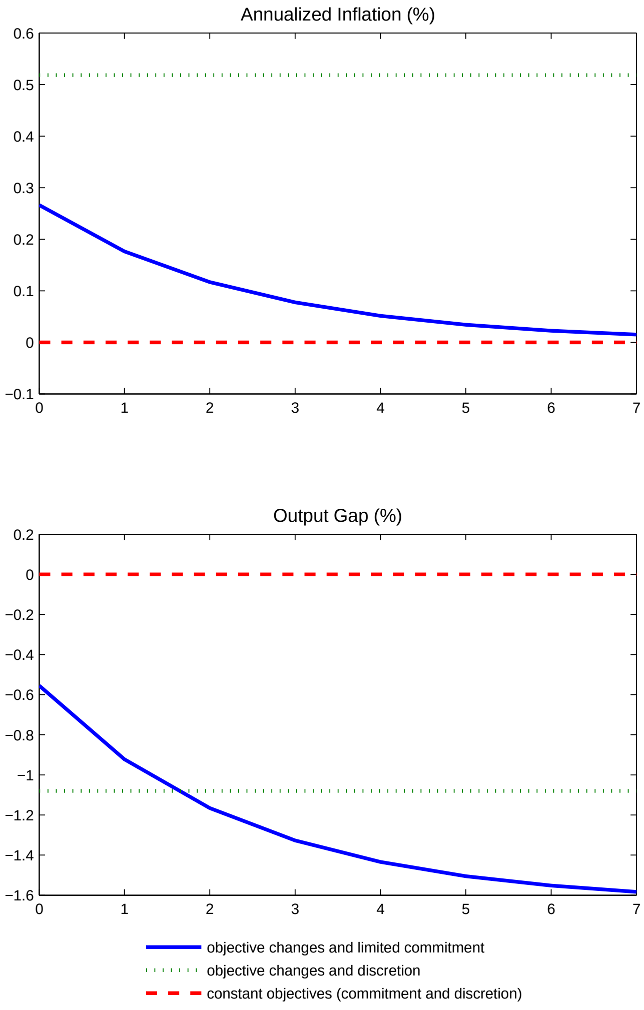
Even independent and conservative central banks, may be subject to some external pressures. Our work emphasizes that even a Rogoff conservative central banker, or a central bank adopting an inflation targeting policy, can benefit from commitment. Besides other considerations, commitment is important to counteract the effects of the possibility of changes in policy objectives.

Another interesting point in our framework, is that the probability that objectives change in the future matters. In this respect, we can emphasize the difference between appointing a conservative central bank and implementing an inflation targeting regime. A conservative central bank puts more weight on inflation, and therefore can be equivalent to an inflation targeting regime, as in the example just shown. However, if an inflation targeting regime has been implemented, changing policy objectives is likely to be harder. It requires institutional reforms that are usually costly and lengthy. When a conservative central bank is in office, without a clearly specified target, changing policy objectives is easier. It is enough that the next chairman is not as conservative as the current one. Therefore, implementing an inflation targeting regime or appointing a conservative central bank is not equivalent. The two cases imply different probabilities that objectives change in the

[18] Our result is still valid if the conservative central banker is one assigning zero weight on output stabilization ($w = 0$) as long as cost-push shocks are added to the NKPC.

[19] Figure 2 plots inflation and output gap for the case of $\widetilde{y}^c = 0$ and $\widetilde{y}^\ell = 0.1$, while keeping all the other parameters as in the previous section.

Figure 2: Changes in objectives and Inflation targeting: Optimal Inflation and Output



Note: The figure plots inflation and the output gap that the conservative central bank (with $\widetilde{y}^c = 0$) implements under several scenarios. The figure plots the case of objective changes and limited commitment (continuous line), objective changes and discretion (dotted line), and no objective changes (dashed line). In this calibration the case of no objective changes with discretion is equivalent to no objective changes with commitment.

future, which immediately affects current outcomes.

3 A model with periodic objective changes

There are several examples of interference in central bank policy in OECD countries. Nevertheless, it may not be entirely plausible to assume that external pressures occur continuously, or that the objectives of the central bank can be changed immediately. In practice, objectives can only be changed with some delay due to institutional features and policy implementation lags. In this section, we add several features that make the model more realistic. We consider the case where the conservative central bank knows that it will decide the monetary policy course for at least T periods. A possible interpretation of this setting is that the central bank is independent and the chairman or the members of the board of advisors are in charge for T periods. At the end of their tenure, policy objectives can change as long as officers with different views are appointed.

Both the conservative central bank and private agents know that in T periods the current conservative objectives can persist (with probability q) or give place to liberal objectives (with probability $1-q$). In the latter case, for simplicity, we assume the central bank will face a symmetric problem, where liberal objectives are unaltered for T periods, and then can change with probability $1-q$. The model in this section, besides being more realistic, allows us to better understand how the possibility of changes in policy objectives affects the conduct of monetary policy.

As before, the central bank can commit to a plan as long as the objectives remain unchanged. In this setting, analytical solutions are not available, and one needs to use the tools of shown in Debortoli and Nunes (2006a) and Debortoli and Nunes (2006b) to solve the model numerically.[20] In the present framework, the problem of the conservative (c) or liberal (ℓ) central bank is the following. Taking as given the sequence of policy $\{\pi_t^j, y_t^j\}_{t=0}^{\infty}$ and the value function V^{ij}, with $j \neq i$, the problem of the central bank can be written as:

$$V^i = \max_{\{\pi_t, y_t\}_{t=0}^{\infty}} E_0 \sum_{m=0}^{\infty} \left(\beta^T q\right)^m \left[-\frac{1}{2} \sum_{t=0}^{T-1} \beta^t \left[\pi_{m+t}^2 + w^i (y_{m+t} - \widetilde{y}^i)^2 \right] + \beta^T (1-q) V^{ij} \right] \tag{14}$$

[20]The present work combines the tools developed in Debortoli and Nunes (2006a) for the probabilistic model and the T-periods model. The probabilistic model was also addressed in Roberds (1987) and Schaumburg and Tambalotti (2007). We also use features of Debortoli and Nunes (2006b), where we considered disagreement among successive policymakers.

$$s.t.\ \pi_{mT+t} = \kappa y_{mT+t} + \beta E_{mT+t}(\pi_{mT+t+1}) \qquad t = 0, 1, ..., T-2 \tag{15}$$

$$\pi_{mT+t} = \kappa y_{mT+t} + (1-q)\beta E_{mT+t}(\pi^{j}_{mT+t+1}) + q\beta E_{mT+t}(\pi^{i}_{mT+t+1}) \quad t = T-1 \tag{16}$$

$$\forall\ m = 0, ..., \infty$$

where m indexes the number of tenures, each lasting for T periods. The objective function reflects the institutional setting the central bank faces. At the end of any tenure (T periods where objectives can not change), current objectives (i) can remain unaltered with probability q. This history is summarized in the outer summation. Within each tenure, plans are made for T periods, as indicated in the inner summation. Finally, the central bank (i) internalizes that with some probability $(1-q)$, at the end of tenure the objectives will be of type (j). In this case, central bank (i) will get the loss function V^{ij}. More formally, define the sequence $\{\pi^i_t, y^i_t\}^{\infty}_{t=0} \equiv \arg\max V^i\ \forall i = \ell, c$. The value function V^{ij}, $\forall i = \ell, c$ and $j \neq i$ is given by

$$V^{ij} \equiv E_0 \sum_{m=0}^{\infty} \left(\beta^T q\right)^m \left[-\frac{1}{2} \sum_{t=0}^{T-1} \beta^t \left[\left(\pi^j_{m+t}\right)^2 + w^i (y^j_{m+t} - \widetilde{y}^i)^2 \right] + \beta^T (1-q) V^{ji} \right] \tag{17}$$

The sequence of constraints (15) and (16) also reflects the institutional setting. Within any tenure m, we can divide the constraints into two groups, depending on how inflation expectations are formed. In the periods $t = 0, ..., T-2$, inflation expectations internalize that in the next period objectives do not change. In the last period of the tenure $(T-1)$, agents recognize that with some probability (q) objectives do not change, while with probability $(1-q)$ objectives do change. We employ the following definition of equilibrium:

Definition 1 *A Markov Perfect Equilibrium with objective changes must satisfy the following condition. For any $i = \ell, c$ and $j \neq i$, given the sequence $\{\pi^j_t, y^j_t\}^{\infty}_{t=0}$:*

1. The value function V^{ij} satisfies equation (17).

2. The sequence $\{\pi^i_t, y^i_t\}^{\infty}_{t=0}$, solves (14) subject to (15) and (16).

In order to solve problem (14), we first write its recursive formulation. To do so we apply the technique of Marcet and Marimon (1998), and we write the problem as a saddle point functional equation that generalizes the usual Bellman equation.

The proof of that result requires considering each tenure as one big period, and then applying the results of Debortoli and Nunes (2006a) to address the probabilistic switch at the end of each tenure. Proposition 1 in the appendix proves this result in detail. As stated in Proposition 2 in the appendix, we can then characterize the policy functions solving our problem as tenure invariant functions of the Lagrange multipliers associated with the constraints (15) and (16). We are not claiming that the policy functions are time-invariant. Indeed, the policy functions change in the different periods within a tenure.

In order to solve our problem we have to find, for both central banks ($i = \ell, c$), the policy functions satisfying the equilibrium conditions stated above. In particular, for each central bank, we need to find as many policy functions as the number of periods within each tenure (T). Moreover, as can be seen in (16), the policies of central bank j enter the problem of central bank i (and viceversa). This implies that we have to solve a fixed point problem in such policy functions. In addition, the implied value functions V^{ij} and V^{ji} also enter the problem and need to be solved for endogenously.[21] We also have to take into account the possibility of default on past promises, an event that occurs whenever there is a change in objectives. The presence of default significantly complicates the numerical procedure, since both the levels and the derivatives of the policy functions enter the first-order conditions of the problem.

We solved the model numerically using the same calibration of the previous section. Regarding the number of periods where objectives remain unchanged (T), and the probability that objectives change, we could have considered several calibrations. We considered that central bank objectives remain unchanged with certainty for four periods ($T = 4$), at that point there is a probability of that objectives change equal to $q = 0.5$. This calibration is convenient for illustrative purposes.[22]

3.1 Results

We first consider the case of disagreement in the output gap target level. Figure 3 plots the optimal policy functions in the case where the conservative central bank may be temporarily replaced by a liberal one (continuous line). One can easily translate the policy functions to certain realizations of events. The conservative central bank implements the policy functions shown in the figure until policy objec-

[21]Note that V^{ij} and V^{ji} are value functions in the presence of disagreement between successive policymakers, therefore unlike in Debortoli and Nunes (2006a) one can not use an envelope result.

[22]This calibration implies that objective changes on average every 8 periods, roughly the same as in the case considered in the previous section.

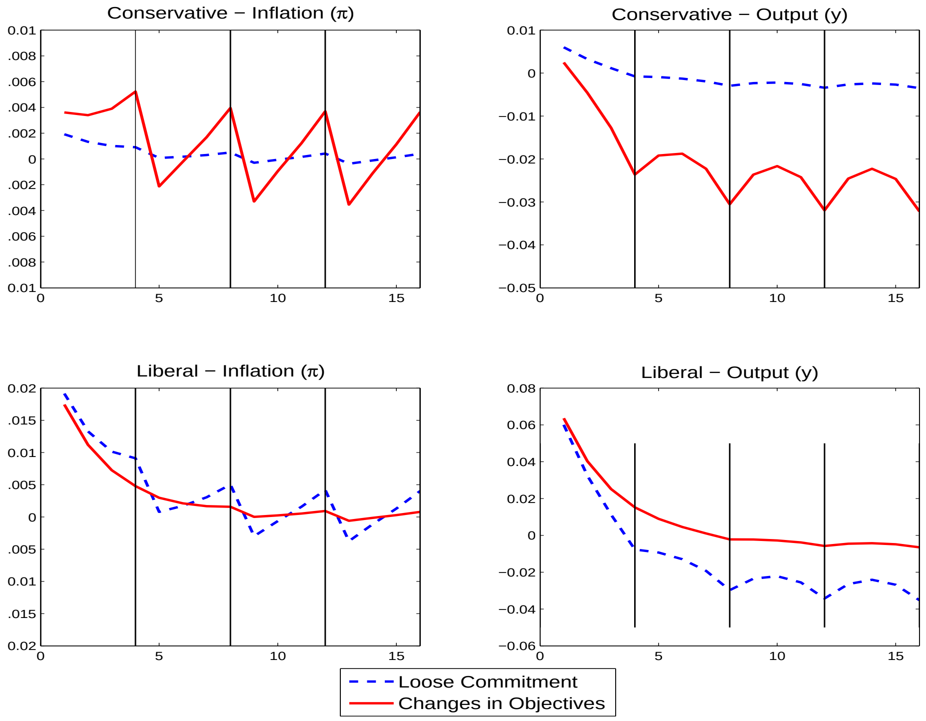
tives are changed. When such change occurs, the liberal policy is implemented until objectives become conservative again. In this model, the possibility that objectives change is only present every four periods - those periods are signaled in the pictures with continuous vertical lines. Therefore, once it is know that the liberal policies are not implemented, the conservative central bank is insulated from any external pressure for four periods. This feature of the model tries to capture the fact that in reality political pressures may not be able to change objectives immediately. The upper panel plots the policy functions of the conservative central bank for inflation and output, the lower panel refers to the liberal policy functions.

For comparison, in figure 3, we also plot the policy functions that occur in a loose commitment setting (dashed line). Loose commitment refers to the fact that the central bank occasionally disregards previous commitments and makes a new plan, but where objectives do not change. In such case, the policy functions with the same objectives are implemented from the beginning until the next reoptimization occurs. The differences between the policy functions with loose commitment and with changes in objectives are only due to the possibility that objectives change. In both cases, there is a common commitment problem.

The possibility that a liberal central banker might be appointed in the future affects the optimal policy functions of the conservative central bank in several ways. First, when the conservative central bank starts (periods 1 to 4), inflation is now higher and output is lower. This is due to an accommodation effect. The possibility that the liberal policies with high inflation are implemented in the future affects current outcomes through inflation expectations. High inflation expectations either imply higher current inflation or lower output. The optimal policy of the conservative implies a combination of higher inflation and lower output.

Second, we also observe that the conservative central bank implements a low inflation level immediately after knowing that the pressures to adopt liberal policies have dissipated (periods 5, 9, 13,...), and that objectives will not change in the following four periods. The rationale of this policy is to anchor inflation expectations. When it is known that in the next period a liberal policy may be implemented, inflation expectations increase. In order to keep inflation expectations low, the conservative central bank finds it optimal to promise to reduce inflation if objectives remain unchanged. This promise regarding future policy affects beneficially the current period through inflation expectations. Note that this promise is extremely time-inconsistent, if there is not a change in objectives, the conservative central bank needs to fulfil its promise of implementing a very low inflation level. This case exemplifies how commitment is used to balance distortions across time and states of nature.

Figure 3: Model with Occasional Changes in Objectives



Note: This figure refers to the model where objectives can not change immediately. The upper two panels plot the policy functions (inflation and output gap) of a conservative central bank, and the lower two panels refer to a liberal central bank. Objectives changes (in the output gap target) can only occur every four periods - marked with continuous vertical lines. The case with objective changes and limited commitment is plotted with a continuous line. The case of no objective changes and limited commitment is plotted with a dashed line. In all panels the horizontal axis refers to the number of periods elapsed after the last change in objectives/reoptimization.

Finally, note that the first period of the conservative central bank (period 1) is fundamentally different from any first period after being confirmed in office (periods 5, 9, 13,...). As we had explained before, periods 5, 9, 13 are characterized by low inflation that was promised in the previous period. In the current setting, we are assuming that past promises are not binding in period 1 because the conservative central bank was not in office before. This is the reason why inflation is relatively high in period 1, when compared, for instance, with period 5.

Summarizing our results, we have found that the possibility that liberal policies are implemented creates three effects in the policies of the conservative. First, the conservative central bank needs to raise inflation to accommodate inflation expectations - the accommodation effect. The accommodation effect is higher the closer is the period where liberal policies may take over. Second, to counteract high inflation expectations, the conservative promises to implement a low inflation if the objectives do not change - the anchoring effect. The anchoring effect only materializes when liberal objectives are not adopted. The combination of the accommodation and anchoring effect explains why the conservative starts with low inflation and then increases it. In the model with one period tenure ($T = 1$), the strength of the anchoring and accommodation effect did not change over time. And in that case, we proved analytically that the possibility that liberal objectives could be adopted induced the conservative to implement higher inflation in every period. Third, the conservative experiences lower output due to the possibility that liberal policies are implemented. This result may come as surprising. If a liberal sector of the economy thinks that current output is too low, and makes pressures for changing the central bank policy, then the outcome is that the economy experiences an even lower output. If the liberal policies are implemented in a later period, then the economy will experience an expansion. However, as long as the objectives do not change, the economy experiences a recession, the opposite outcome of what the pressures for liberal policies may be aiming.

Table 2 reports welfare and the average allocations. The average inflation that the conservative implements is higher relative to the loose commitment case where the conservative is never substituted by a liberal policy. As expected, the overall output average when conservative and liberal objectives coexist is higher than the case where the conservative objectives are unchallenged. But, as we cautioned before, the conservative central bank experiences a lower output due to the pressures of liberal policies. Hence, as long as the central bank's objectives do not change, there is a negative effect on output. In terms of welfare, pressures to adopt liberal objectives create a negative externality on the conservative. Even though, overall output is slightly higher, inflation is further away from target.

Table 2: Inflation and Output in a Model with Occasional Changes in Objectives

	Changes in Objectives			Loose Commitment	
	Average with c	Average with ℓ	Overall	c	ℓ
π	0.2254	0.57	0.3968	0.0723	0.7226
y	-1.6888	1.7822	0.0382	0.0083	0.083
welfare	-0.0036	-0.0272	.	-0.0003	-0.0302

Note: The table reports the average allocations across different simulations of the model.

In a partisan theory of output and inflation, Alesina (1987) considered a classical Phillips curve where current inflation surprises affect current outcomes. In that context, the possibility of a future change in policy does not affect current outcomes. For instance, the possibility that a liberal policy is implemented in period 4, 8, 12 has no consequences on the economy and the optimal policy functions in any other periods. Here, we consider instead a (standard) New Keynesian Phillips curve, where future conditions also influence current outcomes. Therefore, our work has very different mechanisms from the analysis considered in Alesina (1987), where for instance the accommodation effect is simply absent. In addition, in those models it is assumed that the central bank acts with discretion in every period. Given the developments in central bank commitment, we are instead assuming that there is commitment to policies aimed at maximizing the same objectives. Since the anchoring effect found in our model is based on a commitment, that effect is also not present in Alesina's model.

4 Alternative scenarios

In this section, we consider some alternative scenarios where liberal and conservative objectives coexist. We consider a case where the relative weight of inflation stabilization can change, a case with a hybrid Phillips curve, and then a full commitment cooperative setting. In all these cases, the main intuition presented in the baseline case still holds. We finally discuss the reverse scenarios that we examined, the case where future policy objectives may become more conservative.

4.1 Changes on the relative weight of inflation stabilization

Consider the case where both conservative and liberal objectives agree on the output target level, but disagree on the importance of inflation stabilization. In particular, we assume $\widetilde{y}^c = \widetilde{y}^\ell = \widetilde{y}$ and $w^c < w^\ell$. In this context, if policy objectives can change in every period ($T = 1$) with probability $1-q$, we can show that inflation evolves according to

$$\pi_t = \underbrace{\frac{(1+\Phi^\ell)(1-\Phi^c)}{1-\Phi^c\Phi^\ell}}_{\substack{\text{Liberal Objectives} \\ >1}} \underbrace{\left(\frac{\bar{\gamma}_2}{\gamma_2}\right)^t \frac{\bar{\gamma}_2-\beta}{\gamma_2-\beta}}_{\substack{\text{Limited Commitment} \\ >1}} \bar{\pi}_t,$$

where $\Phi^c \equiv \frac{\beta(1-q)}{\gamma_2^c(1-\gamma_1^c)} = \frac{\beta-\beta q}{\gamma_2^c-\beta q} < \Phi^\ell \equiv \frac{\beta(1-q)}{\gamma_2^\ell(1-\gamma_1^\ell)} = \frac{\beta-\beta q}{\gamma_2^\ell-\beta q} < 1$.

Consistent with our analysis in section 2.2, we can conclude that pressures to increase the relative weight of output stabilization generates an inflation bias. This bias increases with the probability of a policy change $(1-q)$.

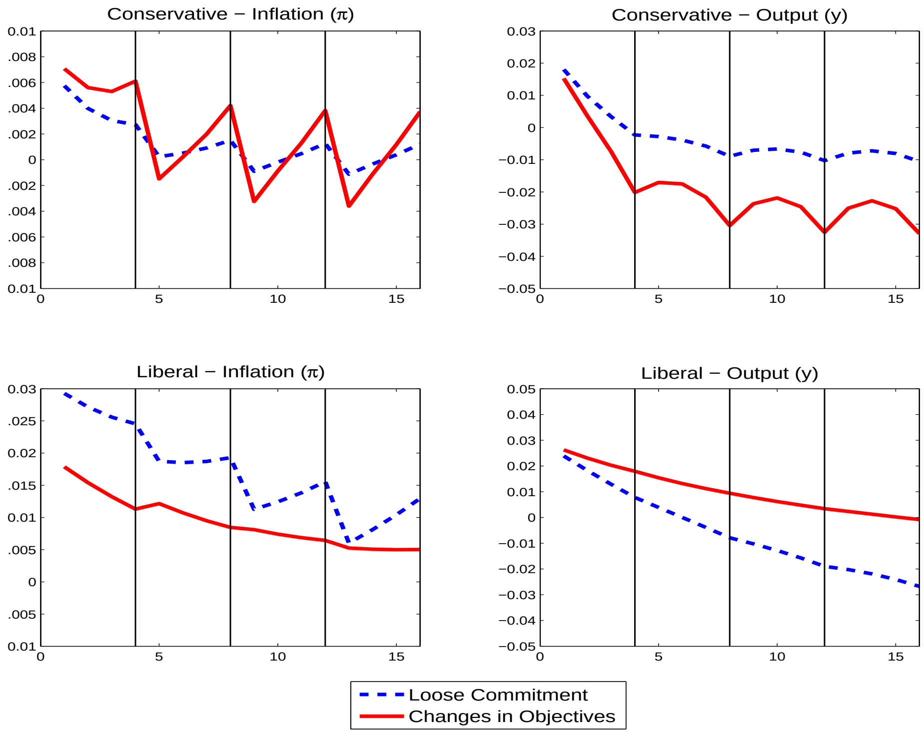
We provide a quantitative example, for the case where the tenure of the central bank lasts for $T = 4$ periods. More specifically, we consider that $\widetilde{y}^c = \widetilde{y}^\ell = 0.03$, $w^c = 0.048$ and $w^\ell = 0.48$. This case is also plausible since it has been argued that the full-commitment microfounded calibration of w is much lower than what policy makers often implement in practice. Figure 4 presents the policy functions and Table 3 presents the average allocations in the economy. All the results and intuition that we mentioned previously remain unaltered.

Table 3: Inflation and Output - Changes in w

	Changes in Objectives			Loose Commitment	
	Average with c	Average with ℓ	Overall	c	ℓ
π	0.336	1.1208	0.7265	0.2168	2.0627
y	-1.3006	1.4567	0.0713	0.0249	0.2137
welfare	-0.007	-0.0329	.	-0.0027	-0.0483

Note: The table reports the average allocations across different simulations of the model.

Figure 4: Alternative Scenario: different weights of inflation stabilization



Note: This figure refers to the model where objectives can not change immediately. The upper two panels plot the policy functions (inflation and output gap) of a conservative central bank, and the lower two panels refer to a liberal central bank. Objectives changes (in weight on output stabilization) can only occur every four periods - marked with continuous vertical lines. The case with objective changes and limited commitment is plotted with a continuous line. The case of no objective changes and limited commitment is plotted with a dashed line. In all panels the horizontal axis refers to the number of periods elapsed after the last change in objectives/reoptimization.

4.2 Hybrid Phillips curve

In this section, following Galí and Gertler (1999), we consider the possibility that the Phillips curve may also include a backward looking term. The hybrid Phillips curve takes the form

$$\pi_t = \kappa y_t + (1-\alpha)\beta E_t \pi_{t+1} + \alpha\beta\pi_{t-1}. \tag{18}$$

The presence of lagged inflation introduces a state variable in the model.[23] The presence of such state variable is relevant for our analysis, since it allows the central bank to influence strategically future decisions. Whether there is a change in objectives or not, current policy will be affected by the past inflation level. Therefore, the central bank can strategically choose an inflation to influence next period decisions, even if objectives change.

For direct comparability purposes, we leave the central banks' objective functions as in the baseline scenario of section 3.[24] This allows us to understand whether the presence of inflation as a state variable changes the interaction patterns between the conservative and liberal policies. Another important point is that the model with the hybrid Phillips curve actually alleviates the time-inconsistency problem. This is because the weight on expected future inflation is smaller. Since a reoptimization will now imply an inflation level that is not as high as before, the conservative central bank necessity to accommodate inflation expectations is not so strong.

In accordance with the empirical evidence in Galí and Gertler (1999), we set α to be 0.3. Figure 5 plots the policy functions.[25] The qualitative features of the effects of the possibility of adopting liberal objectives are the same as in the baseline case. Table 4 reports the relevant statistics. The table mainly confirms the results explained previously.

4.3 A full commitment solution

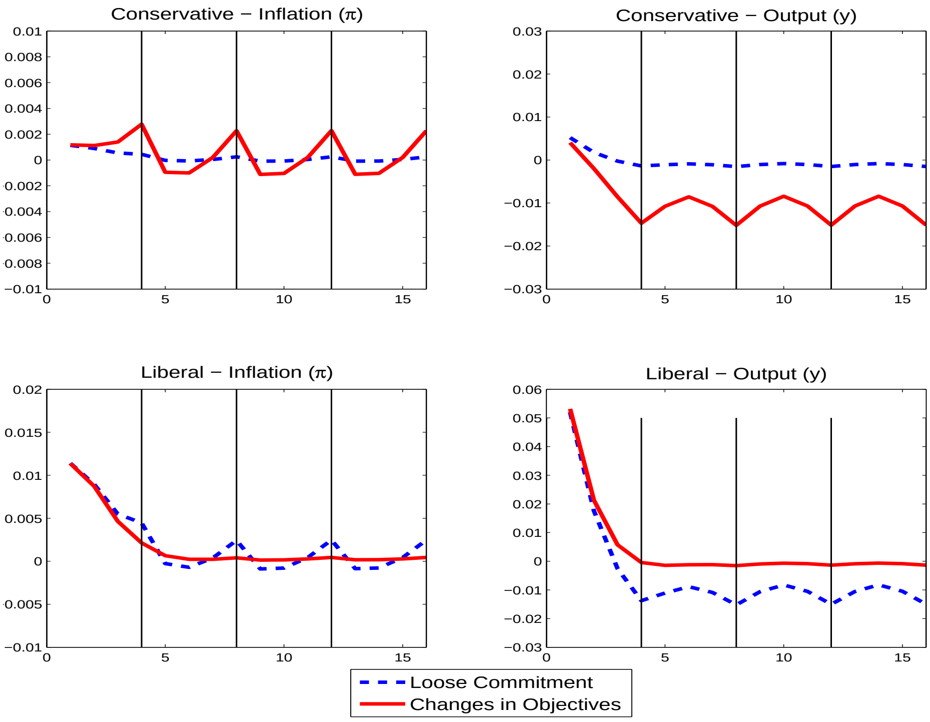
For the sake of realism, we assumed an intermediate level of commitment where the central bank could make binding promises as long as external pressures do not alter policy objectives. This feature reflects the inherent disagreement on different policy objectives. In this section, we want to analyze a setting where the central bank

[23]Under this specification, propositions 1 and 2 in the appendix also apply.

[24]We do not have the goal to characterize policy with indexation, where the functional forms of policy objectives may differ from the ones considered here.

[25]We have set initial inflation to be 0. Considering other values does not affect qualitatively the results.

Figure 5: Alternative Scenario: Hybrid Phillips Curve



Note: This figure refers to the model where objectives can not change immediately, and the Phillips curve also has a backward-looking component. The upper two panels plot the policy functions (inflation and output gap) of a conservative central bank, and the lower two panels refer to a liberal central bank. Objectives changes (in the output gap target) can only occur every four periods - marked with continuous vertical lines. The case with objective changes and limited commitment is plotted with a continuous line. The case of no objective changes and limited commitment is plotted with a dashed line. In all panels the horizontal axis refers to the number of periods elapsed after the last change in objectives/reoptimization.

Table 4: Inflation and Output - Hybrid Phillips Curve

	Changes in Objectives			Loose Commitment	
	Average with c	Average with ℓ	Overall	c	ℓ
π	0.0878	0.3567	0.2216	0.0404	0.4038
y	-0.8503	0.9003	0.0207	0.0045	0.0449
welfare	-0.0017	-0.0253	.	-0.0002	-0.0264

Note: The table reports the average allocations across different simulations of the model.

has full commitment, even though the policy objectives may change over time. One interpretation of this framework is that the structural parameters of the economy, like the degree of nominal rigidities and the degree of firms' monopolistic power, evolve stochastically, thus changing the magnitude of the distortion the central bank aims to correct and the effectiveness of its policy.[26] Another interpretation of this setting is that the central bank itself is subject to preferences shocks. The setting in this section models the central bank has being extremely forward-looking, since it already makes a plan and commits to certain policy actions even if future objectives differ from the current ones. Another essential assumption for the present setting is that there is no disagreement about policy objectives, in which case considering loose commitment would be necessary.

In the baseline setting described in section 3, when external pressures succeed in altering the objectives, policy is reset to achieve the new objectives disregarding previous objectives. In contrast, in the model of this section, policies are chosen to maximize the overall welfare. In fact, since there is no disagreement, we can think that there is a unique policymaker taking decisions.

More specifically, we assume the central bank's policy goals can be $\widetilde{y}^\ell = 0.1$ or $\widetilde{y}^c = 0.01$. For convenience, we still refer to the terminology central bank ℓ or c when the current output gap is respectively $\widetilde{y}^\ell$ or $\widetilde{y}^c$. Every four periods ($T = 4$) the objectives remain unchanged with probability $q = 0.5$, while with probability $1 - q$

[26]This interpretation of the model is partly related to the literature on robust control of Hansen and Sargent (2007), and to the literature about optimal monetary policy in the presence of noisy indicators as in Aoki (2003). In our analysis, however, we focus on the effects of evolving objectives, and assume the structural relationships describing the economy and the exogenous shocks are known and common knowledge.

the objectives do change. The problem of the central bank can be written as:

$$V\left(\widetilde{y}_0\right) = -\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left[\pi_t^2 + w(y_t - \widetilde{y}_t)^2\right]$$
$$s.t. \quad \pi_t = \kappa y_t + \beta E_t \pi_{t+1}$$

where expectations are taken with respect to the variable $\widetilde{y}$. This problem can be written recursively by considering a tenure as a unique period and then applying standard dynamic programming techniques.

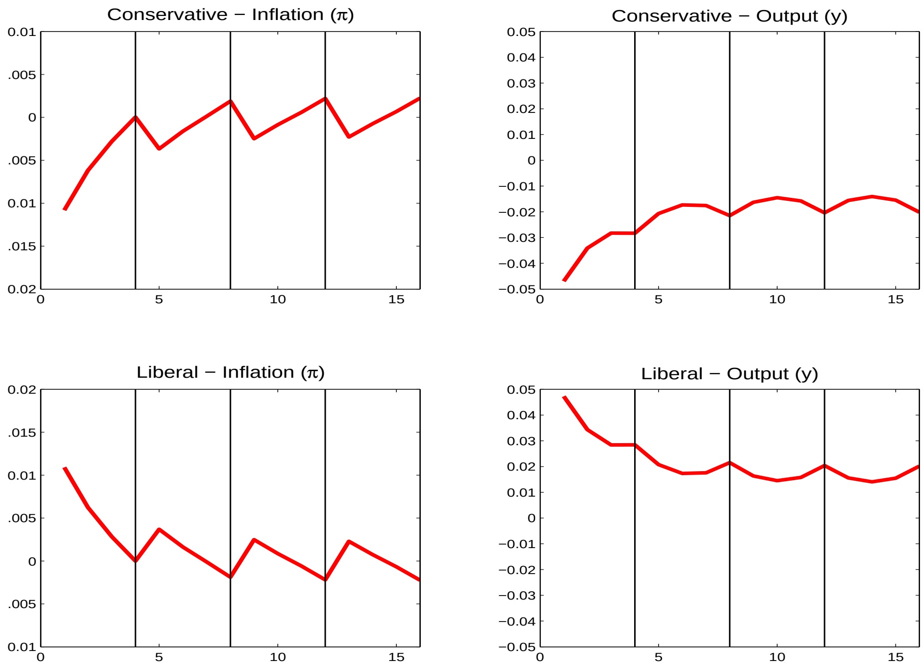
Figure 6 plots the policy functions for both types of objectives.[27] The policy functions of the conservative are extremely similar to the other cases considered previously. There is an accommodation effect, which becomes more visible the closer is the period where the liberal policies may emerge (periods 4, 8, 12 in the graph). In addition, whenever the objectives do not change, and thus the conservative objectives are kept (periods 5, 9, 13), inflation is reduced due to the anchoring effect. Also, as before, the conservative central bank experiences lower output due to the externalities generated by the possibility that more liberal objectives are adopted.[28]

The main difference in the conservative policy between this model and the baseline one in section 3 resides in the initial period. In the present model, inflation in the first four periods is lower. The main reason is that in this model past plans are always fulfilled. Therefore, the anchoring effect is also present in the initial period. The other interesting feature is that this anchoring effect is much stronger when the conservative is newly appointed (period 1 in the graph) rather than when it is reappointed (periods 5, 9, 13). The reason is that in this setup there is no disagreement about policy objectives, and therefore there is cooperation between the liberal and the conservative central bank. The anchoring effect reduces inflation expectations, which allows to increase output for a given inflation rate. When current objectives are liberal, it is specially important to achieve a high output level. Therefore, a

[27]In the model of section 3.1, when policy objectives are changed the lagrange multiplier is reset to zero. Afterwards, the policy functions depend on the evolution of the lagrange multiplier. There, we plotted the policy functions depending on the time spent in office, because there is a unique mapping between the evolution of the lagrange multiplier and the time spent in office. In the setup of this section, the lagrange multiplier is never set to zero. Therefore, even when objectives change, policy functions depend on the entire history that occurred previously. Nevertheless, we found that the qualitative features of the policy functions are not affected by the previous history of events.

[28]The analog of this model with no changes in objectives, is the usual full commitment solution where output converges to zero.

Figure 6: Alternative Scenario: Full Commitment and objective changes



Note: This figure refers to the model where objectives can not change immediately, and the central bank commits to future policy, even if objectives do change. The upper two panels plot the policy functions (inflation and output gap) of a conservative central bank, and the lower two panels refer to a liberal central bank. Objectives changes (in the output gap target) can only occur every four periods - marked with continuous vertical lines. The case with objective changes and full commitment is plotted with a continuous line. The case of no objective changes and full commitment would correspond to zero inflation and zero output gap. In all panels the horizontal axis refers to the number of periods elapsed after the last change in objectives.

liberal central bank promises that inflation will be at a particularly low level when objectives become conservative.

The next table plots the average allocations in the economy. As before, under a conservative central bank, the possibility that liberal policies may be adopted lowers output. The anchoring effect is stronger and is present more frequently in this economy.[29] In fact, when policy objectives may become more liberal, a conservative central bank implement a lower average inflation, which comes at the expense of a deeper recession.[30] This model confirms our findings in previous sections. The possibility that future policy objectives may become less conservative affects considerably the current optimal policy of a conservative central bank. We still observe an anchoring and accommodation effect, while output is lowered.

Table 5: Inflation and Output - Full commitment case.

	Changes in Objectives			Full Commitment	
	Average with c	Average with ℓ	Overall	c	ℓ
π	-0.2429	0.2507	0.0027	0.0005	0.0048
y	-2.5179	2.5466	0.002	0.0018	0.0176

Note: The table reports the average allocations across different simulations of the model.

4.4 The effects of adopting conservative objectives

We have solved our model where both the liberal and conservative policies are set optimally. We have mainly described the effects that external pressures to adopt a liberal policy have on a conservative central bank. This may seem the most reasonable case in the OECD economies, where politicians occasionally exert some influence for more expansionary policies. Nevertheless, our model yields implications for the opposite case, when a central bank expects that more conservative objectives may be adopted in the future. This may be the relevant case for economies where the

[29] In the baseline model of section 3, the anchoring effect is not present in the initial 4 periods. Note that the policies of the conservative from period 1 to 4 are more likely to be implemented than the policies from period 5 to 8, which in turn are more likely to be implemented than subsequent policies.

[30] We do not compare welfare between this case and the full-commitment benchmark, because the utility functions are different, and such comparisons are meaningless.

adoption of more stringent inflation objectives, like a specific low inflation target, is being discussed. Arguably, and subject to interpretation, this may be what it is currently happening in the United States.

In the baseline case, the possible adoption of more conservative objectives makes the liberal to implement a lower inflation rate. In addition, inflation expectations become lower, which allows the liberal central bank to achieve higher output. Both these effects make the liberal to achieve a better welfare outcome. In this sense, the possibility that more conservative objectives are adopted constitutes a positive externality for a liberal central bank. All these conclusions are mainly robust to the other scenarios that we have considered.[31]

5 Conclusions

Both central bankers and politicians frequently discuss whether central bank's objectives should be changed. These discussions may result in institutional reforms, or may influence in a particular direction the appointment of a new chairman or board members of the central bank. This paper analyzes the macroeconomic effects induced by the fact that central bank objectives may change in the future. We analyze optimal policy in such situation and its economic implications.

The paper is not aimed at providing a theoretical basis for partisan economic fluctuations, as for instance in Alesina (1987) and Drazen (2000). In practice, it may be hard to match directly political parties with systematic and successful changes in central bank policy. In this respect, the novelty of our analysis is to show how the possibility of future policy changes already produces effects in earlier periods. Following the recent literature on monetary policy, we model inflation dynamics with a New Keynesian Phillips curve, where expectations about future economic conditions affect current outcomes. Our analysis thus clarifies the theoretical difficulties to find a clear relationship between economic outcomes and policymakers' objectives. We indeed show that if liberal objectives can be adopted in the future, high inflation may be the optimal response of a conservative central bank. We can thus observe a high level of inflation no matter whether liberal objectives are eventually adopted or not.

[31]The only exception is the model with full commitment and no disagreement, as the one considered in section 4.3. There, liberal policies implement an even higher inflation level, and then reduce it over time achieving a specially high output. This policy is only possible because the conservative policies are cooperative, and anchor inflation expectations very firmly by promising a very low inflation level. The main feature that an expansion is obtained still holds in that case.

The most common case in reality is the one where a conservative central bank faces pressures to pursue more expansionary policies. In this circumstances, the optimal response of a conservative central bank is to increase current inflation through an accommodation effect. Simultaneously, the central bank tries to anchor inflation expectations by promising to be even more conservative in the future. Overall, we find that the possibility that policy objectives may become more liberal generates a negative externality for the conservative central bank. More interestingly, we also find that they lead to a contraction in current output, which is precisely the opposite of what pressures to adopt liberal policies may be aiming for. The more likely is the adoption of liberal objectives, the stronger are these effects. Along this dimension, the adoption of an inflation targeting regime seems to be preferable to a conservative central bank a la Rogoff, namely one with higher aversion towards inflation than society. An inflation targeting regime insulates more the central bank from external pressures. Changing policy objectives indeed requires an institutional reform, rather than simply appointing a chairman or advisors with different views.

We have also discussed to which extent credibility matters, in a context where policy objectives can be changed. This is done by taking into account several commitment settings, following the recent contributions of Schaumburg and Tambalotti (2007) and our methods developed in Debortoli and Nunes (2006a). In particular, we show how credible institutions are able to partially counteract the bad externalities generated by the possibility that policy objectives may become more liberal. This result is interesting since it clarifies that having a central bank with sufficient aversion towards inflation, as suggested by Rogoff (1985), or equivalently adopting an inflation targeting policy, does not eliminate the scope for having credible institutions.

Finally, our paper draws conclusions about the reverse case, where the current central bank may perceive that policy objectives may become more conservative in the future. This case may be relevant for countries that are discussing the adoption of inflation targeting regimes, which is, arguably, the case of the United States. In this case, the possibility of more conservative policy in the future creates a positive externality for the liberal central bank. Also, inflation expectations become lower, which is translated into lower current inflation and higher current output.

References

Alesina, A., 1987. Macroeconomic policy in a two-party system as a repeated game. Quarterly Journal of Economics 102 (3), 651–678.

Alesina, A., Roubini, N., Cohen, G., 1997. Political Cycles and the Macroeconomy. The MIT Press.

Aoki, K., 2003. On the optimal monetary policy response to noisy indicators. Journal of Monetary Economics 50, 501–523.

Chappell, H., Havrilesky, T., McGregor, R., 1993. Partisan monetary policies: Presidential influence through the power of appointment. The Quarterly Journal of Economics 108 (1), 185–218.

Davig, T., Leeper, E., 2007. Generalizing the taylor principle. American Economic Review 97 (3), 607–635.

Debortoli, D., Nunes, R., 2006a. Loose commitment. Universitat Pompeu Fabra. Manuscript.

Debortoli, D., Nunes, R., 2006b. Political disagreement, lack of commitment and the level of debt. Universitat Pompeu Fabra. Manuscript.

Drazen, A., 2000. The political business cycle after 25 years. NBER macroeconomics annual.

Faust, J., Irons, J., 1999. Money, politics and the post-war busines cycle. Journal of Monetary Economics 43 (1), 61–89.

Galí, J., 2008. Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework. Princeton University Press.

Galí, J., Gertler, M., 1999. Inflation dynamics: A structural econometric analysis. Journal of Monetary Economics 44 (2), 195–222.

Hansen, L., Sargent, T., 2007. Robustness. Princeton University Press.

Marcet, A., Marimon, R., 1998. Recursive contracts. Universitat Pompeu Fabra. Working Paper.

Roberds, W., 1987. Models of policy under stochastic replanning. International Economic Review 28 (3), 731–755.

Rogoff, K., 1985. The optimal degree if commitment to an intermediate monetary target. Quarterly Journal of Economics 100 (4), 1169–89.

Schaumburg, E., Tambalotti, A., 2007. An investigation of the gains from commitment in monetary policy. Journal of Monetary Economics 54 (2), 302–324.

Sheffrin, S., 1989. Evaluating rational partisan business cycle theory. Economics and Politics 1, 239–259.

Svensson, L., 1997. Optimal inflation contracts, conservative central banks, and linear inflation contracts. American Economic Review 87 (1), 98–114.

Walsh, C., 1995. Optimal contracts for central bankers. American Economic Review 85 (1), 150–167.

Woodford, M., 2003. Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton University Press.

Appendix

A Optimal monetary policy with changes in policy objectives

We derive the solution to the optimal policy problem described in section 2. To do so, we write the first-order necessary conditions of the planner's problem described in (3), given by

$$\pi_t : \quad -\pi_t - \lambda_t + \lambda_{t-1} = 0 \tag{A-1}$$

$$y_t : \quad -w^i(y_t - \widetilde{y}^c) + \lambda_t \kappa = 0 \tag{A-2}$$

$$\lambda_t : \quad \pi_t = \kappa y_t + \beta q \pi_{t+1} + \beta (1-q) \pi_0^\ell \tag{A-3}$$

where λ_t is the Lagrange multiplier associated with the NKPC. Rearranging equations (A-1)-(A-3) we obtain the second-order difference equation

$$\left[\beta q L^{-2} - \left(1 + \beta q + \frac{\kappa^2}{w^c} + 1\right) L^{-1} + 1\right] \lambda_{t-1} = \kappa \widetilde{y}^c + \beta (1-q) \pi_0^\ell$$

whose solution is given by

$$\left(1 - \gamma_1^c L^{-1}\right)\left(1 - \gamma_2^c L^{-1}\right) \lambda_{t-1} = \kappa \widetilde{y}^c + \beta (1-q) \pi_0^j \tag{A-4}$$

where,

$$\gamma_1 = \frac{\left(1 + \beta q + \frac{\kappa^2}{w^i}\right) - \sqrt{\left(1 + \beta q + \frac{\kappa^2}{w^i}\right)^2 - 4\beta q}}{2} \tag{A-5}$$

$$\gamma_2 = \frac{(1 + \beta q + \frac{\kappa^2}{w^i}) + \sqrt{(1 + \beta q + \frac{\kappa^2}{w^i})^2 - 4\beta q}}{2}. \tag{A-6}$$

It is convenient to emphasize that $\gamma_1 \gamma_2 = \beta q$ and $\gamma_1 + \gamma_2 = \left(1 + \beta q + \frac{\kappa^2}{w}\right)$ and $0 < \gamma_1 < 1 < \gamma_2$. Moreover,

$$\frac{\partial \gamma_2}{\partial q} = \frac{\beta}{2}\left(1 + \frac{(\gamma_1 + \gamma_2) - 2}{\gamma_2 - \gamma_1}\right) = \beta \left(\frac{\gamma_2 - 1}{\gamma_2 - \gamma_1}\right) > 0.$$

The unique stable solution to (A-4) is given by the expression

$$\lambda_t = \frac{1}{\gamma_2^c} \lambda_{t-1} - \frac{1}{\gamma_2^c (1 - \gamma_1^c)} \left(\kappa \widetilde{y}^c + \beta (1-q) \pi_0^\ell\right).$$

Solving backward and imposing the initial condition $\lambda_{-1} = 0$, we obtain

$$\lambda_t = \frac{1 - (\gamma_2^c)^{-(t+1)}}{(1 - \gamma_1^c)(1 - \gamma_2^c)} \left[\kappa \widetilde{y}^c + \beta (1 - q) \pi_0^\ell\right].$$

Using (A-1) we obtain the following expression for the evolution of inflation and output

$$\pi_0 = -\lambda_0 = \frac{1}{\gamma_2^c (1 - \gamma_1^c)} \left[\kappa \widetilde{y}^c + \beta (1 - q) \pi_0^\ell\right]$$

and

$$\begin{aligned} \pi_t &= (\gamma_2^c)^{-t} \pi_0 \\ y_t - \widetilde{y}^c &= -\frac{\kappa}{w^c} \frac{1 - (\gamma_2^c)^{-(t+1)}}{1 - (\gamma_2^c)^{-1}} \pi_0 \end{aligned}$$

which corresponds to equations (5)-(7).

For later convenience, we notice that since the liberal central bank is facing a problem that is symmetric with the one described above, using a symmetric expression to (7) we have

$$\pi_0^\ell = \frac{1}{\gamma_2^\ell \left(1 - \gamma_1^\ell\right)} \left[\kappa \widetilde{y}^\ell + \beta (1 - q) \pi_0^c\right] \tag{A-7}$$

A.1 The case of full-commitment and constant policy objectives

The standard case of full-commitment and no uncertainty about policy objectives is a special case of the problem described above where $\widetilde{y}^c = \widetilde{y}^\ell \equiv \widetilde{y}$, $w^c = w^\ell \equiv w$ and $q = 1$. In this case, we have that $\pi_0^\ell = \pi_0^c \equiv \bar{\pi}_0$. Defining $\bar{\gamma}_2$ as the value taken by (A-6) when $q = 1$ we have

$$\bar{\pi}_0 = \frac{1}{\bar{\gamma}_2 - \beta} \kappa \widetilde{y}^c \tag{A-8}$$

where $\bar{\gamma}_2$ is the value taken by γ_2 when $q = 1$. Moreover, from (5) and (6)

$$\bar{\pi}_t = \frac{\bar{\gamma}_2^{-t}}{\bar{\gamma}_2 - \beta} \kappa \widetilde{y}^c \tag{A-9}$$

$$\bar{y}_t - \widetilde{y}^c = -\frac{\kappa}{w} \frac{1 - \bar{\gamma}_2^{-(t+1)}}{1 - \bar{\gamma}_2^{-1}} \frac{1}{\bar{\gamma}_2 - \beta} \kappa \widetilde{y}^c \tag{A-10}$$

A.2 The case of limited commitment

The case of limited commitment is one where policy objectives do not change. However, the monetary authority is not fully credible because at any point in time there is a probability $(1-q)$ that its previous promises are disregarded and that a reoptimization occurs. This case corresponds to one where $\widetilde{y}^c = \widetilde{y}^\ell \equiv \widetilde{y}$, $w^c = w^\ell \equiv w$ and $0 < q < 1$. As in the full-commitment case, we have that $\pi_0^\ell = \pi_0^c \equiv \bar{\pi}_0$. The resulting allocations are given by similar expressions to (A-8)-(A-10), substituting the value of γ_2 as given by (A-6), instead of $\bar{\gamma}_2$. As a consequence, we have that inflation and output are given by

$$\begin{aligned} \pi_t^{LC} &= \frac{\gamma_2^{-t}}{\gamma_2 - \beta}\kappa\widetilde{y}^c \\ y_t^{LC} - \widetilde{y}^c &= -\frac{\kappa}{w}\frac{1-\gamma_2^{-(t+1)}}{1-\gamma_2^{-1}}\frac{1}{\gamma_2-\beta}\kappa\widetilde{y}^c. \end{aligned}$$

Since $\frac{\partial\gamma_2}{\partial q} > 0$ we have that the higher is the probability of commitment, the lower are inflation and output. Finally, equation (8) is obtained dividing the above expression by (A-9).

A.3 The case of changes in the output target

We now analyze the case where there is uncertainty about the output target, i.e. where the current conservative target can be replaced by $\widetilde{y}^\ell > \widetilde{y}^c$, but keeping unchanged the weight on output stabilization, $w^c = w^\ell = w$. Substituting this into (7), and using the fact that in this case, being the output target the only difference among the two types of policymakers, $\gamma_1^c = \gamma_1^\ell \equiv \gamma_1$ and $\gamma_2^c = \gamma_2^\ell \equiv \gamma_2$ we obtain

$$\pi_0^c\left(1 - \frac{\beta^2(1-q)^2}{[\gamma_2(1-\gamma_1)]^2}\right) = \frac{1}{\gamma_2(1-\gamma_1)}\left[\kappa\widetilde{y}^c + \frac{\beta(1-q)}{\gamma_2(1-\gamma_1)}\kappa\widetilde{y}^\ell\right].$$

For convenience, we define $\Phi \equiv \frac{\beta(1-q)}{\gamma_2(1-\gamma_1)} = \frac{\beta-\beta q}{\gamma_2-\beta q} < 1$ and notice that $\frac{\partial\Phi}{\partial q} < 0$.

We thus have

$$\pi_0 = \frac{\Phi}{(1-\Phi)}\frac{\kappa}{\beta(1-q)}\frac{\widetilde{y}^c + \Phi\widetilde{y}^\ell}{(1+\Phi)}$$

Using this expression to substitute for π_0 in (5) and then dividing everything by

(A-9) one obtains

$$\pi_t \quad = \quad \underbrace{\frac{\widetilde{y}^c + \Phi \widetilde{y}^\ell}{\widetilde{y}^c\,(1+\Phi)}}_{\substack{\text{Liberal Objectives} \\ > 1}} \quad \underbrace{\left(\frac{\bar{\gamma}_2}{\gamma_2}\right)^t \frac{\bar{\gamma}_2 - \beta}{\gamma_2 - \beta}}_{\substack{\text{Limited Commitment} \\ > 1}} \quad \bar{\pi}_t,$$

which is the same as (9).

Finally, it easy to see that π_t is increasing in the difference between $\widetilde{y}^\ell$ and $\widetilde{y}^c$. We can also show that it is strictly decreasing in q, indeed

$$\frac{\partial \pi_t}{\partial q} = \frac{\widetilde{y}^c + \Phi \widetilde{y}^\ell}{\widetilde{y}^c\,(1+\Phi)} \frac{\partial \pi_t^{LC}}{\partial q} + \frac{\partial \frac{\widetilde{y}^c + \Phi \widetilde{y}^\ell}{\widetilde{y}^c(1+\Phi)}}{\partial q} \pi_t^{LC} < 0,$$

since both terms of the sum are negative. The first term is negative because of our result in the previous section, while the second term can be written as

$$\frac{\partial \frac{\widetilde{y}^c + \Phi \widetilde{y}^\ell}{\widetilde{y}^c(1+\Phi)}}{\partial q} \pi_t^{LC} = \frac{\partial \frac{\widetilde{y}^c + \Phi \widetilde{y}^\ell}{\widetilde{y}^c(1+\Phi)}}{\partial \Phi} \frac{\partial \Phi}{\partial q} \pi_t^{LC} = \underbrace{\frac{\widetilde{y}^\ell - \widetilde{y}^c}{\widetilde{y}^c(1+\Phi)^2}}_{>0} \underbrace{\frac{\partial \Phi}{\partial q}}_{<0} < 0.$$

A.4 The case of changes in the relative weight of output w

When there is uncertainty about the output weight, we have that the current conservative weight can be replaced by $w^\ell > w^c$, while keeping unchanged the output target $\widetilde{y}^c = \widetilde{y}^\ell = \widetilde{y}$. Substituting (A-7) into (5) one obtains

$$\pi_t = \frac{(\gamma_2^c)^{-t}}{\gamma_2^c\,(1-\gamma_1^c)} \frac{\left(1+\Phi^\ell\right)}{\left(1-\Phi^c \Phi^\ell\right)} \kappa \widetilde{y}$$

which divided by (A-9) delivers the expression in section 4.1.

B Recursive formulation of the problem of section 3

For notational convenience only, we abstract from the presence of uncertainty other than the one regarding the policy objective changes.

Proposition 1 *Being λ the vector of lagrange multipliers associated with the constraints (15) and (16), problem (14) can be written as a saddle point functional equation (SPFE) as follows:*

$$
\begin{aligned}
W(\gamma) &= \min_{\lambda\geq 0} \max_{\{\pi_t,y_t\}_{t=0}^{T-1}} \{h^m(\{\pi_t,y_t\}_{t=0}^{T-1},\lambda,\gamma)\} + \beta(1-q)V^{ij} + \beta q W(\gamma')\} \\
s.t.\ \ & \gamma' = \lambda, \quad \gamma_0 = 0
\end{aligned}
$$

where

$$
\begin{aligned}
h^m(\{\pi_t,y_t\}_{t=0}^{T-1},\lambda,\gamma) &\equiv \ell(\{\pi_t,y_t\}_{t=0}^{T-1}) + \lambda g_1(\{\pi_t,y_t\}_{t=0}^{T-1}) + \gamma g_2(\{\pi_t,y_t\}_{t=0}^{T-1}) \\
\ell(\{\pi_t,y_t\}_{t=0}^{T-1}) &\equiv \sum_{t=0}^{T-1} \beta^t \left[{\pi_t}^2 + w^i(y_t - \widetilde{y})^2\right]
\end{aligned}
$$

$$
\begin{aligned}
g_1(\{\pi_t,y_t\}_{t=0}^{T-1}) &\equiv \begin{bmatrix} \pi_0 - \kappa y_0 - \beta\pi_1 \\ \vdots \\ \pi_{T-2} - \kappa y_{T-2} - \beta\pi_{T-1} \\ \pi_{T-1} - \kappa y_{T-1} - \beta(1-q)\pi_T^j \end{bmatrix} \\
g_2(\{\pi_t,y_t\}_{t=0}^{T-1}) &\equiv \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \pi_0^i \end{bmatrix}
\end{aligned}
$$

Proof. of Proposition 1 Define the real valued function $r(\cdot)$ as follows:

$$
r(\{\pi_t,y_t\}_{t=0}^{T-1}) \equiv -\frac{1}{2}\sum_{t=0}^{T-1} \beta^t \left[\pi_t^2 + w^i(y_t - \widetilde{y}^i)^2\right] + \beta^T(1-q)V^{ij}
$$

Moreover, $g_1(\cdot)$ and $g_2(\cdot)$ are defined as in the second part of the proposition.

Problem (14) is therefore equivalent to:

$$\begin{aligned} V^i = & \max_{\{\pi_t, y_t\}_{t=0}^{\infty}} E_0 \sum_{m=0}^{\infty} (\beta^T q)^m r(\{\pi_{mT+t}, y_{mT+t}\}_{t=0}^{T-1}) \\ s.t. \quad & g_1(\{\pi_{mT+t}, y_{mT+t}\}_{t=0}^{T-1}) + g_2(\{\pi_{(m+1)T+t}, y_{(m+1)T+t}\}_{t=0}^{T-1}) \geq 0 \\ & \forall m = 0, 1, ..., \infty \end{aligned}$$

This formulation fits the definition of Program 1 in Marcet and Marimon (1998). We can therefore write the problem as a saddle point functional equation in the sense that there exists a unique function satisfying:

$$\begin{aligned} & W(\gamma) = \min_{\lambda \geq 0} \max_{\{\pi_t, y_t\}_{t=0}^{T-1}} h(\{\pi_t, y_t\}_{t=0}^{T-1}, \lambda, \gamma) + \beta q W(\gamma')\} \\ & s.t. \quad \gamma' = \lambda, \quad \gamma_0 = 0 \end{aligned}$$

where:

$$h(\{\pi_t, y_t\}_{t=0}^{T-1}, \lambda, \gamma) = r(\{\pi_t, y_t\}_{t=0}^{T-1}) + \lambda g_1(\{\pi_t, y_t\}_{t=0}^{T-1}) + \gamma g_2(\{\pi_t, y_t\}_{t=0}^{T-1})$$

or in a more intuitive formulation define:

$$\begin{aligned} h^m(\{\pi_t, y_t\}_{t=0}^{T-1}, \lambda, \gamma) & \equiv \ell(\{\pi_t, y_t\}_{t=0}^{T-1}) + \lambda g_1(\{\pi_t, y_t\}_{t=0}^{T-1}) + \gamma g_2(\{\pi_t, y_t\}_{t=0}^{T-1}) \\ \ell(\{\pi_t, y_t\}_{t=0}^{T-1}) & \equiv \sum_{t=0}^{T-1} \beta^t \left[{\pi_t}^2 + w^i (y_t - \widetilde{y})^2 \right] \end{aligned}$$

and the saddle point functional equation is:

$$\begin{aligned} & W(\gamma) = \min_{\lambda \geq 0} \max_{\{\pi_t, y_t\}_{t=0}^{T-1}} \{h^m(\{\pi_t, y_t\}_{t=0}^{T-1}, \lambda, \gamma)\} + \beta(1-q)V^{ij} + \beta q W(\gamma')\} \\ & s.t. \quad \gamma' = \lambda, \quad \gamma_0 = 0 \end{aligned}$$

■

Proposition 2 *For any type of policy objectives $i = \ell, c$ the solution of problem (14) is a tenure invariant function $\psi(\gamma)$, such that:*

$$\psi(\gamma) = \arg\min_{\lambda \geq 0} \max_{\{\pi_t, y_t\}_{t=0}^{T-1}} \{h^m(\{\pi_t, y_t\}_{t=0}^{T-1}, \lambda, \gamma)\} + \beta(1-q)V^{ij} + \beta q W(\gamma')\}$$
$$\gamma' = \lambda, \ \gamma_0 = 0$$

Proof. of Proposition 2: Using Proposition 1, this proof follows directly from the results of Marcet and Marimon (1998). ■