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Tax Smoothing in Frictional Labor Markets

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Abstract

We re-examine the optimality of tax smoothing from the point of view of frictional labor markets. Our central result is that whether or not this cornerstone optimal fiscal policy prescription carries over to an environment with labor market frictions depends crucially on the cyclical nature of labor force participation. If the participation rate is exogenous at business-cycle frequencies — as is typically assumed in the literature — we show it is not optimal to smooth tax rates on labor income in the face of business-cycle shocks. However, if households do optimize at the participation margin, then tax-smoothing is optimal despite the presence of matching frictions. To understand these results, we develop a concept of general-equilibrium efficiency in search-based environments, which builds on existing (partial-equilibrium) search-efficiency conditions. Using this concept, we develop a notion of search-based labor-market wedges that allows us to trace the source of the sharply-contrasting fiscal policy prescriptions to the value of adjusting participation rates. Our results demonstrate that policy prescriptions can be very sensitive to the cyclical nature of labor-force participation in search-based environments.

Keywords: labor market frictions, optimal taxation
JEL Classification: E24, E50, E62, E63

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1 Introduction

We have two main aims in this paper. The first is to re-examine a classic issue in the theory of fiscal policy — the optimality of labor-income tax smoothing — from the point of view of frictional labor markets. Since Barro's (1979) partial-equilibrium intuition, Lucas and Stokey's (1983) general-equilibrium analysis, and continuing through to today's quantitative DSGE models used to study optimal fiscal policy, the prescription that governments ought to hold labor tax rates virtually constant in the face of aggregate shocks is well-known to macroeconomists. We show that this cornerstone policy prescription and the intuition underlying it carry over to a general-equilibrium environment in which search and matching frictions exist in labor markets *only if* households optimally adjust their labor-force participation over the business cycle. In contrast, if the labor-force participation rate is exogenous at business cycle horizons, as is almost uniformly assumed in the recent vintages of DSGE labor-search models, then purposeful tax-rate volatility is optimal. The optimal degree of tax-rate variability in this case is one to two orders of magnitude larger than benchmark results in the Ramsey literature. Nevertheless, the degree of cyclical variability in the participation rate implied by our model in this case is well within empirical ranges. Our results demonstrate that a model's policy prescriptions can be very sensitive to its assumed cyclical nature of labor-force participation.

The second aim of our work is to provide a general notion of efficiency for use in search-based general-equilibrium macroeconomic models. As part of the recent surge in popularity in building DSGE models featuring search and matching frictions in labor markets, many studies have focused on the transmission channels of macroeconomic policy and the determination of optimal policy. Efficiency concerns lie at the heart of any model studying optimal policy. In light of this, a contribution of this paper is that we develop a general-equilibrium notion of efficiency for search-based models that not only clearly shows the conditions under which tax-smoothing is and is not optimal, but also may be helpful in casting light on results that are starting to emerge elsewhere in the literature.

In particular, we develop a precise notion of the marginal rate of transformation (MRT) between leisure (being outside the labor force) and output that takes into account search frictions and the long-lived nature of employment relationships that they endogenously generate. With this concept of MRT, we are able to characterize *general-equilibrium efficiency* in a search model using only the basic principle that MRTs should be equated to their associated marginal rates of substitution (MRS). This characterization of efficiency in turn allows us to define a novel type of *labor wedge*, which is explicitly search-based, between MRS and MRT. This view of wedges pinpoints the conditions under which smoothing of labor income tax rates is optimal even though labor markets are always frictional in our model. Our notion of efficiency has as a necessary, but not sufficient, component the well-known Hosios (1990) condition for (partial-equilibrium) search efficiency. Our definition of the MRT nests as a special case the RBC neoclassical-based notion of the MRT between leisure and output. Similarly, our notion of the search-based labor wedge nests as a special case the neoclassical notion of the labor wedge emphasized by Chari, Kehoe, and McGrattan (2007), Shimer (2008), and others. We view as virtues of our definitions of efficiency and wedges that they nest their counterparts in the widely-understood RBC model.

Our baseline search and matching environment is identical to the one that has come into widespread use in recent DSGE modeling efforts. The crucial feature for our analysis is that — in keeping with nearly all of the existing literature — it assumes the size of the labor force is exogenous. We quantitatively demonstrate the optimality of labor tax-rate volatility in this environment and trace the source of this volatility directly to the inability of the labor-force-participation rate to adjust to business-cycle shocks. The inability of the participation rate to adjust is reflected in cyclical variations in the shadow value of the fixed participation margin. Cyclical variations in this shadow value shift the search-based labor wedge that we identify. Time-variation in tax rates is then used to stabilize the overall labor wedge. In a calibrated version of this economy, the optimal degree of labor tax volatility is large: the standard deviation of the optimal tax rate is between 2 and 6 percent, depending on precise parameter values, around a mean of about 20 percent. For comparison, the Ramsey literature's conventional tax-smoothing result entails optimal tax-rate volatility near 0.1 percent or less — for example, see the overview in Chari and Kehoe (1999). Thus, optimal labor tax rates are one to two orders of magnitude more volatile in the widely-used baseline search model than in simple neoclassical-based Ramsey environments.

On the other hand, if participation is endogenous over the business cycle, we show that, despite the presence of search frictions, the labor tax rate is the *only* aspect of the economy that creates a labor wedge. Even though the labor wedge we define is a generalization of the standard notion, the basic prescription that a Ramsey government should keep wedges nearly constant over time applies, which in this case calls for tax-rate smoothing. Thus, if participation adjusts, the analytical insights developed by Werning (2007) and Scott (2007) on the optimality of tax smoothing in a neoclassical labor market also seem to apply to frictional labor markets. More broadly, endogeneity of labor-force participation bridges the gap between a frictionless neoclassical environment and frictional environments (ala Pissarides (2000)) by allowing households a unilateral optimization margin with respect to *some* aspect of their labor-market outcomes. Such an optimization margin is at the core of any neoclassical-based macro model, but is absent in virtually all search-based macro models.

In our model, the optimization margin with respect to labor-market outcomes is the participation decision: optimal policy critically depends on whether or not labor-force participation fluctuates optimally at business cycle horizons. Most of the empirical evidence suggests very small fluctuations along the participation margin at business-cycle timeframes. Elsby, Michaels, and Solon (2008) and many others argue that because cyclical adjustment of participation is so small, assuming no cyclical fluctuations in participation is a useful approximation in order to focus on other labor-market transitions. Empirical arguments such as the latter are often used in theoretical work to justify modeling the size of the labor force as fixed over the business cycle. However, our results show that modeling *small* fluctuations in participation as *zero* fluctuations can lead to sharply contrasting policy prescriptions. In particular, in the version of our model featuring adjustment of participation rates, the equilibrium fluctuations along this margin indeed turn out to be very small: our model predicts that the participation rate varies by about half a percentage point around a mean of 66 percent. This range of variation is very much in line with what is observed in data. Thus, using the narrow range of observed participation rates as a basis for ignoring the participation margin altogether can matter quantitatively and qualitatively for some policy-relevant questions. It is not the narrow range of fluctuations in participation rates that matters for our results; rather, it is the fluctuations in associated shadow prices that matter. Veracierto (2008), while focused on a different range of issues, is another recent example demonstrating that predictions of modern labor-market models can be sensitive to the assumed cyclical nature of labor force participation.

To conduct our analysis, we formulate a DSGE labor-search model in a way that nests both the typical exogenous-labor-force specification as well as the endogenous labor-force specification. We nest these two frameworks by defining a shadow value of adjustment along the participation margin, and this shadow value appears in key equilibrium conditions of the model, including the search wedge described above. If participation always adjusts optimally, this shadow value is by construction zero; if participation does not adjust to shocks, this shadow value fluctuates over time, thus shifting equilibrium conditions. Because our formulation is easy to use, it can be applied to any DSGE search model in order to shed light on the economic forces at work.

Finally, we note that tax volatility in our model is not driven by any incompleteness of government debt markets, which is a well-understood point in Ramsey models since Aiyagari, Marcet, Marimon, and Sargent (2002). Thus, our only point of departure from a neoclassically-based complete-markets Ramsey setup, such as the textbook Chari and Kehoe (1999) presentation, is that we model labor trades as governed by primitive search and matching frictions. While search-based DSGE models have become commonplace in recent years, their ability to shed new insights on optimal fiscal policy has not yet been much explored.[1] Our model features no capital accumulation, in order to highlight the dynamics of labor taxes. We see no reason why our basic intuition and results would not extend to the classic case of Ramsey taxation of both labor and capital income.

[1]To our knowledge, the only work investigating aspects of optimal fiscal policy in labor-search dynamic general equilibrium models is Domeij (2005), Boone and Bovenberg (2002), and our own previous work, Arseneau and Chugh (2006, 2008). In none of these is the focus explicitly on tax smoothing.

The rest of our work is organized as follows. Section 2 lays out the basic search model, in which we are able to capture in nested form both an environment that does feature and an environment that does not feature cyclical adjustment of labor-force participation. In Section 3, we develop our general-equilibrium concepts of MRT and efficiency. In Section 4, we show which features of the decentralized search economy create wedges in the efficiency condition we propose. Section 5 presents the Ramsey problem. Section 6 presents our main results. Section 7 concludes.

2 Model

As many other recent studies have done, we embed the baseline Pissarides (2000) textbook search model into a DSGE framework. The baseline Pissarides (2000) framework features no labor-force participation decision. Virtually all of the recent DSGE models adopting a search-theoretic foundation for labor markets have inherited this structure. We instead consider two variants of the environment facing households, one in which the number of individuals in the labor force (defined as individuals either working or actively seeking employment) is exogenous and one in which it is endogenous. Despite search and matching frictions, the latter setup renders our model close to DSGE models based on neoclassical labor markets, in which the key equilibrium margin is often the labor-force participation margin.[2] Regardless of whether participation is endogenous or exogenous, however, transitions into employment are always subject to search and matching frictions in our model.

We present these two versions of our search model in a unified, nested way. Specifically, rather than construct essentially two different environments, we construct one very general framework that allows for an endogenous participation decision but that, by introducing one additional constraint, can also allow for exogenous participation. As described below, nesting the two environments in this way allows us to clearly isolate the incentives relevant for optimal policy. These incentives turn out to be captured by the shadow value of the constraint that allows us to move between the exogenous- and the endogenous-participation versions of our model.

We present in turn the choice problems of the representative firm, the representative household, the determination of wages, the actions of the government, and the definition of private-sector equilibrium.

[2] Because an RBC model only considers two labor-market states, rather than all three, one can interchangeably refer to the relevant margin there as either a participation margin or, in the more commonly-used language, a labor-*supply* margin. Although the latter terminology has been the most commonly-used, the former characterization seems equally well-justified. For example, Kydland and Prescott (1982, p. 1350-1351) themselves refer to the time allocated to non-work as time allocated to non-*market* activities such as household production. Another example would be education. From the point of view of macroeconomic models, we think it seems natural to consider individuals engaged in non-market activities as outside the labor force.

2.1 Production

The production side of the economy features a representative firm that must open vacancies, which entail costs, in order to hire workers and produce. The representative firm is "large" in the sense that it operates many jobs and consequently has many individual workers attached to it through those jobs.

The firm requires only labor to produce its output. The firm must engage in costly search for a worker to fill each of its job openings. In each job k that will produce output, the worker and firm bargain over the pre-tax real wage w_{kt} paid in that position. Output of any job k is given by $y_{kt} = z_t$, which is subject to a common technology realization z_t.

Any two jobs k_a and k_b at the firm are identical, so from here on we suppress the second subscript and denote by w_t the real wage in any job, and so on. Total output of the firm thus depends on the technology realization and the measure of matches n_t^f that produce,

$$y_t = z_t n_t^f. \tag{1}$$

The total real wage bill of the firm is the sum of wages paid at all of its positions, $n_t^f w_t$.

The firm begins period t with employment stock n_{t-1}^f. Its period-t productive employment stock, n_t^f, depends on its period-t vacancy-posting choices as well as the random matching process described below.[3] With probability $k^f(\theta)$, taken as given by the firm, a vacancy will be filled by a worker. Labor-market tightness is $\theta \equiv v/u$, where u denotes the number of individuals searching for jobs. Matching probabilities for both firms and households depend only on aggregate market tightness given the Cobb-Douglas matching function assumed below.

Our baseline wage-determination mechanism is Nash bargaining, as described below. In the firm's profit maximization problem, the wage-setting protocol is taken as given.[4] We employ Nash bargaining because it has become familiar in DSGE search models, and thus enhances comparability with existing studies. However, as we note below, our results are identical if we employ an

[3] Labor-market matching thus occurs within a period, which, given the quarterly calibration we will pursue, is empirically descriptive of U.S. labor-market flows — see, for example, the evidence of Davis, Faberman, and Haltiwanger (2006). This so-called "instantaneous-hiring" view of labor-market flows has recently become widely used in this class of models, employed by, among others, Blanchard and Gali (2007, 2008) and Krause, Lopez-Salido, and Lubik (2007).

[4] This assumption is without loss of generality in the standard Pissarides-type model because even if the firm believed it could opportunistically manipulate the wages it paid by under- or over-hiring, the fact that labor's marginal product is independent of total employment prevents such opportunistic manipulation of wage-bargaining sets. Thus, in the standard exogenous-productivity Pissarides model, holdup problems are in principle present, but there is no lever by which firms can strategically react to them. If firm output exhibited diminishing marginal product in its total employment level, then the firm would have an incentive to over-hire, and it would not be innocuous to assume that firms take wages as given when choosing how many vacancies to post. See, for example, Smith (1999), Cahuc, Marque, and Wasmer (2008), and Krause and Lubik (2006).

alternative equilibrium concept, *competitive search equilibrium*, in which wages are determined in a decentralized fashion rather than through bilateral negotiations.

The firm thus chooses vacancies to post v_t and a current employment stock n_t^f to maximize discounted profits starting at date t,

$$E_t \sum_{t=0}^{\infty} \beta^t \left\{ \Xi_{t|0} \left[z_t n_t^f - w_t n_t^f - \gamma v_t \right] \right\}, \tag{2}$$

where $\Xi_{t|0}$ is the period-0 value to the representative household of period-t goods, which we assume the firm uses to discount profit flows because households are the ultimate owners of firms.[5] In period t, the firm's problem is thus to choose v_t and n_t^f to maximize (2) subject to a sequence of perceived laws of motion for its employment level,

$$n_t^f = (1-\rho^x) n_{t-1}^f + v_t k^f(\theta_t). \tag{3}$$

Firms incur the real cost γ for each vacancy created, and job separation occurs with exogenous fixed probability ρ^x. Note the timing of events embodied in the law of motion (3). Period t begins with employment stock n_{t-1}^f, some of whom separate from the firm before period-t production occurs; the firm posts vacancies and hires a flow of new employees $k^f(\theta_t)v_t$, which depends on both the firm's decisions and market conditions; the employment stock n_t^f then engages in production.

The firm's first-order conditions with respect to v_t and n_t^f yield a standard job-creation condition

$$\frac{\gamma}{k^f(\theta_t)} = z_t - w_t + (1-\rho^x) E_t \left[\Xi_{t+1|t} \frac{\gamma}{k^f(\theta_{t+1})} \right], \tag{4}$$

where $\Xi_{t+1|t} \equiv \Xi_{t+1|0}/\Xi_{t|0}$ is the household discount factor (again, technically, the real interest rate) between period t and $t+1$. The job-creation condition states that at the optimal choice, the vacancy-creation cost incurred by the firm is equated to the discounted expected value of profits from a match. Profits from a match take into account future marginal revenue product from the match, the wage cost of the match, and the asset value of having a pre-existing relationship with an employee in period $t+1$. This condition is a free-entry condition in the creation of vacancies and is a standard equilibrium condition in a labor search and matching model.

2.2 Households

There is a representative household in the economy. Each household consists of a continuum of measure one of family members, and each individual family member is classified as either inside the labor force or outside the labor force. An individual family member that is outside the labor

[5] Technically, of course, it is the real interest rate with which firms discount profits, and in equilibrium the real interest rate between time zero and time t is measured by $\Xi_{t|0}$. Because there will be no confusion using this equilibrium result "too early," we skip this intermediate level of notation and structure.

force enjoys leisure. An individual family member that is part of the labor force is engaged in one of two activities: working, or not working but actively searching for a job. The convenience of an "infinitely-large" household is that we can naturally suppose that each individual family member experiences the same level of consumption regardless of his personal labor-market status. This tractable way of modeling perfect consumption insurance in general-equilibrium search-theoretic models of labor markets has been common since Andolfatto (1996) and Merz (1995). We use the terms "individual" and "family member" interchangeably from here on. Given the basics of the environment, we also use the terms "leisure" and "outside the labor force" interchangeably from here on.

As noted above, we construct our model in a flexible way to allow for two cases regarding labor-force participation. In the first case, which corresponds to the benchmark Pissarides (2000) model, the size of the labor force is exogenously fixed at the measure $n_t^h + u_t^h = \bar{l} < 1$, with $\bar{l}$ a parameter to be calibrated below. With the labor-force participation rate fixed at $\bar{l}$, the fixed measure $1 - \bar{l} = 1 - u_t^h - n_t^h$ of family members thus enjoy leisure in every period. For reasons that will become clear immediately below, we label this version of our model the *pseudo-labor-force-participation model*, or pseudo-LFP model for short. In the second case, households each period optimally choose the size of $n_t^h + u_t^h$; the labor-force participation rate is thus endogenous in every period t. We label this version of the model the *labor-force-participation model* (LFP model). As the descriptions below show, with the exception of one constraint, the household problem is identical in both the pseudo-LFP model and the LFP model.

2.2.1 Pseudo-LFP Model

Although the labor-force participation rate is fixed in the baseline environment, we formulate the household problem *as if there were free choice over the participation rate*, but impose a constraint that ensures it is always $\bar{l}$. This formulation motivates our label "pseudo" LFP model. Households are fully aware of the constraint $n_t^h + u_t^h = \bar{l}$. The advantage of this formulation of the problem is that the shadow value of this constraint directly sheds light on the mechanism driving our optimal-policy results and makes quite transparent comparisons with the full LFP model described next.

The representative household maximizes expected lifetime discounted utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + g(1 - u_t^h - n_t^h) \right] \tag{5}$$

subject to a sequence of flow budget constraints

$$c_t + b_t = n_t^h (1 - \tau_t^n) w_t + u_t^h \chi + R_t b_{t-1} + (1 - \tau^d) d_t, \tag{6}$$

a sequence of perceived laws of motion for the measure of family members that are employed,

$$n_t^h = (1 - \rho^x) n_{t-1}^h + u_t^h k^h(\theta_t), \tag{7}$$

and the exogenous restriction on the size of the labor force

$$n_t^h + u_t^h = \bar{l}. \tag{8}$$

The functions $u(.)$ and $g(.)$ are standard strictly-increasing and strictly-concave subutility functions over consumption and leisure, respectively.

A utility value of leisure is often what is meant when partial-equilibrium labor-search models refer casually to the "outside benefit of not working." Our model formalizes this idea in a way that is easily comparable to RBC-based models. This way of modeling (part of) the outside benefit of not working is identical to that in Shi and Wen (1999) and Domeij (2005). As emphasized by Blanchard and Gali (2008), the fact that, at the household level, the marginal rate of substitution between consumption and leisure is *not* an exogenous constant also distinguishes our model from partial-equilibrium search environments.

A second notion of the "outside benefit of not working" is represented by χ, which is the flow of unemployment benefits each unemployed individual (actively searching for employment) receives. We assume the unemployment benefit is time-invariant. Because our model explicitly depicts *three* labor-market states, we can meaningfully distinguish between unemployment benefits and the utility of leisure, which are often referred to and modeled interchangeably in partial-equilibrium environments and in even some general-equilibrium search environments.

The rest of the terms in the constraints are as follows. The after-tax real wage rate each employed individual earns is $(1 - \tau_t^n)w_t$. The household takes as given the probability $k^h(\theta)$ that one of its unemployed and searching individuals will find employment. As with matching probabilities for firms, k^h depends only on aggregate labor-market tightness given the assumption of a Cobb-Douglas matching technology. Finally, b_{t-1} is the household's holdings of a state-contingent one-period real government bond at the end of period $t-1$, which has gross state-contingent payoff R_t at the beginning of period t. Important to note is that the government is able to issue fully state-contingent debt; thus, none of our optimal policy results will be driven by an inability on the part of the government to use debt as a shock absorber. Incompleteness of government debt markets can be an important driver of results in Ramsey models — see, for example, Aiyagari, Marcet, Marimon, and Sargent (2002). Of course, because this is a Ramsey-taxation model, there are no lump-sum taxes or transfers between the government and the private sector.

Due to firms' sunk resource and time costs of finding employees, firms earn positive flows of economic profits. These profits are transferred to households at the end of each period in lump-sum fashion: d_t is the household's receipts of firms' flow profits. We permit government taxation of households' receipts of dividends at the fixed tax rate τ^d. As is well-understood in the Ramsey literature, flows of untaxed dividends received by households in and of themselves affect optimal-policy prescriptions.[6] To make our results as comparable as possible to baseline models that prescribe labor-tax-rate smoothing, in which there are zero economic profits/dividends, our main analysis is conducted assuming $\tau^d = 1$. The consequence of this assumption is that any predictions made by our model regarding optimal labor-income taxation cannot be due to incentives to tax profits, either in the long-run or the short-run. In robustness exercises presented in Appendix G, we consider the opposite extreme of $\tau^d = 0$ and show that our main results are unaffected.

The formal analysis of this problem is presented in Appendix A; here we simply describe intuitively the outcome of household optimization. One condition stemming from household optimization is a standard consumption-savings condition,

$$u'(c_t) = E_t\left[\beta u'(c_{t+1})R_{t+1}\right]. \tag{9}$$

As usual, this condition defines the stochastic discount factor, $\Xi_{t+1|t} = \beta u'(c_{t+1})/u'(c_t)$, with which firms, in equilibrium, discount profit flows.

More important for the analysis of the pseudo-LFP model is the household's *pseudo-labor-force participation condition*

$$\begin{aligned}\frac{g'(1-u_t^h-n_t^h)-u'(c_t)\chi}{k^h(\theta_t)} + \frac{\phi_t}{k^h(\theta_t)} &= u'(c_t)(1-\tau_t^n)w_t - g'(1-u_t^h-n_t^h)\\ &- \quad \phi_t + \beta(1-\rho^x)E_t\left\{\frac{g'(1-u_{t+1}^h-n_{t+1}^h)-u'(c_{t+1})\chi}{k^h(\theta_{t+1})} + \frac{\phi_{t+1}}{k^h(\theta_{t+1})}\right\},\end{aligned} \tag{10}$$

which is derived in detail in Appendix A. In (10), ϕ_t denotes the Lagrange multiplier associated with the constraint (8). Suppose for a moment that $\phi_t = 0$, $\forall t$, which means there is no exogenous restriction on the participation rate. With $\phi_t = 0$, condition (10) has a straightforward interpretation: at the optimum, the household each period sends a measure u_t^h of family members to search for jobs until the expected cost of search — the left-hand-side of (10) — is equated to the expected benefit of search — the right-hand-side of (10). The expected cost of search is measured by the marginal utility of leisure (each unit of search involves forgoing one unit of leisure) net of the direct unemployment benefits obtained by searchers (converted into appropriate units using the marginal utility of wealth). The expected benefit of search involves the marginal utility value of after-tax wage income and the marginal disutility of work (the first two terms on the right-hand-side of (10)), along with the asset value to the household of having an additional family member engaged in an ongoing employment relationship (the term in expectations on the right-hand-side of (10)). This asset value reflects the value to the household of sending one fewer family member out to look for a job in the future.

[6] See, for example, Stiglitz and Dasgupta (1971), Jones, Manuelli, and Rossi (1997), Schmitt-Grohe and Uribe (2004), Siu (2004), and Arseneau, Chugh, and Kurmann (2008) for examples in various contexts of this type of taxation incentive.

With $\phi_t = 0$, condition (10) naturally has the interpretation of a free-entry condition into the labor force. However, because in the pseudo-LFP model the size of the labor force is fixed at $\bar{l} < 1$, the shadow value ϕ_t is non-zero. The shadow value ϕ_t measures the value to the household of being able to freely adjust its participation rate; as such, it can be interpreted as the price of participation. In equilibrium, this shadow price always adjusts so that condition (10) results in *no* net entry into or exit from the labor force. Hence the terminology *pseudo* labor-force participation condition to refer to (10). The dynamics of the shadow price ϕ_t are crucial in driving the dynamics of the optimal labor-income tax rate in our analysis below.

We emphasize that in this version of the model, the participation rate $n_t^h + u_t^h$ is indeed fixed at $\bar{l}$ every period. Hence, our pseudo-LFP formulation delivers the same household decisions as if we had taken the more standard approach of specifying the household problem as one of choosing only c_t subject to the budget constraint (6), completely dropping constraints (7) and (8), setting $g(.) = 0$, and setting, as is common in DSGE search models, $n_t^h + u_t^h = \bar{l} = 1 \ \forall t$. It is not the underlying optimal choices of the household that we change by specifying the model this way, it is only the way we cast the household problem that is different. The important gain our formulation brings to the analysis is being able to measure the shadow value ϕ_t.

2.2.2 LFP Model

If households were instead free to choose the labor-force participation rate, the household's decision problem would be exactly as just described, except of course constraint (8) would not bind household behavior. Household optimal choices would thus be characterized by the consumption-savings optimality condition (9) and the "true" labor-force participation condition,

$$\frac{g'(1-u_t^h-n_t^h)-u'(c_t)\chi}{k^h(\theta_t)} = u'(c_t)(1-\tau_t^n)w_t - g'(1-u_t^h-n_t^h) + \beta(1-\rho^x)E_t\left\{\frac{g'(1-u_{t+1}^h-n_{t+1}^h)-u'(c_{t+1})\chi}{k^h(\theta_{t+1})}\right\}, \tag{11}$$

which of course is simply condition (10) with $\phi_t = 0 \ \forall t$.

Despite the search frictions and long-lived nature of employment relationships, the labor-force participation condition (11) has the same interpretation as the labor-supply function in a simple neoclassical labor market, as exists in standard Ramsey models used to study the optimality of tax-smoothing. Indeed, we can recover a neoclassical labor market by setting $\rho^x = 1$ (all employment "relationships" are one-period, spot, transactions), setting $\chi = 0$ (because there is no notion of "unemployment" hence of "unemployment benefits" in a neoclassical market), and fixing the probability of "finding a job" to $k^h(\theta) = 1$ (because in a neoclassical market there of course is no friction in "finding a job"). Imposing these assumptions and logic on (11), we obtain $\frac{g'(.)}{u'(.)} = (1-\tau_t^n)w_t$, which defines the labor-supply function in a neoclassical market.

With matching frictions that create a meaningful separation of the labor force into those individuals that are employed and those individuals that are unemployed, condition (11) defines transitions of individuals from outside the labor force (leisure, in our model) into the pool of searching unemployed, from where the aggregate matching process will pull some individuals into employment. In the pseudo-LFP model, condition (10) plays the same conceptual role, but, because the participation rate is exogenous, what it pins down is the shadow price ϕ_t rather than the participation rate. We thus speak of (11) and (10) as labor-force participation functions, rather than labor-supply functions.

2.3 Wage Bargaining

Our baseline wage-determination mechanism is Nash bargaining. Specifically, we assume that wages of all workers, whether newly-hired or not, are set in period-by-period Nash negotiations. This assumption is common in search-based DSGE models, which is why we use it as our benchmark. A detailed derivation of the wage-bargaining problem is presented in Appendix B for the pseudo-LFP model and in Appendix C for the LFP model. In what follows, we simply present the bargaining outcomes. However, none of our results is sensitive to wages being determined in an explicitly bilateral fashion, as is the case with Nash bargaining. An alternative equilibrium concept for search models, due to Moen (1997), is a *competitive search equilibrium*. In competitive search equilibrium, wages are determined by decentralized forces and taken as given by all market participants. In Appendix H, we show that competitive search delivers exactly the same wage outcome as period-by-period Nash negotiations; hence its policy implications are identical.[7]

Assuming that $\eta \in (0,1)$ is a worker's Nash bargaining power and $1-\eta$ a firm's Nash bargaining power, the Nash wage outcome in the pseudo-LFP model is given by

$$\begin{aligned} w_t = {} & \eta z_t + (1-\eta)\frac{g'(1-u_t^h-n_t^h)}{u'(c_t)(1-\tau_t^n)} + (1-\eta)\frac{\phi_t}{u'(c_t)(1-\tau_t^n)} \\ - {} & \eta\frac{1-\rho^x}{1-\tau_t^n}E_t\left\{\Xi_{t+1|t}(1-\tau_{t+1}^n)\frac{\gamma}{k^f(\theta_{t+1})}\right\} + \eta(1-\rho^x)E_t\left\{\Xi_{t+1|t}\frac{\gamma}{k^f(\theta_{t+1})}\right\}. \end{aligned} \tag{12}$$

The bargained wage is a convex combination of the maximum value to a firm of entering into the marginal employment relationship and the minimum value required for the household to send a family member on the margin into a new employment relationship. The first line of (12) shows that part of the period-t wage payment is a convex combination of the contemporaneous values to the firm and the household, given, respectively, by the marginal product of a new employment match

[7]This equivalence is a DSGE extension of the equivalence shown by Moen (1997) between competitive search outcomes and Nash bargaining under the well-known Hosios (1990) condition. As we discuss below, all of our analysis of the bargaining economy satisfies the Hosios (1990) condition. See also Rogerson, Shimer, and Wright (2005) for more on competitive search.

z_t and the after-tax MRS between consumption and participation. This after-tax MRS includes the usual MRS $g'(.)/u'(.)$ but also a second MRS-like term that reflects the price the household would be willing to pay to be able to send additional members to search for jobs. This value is reflected in the shadow price ϕ_t. The second line of (12) captures the forward-looking, relationship, aspect of employment, whose value is also capitalized in the period-t wage payment. Apart from the fact that expectations of future (period $t+1$) tax rates appear in this forward-looking term, this forward-looking aspect of wages is standard in search models that use Nash bargaining.

If instead participation rates adjust optimally over the business cycle, the Nash wage outcome is given by

$$\begin{aligned} w_t &= \eta z_t + (1-\eta)\frac{g'(1-u_t^h-n_t^h)}{u'(c_t)(1-\tau_t^n)} \\ &- \eta\frac{1-\rho^x}{1-\tau_t^n}E_t\left\{\Xi_{t+1|t}(1-\tau_{t+1}^n)\frac{\gamma}{k^f(\theta_{t+1})}\right\} + \eta(1-\rho^x)E_t\left\{\Xi_{t+1|t}\frac{\gamma}{k^f(\theta_{t+1})}\right\}, \end{aligned} \tag{13}$$

which of course is simply (12) with $\phi_t = 0$. The wage rules (12) and (13) thus differ only in whether or not the shadow price of the participation restriction affects the bargained wage.

Finally, note that if labor taxes were constant at $\tau_t^n = \bar{\tau}^n\ \forall t$, the wage outcome would be $w_t = \eta z_t + (1-\eta)\frac{g'(1-u_t-n_t)}{u'(c_t)(1-\bar{\tau}^n)} + (1-\eta)\frac{\phi_t}{u'(c_t)(1-\bar{\tau}^n)}$. In this case, the presence of the labor tax only changes firms' effective bargaining power in a static manner: $\bar{\tau}^n > 0$ causes $(1-\eta)/(1-\bar{\tau}^n) > 1-\eta$. This kind of static bargaining wedge underpins the results in Arseneau and Chugh (2006); in our model here, this purely static effect would be unable to offset cyclical variations in the shadow prices of the participation restriction.

2.4 Government

The government finances an exogenous stream of spending $\{g_t\}$ by collecting labor income taxes, dividend income taxes, and issuing real state-contingent debt. The period-t government budget constraint is

$$\tau_t^n w_t n_t + \tau^d d_t + b_t = g_t + R_t b_{t-1} + u_t\chi. \tag{14}$$

As noted when we presented the problem of the representative household, the fact that the government is able to issue fully state-contingent real debt means that none of our results is driven by incompleteness of debt markets.

We include payment of unemployment benefits as a government activity for two reasons. First, we think it empirically descriptive to view the government as providing such insurance. Second, from a technical standpoint, including $u_t\chi$ in the government budget constraint means that χ does *not* appear in the economy-wide resource constraint (presented below). In DSGE labor-search models, it is common to include unemployment benefits in the household budget constraint but yet exclude them from the economy-wide resource constraint — see, for example, Krause and Lubik (2007) or Faia (2008). In such models, the government budget constraint is a residual object due to the presence of a lump-sum tax. In contrast, we rule out lump-sum taxes in order to conduct our Ramsey analysis and thus cannot treat the government's budget as residual. A Ramsey problem requires specifying both the resource constraint and either the government or household budget constraint as equilibrium objects, and this requires us to take a more precise stand on the source of unemployment benefits than usually taken in the literature. To make our model setup as close as possible to existing ones that study tax smoothing, we must assert that payment of unemployment benefits is a transfer between the government and households.

2.5 Matching Technology

In equilibrium, $n_t = n_t^f = n_t^h$, so we now refer to employment simply as n_t. Matches between unemployed individuals searching for jobs and firms searching to fill vacancies are formed according to a constant-returns matching technology, $m(u_t, v_t)$, where u_t is the number of searching individuals and v_t is the number of posted vacancies. A fraction ρ^x of matches that produced in period $t-1$ are exogenously destroyed before period t. All newly-formed matches produce at least once before possibly dissolving. The evolution of aggregate employment is thus given by

$$n_t = (1-\rho^x)n_{t-1} + m(u_t, v_t). \tag{15}$$

2.6 Private-Sector Equilibrium

Whether considering the pseudo-LFP model or the LFP model, a symmetric private-sector equilibrium is made up of endogenous processes $\{c_t, w_t, n_t, v_t, u_t, R_t, b_t\}_{t=0}^{\infty}$ that satisfy the vacancy-posting condition (4), the consumption-savings optimality condition (9), the government budget constraint (14), the law of motion for the aggregate stock of employment (15), and the aggregate resource constraint of the economy

$$c_t + g_t + \gamma v_t = z_t n_t. \tag{16}$$

In (16), total costs of posting vacancies γv_t are a resource cost for the economy. As discussed above, unemployment benefits χ do not absorb any part of market output.

In the LFP model, the labor-force participation condition (11) and the period-by-period Nash wage outcome (13) complete the set of conditions characterizing equilibrium. In the pseudo-LFP model, the pseudo-LFP condition (10) and the Nash wage outcome (12) are instead part of the set of private-sector equilibrium conditions, as is the exogenous restriction on the total size of the labor force

$$n_t + u_t = \bar{l}. \tag{17}$$

In the pseudo-LFP model, the endogenous process of shadow prices $\{\phi_t\}_{t=0}^{\infty}$ is also added to the list of endogenous equilibrium processes. Finally, in either the pseudo-LFP model or the LFP model, the private sector takes as given stochastic processes $\{z_t, g_t, \tau_t^n\}_{t=0}^{\infty}$.

3 Search Efficiency in General Equilibrium

To understand the optimal tax results that emerge from the Ramsey problem, it is useful to first present the conditions that characterize the constrained-efficient allocation that would be chosen by a Social Planner that is restricted by the matching technology. Our focus is on developing a general-equilibrium search-theoretic notion of efficiency along the consumption-leisure margin. Although Hosios-efficiency is a necessary component of it, the efficiency condition we present below is different from the well-known Hosios (1990) condition, which, because it describes outcomes only in the labor market, is a partial-equilibrium efficiency condition. Our efficiency condition links outcomes across markets. Furthermore, for our general-equilibrium notion of efficiency, the unemployment benefit χ is irrelevant because, as discussed in Section 2.4, we assume that unemployment benefits do *not* appear in the economy-wide resource frontier. As in a model with neoclassical markets, efficiency only takes into consideration preferences and technologies; in our model, χ is a feature of neither preferences nor technology, but rather simply a transfer between the government and households. We assume throughout this discussion (and in the remainder of the paper) that the matching technology of the economy is Cobb-Douglas, $m(u,v) = u^{\xi_u} v^{1-\xi_u}$, with $\xi_u \in (0,1)$.

The Social Planner's problem is described by

$$\max_{\{c_t, n_t, u_t, v_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + g(1 - u_t - n_t) \right] \tag{18}$$

subject to the sequence of laws of motion for the employment stock (15) and resource constraints (16). We do not view the Social Planner here as being restricted by the exogenous participation limit $n_t + u_t = \bar{l}$: we think it most natural and informative to consider a Social Planner that can freely and efficiently allocate the total number of individuals in the economy between the labor force and leisure. The full solution of this problem appears in Appendix D; here, we focus only on efficiency along the consumption-leisure margin, as summarized by the following proposition.

Proposition 1. *There exists a constrained-efficient (first-best) allocation in which efficiency along the consumption-leisure margin is characterized by*

$$\frac{g'(1 - u_t - n_t)}{u'(c_t)} = \gamma \theta_t \frac{\xi_u}{1 - \xi_u}. \tag{19}$$

The formal proof of this proposition is detailed in Appendix D. Here, we simply provide intuition by reconstructing condition (19) from the basic tenet that under consumption-leisure efficiency, the marginal rate of substitution (MRS) between consumption and leisure (outside the labor force) must equal the corresponding marginal rate of transformation (MRT). That the left-hand-side of (19) is the household's MRS between consumption and leisure (outside the labor force) is obvious. The right-hand-side of (19) must therefore have the interpretation of the MRT between consumption and leisure.

Rather than take (19) as prima facie evidence that the right-hand-side must be the MRT, we can describe the MRT conceptually from first principles as follows. Consider the economy's resource frontier (16) and the identity limiting the size of the population (household), $n_t = 1 - u_t - \varrho_t$, where ϱ_t is the measure of individuals outside the labor force and hence enjoying leisure. There are two ways the economy can transform a unit of leisure in period t into a unit of output, and hence consumption, in period t. We trace the MRT in these two possible ways, and then connect them.

One way for the economy to achieve transformation of ϱ_t into output (and hence c_t) is to first trade off one unit of ϱ_t for vacancy postings v_t; doing so yields $1/\gamma$ additional vacancy postings, as (16) shows.[8] This marginal addition to the flow of vacancy postings increases the number of aggregate labor-market matches by $(1/\gamma)\partial m(u_t, v_t)/v_t \equiv m_{vt}/\gamma$. Because labor-market matches are long-lived, this marginal additional to the stock of period-t employment increases output by $\left\{z_t + (1-\rho^x)E_t\left[z_{t+1} + (1-\rho^x)z_{t+2} + (1-\rho^x)^2 z_{t+3} + ...\right]\right\} m_{vt}/\gamma$, which can be expressed compactly as

$$MRT^{l\to v} = \left[z_t + (1-\rho^x)E_t\sum_{s=0}^{\infty} z_{t+1+s}\right]\frac{m_{vt}}{\gamma}. \qquad (20)$$

Alternatively, the economy can achieve transformation of ϱ_t into output by first trading off one unit of ϱ_t for one unit of u_t, which follows simply from the restriction $n_t + u_t + \varrho_t = 1$. This marginal addition to the flow of searching unemployed individuals increases the number of aggregate labor-market matches by $\partial m(u_t, v_t)/\partial u_t \equiv m_{ut}$. In turn, because labor-market matches are long-lived, this marginal additional to the stock of period-t employment increases output by $\left\{z_t + (1-\rho^x)E_t\left[z_{t+1} + (1-\rho^x)z_{t+2} + (1-\rho^x)^2 z_{t+3} + ...\right]\right\} m_{ut}$, which can be expressed compactly as

$$MRT^{l\to u} = \left[z_t + (1-\rho^x)E_t\sum_{s=0}^{\infty} z_{t+1+s}\right] m_{ut}. \qquad (21)$$

For the economy to be on its production possibilities frontier, the two ways of transforming leisure into output must be equivalent. This is satisfied if $m_{ut} = m_{vt}/\gamma$. Because the matching technology is Cobb-Douglas, we have $m_{ut}/m_{vt} = \theta_t\xi_u/(1-\xi_u)$, where, as mentioned above, $\xi_u \in (0,1)$ is the elasticity of the matching technology with respect to u_t and, recall, $\theta_t \equiv v_t/u_t$. Using

[8] To see this, first note that $z_t n_t = z_t(1 - u_t - \varrho_t)$ Furthermore, because the first time a newly-formed match produces output is in the period of match-creation, omitted from this transformation is z_t; z_t only results from a successful new match formed in t.

this fact and the efficiency requirement that the two ways of transforming leisure into output must be identical, we have

$$MRT = \gamma\theta_t \frac{\xi_u}{1-\xi_u}, \tag{22}$$

which indeed is the right-hand-side of (19).

This conceptualization of the MRT between output (consumption) and leisure (outside the labor force) is novel. It compactly describes the *two* technologies — the matching technology $m(u_t, v_t)$ and the production technology $z_t n_t$ — that must operate for leisure to be transformed into output and hence consumption. As search-theoretic frameworks become increasingly popular in general equilibrium models, it is useful to be able to describe transformation frontiers and the MRTs implied by them in general ways.[9] Our notion of the MRT between consumption and leisure encompasses that in a standard RBC model. As noted above, one can recover a standard neoclassical labor market by assuming vacancy-creation costs are zero ($\gamma = 0$) (in which case, because they are costless, the number of vacancies posted is infinite) and that all "employment relationships" last only one period ($\rho^x = 1$). With Cobb-Douglas matching and an infinity of vacancy postings, $m_{vt} = 0$ and $m_{ut} = 1$. Imposing these assumptions and results in the preceding logic, we obtain $MRT = z_t$, obviously identical to that in a simple DSGE model featuring linear-in-labor production.

For the purposes of our Ramsey analysis, we show in the next section how three particular features of our model's decentralized economy — proportional labor taxation, the potential inability by households to freely choose the labor-force participation rate, and, of a bit less importance for our optimal-policy results, transfers of unemployment benefits from the government to households — disrupt efficiency along the consumption-leisure margin.

4 Search-Based Labor Wedge

In any Ramsey problem, the basic tension is between raising revenue for the government via proportional taxation and creating wedges between MRS/MRT pairs. We prove the following in Appendix E:

Proposition 2. *In the decentralized economy with Nash bargaining, proportional labor taxation, and unemployment benefits, the equilibrium consumption-leisure margin can be expressed as*

$$\frac{g'(1-u_t-n_t)}{u'(c_t)} - \chi + \frac{\phi_t}{u'(c_t)} = (1-\tau_t^n)\gamma\theta_t \frac{\eta}{1-\eta}. \tag{23}$$

Comparing (27) with the efficiency condition (19), it is clear that for the decentralized economy with Nash bargaining to achieve efficiency, four conditions must be satisfied:

[9] In a general-equilibrium search-model of physical capital markets and in a search-theoretic model of monetary exchange, Arseneau, Chugh, and Kurmann (2008) and Aruoba and Chugh (2008), respectively, develop analogous search-based notions of intertemporal MRTs.

- The decentralized economy must feature $\eta = \xi_u$, which corresponds to the Hosios (1990) condition for search efficiency;
- The transfer of unemployment benefits from the government to households must be zero — that is, $\chi = 0$;
- Households must be able to optimally choose the labor-force participation rate in every period, so that $\phi_t = 0 \ \forall t$;
- Proportional labor income taxation must be zero — that is, $\tau_t^n = 0 \ \forall t$.

The first condition is the usual Hosios (1990) condition. The standard Hosios efficiency condition is a partial-equilibrium statement because it focuses only on the labor market and takes as given outcomes in goods markets and intertemporal markets. Our efficiency condition is a general-equilibrium one, linking outcomes across markets. Thus, Hosios efficiency is a necessary, though not sufficient, condition for our general-equilibrium notion of search efficiency. Also required is zero-taxation of labor income and the absence of any transfers of unemployment benefits between the government and households, because unemployment benefits are not part of the technology of the economy. In a Ramsey taxation problem, $\tau^n = 0$ of course cannot occur.[10] Given this, most important for our results is that for efficiency, households must be able to optimally choose the participation rate $n_t + u_t$, which would render the shadow value $\phi_t = 0$. If $\phi_t \neq 0$, as occurs in the pseudo-LFP model because the participation rate is exogenous, efficiency is disrupted.

In all of our analysis, we assume the Hosios (1990) condition is satisfied. Also, for simplicity, assume for a moment that $\chi = 0$.[11] In this case, we can express (27) as

$$\frac{g'(1-u_t-n_t)}{u'(c_t)} = \gamma\theta_t \frac{\xi_u}{1-\xi_u}\left[(1-\tau_t^n) - \frac{\phi_t}{u'(c_t)}\frac{1-\xi_u}{\xi_u}\frac{1}{\gamma\theta_t}\right]. \tag{24}$$

The term in square brackets is the equilibrium wedge between the MRS between consumption and leisure and the MRT between consumption and leisure, both which we defined in Section 3. Clearly, if households can freely choose the labor-force participation rate every period — which implies the shadow value $\phi_t = 0 \ \forall t$ — then the only wedge between MRS and MRT is the labor-income tax. Although our model is not a neoclassical one, the basic optimal-taxation principle of (nearly) equating deadweight losses across (intertemporal) markets (which is the essence of the widely-known dynamic tax-smoothing result) leads to the natural conjecture that in this case tax-smoothing is optimal.

[10] Unless the initial assets of the government were so large, either by assumption or via an effective initial lump-sum levy on existing assets, that it never needed to impose labor taxes. As usual in the Ramsey literature, we rule out these possibilities because they assume away the nature of the Ramsey problem.

[11] Our numerical work is conducted assuming two alternative values for χ: $\chi = 0$ and a χ such that unemployment transfers represent 40 percent of after-tax wages, which corresponds to Shimer's (2005) calibration.

On the other hand, if households are restricted in their labor-force participation, we would expect ϕ_t in general to be time-varying. In this case, fluctuations in the second term in brackets would require offsetting fluctuations in τ_t^n in order to keep the *total* deadweight loss (nearly) constant over time. As χ rises above zero, the above modifies slightly to

$$\frac{g'(1-u_t-n_t)}{u'(c_t)} = \gamma\theta_t \frac{\xi_u}{1-\xi_u}\left[(1-\tau_t^n) - \left(\frac{\phi_t}{u'(c_t)} - \chi\right)\frac{1-\xi_u}{\xi_u}\frac{1}{\gamma\theta_t}\right], \tag{25}$$

but all of the logic still applies.

The term

$$\left[(1-\tau_t^n) - \left(\frac{\phi_t}{u'(c_t)} - \chi\right)\frac{1-\xi_u}{\xi_u}\frac{1}{\gamma\theta_t}\right] \tag{26}$$

is thus a novel notion of a *labor wedge*. In particular, it is a more general notion of a labor wedge than that measured by Chari, Kehoe, and McGrattan (2007), Shimer (2008), Ohanian, Raffo, and Rogerson (2008), and others. Future empirical work may be able to exploit this way of formulating the labor wedge. Premised as it is on the idea that employment experiences are fundamentally long-lived economic phenomena, rather than spot transactions, it offers a new way to understand what Chari, Kehoe, and McGrattan (2007), Smets and Wouters (2007), Shimer (2008), and others have identified as perhaps the most important area in macroeconomics where much deeper empirical and theoretical understanding is required. We leave such empirical investigation to future work.

We mentioned above that our results are invariant to whether wages are determined through Nash bargaining (at the Hosios condition) or through competitive search. We prove the following in Appendix H:

Proposition 3. *In the decentralized economy with competitive search, proportional labor taxation, and unemployment benefits, the equilibrium consumption-leisure margin can be expressed as*

$$\frac{g'(1-u_t-n_t)}{u'(c_t)} - \chi + \frac{\phi_t}{u'(c_t)} = (1-\tau_t^n)\gamma\theta_t\frac{\xi_u}{1-\xi_u}. \tag{27}$$

Hence, under competitive search, the labor wedge is identical to that presented in (26). Using this notion of a labor wedge, we now turn to a quantitative assessment of our conjectures regarding the optimality of tax smoothing. Specifically, we conjecture that in the pseudo-LFP model, tax-smoothing will not be optimal, whereas in the LFP model tax-smoothing will be optimal. We first present the Ramsey problem and then present results.

5 Ramsey Problem

A standard approach in Ramsey models based on neoclassical markets is to capture in a single, present-value implementability constraint (PVIC) all equilibrium conditions of the economy apart from the resource frontier. The PVIC is the key constraint in any Ramsey problem because it governs the welfare loss of using non-lump-sum taxes to finance government expenditures.[12] The PVIC is the household (equivalently, government) budget constraint expressed in intertemporal form with all prices and policies substituted out using equilibrium conditions. In relatively simple models, the PVIC encodes all the equilibrium conditions that must be respected by Ramsey allocations in addition to feasibility. In complicated environments that deviate substantially from neoclassical markets, however, such as Schmitt-Grohe and Uribe (2005), Chugh (2006), and Arseneau and Chugh (2008), it is not always possible to construct such a single constraint, meaning that, in principle, all of the equilibrium conditions must be imposed explicitly as constraints on the Ramsey problem.

Our model presents an environment in which it is instructive to construct a PVIC but nonetheless leave some equilibrium conditions as separate constraints on the Ramsey planner. As we show in Appendix F, we can construct a PVIC starting from the household flow budget constraint (6) and using the household optimality conditions (9) and (10). The PVIC for our model is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u'(c_t)c_t - \left[g'(1-u_t-n_t)+\phi_t\right](n_t+u_t^h) - u'(c_t)(1-\tau^{pr})d_t \right\} = A_0, \tag{28}$$

where the time-zero assets of the private-sector are given by

$$A_0 \equiv u'(c_0)R_0 b_{-1} + (1-\rho^x)\left[\frac{g_0' - u_0'\chi}{k^h(\theta_0)} + \frac{\phi_0}{k^h(\theta_0)}\right] n_{-1}. \tag{29}$$

Several observations about this PVIC are in order. First, because employment is a state variable, the household's "ownership" of the initial stock of employment relationships, n_{-1}, is part of its time-zero assets, as shown in A_0. Second, with free choice by households regarding labor-force participation, we would have $\phi_t = 0 \ \forall t$. Third, if labor markets were neoclassical, the sum $n_t + u_t$ would be interpreted as total *labor* in the economy, because a neoclassical environment of course features no notion of unemployment. Fourth, as we mentioned above, a spot, neoclassical labor market can be interpreted as featuring $\rho^x = 1$ because there is no long-lived aspect to labor-market transactions. Fifth, with a constant-returns production technology in a neoclassical environment, $d_t = 0 \ \forall t$. Imposing the last four of these conditions in our model collapses the PVIC (28), as well as the initial assets A_0, to that in a standard Ramsey model based on neoclassical markets.[13]

However, unlike in a neoclassical model, the PVIC (28) does not capture all equilibrium conditions of the decentralized economy. In particular, Ramsey allocations must also respect the vacancy-posting condition (4), the Nash wage outcome ((13) for the LFP model or (12) for the pseudo-LFP model), the law of motion for the aggregate employment stock (15), and, for the

[12] See, for example, Ljungqvist and Sargent (2004, p. 494) for more discussion.

[13] In particular, we would have $E_0 \sum_{t=0}^{\infty} \beta^t \left[u'(c_t)c_t - g'(1-l_t)l_t\right] = u'(c_0)R_0 b_{-1}$, where $l_t \equiv n_t + u_t$. This PVIC is identical to that in Chari and Kehoe (1999) for an environment without physical capital.

pseudo-LFP model, the participation restriction (17). None of these restrictions is encoded in the PVIC (28).

The Ramsey problem is thus to choose state-contingent processes $\{c_t, n_t, u_t, \theta_t, w_t, \tau_t^n, \phi_t\}_{t=0}^{\infty}$ to maximize (5) subject to the PVIC (28), the vacancy-posting condition (4), the Nash wage outcome (either (13) or (12)), the law of motion for the aggregate employment stock (15), the aggregate resource constraint (16), and, if the pseudo-LFP environment is under consideration, the restriction on the size of the labor force (17). If the LFP environment is under consideration, the constraint (17) is dropped and the process of shadow values $\{\phi_t\}$ is trivially set to zero. For computational convenience, we leave both the bargained real wage and the labor tax rate as explicit Ramsey choice variables. We instead could have inverted the Nash wage equation to eliminate one of them; in any case, however, we would still be left with the other as an explicit Ramsey choice variable.

As is standard in Ramsey taxation problems, we assume full commitment. Thus, we emphasize that none of our results is driven by the use of a discretionary policy. Finally, throughout our analysis, we assume that the first-order conditions of the Ramsey problem are necessary and sufficient and that all allocations are interior.

6 Optimal Taxation

We characterize both the Ramsey steady state and dynamics numerically. Before presenting quantitative results, we describe our parameterization.

6.1 Parameterization and Solution Strategy

Parameters are set so that the long-run (deterministic) Ramsey steady state is identical in both the LFP model and the pseudo-LFP model, which allows us to cleanly focus just on the differential business-cycle responses of policy in the two cases. For utility, standard functional forms are used,

$$u(c_t) = \ln c_t \tag{30}$$

and

$$g(x_t) = \frac{\kappa}{1-\iota} x_t^{1-\iota}. \tag{31}$$

We set $\iota = 1$ to deliver a unit elasticity of labor-force participation. This value is a common compromise in DSGE models because it lies between macro evidence that suggests a very high elasticity of labor supply and micro evidence that suggests a very low elasticity of labor supply. We emphasize, however, that ι measures the elasticity of labor-force participation in our environment, not the elasticity of labor supply per se, due to the search frictions. We nonetheless set $\iota = 1$

to achieve some comparability with standard DSGE models that study optimal policy, but our qualitative results are not driven by this choice.

The average labor-force participation rate in the U.S. during the period 1988-2007 was 66 percent, so we set $\bar{l} = 0.66$ as the measure of individuals in the representative household that are in the labor force on average. Thus, in the pseudo-LFP model, the participation rate by construction is always 66 percent even as the economy is hit by shocks. The parameter κ is then chosen so that $n + u = \bar{l} = 0.66$ in the Ramsey steady-state of the LFP model. The resulting value is $\kappa = 0.51$. Because this setting for κ makes $\bar{l} = 0.66$ endogenously optimal in the LFP model, it has the consequence that in the pseudo-LFP model, in which we exogenously impose $n + u = \bar{l}$, the multiplier ϕ is exactly zero. It is thus this precise setting for κ that, in concert with the rest of the calibration, makes the Ramsey steady-states identical in the two cases.

Well-understood since Hagedorn and Manovskii (2008) is that the (net-of-tax) social flow gain from employment is important for dynamics in search models. In partial equilibrium search models, the flow gain is governed by the difference between the marginal output of a match and a typically-exogenous "outside benefit" that is interchangeably referred to as "unemployment benefits," "the value of leisure," and "the value of home production." Because the basic Pissarides (2000) model, on which the analyses of Shimer (2005), Hall (2005), and Hagedorn and Manovskii (2008) are based, is partial equilibrium, a precise stand on the nature of the outside benefit is unnecessary. In our general-equilibrium model, we have well-defined notions of government-provided unemployment benefits and the value of leisure.[14] Measured in terms of goods, the former is χ and the latter is $g'(1 - u_t - n_t)/u'(c_t)$, the MRS between consumption and leisure. Of course, because we have *three* groups of individuals — the employed, the searching unemployed, and those outside the labor force — these outside benefits accrue to different groups. The unemployment benefit χ is received by a searching unemployed individual, while the MRS $g'(.)/u'(.)$ is received by an individual outside the labor force.

We use as our baseline parameter setting $\chi = 0$, so that only individuals outside the labor force receive an outside benefit. This setting makes our results comparable to RBC models in which it is only the value of leisure — being outside the labor force — that an individual obtains by not working. Given the rest of our calibration, setting $\chi = 0$ delivers an outside benefit of nearly 90 percent of the after-tax wage rate. Specifically, in the Ramsey steady state (of either the pseudo-LFP model or the LFP model), setting $\chi = 0$ delivers $\frac{g'(.)/u'(c)}{(1-\tau^n)w} = 0.89$, which is close to Hagedorn and Manovskii's (2008) preferred calibration of the outside benefit. We also conduct experiments setting χ at a positive value in the Ramsey steady state such that $\frac{\chi}{(1-\tau^n)w} = 0.40$, the value suggested by Shimer (2005). The important point is that because our environment explicitly

[14] We of course have not modeled home production; it may be interesting to extend our model to allow this feature.

models *both* the value of leisure *and* the value of government-provided unemployment benefits, we can accommodate both the Shimer (2005) view and/or the Hagedorn and Manovskii (2008) view. Our main results do not hinge on adopting one or the other calibration,

The rest of our calibration is relatively standard in this class of models. We assume a quarterly subjective discount factor $\beta = 0.99$. The matching function is Cobb-Douglas, $m(u, v) = \psi u_t^{\xi_u} v_t^{1-\xi_u}$, with $\xi_u = 0.4$, in line with the evidence in Blanchard and Diamond (1989), and ψ chosen so that the quarterly job-finding rate of a searching individual is 90 percent in the model with zero government-provided unemployment benefits. The resulting value is $\psi = 0.9$. Given our Cobb-Douglas matching specification, this also directly fixes the matching rate for an open vacancy at the same value, $k^f(\theta) = 0.90$. The fixed cost γ of opening a vacancy is set so that posting costs absorb 5 percent of total output in the Ramsey steady state with zero unemployment benefits; the resulting value is $\gamma = 1.15$.

We fix the Nash bargaining weight at $\eta = 0.40$ so that it ostensibly satisfies the well-known Hosios (1990) condition $\eta = \xi_u$ for partial-equilibrium search efficiency. We use this as our main guidepost, although we point out that because $\tau^n \neq 0$ in the Ramsey equilibrium, the naive calibration $\eta = \xi_u$ actually does not deliver search efficiency. This point was discussed in Section 4. Rather, the proper setting must also take into account the steady-state distortionary tax rate.[15] Our main focus, though, is not on steady-state inefficiencies, but rather on cyclical policy responses.

Finally, the exogenous productivity and government spending shocks follow AR(1) processes in logs,

$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_t^z, \tag{32}$$

$$\ln g_t = (1 - \rho_g) \ln \bar{g} + \rho_g \ln g_{t-1} + \epsilon_t^g, \tag{33}$$

where $\bar{g}$ denotes the steady-state level of government spending, which we calibrate in our baseline model with a 40-percent replacement rate to constitute 19 percent of steady-state output in the Ramsey allocation. The resulting value is $\bar{g} = 0.11$, which we hold constant across all experiments and all specifications of our model. The innovations ϵ_t^z and ϵ_t^g are distributed $N(0, \sigma_{\epsilon^z}^2)$ and $N(0, \sigma_{\epsilon^g}^2)$, respectively, and are independent of each other. We choose parameters $\rho_z = 0.95$, $\rho_g = 0.97$, $\sigma_{\epsilon^z} = 0.006$, and $\sigma_{\epsilon^g} = 0.027$, consistent with the RBC literature and Chari and Kehoe (1999). Also regarding policy, we assume that the steady-state government debt-to-GDP ratio (at an annual frequency) is 0.4, in line with evidence for the U.S. economy and with the calibrations of Schmitt-Grohe and Uribe (2005), Chugh (2006), and Arseneau and Chugh (2008).

[15] Setting a particular $\eta > \xi_u$ would restore partial-equilibrium search efficiency, but this value of η is endogenous to the Ramsey policy. There is little justification for endogenizing the Nash parameter in this way, so we think this is an uninteresting very special case.

We use a nonlinear numerical solution algorithm to compute the Ramsey deterministic steady-state equilibrium. To study dynamics, we perform a second-order approximation of the Ramsey first-order conditions for time $t > 0$ in levels around the non-stochastic steady-state of these conditions. We conduct a second-order approximation, rather than a more commonly-employed first-order approximation, to guard against the computational inaccuracies of Ramsey-optimal policies pointed out by Chari, Kehoe, and Christiano (1995, p. 383) in a baseline neoclassical model. They demonstrated that inaccuracies of first-order approximations tend to be more severe for policy variables than for Ramsey allocation variables. Because our main focus is on the dynamics of optimal policy — the labor tax rate — we mitigate this computational inaccuracy by approximating the Ramsey equilibrium to second-order.[16] Our numerical method is our own implementation of the perturbation algorithm described by Schmitt-Grohe and Uribe (2004).

We use our second-order accurate decision rules to simulate time-paths of the Ramsey equilibrium in the face of TFP and government spending realizations, the shocks to which we draw according to the parameters of the laws of motion described above. As is common when focusing on asymptotic policy dynamics, we assume that the initial state of the economy is the asymptotic Ramsey steady state, thus adopting the timeless perspective. As we mentioned above, we assume throughout, as is also typical in the literature, that the first-order conditions of the Ramsey problem are necessary and sufficient and that Ramsey allocations are interior. We conduct 1000 simulations, each 200 periods long. For each simulation, we then compute first and second moments and report the medians of these moments across the 1000 simulations.

6.2 Main Results

The main results are presented in Table 1. The top panel presents summary statistics from simulations from the pseudo-LFP model, and the bottom panel presents the same information for the LFP model. As discussed above, the two models are calibrated such that the long-run Ramsey equilibria are identical. This allows for sharp focus on the cyclical properties of optimal policy. Accordingly, the mean realizations for the simulated policy and allocation variables in the first column of Table 1 are identical in the top and bottom panels, with differences only due to numerical differences in approximation.

As comparison of the first row of each panel of the table shows, the optimal labor-income tax rate is much more volatile — over an order of magnitude more volatile — in the pseudo-LFP model than in the LFP model. If there is no adjustment at the participation margin — as is the case in

[16] However, the results using a first-order approximation were extremely similar to the ones reported here. Moreover, the deterministic steady state, which is the point around which we conduct the second-order approximation, turned out to not be very far from the stochastic steady state to which the equilibrium converged using the second-order accurate decision rules.

Variable	Mean	Std. Dev.	Auto corr.	Corr(x,Y)	Corr(x,Z)	Corr(x,G)
			Pseudo-labor-force participation			
τ^n	0.2055	0.0183	0.9136	-0.1818	-0.1871	0.9804
gdp	0.5940	0.0092	0.9047	1	0.9941	-0.0407
c	0.4078	0.0062	0.9196	0.7204	0.7257	-0.6597
N	0.5936	0.0006	0.9632	0.8024	0.7842	0.0257
u	0.0661	0.0006	0.9631	-0.8912	-0.8760	-0.0313
$N+u$	0.6600	0	—	—	—	—
w	0.8606	0.0143	0.9077	0.9885	0.9882	-0.1052
θ	0.9974	0.0193	0.9046	0.9143	0.9022	0.0361
v	0.0659	0.0007	0.8125	0.8920	0.8835	0.0399
ϕ	-0.0040	0.0393	0.9142	0.1826	0.1880	-0.9795
$\phi/u'(c)$	-0.0013	0.0160	0.9142	0.1829	0.1884	-0.9797
			Full labor-force participation			
τ^n	0.2041	0.0014	0.7638	0.1851	-0.0063	0.6292
gdp	0.5937	0.0092	0.9001	1	0.9383	0.2215
c	0.4082	0.0048	0.8603	0.6091	0.8007	-0.5246
N	0.5932	0.0027	0.9456	0.2970	0.0329	0.8617
u	0.0666	0.0016	0.7082	-0.4540	-0.5659	0.4761
$N+u$	0.6599	0.0036	0.8985	0.0710	-0.1932	0.9679
w	0.8613	0.0142	0.8927	0.9179	0.9818	-0.1039
θ	0.9823	0.0340	0.8874	0.7268	0.7495	-0.1128
v	0.0653	0.0014	0.6662	0.5938	0.5260	0.2773
ϕ	0	0	—	—	—	—

Table 1: Ramsey dynamics with zero government-provided unemployment benefits.

the vast majority of the existing literature studying policy in DSGE labor search models — the standard deviation of the optimal tax rate is near 2 percent. In contrast, if participation adjusts cyclically, the standard deviation of the tax rate is around 0.1 percent. This latter figure fits right into the benchmark results of Chari and Kehoe (1999), Werning (2007), and Scott (2007) regarding the optimal degree of variability in tax rates in neoclassical markets, even though our environment always features matching frictions. Thus, whether or not cyclical adjustment occurs along the participation margin can matter a great deal for optimal policy prescriptions.

The bottom two rows of the top panel of Table 1 quantify the cyclical fluctuations in the value of the binding participation margin in the pseudo-LFP model: the next-to-last row displays the dynamics of the multiplier ϕ_t itself, while the last row converts this into goods by deflating by $u'(c_t)$. It is the goods value of the binding participation restriction, and in particular its fluctuating nature, that has direct interpretation. Indeed, recall from (12) that it is $\phi_t/u'(c_t)$ that affects the bargained wage and that enters the wedge condition (26).

The shadow value of the binding participation constraint clearly fluctuates substantially. It is this variation in the shadow value of participation that gives rise to a role for cyclical variation in the labor tax rate. Referring back to the labor wedge (26) that we developed, any variation in τ_t^n is designed to offset variations in the second term of (26), so that the *overall* labor wedge is smoothed over time. At its core, smoothing of labor wedges over time is what the basic Ramsey prescription of tax-smoothing is designed to achieve.[17] In a simple neoclassical model of fiscal policy, it is typically only the labor tax that enters this wedge — wedge-smoothing thus implies tax smoothing in neoclassical Ramsey environments. Werning (2007) and Scott (2007) recently shed new analytical insight on this long-established result. In contrast, here, the binding participation restriction endogenously causes shifts in the labor wedge. To offset this time-variation in labor wedges, the Ramsey government moves tax rates in offsetting ways. To see this last point clearly, notice that the contemporaneous correlations between τ_t^n and GDP, the TFP realization, and the government spending realization are all *exactly opposite* to the contemporaneous correlations between $\phi_t/u'(c_t)$ and these aggregates. For example, the correlation between the tax rate and GDP is -0.193, while the correlation between the shadow value $\phi_t/u'(c_t)$ and GDP is 0.193.

As further evidence that the variations in the labor tax rate are designed to offset movements in the shadow value of the participation constraint, Figure 1 displays the response of the Ramsey policy to one-standard-deviation positive impulses to government spending and TFP. The countercyclical nature of the optimal tax rate to the shadow value is obvious; indeed, the correlation between the two is virtually -1 in the face of either a shock to government spending or to TFP. Corroborating this evidence is that, in our simulated economies, the median contemporaneous correlation between τ_t^n and $\phi_t/u'(c_t)$ was -0.998.

The intuition behind the tax volatility result is simple. Suppose a shock hits the economy that, if there were no restriction on participation, would cause the participation rate to rise. Because of institutional or other restrictions, however, participation cannot respond, which causes the shadow value of the constraint to rise instead. This lowers the value of leisure relative to the value of participating in the formal labor market. From an efficiency standpoint, this misalignment of values is suboptimal. The Ramsey government can realign these values by raising the reward for work (the after-tax wage) by lowering the tax rate. In contrast, if there is no institutional impediment to adjusting participation, the shadow value is of course always zero and the optimality of tax-smoothing re-emerges.

[17] Indeed, tax-smoothing in simple Ramsey models is nothing more than an intertemporal application of the basic optimal-policy prescription to, under certain preference specifications, smooth tax rates across goods in a static setting.

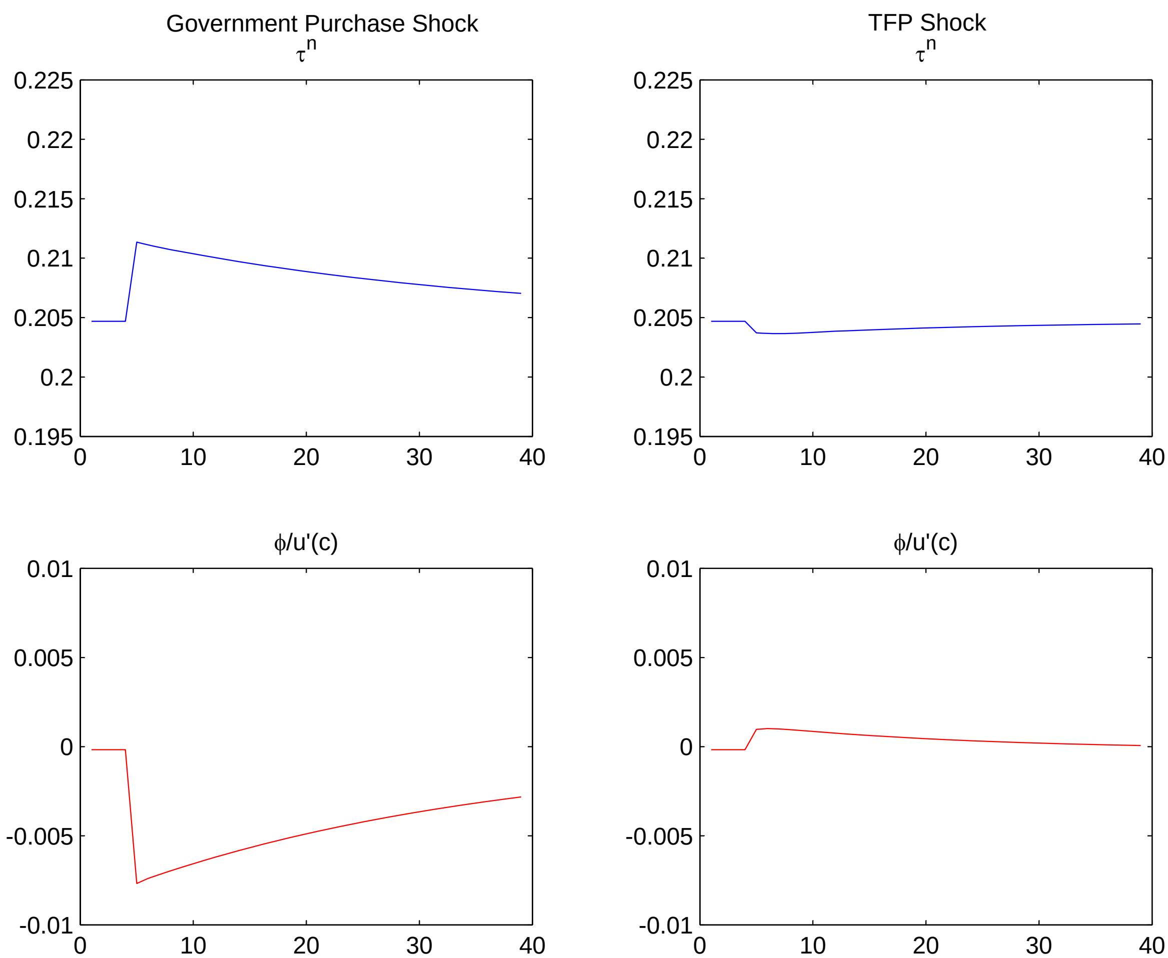

Figure 1: Impulse responses of optimal policy to one-standard-deviation positive shock to government spending (left panels) and TFP (right panels).

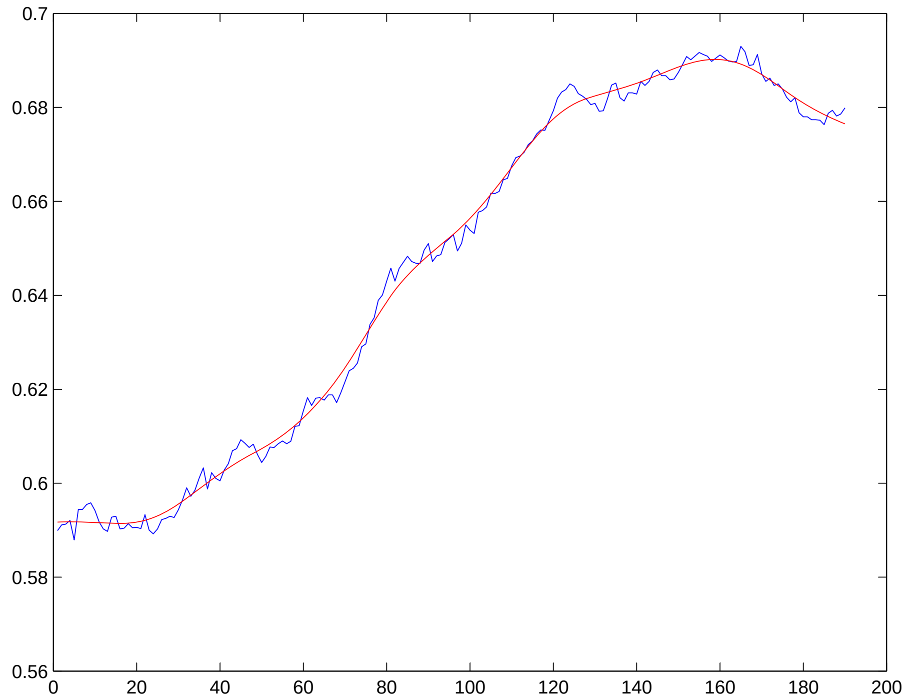

Figure 2: U.S. labor force participation rate, 1959:1 - 2006:2. Blue line is actual participation rate, red line is HP filter trend (obtained with smoothing parameter 1600). The standard deviation of the cyclical component of the participation rate is 0.0032.

We mentioned at the outset the conventional empirical view, as recently described by Elsby, Michaels, and Solon (2008), that cyclical variation in participation is "...miniscule relative to the cyclical variation in unemployment" (Elsby et al). For our LFP model, the lower panel of Table 1 shows that the participation rate fluctuates very little. Across our simulations, the median standard deviation of $N+u$ is 0.0035 around a mean of 0.66. Thus, over the business cycle and given that the optimal fiscal policy is in place, two-thirds of the observations on the participation rate lie between 65.65 percent and 66.35 percent under the optimal policy. Although our model is not calibrated to match the observed cyclical volatility of the labor-force participation rate, it matches extremely well with the data. Shown in Figure 2 is quarterly participation rates (as a fraction of the entire population) in the U.S. between 1959:1 and 2006:2 along with its HP filter (smoothing parameter 1600) trend. The standard deviation of the cyclical component of the participation rate is 0.0032, very close to the volatility implied by our model. Our results are thus not driven by counterfactually large cyclical fluctuations in participation.

Nevertheless, in our model, very small fluctuations in participation lead to a very different policy prescription than if there is literally zero adjustment at the participation margin. Our results thus offer a cautionary note for policy analysis in search-based models: on empirical grounds, modeling

Variable	Mean	Std. Dev.	Auto corr.	Corr(x, Y)	Corr(x, Z)	Corr(x, G)
			Pseudo-labor-force-participation			
τ^n	0.2520	0.0601	0.9407	-0.6559	-0.4033	0.8162
gdp	0.5735	0.0122	0.9291	1.0000	0.9403	-0.3340
c	0.4035	0.0066	0.9565	0.8961	0.7066	-0.6701
N	0.5730	0.0050	0.9629	0.7762	0.5548	-0.6829
u	0.0867	0.0050	0.9630	-0.8323	-0.6137	0.7472
$N + u$	0.6600	0	—	—	—	
w	0.8871	0.0129	0.8127	0.5472	0.7297	0.4262
θ	0.5976	0.0716	0.8911	0.8267	0.6288	-0.7599
v	0.0512	0.0034	0.7888	0.7568	0.5924	-0.7139
ϕ	0.0099	0.1829	0.9506	0.7083	0.4607	-0.8525
$\phi/u'(c)$	0.0060	0.0738	0.9506	0.7084	0.4609	-0.8522
			Full labor-force-participation			
τ^n	0.2541	0.0086	-0.6462	0.0713	0.2069	-0.0611
gdp	0.5735	0.0083	0.8730	1.0000	0.8802	0.3615
c	0.4038	0.0046	0.8541	0.4923	0.8020	-0.5670
N	0.5731	0.0038	0.9407	0.1906	-0.2192	0.8507
u	0.0869	0.0027	0.1992	-0.3445	-0.5610	0.3649
$N + u$	0.6600	0.0056	0.6609	-0.0333	-0.4295	0.7974
w	0.8873	0.0166	0.4639	0.7856	0.8731	-0.1212
θ	0.5953	0.0158	0.6522	0.7667	0.9061	-0.0573
v	0.0517	0.0010	0.3384	0.5158	0.3462	0.5285
ϕ	0	0	—	—	—	—

Table 2: Ramsey dynamics with government-provided unemployment benefits 40 percent of after-tax wages.

participation as completely exogenous over the cycle may seem justified. However, even very small equilibrium adjustments can qualitatively affect the policy insights a model delivers. Given the simple nested formulations of the seemingly very different models we have developed, it seems at least worthwhile to check sensitivity of results along this dimension.

As Table 1 shows and as we have been discussing, tax volatility arises in the environment in which the participation rate is inelastic. A natural conjecture is that if labor-supply were inelastic in a neoclassical labor market and, as in our model, labor were the only input to production, tax-rate volatility would be optimal there, as well. We agree with this conjecture. However, in that environment, the labor-income tax would be akin to a lump-sum tax because the *equilibrium* quantity of labor is inelastic, not simply households' supply of labor. Such an environment renders Ramsey analysis of "optimal taxation" moot because clearly there would be no distortions from *any* time-pattern of tax rates. In contrast, in a search and matching environment that acknowledges there are *three* labor-market states, not just two, even if participation is inelastic, the equilibrium quantity of labor *is* elastic and is governed by search and matching. Thus, while it is true that an RBC model with inelastic labor would deliver a policy prescription of tax-rate volatility, it would do so for somewhat trivial reasons.

Finally, Table 2 shows that the main results remain intact if the government provides unemployment benefits. For the experiments presented in Table 2, we choose, as we described above, a χ so that in the deterministic Ramsey steady state, $\frac{\chi}{(1-\tau^n)w} = 0.4$, which is consistent with the calibration of Shimer (2005). All other parameter settings are held constant at the values underlying the results in Table 2. Because unemployment benefits are government-financed, the mean labor tax rate must rise in both the pseudo-LFP model and the LFP model; as the table shows, the mean tax rate rises to about 25 percent, compared to about 20 percent if there are zero unemployment benefits as in Table 1. Unlike the case with zero unemployment benefits, the contemporaneous correlations between τ_t^n and (gdp_t, g_t, z_t) is not *exactly* the mirror image of the correlations between $\phi_t/u'(c_t)$ and these aggregates. Nonetheless, the correlation between τ_t^n and $\phi_t/u'(c_t)$ is still highly negative: averaging over our simulated economies, the median contemporaneous correlation between them was -0.96. Hence, as the wedge condition (26) shows, χ does open up a gap between the shadow value and the tax rate, but it is still clearly fluctuations in the shadow value that tax-rate volatility is designed to offset. And, once again, if participation can adjust cyclically, the optimality of tax-rate smoothing is restored, as the first row of the lower panel of Table 2 shows.

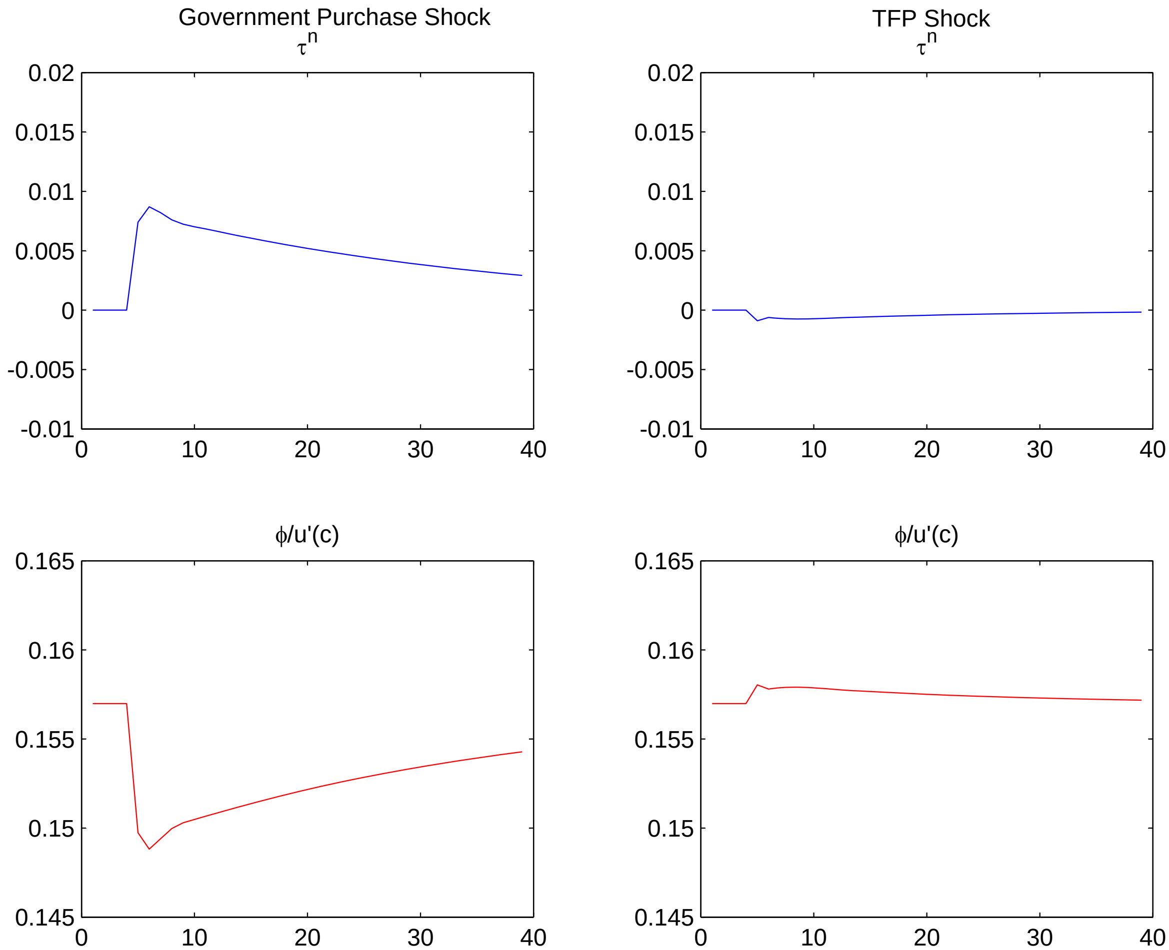

Figure 3: In economy with lump-sum taxes, impulse responses of optimal policy to one-standard-deviation positive shock to government spending (left panels) and TFP (right panels).

6.3 Efficiency Considerations and Tax Volatility

We have explained our results regarding optimal tax-rate dynamics in terms of the labor wedge defined by (26). To demonstrate that the lack of tax-smoothing in our pseudo-LFP model is indeed fundamentally driven by fluctuations in this wedge and not by some interactions between search frictions and the Ramsey financing problem itself, we can introduce a lump-sum tax into our environment. The presence of a lump-sum tax makes identically zero the Ramsey multiplier on the PVIC (28), which thus means that public-finance considerations are absent.[18] Performing our experiments in the presence of lump-sum taxes therefore isolates the efficiency concerns underlying the dynamics of tax rates.

[18]See again Ljungqvist and Sargent (2004, p. 494) for more discussion on this.

Figure 3 reports the results of the same impulse experiments underlying Figure 1, except with a lump-sum tax assumed available. All parameters are held constant at their benchmark values, and we assume $\chi = 0$ for this demonstration. With lump-sum taxation available, there is no reason to use labor-income taxation, hence in the long-run $\tau^n = 0$, as the upper row of Figure 3 shows. Following a shock to either government spending or TFP, however, the labor tax rate deviates from zero. Although the effect is more dramatic for a government spending impulse than for a TFP impulse, comparison of Figure 3 with Figure 1 shows that the patterns of responses are the same, and, in particular, movements in the tax rate are designed to stabilize movements in the overall wedge. Furthermore, in simulation exercises we conducted with a lump-sum tax assumed available, we found that the standard deviation of the labor-income tax rate was 2.27 percent, which is comparable to the 1.83 percentage point standard deviation reported in the first row of Table 1 for the full Ramsey financing problem.

We conclude from these experiments that it is indeed movements in the search labor wedge (26) that tax-rate fluctuations are designed to mitigate when the labor-force participation rate is fixed over the business cycle. This cyclical efficiency concern is not blurred by consideration of the full Ramsey public-financing problem.

6.4 Alternative Policy Instruments

Finally, one may reasonably ask whether the presence of other policy instruments might reinstate the optimality of tax-smoothing even if participation is exogenous. Simply inspecting the wedge (26), it is clear that if the unemployment benefit χ were time-varying and always equal to $\phi_t/u'(c_t)$, the second part of this wedge condition would drop out, leaving only the labor income tax rate. Smoothing τ_t^n then follows from standard Ramsey principles.

Alternatively, suppose the government had access to a time-varying proportional vacancy subsidy, which lowers for firms the cost of searching workers. Denoting by $\tau_t^s > 0$ the proportional vacancy-posting subsidy rate, this subsidy would make the cost faced by the firm for each vacancy created $(1-\tau_t^s)\gamma < \gamma$. It is straightforward to show that in all equilibrium conditions (apart from the aggregate resource constraint) where γ appears, $(1-\tau_t^s)\gamma$ would instead appear. In particular, the wedge presented in (26) would modify to

$$\left[(1-\tau_t^n) - \left(\frac{\phi_t}{u'(c_t)} - \chi_t\right)\frac{1-\xi_u}{\xi_u}\frac{1}{(1-\tau_t^s)\gamma\theta_t}\right], \tag{34}$$

in which we have also, based on the immediately preceding discussion, also supposed that the unemployment benefit χ can be time-varying.

With all three instruments τ_t^n, χ_t, and τ_t^s appearing in the single wedge (34), there is an infinite number of potential decentralizations for any given allocation. To achieve a given target for the overall wedge, one decentralization involves fixing both χ and τ^s at time-invariant rates and allowing fluctuations in τ_t^n to do the job. This is the decentralization we pursued in our main analysis. Another decentralization is to keep τ_t^n constant ("tax-smoothing") and allow some appropriate combination of fluctuations in χ_t and τ_t^s to stabilize the overall wedge.

If one were to prefer the latter way of "restoring the optimality of tax-smoothing," it must be driven by considerations outside the scope of the model. This is because, within the scope of our model, our baseline specification of policy instruments was already *complete*, in the sense understood in the literature on optimal taxation.[19] When a tax system is complete, introducing additional policy instruments does not at all change the Ramsey *allocation* problem; instead, as in the example just described, all it does is create indeterminacies about *which* instruments should be used in which configurations to support the Ramsey allocation. The model does not provide any basis for preferring one decentralization over another, which is a well-understood point in Ramsey models of optimal taxation. Hence, loading redundant policy instruments onto the same, single, wedge (34) is an uninteresting way of restoring labor-tax-smoothing.

7 Conclusion

We have shown that the optimality of smoothing labor-income tax rates over time, a basic result in the theory of optimal fiscal policy, does not carry over to the most common DSGE specification of a search and matching model of labor markets. The crucial missing margin of adjustment in much of the existing work is household optimization with respect to labor-market outcomes. Such a margin is at the core of any standard macro model, but absent in a baseline search-based macro model. By formulating the standard exogenous-participation search model in way that makes it seem as if a participation margin does exist, we have shown that cyclical fluctuations in the shadow value associated with the participation restriction disrupt labor-market efficiency. Purposeful tax-rate volatility is then optimal because it offsets shifts in labor-market wedges. In contrast, if there is no impediment to adjusting participation at business-cycle frequencies, tax-smoothing is optimal despite the presence of search and matching frictions.

Clearly, more empirical resolution is needed about the cyclical nature of labor-force participation. There are many types of individuals for whom participation is difficult to adjust quickly: a person engaged in child-rearing or education may not be able to make himself available for work within the few quarters or few years over which business-cycle shocks have their impact. On the other hand, there are many types of individuals for whom participation may be easy to ad-

[19] As Chari and Kehoe (1999, p. 1679-1680) define it, an *incomplete* tax system is in place if, for at least one pair of goods in the economy, the government has *no* policy instrument that drives a wedge between its MRS and its MRT. If this is not the case, then the tax system is instead said to be *complete*.

just quickly: a person engaged in short-term schooling or short-term (re-)training may be able to (re-)join the labor force within weeks or months. With participation endogenous, our quantitative results show that it may nonetheless fluctuate very little, which in turn may lead one to favor the exogeneity assumption on the basis of simplicity. But optimal-policy prescriptions are dramatically different under an exogenous-participation view than under an endogenous-participation view even though equilibrium fluctuations in this margin may be small. In the end, though, which view of cyclical fluctuations in participation is more relevant is an open question. Our work raises an important cautionary note for typical formulations of DSGE labor-search models, especially as they are increasingly applied to study policy questions.

To understand our optimal-policy results, we have developed a notion of general-equilibrium efficiency in search-based models. This view of efficiency encompasses the Hosios (1990) condition for search efficiency, but is especially intended for general-equilibrium analysis. As search-based quantitative general-equilibrium models become increasingly popular for studying policy questions, the general notion of efficiency developed here may be useful for gaining insights into existing and newly-emerging results. We have in mind in particular the growing body of policy analysis conducted in search-based New Keynesian monetary models.

A Deriving the Pseudo-Labor-Force Participation Condition

The household optimization problem is to choose state-contingent processes for $\{c_t\}$, $\{b_t\}$, $\{u_t^h\}$, and $\{n_t^h\}$ to maximize

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + g(1 - u_t^h - n_t^h) \right] \tag{35}$$

subject to sequences of flow budget constraints

$$c_t + b_t = (1 - \tau_t^n) w_t n_t^h + u_t^h \chi + R_t b_{t-1} + d_t, \tag{36}$$

perceived laws of motion for the employment stock

$$n_t^h = (1 - \rho^x) n_{t-1}^h + u_t^h k^h(\theta_t), \tag{37}$$

and the restriction that the size of the labor force is fixed

$$n_t^h + u_t^h = \bar{l}. \tag{38}$$

Denote by $\{\lambda_t\}$, $\{\mu_t^h\}$, and $\{\phi_t\}$ the sequences of Lagrange multipliers on the sequences of these constraints, respectively. The first-order conditions with respect to c_t, b_t, u_t^h and n_t^h; they are, respectively,

$$u'(c_t) - \lambda_t = 0, \tag{39}$$

$$-\lambda_t + \beta R_t E_t \lambda_{t+1} = 0, \tag{40}$$

$$-g'(1 - u_t^h - n_t^h) + \lambda_t \chi + \mu_t^h k^h(\theta_t) - \phi_t = 0, \tag{41}$$

and

$$-\mu_t^h + \lambda_t (1 - \tau_t^n) w_t - g'(1 - u_t^h - n_t^h) - \phi_t + \beta (1 - \rho^x) E_t \mu_{t+1}^h = 0. \tag{42}$$

Conditions (39) and (40) clearly yield a standard bond-Euler equation, which is expression (9) in the main text.

To obtain the pseudo-labor-force-participation (LFP) condition, start with (41) and (42). Solving (41) for μ_t^h,

$$\mu_t^h = \frac{g'(1 - u_t^h - n_t^h) - u'(c_t)\chi}{k^h(\theta_t)} + \frac{\phi_t}{k^h(\theta_t)}, \tag{43}$$

in which we have used the result $\lambda_t = u'(c_t)$, which follows from (39). Using this expression and its period $t+1$ analog in (42), we have

$$\begin{aligned} \frac{g'(1 - u_t^h - n_t^h) - u'(c_t)\chi}{k^h(\theta_t)} + \frac{\phi_t}{k^h(\theta_t)} &= u'(c_t)(1 - \tau_t^n) w_t - g'(1 - u_t^h - n_t^h) \\ - \quad \phi_t + \beta(1 - \rho^x) E_t &\left\{ \frac{g'(1 - u_{t+1}^h - n_{t+1}^h) - u'(c_{t+1})\chi}{k^h(\theta_{t+1})} + \frac{\phi_{t+1}}{k^h(\theta_{t+1})} \right\}, \end{aligned} \tag{44}$$

which is the pseudo-LFP condition presented in condition (10) in the main text.

B Nash Bargaining in Model with Pseudo-Labor-Force Participation Margin

Starting with the solution to the household problem obtained in Appendix A, we next use the envelope conditions of the household's optimization problem to define the value equations relevant for the Nash bargaining problem. Note that because we refrain from making the substitution $n_t^h + u_t^h = \bar{l}$, both n_{t-1}^h and u_{t-1}^h are state variables in the formulation of the household optimization problem offered in Appendix A.

Let $\mathbf{V}(n_{t-1}^h, u_{t-1}^h)$ denote the value function associated with the solution to this problem. The envelope conditions are thus $\mathbf{V}_u(n_{t-1}^h, u_{t-1}^h) = 0$ and

$$\mathbf{V}_n(n_{t-1}^h, u_{t-1}^h) = (1-\rho^x)\mu_t^h. \tag{45}$$

Recognizing the relationship at the optimal solution between μ_t^h and allocations defined by (43), and substituting $\mathbf{V}_n(n_{t-1}^h, u_{t-1}^h)$ into (44), we can write

$$\frac{\mathbf{V}_n(n_{t-1}^h, u_{t-1}^h)}{1-\rho^x} = u'(c_t)(1-\tau_t^n)w_t - g'(1-u_t^h-n_t^h) - \phi_t + (1-\rho^x)E_t\left\{\frac{\beta \mathbf{V}_n(n_t^h, u_t^h)}{1-\rho^x}\right\}. \tag{46}$$

To express things in terms of goods, recognize that $\lambda_t = u'(c_t)$ at the optimum and define $\mathbf{W_t}$ as

$$\begin{aligned}
\mathbf{W_t} \equiv \frac{\mathbf{V}_n(n_{t-1}^h, u_{t-1}^h)}{\lambda_t(1-\rho^x)} &= (1-\tau_t^n)w_t - \frac{g'(1-u_t^h-n_t^h)}{\lambda_t} - \frac{\phi_t}{\lambda_t} + (1-\rho^x)E_t\left\{\frac{\beta \mathbf{V}_n(n_t^h, u_t^h)}{\lambda_t(1-\rho^x)}\right\} \\
&= (1-\tau_t^n)w_t - \frac{g'(1-u_t^h-n_t^h)}{\lambda_t} - \frac{\phi_t}{\lambda_t} + (1-\rho^x)E_t\left\{\Xi_{t+1|t}\mathbf{W_{t+1}}\right\}.
\end{aligned}\tag{47}$$

In the second line, we have made use of the definition of the one-period-ahead stochastic discount factor, $\Xi_{t+1|t} \equiv \beta\lambda_{t+1}/\lambda_t$. We have $\mathbf{U_t} \equiv \mathbf{V}_u(n_{t-1}^h, u_{t-1}^h)/\lambda_t = 0$. From the point of view of the household, it is the surplus $\mathbf{W_t} - \mathbf{U_t}$ over which it is bargaining.

On the firm side, we have the surplus to a firm of the marginal worker is

$$\mathbf{J_t} = z_t - w_t + (1-\rho^x)E_t\left\{\Xi_{t+1|t}\mathbf{J_{t+1}}\right\}; \tag{48}$$

for use below, note that $\mathbf{J_t} = \frac{\gamma}{k^f(\theta_t)}$.

In generalized Nash bargaining, the parties choose w_t to maximize

$$(\mathbf{W_t} - \mathbf{U_t})^\eta \, \mathbf{J_t}^{1-\eta}. \tag{49}$$

The solution to this problem gives the time-t generalized Nash sharing rule, $\frac{\mathbf{W_t}}{1-\tau_t^n} = \frac{\eta}{1-\eta}\mathbf{J_t}$.

Now proceed to derive an explicit expression for w_t. Inserting the definition of $\mathbf{W_t}$ into the Nash sharing rule,

$$-\frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau_t^n)} + w_t - \frac{\phi_t}{\lambda_t(1-\tau_t^n)} + \frac{1-\rho^x}{1-\tau_t^n}E_t\left\{\Xi_{t+1|t}\mathbf{W_{t+1}}\right\} = \frac{\eta}{1-\eta}\mathbf{J_t}, \tag{50}$$

and then using the time-$t+1$ Nash sharing rule,

$$-\frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau_t^n)}+w_t-\frac{\phi_t}{\lambda_t(1-\tau_t^n)}+\frac{1-\rho^x}{1-\tau_t^n}E_t\left\{\Xi_{t+1|t}(1-\tau_{t+1}^n)\frac{\eta}{1-\eta}\mathbf{J_{t+1}}\right\}=\frac{\eta}{1-\eta}\mathbf{J_t}. \quad (51)$$

Make the substitution $\mathbf{J_t}=\frac{\gamma}{k^f(\theta_t)}$, and similarly for $\mathbf{J_{t+1}}$, which yields

$$-\frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau_t^n)}+w_t-\frac{\phi_t}{\lambda_t(1-\tau_t^n)}+\frac{1-\rho^x}{1-\tau_t^n}E_t\left\{\Xi_{t+1|t}(1-\tau_{t+1}^n)\frac{\eta}{1-\eta}\frac{\gamma}{k^f(\theta_{t+1})}\right\}=\frac{\eta}{1-\eta}\frac{\gamma}{k^f(\theta_t)}. \quad (52)$$

Next, use the job-creation condition $\frac{\gamma}{k^f(\theta_t)}=z_t-w_t+(1-\rho^x)E_t\left\{\Xi_{t+1|t}\frac{\gamma}{k^f(\theta_{t+1})}\right\}$ to substitute on the right-hand-side, which gives

$$-\frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau_t^n)}+w_t-\frac{\phi_t}{\lambda_t(1-\tau_t^n)}+\frac{1-\rho^x}{1-\tau_t^n}E_t\left\{\Xi_{t+1|t}(1-\tau_{t+1}^n)\frac{\eta}{1-\eta}\frac{\gamma}{k^f(\theta_{t+1})}\right\}=\frac{\eta}{1-\eta}\left[z_t-w_t+(1-\rho^x)E_t\left\{\Xi_{t+1|t}\frac{\gamma}{k^f(\theta_{t+1})}\right\}\right]. \quad (53)$$

Grouping terms involving w_t,

$$\begin{aligned} w_t\left[1+\frac{\eta}{1-\eta}\right] &= \frac{\eta}{1-\eta}z_t+\frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau_t^n)}+\frac{\phi_t}{\lambda_t(1-\tau_t^n)} \\ &- \frac{\eta}{1-\eta}\frac{1-\rho^x}{1-\tau_t^n}E_t\left\{\Xi_{t+1|t}(1-\tau_{t+1}^n)\frac{\gamma}{k^f(\theta_{t+1})}\right\}+\frac{\eta}{1-\eta}(1-\rho^x)E_t\left\{\Xi_{t+1|t}\frac{\gamma}{k^f(\theta_{t+1})}\right\}. \end{aligned} \quad (54)$$

Finally, multiplying by $1-\eta$ gives the wage equation

$$\begin{aligned} w_t &= \eta z_t+(1-\eta)\frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau_t^n)}+(1-\eta)\frac{\phi_t}{\lambda_t(1-\tau_t^n)} \\ &- \eta\frac{1-\rho^x}{1-\tau_t^n}E_t\left\{\Xi_{t+1|t}(1-\tau_{t+1}^n)\frac{\gamma}{k^f(\theta_{t+1})}\right\}+\eta(1-\rho^x)E_t\left\{\Xi_{t+1|t}\frac{\gamma}{k^f(\theta_{t+1})}\right\}, \end{aligned} \quad (55)$$

which is expression (12) in the main text.

If tax rates never fluctuated, so that $\tau_{t+1}^n=\tau_t^n=\tau^n$, the last two terms would cancel, leaving

$$w_t=\eta z_t+(1-\eta)\frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau^n)}+(1-\eta)\frac{\phi_t}{\lambda_t(1-\tau^n)}. \quad (56)$$

The last term reflects the value to the household of not being able to resize the labor force, and this value is shared between firms and households via bargaining.

C Nash Bargaining in Model with Labor-Force Participation Margin

With endogenous labor-force-participation and instantaneous hiring, the household problem is as described above, except the perceived law of motion is now

$$n_t^h = (1-\rho^x)n_{t-1}^h + k^h(\theta_t)u_t^h. \tag{57}$$

We again define the bargaining-relevant value equations using the household-level envelope condition. Define $\mathbf{V}(n_{t-1}^h)$ as the value function associated with the optimal plan that solves the household problem. The envelope condition is thus $\mathbf{V}'(n_{t-1}^h) = (1-\rho^x)\mu_t^h$, where μ_t^h is the Lagrange multiplier associated with the time-t perceived law of motion. Next, use the household's optimality condition on n_t, which is $\mu_t^h = \lambda_t(1-\tau_t^n)w_t - g'(1-u_t^h-n_t^h) + \beta(1-\rho^x)E_t\mu_{t+1}^h$, to express the envelope condition as

$$\frac{\mathbf{V}'(n_{t-1}^h)}{1-\rho^x} = \lambda_t(1-\tau_t^n)w_t - g'(1-u_t^h-n_t^h) + \beta(1-\rho^x)E_t\left\{\frac{\mathbf{V}'(n_t^h)}{1-\rho^x}\right\}, \tag{58}$$

in which we have normalized by $1-\rho^x$ due to the timing of events.

To express things in units of goods, define $\mathbf{W_t}$ as

$$\begin{aligned}\mathbf{W_t} \equiv \frac{\mathbf{V}'(n_{t-1}^h)}{\lambda_t(1-\rho^x)} &= -\frac{g'(1-u_t^h-n_t^h)}{\lambda_t} + (1-\tau_t^n)w_t + (1-\rho^x)E_t\left\{\frac{\beta\mathbf{V}'(n_{t-1}^h)}{\lambda_t(1-\rho^x)}\right\} \\ &= -\frac{g'(1-u_t^h-n_t^h)}{\lambda_t} + (1-\tau_t^n)w_t + (1-\rho^x)E_t\left\{\Xi_{t+1|t}\mathbf{W_{t+1}}\right\}. \end{aligned} \tag{59}$$

As before, the second line makes use of the definition of the one-period-ahead stochastic discount factor, $\Xi_{t+1|t} \equiv \beta\lambda_{t+1}/\lambda_t$.

On the firm side, we still have

$$\mathbf{J_t} = z_t - w_t + (1-\rho^x)E_t\left\{\Xi_{t+1|t}\mathbf{J_{t+1}}\right\}; \tag{60}$$

and, for use below, note that $\mathbf{J_t} = \frac{\gamma}{k^f(\theta_t)}$.

As above, we have $\mathbf{U_t} \equiv \left(\partial\mathbf{V_t}/\partial u_t^h\right)/\lambda_t = 0$; and, as above, the time-t generalized Nash sharing rule is $\frac{\mathbf{W_t}}{1-\tau_t^n} = \frac{\eta}{1-\eta}\mathbf{J_t}$.

Now proceed to derive an explicit expression for w_t. Inserting the definition of $\mathbf{W_t}$ into the Nash sharing rule,

$$-\frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau_t^n)} + w_t + \frac{1-\rho^x}{1-\tau_t^n}E_t\left\{\Xi_{t+1|t}\mathbf{W_{t+1}}\right\} = \frac{\eta}{1-\eta}\mathbf{J_t}, \tag{61}$$

and then using the time-$t+1$ Nash sharing rule,

$$-\frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau_t^n)} + w_t + \frac{1-\rho^x}{1-\tau_t^n}E_t\left\{\Xi_{t+1|t}(1-\tau_{t+1}^n)\frac{\eta}{1-\eta}\mathbf{J_{t+1}}\right\} = \frac{\eta}{1-\eta}\mathbf{J_t}. \tag{62}$$

Make the substitution $\mathbf{J_t} = \frac{\gamma}{k^f(\theta_t)}$, and similarly for $\mathbf{J_{t+1}}$, which yields

$$-\frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau_t^n)} + w_t + \frac{1-\rho^x}{1-\tau_t^n}E_t\left\{\Xi_{t+1|t}(1-\tau_{t+1}^n)\frac{\eta}{1-\eta}\frac{\gamma}{k^f(\theta_{t+1})}\right\} = \frac{\eta}{1-\eta}\frac{\gamma}{k^f(\theta_t)}. \tag{63}$$

Next, use the job-creation condition $\frac{\gamma}{k^f(\theta_t)} = z_t - w_t + (1-\rho^x)E_t\left\{\Xi_{t+1|t}\frac{\gamma}{k^f(\theta_{t+1})}\right\}$ to substitute on the right-hand-side, which gives

$$-\frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau_t^n)} + w_t + \frac{1-\rho^x}{1-\tau_t^n}E_t\left\{\Xi_{t+1|t}(1-\tau_{t+1}^n)\frac{\eta}{1-\eta}\frac{\gamma}{k^f(\theta_{t+1})}\right\} = \frac{\eta}{1-\eta}\left[z_t - w_t + (1-\rho^x)E_t\left\{\Xi_{t+1|t}\frac{\gamma}{k^f(\theta_{t+1})}\right\}\right]. \tag{64}$$

Grouping terms involving w_t,

$$\begin{aligned} w_t\left[1+\frac{\eta}{1-\eta}\right] &= \frac{\eta}{1-\eta}z_t + \frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau_t^n)} \\ &- \frac{\eta}{1-\eta}\frac{1-\rho^x}{1-\tau_t^n}E_t\left\{\Xi_{t+1|t}(1-\tau_{t+1}^n)\frac{\gamma}{k^f(\theta_{t+1})}\right\} + \frac{\eta}{1-\eta}(1-\rho^x)E_t\left\{\Xi_{t+1|t}\frac{\gamma}{k^f(\theta_{t+1})}\right\}. \end{aligned} \tag{65}$$

Finally, multiplying by $1-\eta$ gives the wage equation

$$\begin{aligned} w_t &= \eta z_t + (1-\eta)\frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau_t^n)} \\ &- \eta\frac{1-\rho^x}{1-\tau_t^n}E_t\left\{\Xi_{t+1|t}(1-\tau_{t+1}^n)\frac{\gamma}{k^f(\theta_{t+1})}\right\} + \eta(1-\rho^x)E_t\left\{\Xi_{t+1|t}\frac{\gamma}{k^f(\theta_{t+1})}\right\}, \end{aligned} \tag{66}$$

which is expression (13) in the main text. As in the case of model with a one-period delay before new matches become productive, if $\tau_{t+1}^n = \tau_t^n = \tau^n$, the last two terms cancel with each other and the wage collapses to a simple static split,

$$w_t = \eta z_t + (1-\eta)\frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau^n)} \tag{67}$$

D Social Planner Problem

Here we prove the efficiency condition presented in Proposition 1. A Social Planner in our economy optimally allocates the measure one of individuals in the representative household to leisure and the labor force. Denote by ϱ_t the measure of individuals that enjoy leisure in period t. That is, $\varrho_t = 1 - u_t - n_t$ is the measure of individuals outside the labor force, neither working nor actively in the pool of searching unemployed. Our main interest in the social-planning problem is on developing a search-theoretic notion of static efficiency between leisure (search) and consumption. To emphasize static variables, and move the focus away from the endogenously-evolving stock of employment n_t, we cast the social planning problem as one of choosing u_t and ϱ_t, rather than as one of choosing n_t. Of course, a choice of u_t and ϱ_t implies a choice of n_t.

The social-planning problem is thus

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + g(\varrho_t) \right] \tag{68}$$

subject to the sequence of resource constraints

$$c_t + g_t + \gamma v_t = z_t(1 - \varrho_t - u_t) \tag{69}$$

and laws of motion for the employment stock

$$(1 - \varrho_t - u_t) = (1 - \rho^x)(1 - \varrho_{t-1} - u_{t-1}) + m(u_t, v_t). \tag{70}$$

As noted above, because our focus is on the margin within a period between c_t and u_t, we have expressed n_t in the constraints as $1 - \varrho_t - u_t$.

Denote by λ_t^1 and λ_t^2 the Lagrange multipliers on these two constraints, respectively. The first-order conditions with respect to c_t, v_t, u_t, and ϱ_t are thus

$$u'(c_t) - \lambda_t^1 = 0, \tag{71}$$

$$-\gamma \lambda_t^1 + \lambda_t^2 m_v(u_t, v_t) = 0, \tag{72}$$

$$-\lambda_t^1 z_t + \lambda_t^2 m_u(u_t, v_t) + \lambda_t^2 - (1 - \rho^x)\beta E_t \lambda_{t+1}^2 = 0, \tag{73}$$

and

$$g'(\varrho_t) - \lambda_t^1 z_t + \lambda_t^2 - (1 - \rho^x)\beta E_t \lambda_{t+1}^2 = 0. \tag{74}$$

Conditions (73) and (74) imply

$$g'(\varrho_t) = \lambda_t^2 m_u(u_t, v_t). \tag{75}$$

Using (72), this can be expressed as

$$g'(\varrho_t) = \gamma \lambda_t^1 \frac{m_u(u_t, v_t)}{m_v(u_t, v_t)}. \tag{76}$$

Using (71), this can be expressed as

$$\frac{g'(\varrho_t)}{u'(c_t)} = \gamma \frac{m_u(u_t, v_t)}{m_v(u_t, v_t)}. \tag{77}$$

Given a Cobb-Douglas matching technology $m(u,v) = u^{\xi_u} v^{1-\xi_u}$, we have that $\frac{m_u}{m_v} = \frac{\xi_u}{1-\xi_u}\theta$ (where $\theta \equiv v/u$).

We thus can express static efficiency along the consumption-leisure margin as

$$\frac{g'(1 - u_t - n_t)}{u'(c_t)} = \gamma\theta_t \frac{\xi_u}{1-\xi_u}, \tag{78}$$

in which we have re-inserted the identity $\varrho_t = 1 - u_t - n_t$. Condition (78) characterizes efficiency along the static consumption-leisure margin. The left-hand-side is clearly the marginal rate of substitution (MRS) for the household between consumption and leisure. Under the basic tenet that efficiency occurs where MRS is equated to the associated marginal rate of transformation (MRT), the right-hand-side of (78) is thus the MRT between consumption and leisure in a search-based environment.

E General Equilibrium Consumption-Leisure Wedges

Here we prove Proposition 2. To derive the equilibrium margin between consumption and leisure and to see which features of the decentralized economy disrupt consumption-leisure efficiency, start with the pseudo-labor-force participation condition

$$\frac{g'(1-u_t-n_t)-u'(c_t)\chi}{k^h(\theta_t)}+\frac{\phi_t}{k^h(\theta_t)}=u'(c_t)(1-\tau_t^n)w_t-g'(1-u_t-n_t)$$
$$-\quad \phi_t+\beta(1-\rho^x)E_t\left\{\frac{g'(1-u_{t+1}-n_{t+1})-u'(c_{t+1})\chi}{k^h(\theta_{t+1})}+\frac{\phi_{t+1}}{k^h(\theta_{t+1})}\right\}. \tag{79}$$

and the job-creation condition

$$\frac{\gamma}{k^f(\theta_t)}=z_t-w_t+(1-\rho^x)E_t\left\{\Xi_{t+1|t}\frac{\gamma}{k^f(\theta_{t+1})}\right\}. \tag{80}$$

Divide (79) by (80) to get

$$\frac{k^f(\theta_t)\left[\frac{g'(1-u_t-n_t)-u'(c_t)\chi}{k^h(\theta_t)}+\frac{\phi_t}{k^h(\theta_t)}\right]}{\gamma}$$
$$=\quad\frac{u'(c_t)(1-\tau_t^n)w_t-g'(1-u_t-n_t)-\phi_t+\beta(1-\rho^x)E_t\left\{\frac{g'(1-u_{t+1}-n_{t+1})-u'(c_{t+1})\chi}{k^h(\theta_{t+1})}+\frac{\phi_{t+1}}{k^h(\theta_{t+1})}\right\}}{z_t-w_t+(1-\rho^x)E_t\left\{\Xi_{t+1|t}\frac{\gamma}{k^f(\theta_{t+1})}\right\}} \tag{81}$$

Using the result that $k^f(\theta_t)/k^h(\theta_t)=\theta_t^{-1}$ due to the assumption of Cobb-Douglas matching, we can re-arrange the left-hand-side as

$$\frac{g'(1-u_t-n_t)-u'(c_t)\chi+\phi_t}{\gamma\theta_t}$$
$$=\quad\frac{u'(c_t)(1-\tau_t^n)w_t-g'(1-u_t-n_t)-\phi_t+\beta(1-\rho^x)E_t\left\{\frac{g'(1-u_{t+1}-n_{t+1})-u'(c_{t+1})\chi}{k^h(\theta_{t+1})}+\frac{\phi_{t+1}}{k^h(\theta_{t+1})}\right\}}{z_t-w_t+(1-\rho^x)E_t\left\{\Xi_{t+1|t}\frac{\gamma}{k^f(\theta_{t+1})}\right\}} \tag{82}$$

Next, note from our work in Appendix B that the numerator on the right-hand-side is $u'(c_t)\left[\mathbf{W}_t-\mathbf{U}_t\right]=u'(c_t)\mathbf{W}_t$, the surplus to the household (expressed in terms of utility) of having the marginal member enter into an employment relationship. Also note that the denominator on the right-hand-side of the previous expression is $\mathbf{J}_t$, the surplus to the firm of entering into an employment relationship with one additional worker.

Thus, the last expression can be simplified to

$$\frac{g'(1-u_t-n_t)-u'(c_t)\chi+\phi_t}{\gamma\theta_t}=\frac{u'(c_t)\mathbf{W}_t}{\mathbf{J}_t}. \tag{83}$$

Using in this expression the private economy's Nash-bargaining outcome $\frac{\mathbf{W}_t}{1-\tau_t^n}=\frac{\eta}{1-\eta}\mathbf{J}_t$, we have

$$\frac{g'(1-u_t-n_t)-u'(c_t)\chi+\phi_t}{\gamma\theta_t}=u'(c_t)\frac{\eta}{1-\eta}(1-\tau_t^n). \tag{84}$$

Rearranging, we have that in the decentralized Nash-bargaining economy with taxes, unemployment benefits, and no ability for households to enter or exit the labor force,

$$\frac{g'(1-u_t-n_t)-u'(c_t)\chi+\phi_t}{u'(c_t)}=(1-\tau_t^n)\gamma\theta_t\frac{\eta}{1-\eta}, \tag{85}$$

in which, recall, $\eta \in (0,1)$ is the Nash bargaining power of households.

Comparing (85) with the efficiency condition (78), it is clear that four things are needed in the decentralized economy in order for consumption-leisure efficiency to be achieved:

- The decentralized economy must feature $\eta = \xi_u$, which corresponds to the Hosios (1990) condition for search efficiency
- The transfer of unemployment benefits from the government to households must be zero — that is, $\chi = 0$
- Households must be able to optimize with respect to the size of the total labor force every period — that is, $\phi_t = 0\ \forall t$
- Proportional labor income taxation must be zero — that is, $\tau_t^n = 0\ \forall t$

F Derivation of Implementability Constraint

The derivation of the implementability constraint follows that laid out in Lucas and Stokey (1983) and Chari and Kehoe (1999). The derivation presented here nests that for the both the pseudo-LFP model and the LFP model — obtaining the implementability constraint for the latter from the former requires setting the shadow value on the exogenous labor-force size restriction to zero, as has been discussed in the text.

From the pseudo-labor-force participation model, we will be able to use the household's (pseudo) perceived law of motion for employment,

$$n_t = (1-\rho^x)n_{t-1} + k^h(\theta_t)u_t^h, \tag{86}$$

the size restriction

$$n_t + u_t^h = \bar{l}, \tag{87}$$

and the pseudo-labor-force-participation condition

$$\begin{aligned}\frac{g'(1-u_t^h-n_t^h)-u'(c_t)\chi}{k^h(\theta_t)} + \frac{\phi_t}{k^h(\theta_t)} &= u'(c_t)(1-\tau_t^n)w_t - g'(1-u_t^h-n_t^h)\\ - \quad \phi_t + \beta(1-\rho^x)E_t&\left\{\frac{g'(1-u_{t+1}^h-n_{t+1}^h)-u'(c_{t+1})\chi}{k^h(\theta_{t+1})} + \frac{\phi_{t+1}}{k^h(\theta_{t+1})}\right\}.\end{aligned} \tag{88}$$

Start with the household flow budget constraint in equilibrium

$$c_t + b_t = (1-\tau_t^n)w_t n_t + u_t^h\chi + R_t b_{t-1} + (1-\tau^d)d_t. \tag{89}$$

Multiply by $\beta^t u'(c_t)$ and sum over dates and states starting from $t=0$,

$$\begin{aligned}E_0\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t + E_0\sum_{t=0}^{\infty}\beta^t u'(c_t)b_t &= E_0\sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau_t^n)w_t n_t\\ +E_0\sum_{t=0}^{\infty}\beta^t u'(c_t)u_t^h\chi &+ E_0\sum_{t=0}^{\infty}\beta^t u'(c_t)R_t b_{t-1} + E_0\sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau^d)d_t.\end{aligned}$$

Use the household's Euler equation, $u'(c_t) = E_t\left[\beta u'(c_{t+1})R_{t+1}\right]$, to substitute for $u'(c_t)$ in the term on the left-hand-side involving b_t,

$$\begin{aligned}E_0\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t + E_0\sum_{t=0}^{\infty}\beta^{t+1} u'(c_{t+1})R_{t+1}b_t &= E_0\sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau_t^n)w_t n_t\\ +E_0\sum_{t=0}^{\infty}\beta^t u'(c_t)u_t^h\chi &+ E_0\sum_{t=0}^{\infty}\beta^t u'(c_t)R_t b_{t-1} + E_0\sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau^d)d_t.\end{aligned}$$

From here on, we suppress the E_0 operator to conserve on notation.

Canceling terms in the second summation on the left-hand-side with the third summation on the right-hand-side leaves only the time-zero bond position,

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t = \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau_t^n)w_t n_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)u_t^h\chi + \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau^d)d_t + u'(c_0)R_0 b_{-1}. \tag{90}$$

Next, use (88) to substitute for the sequence of terms $u'(c_t)(1-\tau_t^n)w_t$ in the first summation on the right-hand-side, which gives

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t = \sum_{t=0}^{\infty}\beta^t\left[\frac{g_t' - u'(c_t)\chi}{k^h(\theta_t)} + \frac{\phi_t}{k^h(\theta_t)}\right]n_t + \sum_{t=0}^{\infty}\beta^t\left[g_t' + \phi_t\right]n_t \tag{91}$$

$$- \quad (1-\rho^x)\sum_{t=0}^{\infty}\beta^{t+1}\left[\frac{g_{t+1}' - u'(c_{t+1})\chi}{k^h(\theta_{t+1})} + \frac{\phi_{t+1}}{k^h(\theta_{t+1})}\right]n_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)u_t^h\chi + \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau^d)d_t + u'(c_0)R_0 b_{-1} \tag{92}$$

To further conserve on notation, we now use g_t' to stand for $g'(1-u_t^h-n_t^h)$.

Next, use $n_t = (1-\rho^x)n_{t-1} + k^h(\theta_t)u_t^h$ to substitute for the sequence of n_t terms that appear in the first summation on the right-hand-side, which gives

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t = (1-\rho^x)\sum_{t=0}^{\infty}\beta^t\left[\frac{g_t' - u'(c_t)\chi}{k^h(\theta_t)} + \frac{\phi_t}{k^h(\theta_t)}\right]n_{t-1} \tag{93}$$

$$+ \quad \sum_{t=0}^{\infty}\beta^t\left[\frac{g_t' - u'(c_t)\chi}{k^h(\theta_t)} + \frac{\phi_t}{k^h(\theta_t)}\right]k^h(\theta_t)u_t^h + \sum_{t=0}^{\infty}\beta^t\left[g_t' + \phi_t\right]n_t \tag{94}$$

$$- \quad (1-\rho^x)\sum_{t=0}^{\infty}\beta^{t+1}\left[\frac{g_{t+1}' - u'(c_{t+1})\chi}{k^h(\theta_{t+1})} + \frac{\phi_{t+1}}{k^h(\theta_{t+1})}\right]n_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)u_t^h\chi + \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau^d)d_t + u'(c_0)R_0 b_{-1} \tag{95}$$

The first summation on the right-hand-side cancels with the fourth summation on the right-hand-side, leaving only the time-zero term:

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t = \sum_{t=0}^{\infty}\beta^t\left[\frac{g_t' - u'(c_t)\chi}{k^h(\theta_t)} + \frac{\phi_t}{k^h(\theta_t)}\right]k^h(\theta_t)u_t^h + \sum_{t=0}^{\infty}\beta^t\left[g_t' + \phi_t\right]n_t \tag{96}$$

$$+ \quad \sum_{t=0}^{\infty}\beta^t u'(c_t)u_t^h\chi + \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau^d)d_t + u'(c_0)R_0 b_{-1} + (1-\rho^x)\left[\frac{g_0' - u'(c_0)\chi}{k^h(\theta_0)} + \frac{\phi_0}{k^h(\theta_0)}\right]n_{-1} \tag{97}$$

Expanding and rearranging the first summation on the right-hand-side,

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t = \sum_{t=0}^{\infty}\beta^t\left[g_t' + \phi_t\right]u_t^h + \sum_{t=0}^{\infty}\beta^t\left[g_t' + \phi_t\right]n_t - \sum_{t=0}^{\infty}\beta^t u'(c_t)u_t^h\chi \tag{98}$$

$$+ \quad \sum_{t=0}^{\infty}\beta^t u'(c_t)u_t^h\chi + \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau^d)d_t + u'(c_0)R_0 b_{-1} + (1-\rho^x)\left[\frac{g_0' - u'(c_0)\chi}{k^h(\theta_0)} + \frac{\phi_0}{k^h(\theta_0)}\right]n_{-1} \tag{99}$$

Finally, canceling the third and fourth summations on the right-hand-side and re-introducing the conditional expectation E_0, we have the present-value implementability constraint

$$E_0\sum_{t=0}^{\infty}\beta^t\left\{u'(c_t)c_t - \left[g'(1-u_t^h-n_t) + \phi_t\right](n_t+u_t^h) - u'(c_t)(1-\tau^d)d_t\right\} = A_0, \tag{100}$$

where

$$A_0 \equiv u'(c_0)R_0 b_{-1} + (1-\rho^x)\left[\frac{g_0' - u'(c_0)\chi}{k^h(\theta_0)} + \frac{\phi_0}{k^h(\theta_0)}\right]n_{-1}. \tag{101}$$

G Zero Taxation of Dividend Income

We briefly explore the sensitivity of our results to the assumption of zero dividend taxation. All the quantitative results presented thus far have assumed 100-percent taxation of the dividend income received by households. As we noted in Section 2.2, it is well-understood in Ramsey models that the unavailability of profit taxes can affect policy prescriptions. This is because, following production, profit flows (which is what households' dividend income reflects) represent an inelastic source of revenue which the government would like to tax heavily. A 100-percent profit/dividend tax is thus trivially optimal, and this is the case on which we have focused. To demonstrate the robustness of our main results to the absence of a profit tax, we report in Table 3 simulation-based results under the polar opposite assumption of zero dividend taxation, $\tau^d = 0$. As comparison of the results in Table 3 with those reported in Table 1 shows, the cyclical properties of optimal policy in both the pseudo-LFP model and the LFP model are virtually identical under zero or full taxation of dividend income. Table 3 tabulates results for the case of zero government-provided unemployment benefits. For the case of positive government-provided unemployment benefits, we find results very nearly the same as those reported in Table 2; for brevity, though, we do not report these results.

Variable	Mean	Std. Dev.	Auto corr.	Corr(x, Y)	Corr(x, Z)	Corr(x, G)
			Pseudo-labor-force-participation			
τ^n	0.2176	0.0179	0.8324	-0.2243	-0.2263	0.9660
gdp	0.5942	0.0092	0.9059	1.0000	0.9998	-0.0496
c	0.4080	0.0060	0.9133	0.7354	0.7358	-0.6639
N	0.5937	0.0006	0.9580	0.8623	0.8532	-0.0363
u	0.0659	0.0006	0.9580	-0.9585	-0.9527	0.0395
$N+u$	0	—	—	—	—	
w	0.8612	0.0150	0.7677	0.9647	0.9675	-0.1142
θ	1.0007	0.0180	0.8824	0.9781	0.9756	-0.0385
v	0.0660	0.0006	0.7659	0.9454	0.9463	-0.0349
ϕ	-0.0245	0.0371	0.8580	0.2426	0.2442	-0.9664
			Full labor-force-participation			
τ^n	0.2194	0.0009	0.0392	0.6254	0.5896	0.1612
gdp	0.5906	0.0089	0.9008	1.0000	0.9237	0.2848
c	0.4049	0.0050	0.8766	0.5692	0.8141	-0.5594
N	0.5902	0.0032	0.9634	0.2444	-0.0848	0.8942
u	0.0655	0.0013	0.5554	-0.3980	-0.5849	0.5341
$N+u$	0.6557	0.0040	0.8761	0.0750	-0.2569	0.9679
w	0.8611	0.0142	0.9096	0.9006	0.9957	-0.1073
θ	1.0008	0.0189	0.8511	0.8724	0.9777	-0.1541
v	0.0656	0.0009	0.5621	0.6033	0.4653	0.5650
ϕ	0	0	—	—	—	—

Table 3: Ramsey dynamics with zero government-provided unemployment benefits and zero taxation of dividend income ($\tau^d = 0$).

H Competitive Search Equilibrium

While Nash bargaining has become the standard wage-determination mechanism in DSGE search models, many other models of wage determination have been developed and usefully employed in the literature on labor-market theory. One particularly appealing alternative is competitive search equilibrium, which entails decentralized determination of wages which are taken as given by both firms and households in their optimal search behavior. From the point of view of standard DSGE macroeconomic models, this equilibrium concept is appealing because there are no bilateral negotiations whatsoever; wages are always determined in a market-clearing fashion. Because competitive search has not yet found its way to mainstream DSGE macro models, we show here how to implement a competitive search equilibrium. Our treatment adapts Moen's (1997) implementation for a full general-equilibrium environment. As we show, the condition characterizing equilibrium along the consumption-leisure (outside the labor force) margin that arises from competitive search is identical to that which arises under Nash bargaining under the Hosios (1990) parameterization for bargaining power. Because the Hosios parameterization is the case for which our main results are obtained, all of our optimal-policy results are thus identical under competitive search. Indeed, the derivations below can be viewed as extending the equivalence shown by Moen (1997) between competitive search equilibrium and Nash-Hosios bargaining.

To implement competitive search equilibrium, we must first define payoff functions to search for both firms and households. In a particular labor submarket i, any firm j that pays the vacancy-posting cost γ has expected payoff of matching with a worker

$$k^f(\theta_{ijt})\left[z_t - w_{ijt} + (1-\rho^x)E_t\left\{\Xi_{t+1|t}\frac{\gamma}{\theta_{it+1}}\right\}\right]. \tag{102}$$

As in Moen (1997), the matching probability in period t, $k^f(\theta_{ijt})$, and the wage payment in period t, w_{ijt}, are firm ij-specific. The continuation value captured by $\gamma/k^f(\theta_{it+1})$, however, is a sub-market-specific value, reflecting the replacement value of a given worker at sub-market i prices (because there are no match-specific idiosyncracies regarding employment in our model).

For the representative household, payoff functions, defined in terms of goods, for searching for ("applying to") a job at firm j in labor submarket i are given by

$$k^h(\theta_{ijt})\left[-\frac{g'(1-u_t^h-n_t^h)}{\lambda_t} + (1-\tau_t^n)w_{ijt} + (1-\rho^x)E_t\left\{\Xi_{t+1|t}\mathbf{W_{t+1}}\right\}\right] \tag{103}$$

if match-formation at firm ij is successful, which occurs with probability $k^h(\theta_{ijt})$, and

$$(1-k^h(\theta_{ijt}))\,[0] \tag{104}$$

if match-formation is unsuccessful, which occurs with probability $1-k^h(\theta_{ijt})$. Note that these payoffs are from the point of view of the household — i.e., they reflect the gain to the household of having one more household member search for a job at firm ij — and are straightforward generalizations of the payoff functions defined in Appendix C. As derived in Appendix C, there is zero payoff to the household if search is unsuccessful.

For the concept of competitive search equilibrium, *directed* search, rather than random search, is a critical component.[20] The consequence of optimally-directed job search at firm ij is thus that

$$k^h(\theta_{ijt})\mathbf{W_{ijt}} + (1 - k^h(\theta_{ijt}))\,[0] = X, \tag{105}$$

where X is the expected payoff of searching for a job at a firm different from ij (at either a different firm in labor submarket i or in another labor submarket altogether) and hence is independent of firm ij outcomes. In writing (105), we have used $\mathbf{W_{ijt}} = -\frac{g'(1-u_t^h-n_t^h)}{\lambda_t} + (1-\tau_t^n)w_{ijt} + (1-\rho^x)E_t\left\{\Xi_{t+1|t}\mathbf{W_{t+1}}\right\}$ from Appendix C. The competitive-search wage w_{ijt} and tightness θ_{ijt} are the solutions to maximization of (102) subject to (105).

Denoting by φ_{ijt} the multiplier on the constraint, the first-order condition with respect to w_{ijt} yields

$$\varphi_{ijt} = \frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}\frac{1}{1-\tau_t^n}. \tag{106}$$

Given constant-returns-to-scale matching, this reduces to

$$\varphi_{ijt} = \frac{1}{\theta_{ijt}}\frac{1}{1-\tau_t^n}. \tag{107}$$

Then, using this expression in the first-order condition with respect to θ_{ijt}, we have

$$\frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}}\left[z_t - w_{ijt} + (1-\rho^x)E_t\left\{\Xi_{t+1|t}\frac{\gamma}{k^f(\theta_{it+1})}\right\}\right] = -\frac{1}{\theta_{ijt}}\frac{1}{1-\tau_t^n}\frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}}\mathbf{W_{ijt}}. \tag{108}$$

We now restrict attention to equilibria that are symmetric across firms in a given submarket and across submarkets, so we drop ij indexes. Given Cobb-Douglas matching $m(u,v) = u^{\xi_u}v^{1-\xi_u}$, we have $(\partial k^h(\theta)/\partial\theta)/(\partial k^f(\theta)/\partial\theta) = -\theta(1-\xi_u)/\xi_u$. Also, using the symmetric-equilibrium version of the vacancy-creation condition, we can replace the term in brackets on the left hand side of the previous expression with $\gamma/k^f(\theta_t)$; making these substitutions,

$$\frac{\gamma}{k^f(\theta_t)} = \frac{1-\xi_u}{\xi_u}\frac{1}{1-\tau_t^n}\mathbf{W_t}. \tag{109}$$

Next, from optimal household labor-force-participation and the definition of $\mathbf{W_t}$ above, this can be written as

$$\gamma\frac{k^h(\theta_t)}{k^f(\theta_t)}\frac{\xi_u}{1-\xi_u}(1-\tau_t^n) = \frac{g'(1-u_t-n_t) - u'(c_t)\chi + \phi_t}{u'(c_t)}. \tag{110}$$

Once again recognizing that $\frac{k^h(\theta)}{k^f(\theta)} = \theta$ because of constant-returns matching, we have that period-t equilibrium outcomes under competitive search are described by

$$\frac{g'(1-u_t-n_t) - u'(c_t)\chi + \phi_t}{u'(c_t)} = (1-\tau_t^n)\gamma\theta_t\frac{\xi_u}{1-\xi_u}. \tag{111}$$

[20] See Rogerson, Shimer, and Wright (2005, p. 972-976) for more discussion.

Comparing this with the outcome in the decentralized bargaining economy (under the Hosios parameterization $\eta = \xi_u$) presented in condition (27) shows that they are identical. Hence, the search-based labor wedge in the competitive search economy is identical to the wedge in the bargaining economy, which necessarily implies that optimal policy in the competitive search economy is identical to that in the bargaining economy.

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