Board of Governors of the Federal Reserve System

International Finance Discussion Papers

Number 988

December 2009

Asset Returns with Earnings Management

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Abstract

The paper investigates stock return dynamics in an environment where executives have an incentive to maximize their compensation by artificially inflating earnings. A principal-agent model with financial reporting and managerial effort is embedded in a Lucas asset-pricing model with periodic revelations of the firm's underlying profitability. The return process generated from the model is consistent with a range of financial anomalies observed in the return data: volatility clustering, asymmetric volatility, and increased idiosyncratic volatility. The calibration results further indicate that earnings management by individual firms does not only deliver the observed features in their own stocks, but can also be strong enough to generate market-wide patterns.

Keywords: Earnings management, Stock returns, Financial anomalies, Volatility clustering, GARCH, Optimal contract

JEL Classifications: E44, D82, D83, G12

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*I gratefully acknowledge Eric Young and Chris Otrok for their inspiration and support. A special debt of gratitude is owed to Toshi Mukoyama for meticulous guidance and to Mark Carey for enormous help. I am also greatly indebted to David Bowman, Antonio Falato, Pingyang Gao, Marvin Goodfriend, Jon Glover, Rick Green, Denis Gromb, Zhiguo He, Bob King, Pete Kyle, Nathan Larson, Richard Leftwich, Pierre Liang, Yohei Okawa, Michael Palumbo, Nagpurnanand R. Prabhala, Adriano Rampini, Steve Sharpe, Cathy Schrand, Abbie Smith, Xuan Tam, John Weinberg, Wei Xiong, and Stan Zin for their helpful comments. I sincerely thank the seminar participants at Boston University, Carnegie Mellon University, Cornell University, Darden School of Business, Federal Reserve Board of Governors, Federal Reserve Bank of Cleveland, Federal Reserve Bank of Richmond, Georgetown University, INSEAD, Peking University, University of Chicago Booth School of Business, University of Maryland, University of Virginia, Wharton School of Business, World Bank, and Tsinghua University, for their helpful comments. In addition, I thank University of Virginia, John M. Olin Foundation, and Federal Reserve Board for financial support. The views expressed herein are the author's and do not necessarily reflect the opinions of the Board of Governors of the Federal Reserve System.

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1 Introduction

Executives' desire to use financial reports, especially bottom-line earnings, to pursue their own financial interests gives rise to the phenomenon of *earnings management*, which is defined as intentional manipulation of reported earnings by knowingly choosing accounting methods and estimates that do not accurately reflect the firm's underlying fundamentals. The accounting irregularities at Enron and WorldCom that precipitated the stock market downturn of 2002 and the corporate scandals that triggered the financial meltdown in 2008, notably Freddie Mac and AIG,[1] indicate that such behavior can engender significant economic consequences, especially in the financial markets. This paper explicitly examines the asset pricing implications of earnings management.

This intentional manipulation of financial information must be reflected in the pricing of stocks, since it affects the inference of the investors who value the stock of a firm. Empirical studies (e.g., Turner et al. [2001], Wu [2002], and Palmrose et al. [2004]) suggest that distorted information flow can cause adverse capital market reactions. In these studies, on average, stock returns fall by about 10% on the days around earnings restatement announcements. Figure 1, reproduced from Wu [2002], documents how stock returns react to restatements.[2] However, due to the lack of theoretical guidance and difficulty of detecting earnings management with accuracy, comparatively little is known about the potential systematic impact of earnings management on stocks.

The objective of the present study is to analyze the implications of earnings management

[1]Morgan Stanley determined the accounting tactics, while legal, enabled Freddie Mac, and to a lesser extent Fannie Mae, to overstate the value of their reserves. Both companies also pushed inevitable losses into future by sharply curtailing their repurchase of soured mortgages out of the securitizations they have guaranteed. "Fannie Mae and Freddie Mac were 'playing games with their accounting' to meet reserve requirements, prompting the government to seize control of the companies," U.S. Senator Richard Shelby said (Bloomberg [September 9, 2008]). In the case of AIG, PricewaterhouseCoopers prompted an announcement about the material accounting weaknesses related to the valuation of AIG's derivatives holdings. Prosecutors insisted that five former executives from the American International Group deliberately mounted a fraud to manipulate its financial statements, after a string of AIG scandals early this decade."Accounting flaws at American International Group significantly understated the insurance giant's losses on complex financial instruments linked to mortgages and corporate debt," AIG said in an official public statement (The New York Times [February 12, 2008]).

[2]I thank Min Wu for providing Figure 3 of Wu [2002], which is reproduced as Figure 1 in the current paper.

Figure 1: Cumulative abnormal returns around restatements: day (-125,+125)

This figure displays the mean of cumulative abnormal returns of restating firms from 1977 to 2001. Day 0 is the restatement announcement date. Source: Wu [2002]

for dynamic patterns of asset returns. In particular, this paper shows that earnings management is a possible explanation for a number of stylized financial facts, namely, volatility clustering, asymmetric volatility, and increased idiosyncratic volatility. These results underscore why earnings management is of central importance in pricing financial assets, in understanding the risk implied by empirical financial anomalies, and in contemplating the ongoing debate on regulations of financial markets and executive compensation.

I conduct this exercise within a Lucas asset-pricing model that is standard in all aspects, except that the investors hire a manager to operate the firm and report the firm's earnings. In particular, a principal-agent model with financial reporting and productive effort is embedded in a simple variant of the Lucas asset-pricing model. The investors engage in a (single-period) contractual relationship with a newly hired manager in every period and pay the manager a fraction of the reported earnings as compensation. The manager exerts an unobserved effort that affects the production, and possibly has discretion over the quantity of apples reported to the investors. The reported earnings are paid to the investors as dividends. The key feature I focus on here is the manager's ability to manipulate earnings reports. Earnings management occurs in the model when the reported apple harvest (earnings) differs from the true amount.[3]

There are periodic investigations concerning the underlying true earnings of the firm. In the final period of each revelation cycle, the uncertainty about true earnings is resolved, and the investors bear monetary costs in the event that earnings management is detected.[4] The investors are assumed to be risk-neutral; thus the price of the firm in each period is given by the discounted expected future dividends net of the labor wage and the financial loss associated with earnings management.

The return sequences generated from the model mimic a set of stylized facts in stock return data. First and foremost, the model returns exhibit volatility clustering. Because earnings management patterns vary with underlying true performance, certain levels of earnings lead to higher frequency of restatements than others,[5] creating larger swings in the return sequence. Return volatility becomes state-dependent in the model. As the state (that is, actual earnings) exhibits persistence over time, return volatility is time-varying and persistent. In addition to the direct impact due to possible *future* manipulation, an indirect effect reflecting suspicion of *previous* misreporting amplifies the persistence in volatility. The possibility of earnings management creates a range of reports that are associated with belief revision and intense suspicion of manipulation. The anticipation of restatements increases uncertainty and hence volatility. The volatility persists as reported earnings persist. Although

[3]The modeling technique presented here bears some similarities with Shorish and Spear [2005]. The similarities and differences between their paper and this paper will be discussed later in this section.

[4]This analysis does not make a distinction between earnings management and fraud. While the accounting choices that explicitly violate Generally Accepted Accounting Principles (GAAP) clearly constitute both earnings management and fraud, according to the SEC, systematic choices made within the boundaries of GAAP can constitute earnings management as long as they are used to obscure the true performance of a firm and will lead to adverse consequences for the firm in the same way as fraud.

Following this notion, there is no economic difference between fraud and earnings management in the model: in both cases the reported number is different from the true amount, and such behavior hurts the firm's future prospects.

[5]The notion of "restatements" in the paper does not necessarily imply actual restatement announcements but rather the broadly-defined adverse consequences of earnings management. The periodic revelations of true earnings in the model hence capture the negative consequences of earnings management that periodically show up in returns and can be understood as reflecting the reversing nature of earnings management.

the conditional heteroskedasticity observed in many financial markets has led to ARCH and GARCH models that are intensively used in analyzing stock returns, the underlying microeconomic motives are still not well understood. This paper presents the persistence in earnings management behavior as a likely source of the persistence in stock return volatility.

The model data capture another stylized fact in the finance literature: asymmetric volatility in stock returns. The mechanism is also twofold. First, earnings management goes hand-in-hand with weak economic performance, due to stronger financial incentives to inflate earnings when the performance is weaker. Because current low earnings lead to more frequent future earnings manipulation and resultant drastic consequences, low returns are associated with high volatility in the subsequent periods. Second, earnings reports within certain range are viewed as symptomatic of intentional misstatement. The inference of earnings management reduces the current price and increases the uncertainty over subsequent outcomes, thereby intensifying asymmetric volatility.[6] The existing literature on asymmetric volatility falls into two categories: the leverage effect proposed by Black and Scholes [1973], Merton [1974], and Black [1976] and the volatility feedback effect put forward by French et al. [1987] and Campbell and Hentschel [1992]. However, Christie [1982] and Schwert [1989] find that the leverage effect is too small to account for the asymmetry in volatility, and Campbell and Hentschel [1992] find that the volatility feedback effect normally has little impact on returns. This paper shows that the asymmetric association of earnings management to true earnings contributes to the observed asymmetric behavior in stock returns. The calibration results further suggest that this channel can be quantitatively important.

Last but not least important, as earnings management becomes more likely in the model, asset returns exhibit greater volatility. The dramatic consequences of earnings management generate active fluctuations in the return sequence and thus intensify return volatility. This

[6] Following Shin [2003], Rogers et al. [2007] empirically document that strategic disclosure, defined as the reporting of good news and the withholding of bad news, provides an explanation for asymmetric return volatility. They find that asymmetric volatility is more pronounced in the return series of individual firms that are more likely to disclose strategically as measured by their litigation risk incentives. Patterns in return volatility in market indices are also consistent with strategic disclosure as an explanation. As earnings management represents strategic decisions in mandatory reporting, different from strategic disclosure with verifiable reports, I do not present their findings as direct empirical evidence for this model. However, their paper suggests that financial reporting decisions can matter in generating the observed patterns in stock returns, in line with the prediction of the current model.

work adds to a growing literature that studies individual stock return volatility. Campbell et al. [2001] document that the level of average stock return volatility increased considerably from 1962 to 1997 in the United States. Furthermore, most of this increase is attributable to idiosyncratic stock return volatility as opposed to the volatility of the stock market indices. Rajgopal and Venkatachalam [2008] explore whether deteriorating financial reporting quality, as measured by earnings quality and dispersion in analyst forecasts of future earnings, can plausibly explain the increase in idiosyncratic volatility over the past four decades. Their results from cross-sectional and time-series regressions indicate a strong association between idiosyncratic return volatility and financial reporting quality. The current model replicates the positive relationship between the likelihood of earnings management and the volatility of individual returns, and thus contributes to the theoretical explanations of the data.

In this paper, the contracting system in a principal-agent model with financial reporting and moral hazard is first examined as a point of departure. This principal-agent model is developed and analyzed in greater detail in Sun [2008]. The purpose of this step is to provide the underlying economic motive for earnings management in the model, to understand how motives to induce managerial effort and to motivate truthful reports differentially affect the optimal contract, and to identify how earnings management decisions vary with actual economic performance. This principal-agent model lays out a micro-foundation for asset pricing in that it generates a set of earnings reports that may or may not be systematically biased. This model of managerial reporting under moral hazard is built on Dye [1988]. The message space is limited to a single-dimensional signal while the privately informed agent receives two dimensions of private information; therefore the Revelation Principle is not applicable.[7]

[7] A recent paper by Crocker and Slemrod [2007] considers an alternative environment where the Revelation Principle can be applied. In solving the model, they assume a monotonically increasing reporting function; actual earnings can therefore be recovered by inverting the reporting function. In their setting, the principal knows the exact amount of actual earnings as a function of the report, while in the current model the principal faces uncertainty over whether earnings management occurs. The manager possesses a second dimension of private information in this model, and hence the reporting function is no longer invertible. As a model that constructs an explanation for earnings management, the current contract work can be viewed as complementary to theirs. As a microeconomic foundation for the investigation into asset pricing with earnings management, their model would generate prices that are fully revealing in the equilibrium; whereas the investors in this model try to infer the true outcomes through Bayesian learning, but cannot perfectly see through earnings management.

In order to highlight the role that earnings management plays in price formulation, the principal-agent model with reporting choices is embedded into an otherwise standard Lucas asset-pricing model. In particular, by switching on and off the measure for earnings management in the model, I maintain the focus on earnings management and make the comparison with the standard asset-pricing model transparent. This modeling approach is related to Shorish and Spear [2005], where the owner of the firm hires a manager to maximize the firm's value, and there is asymmetric information about the manager's effort level between the owner and the manager. Along this line of agency-based asset pricing, Gorton and He [2006] show that when compensation depends on the firm's market performance, stock prices are set to induce the optimal effort level. In contrast with these papers, the current paper focuses on earnings management incentive in the contractual relationship and price formulation by assuming additional asymmetric information regarding output realizations.

This analysis also relates to the literature on asset pricing under asymmetric information, such as Detemple [1986], Wang [1993], and Cecchetti et al. [2000]. In particular, Wang [1993] presents a dynamic asset-pricing model in which the investors can be either informed or uninformed: the informed investors know the future dividend growth rate, while the uninformed investors do not. He finds that the existence of uninformed investors can lead to risk premia much higher than those under symmetric and perfect information. Distinguished from previous studies that examine the impact of information asymmetry and heterogeneous beliefs among investors, the study reported in this paper analyzes information asymmetry between corporate executives and outside investors as a whole.

There have not been many theoretical studies that examine the economic impact of earnings management. Fischer and Verrecchia [2000] is an early and notable exception. They show that more bias in the report reduces the correlation between share price and reported earnings, and they also study how the cost to the manager of biasing the report and the market's uncertainty about the manager's objective affect the slope and the intercept term in a regression of market price on earnings reports. Subsequently, Guttman et al. [2006] use a signaling model similar to Fischer and Verrecchia [2000] to explain the discontinuity observed in the distribution of earnings reports. While these papers do not model the contractual relationship between shareholders and the manager, Kwon and Yeo [2008] consider a single-period model where the principal takes into account how compensation affects productive effort and market expectations when designing the optimal contract. In their paper, a rational market can simply recalibrate or discount the reported performance when the manager overstates earnings, and correctly guess the true performance. They show that such rational market discounting leads to less productive effort by the manager and less performance pay by the principal. In contrast with the studies presented in these papers, the current study considers stock returns under earnings management in a dynamic setting, with a central focus on the return properties beyond the first moment. This study further provides a quantitative evaluation of the model.

Existing studies have analyzed earnings management behavior and stylized financial facts in isolation, and a systematic investigation into the link between earnings management and financial anomalies has not yet been undertaken. By incorporating earnings management into an otherwise standard asset-pricing model, this paper presents a mechanism through which financial misrepresentation may lead to a set of stylized financial facts. This paper suggests that there may be a unifying cause for these empirical regularities in the financial markets. In addition, the calibration results indicate that earnings management can be quantitatively important in explaining dynamic return patterns. This quantitative analysis further suggests that earnings management by individual firms may not only generate patterns in their own stock returns, but also be powerful enough to drive the observed effects in stock market indices.

The remainder of this paper proceeds as follows. Section 2 lays out the setup of the model. Section 3 discusses the general results, and presents the properties of simulated returns from the model. As one step toward calibration, Section 4 extends the model to continuous earnings. Section 5 presents a quantitative evaluation of the model. Section 6 checks the robustness of the model dynamics by adopting an alternative calibration strategy and incorporating stochastic investigation. Section 7 contains concluding remarks.

2 Model

The core of this paper is based on a Lucas asset-pricing model in which the investors hire a manager to operate the firm and report the firm's earnings. The investors design a contract that controls the manager's effort decision and reporting choice. In every period, the principal (investors) offers a newly hired manager a single-period contract. Earnings y are stochastic and take two possible values, $y \in \{l, h\}$, where $l < h$. The firm's production is associated with a simple Markov process:

$$\Pr(y_{t+1} = j | y_t = i) = \pi_{ij}, \qquad \forall i \in \{l, h\}, \quad \forall j \in \{l, h\}$$

The manager makes earnings announcements, and the reported earnings $R(y)$ are then paid out as dividends to the investors.[8] The underlying true earnings are periodically revealed.[9] For the purpose of illustration as well as tractability, it is assumed that after every two periods the uncertainty about the underlying earnings in the past two periods is resolved, and the investors bear financial losses if earnings management is detected. The investors know the revelation periodicity. The price of the firm in each period is given by discounted expected future dividends net of the executive compensation and monetary costs of earnings management.

One interpretation of the model is that the manager finances the discrepancy in the report from a market outside the economy, and the firm's owner (the investors) must repay a large amount of money at the time earnings management is detected (this is a part of the monetary loss that the investors have to bear upon detection of manipulation). Because the current manager is replaced in the next period, the significant repayment burden imposed on the investors does not directly affect the manager's incentive.[10] Another, and much broader,

[8]This analysis does not explicitly model how the manager finances the discrepancy in the reports. In reality, the manager can obtain funds from the firm's suppliers by talking them into some sham transactions or borrow money from banks. Although without the active help from suppliers and banks, companies could not have deceived investors and analysts alike, a recent Supreme Court ruling shields third parties, including suppliers and banks, from being held responsible for knowingly participating in financial data manipulation. The source of funds is therefore chosen to be left outside the model for simplification without causing any modeling inconsistencies.

[9]The model results do not hinge on the particular time structure of information disclosure. The model results are robust to a stochastic nature of investigation.

[10]The model lets the financial cost of earnings management almost entirely fall on the investors by dis-

Figure 2: Timeline of contracting within each period

Contract is offered	Manager exerts effort $e \in \{L, H\}$	Manager privately learns of earnings management opportunity	Earnings realize. Manager privately observes earnings	Manager makes a report	Manager is compensated based on his report

interpretation of the model is that the manager may engage in activities that boost current earnings at the expense of future (long-term) benefits. In particular, the manager may follow myopic strategies and take economically suboptimal actions to inflate current earnings, such as forsaking profitable investment and postponing R&D and capital spending plans.[11] This interpretation corresponds to a more general notion of earnings management this model captures, which is an overstatement of current earnings that has negative consequences for the firm's future prospects.[12]

2.1 Optimal contract

The contractual environment follows Sun [2008]. A risk-neutral principal (investors) hires a risk-averse agent (manager) for one period. Figure 2 details the timeline of the contracting arrangement between the principal and the manager. In the beginning of each period, the manager accepts the take-it-or-leave-it contract offered by the principal for one period. Earnings are stochastic and influenced by the manager's effort. The unobserved effort level of the manager, e, can take two values, low (L) and high (H). The manager incurs disutility from exerting effort, denoted by the cost function $a(e)$. In particular, high effort is associated

missing the manager before the investigation, which reflects a general view that investors suffer most from earnings manipulation (please see footnote 20 for more details) whereas executives tend to absorb personal gains. While a dynamic contract with auditing technology can be an interesting extension of the model, the current version suffices in delivering overstatement in the equilibrium so as to derive its asset pricing implication.

[11] Grahama et al. [2005] document that 78% of executives in the survey admit to sacrificing long-term value to maintain predictability in earnings.

[12] Both as a proxy for current economic performance and an indicator of the firm's future productivity, a distinction between earnings and cash flows is not necessary in the paper. In particular, the manager inflates the reported performance, which could be earnings or, less commonly, cash flows; such manipulative behavior comes at the expense of longer term benefits, decreasing the value of future earnings and cash flows.

with a cost of $a(H) = c$, and low effort involves no cost: $a(L) = 0$. Earnings take two possible values, represented by $y \in \{l, h\}$, where $l < h$. Let p_e be the probability that earnings are h when the effort is e, with $p_H > p_L$. After exerting effort, the manager privately learns whether he has the opportunity to manage earnings. With probability x, the manager has discretion over how much earnings to report.[13] With probability $(1 - x)$, the manager is prohibited from manipulating earnings. Then the manager privately observes the earnings, and makes an earnings announcement.

If the manager produces an inaccurate report, the manager incurs a personal cost, denoted by $\phi(\cdot)$. ϕ is a function of the discrepancy between true earnings and reported earnings. When the manager reports honestly, he incurs no cost: $\phi(0) = 0$.[14] When the manager overstates earnings, there is a positive cost $\phi(h - l) = \psi > 0$. Earnings management occurs in the model when the reported earnings differ from true earnings. More specifically, earnings management emerges in this environment if the manager announces that high earnings (h) have been achieved when the actual realization of earnings is low (l).

As the contract must be designed based on mutually observed variables, the manager's compensation can be based only on the earnings report. As long as the manager's reported earnings fall in the set $\{l, h\}$, the principal cannot directly detect whether the manager has

[13]Here, whether the manager has the opportunity of managing earnings is assumed to be a random event, and the outcome is the manager's private information. Generally Accepted Accounting Principles (GAAP) provide guidelines on how to record and summarize each type of economic transaction, and hence define the accounting latitude available to senior management in financial reporting. In practice, certain economic activities, those where there is no hard-and-fast rule for which accounting method to use, lead to more discretion than others. In any particular period, economic transactions of this type may or may not take place. By virtue of being closer to the operations process, only the manager knows the extent of these activities and hence the degree of reporting latitude available.

In modeling language, the stochastic opportunity to manage earnings adds an additional noise in financial reports that investors cannot perfectly filter out. Due to the additional uncertainty, the investors need to make inferences as to whether earnings management occurs, and earnings management is not fully unraveled in the equilibrium. Alternatively, an environment where the opportunity to manipulate reports realizes with certainty while the size of manipulation is stochastic would be essentially identical to the current model.

[14]There are two frictions in the model that restrain earnings management: earnings management opportunity that realizes with probability x and the cost involved in misstating earnings ϕ. This model can be also considered with only one friction: the cost of manipulation with a simple stochastic structure. The manipulation cost now in the model follows a binary distribution with two possible realizations ∞ and ψ. The cost of manipulating earnings includes the educational cost of learning how to modify certain components of earnings without getting detected, the costs involved in bribing auditors not to report a discrepancy in the earnings report, and expected reputation damage in case of being caught.

misstated earnings. For notation convenience, high and low reported earnings are denoted by $\tilde{h}$ and by $\tilde{l}$, to distinguish from high and low actual earnings.[15] It is also assumed that the manager is essential to the operation of the firm, so the contract must be such that the manager (weakly) prefers to work for the principal regardless of whether the manager gains the opportunity to manage earnings.

The contract between the risk-neutral principal and the risk-averse agent includes a set of wages contingent on the reports, which can be alternatively characterized as a set of contingent utilities. The manager's utility level corresponding to compensation level w_i, $i \in \{\tilde{l}, \tilde{h}\}$, is denoted as $U(w_i) = u_i$, where $U(\cdot)$ is a strictly increasing and strictly concave utility function. Let $U^{-1}(\cdot) = V(\cdot)$. Then $V(u_i)$ is the cost to the principal of providing the agent with utility u_i. Because $U(\cdot)$ is a strictly increasing and strictly concave function, $V(\cdot)$ is a strictly increasing and strictly convex function.

In this environment, the contract must not only induce effort but also control for the manager's reporting incentive. This study assumes that the difference in the earnings is large enough that the principal always wants to implement high effort. The objective of the manager is to maximize utility by choosing a level of effort and a reporting strategy represented by $R(y)$, subject to the contract offered. When the manager has no discretion, we denote the report by $\bar{R}(h)$. By assumption, $\bar{R}(h) = \tilde{h}, \bar{R}(l) = \tilde{l}$. The manager's utility is of the form $U_m(e, R(y)) = xE[u_{R(y)} - \phi(R(y) - y) - a(e)] + (1 - x)E[u_{\bar{R}(y)} - a(e)]$. The first term is the manager's expected utility if the manager has sufficient discretion over reporting. The second term is the manager's expected utility if the manager has to truthfully report. The principal chooses the utility values u_i, $i \in \{\tilde{l}, \tilde{h}\}$, and recommended reporting choices $R(y)$ for each realization of earnings that minimize the expected cost of inducing effort.[16]

[15]Following Dye [1988], the model presented in this section places restrictions on the manager's ability to communicate the truth. In addition to the unobserved effort level, the manager observes two dimensions of information, the value of actual earnings and the realization of misstatement opportunity. However, the manager is permitted to communicate only a one-dimensional signal, which is an earnings announcement. Communication is restricted in that the manager cannot fully communicate the full dimensionality of his information, and hence the Revelation Principle is not applicable.

[16]As in the standard principal-agent model, the principal is the residual claimant, and hence entitled to receive the firm's earnings. The one-step departure from the standard model here is that the principal in this model does not observe the true earnings when the principal has to compensate the manager.

Formally, the optimal contract solves

$$\min_{u_{\tilde{h}}, u_{\tilde{l}}, R(h), R(l)} \quad E[V(u)|H]$$
$$= x[p_H V(u_{R(h)}) + (1-p_H)V(u_{R(l)})] + (1-x)[p_H V(u_{\tilde{h}}) + (1-p_H)V(u_{\tilde{l}})]$$

subject to

$$H = \arg\max_{e\in\{L,H\}} xE[u_{R(y)} - \phi(R(y)-y) - a(e)] + (1-x)E[u_{\bar{R}(y)} - a(e)], \qquad \forall y \in \{l,h\}. \quad (1)$$

$$E[u|H] = xE[u_{R(y)} - \phi(R(y)-y) - a(e)|H] + (1-x)E[u_{\bar{R}(y)} - a(e)|H] \geq \bar{U}. \qquad (2)$$

The objective function is the expected cost for the principal to motivate high effort. The first term is the cost of implementing high effort when the manager has an opportunity to manage earnings, and the second term is the cost if the manager does not have the opportunity. The first constraint is the incentive constraint for the manager's effort choice — here, it is assumed that the principal wants to induce high effort. The second is the participation constraint, where $\bar{U}$ is the manager's outside option. In addition to these constraints, when the manager has an opportunity to misstate earnings, the principal faces another constraint. As the reporting decision has been necessarily delegated to the manager, the "recommended reporting strategy" has to be voluntarily followed by the manager:

$$R(y) = \arg\max_{r\in\{\tilde{l},\tilde{h}\}} u_r - \phi(r-y) \qquad \forall y \in \{l,h\}. \qquad (3)$$

The optimal contract includes a set of utility promises $\{u_{\tilde{h}}, u_{\tilde{l}}\}$ and the recommended action $\{e^*, R(y)\}$. Following the convention, it is assumed that the principal wants to induce high effort, so $e^* = H$. Figure 3 summarizes the main results. The optimal contract is described as the curve ABC, which depicts how the wedge between promised utilities assigned to reports of high and low earnings varies with different values of manipulation cost ψ. The shaded area below the 45° line shows the combination of the compensation differential and manipulation cost that induces truthful reporting. Below I restate the relevant results shown in Sun [2008].

Proposition 1 *$\psi < c/(p_H - p_L)$ is the necessary and sufficient condition for earnings management to occur under the optimal contract.*

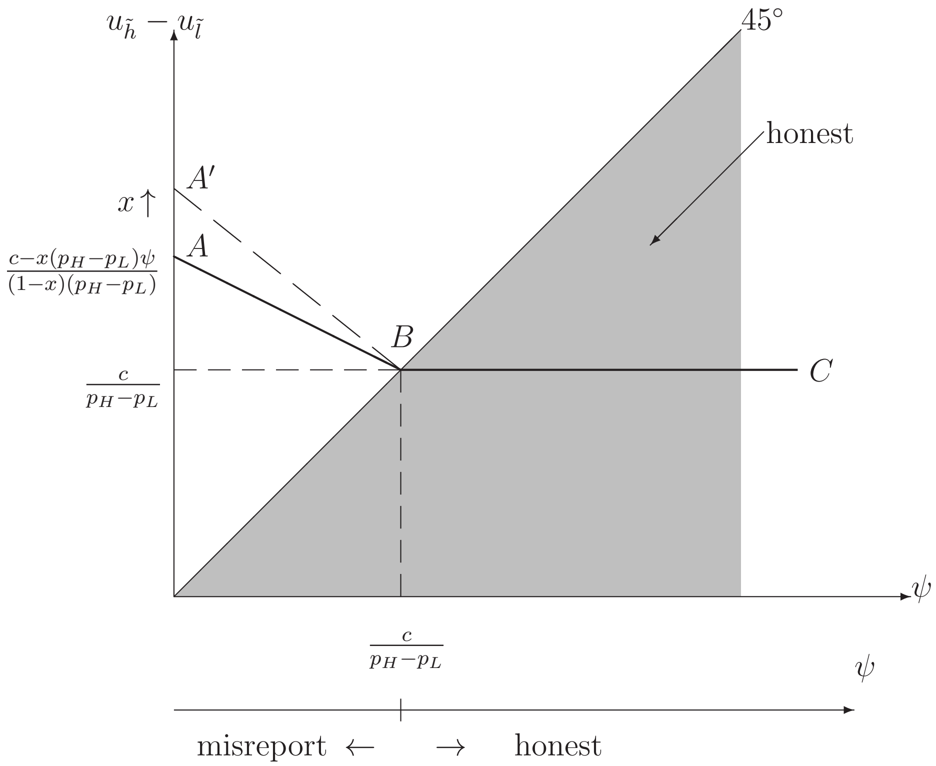


Figure 3: Main results

Lemma 1 *If $\psi < c/(p_H - p_L)$ holds, the optimal contract satisfies*

$$u_{\tilde{h}} = \bar{U} + \frac{c(1-p_L)}{(p_H - p_L)}, \tag{4}$$

$$u_{\tilde{l}} = \bar{U} + \frac{c(1-p_L)}{(p_H - p_L)} - \frac{c - x(p_H - p_L)\psi}{(1-x)(p_H - p_L)}. \tag{5}$$

The contract model illustrates the necessary and sufficient condition for earnings management to occur, and it yields a number of empirical implications of how earnings management affects executive compensation that are in line with empirical findings, which are detailed in Sun [2009]. In the current paper, this principal-agent model derives the manager's motive to manage earnings and also serves as a micro-foundation for asset pricing. Given a sequence of true earnings, the contract model generates a set of reports that may or may not be systematically biased. Because the realization of manipulation opportunity is stochastic, the investors are not able to make perfect inferences as to whether a report has been manipulated. As a micro-foundation for asset pricing, the central features this contract model boils down to are (1) the investors' inability to see through earnings management and (2) a focus

Figure 4: Model timeline

← t (1st period) →		← $t+1$ (2nd period) →			
Price $q_1(y_{t-1})$	Manager$_t$ makes a report r_t and is paid	Price $q_2(y_{t-1}, r_t)$	Manager$_{t+1}$ makes a report r_{t+1} and is paid	y_t and y_{t+1} are revealed	Investors bear financial costs F_1 or F_2

In this figure, q_1 and q_2 are the pricing functions in period 1 and 2 of each revelation cycle respectively. y_{t-1} is the actual earnings in period 2 of the previous revelation cycle. y_t and y_{t+1} are actual earnings in period 1 and period 2 in the current revelation cycle. F_1 and F_2 are the amount of financial loss investors bear if the manager manipulates earnings in one period and that if the manager manipulates earnings in both periods in the current revelation cycle.

on the upward manipulation of earnings.[17] The analysis below assumes that the condition for earnings management to occur is met so that the manager always overstates earnings when the earnings are low and the earnings management opportunity arises.

2.2 Asset prices

Now, this contract model is embedded into a dynamic model of asset pricing. It is assumed that the earnings process is persistent: the true earnings at time t, y_t, depend on y_{t-1} in addition to the manager's current effort. In particular, under the high effort by the manager (which is always the case in the equilibrium I consider), I assume that the true earnings follow a Markov process with transition probability $\pi_{yy'}$, where y is the earnings at time $t-1$ and y' is the earnings at time t. The asset price is determined as the present value of dividends, which are reported earnings net of the compensation and financial losses associated with

[17]This paper has a central focus on misreporting on upside. The reason is that overstatement of earnings is more widespread than understatement in the data and more problematic in general. Empirical work on SEC enforcement actions aimed at violations of Generally Accepted Accounting Principles suggests that over-reporting is the more frequent source of firm-wide financial misrepresentation (Feroz et al. [1991]). The average amount of restated earnings is hugely negative, and over 75% of restating firms restated their earnings downwards, indicating a strong drive to appear more productive than they actually are. Burgstahler and Dichev [1997] also estimate that 8-12 percent of the firms with small pre-managed earnings decreases manipulate earnings to achieve earnings increases, and 30-44 percent of the firms with small pre-managed losses manage earnings to create positive earnings. As long as the asymmetry between overstatement and understatement remains, in other words, the magnitude, frequency and consequences of overstatement are not exactly identical to those of understatement, the model results and intuition hold.

earnings management. Figure 4 chronicles the timeline of the model. It describes the timing of the events in two consecutive periods t and $t+1$, and this two-period revelation cycle repeats over time. Because the model is stationary, all the relevant past information is summarized in the previously revealed earnings and current reported earnings.

In the first period of the two-period revelation cycle (hereafter, period 1), the price of the firm $q_1(y_{t-1})$ is determined based on the revelation of the previous period's earnings y_{t-1}. Having the manager's reporting incentive in mind, the investors form their expectations about future dividend income based on the revelation of the firm's previous earnings y_{t-1}. In the second period of each cycle (hereafter, period 2), given the earnings report in the first period r_t and the true outcome in the ending period of the last cycle y_{t-1}, the firm is priced as $q_2(y_{t-1}, r_t)$. After the manager reports the earnings and pays them out entirely to the investors, the investigation takes place. When the investigation is conducted, the true realization of earnings in each period of the cycle is revealed, and the investors bear financial costs associated with any misstatement of earnings that occurs during the cycle. If the report is inflated in one of the two periods, the investors incur an amount of financial losses F_1. If earnings management occurs in both periods, the investors must pay an amount of monetary costs F_2, where $F_2 \geq 2F_1$.

I assume that the investors have linear utility and maximize the sum of the expected dividends. Then the value of the firm can be formulated as follows. In the beginning of an revelation cycle, given the revelation of the true outcome in the end of the last cycle y_{t-1}, the price of the firm $q_1(y_{t-1})$ is given by the expected sum of the net dividends and asset price in the next period (the time subscript is dropped when the timing is clear):

$$\begin{aligned} q_1(h) =& \pi_{hh}[d_{\tilde{h}} + \beta q_2(h, \tilde{h})] + \pi_{hl} x [d_{\tilde{h}} + \beta q_2(h, \tilde{h})] \\ &+ \pi_{hl}(1-x)[d_{\tilde{l}} + \beta q_2(h, \tilde{l})], \end{aligned} \tag{6}$$

and

$$\begin{aligned} q_1(l) =& \pi_{lh}[d_{\tilde{h}} + \beta q_2(l, \tilde{h})] + \pi_{ll} x [d_{\tilde{h}} + \beta q_2(l, \tilde{h})] \\ &+ \pi_{ll}(1-x)[d_{\tilde{l}} + \beta q_2(l, \tilde{l})], \end{aligned} \tag{7}$$

where d_r is the net dividend income and β is the investors' discount factor. The net dividend income equals the reported earnings less the compensation, that is, $d_r = r - w(r)$, where $r \in \{\tilde{l}, \tilde{h}\}$.

Regardless of the revelation of y_{t-1} in period 1, the investors may encounter three possible states in period 2. The first term in (6) and (7) is the expected net dividend income if the manager sends an honest report of high earnings in the next period. The second term in (6) and (7) represents the case in which the actual realization of earnings is low, but the manager makes an overstatement of earnings. The third term in the prices is the case in which the manager truthfully reports low earnings.

Given the first-period report r_t and the previously revealed outcome y_{t-1}, the investors update their belief about the true state in period 1. If the first-period report is low, it is for certain an honest report. If the report sent by the manager is high, it may be an overstated report that leads to immediate penalties. The posterior belief of the first-period report being truthful is derived following Bayes' Rule. If the previously revealed outcome is high, the conditional probability of $y_t = h$, denoted by γ_1, is

$$\begin{aligned}
\gamma_1 &= \Pr(y_t = h | r_t = \tilde{h}, y_{t-1} = h) \\
&= \frac{\Pr(y_t = h, r_t = \tilde{h} | y_{t-1} = h)}{\Pr(r_t = \tilde{h} | y_{t-1} = h)} \\
&= \frac{\Pr(r_t = \tilde{h} | y_t = h, y_{t-1} = h) \Pr(y_t = h | y_{t-1} = h)}{\Pr(r_t = \tilde{h} | y_{t-1} = h)} \\
&= \frac{\pi_{hh}}{\pi_{hh} + \pi_{hl} x},
\end{aligned}$$

If the previously revealed outcome is low, the conditional probability of $y_t = h$, denoted by γ_2, is

$$\begin{aligned}
\gamma_2 &= \Pr(y_t = h | r_t = \tilde{h}, y_{t-1} = l) \\
&= \frac{\Pr(y_t = h, r_t = \tilde{h} | y_{t-1} = l)}{\Pr(r_t = \tilde{h} | y_{t-1} = l)} \\
&= \frac{\Pr(r_t = \tilde{h} | y_t = h, y_{t-1} = l) \Pr(y_t = h | y_{t-1} = l)}{\Pr(r_t = \tilde{h} | y_{t-1} = l)} \\
&= \frac{\pi_{lh}}{\pi_{lh} + \pi_{ll} x}.
\end{aligned}$$

The price of the firm $q_2(y_{t-1}, r_t)$ is determined using these posterior probabilities. There are two cases. First, if period 1's report is low, the investors know that the realization of earnings is low.

$$q_2(l,\tilde{l}) = q_2(h,\tilde{l}) =$$
$$\pi_{lh}\left[d_{\tilde{h}} + \beta q_1(h)\right] + \pi_{ll} x \left[d_{\tilde{h}} - F_1 + \beta q_1(l)\right] + \pi_{ll}(1-x)\left[d_{\tilde{h}} + \beta q_1(l)\right]. \tag{8}$$

Because actual earnings follow a Markov process, the most recent realization of earnings is the only useful information for predicting future earnings. The price in response to a low report (which implies a realization of low earnings) is thus independent of the previous revelation of earnings, equal to the expected payoff over three possible states in the next period. The first term in (8) is the expected net dividend income if the manager sends an honest report of high earnings in the current period. The second term in (8) represents the case in which the manager makes an overstatement of earnings that leads to immediate financial losses. The third term in prices is associated with the situation in which the manager truthfully reports low earnings.

If the report just sent by the manager in period 1 is high, the report may or may not be truthful. Prices are determined as follows:

$$q_2(h,\tilde{h}) = \tag{9}$$
$$\gamma_1\left\{\pi_{hh}\left[d_{\tilde{h}} + \beta q_1(h)\right] + \pi_{hl} x\left[d_{\tilde{h}} - F_1 + \beta q_1(l)\right] + \pi_{hl}(1-x)\left[d_{\tilde{l}} + \beta q_1(l)\right]\right\}$$
$$+ (1-\gamma_1)\left\{\pi_{lh}\left[d_{\tilde{h}} - F_1 + \beta q_1(h)\right] + \pi_{ll} x\left[d_{\tilde{h}} - F_2 + \beta q_1(l)\right] + \pi_{ll}(1-x)\left[d_{\tilde{l}} - F_1 + \beta q_1(l)\right]\right\},$$

$$q_2(l,\tilde{h}) = \tag{10}$$
$$\gamma_2\left\{\pi_{hh}\left[d_{\tilde{h}} + \beta q_1(h)\right] + \pi_{hl} x\left[d_{\tilde{h}} - F_1 + \beta q_1(l)\right] + \pi_{hl}(1-x)\left[d_{\tilde{l}} + \beta q_1(l)\right]\right\}$$
$$+ (1-\gamma_2)\left\{\pi_{lh}\left[d_{\tilde{h}} - F_1 + \beta q_1(h)\right] + \pi_{ll} x\left[d_{\tilde{h}} - F_2 + \beta q_1(l)\right] + \pi_{ll}(1-x)\left[d_{\tilde{l}} - F_1 + \beta q_1(l)\right]\right\}.$$

The first term in (9) and (10) corresponds to the case where the first-period report is honest. In this case, there are three possible situations in the next period. In particular, if the realization of the second-period earnings is low and the manager has an opportunity to inflate earnings, the manager will report high. An amount of monetary penalties F_1 will be charged and thus subtracted in the pricing equation. The second term in (9) and (10) represents the case in which the first-period report is false. There are again three possible states in the second period. The investors suffer from an amount of financial losses F_1 if the manager truthfully presents earnings in period 2 and an amount F_2 if the manager manipulates earnings in period 2.

The manager's overstatement of earnings enables the investors to enjoy a higher level of current period consumption than they would in the absence of earnings management; however, this practice also exposes the investors to the loss from earnings restatement risk, that is, the subsequent financial cost after the periodic investigations. The net dividends in period 1 equal the reported earnings net of the compensation, that is, $d_r = y - w(r)$, where $r \in \{\tilde{l}, \tilde{h}\}$.[18] If $\beta F_1 > (d_{\tilde{h}} - d_{\tilde{l}})$, the cost of financial misreporting overwhelms the benefit. Everything else constant, all the prices decrease as x rises. I restrict my attention to this case throughout this analysis.[19]

2.3 Comparative statics

The price differential between $q_1(h)$ and $q_1(l)$ measures how sensitive the firm's price $q_1(y_{t-1})$ is in response to the investigation results y_{t-1}. How does $q_1(h) - q_1(l)$ change as the opportunity of earnings management, x, changes? To examine this, let us first ignore that the wage of the manager actually changes with x. It can be shown that as long as the firm's stochastic production process is persistent, that is, $\pi_{hh} > \pi_{lh}$, the price becomes more responsive to investigation results as x increases. Under the condition that $\beta F_1 > (d_{\tilde{h}} - d_{\tilde{l}})$,

[18] If $\beta F_1 = (d_{\tilde{h}} - d_{\tilde{l}})$, the cost of earnings management is offset by its benefit exactly, and it is straightforward to determine that prices become independent of x. If $\beta F_1 < (d_{\tilde{h}} - d_{\tilde{l}})$, the benefit of earnings management overwhelms its cost, then earnings management is not only beneficial to the manager, but also to the investors. The prices increase with the frequency of earnings management.

[19] This is a better description of reality than the other two cases, as indicated in the calibration exercise. The Securities and Exchange Commission has collected over $10 billion penalties in fraud cases since 2002, and the amount of settlement fines has been growing over time. In addition, as a typical yet somewhat extreme example, the meltdown of Enron caused over 4,500 employees to lose their jobs and pension funds worth over $1 billion. The stock's value plummeted from $90 to below 50 cents, wiping out $60 billion of shareholders' assets. The loss of confidence in corporate financial reporting could also hurt business and investment opportunities. Furthermore, the reduced availability and higher cost of capital may as well cause firms to postpone capital spending plans and accelerate layoffs. Although the production inefficiency due to earnings restatements, including a declaration of bankruptcy and the lack of investment caused by reputation damage, is not specifically modeled in this framework, it is implicitly included in the monetary losses F_1 and F_2 that are incurred during the periodic investigations.

both $q_1(h)$ and $q_1(l)$ fall as x escalates. However, $q_1(l)$ diminishes faster than $q_1(h)$, because a low previous output implies that future outputs tend to be low as well, imposing greater exposure to earnings restatement risk.

The analysis above does not consider that wages and thus net dividend income change with x. However, the same qualitative result holds even if the change in the compensation is taken into account. The optimal contract in this environment is characterized by (4) and (5). It can be seen that the compensation for the report of high earnings is independent of x, and the compensation for low earnings reports decreases as x expands. Therefore, as x becomes greater, the net dividend income from a report of high earnings, that is, $d_{\tilde{h}} = \tilde{h} - w(\tilde{h})$, remains the same, whereas the net dividend from a low earnings report, $d_{\tilde{l}} = \tilde{l} - w(\tilde{l})$, increases, resulting in a smaller dividend differential between high and low reports. Assuming that the monetary costs F_1 and F_2 do not vary with x, as the financial gain from earnings management, represented by $d_{\tilde{h}} - d_{\tilde{l}}$, diminishes, earnings management becomes more financially costly to the investors. The prices thus drop more as x rises. The change in the compensation schedule in response to the change of x internalizes the financial gain from earnings management, and it reinforces the amplification of the price differential and hence the price volatility.

Keeping the revelation of previous earnings constant, the price wedge in response to different reports in the ending period of one cycle, as measured by $q_2(h, \tilde{h}) - q_2(h, \tilde{l})$, does not necessarily have a monotonic relationship with x. To see this in a relatively straightforward manner, let us first ignore the effect of x on the manager's wages. $q_2(h, \tilde{h})$ is decreasing in x because of two forces that reinforce each other. First, as x rises, it is more likely to have false reports in future. These falsified reports lead to the investors' financial losses. Second, it is also more likely that the previous report r_t is a false report, resulting in penalties waiting to be paid. Because $\tilde{l}$ in $q_2(h, \tilde{l})$ is surely an honest report, the second force is absent. However, we do not necessarily obtain a smaller gap between $q_2(h, \tilde{h})$ and $q_2(h, \tilde{l})$ as x increases. Because of the high persistence in the earnings process, the first force works stronger for $q_2(h, \tilde{l})$ than for $q_2(h, \tilde{h})$. The impact of changes in x on the price volatility remains ambiguous in this case.

There are additional effects to consider if we take into account the impact of x on compensation schedule. Recall that the compensation structure in this environment exhibits the property that as earnings management becomes more likely, the compensation wedge is magnified, leading to a smaller dividend differential. As earnings management becomes more costly to the investors, prices decline more when x increases. This response of the wage payment to changes in x strengthens the first mechanism that is at work for both $q_2(h, \tilde{h})$ and $q_2(h, \tilde{l})$ without affecting the other mechanism that works only for $q_2(h, \tilde{h})$. Although the net effect of x on the price volatility could spin either way in the second period of one cycle, incorporating the change in the compensation scheme generates higher price volatility than otherwise.

From this point forward in this paper, I will ignore the wage values in the price calculation, so as not to complicate the mechanism and conflate with the main argument. The channel that earnings management influences returns through wages should be quantitatively weak, because executive compensation, although sizable and growing, does not constitute a substantial fraction of firms' earnings.[20]

The asset return is calculated as the sum of the current period price and dividends divided by the previous period price and then subtracted by one. The return volatility in the model is measured as the average return volatility in each period. When earnings management becomes likely, restatement risk amplifies the movement of returns and thus raises return volatility. Analogously, in order to compare the conditional volatility difference in response to earnings revelations, I use the difference between the average return volatility following a revelation of high earnings and that following a revelation of low earnings. Earnings management risk increases the volatility difference, because low earnings generate financial incentives for the manager to overstate earnings while high earnings do not. In particular, a previous low output equalizes the distribution of current reports and hence raises uncertainty in period 1, and it also leads to a greater likelihood and amount of financial loss in period 2, magnifying the return volatility in both periods. Now that return volatility depends on the true state of the firm, given a persistent state evolution process, the volatility is also persistent.

[20]CEOs of public companies earn a mean of $600,000 this decade, which is about 0.5% of the firms' average earnings.

Table 1: Parameter values in the numerical example with binary earnings

Parameter	Description	Value
h	Level of high earnings	50
l	Level of low earnings	0
π_{hh}	Transition probability: $\Pr[y' = h \mid y = h]$	0.8
π_{ll}	Transition probability: $\Pr[y' = l \mid y = l]$	0.8
β	Discount factor	0.95
F_1	Monetary loss for one restatement	$1.2(h-l)/\beta$
F_2	Monetary loss for two restatements	$2F_1$

3 Results

In this section, I solve the model numerically and present the results from model simulations. Table 1 shows the parameter values in the numerical example.[21] The primary purpose in this section is to illustrate that earnings management can generate a number of stylized financial facts. The quantitative results will be presented in Section 5.[22]

[21]There can be asymmetry in volatility due to a denominator effect in discrete state models, which is quantitatively insignificant in the numerical example. As shown in the comparative static analysis and continuous case, the mechanism is *not* a result of the binary structure.

[22]It is worth noting that the asset pricing model is consistent with the contract model in the sense that it is optimal for the investors to implement high effort when designing executive compensation, although earnings management leads to monetary penalties imposed on the investors. Recall that in the contract model with two-earnings-level specification, the principal always wants to induce high effort. In the following analysis, wage values are assumed to be negligibly small relative to firms' earnings. In a standard principal-agent model without earnings management, high effort is desirable as long as high earnings are different enough from low earnings. With the possibility of earnings management and revelations, it is still beneficial for the principal to induce high effort if the value of high effort outweighs the possible monetary loss associated with earnings management. That is,

$$[p_H h + (1 - p_H)l] - [p_L h + (1 - p_L)l] > xF_1 \tag{11}$$

And recall that for earnings management to exert influence on stock returns, the discounted monetary penalties associated with earnings management must be different from the amount of overstatement, and this analysis focuses on the case that earnings management is costly to the investors. That is,

$$\beta F_1 > h - l \tag{12}$$

The numerical example used here satisfies both (11) and (12). The assumption that high effort is desirable for the principal remains valid, after taking into account the negative consequence of earnings management.

3.1 Volatility clustering and asymmetric volatility

For the illustrative purpose, I use $x = 0$ and $x = 0.1$ as an example to demonstrate the impact of earnings management throughout this section. The simulated return sequence from the model captures the stylized facts of conditional volatility: first, conditional volatility exhibits persistence; second, stock returns are negatively correlated with the volatility of subsequent returns.

The EGARCH (1,1) model of the return series is estimated using Maximum Likelihood method with 10,000 artificially generated observations. The EGARCH (1,1) model used is $\log \sigma_t^2 = K + G \log \sigma_{t-1}^2 + A[|\epsilon_{t-1}|/\sigma_{t-1} - E\{|\epsilon_{t-1}|/\sigma_{t-1}\}] + L[\epsilon_{t-1}/\sigma_{t-1}]$, where E is the expectation operator, ϵ_t is the innovation, and σ_t is the conditional variance of the innovation. The G term captures volatility clustering (that is, persistence of volatility). A positive value of the A term in the equation implies that a deviation of the standardized innovation from its expected value causes the variance to be larger than otherwise. The L coefficient allows this effect to be asymmetric.[23]

Table 2 presents the results. The upper panel presents the case without earnings management, that is, $x = 0$. In this case, there is no GARCH or ARCH effect present in the simulated return data. As x becomes positive, return volatility becomes serially correlated. Before estimation, the Lagrange Multiplier (LM) test is applied to the return data, and the LM test strongly rejects the i.i.d. residual hypothesis at the 95% confidence level. The coefficients of the EGARCH (1,1) model are all statistically significant beyond the 95% confidence level. In addition, the conditional variance process is strongly persistent (with G coefficient $= 0.60$). The negative value of the coefficient L shows evidence of asymmetry in the model return behavior — negative surprises increase volatility more than positive surprises.

The persistence and asymmetry in the conditional volatility of stock returns in the model are generated by earnings management incentive together with a persistent earnings process. When true earnings are revealed to be low, the persistence in the earnings-generating process

[23] If $L = 0$, then a positive surprise ($\epsilon_{t-1} > 0$) has the same effect on volatility as a negative surprise of the same magnitude. If $-1 < L < 0$, a positive surprise increases volatility less than a negative surprise. If $L < -1$, a positive surprise actually reduces volatility while a negative surprise increases volatility. For further reference, see Hamilton [1994, p. 668].

Table 2: EGARCH(1,1) estimation results (binary earnings)

x=0	Coefficient	Std.Error	T-statistic
K	-5.0000	0.4153	-12.0387
G	-0.0001	0.6829	0.0001
A	0.0000	0.0087	0.0000
L	0.0009	0.0092	0.1049

x=0.1	Coefficient	Std.Error	T-statistic
K	-1.8621	0.3136	-5.9380
G	0.5999	0.0663	9.0545
A	0.0407	0.0058	6.9856
L	-0.1125	0.0278	-4.0553

This table reports the estimates of the EGARCH coefficients in the binary example. Maximum likelihood is used to estimate the coefficients needed to fit the following EGARCH model to the model return series: $\log \sigma_t^2 = K + G \log \sigma_{t-1}^2 + A[|\epsilon_{t-1}|/\sigma_{t-1} - E\{|\epsilon_{t-1}|/\sigma_{t-1}\}] + L[\epsilon_{t-1}/\sigma_{t-1}]$, where ϵ_t is an innovation and σ_t is the conditional variance of the innovation. The model return is simulated for 10,000 periods. x is the probability that the manager is able to manipulate earnings in one period.

implies that earnings tend to stay low for a while, so earnings management is likely to occur in the current and future periods. A higher frequency of occurrence of earnings management increases future return volatility. If the previous earnings are revealed to be high, the current and future earnings are likely to remain high. Overstatement of earnings has little chance of occurring; thereby future returns are relatively stable in this case. As a result, the volatility of the return series is persistent, and returns are negatively correlated with the subsequent volatility.[24]

3.2 Return volatility

Table 3 presents the volatility of the simulated returns. Monetary losses that incur during revelations generate large swings in the return sequence and hence produce volatility. When earnings management and earnings restatements occur more frequently, returns become more volatile. Campbell et al. [2001] document that idiosyncratic stock return volatility increased considerably from 1962 to 1997 in the United States. Rajgopal and Venkatachalam [2008]

[24]The core intuition does not hinge upon the two-period time structure of information disclosure. The mechanism that drives EGARCH property stays in effect when the model incorporates additional periods and stochastic investigation.

Table 3: Volatility of the model returns (binary earnings)

x	Standard Deviation
0	0.0954
0.1	0.1015
0.2	0.1086

This table reports the standard deviation of returns in the numerical example with binary earnings. x is the probability that the manager is able to manipulate earnings in one period.

report a strong association between idiosyncratic return volatility and financial reporting quality, as measured by both earnings quality and forecast dispersion, in both cross-sectional and time-series regressions. In line with the empirical findings, as x increases in the model, implying that the informativeness of earnings reports becomes weakened, the returns exhibit greater volatility.

4 Extension to continuous earnings

In this section,the model is extended to the case with a continuum of earnings. This model is used for the quantitative analysis in the next section. In the continuous case, I assume that earnings follow an AR(1) process: $y' = \rho y + k + \epsilon$, where $\rho < 1$, k is a constant, and ϵ is a white noise process with zero mean and standard deviation σ.

4.1 Optimal contract

Analogous to the binary model elaborated above, a risk-neutral principal (investors) hires a risk-averse agent (manager) for one period. Expending high effort incurs a utility cost, that is, c, to the manager, whereas low effort involves no cost. The manager's effort decision and an exogenous state realization together determine the firm's economic earnings, which are privately observed by the manager. The conditional distributions of earnings given high and low effort follow normal distributions: $f(y|e = H) \sim N(\mu_H, \sigma_H)$ and $f(y|e = L) \sim N(\mu_L, \sigma_L)$, where $\mu_H > \mu_L$. After exerting effort, the manager privately learns whether an opportunity is available to inflate earnings in the manager's favor. With probability x, the manager has discretion to overstate earnings by a constant amount a, and a utility cost

$\phi(R(y) - y)$ is involved in such earnings manipulation. In particular, $\phi(a) = \psi > 0$. With probability $(1 - x)$, the applicable accounting rules are so hard-and-fast that the manager has no option but truthfully present earnings. The manager's outside option is $\bar{U}$.

The model is extended to the case with continuous earnings by characterizing the optimal wage function contingent on the earnings reports. The optimal wage schedule is numerically computed in Sun [2008], utilizing Simulated Annealing algorithm with Gauss Hermite quadrature. In the numerical implementation, it is always the case that under the optimal contract, there exists a threshold level of earnings y^*, above which the manager does not find it worthwhile to manipulate earnings and truth-telling strategy is thus maintained. Below this threshold, the manager achieves personal gains from manipulation and inflates earnings whenever possible. Thereafter, this paper focuses on this threshold-style of reporting behavior.

The intuition behind the existence of the threshold earnings that separates truthful reporting and earnings management is as follows. Given that the manager is risk averse, a wage function that is not too convex translates into a set of concave utility promises. As actual earnings expand, the manager faces a decreasing utility gain but a constant utility cost from overstating earnings. As a consequence, earnings management occurs when the realized earnings are relatively low, and a truthful reporting strategy is sustained if actual earnings are high.

4.2 Asset prices

The pricing formulation is extended to the continuous case as follows.[25] Based on the revelation of previous earnings, the price in period 1 is determined as the expected sum of the dividends and price in the next period:

$$\begin{aligned} q_1(y) = &\Pr[y' \geq y^*|y]E\left[(\rho y + k + \epsilon) + \beta q_2(y, \rho y + k + \epsilon)|y' \geq y^*\right] \\ &+ \Pr[y' < y^*|y]xE\left[(\rho y + k + \epsilon + a) + \beta q_2(y, \rho y + k + \epsilon + a)|y' < y^*\right] \\ &+ \Pr[y' < y^*|y](1 - x)E\left[(\rho y + k + \epsilon) + \beta q_2(y, \rho y + k + \epsilon)|y' < y^*\right]. \end{aligned} \tag{13}$$

[25] Again, the labor wage is assumed to be negligibly small compared with the firm's earnings, therefore compensation does not affect net dividends or asset prices.

The first term in the pricing function represents the case when the actual earnings in the next period exceed the threshold level of earnings that elicits the truth, and therefore the manager reports honestly. The second term in (13) is the case when the next period's actual earnings fall below the threshold earnings, and the manager has an opportunity to manage earnings. The manager in this case overstates earnings. In particular, the next period's report is $r = \rho y + k + \epsilon + a$. The third term in (13) represents the situation in which the next period's earnings are below the threshold earnings, but the manager does not have the earnings management opportunity. In this case, the manager has to truthfully represent the earnings.

The price in period 2 is a function of the previously revealed earnings and the earnings report in period 1.

$$q_2(y, r) = p\Omega + (1 - p)\tilde{\Omega},$$

where

$$\begin{aligned}\Omega \equiv & \Pr[y'' \geq y^*|y' = r]E\left[(\rho r + k + \epsilon) + \beta q_1(\rho r + k + \epsilon)|y'' \geq y^*\right] \\ & + \Pr[y'' < y^*|y' = r]xE\left[(\rho r + k + \epsilon + a) - F_1 + \beta q_1(\rho r + k + \epsilon)|y'' < y^*\right] \\ & + \Pr[y'' < y^*|y' = r]\,(1 - x)\,E\left[(\rho r + k + \epsilon) + \beta q_1(\rho r + k + \epsilon)|y'' < y^*\right],\end{aligned}$$

and

$$\begin{aligned}\tilde{\Omega} \equiv & \Pr[y'' \geq y^*|y' = r - a]E\left[(\rho(r - a) + k + \epsilon) - F_1 + \beta q_1\left(\rho(r - a) + k + \epsilon\right)|y'' \geq y^*\right] \\ & + \Pr[y'' < y^*|y' = r - a]xE\left[(\rho(r - a) + k + \epsilon + a) - F_2 + \beta q_1\left(\rho(r - a) + k + \epsilon\right)|y'' < y^*\right] \\ & + \Pr[y'' < y^*|y' = r - a](1 - x)E\left[(\rho(r - a) + k + \epsilon) - F_1 + \beta q_1\left(\rho(r - a) + k + \epsilon\right)|y'' < y^*\right].\end{aligned}$$

Here, Ω is the expected present value of the dividends when the first-period report is truthful, and $\tilde{\Omega}$ corresponds to the case where the first-period report is false. Similar to the pricing function in period 1, the first term in Ω and $\tilde{\Omega}$ represents the case when the second-period earnings are higher than the threshold earnings, and the reported earnings are truthful. In $\tilde{\Omega}$, F_1 is subtracted because investors must bear monetary penalties for the earnings management practice in period 1 of this revelation cycle. The second term in Ω and $\tilde{\Omega}$ represents the case when the actual earnings in period 2 are lower than the threshold earnings, and the manager has discretion to inflate earnings by a. In this case, the investors pay F_1 for the overstatement if the first-period report is honest (as in Ω) and F_2 if the first-period report is also falsified (as in $\tilde{\Omega}$). The third term is the case when the manager does not have sufficient discretion over reporting in period 2 and has to truthfully report the earnings that fall below the threshold earnings. In $\tilde{\Omega}$, the deduction of F_1 is due to the earnings overstatement by the manager in period 1.

The posterior belief of having an accurate report in period 1, that is, $p = \Pr[y' = r|y]$, is derived following Bayes' Rule,

$$p = \begin{cases} 1 & \text{if } r \in [y^* + a, \infty), \\ \dfrac{f(r-k-\rho y)}{f(r-k-\rho y) + xf(r-a-k-\rho y)} & \text{if } r \in (y^*, y^* + a), \\ \dfrac{(1-x)f(r-k-\rho y)}{(1-x)f(r-k-\rho y) + xf(r-a-k-\rho y)} & \text{if } r \in (-\infty, y^*]. \end{cases} \tag{14}$$

Note that the compensation contract endogenously determines the threshold level y^* that elicits the truth. As actual earnings follow an AR(1) process, the implied conditional distributions of earnings given effort change over time, leading to changes of compensation contracts and hence threshold levels. In the simulation of prices and returns, the endogeneity of y^* requires calculations of the optimal contract for each possible earnings distribution implied by previous earnings. Sun [2008] specifies the parameterization of the principal-agent model such that the threshold level equals the conditional mean of actual earnings given high effort. The following proposition states the conditions under which the wage schedule shifts in a parallel manner when the earnings distribution moves. More specifically, the optimal contract and the underlying earnings distribution move together in the same direction by an equal amount. Therefore, the threshold level is always equal to the mean of earnings given high effort, even when the mean level itself varies over time.[26]

Proposition 2 *Suppose that the values of the parameters* $(a, \psi, c, \bar{U}, \sigma_H, \sigma_L)$ *are fixed, and* $f(y|e = H)$ *and* $f(y|e = L)$ *shift in a parallel manner by* δ*, keeping* $(\mu_H - \mu_L)$ *fixed. Then*

[26] A possible alternative interpretation of the existence of threshold level outside the model is that executives strive to beat the consensus earnings forecast by financial analysts, and the best forecast is the conditional mean of earnings given the previous earnings reports.

a parallel shift of the wage function $w(r)$ by δ is a solution to the principal's problem, and therefore the threshold level y^* will shift by δ as well.

Proof: See Appendix.

Below, I restrict the attention to the parameterization specified in Sun [2008] and the conditions stated above. In the first period of each revelation cycle, the investors have perfect knowledge of the value of y^* given the revelation of previous earnings. In the second period, they form an expectation of actual earnings in period 1 based on the report in period 1 and the previously revealed earnings. The investors use this expectation to infer the current distribution of earnings for both compensation design purposes and firm valuation purposes.

The threshold level y^* can be derived as follows:

$$y^* = \begin{cases} \rho y + k & \text{in period 1,} \\ \rho\,[pr + (1-p)(r-a)] + k & \text{in period 2.} \end{cases}$$

For the baseline case without earnings management ($x = 0$), reported earnings are always truthful, and the pricing function can be derived analytically. In this case, there is no difference between the reporting period (that is, period 1 of each revelation cycle) and the revelation period (that is, period 2 of each revelation cycle). The pricing equations in each period thus coincide with each other, equal to the sum of discounted expected future earnings.

$$\begin{aligned} q(y) &= E\Big\{(\rho y + k + \epsilon) + \beta\,[\rho(\rho y + k + \epsilon) + k + \epsilon] + \beta^2\{\rho\,[\rho(\rho y + k + \epsilon) + k + \epsilon] + k + \epsilon\} + \cdots\Big\} \\ &= \lim_{n\to\infty} \frac{\rho\,[1-(\beta\rho)^n]}{(1-\beta\rho)} y + \lim_{n\to\infty} \sum_n \frac{\beta^{n-1}k}{(1-\rho)} - \lim_{n\to\infty} \sum_n \frac{\beta^{n-1}\rho^n k}{(1-\rho)} \\ &= \frac{\rho y}{(1-\beta\rho)} + \frac{k}{(1-\beta)(1-\beta\rho)}. \end{aligned} \tag{15}$$

Since actual earnings follow $y' = \rho y + k + \epsilon$, we can lag and substitute (15) into the earnings process to yield

$$q(y') = \rho q(y) + \frac{(1-\rho)k}{(1-\beta)(1-\beta\rho)} + \frac{\rho}{(1-\beta\rho)}(k+\epsilon).$$

The price follows an AR(1) process with the same autoregressive parameter as the earnings process but with different mean and variance.

Table 4: Parameter values in the numerical example with continuous earnings

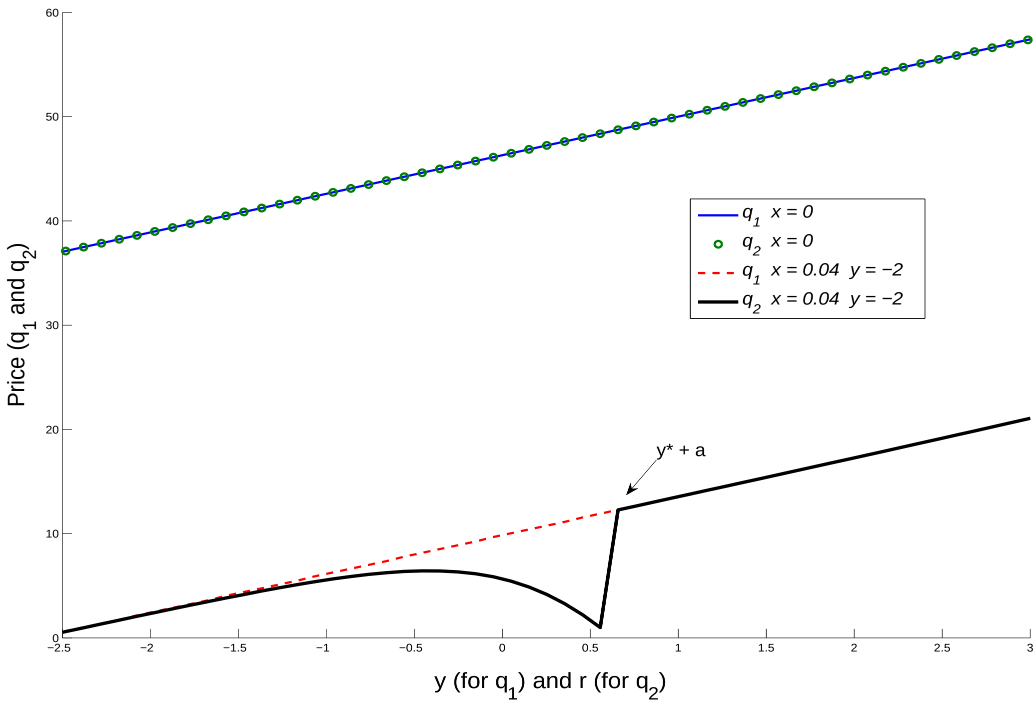
Parameter	Description	Value
ρ	Autoregressive parameter	0.77
k	Constant term	0.23
a	Amount of overstatement	2.1
β	Discount factor	0.98
F_1	Monetary loss for one restatement	31.8
F_2	Monetary loss for two restatements	$2F_1$

The system of integral equations that characterizes the asset prices with earnings management does not yield an analytical solution. Instead, the prices are computed using Monte Carlo integration. Here, a numerical example is presented to illustrate how earnings management affects asset prices. Table 4 shows the parameter values specified in the price computation. With a couple of exceptions, most of the parameter values are taken from the calibration implemented in the next section. For the purpose of illustration, I enlarge the value of x and F_1, compared with the value calibrated in the next section, to demonstrate the impact of earnings management on price dynamics.

Figure 5 shows how period 1's price varies with revealed previous earnings and how period 2's price varies with reported earnings, keeping previously revealed earnings fixed. The dotted line and the light line that overlap with each other represent the price of period 1 (as a function of y) and that of period 2 (as a function of r) in the baseline case. The dashed line is period 1's price (as a function of y) with earnings management, and the dark line is period 2's price (as a function of r) for a given level of previous earnings y. Compared to the baseline case, a positive value of x makes the prices in both periods lower for a given level of previous earnings and earnings report. The price is discounted to reflect future monetary losses because of a possibly manipulated report in the current period. The shift of prices is parallel (except for some deviation in period 2), because the possibility of having a false report in the current period is independent of y under the current assumptions.

With earnings management opportunity, the price of period 1 and that of period 2 differ only to reflect the additional information coming from the comparison between y and r. In period 2, the comparison between y and r reveals some information about the possibility

Figure 5: Pricing function with continuous earnings



This figure displays the pricing functions in the numerical example with continuous earnings, computed using Simulated Annealing. The horizontal axis is revealed previous earnings y for period 1's price q_1 and first-period earnings report r for period 2's price q_2. x is the probability that the manager is able to manipulate earnings in one period. y is the actual earnings in period 2 of the last revelation cycle.

that r is a false report, as shown in (14). Note that y^* is the conditional mean of the true earnings, which is a function of y. If r is very small, it is unlikely that the report has been inflated. If r is very large, it cannot be a manipulated report because there is no incentive to manage earnings when true earnings are greater than y^*. In particular, if $r > y^* + a$, the investors can infer (with probability 1) that r is a truthful report. In the medium range of r, the probability is large that r is a false report.

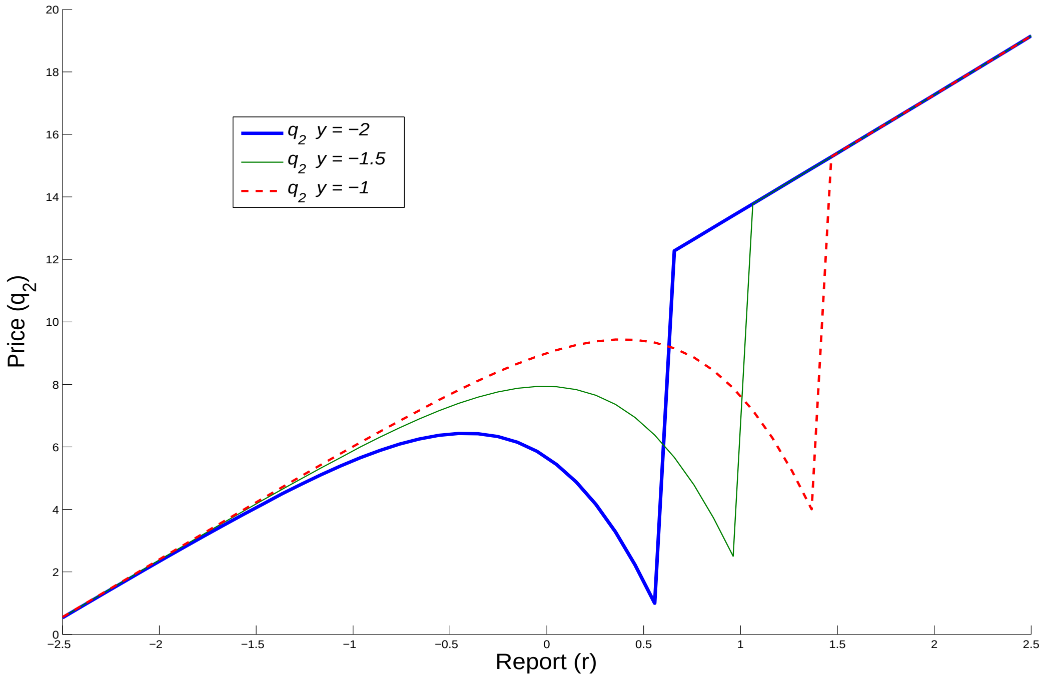
In the particular case with normal distributions of earnings, the following result holds.

Lemma 2 *if* $r \in (-\infty, y^*]$ *or* $r \in (y^*, y^* + a)$, p *is strictly decreasing in* y.

Proof: See Appendix.

In Figure 6, period 2's price is plotted as a function of reported earnings for different levels of previously revealed earnings y. The dark, light, and dashed line represent a relatively low,

Figure 6: Pricing function in period 2



This figure displays the period-2 pricing function $q_2(y, r)$ for different values of y. The prices are computed using Simulated Annealing. y is the actual earnings in period 2 of the last revelation cycle. r is the manager's report in period 1 of the current revelation cycle.

medium, and high level of previous earnings respectively. If the previously revealed earnings are higher, the threshold level that induces truthful reporting is thus higher. The sharp drop-off of prices occurs at a higher level of reports.

5 Quantitative results

In this section I describe how I calibrate the model. Because this model describes individual stock returns, the calibration strategy is to simulate realizations of productivity shocks and earnings management opportunities for a large number of individual firms, gather the return sequences together, and then set the parameter values so as to match the aggregate targets.

To capture fluctuations in stock market indices, the calibrated model incorporates aggregate uncertainty: an aggregate productivity shock. The production process that individual firms follow is thus specified as $y' = \rho y + \epsilon_a + \epsilon_i$, where $\epsilon_a \sim N(0, \sigma_a^2)$ and $\epsilon_i \sim N(0, \sigma_i^2)$.

Here, ϵ_a and ϵ_i represent aggregate productivity shock and idiosyncratic productivity shock respectively, and they are independent. Aggregate productivity shock is assumed to be observable to both managers and investors. In doing so, I maintain the focus on the asymmetric information between managers and investors regarding idiosyncratic performance, without causing additional inference problems.[27]

In the rest of this section, I first calibrate the model using Compustat industrial quarterly data after restatement corrections as actual earnings process, and investigate the statistical properties of returns generated from the model.[28] This case represents the benchmark calibration. Second, counterfactual experiments are conducted by considering different levels of earnings management prevalence to assess the impact of earnings management in financial markets.

5.1 Benchmark calibration

Table 5 contains the benchmark parameter values. The period length is set to be half a year. The annual periodicity of restatements is thus in accordance with the empirical finding that the average number of restated fiscal quarters is about four (Wu, 2002).[29] The discount factor β is chosen to be 0.98 so that the implied semiannual real interest rate is 2 percent.

The autoregressive parameter ρ, the constant drift k, and standard deviations of productivity shocks σ_a and σ_i are calibrated using Compustat data. I include all available observations on the quarterly industrial Compustat database from Q1 1971 to Q4 2006 to study firms' earnings. Compustat quarterly files provide data on a restated basis. When a

[27] If aggregate productivity shock is unobservable to investors, earnings reports from all the firms in the economy convey information regarding the aggregate state of the economy. In pricing individual firms, investors should utilize earnings reports from all the firms to filter out aggregate shock and then make inference about individual outcomes. As earnings management is considered as a phenomenon arising from asymmetric information about idiosyncratic performance, the possible information asymmetry regarding aggregate economy is beyond the scope of this paper.

[28] There is a possibility that earnings management may be more prevalent than earnings restatements, and there can be a potential discrepancy between restated earnings and true earnings. However, the model in this paper is designed to examine the impact of earnings management behavior that leads to SEC enforcement actions or earnings restatements. Thus, earnings management practice that goes unnoticed over the firm's entire life cycle is outside the scope of the model. I will also consider matching the moments of unrestated earnings later in the next section to check the robustness of model properties.

[29] Wu (2002) analyzes 932 earnings restatements from Jan 1997 through Dec 2001. The restated period varies from one quarter to eight years, with an average of 4.2 quarters in the sample.

Table 5: Benchmark parameterization

Parameter	Description	Value
β	Discount factor	0.98
ρ	Autoregressive parameter	0.77
k	Constant term	0.23
σ_a	Std.Dev of aggregate productivity shock	0.07
σ_i	Std.Dev of idiosyncratic productivity shock	0.11
x	Earnings management prevalence	0.04
a	Amount of overstatement	0.07
F_1	Monetary loss for one restatement	1.06
F_2	Monetary loss for two restatements	2.12

company reports for a new quarter and at the same time reports different data than originally reported for the corresponding quarter of the prior year, that data for the corresponding quarter of the prior year is changed and said to be restated.[30] In this benchmark calibration, the net income process from Compustat is taken as actual earnings process.

In the results reported here, I use the sum of net income over both quarters (Compustat quarterly data item #69) to study firms' earnings. The results are also computed using earnings before extraordinary items (Compustat quarterly data item #8), and the results are generally consistent for these two alternative measures of earnings. The earnings data are drawn from a broad spectrum of firm sizes, and are therefore scaled following the approach in the literature. The earnings variable is scaled by beginning-of-the-period market value of common equity, computed as the close price in the end of the previous period multiplied by the number of common shares outstanding (i.e., [one-period-lagged Compustat quarterly data item #14] $\times$ [Compustat quarterly data item #61]). Following the convention, I also winsorize the data at 1 percent extreme values from each tail to reduce the impact of outliers and data errors.

The descriptive statistics of semiannual earnings in the sample are presented in Table 6. I normalize the steady-state level of actual earnings to be one, that is, $\bar{y} = \dfrac{k}{1-\rho} = 1$. The value of ρ is chosen to match with the average autocorrelation of firms' earnings, which is the third entry in Table 6. This gives $\rho = 0.77$, and $k = 1 - \rho = 0.23$. The standard deviation of

[30] These restatements can be due to mergers, acquisitions, discontinued operations, and accounting changes.

Table 6: Moments of semi-annual scaled earnings

	Mean	Std.Dev	Autocorr	Std.Dev of avg. earnings
Scaled earnings	0.06	0.21	0.77	0.12

This table reports the descriptive statistics of semi-annual earnings calculated as net income scaled by beginning-of-the-period market value of common equity. The sample period spans from 1971 to 2006. The data is winsorized at 1 percent extreme values from each tail. The first entry is the mean, the second entry is the total standard deviation, the third entry is the autocorrelation of the pooled sample, and the last entry is the standard deviation of the average earnings across firms.

aggregate productivity shock σ_a is set to be 0.07 to match with the time variation of average earnings across firms, shown in the fourth column in Table 6. As aggregate productivity shock and idiosyncratic productivity shock are independent of one another, given the variance of aggregate productivity shock, the standard deviation of idiosyncratic productivity shock is calculated to be $\sigma_i = 0.11$.

The parameter x is calibrated to be 0.04, yielding an overall earnings restatement rate 2 percent. This feature is in line with the average frequency of restatement announcements among publicly traded companies over the period of Jan 1997 to Sep 2005 (GAO, 2002 and GAO, 2006).[31] Wu (2002) documents that the average amount of restated earnings in her sample is $-\$9.8$ million, while the average number of restated quarters is 4.2.[32] As the model is calibrated on a semiannual basis, I choose the amount of overstatement to be half of \$9.8 million in each period, that is, \$4.9 million. After scaled by average market value of listed companies and then normalized by average scaled earnings, a is 0.07.

To measure the monetary loss that the investors incur in the event of earnings restatements in the model, the current paper focuses on the average immediate market-adjusted loss in market capitalization of restating companies, that is, \$75.5 million for each restatement

[31] To identify and collect financial statements, GAO (2002, 2006) use Lexis-Nexis, an online periodical database, to conduct an intensive keyword search using variations of the word “restate.” They include only announced restatements that were being made to correct previous material misstatements of financial results, while exclude announcements involving stock splits, changes in accounting principles, and other financial statement restatements that were not made to correct mistakes in the application of accounting standards.

[32] Wu (2002) analyzes 932 earnings restatements from Jan 1997 through Dec 2001. The raw restated earnings magnitude runs from \$1.1 billion downward to \$470 million upward.

Table 7: Comparison of data volatility (benchmark calibration)

	Standard Deviation
Model	0.0714
Data	0.3789

This table reports the the average standard deviation of returns in the model with benchmark calibration and that in the CRSP data files from 1931 to 2007.

announcement (GAO, 2002 and GAO, 2006).[33] I choose the three-trading-day window to focus regarding the market response to the exclusion of other factors. This measure provides a lower bound for the financial losses the investors suffer from restatements, and the associated result serves as a lower bound for evaluating the importance of earnings management in financial markets. The scaled and normalized measure for the financial loss associated with each restatement is $F_1 = 1.06$. F_2 is then set to be 2.12.

5.2 Results

I report the simulation results on the parsimoniously parameterized model using the benchmark calibration for 500 firms and compare the statistical properties with S&P 500 index returns data. To get compound semiannual returns, I obtain S&P 500 quarterly returns from CRSP quarterly files from Jan 1931 to Dec 2007.[34]

Table 7 shows that relative to S&P 500 Index data, the volatility of the model-generated data is moderately lower. Table 8 compares EGARCH estimation results from the model returns and S&P 500 Index returns. The coefficients of the EGARCH (1,1) model are all statistically significant beyond the 95% confidence level. Consistent with the data, the conditional variance process is strongly persistent, although the magnitude of G coefficient is not as much as the data show. Since the coefficient L has a negative value, the model displays asymmetric volatility — negative surprises increase volatility more than positive

[33]To determine the immediate impact on stock prices, GAO (2002) analyzes 689 earnings restatements that were announced from January 1997 to March 2002. GAO (2006) examines 1061 restatement announcements from July 2002 to September 2005. For each of these cases, they examine the company's stock price on the trading days before, of, and after the announcement date to assess the immediate impact and calculate the change in market capitalization. I take an average of the immediate market-adjusted loss in market capitalization in the two samples.

[34]I consider a longer-period sample for stock returns than company earnings, excluding the 1929 stock market crash. The longer time span is chosen due to the semiannual frequency of the model.

Table 8: Comparison of EGARCH(1,1) estimation results (benchmark calibration)

Model data	Coefficient	Std.Error	T-statistic
K	-5.0000	2.5890	-1.9312
G	0.5260	0.2454	2.1436
A	0.0529	0.0235	2.2474
L	-0.0234	0.0139	-1.6784

S&P 500 data	Coefficient	Std.Error	T-statistic
K	-1.2262	0.5087	-2.4105
G	0.7034	0.1239	5.6773
A	0.4469	0.1806	2.4742
L	-0.1801	0.1030	-1.7479

This table reports the estimates of the EGARCH coefficients in the benchmark calibration. Maximum likelihood is used to estimate the coefficients needed to fit the following EGARCH model to the model return series: $\log \sigma_t^2 = K + G \log \sigma_{t-1}^2 + A[|\epsilon_{t-1}|/\sigma_{t-1} - E\{|\epsilon_{t-1}|/\sigma_{t-1}\}] + L[\epsilon_{t-1}/\sigma_{t-1}]$, where ϵ_t is an innovation and σ_t is the conditional variance of the innovation. The model return is simulated for 10,000 periods. The S&P 500 index returns are compounded using CRSP quarterly files from 1931 to 2007.

surprises.

The intuition for the EGARCH effect in the binary example with two levels of earnings can be extended to the current model with a continuum of earnings. The general unifying story is that earnings management goes hand-in-hand with weak performance, because the financial incentive to artificially inflate earnings is strong when the earnings realization is poor. Relatively low earnings lead to more frequent future restatements than high earnings, generating greater movements in the return data. The return volatility becomes state-dependent, and the state (actual earnings) is persistent. Return volatility is thus persistent and asymmetric. In addition to this direct impact, an indirect effect due to *suspicion* of earnings management amplifies the persistence and asymmetry in return volatility. As shown in Figure 6, the possibility of earnings management creates a region of reports at the lower end that cause active learning and intensive suspicion of misstatement. Investors lower the price in anticipation of restatements. The uncertainty regarding the firm's fundamental value and subsequent outcomes is increased in this case, and some of the earnings reports under suspicion are associated with subsequent restatements and market fluctuations. Because the reported numbers tend to persist, the volatility also persists and exhibits asymmetry.

Although the model is consistent with volatility clustering and asymmetric volatility in the data, the magnitude is somewhat smaller. The A coefficient and L coefficient in S&P 500 Index returns are an order of magnitude greater than can be reproduced in the model. In light of the difficulties in measuring monetary losses in the event of earnings restatements, the discrepancy is not as large as it appears. For example, GAO (2002) and GAO (2006) show that restatement announcements have a negative effect on stock prices beyond their immediate impact. They find persistent market capitalization declines for restating companies. After controlling for the movement in the overall market, they report an average of $79.3 million loss in market value from 20 trading days before through 20 trading days after a restatement announcement (the intermediate impact) and an average of $136.1 million loss in market value from 60 trading days before through 60 trading days after the announcement (the longer-term impact). In addition, the use of market capitalization loss as a proxy for monetary loss that the investors incur precludes other potentially important factors.[35] The effects of such errors would be to bias the financial loss downwards, a correction of which would result in the model moving closer to the data. Measurement errors in the frequency of earnings management would have a similar effect on dynamic return patterns. Another plausible explanation for the discrepancy between model prediction and observational data is the oversimplicity of the model. Thus, although the overall fit of the model is good, it is not surprising, given the level of abstraction, that there are elements of the fine structure of returns the model is not designed to capture.

5.3 Counterfactual experiment

GAO (2002) and GAO (2006) document a significant upward trend in the number of restatements over time. To gain insight on policy-related issues, it is of interest to examine how the magnitude of financial anomalies varies with the extent of earnings management. Here, I consider the economies with different levels of earnings management prevalence. Specifically, I consider various values of x to assess the importance of earnings management. In these

[35]For example, the loss of confidence in the corporate financial reporting could also hurt business and investment opportunities. Furthermore, the reduced availability and higher cost of capital may as well cause firms to postpone capital spending plans and accelerate layoffs. How to accurately measure the efficiency loss associated with earnings management is a question that warrants further research.

Table 9: EGARCH(1,1) estimation results with different levels of x

x=0	Coefficient	Std.Error	T-statistic
K	-4.9882	0.6962	-7.1651
G	0.0028	0.7555	0.0037
A	0.0116	0.0279	0.4133
L	0.0009	0.0007	1.2500

x=0.04	Coefficient	Std.Error	T-statistic
K	-5.0000	2.5890	-1.9312
G	0.5260	0.2454	2.1436
A	0.0529	0.0235	2.2474
L	-0.0234	0.0139	-1.6784

x=0.1	Coefficient	Std.Error	T-statistic
K	-2.9453	1.6996	-1.7330
G	0.6786	0.1855	3.6589
A	0.0353	0.0203	1.7393
L	-0.0255	0.0129	-1.9729

This table reports the estimates of the EGARCH coefficients for the model with different values of x. The other parameters are recalibrated to match the same targets. Maximum likelihood is used to estimate the coefficients needed to fit the following EGARCH model to the model return series: $\log \sigma_t^2 = K + G \log \sigma_{t-1}^2 + A[|\epsilon_{t-1}|/\sigma_{t-1} - E\{|\epsilon_{t-1}|/\sigma_{t-1}\}] + L[\epsilon_{t-1}/\sigma_{t-1}]$, where ϵ_t is an innovation and σ_t is the conditional variance of the innovation. The model return is simulated for 10,000 periods. x is the probability that the manager is able to manipulate earnings in one period.

economies with different values of x, the other parameters are chosen to match the same aggregate targets as in the benchmark calibration.

Table 9 presents the results. The extreme case of $x = 0$ in this model, shown in the first panel, corresponds to the standard Lucas asset-pricing model. In this case, earnings management does not exist. The estimated EGARCH coefficients are substantially reduced and insignificant. No long-memory persistence or asymmetric behavior is present in the model data.

As x is increased to 0.04 as in the calibrated model, the EGARCH estimation results on the simulated return data demonstrate the presence of strong persistence and asymmetry in volatility. When $x = 0.1$, G and L coefficients become larger in magnitude and more significant. These are strong indications that incorporating earnings management intensifies

Table 10: Volatility of the model returns with different level of x

x	Standard Deviation
0	0.0424
0.04	0.0714
0.1	0.1044

This table reports the standard deviation of the calibrated model returns for different x. x is the probability that the manager is able to manipulate earnings in one period.

both persistence and asymmetry in return volatility.

Table 10 contains the standard deviation of returns in the simulated data. Consistent with the empirical studies mentioned in Section 1 and Section 3.2, as x increases (implying that the informativeness of earnings reports becomes weakened), the returns exhibit greater volatility. Monetary penalties charged upon restatement announcements generate large swings in the return sequence, and hence raise volatility.

Models such as the one considered in this paper can be used to predict the consequence of a particular corporate governance rule on financial reporting. The comparison of the financial returns dynamics with different prevalence of earnings management underscores why earnings management is of central importance in pricing of financial assets, in understanding the risk implied by empirical anomalies, and in the current debate about advantages of strict implementation of corporate governance policy, such as the Sarbanes-Oxley Act.

6 Robustness check

In this section, robustness check of the baseline model is conducted, both in terms of quantitative evaluations and model specifications. First, following an alternative calibration strategy, I recalibrate the model to Compustat Unrestated data, and study the return patterns. Second, I consider a setting in which investigations are conducted stochastically, and check whether model dynamics are robust to a stochastic feature of revelations.

6.1 Alternative calibration

Of particular interest is the sensitivity of the quantitative results to the specification of restated data as true earnings. An alternative to the benchmark calibration strategy is

Table 11: Moments of semi-annual scaled reports

	Mean	Std.Dev	Autocorr	Std.Dev of avg.	Avg. of Std.Dev
Scaled reports	0.10	0.22	0.82	0.03	0.15

The table reports the descriptive statistics of reported earnings in the Compustat Unrestated dataset from 1987 to 2006. The data is winsorized at 1 percent extreme values from each tail. The net income is scaled by beginning-of-the-period market value of common equity. The first entry is the mean, the second entry is the total standard deviation, the third entry is the autocorrelation of the pooled sample, the fourth entry is the standard deviation of the average reports across firms, and the last entry is the average standard deviation of the reports within the firms.

Table 12: Alternative parameterization

Parameter	Description	Value
ρ	Autoregressive parameter	0.82
k	Constant term	0.18
σ_a	Std.Dev of aggregate productivity shock	0.02
σ_i	Std.Dev of idiosyncratic productivity shock	0.08
β	Discount factor	0.98
x	Earnings management prevalence	0.04
a	Amount of overstatement	0.03
F_1	Monetary loss for one restatement	0.49
F_2	Monetary loss for two restatements	0.98

to take unrestated data and match them with the reported earnings generated from the model. In contrast to the conventional Compustat quarterly dataset that contains restated statements, Compustat Unrestated dataset covers the initial 10Q filing for a quarter that may be subject to SEC filings and earnings restatements in subsequent quarters. Here, I recalibrate the model using the Compustat Unrestated dataset.

The Compustat Unrestated dataset starts in 1987 for U.S. companies, covering a shorter time span than the Compustat restated dataset. Table 11 presents the moments of semi-annual reported earnings scaled by beginning-of-the-period market value. Here, ρ, σ_a, and σ_i are calibrated to match the average autocorrelation of firms' earnings, time variation of average reports across firms, and average time variation of reports within firms, shown in the third, fourth, and fifth entry respectively in Table 11. This gives $\rho = 0.82, \sigma_a = 0.02$, and $\sigma_i = 0.08$. As the steady-state report is normalized to 1, k is then set to be 0.18.

Table 13: Comparison of EGARCH(1,1) estimation results (alternative calibration)

Model data	Coefficient	Std.Error	T-statistic
K	-2.7503	1.4893	-1.8467
G	0.8049	0.1056	7.6225
A	0.0339	0.0166	2.0492
L	-0.0231	0.0106	-2.1911

S&P 500 data	Coefficient	Std.Error	T-statistic
K	-1.2262	0.5087	-2.4105
G	0.7034	0.1239	5.6773
A	0.4469	0.1806	2.4742
L	-0.1801	0.1030	-1.7479

This table reports the estimates of the EGARCH coefficients in the alternative calibration. Maximum likelihood is used to estimate the coefficients needed to fit the following EGARCH model to the model return series: $\log\sigma_t^2 = K + G\log\sigma_{t-1}^2 + A[|\epsilon_{t-1}|/\sigma_{t-1} - E\{|\epsilon_{t-1}|/\sigma_{t-1}\}] + L[\epsilon_{t-1}/\sigma_{t-1}]$, where ϵ_t is an innovation and σ_t is the conditional variance of the innovation. The model return is simulated for 10,000 periods. The S&P 500 index returns are compounded using CRSP quarterly files from 1931 to 2007.

The rest of the parameters are chosen to match the same targets as in the benchmark calibration, and that gives $\beta = 0.98, x = 0.04, a = 0.03, F_1 = 0.49$, and $F_2 = 0.98$, as presented in Table 12. Some values are different from the benchmark calibration because of the normalization of reported earnings to unity, compared with the normalization of restated earnings to unity.

Table 13 contains measures of EGARCH effect for the model returns and S&P 500 Index returns. The results are similar to those with the benchmark parameterization, except that the G coefficient somewhat overshoots. The stronger persistence in volatility than in the benchmark calibration is attributable to the higher persistence in firms' earnings. This result confirms that most of the volatility clustering in the model has to come from the persistent component in earnings management, which directly stems from the persistent component in earnings. This element of the model is crucial in making it consistent with the observed heteroskedasticity. The finding that EGARCH effect is quite similar for different calibration strategies suggests that, even though the parameters may differ across economies, the nature of return dynamics can still be quite similar.

Table 14 compares the volatility of the model and the data. Compared with the bench-

Table 14: Comparison of data volatility (alternative calibration)

	Standard Deviation
Model	0.0300
Data	0.3789

This table reports the the average standard deviation of returns in the model with benchmark calibration and that in the CRSP data files from 1931 to 2007.

mark parameterization, the model volatility is reduced. The reason is that the value of monetary loss associated with earnings management is calibrated to be lower (in particular, less than half in size), leading to a more moderate reaction of asset returns to restatement announcements. A smaller fluctuation of the returns during restatements produces lower volatility.

6.2 Stochastic investigation

In the baseline model, the periodic investigation is conducted deterministically every two periods. To examine how this assumption affects the results, here I consider a setting where investigations take place stochastically. As in Section 2 and Section 3, there are two levels of earnings: $y \in \{l, h\}$. Actual earnings follow a Markov process

$$\Pr(y_{t+1} = j | y_t = i) = \pi_{ij}, \qquad \forall i \in \{l, h\}, \quad \forall j \in \{l, h\}.$$

The investigation regarding financial reporting is now assumed to be stochastic, and occurs with probability λ every period. If the investigation takes place, all the previous earnings since the most recent investigation are revealed. The financial statements in the corresponding periods when earnings management occurs have to be restated, and the investors bear monetary penalties. More specifically, the amount of financial charges upon restatement announcements is a strictly increasing function of the number of periods in which the manager manipulates earnings. The timeline of the model events in each period is described in Figure 7.

Note that the derivation of the posterior probability of having a false report at each point in time requires utilizing the entire history of reports since the most recent investigation up

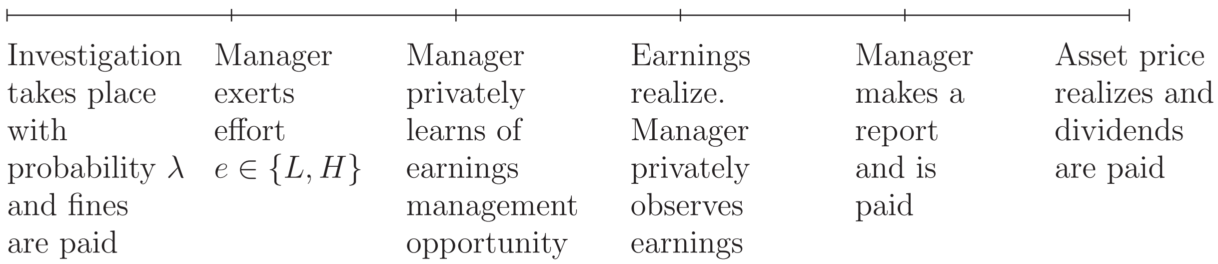


Figure 7: Model timeline with stochastic investigation

to the current report. In particular, when the manager makes an earnings announcement every period, the investors not only infer the current realization and predict future earnings, but also revise their expectation on each previous report in history.

Fortunately, in this setting all the relevant information in the reporting history can be summarized with a small set of state variables. In what follows, the problem is reduced to a variational problem in which history dependence can be summarized and asset price can be characterized by the following five state variables.[36]

- γ: the conditional probability (with the information from the current report) that the current true earnings are high;
- Z: the expected number of periods involving earnings management since the last investigation until the most recent low report ($Z = 0$ if there is no low report since the last investigation until the previous period);
- N: the number of consecutive high reports until the previous period since the last low report or the last investigation, whichever is more recent;
- r: the current earnings report, $r \in \{\tilde{l}, \tilde{h}\}$;
- $\bar{y}$: the true earnings before the series of consecutive N high reports starts.

Given the earnings management incentive in this binary setting, the current true earnings are revealed under two circumstances. The first is when the investigation regarding financial reporting takes place. In this case, the entire history of earnings realizations is revealed.

[36]For detailed examples of what each state variable represents, see Appendix B.

The second is when the manager sends a low report. If the reported earnings are low, although the credibility of financial statements in prior periods remains ambiguous, the current earnings are low with certainty. In the following, I derive the pricing functions that describe a stationary solution to the problem using these state variables. The stock price at time t is denoted by $q_t = P(\gamma_t, Z_t, N_t, r_t, \bar{y}_t)$.

Let the monetary penalties charged for earnings management be a linear function of the number of restating periods upon investigation. Specifically, the fines $F = \kappa n$, where κ is a constant and n is the number of periods involving earnings management since the most recent investigation. As the investors update their beliefs in the standard Bayesian fashion, γ' evolves following Bayes' Rule:

$$\gamma' = \begin{cases} \dfrac{\gamma\pi_{hh} + (1-\gamma)\pi_{lh}}{\gamma\pi_{hh} + (1-\gamma)\pi_{lh} + \gamma(1-\pi_{hh})x + (1-\gamma)(1-\pi_{lh})x}, & r = \tilde{h} \text{ at } t+1, \\ 0, & r = \tilde{l} \text{ at } t+1. \end{cases}$$

First, the price associated with a high report, $P(\gamma, Z, N, \tilde{h}, \bar{y})$, is derived.[37]

$$P(\gamma, Z, N, \tilde{h}, \bar{y}) = \tilde{h} + \beta\left[(1-\lambda)W_n^{\tilde{h}} + \lambda W_i^{\tilde{h}}\right]. \tag{16}$$

Here, β is the discount factor. $W_n^{\tilde{h}}$ represents the expected price if the investigation does not occur in the beginning of the next period, and $W_i^{\tilde{h}}$ represents the expected price if the investigation occurs. Both prices are conditional on a current high report.

If the investigation does not take place in the beginning of the next period, the expected price is

$$W_n^{\tilde{h}} = \mu P(\gamma', Z, N+1, \tilde{h}, \bar{y}) + (1-\mu)P(0, Z, N+1, \tilde{l}, \bar{y}). \tag{17}$$

The first term in (17) is the expected price if the next report is high. The second term is the expected price when the report in the next period is low. Note that a low report is always truthful, and thus γ is updated to 0. μ denotes the conditional probability that the manager makes a high report in the next period:

$$\mu = \gamma\pi_{hh} + \gamma(1-\pi_{hh})x + (1-\gamma)\pi_{lh} + (1-\gamma)(1-\pi_{lh})x$$

[37]Again, the impact of wage values in price calculations is not considered in the current analysis.

If the investigation takes place in the next period, the expected price is

$$\begin{aligned}W_i^{\tilde{h}} =& -\kappa[Z + f(N+1;\bar{y}))] \\ &+ \gamma\left[\xi_1 P\left(\frac{\pi_{hh}}{\xi_1}, 0, 0, \tilde{h}, h\right) + (1-\xi_1)P(0,0,0,\tilde{l},h)\right] \\ &+ (1-\gamma)\left[\xi_2 P\left(\frac{\pi_{lh}}{\xi_2}, 0, 0, \tilde{h}, l\right) + (1-\xi_2)P(0,0,0,\tilde{l},l)\right]. \end{aligned} \tag{18}$$

where ξ_1 represents the conditional probability of having a high report in the next period, given the current true earnings are high. ξ_2 is the probability of having a high report conditional on that the current true earnings are low.

$$\begin{aligned}\xi_1 &= \pi_{hh} + (1-\pi_{hh})x \\ \xi_2 &= \pi_{lh} + (1-\pi_{lh})x\end{aligned}$$

The first term in (18) is the expected amount of financial penalties for earnings management. $f(N+1;\bar{y})$ denotes the expected number of falsified reports among the $(N+1)$ consecutive reports of high earnings since the last low report or the last investigation, whichever is more recent. The function $f(N+1;\bar{y})$ is calculated from the model fundamental in a recursive manner, and the method is illustrated in Appendix C. The number of the expected restating periods is thus the sum of $f(N+1;\bar{y})$ and the expected number of periods involving earnings management from the last investigation through the most recent low report, Z. Recall that γ is the conditional probability that the current high report is truthful. The second term in (18) thus represents the expected price if the current high report is truthful. The third term is the case in which the current earnings are low and have been overstated.

Now let us consider the asset price if the current report is low.

$$P(0, Z, N, \tilde{l}, \bar{y}) = \tilde{l} + \beta\left[(1-\lambda)W_n^{\tilde{l}} + \lambda W_i^{\tilde{l}}\right]. \tag{19}$$

where $W_n^{\tilde{l}}$ and $W_i^{\tilde{l}}$ represent the expected price if the investigation does not occur in the next period and the expected price if the investigation occurs, respectively, conditional on a current low report.

If the investigation does not take place in the next period, the expected price is

$$W_n^{\tilde{l}} = \xi P\left(\frac{\pi_{lh}}{\xi}, Z, 0, \tilde{h}, l\right) + (1-\xi)P(0, Z, 0, \tilde{l}, l)$$

Parameter	Value
h	20
l	10
π_{hh}	0.8
π_{ll}	0.8
β	0.98
κ	15
λ	0.5

Table 15: Parameter values in the numerical example with binary earnings

where ξ denotes the conditional probability that the manager makes a high report in the next period:

$$\xi = \pi_{lh} + (1 - \pi_{lh})x$$

If the investigation takes place in the next period:

$$\begin{aligned} W_i^{\tilde{l}} = & -\kappa[Z + f(N; \bar{y})] \\ & + \xi P\left(\frac{\pi_{lh}}{\xi}, 0, 0, \tilde{h}, l\right) \\ & + (1 - \xi)P\left(0, 0, 0, \tilde{l}, l\right) \end{aligned} \tag{20}$$

The first term in (20) is the expected monetary charges for earnings management, which is a linear function of the expected number of restating periods. The second term is the expected price if the realization of actual earnings is high in the next period, and the third term corresponds to the case in which the realization is low. Thus, from (16) and (19), the price in each period can be solved recursively.

Table 15 contains the parameter values. The pricing functions are computed numerically. Figure 8 displays $f(N, \bar{y})$, the shape of which may vary with parameterizations. Figure 9 and Figure 10 show how the prices associated with a high report change with γ and N. As the monetary penalties associated with earnings management is a linear function of the number of restated financial statements, the price in response to a high report is linearly increasing in γ and linearly decreasing in Z. As shown in Figure 11, the price in response to a low report is also linearly decreasing in both Z, with γ updated to 0.

The model is simulated for 10,000 periods. in a numerical example. In order to illustrate

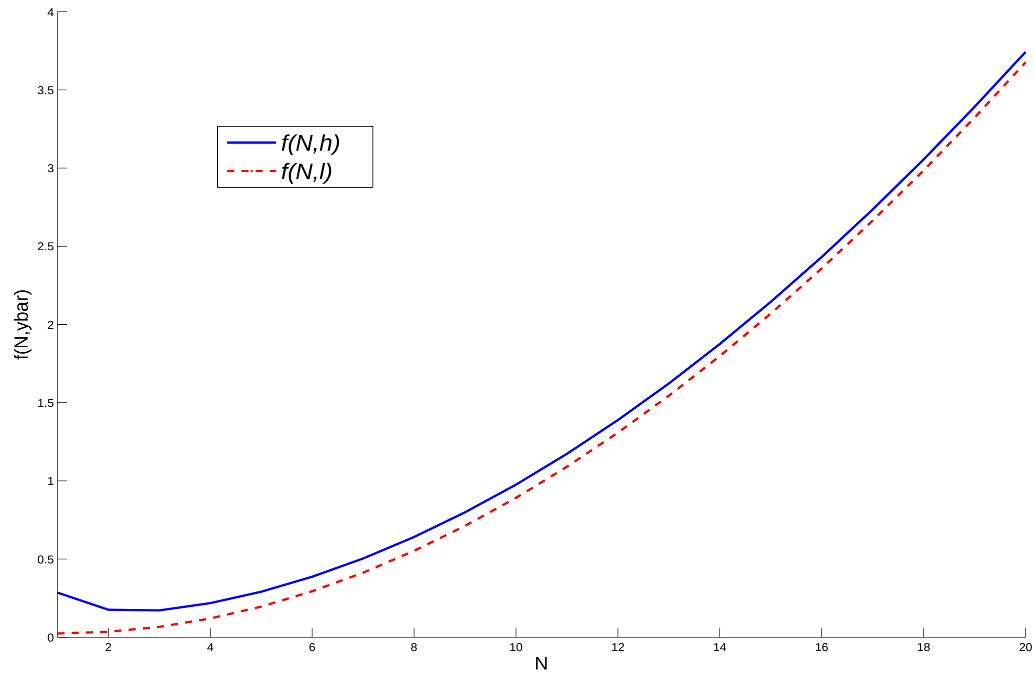


Figure 8: The expected number of inflated reports among N consecutive high reports $f(N, \bar{y})$

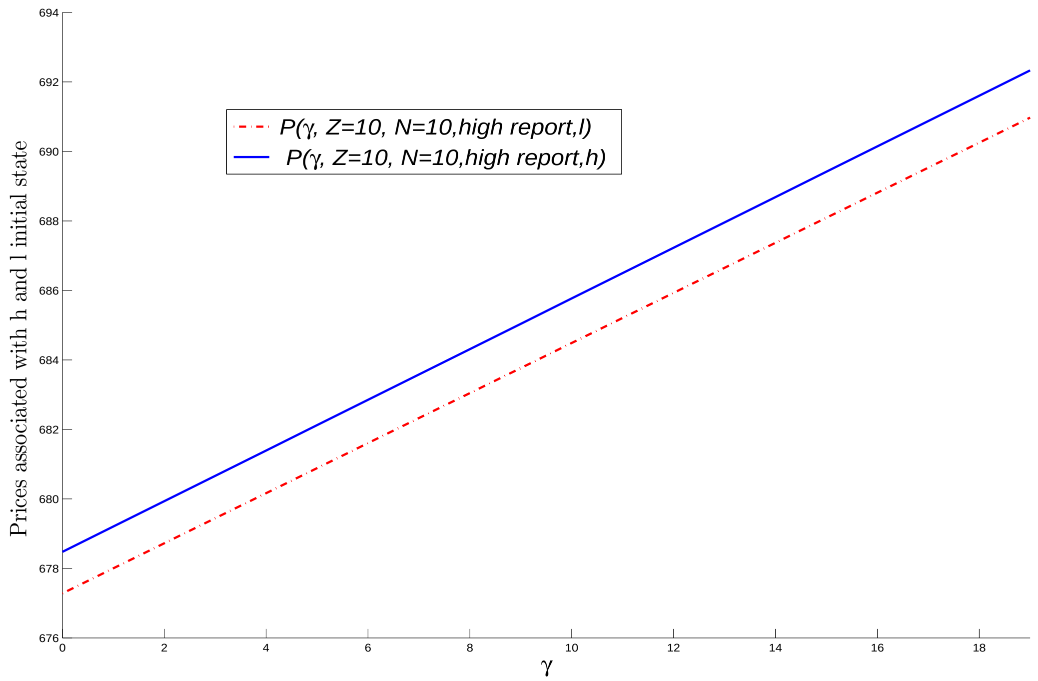


Figure 9: Price for a high report as a function of γ

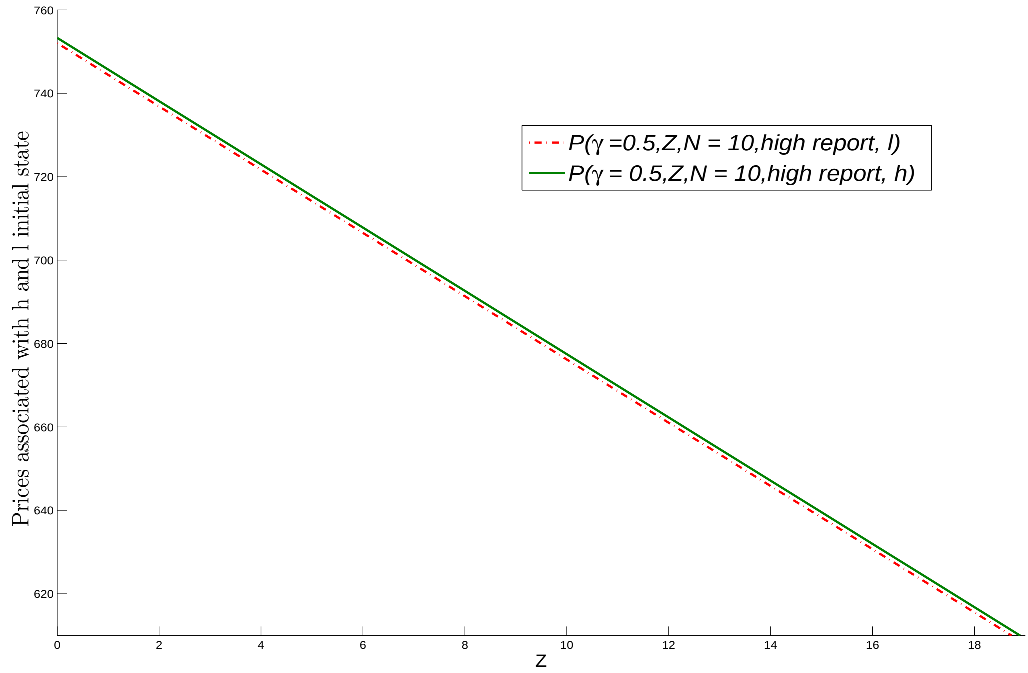


Figure 10: Price for a high report as a function of Z

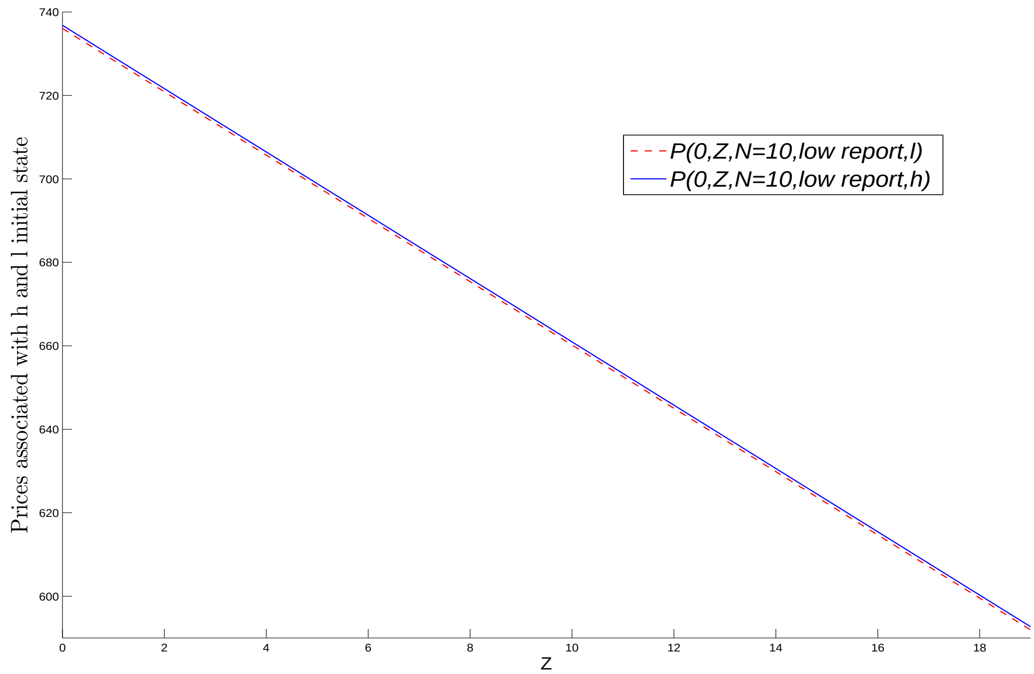


Figure 11: Price for a low report as a function of Z

x=0	Coefficient	Std.Error	t-statistic
K	-5.0000	12.8300	-0.3897
G	0.0576	0.0880	0.6552
A	0.0033	0.0119	0.2838
L	0.0041	0.0066	0.6195

x=0.1	Coefficient	Std.Error	t-statistic
K	-2.0291	0.2979	-6.8092
G	0.7441	0.0376	19.7951
A	0.1068	0.0207	5.1616
L	-0.0841	0.0197	-4.2789

Table 16: EGARCH(1,1) estimation results

Variance equation: $\log \sigma_t^2 = K + G \log \sigma_{t-1}^2 + A[|\epsilon_{t-1}|/\sigma_{t-1} - E\{|\epsilon_{t-1}|/\sigma_{t-1}\}] + L[\epsilon_{t-1}/\sigma_{t-1}]$

x	Standard Deviation
0	0.0134
0.1	0.0193
0.2	0.0201

Table 17: Volatility of the model returns

the influence of earnings management incentive on dynamic return patterns, I compare the model returns with $x = 0$ and those with $x = 0.1$. Table 16 presents the EGARCH estimation results on the model returns. In a model without earnings management ($x = 0$), there is no persistence in return volatility (shown in the upper panel). As earnings management becomes possible, the coefficients of the EGARCH model are all statistically significant. Persistence and asymmetry are present in the model return volatility. In addition, Table 17 shows that the model returns become more volatile as x increases. The same set of results and intuition from the model with deterministic monitoring carry through.

This model of stochastic investigation assumes a constant exogenous probability of monitoring in every period. However, with a positive monitoring cost, it is natural to argue that monitoring would occur with a higher probability in bad times, since there tends to be little interest in investigating when the market is booming. Accounting fraud does come in waves, and is detected more intensively during market collapses. As monitoring occurs more often when the aggregate state of the economy is bad and earnings management is more prevalent, the asymmetric behavior in stock returns tends to be more pronounced. An monitoring probability that varies with the aggregate economic prospects would strengthen the mechanism illustrated in this paper, and intensify these observed features of asset returns.

7 Conclusion

This paper examines dynamic asset return patterns in an economy in which information about underlying profitabilities is obscured. An important ingredient in the current formulation of the asset-pricing problem is that executives intentionally manipulate financial information in their own best interests. Executives possess two dimensions of private information: realizations of actual earnings and realizations of earnings management opportunity. Because different combinations of these two could generate identical earnings reports, there is no strict monotonicity and hence no invertibility of the reporting function. Although the investors are fully rational, and they learn in a standard Bayesian fashion, they cannot perfectly filter out the manipulation component in the reports. Therefore, earnings management causes a pricing distortion — honest firms are undervalued, while firms that manipulate their accounting numbers are overpriced.

This study shows that an asset-pricing model with earnings management delivers the observed features of asset return data: volatility clustering, asymmetric conditional volatility, and excess individual volatility. To the best of my knowledge, such features are not replicated by one representative-agent model without introducing complex preference structures. Formal modeling of the implication of endogenously determined earnings management behavior for dynamic return patterns is rather limited. The goal is to point out that incorporating corporate misconduct in an otherwise standard asset-pricing model can mimic a number of stylized financial facts, and earnings management may play a crucial role in price formulations in financial markets.

The quantitative analysis indicates that, in addition to generating patterns in their own stocks, earnings management by individual firms may also create the observed patterns in stock market indices regardless of the covariance effects of aggregation of reporting decisions across firms. Importantly, these effects are symptoms of inefficiency and risk, and they are likely to be more pronounced during episodes of weak economic performance when the financial incentive to inflate earnings is particularly strong. The mechanism illustrated in this paper also presents a likely source of non-fundamental volatility and financial risk.

The current model of shareholders-manager behavior in a financial market is a simplified one. In particular, the current analysis does not explicitly model how the manager finances the discrepancy in the reports. As elaborated in section 2, leaving the source of funds outside the model is for the purpose of simplification without causing a modeling inconsistency. This formulation can also be viewed as a simple way of illustrating the idea that the manager can divert resources from profitable investment to current payout. Formulating this idea explicitly requires a production economy with investment, and I take the current framework as the first step towards the ultimate goal. A full understanding of the welfare consequences is a task for future research.

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Appendix

A Proofs

Proof of Proposition 2:

After the parallel shift of $f(y|e=H)$ and $f(y|e=L)$ by δ, the conditional distribution of actual earnings given effort is denoted by $g(y|e=i) = f(y-\delta|e=i)$, $\forall i \in \{L, H\}$. The principal has a utility function given by $V(y-w)$.

The Lagrangian for the principal's problem in this case is

$$\mathcal{L} = \int_{\underline{y}+\delta}^{\bar{y}+\delta} \Big\{ V\left[y - w(r(y))\right] g(y|e=H) + \lambda \left[u(w)g(y|e=H) - \bar{U}\right] \\ + \mu \left[u(w)g(y|e=H) - u(w)g(y|e=L) - c\right] \Big\} dy$$

The reporting function $r(y)$ is given by

$$r(y) = \begin{cases} y + a & \text{if } u\left[w(y+a)\right] - u\left[w(y)\right] > \psi \text{, and earnings management opportunity realizes} \\ y & \text{otherwise.} \end{cases}$$

Differentiating with respect to $w(r)$ inside the integral sign, we obtain the first-order condition. Assuming that it is optimal to elicit high effort, an optimal incentive compensation scheme $w(r)$ satisfies

$$\frac{V'[y - w(r)]}{u'[w(r)]} = \lambda + \mu \left[1 - \frac{g\left(y(r)|e=L\right)}{g\left(y(r)|e=H\right)}\right], \tag{21}$$

Assume that the principal is risk-neutral, and the manager's utility function takes the logarithm form given by $u(w) = \log(w)$. (21) simplifies to

$$\begin{aligned} w(r) &= \lambda + \mu \left[1 - \frac{g\left(y(r)|e=L\right)}{g\left(y(r)|e=H\right)}\right] \\ &= \lambda + \mu \left[1 - \frac{f\left(y(r) - \delta|e=L\right)}{f\left(y(r) - \delta|e=H\right)}\right] \end{aligned} \tag{22}$$

The solutions also satisfy the complementary slackness conditions

$$\lambda \int_{\underline{y}+\delta}^{\bar{y}+\delta} \left\{u(w)g(y|e=H) - \bar{U}\right\} dy = 0,$$

$$\mu \int_{\underline{y}+\delta}^{\bar{y}+\delta} \{u(w)g(y|e = H) - u(w)g(y|e = L) - c\}\, dy = 0.$$

which can be rewritten as

$$\lambda \int_{\underline{y}+\delta}^{\bar{y}+\delta} \left\{u(w)f(y - \delta|e = H) - \bar{U}\right\} dy = 0, \tag{23}$$

$$\mu \int_{\underline{y}+\delta}^{\bar{y}+\delta} \{u(w)f(y - \delta|e = H) - u(w)f(y - \delta|e = L) - c\}\, dy = 0. \tag{24}$$

The following inequalities should also be satisfied

$$\lambda \geq 0, \quad \mu \geq 0. \tag{25}$$

Let $w^*(r)$ be the solution to the principal's problem before the parallel shift of $f(y|e = H)$ and $f(y|e = L)$. λ^* and μ^* are the corresponding Lagrangian multipliers. Then $w^*(r)$, λ^*, and μ^* satisfy the first-order condition

$$w^*(r) = \lambda^* + \mu^* \left[1 - \frac{f\,(y(r)|e = L)}{f\,(y(r)|e = H)}\right]$$

together with the complementary slackness conditions

$$\lambda^* \int_{\underline{y}}^{\bar{y}} \left\{u(w)f(y|e = H) - \bar{U}\right\} dy = 0,$$

$$\mu^* \int_{\underline{y}}^{\bar{y}} \{u(w)f(y|e = H) - u(w)f(y|e = L) - c\}\, dy = 0.$$

and the inequalities

$$\lambda^* \geq 0, \quad \mu^* \geq 0.$$

It follows that

$$w^*(r - \delta) = \lambda^* + \mu^* \left[1 - \frac{f\,(y(r) - \delta|e = L)}{f\,(y(r) - \delta|e = H)}\right]$$

$$\lambda^* \int_{\underline{y}+\delta}^{\bar{y}+\delta} \left\{u\left[w^*(r - \delta)\right] f(y - \delta|e = H) - \bar{U}\right\} dy = 0,$$

$$\mu^* \int_{\underline{y}+\delta}^{\bar{y}+\delta} \{u\left[w^*(r - \delta)\right] f(y - \delta|e = H) - u\left[w^*(r - \delta)\right] f(y - \delta|e = L) - c\}\, dy = 0.$$

$$\lambda^* \geq 0, \quad \mu^* \geq 0.$$

It is straightforward to determine that $w(r) = w^*(r - \delta)$, $\lambda = \lambda^*$, and $\mu = \mu^*$ satisfy (22), (23), (24), and (25). The reporting choice $r(y)$ remains unchanged in this case. Therefore, a parallel shift of the wage function by δ solves the principal's problem. □

The idea underlying this analysis is that given a parallel shift of conditional distributions of output, a parallel shift of the wage payment schedule by the same amount provides the same incentive to the manager and same marginal value of effort to the risk-neutral principal. First, because the distribution of wage payment remains unchanged after parallel shifts of the wage function and output distribution by a same amount, the manager does not have an incentive to deviate from the recommended effort and reporting choice.

Second, the risk-neutral principal designs the compensation based on the monetary value of high effort relative to low effort, which is the difference in the residuals. The residual is the expected earnings net of compensation payment, conditional on high and low effort. The monetary value of effort can be denoted by $\Big[$(expected earnings given high effort - expected payment given high effort) $-$ (expected earnings given high effort $-$ expected payment given low effort)$\Big]$. It can be rewritten as $\Big[$(expected earnings given high effort $-$ expected earnings given low effort) $-$ (expected payment given high effort $-$ expected payment given low effort)$\Big]$. As long as $(\mu_H - \mu_L)$ and the wage distribution remain constant, the principal does not have any incentives to change the shape of incentive schemes.

A parallel shift of the wage schedule by an equal amount as the shift of output distributions provides the manager with the same incentive and the principal with the same value, and therefore is an optimal contract in this case.

Proof of Lemma 4:

If $r \in (y^*, y^* + a)$,

$$\begin{aligned} p &= \Pr[y' = r|y] \\ &= \frac{f(r-k-\rho y)}{f(r-k-\rho y) + xf(r-a-k-\rho y)} \\ &= \frac{1}{1 + x\left[\frac{f(r-a-k-\rho y)}{f(r-k-\rho y)}\right]} \\ &= \frac{1}{1 + x\exp\left[\frac{1}{2\sigma}(r-k-\rho y)^2 - \frac{1}{2\sigma}(r-a-k-\rho y)^2\right]} \\ &= \frac{1}{1 + x\exp\left[\frac{a}{2\sigma}(2r-2k-2\rho y-a)\right]} \end{aligned}$$

Using the same property of normal distributions, it is straightforward to check that p is decreasing in r when $r < y^*$.

$$p = \frac{1}{1 + (1-x)\exp\left[\frac{a}{2\sigma}(2r-2k-2\rho y-a)\right]}.$$

□

B Examples of state variables in the model with stochastic investigation

As the monetary penalties upon investigation depends on the number of restated financial statements, the expected number of periods in which the manager inflates earnings since the most recent realization up to now is necessary in characterizing the prices. If there are N consecutive high reports and no low reports after the most recent investigation, a function of $f(N;\bar{y})$ determines the expected number of periods involving earnings management until the last period. If there is any low report after the last investigation, the sum of Z and $f(N;\bar{y})$ summarizes the history. In addition, γ and r incorporate the information regarding the current true state conveyed by the current report.

To be clear on what each variable represents, a set of clarifying examples is provided in the following. Now let today be $t = 10$ and let the last investigation happen at the beginning of $t = 5$. Suppose that the true state of $t = 4$ is revealed to be y_4.

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{l}, \tilde{h}, \tilde{h}\}$, then, at $t = 10$, Z is the expected number of inflated reports during periods 5, 6, and 7; $N = 1$ (it does not include the current period); and $r = \tilde{h}$. $\bar{y} = l$, because the true state in period 8 is known to be low (recall that all the low reports are honest reports).

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}\}$, then, at $t = 10$, $Z = 0$ (there is not any low report after the last investigation until the previous period); $N = 5$ (it does not include the current period); and $r = \tilde{h}$. $\bar{y} = y_4$, because it is the known true state before the consecutive high reports.

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{l}\}$, then, at $t = 10$, $Z = 0$ (there is not any low report after the last investigation until the previous period); $N = 5$; and $r = \tilde{l}$. $\bar{y} = y_4$, because it is the known true state before the consecutive high reports. Note that $\gamma = 0$ at $t = 10$, because the current low report is an honest one.

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{l}, \tilde{h}, \tilde{l}, \tilde{h}\}$, then, at $t = 10$, Z is the expected number of inflated reports during periods 5, 6, and 8; $N = 0$ (it does not include the current period); and $r = \tilde{h}$. $\bar{y} = l$, because the true state in period 9 is known to be low (all the low reports are honest reports). Note that in the case of $N = 0$, $\bar{y}$ is set to be y_{t-1} ($N = 0$ occurs only when the report at $(t-1)$ is low or the investigation happens at the beginning of t).

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{l}, \tilde{h}\}$, then, at $t = 10$, Z is the expected number of inflated reports during periods 5, 6, 7, and 8; $N = 0$; and $r = \tilde{h}$. $\bar{y} = l$, because the true state in period 9 is known to be low (Again, all the low reports are honest reports).

Let today be $t = 5$ and let the investigation happen at the beginning of $t = 5$.

- If $r_5 = \tilde{h}$, then $Z = 0$, $N = 0$, $r = \tilde{h}$, and $\bar{y} = y_4$.

- If $r_5 = \tilde{l}$, then $Z = 0$, $N = 0$, $r = \tilde{l}$, and $\bar{y} = y_4$.

C Calculation of $f(N;\bar{y})$ in the model with stochastic investigation

Let the information set $\mathcal{R}_N^{\bar{y}} \equiv \{\bar{y}, r_1 = \tilde{h}, r_2 = \tilde{h}, \cdots, r_N = \tilde{h}\}$. y_n represents the true earnings in period n, $\forall n \in \{1, 2, \cdots, N\}$. Thus $f(N;\bar{y})$ can be written as

$$\begin{aligned} f(N;\bar{y}) =& \Pr[y_1 = l|\mathcal{R}_N^{\bar{y}}] + \Pr[y_2 = l|\mathcal{R}_N^{\bar{y}}] + \cdots \\ &+ \Pr[y_n = l|\mathcal{R}_N^{\bar{y}}] + \cdots + \Pr[y_N = l|\mathcal{R}_N^{\bar{y}}] \end{aligned}$$

The problem of deriving $f(N;\bar{y})$ in a recursive way is transformed into an equivalent problem, that is, to recursively derive

$$\Pr[y_n = l|\mathcal{R}_N^{\bar{y}}] = 1 - \Pr[y_n = h|\mathcal{R}_N^{\bar{y}}], \qquad \forall n \in \{1, 2, \cdots, N\}.$$

Note that

$$\begin{aligned} \mathcal{R}_N^h &\equiv \{h, r_1 = \tilde{h}, r_2 = \tilde{h}, \cdots, r_N = \tilde{h}\} \\ \mathcal{R}_N^l &\equiv \{l, r_1 = \tilde{h}, r_2 = \tilde{h}, \cdots, r_N = \tilde{h}\} \end{aligned}$$

The proof includes two steps. In step 1, $\Pr[y_1 = h|\mathcal{R}_1^l]$ and $\Pr[y_1 = h|\mathcal{R}_1^h]$ are calculated. In step 2, I show that $\Pr[y_n = h|\mathcal{R}_{N+1}^l]$ and $\Pr[y_n = h|\mathcal{R}_{N+1}^h]$, $\forall n \in \{1, 2, \cdots, N+1\}$, can be calculated using $\Pr[y_n = h|\mathcal{R}_N^l]$ and $\Pr[y_n = h|\mathcal{R}_N^h]$, $\forall n \in \{1, 2, \cdots, N\}$.

As the first step, $\Pr[y_1 = h|\mathcal{R}_1^l]$ and $\Pr[y_1 = h|\mathcal{R}_1^h]$ are derived as follows.

$$\begin{aligned} \Pr[y_1 = h|\mathcal{R}_1^l] &= \Pr[y_1 = h|\bar{y} = l, r_1 = \tilde{h}] \\ &= \frac{\Pr[y_1 = h, r_1 = \tilde{h}|\bar{y} = l]}{\Pr[r_1 = \tilde{h}|\bar{y} = l]} \\ &= \frac{\pi_{lh}}{\pi_{lh} + (1 - \pi_{lh})x}, \\ \Pr[y_1 = h|\mathcal{R}_1^h] &= \Pr[y_1 = h|\bar{y} = h, r_1 = \tilde{h}] \\ &= \frac{\Pr[y_1 = h, r_1 = \tilde{h}|\bar{y} = h]}{\Pr[r_1 = \tilde{h}|\bar{y} = h]} \\ &= \frac{\pi_{hh}}{\pi_{hh} + (1 - \pi_{hh})x}. \end{aligned}$$

In step 2, I first show that $\Pr[y_n = h|\mathcal{R}^l_{N+1}]$ can be calculated if $\Pr[y_n = h|\mathcal{R}^l_N]$ is known. For $n \in \{1, 2, \cdots, N+1\}$,

$$\Pr[y_n = h|\mathcal{R}^l_N, r_{N+1} = \tilde{h}] = \frac{\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}^l_N]}{\Pr[r_{N+1} = \tilde{h}|\mathcal{R}^l_N]}. \tag{26}$$

The denominator in (26), $\Pr[r_{N+1} = \tilde{h}|\mathcal{R}^l_N]$, is derived as the following.

$$\begin{aligned}\Pr[r_{N+1} = \tilde{h}|\mathcal{R}^l_N] =& \Pr[r_{N+1} = \tilde{h}, y_{N+1} = h|\mathcal{R}^l_N] + \Pr[r_{N+1} = \tilde{h}, y_{N+1} = l|\mathcal{R}^l_N] \\ =& \Pr[r_{N+1} = \tilde{h}|y_{N+1} = h, \mathcal{R}^l_N] \times \Pr[y_{N+1} = h|\mathcal{R}^l_N] \\ &+ \Pr[r_{N+1} = \tilde{h}|y_{N+1} = l, \mathcal{R}^l_N] \times \Pr[y_{N+1} = l|\mathcal{R}^l_N] \\ =& \Pr[y_{N+1} = h|\mathcal{R}^l_N] + x\left[1 - \Pr[y_{N+1} = h|\mathcal{R}^l_N]\right],\end{aligned}$$

where

$$\begin{aligned}\Pr[y_{N+1} = h|\mathcal{R}^l_N] =& \Pr[y_{N+1} = h, y_N = h|\mathcal{R}^l_N] + \Pr[y_{N+1} = h, y_N = l|\mathcal{R}^l_N] \\ =& \Pr[y_{N+1} = h|y_N = h, \mathcal{R}^l_N] \times \Pr[y_N = h|\mathcal{R}^l_N] \\ &+ \Pr[y_{N+1} = h|y_N = l, \mathcal{R}^l_N] \times \Pr[y_N = l|\mathcal{R}^l_N] \\ =& \pi_{hh} \Pr[y_N = h|\mathcal{R}^l_N] + \pi_{lh}\left[1 - \Pr[y_N = h|\mathcal{R}^l_N]\right].\end{aligned} \tag{27}$$

As $\Pr[y_N = h|\mathcal{R}^l_N]$ is known from the supposition, this can be calculated. The denominator is obtained

$$\begin{aligned}\Pr[r_{N+1} = \tilde{h}|\mathcal{R}^l_N] =& \pi_{hh} \Pr[y_N = h|\mathcal{R}^l_N] + \pi_{lh}\left[1 - \Pr[y_N = h|\mathcal{R}^l_N]\right] \\ &+ x\{1 - \pi_{hh} \Pr[y_N = h|\mathcal{R}^l_N] - \pi_{lh}\left[1 - \Pr[y_N = h|\mathcal{R}^l_N]\right]\}.\end{aligned} \tag{28}$$

Now let us consider the numerator in (26). For $n = N+1$, $\Pr[y_{N+1} = h, r_{N+1} = \tilde{h}|\mathcal{R}^l_N]$ can be rewritten as

$$\begin{aligned}\Pr[y_{N+1} = h, r_{N+1} = \tilde{h}|\mathcal{R}^l_N] &= \Pr[r_{N+1} = \tilde{h}|y_{N+1} = h, \mathcal{R}^l_N] \times \Pr[y_{N+1} = h|\mathcal{R}^l_N] \\ &= \Pr[y_{N+1} = h|\mathcal{R}^l_N],\end{aligned}$$

where $\Pr[y_{N+1} = h|\mathcal{R}^l_N]$ is derived in (27).

For $n \in \{1, 2, \cdots, N\}$, the numerator $\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}^l_N]$ can be rewritten as

$$\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}^l_N] = \Pr[r_{N+1} = \tilde{h}|y_n = h, \mathcal{R}^l_N] \times \Pr[y_n = h|\mathcal{R}^l_N].$$

Here, $\Pr[y_n = h|\mathcal{R}_N^l]$ is known from the supposition. Now we only need to check if $\Pr[r_{N+1} = \tilde{h}|y_n = h, \mathcal{R}_N^l]$ can be calculated. I rewrite

$$\Pr[r_{N+1} = \tilde{h}|y_n = h, \mathcal{R}_N^l] = \Theta + \Lambda,$$

where

$$\begin{aligned}
\Theta =& \Pr[r_{N+1} = \tilde{h}, y_{N+1} = h|y_n = h, \mathcal{R}_N^l] \\
=& \Pr[r_{N+1} = \tilde{h}|y_{N+1} = h, y_n = h, \mathcal{R}_N^l] \times \Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l] \\
=& 1 \times \Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l] \\
=& \Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l], \qquad (29)
\end{aligned}$$

$$\begin{aligned}
\Lambda =& \Pr[r_{N+1} = \tilde{h}, y_{N+1} = l|y_n = h, \mathcal{R}_N^l] \\
=& \Pr[r_{N+1} = \tilde{h}|y_{N+1} = l, y_n = h, \mathcal{R}_N^l] \times \Pr[y_{N+1} = l|y_n = h, \mathcal{R}_N^l] \\
=& x\left[1 - \Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l]\right] \\
=& x[1 - \Theta]. \qquad (30)
\end{aligned}$$

If $n = N$, it is straightforward to determine that

$$\Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l] = \pi_{hh}.$$

Now let us consider $\Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l]$ if $n < N$. Because actual earnings y follow a Markov process, all the past information is fully summarized in the most recent realization, and the prior realizations are informationally irrelevant. Thus,

$$\begin{aligned}
\Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l] &= \Pr[y_{N+1} = h|y_n = h, \bar{y} = l, r_1 = \tilde{h}, \cdots, r_N = \tilde{h}], \\
&= \Pr[y_{N+1} = h|\bar{y} = h, r_{n+1} = \tilde{h}, \cdots, r_N = \tilde{h}]
\end{aligned}$$

and

$$\Pr[y_{N+1} = h|\bar{y} = h, r_{n+1} = \tilde{h}, \cdots, r_N = \tilde{h}] = \Pr[y_{N-n+1}|\bar{y} = h, r_1 = \tilde{h}, \cdots, r_{N-n} = \tilde{h}].$$

Recall that $\mathcal{R}_{N-n}^h \equiv \{\bar{y} = h, r_1 = \tilde{h}, \cdots, r_{N-n} = \tilde{h}\}$. Therefore,

$$\Pr[y_{N+1} = h|y_n = h, \mathcal{R}_N^l] = \begin{cases} \Pr[y_{N-n+1} = h|\mathcal{R}_{N-n}^h] & \text{if } n < N, \\ \pi_{hh} & \text{if } n = N. \end{cases} \qquad (31)$$

and

$$\begin{aligned}\Pr[y_{N-n+1} = h|\mathcal{R}^h_{N-n}] =& \Pr[y_{N-n+1} = h, y_{N-n} = h|\mathcal{R}^h_{N-n}] + \Pr[y_{N-n+1} = h, y_{N-n} = l|\mathcal{R}^h_{N-n}] \\ =& \Pr[y_{N-n+1} = h|y_{N-n} = h, \mathcal{R}^h_{N-n}] \times \Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}] \\ &+ \Pr[y_{N-n+1} = h|y_{N-n} = l, \mathcal{R}^h_{N-n}] \times \Pr[y_{N-n} = l|\mathcal{R}^h_{N-n}] \\ =& \pi_{hh} \Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}] + \pi_{lh} \left[1 - \Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}]\right],\end{aligned}$$

where $\Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}]$ is known from the supposition, since $N - n < N$. Therefore, Θ and Λ can be both calculated. Hence, the numerator in (26) can be derived following this procedure. The numerator is obtained

$$\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}^l_N] = \begin{cases} \pi_{hh} \Pr[y_N = h|\mathcal{R}^l_N] + \pi_{lh} \left[1 - \Pr[y_N = h|\mathcal{R}^l_N]\right] & \text{if } n = N+1, \\ \Pr[y_N = h|\mathcal{R}^l_N] \left[\pi_{hh} + x(1-\pi_{hh})\right] & \text{if } n = N, \\ \Pr[y_n = h|\mathcal{R}^l_N] \Big\{ \pi_{hh} \Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}] + & \text{if } n < N. \\ \pi_{lh} \left[1 - \Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}]\right] & \\ +x\{1 - \pi_{hh} \Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}] - \pi_{lh} \left[1 - \Pr[y_{N-n} = h|\mathcal{R}^h_{N-n}]\right]\} \Big\} & \end{cases} \tag{32}$$

Now combining the expressions (28) and (32), it has been shown that $\Pr[y_n = h|\mathcal{R}^l_N, r_{N+1} = \tilde{h}]$ can be calculated using $\Pr[y_n = h|\mathcal{R}^l_N, r_N = \tilde{h}]$. The same procedure can be repeated for $\Pr[y_n = h|\mathcal{R}^h_N, r_{N+1} = \tilde{h}]$ as follows.

$$\Pr[y_n = h|\mathcal{R}^h_N, r_{N+1} = \tilde{h}] = \frac{\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}^h_N]}{\Pr[r_{N+1} = \tilde{h}|\mathcal{R}^h_N]}.$$

where the denominator is

$$\begin{aligned}\Pr[r_{N+1} = \tilde{h}|\mathcal{R}^h_N] =& \pi_{hh} \Pr[y_N = h|\mathcal{R}^h_N] + \pi_{lh} \left[1 - \Pr[y_N = h|\mathcal{R}^h_N]\right] \\ &+ x\{1 - \pi_{hh} \Pr[y_N = h|\mathcal{R}^h_N] - \pi_{lh} \left[1 - \Pr[y_N = h|\mathcal{R}^h_N]\right]\}.\end{aligned}$$

and the numerator is

$$\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}_N^h] =$$
$$\begin{cases} \pi_{hh}\Pr[y_N = h|\mathcal{R}_N^h] + \pi_{lh}\left[1 - \Pr[y_N = h|\mathcal{R}_N^h]\right] & \text{if } n = N+1, \\ \\ \Pr[y_N = h|\mathcal{R}_N^h]\left[\pi_{hh} + x(1-\pi_{hh})\right] & \text{if } n = N, \\ \\ \Pr[y_n = h|\mathcal{R}_N^h]\Big\{\pi_{hh}\Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h] + & \text{if } n < N. \\ \pi_{lh}\left[1 - \Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h]\right] & \\ +x\{1 - \pi_{hh}\Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h] - \pi_{lh}\left[1 - \Pr[y_{N-n} = h|\mathcal{R}_{N-n}^h]\right]\}\Big\} & \end{cases}$$